

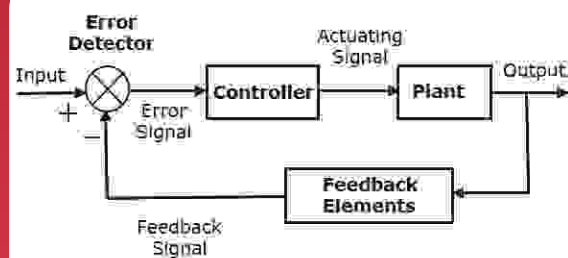
GATE | PSUs



ELECTRICAL ENGINEERING

Control Systems

Text Book : Theory with worked out Examples
and Practice Questions



Control Systems

(Solutions for Volume-1 Class Room Practice Questions)

1. Basics of Control Systems

01. Ans: (c)

$$\text{Sol: } 2 \frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 4y(t) = r(t) + 2r(t-1)$$

Apply LT on both sides

$$2s^2 Y(s) + 3sY(s) + 4Y(s) = R(s) + 2e^{-s}R(s)$$

$$Y(s)(2s^2 + 3s + 4) = R(s)(1 + 2e^{-s})$$

$$\frac{Y(s)}{R(s)} = \frac{1 + 2e^{-s}}{2s^2 + 3s + 4}$$

02. Ans: (b)

$$\text{Sol: } I.R = 2 \cdot e^{-2t} u(t)$$

$$\text{Output response } c(t) = (1 - e^{-2t}) u(t)$$

$$\text{Input response } r(t) = ?$$

$$T.F = \frac{C(s)}{R(s)}$$

$$T.F = L(I.R) = \frac{2}{s+2}$$

$$R(s) = \frac{C(s)}{T.F} = \frac{\frac{1}{s} - \frac{1}{s+2}}{\frac{2}{s+2}} = \frac{1}{s}$$

$$R(s) = \frac{1}{s}$$

$$r(t) = u(t)$$

03. Ans: (b)

Sol: Unit impulse response of unit-feedback control system is given

$$c(t) = t \cdot e^{-t}$$

$$T.F = L(I.R)$$

$$= \frac{1}{(s+1)^2}$$

$$\text{Open Loop T.F} = \frac{\text{Closed Loop T.F}}{1 - \text{Closed Loop T.F}}$$

$$= \frac{1}{(s+1)^2} = \frac{1}{s^2 + 2s}$$

04. Ans: (a)

Sol: G changes by 10%

$$\Rightarrow \frac{\Delta G}{G} \times 100 = 10\%$$

$$C_1 = 10\%$$

[∴ open loop] whose sensitivity is 100%]

$$\%G \text{ change} = 10\%$$

$$\frac{\% \text{ of change in } M}{\% \text{ of change in } G} = \frac{1}{1 + GH}$$

$$\% \text{ of change in } M = \frac{10\%}{1 + (10)1} = 1\%$$

% change in C_2 by 1%

05.

Sol: $M = C/R$

$$\frac{C}{R} = M = \frac{GK}{1 + GH}$$

$$S_K^M = \frac{\partial M}{\partial K} \times \frac{K}{M} = 1$$

[∴ K is not in the loop ⇒ sensitivity is 100%]

$$S_H^M = \frac{\partial M}{\partial H} \times \frac{H}{M} = \frac{\partial}{\partial H} \left(\frac{GK}{1 + GH} \right) \frac{H}{M}$$

$$= \left(\frac{GK(-G)}{(1+GH)^2} \right) \left[\frac{H}{GK} \right]_{1+GH}$$

$$S_H^M = \frac{-GH}{(1+GH)}$$

06.

Sol: Given data

$$G = 2 \times 10^3, \partial G = 100$$

$$\% \text{ change in } G = \frac{\partial G}{G} \times 100 = 5\%$$

$$\% \text{ change in } M = 0.5\%$$

$$\frac{\% \text{ of change in } M}{\% \text{ of change in } G} = \frac{1}{1+GH}$$

$$\frac{0.5\%}{5\%} = \frac{1}{1+2 \times 10^3 H}$$

$$1+2 \times 10^3 H = 10$$

$$H = 4.5 \times 10^{-3}$$

07. **Ans: (b)**

Sol: $K = \frac{\text{output}}{\text{input}} = \frac{c(t)}{r(t)} = \frac{\text{mm}}{0^\circ\text{C}}$

08. **Ans: (d)**

Sol: Introducing negative feedback in an amplifier results, increases bandwidth.

2. Signal Flow Graph and Block Diagram

01. **Ans: (d)**

Sol: No. of loops = 3

$$\text{Loop1: } -G_1 G_3 G_4 H_1 H_2 H_3$$

$$\text{Loop2: } -G_3 G_4 H_1 H_2$$

$$\text{Loop3: } -G_4 H_1$$

$$\text{No. of Forward paths} = 3$$

$$\text{Forward Path1: } G_1 G_3 G_4$$

$$\text{Forward Path 2: } G_2 G_3 G_4$$

$$\text{Forward Path 3: } G_2 G_4$$

$$= \frac{G_1 G_3 G_4 + G_2 G_3 G_4 + G_2 G_4}{1 + G_1 G_3 G_4 H_1 H_2 H_3 + G_3 G_4 H_1 H_2 + G_4 H_1}$$

02. **Ans: (a)**

Sol: Number of forward paths = 2

Number of loops = 3

$$Y(s) = \frac{\frac{1}{s} \times \frac{1}{s} \times \frac{1}{s} [1-0] + \frac{1}{s}}{R(s)}$$

$$R(s) = 1 - \left[\frac{1}{s} \times (-1) \left(\frac{1}{s} \right) (-1) + \frac{1}{s} \times \frac{1}{s} (-1) + \left(\frac{1}{s} \times \frac{1}{s} (-1) \right) \right]$$

$$= \frac{\frac{1}{s^3} + \frac{1}{s}}{1 - \left[\frac{1}{s^2} - \frac{1}{s^2} - \frac{1}{s^2} \right]} = \frac{1+s^2}{s^3} = \frac{1+s^2}{s^3}$$

$$= \frac{1+s^2}{s} \times \frac{1}{s^2+1} = \frac{1}{s}$$

03.

Sol: Number of forward paths = 2

Number of loops = 5,

Two non touching loops=4

$$TF = \frac{24[1-(-0.5)] + 10[1-(-3)]}{1 - [-24 - 3 - 4 + (5 \times 2 \times (-1)) + (-0.5)] + [30 + 1.5 + 2] + \left(\left(\frac{-1}{2} \right) \times (-24) \right)}$$

$$= \frac{76}{88} = \frac{19}{22}$$

04.

Sol: Number of forward paths = 2

Number of loops = 5

$$T.F = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_2 G_3 H_2 + G_1 G_2 H_1 + G_1 G_2 G_3 + G_4 H_2 + G_1 G_4}$$

05. Ans: (c)

Sol: From the network

$$V_o(s) = \frac{1}{sC} I(s) \dots\dots\dots(1)$$

$$-V_i(s) + RI(s) + V_o(s) = 0$$

$$I(s) = \frac{1}{R} V_i(s) + \left(\frac{-1}{R}\right) V_o(s) \dots\dots\dots(2)$$

From SFG

$$V_o(s) = x.I(s) \dots\dots\dots(3)$$

$$I(s) = \frac{1}{R} V_i(s) + y V_o(s) \dots\dots\dots(4)$$

From equ(1) and (3)

$$x = \frac{1}{sC}$$

From equ(2) and (4)

