

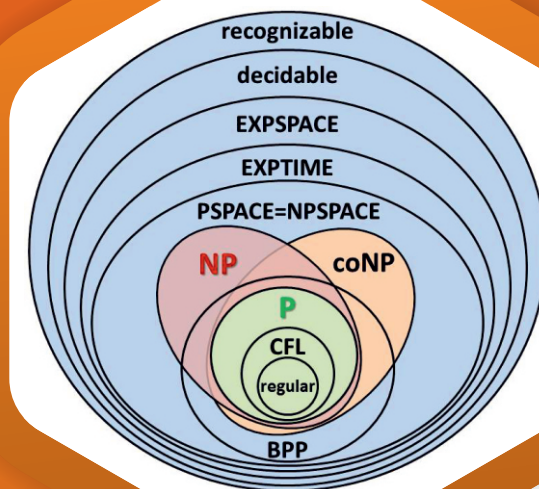
GATE | PSUs



COMPUTER SCIENCE & INFORMATION TECHNOLOGY

Theory of Computation

Text Book : Theory with worked out Examples
and Practice Questions



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Theory of Computation

(Solutions for Text Book Practice Questions)

1. Introduction

01. Ans: (d)

Sol: (a) $\{x|x \geq 10 \text{ or } x \leq 5\}$ is infinite set

(b) $\{x|x \geq 10 \text{ or } x \leq 100\}$ is infinite set

(c) $\{x|x \leq 100 \text{ or } x \geq 200\}$ is infinite set

02. Ans: (b)

Sol:

(a) Set of real numbers between 10 and 100 is uncountable

(b) $\{x|x \geq 10 \text{ or } x \leq 100\}$ is finite set. So countable

(c) Set of real numbers between 0 and 1 is uncountable

03. Ans: (d)

Sol: (a) $|\epsilon| = 0$

(b) $|\{\} | = 0$

(c) $|\{\epsilon\}| = 1$

04. Ans: (b)

Sol: $\Sigma = \{0,1\}$

00, 01, 10, 11 are 2 length strings

05. Ans: (b)

Sol: $w = abc$

Prefix(w) = $\{\epsilon, a, ab, abc\}$

06. Ans: (b)

Sol: $w = abc$

Suffix(w) = $\{\epsilon, c, bc, abc\}$

07. Ans: (d)

Sol: $w = abc$

Substring(w) = $\{\epsilon, a, b, c, ab, bc, abc\}$

08. Ans: (a)

Sol: Language accepted by finite automata is called as Regular language.

09. Ans: (d)

Sol: Every recursive language is REL but REL need not be recursive language.

10. Ans: (b)

Sol: Every regular grammar is CFG but CFG need not be regular grammar.

2. Regular Languages

(Finite automata, Regular expression, regular grammar)

01. Ans: (a) & (c)

Sol: Regular Languages are closed under

i) string reversal

ii) intersection with finite sets

02. Ans: (c)

Sol: A minimal DFA that is equivalent to a NFA with n states has atmost 2^n states.

03. Ans: (a)

Sol: (a) $(1+01)^* (\epsilon+0)$ generates all strings not containing '00'

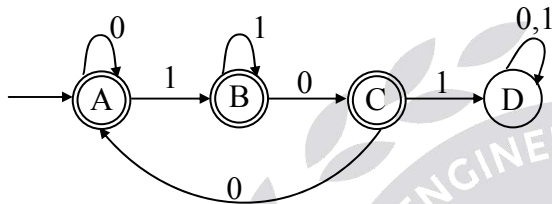
(b) $(0+10)^* (\epsilon+1)$ generates invalid string '00'

(c) $(1+01)^*$ cannot generate '0'

(d) $(\epsilon+0) (101)^* (\epsilon+0)$ generates invalid string '00'

04. Ans: (a)

Sol:

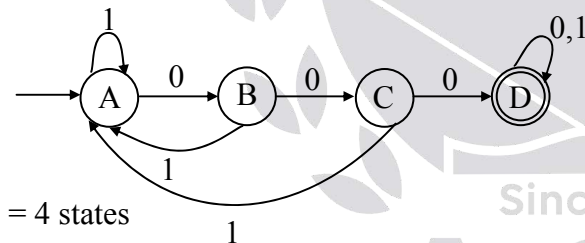


05. Ans: (d)

Sol: Given grammar generating all strings ending in '00'

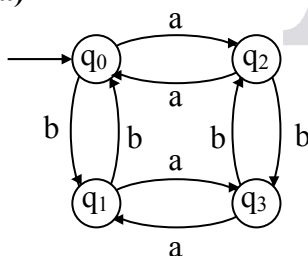
06. Ans: (a)

Sol:



07. Ans: (a)

Sol:



q_0 : Even a's and Even b's

q_1 : Even a's and odd b's

q_2 : Odd a's and Even b's

q_3 : Odd a's and Odd b's

q_1 should be final state.

08. Ans: (b)

Sol: Concatenation of two infinite languages is also infinite. So, infinite languages closed under concatenation.

09. Ans: (c)

Sol: $\{wxw^R \mid x, w \in (0+1)^+\} = 0(0+1)^+0 + 1(0+1)^+1$
 \therefore It is regular language

10. Ans: (a)

Sol: (I) NFA with many final states can be converted to NFA with only one final state with the help of ϵ -moves.

(II) Regular sets are not closed under infinite union

(III) Regular sets are not closed under infinite intersection

(IV) Regular languages are closed under substring operation

\therefore I and IV are correct.