$$y = -\frac{1}{R}$$

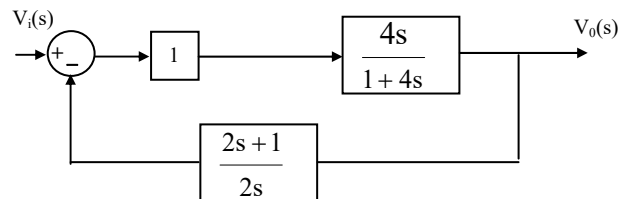
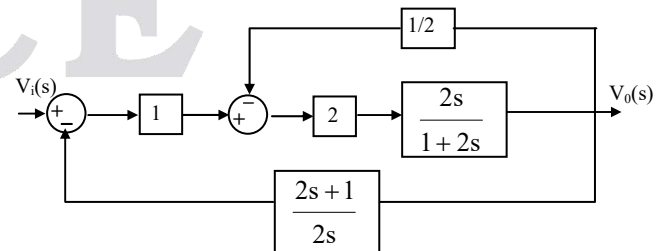
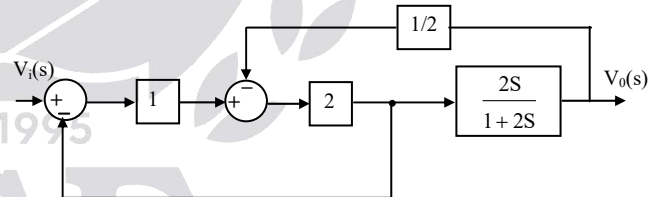
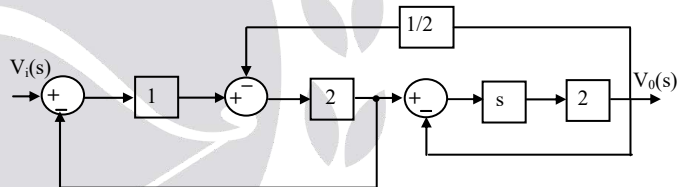
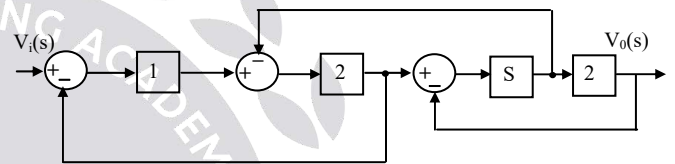
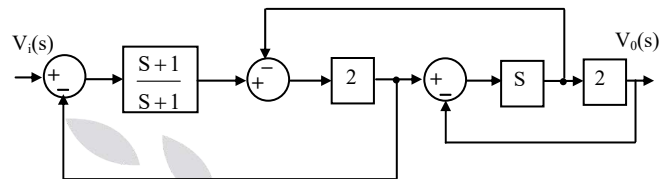
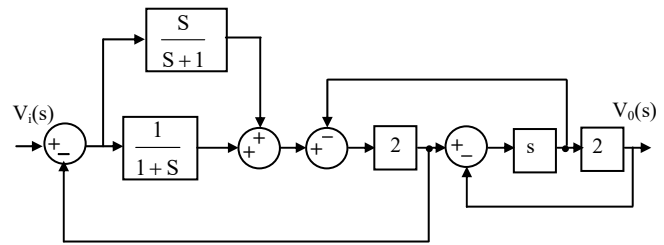
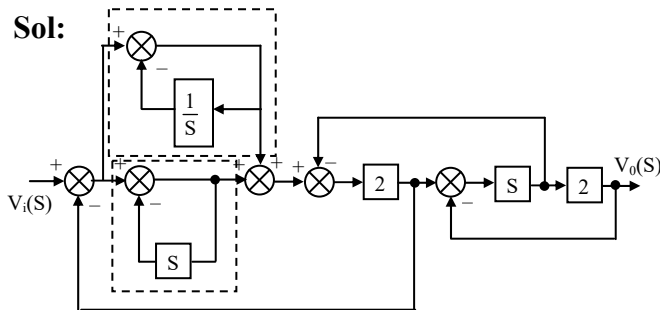
06. Ans: (a)

Sol: Use gain formula

$$\begin{aligned} \text{transfer function} &= \frac{G(s)}{1 - \left(G(s) \frac{1}{G(s)} + G(s) \right)} \\ &= \frac{G(s)}{1 - 1 - G(s)} = -1 \end{aligned}$$

07.

Sol:



$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{4s}{1 + \frac{2(2s+1)}{1+4s}} = \frac{4s}{8s+3}$$

08.

Sol: Apply Mason's Gain formula

$$M = \frac{Y_{out}}{Y_{in}} = \frac{\sum_{k=1}^N M_k \Delta_k}{\Delta}$$

No. of forward paths = 2

First forward path gain = $G_1 G_2 G_3 G_4$

Second forward path gain = $G_5 G_6 G_7 G_8$

No. of loops = 4

First loop gain = $-G_2 H_2$

Second loop gain = $-G_6 H_6$

Third loop gain = $-G_3 H_3$

Fourth loop gain = $-G_7 H_7$

Non touching loops = 4

Loop gains $\rightarrow G_2 H_2 G_6 H_6$

$\rightarrow G_2 H_2 G_7 H_7$

$\rightarrow G_6 H_6 G_7 H_7$

$\rightarrow G_2 H_2 G_3 H_3$

Transfer function =

$$\frac{G_1 G_2 G_3 G_4 (1 + G_6 H_6 + G_7 H_7) + G_5 G_6 G_7 G_8 (1 + G_2 H_2 + G_3 H_3)}{1 + G_2 H_2 + G_3 H_3 + G_6 H_6 + G_7 H_7 + G_2 H_2 G_6 H_6 + G_2 H_2 G_7 H_7 + G_3 H_3 G_6 H_6 + G_3 H_3 G_7 H_7}$$

3. Time Response Analysis

01. Ans: (a)

Sol: $\frac{C(s)}{R(s)} = \frac{1}{1+sT}$, $R(s) = \frac{8}{s}$

$$C(s) = \frac{8}{s(1+sT)} \Rightarrow c(t) = 8(1 - e^{-t/T})$$

$$3.6 = 8 \left(1 - e^{\frac{-0.32}{T}} \right)$$

$$0.45 = 1 - e^{\frac{-0.32}{T}}$$

$$0.55 = e^{\frac{-0.32}{T}}$$

$$-0.59 = \frac{-0.32}{T}$$

$$T = 0.535 \text{ sec}$$

02. Ans: (c)

Sol: $\cos \phi = \xi$

$$\cos 60 = 0.5$$

$$\cos 45 = 0.707$$

Poles left side $0.5 \leq \xi \leq 0.707$

Poles right side $-0.707 \leq \xi \leq -0.5$

$$\therefore 0.5 \leq |\xi| \leq 0.707$$

$$3 \text{ rad/s} \leq \omega_n \leq 5 \text{ rad/s}$$

03. Ans: (c)

Sol: For R-L-C circuit:

$$T.F = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{1}{Cs} I(s)$$

$$= \frac{1}{Cs} \frac{V_i(s)}{R + Ls + \frac{1}{Cs}}$$

$$\text{T.F} = \frac{V_o(s)}{V_i(s)} = \frac{1}{RCs + LCs^2 + 1}$$

$$= \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}} \quad 2\xi\omega_n = \frac{R}{L}$$

$$\xi = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$\xi = \frac{10}{2} \sqrt{\frac{10 \times 10^{-6}}{1 \times 10^{-3}}} = 0.5$$

$$\text{M.P} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$= 16.3\% \approx 16\%$$

04. Ans: (b)

$$\text{Sol: TF} = \frac{8/s(s+2)}{1 - \left(\frac{-8as}{s(s+2)} - \frac{8}{s(s+2)} \right)}$$

$$= \frac{8}{s(s+2) + 8as + 8}$$

$$= \frac{8}{s^2 + 2s + 8as + 8}$$

$$= \frac{8}{s^2 + (2+8a)s + 8}$$

$$\omega_n^2 = 8 \Rightarrow \omega_n = 2\sqrt{2}$$

$$2\xi\omega_n = 2 + 8a$$

$$\xi = \frac{1+4a}{2\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1+4a}{2\sqrt{2}} \Rightarrow a = 0.25$$

05. Ans: 4 sec

$$\text{Sol: T.F} = \frac{100}{(s+1)(s+100)}$$

$$= \frac{100}{s^2 + 101s + 100}$$

$$\omega_n^2 = 100$$

$$\omega_n = 10$$

$$2\xi\omega_n = 101$$

$$\xi = \frac{101}{20}$$

$\xi > 1 \rightarrow$ system is over damped i.e., roots are real & unequal.

Using dominate pole concept,

$$\text{T.F} = \frac{100}{100(s+1)} = \frac{1}{s+1}, \text{ Here } \tau = 1 \text{ sec}$$

$$\therefore \text{Setting time for 2\% criterion} = 4\tau$$

$$= 4 \text{ sec}$$

$$\text{06. } M_p = \frac{C(t_p) - C(\infty)}{C(\infty)}$$

$$= \frac{1.254 - 1.04}{1.04} = 0.2$$

$$\xi = \sqrt{\frac{(\ln M_p)^2}{(\ln M_p)^2 + \pi^2}}$$

$$M_p = 0.2 ; \xi = 0.46$$

07. Ans: (d)

Sol: Given data: $\omega_n = 2, \zeta = 0.5$

Steady state gain = 1

$$OLTF = \frac{K_1}{s^2 + as + 2} \text{ and } H(s) = K_2$$

$$CLTF = \frac{G(s)}{1+G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1}{s^2 + as + 2 + K_1K_2}$$

DC or steady state gain from the TF

$$\frac{K_1}{2 + K_1K_2} = 1$$

$$K_1(1 - K_2) = 2 \quad \dots\dots\dots (1)$$

$$CE \text{ is } s^2 + as + 2 + K_1K_2 = 0$$

$$\omega_n = \sqrt{2 + K_1K_2} = 2$$

$$4 = (2 + K_1K_2)$$

$$K_1K_2 = 2 \quad \dots\dots\dots (2)$$

Solving equations (1) & (2) we get

$$K_1 = 4, \quad K_2 = 0.5$$

$$2\zeta \omega_n = a$$

$$2 \times \frac{1}{2} \times 2 = a$$

$$a = 2$$

08. Ans: A – T, B – S, C- P, D – R, E – Q

Sol:

(A) If the poles are real & left side of s-plane, the step response approaches a steady state value without oscillations.

(B) If the poles are complex & left side of s-plane, the step response approaches a steady state value with the damped oscillations.

(C) If poles are non-repeated on the $j\omega$ axis, the step response will have fixed amplitude oscillations.

(D) If the poles are complex & right side of s-plane, response goes to ‘∞’ with damped oscillations.

(E) If the poles are real & right side of s-plane, the step response goes to ‘∞’ without any oscillations.

09. Ans: (c)

Sol: If R ↑ damping ↑

$$\Rightarrow \xi = \frac{R}{2} \sqrt{\frac{C}{L}}$$

(i) If R ↑, steady state voltage across C will be reduced (wrong)

(Since steady state value does not depend on ξ)

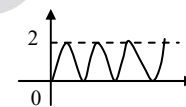
If $\xi \uparrow$, $C(\infty)$ = remain same

(ii) If $\xi \uparrow$, $\omega_d \downarrow$ ($\omega_d = \omega_n \sqrt{1 - \xi^2}$)

(iii) If $\xi \downarrow$, $t_s \uparrow \Rightarrow 3^{rd}$

Statement is false

(iv) If $\xi = 0$



True

$\Rightarrow 2$ and 4 are correct

10.

Sol: (i) Unstable system

$$\therefore \text{error} = \infty$$

$$(ii) G(s) = \frac{10(s+1)}{s^2}$$

$$\text{Step} \rightarrow R(s) = \frac{1}{s}$$

$$k_p = \infty$$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+\infty} = 0$$

Parabolic $\Rightarrow k_a = 10$

$$e_{ss} = \frac{1}{10} = 0.1$$

11.

Sol: $G(s) = 10/s^2$ (marginally stable system)

\therefore Error can't be determined

12.

Sol: $e_{ss} = \frac{1}{11}$, $R(s) = \frac{1}{s}$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+k_p} = \frac{1}{11} = \frac{1}{1+10}$$

$$k_p = \lim_{s \rightarrow 0} s G(s)$$

$$10 = \lim_{s \rightarrow 0} s G(s)$$

$$k = 10$$

$$R(s) = \frac{1}{s^2} \text{ (ramp)}$$

$$e_{ss} = \frac{A}{k_v} = \frac{1}{k_v} = \frac{1}{10}$$

(System is increased by 1)

$$\Rightarrow e_{ss} = 0.1$$

13. Ans: (a)

Sol: $T(s) = \frac{(s-2)}{(s-1)(s+2)^2}$ (unstable system)

14. Ans: (b)

Sol: Given data: $r(t) = 400tu(t)$ rad/sec

Steady state error $= 10^\circ$

$$\text{i.e., } e_{ss} = \frac{\pi}{180^\circ} (10^\circ) \text{ radians}$$

$$G(s) = \frac{20K}{s(1+0.1s)} \text{ and } H(s) = 1$$

$$r(t) = 400tu(t) \Rightarrow 400/s^2$$

$$\text{Error } (e_{ss}) = \frac{A}{K_v} = \frac{400}{K_v}$$

$$K_v = \lim_{s \rightarrow 0} s G(s)$$

$$K_v = \lim_{s \rightarrow 0} \frac{20K}{s(1+0.1s)}$$

$$K_v = 20K$$

$$e_{ss} = \frac{400}{20K}$$

$$e_{ss} = \frac{20}{K} = \frac{\pi}{18}$$

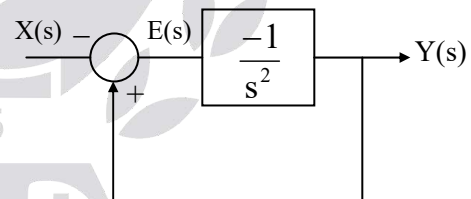
$$K = 114.5$$

15. Ans: (d)

Sol: $\frac{d^2y}{dt^2} = -e(t)$

$$s^2 Y(s) = -E(s)$$

$$x(t) = t u(t) \Rightarrow X(s) = \frac{1}{s^2}$$



$$Y(s) = \frac{-1}{s^2} E(s)$$

$$\frac{Y(s)}{E(s)} = \frac{-1}{s^2}$$

$$\frac{E(s)}{X(s)} = \frac{-1}{1 + \frac{1}{s^2}}$$

$$E(s) = \frac{-s^2}{1+s^2} X(s)$$

$$= \frac{-s^2}{1+s^2} \times \frac{1}{s^2} = \frac{-1}{1+s^2}$$

$$= L^{-1} \left[\frac{-1}{1+s^2} \right] = -\sin t$$

16. Ans: (a)

Sol: $e_{ss} = 0.1$ for step input

For pulse input = 10

time = 1 sec

error is function of input

$t \rightarrow \infty$ input = 0

\therefore Error = zero

17. Ans: (c)

Sol:

$$\frac{C(s)}{R(s)} = \frac{100}{(s+1)(s+5)}$$

$$1 + \frac{100 \times 0.2}{(s+1)(s+5)}$$

$$= \frac{100}{(s+1)(s+5) + 20}$$

$$= \frac{100}{s^2 + 6s + 5 + 20}$$

$$= \frac{100}{s^2 + 6s + 25}$$

$$\omega_n^2 = 25, \omega_n = 5$$

$$2\xi\omega_n = 6$$

$$\xi = \frac{6}{10} = \frac{3}{5}$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 5 \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= 5 \times \frac{4}{5} = 4 \text{ rad/sec}$$

18. Ans: (c)

Sol: $f(t) = \frac{Md^2x}{dt^2} + B \frac{dx}{dt} + Kx(t)$

Applying Laplace transform on both sides,
with zero initial conditions

$$F(s) = Ms^2X(s) + BsX(s) + KX(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Characteristic equation is $Ms^2 + Bs + K = 0$

$$s^2 + \frac{B}{M}s + \frac{K}{M} = 0$$

Compare with $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$$2\xi\omega_n = \frac{B}{M}$$

$$\xi = \frac{B}{2\sqrt{MK}} \quad \omega_n = \sqrt{\frac{K}{M}}$$

$$\text{Time constant } T = \frac{1}{\xi\omega_n}$$

$$= \frac{1}{B} \times 2M$$

$$T = \frac{2M}{B}$$

Hence, statements 2 & 3 are correct

19. Ans: (c)

Sol: type 1 system has a infinite positional error constant.

20. Ans: (a)

Sol: Given $G(s) = \frac{1}{s(1+s)(s+2)}, H(s) = 1$.

It is type-I system

Positional error constant $k_p = \lim_{s \rightarrow 0} G(s)H(s)$

$$k_p = \lim_{s \rightarrow 0} \frac{1}{s(1+s)(s+2)}$$

$$= \infty$$

Steady state error due to step input

$$= \frac{1}{1+k_p} = 0$$

21. Open loop T/F $G(s) = \frac{A}{S(S+P)}$

$$\text{C.L T/F} = \frac{A}{S^2 + SP + A}$$

$$\omega_n = \sqrt{A}$$

$$\text{Setting time} = 4/\xi\omega_n = 4$$

$$2\xi\omega_n = P \quad \therefore \frac{4}{P/2} = 4$$

$$\xi\omega_n = P/2 \quad \Rightarrow P = \frac{8}{4} = 2$$

$$e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.1 \Rightarrow \frac{\pi\xi}{\sqrt{1-\xi^2}} = \ln 10$$

$$= 2.3$$

$$\Rightarrow \frac{\xi^2}{1-\xi^2} = 0.5373$$

$$\Rightarrow 1.5373 \xi^2 = 0.5373$$

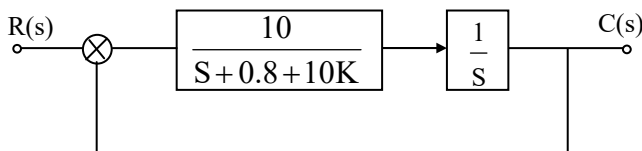
$$\xi = 0.59$$

$$\xi\omega_n = 1$$

$$\Rightarrow \omega_n = 1.694 \Rightarrow A = \omega_n^2 = 2.87$$

22.

Sol:



$$\frac{C(s)}{R(s)} = \frac{10}{s(s+0.8+10K)+10}$$

$$= \frac{10}{s^2 + s(0.8+10K)+10}$$

$$\omega_n = \sqrt{10} \quad 2\xi\omega_n = 0.8 + 10K$$

$$\Rightarrow 2 \times \frac{1}{2} \times \sqrt{10} = 0.8 + 10K$$

$$\Rightarrow K = 0.236$$

$$t_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - \cos^{-1}(\xi)}{\omega_n \sqrt{1-\xi^2}}$$

$$= \frac{\pi - \pi/3}{2.88} = 0.74 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} = 1.1 \text{ sec}$$

$$\%M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} = 0.163 \times 100 = 16.3\%$$

$$t_s \text{ (for 2\%)} = \frac{4}{\xi\omega_n} = \frac{4}{0.5 \times \sqrt{10}} = 2.53 \text{ sec}$$

4. Stability

01. CE = $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$

s^5	1	8	7	
s^4	4(1)	8(2)	4(1)	
s^3	6(1)	6(1)	0	
s^2	1	1	0	→ Row of AE
s^1	0(2)	0	0	→ Row of zero
s^0	1			

No. of AE roots = 2	No. of CE roots = 5
No. of sign changes	No. of sign changes
Below AE = 0	in 1 st column = 0
No. of RHP = 0	∴ No. of RHP = 0
No. of LHP = 0	No. of jωp = 2
No. of jωp = 2	⇒ No. of LHP = 3

System is marginally stable.

(ii) $s^2 + 1 = 0$

$$s = \pm 1j = \pm j\omega_n$$

$$\omega_n = 1 \text{ rad/sec}$$

Oscillating frequency $\omega_n = 1 \text{ rad/sec}$

02. (i) $s^5 + s^4 + s^3 + s^2 + s + 1 = 0$

$+s^5$	1	1	1
$+s^4$	1	1	1
$+s^3$	0(2)	0(1)	0
$+s^2$	$\frac{1}{2}$	1	
(1) $-s^1$	-3	0	
(2) $+s^0$	1		

$$AE(1) = s^4 + s^2 + 1 = 0$$

$$\frac{d(AE)}{ds} = 4s^3 + 2s = 0$$

$$\Rightarrow 2s^3 + s = 0$$

AE

No. of sign changes below AE = 2

No. of AE roots = 4

No. of RHP = 2

No. of LHP = 2

No. of jωp = 0

CE

No. of sign changes in 1st column = 2

No. of CE roots = 5

No. of RHP = 2

No. of LHP = 3

No. of jωp = 0

System is unstable

(ii) $s^6 + 2s^5 + 2s^4 + 0s^3 - s^2 - 2s - 2 = 0$

s^6	1	2	-1	-2
s^5	2(1)	0	-2(-1)	0
s^4	2(1)	+0	-2(-1)	0
s^3	0(4)	0	0	0
s^2	0(ε)	-1	0	0
s^1	4/ε			
$-s^0$	-1			

$$AE = s^4 - 1 = 0$$

$$\frac{dAE}{ds} = 4s^3 + 0 = 0$$

CE

No. of CE roots = 6

No. of sign changes in the 1st column = 1

No. of RHP = 1

No. of LHP = 3

No. of jωp = 2

AE

No. of AE roots = 4

No. of sign changes below AE = 1

No. of RHP = 1

No. of jωp = 2

No. of LHP = 1

03. CE = $s^3 + 20s^2 + 16s + 16K = 0$

s^3	1	16
s^2	20	16K
s^1	$\frac{20(16) - 16K}{20}$	0
s^0	16K	

(i) For stability $\frac{20(16) - 16K}{20} > 0$

$$\Rightarrow 20(16) - 16K > 0$$

$$\Rightarrow K < 20 \text{ and } 16K > 0 \Rightarrow K > 0$$

$$\text{Range of } K \text{ for stability } 0 < K < 20$$

(ii) For the system to oscillate with ω_n it must be marginally stable

i.e., s^1 row should be 0

s^2 row should be A.E

$$\therefore \text{A.E roots} = \pm j\omega_n$$

$$\therefore s^1 \text{ row} \Rightarrow 20(16) - 16K = 0$$

$$\Rightarrow K = 20$$

$$\text{A.E is } 20s^2 + 16K = 0$$

$$20s^2 + 16(20) = 0$$

$$\Rightarrow s = \pm j4$$

$$\omega_n = 4 \text{ rad/sec}$$

04.

Sol: CE = $1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$

$$s^3 + as^2 + (K+2)s + K + 1 = 0$$

$$s^3 + as^2 + (K+2)s + (K+1) = 0$$

s^3	1	$K+2$
s^2	a	$K+1$
s^1	$\frac{a(K+2) - (K+1)}{a}$	0
s^0	$K+1$	

Given,

$$\omega_n = 2$$

$$\Rightarrow s^1 \text{ row} = 0$$

s^2 row is A.E

$$a(K+2) - (K+1) = 0$$

$$a = \frac{K+1}{K+2}$$

$$\text{A.E} = as^2 + K + 1 = 0$$

$$= \frac{K+1}{K+2} s^2 + K + 1 = 0$$

$$(k+1) \left(\frac{s^2}{k+2} + 1 \right) = 0$$

$$s^2 + k + 2 = 0$$

$$s = \pm j\sqrt{(k+2)}$$

$$\omega_n = \sqrt{k+2} = 2$$

$$k = 2$$

$$a = \frac{k+1}{k+2} = \frac{3}{4} = 0.75$$

05.

Sol: $s^3 + ks^2 + 9s + 18$

s^3	1	9
s^2	K	18
s^1	$\frac{9K-18}{K}$	0
s^0	18	

Given that system is marginally stable,

Hence

$$s^1 \text{ row} = 0$$

$$\frac{9K - 18}{K} = 0$$

$$9K = 18 \Rightarrow K = 2$$

$$\text{A.E is } 9s^2 + 18 = 0$$

$$Ks^2 + 18 = 0,$$

$$2s^2 + 18 = 0$$

$$2s^2 = -18$$

$$s = \pm j3$$

$$\therefore \omega_n = 3 \text{ rad/sec.}$$

06. Ans: (d)

Sol: Given transfer function $G(s) = \frac{k}{(s^2 + 1)^2}$

Characteristic equation $1 - G(s).H(s) = 0$

$$1 - \frac{k}{(s^2 + 1)^2} = 0$$

$$s^4 + 2s^2 + 1 - k = 0 \dots (1)$$

RH criteria

s^4	1	2	$1-K$
s^3	4	4	-
s^2	1	$1-K$	
s^1	$4K$		
s^0	$1-K$		

$$AE = s^4 + 2s^2 + 1 - K$$

$$\frac{d}{ds}(AE) = 4s^3 + 4s$$

$1 - K > 0$ no poles are on RHS plane and LHS plane.

All poles are on $j\omega$ - axis

$\therefore 0 < K < 1$ system marginally stable

07. Ans: (d)

Sol: Assertion: FALSE

Let the TF = s. "s" is the differentiator

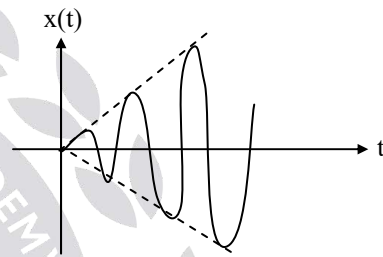
Impulse response $L^{-1}[TF] = L^{-1}[s] = \delta'(t)$

$$\lim_{t \rightarrow \infty} \delta'(t) = 0$$

\therefore It is BIBO stable

Reason: True

$$x(t) = t \sin t$$



$$\lim_{t \rightarrow \infty} x(t) = \lim_{t \rightarrow \infty} t \sin t \text{ is unbounded}$$

08. Ans: (a)

Sol: Assertion: TRUE

If feedback is not properly utilized the closed loop system may become unstable.

Reason: True

Feedback changes the location of poles

$$\text{Let } G(s) = \frac{-2}{s+1} \quad H(s) = 1$$

Open loop pole $s = -1$ (stable)

$$CLTF = \frac{\frac{-2}{s+1}}{1 + \frac{-2}{s+1}} = \frac{-2}{s-1}$$

Closed loop pole is at $s = 1$ (unstable)

\therefore After applying the feedback no more system is open loop. It becomes closed loop system. Hence poles are affected.

5. Root Locus Diagram

01. Ans: (a)

Sol: $s_1 = -1 + j\sqrt{3}$

$s_2 = -3 - j\sqrt{3}$

$$G(s).H(s) = \frac{K}{(s+2)^3}$$

$s_1 = -1 + j\sqrt{3}$

$$G(s).H(s) = \frac{K}{(-1 + j\sqrt{3} + 2)^3}$$

$$= \frac{K}{(1 + j\sqrt{3})^3}$$

$$= -3 \tan^{-1}(\sqrt{3})$$

$$= -180^\circ$$

It is odd multiples of 180° , Hence s_1 lies on Root locus

$s_2 = -3 - j\sqrt{3}$

$$G(s).H(s) = \frac{K}{(-3 - j\sqrt{3} + 2)^3}$$

$$= \frac{K}{(-1 - j\sqrt{3})^3}$$

$$= -3 [180^\circ + 60^\circ] = -720^\circ$$

It is not odd multiples of 180° , Hence s_2 is not lies on Root locus.

02. Ans: (a)

Sol: Over damped – roots are real & unequal

$\Rightarrow 0 < k < 4$

(b) $k = 4$ roots are real & equal

\Rightarrow Critically damped $\xi = 1$

(c) $k > 4 \Rightarrow$ roots are complex

$0 < \xi < 1 \Rightarrow$ under damped

03. Ans: (a)

Sol: Asymptotes meeting point is nothing but centroid

$$\text{centroid } \sigma = \frac{\sum \text{poles} - \sum \text{zeros}}{p - z}$$

$$= \frac{-3 - 0}{3 - 0} = -1$$

centroid = (-1, 0)

04. Ans: (b)

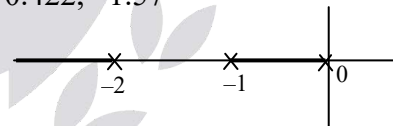
Sol: break point = $\frac{dK}{ds} = 0$

$$\frac{d}{ds} (G_1(s).H_1(s)) = 0$$

$$\frac{d}{ds} [s(s+1)(s+2)] = 0$$

$$3s^2 + 6s + 2 = 0$$

$$s = -0.422, -1.57$$



But $s = -1.57$ do not lie on root locus

So, $s = -0.422$ is valid break point.

Point of intersection wrt $j\omega$ axis

$$s^3 + 3s^2 + 2s + k = 0$$

$$\begin{array}{r|l} s^3 & 1 & 2 \\ s^2 & 3 & k \\ s^1 & \frac{6-k}{3} & 0 \\ s^0 & k & \end{array}$$

As s^1 Row = 0

$$k = 6$$

$$3s^2 + 6 = 0$$

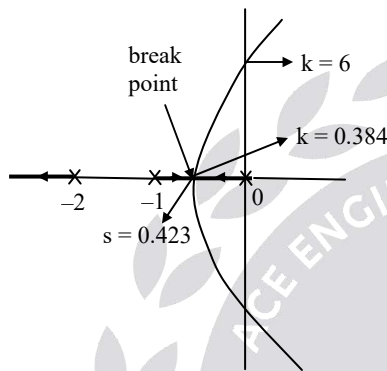
$$s^2 = -2$$

$$s = \pm j\sqrt{2}$$

point of inter section: $s = \pm j\sqrt{2}$

05. Ans: (b)

Sol:



$$\frac{K}{s(s+1)(s+2)}$$

substitute $s = -0.423$ and apply the magnitude criteria.

$$\left| \frac{K}{(-0.423)(-0.423+1)(-0.423+2)} \right| = 1$$

$$K = 0.354$$

when the roots are complex conjugate then the system response is under damped.

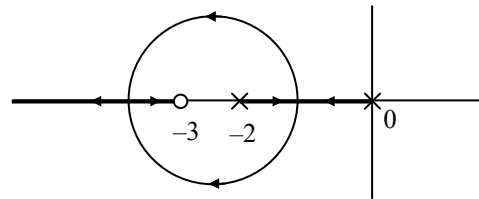
From $K > 0.384$ to $K < 6$ roots are complex conjugate then system to be under damped the values of k is $0.384 < K < 6$.

06. Ans: (c)

Sol: If the roots are lies on the real axis then system exhibits the non-oscillatory response. from $K \geq 0$ to $K \leq 0.384$ roots lies on the real axis. Hence for $0 \leq K \leq 0.384$ system exhibits the non-oscillatory response.

07. Ans: (a)

Sol:



$$\frac{d}{ds} [G(s).H(s)] = \frac{d}{ds} \left[\frac{k(s+3)}{s(s+2)} \right]$$

$$s^2 + 6s + 6 = 0$$

break points $-1.27, -4.73$

$$\text{radius} = \frac{4.73 - 1.27}{2} = 1.73$$

center $= (-3, 0)$

08. Ans: (c)

$$\text{Sol: } G(s).H(s) = \frac{K(s+3)}{s(s+2)}$$

$$k|_{s=-4} = \left| \frac{(-4)(-4+2)}{(-4+3)} \right| = \left| \frac{(-4)(-2)}{(-1)} \right| = 8$$

09. Ans: (a)

Sol: $s^2 - 4s + 8 = 0 \Rightarrow s = 2 \pm 2j$ are two zeroes

$s^2 + 4s + 8 = 0 \Rightarrow s = -2 \pm 2j$ are two poles

$$\phi_A = 180 - \angle GH|_{s=2+2j}$$

$$GH = \frac{k[s - (2+2j)][s - (2-2j)]}{[s - (-2+2j)][s - (-2-2j)]}$$

$$\angle GH|_{s=2+2j} = \frac{\angle k \angle 4j}{\angle 4 \angle 4+4j}$$

$$= 90^\circ - 45^\circ = 45^\circ$$

$$\phi_A = 180^\circ - 45^\circ = \pm 135^\circ$$

10. Ans: (b)

Sol: $s^2 - 4s + 8 = 0 \Rightarrow s = 2 \pm 2j$ are two zeroes

$s^2 + 4s + 8 = 0 \Rightarrow s = -2 \pm 2j$ are two poles

$$\phi_d = 180^\circ + \angle GH \Big|_{s=-2 \pm 2j}$$

$$\begin{aligned} \angle GH \Big|_{s=-2 \pm 2j} &= \angle \frac{k[s - (2 + 2j)][s - (2 - 2j)]}{[s - (-2 + 2j)][s - (-2 - 2j)]} \Big|_{s=-2 \pm 2j} \\ &= \frac{\angle k(-4)(-4 + 4j)}{\angle 4j} \end{aligned}$$

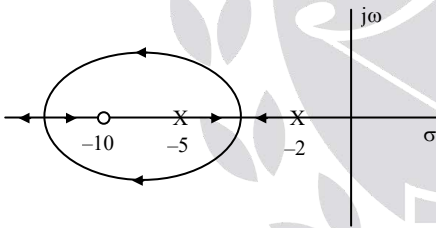
$$= 180^\circ + 180^\circ - 45^\circ - 90^\circ = 225^\circ$$

$$\phi_d = 180^\circ + 225^\circ = 405^\circ$$

$$\therefore \phi_d = \pm 45^\circ$$

11. Ans: (d)

Sol: Poles $s = -2, -5$; Zero $s = -10$



\therefore Breakaway point exist between -2 and -5

12.

Sol: Refer Pg No: 77, Vol-1 Ex: 8

6. Frequency Response Analysis

01. Ans: (c)

Sol: $G(s).H(s) = \frac{100}{s(s+4)(s+16)}$

Phase crossover frequency (ω_{pc}):

$$\angle G(j\omega).H(j\omega) / \omega = \omega_{pc} = -180^\circ$$

$$-90^\circ - \tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -180^\circ$$

$$-\tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -90^\circ$$

$$\tan[\tan^{-1}(\omega_{pc}/4) + \tan^{-1}(\omega_{pc}/16)] = \tan(90^\circ)$$

$$\frac{\frac{\omega_{pc}}{4} + \frac{\omega_{pc}}{16}}{1 - \frac{\omega_{pc}}{4} \cdot \frac{\omega_{pc}}{16}} = \frac{1}{0}$$

$$\omega_{pc}^2 = 16 \times 4 \Rightarrow \omega_{pc} = 8 \text{ rad/sec}$$

02. Ans: (d)

Sol: $G(s).H(s) = \frac{100}{s(s+2)(s+16)}$

$$\text{Gain margin (G.M)} = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$$

$$\begin{aligned} |G(j\omega).H(j\omega)|_{\omega=\omega_{pc}} &= \frac{100}{\omega_{pc} \sqrt{\omega_{pc}^2 + 16} \sqrt{\omega_{pc}^2 + 16^2}} \\ &= \frac{5}{64} \end{aligned}$$

$$\text{G.M} = \frac{64}{5} = 12.8$$

03. Ans: (c)

Sol: $G(s).H(s) = \frac{2e^{-0.5s}}{(s+1)}$

gain crossover frequency,

$$\omega_{gc} = |G(j\omega).H(j\omega)|_{\omega=\omega_{gc}} = 1$$

$$\frac{2}{\sqrt{\omega_{gc}^2 + 1}} = 1$$

$$\omega_{gc}^2 + 1 = 4 \Rightarrow \omega_{gc} = \sqrt{3} \text{ rad/sec}$$

04. Ans: (b)

Sol: $\omega_{gc} = \sqrt{3} \text{ rad/sec}$

$$P.M = 180^\circ + \angle G(j\omega).H(j\omega) / \omega = \omega_{gc}$$

$$\angle G(j\omega).H(j\omega) / \omega = \omega_{gc} = -0.5 \omega_{gc} - \tan^{-1}(\omega_{gc})$$

$$= -109.62^\circ$$

$$P.M = 70.39^\circ$$

05. Ans: (a)

Sol: $M_r = 2.5 = \frac{1}{2\xi\sqrt{1-\xi^2}}$

$$2\xi\sqrt{1-\xi^2} = \frac{1}{2.5}$$

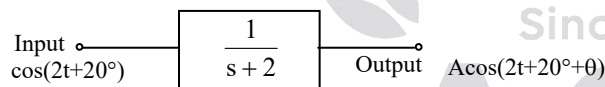
$$\xi^4 - \xi^2 + 0.04 = 0$$

$$\xi^2 = 0.958 \quad \xi^2 = 0.0417$$

$$\xi = 0.204 \quad (M_r > 1)$$

06. Ans: (a)

Sol: Closed loop T.F = $\frac{1}{s+2}$



$$A = \frac{1}{\sqrt{\omega^2 + 4}} = \frac{1}{\sqrt{4 + 4}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

$$\phi = -\tan^{-1}\omega/2$$

$$= -\tan^{-1}2/2$$

$$\Rightarrow \phi = -\tan^{-1}(1) = -45^\circ$$

$$\text{output} = \frac{1}{2\sqrt{2}} \cos(2t + 20^\circ - 45^\circ)$$

$$= \frac{1}{2\sqrt{2}} \cos(2t - 25^\circ)$$

07. Ans: (c)

Sol: Initial slope = -40 dB/dec

Two integral terms $\left(\frac{1}{s^2}\right)$

$$\therefore \text{Part of TF} = G(s)H(s) = \frac{K}{s^2}$$

at $\omega = 0.1$

$$\begin{aligned} \text{Change in slope} &= -20 - (-40) \\ &= 20^\circ \end{aligned}$$

$$\text{Part of TF} = G(s)H(s) = \frac{K\left(1 + \frac{s}{0.1}\right)}{s^2}$$

At $\omega = 10$ slope changed to -60 dB/dec

$$\begin{aligned} \text{Change in slope} &= -60 - (-20) \\ &= -40 \text{ dB/dec} \end{aligned}$$

$$\text{TF } (G(s)H(s)) = \frac{K\left(1 + \frac{s}{0.1}\right)}{s^2\left(\frac{s}{10} + 1\right)^2}$$

$$20 \log K - 2(20 \log 0.1) = 20 \text{ dB}$$

$$20 \log K = 20 - 40$$

$$20 \log K = -20$$

$$K = 0.1$$

$$G(s)H(s) = \frac{(0.1)\left(1 + \frac{s}{0.1}\right)}{s^2\left(1 + \frac{s}{10}\right)^2}$$

$$= \frac{(0.1) \times 10^2 (s + 0.1)}{(0.1)s^2 (s + 10)^2}$$

$$G(s)H(s) = \frac{100(s + 0.1)}{s^2 (s + 10)^2}$$

08. Ans: (b)

$$\text{Sol: } G(s)H(s) = \frac{Ks}{\left(1 + \frac{s}{2}\right)\left(1 + \frac{s}{10}\right)}$$

$$12 = 20 \log K + 20 \log 0.5$$

$$12 = 20 \log K + (-6)$$

$$20 \log K = 18 \text{ dB} = 20 \log 2^3$$

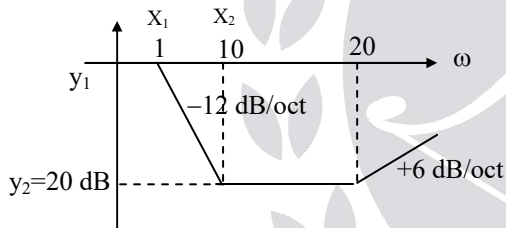
$$K = 8$$

$$G(s)H(s) = \frac{8s \times 2 \times 10}{(2+s)(10+s)}$$

$$G(s)H(s) = \frac{160s}{(2+s)(10+s)}$$

09. Ans: (b)

Sol:



$$G(s)H(s) = \frac{K \left(1 + \frac{s}{10}\right)^2 \left(1 + \frac{s}{20}\right)}{(1+s)^2}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = -40 \text{ dB/dec}$$

$$\frac{20 - y_1}{\log 10 - \log 1} = -40$$

$$y_1 = +60 \text{ dB} \Big|_{\omega \leq 1}$$

$$\Rightarrow 20 \log K = 60$$

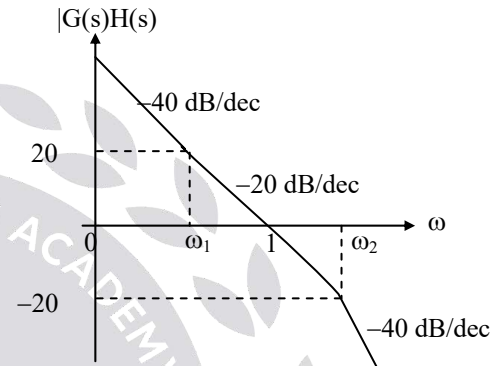
$$K = 10^3$$

$$G(s)H(s) = \frac{10^3 (s+10)^2 (s+20)}{10^2 \times 20 \times (s+1)^2}$$

$$= \frac{(s+10)^2 (s+20)}{2(s+1)^2}$$

10. Ans: (d)

Sol:



ω_1 calculation:

$$\frac{0 - 20}{\log 1 - \log \omega_1} = -20 \text{ dB/dec}$$

$$\omega_1 = 0.1$$

ω_2 calculation:

$$\frac{-20 - 0}{\log \omega_2 - \log 1} = -20 \text{ dB/dec}$$

$$\omega_2 = 10$$

$$G(s)H(s) = \frac{K \left(1 + \frac{s}{0.1}\right)}{s^2 \left(1 + \frac{s}{10}\right)}$$

$$20 \log K - 2(20 \log 0.1) = 20$$

$$20 \log K = 20 - 40$$

$$K = 0.1$$

$$G(s)H(s) = \frac{0.1 \times \frac{1}{0.1} (0.1 + s)}{s^2 \frac{1}{10} (10 + s)}$$

$$= \frac{10(0.1 + s)}{s^2 (10 + s)}$$

11.

Sol: $\frac{200}{s(s+2)} = \frac{100}{s\left(1 + \frac{s}{2}\right)}$

$$x = -KT \Rightarrow -(100) \times \frac{1}{2} = x = -50$$

12. Ans: (c)

Sol: For stability $(-1, j0)$ should not be enclosed by the polar plot.

For stability

$$1 > 0.01 K$$

$$\Rightarrow K < 100$$

13.

Sol: $GM = -40 \text{ dB}$

$$20 \log \frac{1}{a} = -40 \Rightarrow a = 10^2$$

$$POI = 100$$

14.

Sol: (i) $GM = \frac{1}{0.1} = +10 = 20 \text{ dB}$

$$PM = 180^\circ - 140^\circ = 40^\circ$$

(ii) $PM = 180 - 150^\circ = 30^\circ$

$$GM = \frac{1}{0} = \infty \quad POI = 0$$

(iii) ω_{PC} does not exist

$$GM = \frac{1}{0} = \infty \quad PM = 180^\circ + 0^\circ = 180^\circ$$

(iv) ω_{gc} not exist

$$\omega_{pc} = \infty$$

$$GM = \frac{1}{0} = \infty$$

$$PM = \infty$$

(v) $GM = \frac{1}{0.5} = 2$

$$PM = 180 - 90 = 90^\circ$$

15. Ans: (d)

Sol: For stability $(-1, j0)$ should not be enclosed by the polar plot. In figures (1) & (2) $(-1, j0)$ is not enclosed.

\therefore Systems represented by (1) & (2) are stable.

16. Ans: (b)

Sol: Open loop system is stable, since the open loop poles are lies in the left half of s-plane

$$\therefore P = 0.$$

From the plot $N = -2$.

$$\text{No. of encirclements } N = P - Z$$

$$N = -2, P = 0 \text{ (Given)}$$

$$\therefore N = P - Z$$

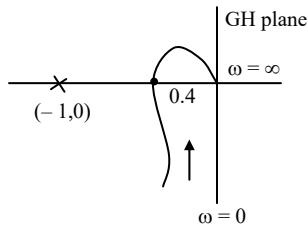
$$-2 = 0 - Z$$

$$Z = 2$$

Two closed loop poles are lies on RH of s-plane and hence the closed loop system is unstable.

17. Ans: (c)

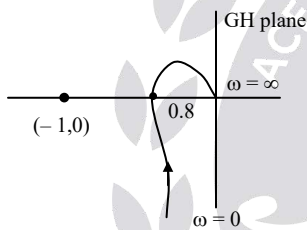
Sol:



$$\frac{K_c}{K} = 0.4 \quad \text{When } K = 1$$

Now, K double, $\frac{K_c}{K} = 0.4$

$$K_c = 0.4 \times 2 = 0.8$$



Even though the value of K is double, the system is stable (negative real axis magnitude is less than one)

Oscillations depends on 'ξ'

$$\xi \propto \frac{1}{\sqrt{K}} \quad \text{as } K \text{ is increased } \xi \text{ reduced, then}$$

more oscillations.

18. Ans: (a)

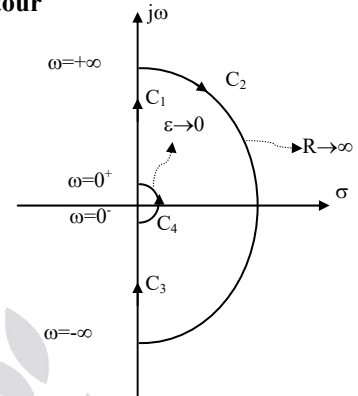
Sol: Given system $G(s) = \frac{10(s-12)}{s(s+2)(s+3)}$

It is a non minimum phase system since $s = 12$ is a zero on the right half of s-plane

19.

Sol: Given that $G(s)H(s) = \frac{10(s+3)}{s(s-1)}$

s-plane
Nyquist Contour



- Nyquist plot is the mapping of Nyquist contour(s-plane) into $G(s)H(s)$ plane.
- The Nyquist contour in the s-plane enclosing the entire right half of S-plane is shown figure.

The Nyquist Contour has four sections C_1 , C_2 , C_3 and C_4 . These sections are mapped into $G(s)H(s)$ plane

Mapping of section C_1 : It is the positive imaginary axis, therefore sub $s = j\omega$, ($0 \leq \omega \leq \infty$) in the TF $G(s)H(s)$, which gives the polar plot

$$G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$

Let $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{10(j\omega+3)}{j\omega(j\omega-1)}$$

$$G(j\omega)H(j\omega) = \frac{10\sqrt{\omega^2+9}}{\omega\sqrt{\omega^2+1}} \angle \left\{ \tan^{-1}\left(\frac{\omega}{3}\right) - [90^\circ + 180^\circ - \tan^{-1}(\omega)] \right\}$$

At $\omega = 0 \Rightarrow \infty \angle -270^\circ$

At $\omega = \omega_{pc} = \sqrt{3} \Rightarrow 10 \angle -180^\circ$

At $\omega = \infty \Rightarrow 0 \angle -90^\circ$

point of intersection of the Nyquist plot with respect to negative real axis is calculated below

$$\begin{aligned} \text{Arg}G(j\omega)H(j\omega) &= \arg \frac{10(j\omega+3)}{j\omega(j\omega-1)} \\ &= -180^\circ \text{ will give the } \omega_{pc} \end{aligned}$$

Magnitude of $G(j\omega)H(j\omega)$ gives the point of intersection

$$\begin{aligned} \angle \tan^{-1}\left(\frac{\omega}{3}\right) - [90^\circ + 180^\circ - \tan^{-1}(\omega)] \\ = -180^\circ \Big|_{\omega = \omega_{pc}} \\ \angle \tan^{-1}\left(\frac{\omega_{pc}}{3}\right) - [90^\circ + 180^\circ - \tan^{-1}(\omega_{pc})] = -180^\circ \end{aligned}$$

$$\tan^{-1}\left(\frac{\omega_{pc}}{3}\right) + \tan^{-1}(\omega_{pc}) = 90^\circ$$

Taking “tan” both the sides

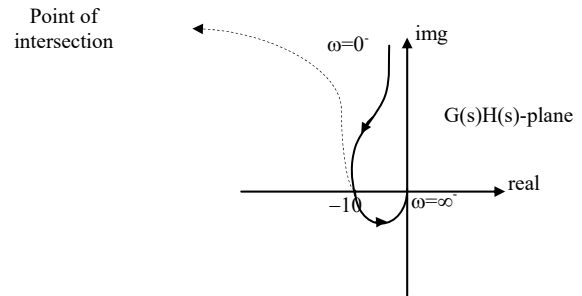
$$\frac{\frac{\omega_{pc}}{3} + \omega_{pc}}{1 - \frac{(\omega_{pc})^2}{3}} = \tan 90^\circ = \infty$$

$$1 - \frac{\omega_{pc}^2}{3} = 0$$

$$\omega_{pc} = \sqrt{3} \text{ rad/sec}$$

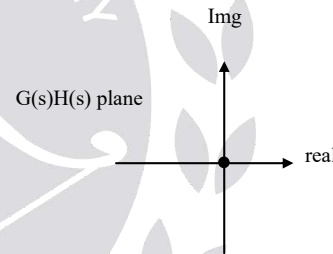
Therefore the point of intersection is

$$|G(j\omega)H(j\omega)| \text{ at } \omega_{pc} = \frac{10\sqrt{\omega_{pc}^2 + 3^2}}{\omega_{pc}\sqrt{1 + \omega_{pc}^2}} = 10$$



The mapping of the section C_1 is shown figure.

Mapping of section C_2 : It is the radius ‘R’ semicircle, therefore sub $s = \lim_{R \rightarrow \infty} R e^{j\theta}$ (θ is from 90° to 0° to -90°) in the TF $G(s)H(s)$, which merges to the origin in $G(s)H(s)$ plane.

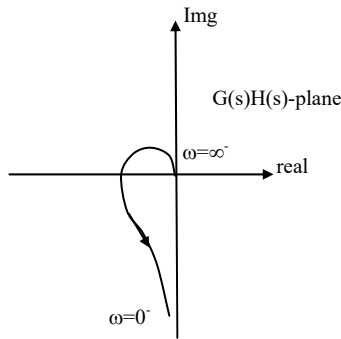


$$G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$

$$G(Re^{j\theta})H(Re^{j\theta}) = \frac{2(Re^{j\theta} + 3)}{Re^{j\theta}(Re^{j\theta} - 1)} \approx 0$$

The plot is shown in figure.

Mapping of section C_3 : It is the negative imaginary axis, therefore sub $s = j\omega$, ($-\infty \leq \omega \leq 0$) in the TF $G(s)H(s)$, which gives the mirror image of the polar plot and is symmetrical with respect to the real axis, The plot is shown in figure.



Mapping of section C4: It is the radius 'ε' semicircle, therefore $\text{subs} = \lim_{\epsilon \rightarrow 0} \epsilon e^{j\theta}$ ($-90^\circ \leq \theta \leq 90^\circ$) in the TF $G(s)H(s)$, which gives clockwise infinite radius semicircle in $G(s)H(s)$ plane.

The plot is shown below

$$G(\epsilon e^{j\theta})H(\epsilon e^{j\theta}) = \frac{10(\epsilon e^{j\theta} + 3)}{\epsilon e^{j\theta}(\epsilon e^{j\theta} - 1)}$$

$$G(\epsilon e^{j\theta})H(\epsilon e^{j\theta}) \approx \frac{10 \times 3}{-\epsilon e^{j\theta}} = \infty \angle 180^\circ - \theta$$

When, $\theta = -90^\circ \infty \angle 270^\circ$

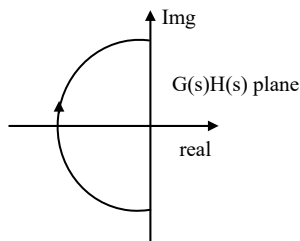
$\theta = -40^\circ \infty \angle 220^\circ$

$\theta = 0^\circ \infty \angle 0^\circ$

$\theta = 40^\circ \infty \angle 140^\circ$

$\theta = 90^\circ \infty \angle 90^\circ$

It is clear that the plot is clockwise '∞' radius semicircle centred at the origin



Combining all the above four sections, the

$$\text{Nyquist plot of } G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$

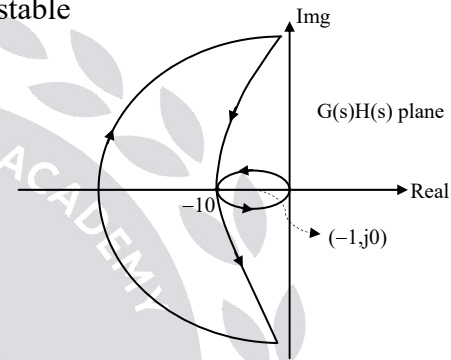
is shown in figure below

From the plot $N=1$

Given that $P=1$

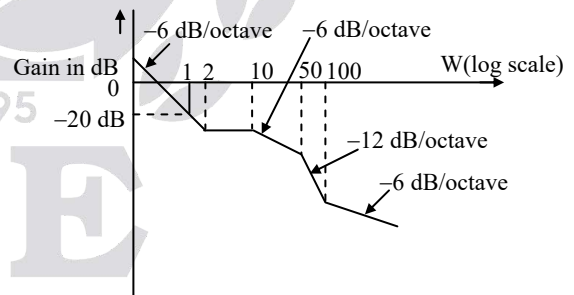
$$N=P-Z$$

$Z = P - N = 1 - 1 = 0$, therefore system is stable



20.

Sol: The given bode plot is shown below.



Initial slope = - 6 db/octave.

i.e., there is one pole at origin (or) one integral term.

portion of transfer function

$$G(s) = \frac{K}{s}$$

At $\omega = 2$ rad/sec, slope is changed to 0dB/octave.

∴ change in slope = present slope – previous slope

$$= 0 - (-6) = 6 \text{ dB/octave}$$

∴ There is a real zero at corner frequency

$$\omega_1 = 2.$$

$$(1 + sT_1) = \left(1 + \frac{s}{\omega_1}\right) = \left(1 + \frac{s}{Z}\right)$$

At $\omega = 10$ rad/sec, slope is changed to -6 dB/octave.

∴ change in slope = $-6 - 0 = -6$ dB/octave.

∴ There is a real pole at corner frequency

$$\omega_2 = 2.$$

$$\frac{1}{1 + sT_2} = \frac{1}{\left(1 + \frac{s}{\omega_2}\right)} = \frac{1}{\left(1 + \frac{s}{10}\right)}$$

At $\omega = 50$ rad/sec, slope is changed to -12 dB/octave.

∴ change in slope = $-12 - (-6) = -6$ dB/octave

∴ There is a real pole at corner frequency

$$\omega_3 = 50 \text{ rad/sec.}$$

$$\frac{1}{1 + sT_3} = \frac{1}{\left(1 + \frac{s}{\omega_3}\right)} = \frac{1}{\left(1 + \frac{s}{50}\right)}$$

At $\omega = 100$ rad/sec, the slope changed to -6 dB/octave.

∴ change in slope = $-6 - (-12) = 6$ dB/octave.

∴ There is a real zero at corner frequency

$$\omega_4 = 100 \text{ rad/sec.}$$

$$\therefore (1 + sT_4) = \left(1 + \frac{s}{\omega_4}\right) = \left(1 + \frac{s}{100}\right)$$

$$\therefore \text{Transfer function} = \frac{K \left(1 + \frac{s}{2}\right) \left(1 + \frac{s}{100}\right)}{s \left(1 + \frac{s}{50}\right) \left(1 + \frac{s}{10}\right)}$$

$$= \frac{K(s+2)(s+100)}{s(s+50)(s+10)} \cdot \frac{1}{2} \cdot \frac{1}{100}$$

$$= \frac{2.5K(s+2)(s+100)}{s(s+10)(s+50)}$$

$$= \frac{2.5K(s+2)(s+100)}{s(s+10)(s+50)}$$

In the given bode plot,

at $\omega = 1$ rad/sec, Magnitude = -20 dB.

$$-20 \text{ dB} = 20 \log K - 20 \log \omega + 20 \sqrt{1 + \left(\frac{\omega}{2}\right)^2} + 20 \sqrt{1 + \left(\frac{\omega}{100}\right)^2}$$

$$-20 \log \sqrt{1 + \left(\frac{\omega}{50}\right)^2} - 20 \log \sqrt{1 + \left(\frac{\omega}{10}\right)^2}$$

At $\omega = 1$ rad/sec,

$$-20 = 20 \log K - 20 \log \omega / \omega = 1 \text{ rad/sec}$$

[∴ Remaining values eliminated]

$$-20 = 20 \log K$$

$$\Rightarrow K = 0.1$$

∴ Transfer function

$$\frac{C(s)}{R(s)} = \frac{0.25(s+2)(s+100)}{s(s+10)(s+50)}$$

7. Controllers & Compensators

01. Ans: (a)

$$\text{Sol: } G_C(s) = (-1) \left(-\frac{Z_2}{Z_1} \right)$$

$$= (-1)(-1) \left(\frac{R_2 + \frac{1}{sC}}{R_1} \right)$$

$$G_c(s) = \frac{(100 \times 10^3) + \frac{1}{s \times 10^{-6}}}{10^6}$$

$$G_c(s) = \frac{1 + 0.1s}{s}$$

02. Ans: (c)

Sol: CE $\Rightarrow 1 + G_c(s) G_p(s) = 0$

$$= 1 + \frac{1 + 0.1s}{s} \times \frac{1}{(s+1)(1+0.1s)}$$

$$= 1 + \frac{1 + 0.1s}{s(s+1)(1+0.1s)} = 0$$

$$\Rightarrow s^2 + s + 1 = 0 \Rightarrow \omega_n = 1,$$

$$e^{\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}} \right]_{\xi=0.5}} = 0.163$$

$$M_p = 16.3\%$$

03. Ans: (b)

Sol: T.F = $\frac{k(1+0.3s)}{1+0.17s}$

$$T = 0.17, aT = 0.3 \Rightarrow a = \frac{0.3}{0.17}$$

$$C = 1 \mu F$$

$$T = \frac{R_1 R_2}{R_1 + R_2} C, a = \frac{R_1 + R_2}{R_2}$$

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{0.17}{1 \times 10^{-6}} = 170000$$

$$\frac{R_1 + R_2}{R_2} = 1.764$$

$$aT = R_1 C$$

$$R_1 = \frac{aT}{C} = \frac{0.3}{C} = (0.3)(10^6)$$

$$= 300 \text{ k}\Omega$$

Bv

$$300 \text{ k} + R_2 - 1.76 R_2 = 0$$

$$R_2 = \frac{300}{0.70} = 394.736$$

$$= 400 \text{ k}\Omega$$

04. Ans: (d)

Sol: PD controller improves transient stability and PI controller improves steady state stability. PID controller combines the advantages of the above two controllers.

05.

Sol: For $K_I = 0 \Rightarrow$

$$\frac{C(s)}{R(s)} = \frac{(K_p + K_D s)}{s(s+1) + (K_p + K_D s)}$$

$$= \frac{K_p + K_D s}{s^2 + (1 + K_D)s + K_p}$$

$$\omega_n = \sqrt{K_p}$$

$$2\xi\omega_n = 1 + K_D$$

$$\Rightarrow 2(0.9) \sqrt{K_p} = 1 + K_D$$

$$\Rightarrow 1.8 \sqrt{K_p} = 1 + K_D \quad \dots\dots\dots (1)$$

$$\text{Dominant time constant } \frac{1}{\xi\omega_n} = 1$$

$$\Rightarrow \omega_n = \frac{1}{0.9} = 1.111$$

$$K_p = \omega_n^2 = 1.11^2 = 1.234$$

From eq. (1),

$$\Rightarrow 1.8 \times \frac{1}{0.9} = 1 + K_D$$

$$\Rightarrow K_D = 1$$

8. State Space Analysis

01. Ans: (a)

Sol: TF = $\frac{1}{s^2 + 5s + 6}$

$$= \frac{1}{(s+2)(s+3)}$$

$$= \frac{1}{s+2} + \frac{-1}{s+3}$$

$$\therefore A = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$C = [1 \quad 1]$$

02. Ans: (c)

Sol: Given problem is Controllable canonical form.

(or)

$$TF = C[sI - A]^{-1}B + D$$

$$= [6 \quad 5 \quad 1] \begin{bmatrix} s & 1 & 0 \\ 0 & s & 1 \\ -5 & -3 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$= \frac{3s^2 + 15s + 18}{s^3 + 6s^2 + 3s + 5}$$

03. Ans: (d)

Sol: $\frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y = u(t)$

2nd order system hence two state variables are chosen

Let $x_1(t), x_2(t)$ are the state variables

CCF – SSR

Let $x_1(t) = y(t) \dots\dots\dots (1)$

$x_2(t) = \dot{y}(t) \dots\dots\dots (2)$

Differentiating (1)

$$\dot{x}_1(t) = \dot{y}(t) = x_2(t) \dots\dots\dots (3)$$

$$\begin{aligned} \dot{x}_2(t) &= \ddot{y}(t) = u(t) - 3\dot{y}(t) - 2y(t) \\ &= u(t) - 3x_2(t) - 2x_1(t) \dots\dots\dots (4) \end{aligned}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

A B

From equation 1. The output equation in matrix form

$$y(t) = [1 \quad 0] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, D = 0$$

04. Ans: (b)

Sol: OCF – SSR

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

05. Ans: (c)

Sol: Normal form – SSR

$$TF = \frac{Y(s)}{G(s)} = \frac{1}{s^2 + 3s + 2} = \frac{1}{(s+1)(s+2)}$$

⇒ Diagonal canonical form

The eigen values are distinct i.e., -1 & -2.

∴ Corresponding normal form is called as diagonal canonical form

DCF – SSR

$$\frac{Y(s)}{U(s)} = \frac{b_1}{s+1} + \frac{b_2}{s+2}$$

$$b_1 = 1, b_2 = -1$$

$$Y(s) = \underbrace{\frac{b_1}{s+1}}_{x_1} U(s) + \underbrace{\frac{b_2}{s+2}}_{x_2} U(s)$$

Let $Y(s) = X_1(s) + X_2(s)$

Where $y(t) = x_1(t) + x_2(t)$ (1)

Where $X_1(s) = \frac{b_1}{s+1} U(s)$

$s X_1(s) + X_1(s) = b_1 U(s)$

Take Laplace Inverse

$\dot{x}_1 + x_1 = b_1 u(t)$ (2)

$X_2(s) = \frac{b_2}{s+2} U(s)$

$s X_2(s) + 2 X_2(s) = b_2 U(s)$

Laplace Inverse

$\dot{x}_2 + 2x_2 = b_2 u(t)$

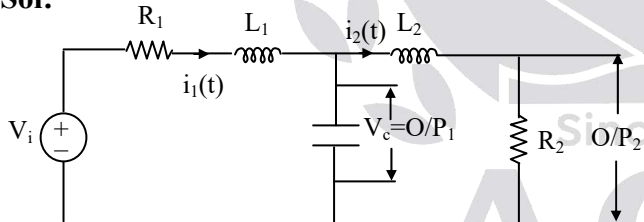
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

From (1) output equation.

$$y(t) = [1 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

06. Ans: (c)

Sol:



$OP_1 \Rightarrow y_1 = V_c$

$OP_2 \Rightarrow y_2 = R_2 i_2$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} V_c \\ i_1 \\ i_2 \end{bmatrix}$$

$y = C X$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix}$$

07. Ans: (a)

Sol: T.F = $C[sI-A]^{-1}B + D$

$$\begin{aligned} &= [1 \ 0] \begin{bmatrix} s+4 & 1 \\ 3 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= [1 \ 0] \frac{1}{s^2+5s+1} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \frac{1}{s^2+5s+1} [1 \ 0]_{1 \times 2} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2 \times 1} \\ &= \frac{1}{s^2+5s+1} [s+1 \ -1]_{1 \times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2 \times 1} \\ &= \frac{1}{s^2+5s+1} [s+1-1] \\ &= \frac{s}{s^2+5s+1} \end{aligned}$$

08. Ans: (c)

Sol: State transition matrix $\phi(t) = L^{-1}[(sI-A)^{-1}]$

$$sI - A = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$L^{-1}[[sI - A]^{-1}] = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

09. Ans: (b)

Sol: Controllability

$$[M] = [B \ AB \ A^2B \ \dots \ A^{n-1}B]$$

$$AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$$|M| = -1 \neq 0 \text{ (Controllable)}$$

Observability

$$[N] = [C^T \ A^T C^T \ \dots \ (A^T)^{n-1} C^T]$$

$$A^T C^T = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$$|N| = 0 \text{ (Not observable)}$$

10. Ans: (c)

Sol: According to Gilberts test the system is controllable and observable.

11. Ans: (c)

Sol:
$$\frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$

at node \dot{x}_1

$$\dot{x}_1 = -a_1 x_1 - a_2 x_2 - a_3 x_3$$

at $\dot{x}_2 = x_1$ & $\dot{x}_3 = x_2$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

12.

Sol: The given state space equations:

$$\dot{X} = X_2$$

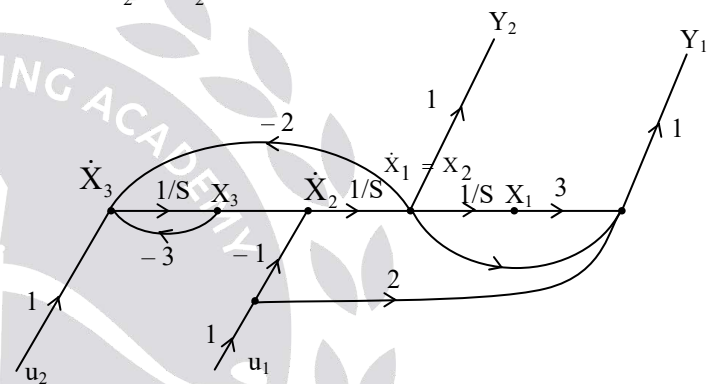
$$\dot{X}_2 = X_3 - u_1$$

$$\dot{X}_3 = -2X_2 - 3X_3 + u_2$$

and output equations are :

$$Y_1 = X_1 + 3X_2 + 2u_1$$

$$Y_2 = X_2$$



The given state space equations in matrix for

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}_{3 \times 3} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}_{3 \times 1} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}_{3 \times 2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2 \times 1}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{2 \times 3} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}_{3 \times 1} + \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}_{2 \times 2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2 \times 1}$$

Where A: State matrix

B: Input matrix

C: Output matrix

D: Transition matrix

Characteristic equation

$$|sI - A| = 0$$

$$\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 2 & s+3 \end{vmatrix} = 0$$

$$\Rightarrow s[s(s+3)+2]+1(0) = 0$$

$$\Rightarrow s(s^2 + 3s + 2) = 0$$

$$\Rightarrow s(s+1)(s+2) = 0$$

The roots are 0, -1, -2.