11. Ans: (d)

Sol: $r = (0+1)^* 00(0+1)^*$

$A \rightarrow 0B \mid 0A \mid 1A$

$B \rightarrow 0C \mid 0$

$C \rightarrow 0C \mid 1C \mid 0 \mid 1$

12. Ans: (a)

Sol: $A_n = \{a^k \mid k \text{ is a multiple of } n\}$

For some n ,

A_n is regular

Let $n = 5$,

$A_n = A_5 = \{a^k | k \text{ is multiple of } 5\}$
= regular.

13. Ans: (d)

Sol: $L = \{a^m b^n | m \geq 1, n \geq 1\} = a^+ b^+$ is regular.

14. Ans: (c)

Sol: DFA accepts L and has m states

It has 2 final states. It implies $(m-2)$ non-final states.

DFA that accepts complement of L also has m states but it has $(m-2)$ final states and 2 non-final states.

15. Ans: (d)

Sol: (a) $0^* (1+0)^*$; It generates invalid string '100'

(b) $0^* 1010^*$; It cannot generate valid string ' ϵ '

(c) $0^* 1^* 01^*$; It cannot generate valid string ' ϵ '

(d) $0^* (10+1)^*$; It generates all strings not containing '100' as substring

16. Ans: (a)

Sol: P1: Membership problem for FA is decidable

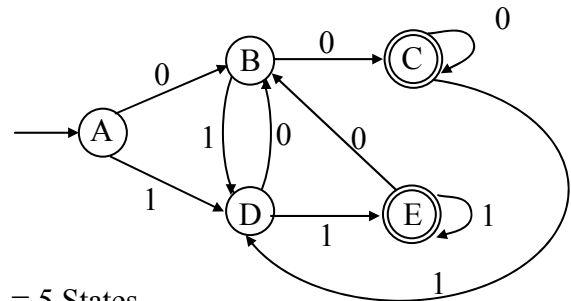
P2 : Infiniteness problem for CFG is decidable

For P1, CYK algorithm exist

For P2, Dependency graph exist

17. Ans: (b)

Sol: L = set of all binary strings whose last 2 symbols are same.



= 5 States.

18. Ans: (a)

Sol: $L = a^n b^n$ is not regular

It can be proved using Pumping Lemma
 L does not satisfy Pumping Lemma

19. Ans: (c)

Sol: It requires 29099 remainders to represent the binary numbers of the given language.
So, 29099 states required.

20. Ans: (d)

Sol: The following problems are decidable for regular languages. Equivalence, Finiteness, Emptiness, infiniteness, totality, containment, Emptiness of complement, Emptiness of intersection, Emptiness of complement of intersection.

21. Ans: (a)

Sol: I. $\{a^n b^{2m} | n \geq 0, m \geq 0\} \Rightarrow$ Regular

II. $\{a^n b^m | n = 2m\} \Rightarrow$ not regular

III. $\{a^n b^m | n \neq m\} \Rightarrow$ not regular

IV. $\{x \prec y | x, y \in \{a, b\}^*\} \Rightarrow$ Regular

So, I & IV are correct.

22. Ans: (c)

Sol: Let $n = 3$

If $w = abc$,

Substrings of $w = \{\epsilon, a, b, c, ab, bc, abc\}$

non empty substrings of $w = \{a, b, c, ab, bc, abc\}$

number of substrings of w of length n is $\leq (\Sigma n) + 1$

number of non empty substrings of w of length $n \leq (\Sigma n)$.

23. Ans: (c)

Sol:

δ	a	b
$\rightarrow A$	3 choices	3 choices
B	3	3
C	3	3

$3 \times 3 \times 3 \times \dots \times 6$ times $= 3^6$ machines possible with 'A' as initial state.

Final states can be any of subset of $\{A, B, C\}$

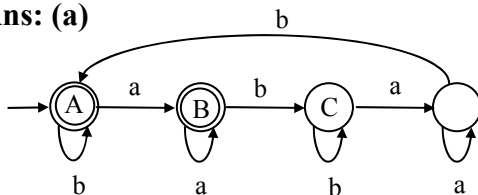
So, 2^3 possible final states combination.

Total 8×3^6 DFAs.

Number of DFAs with atleast 2 final states $= 4 \times 3^6$.

24. Ans: (a)

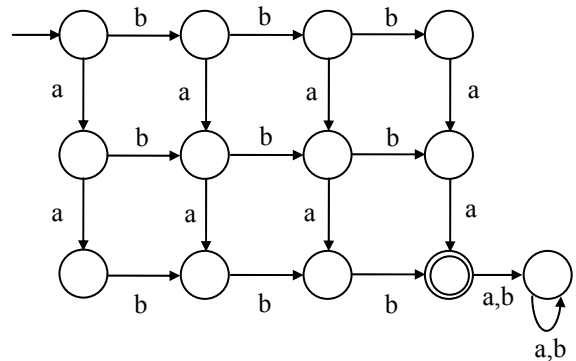
Sol:



$= 4$ states

25. Ans: (b)

Sol:



$= 13$ states

26. Ans: (b)

Sol: (i) $\left. \begin{array}{l} S \rightarrow aAB \\ A \rightarrow b \end{array} \right\} L = \phi$

(ii) $\left. \begin{array}{l} S \rightarrow aA \mid bB \\ A \rightarrow a \end{array} \right\} L = \{aa\}$

(iii) $\left. \begin{array}{l} S \rightarrow aA \\ A \rightarrow aA \\ B \rightarrow b \end{array} \right\} L = \phi$

(iv) $\left. \begin{array}{l} S \rightarrow aA \mid bB \\ B \rightarrow b \end{array} \right\} L = \{bb\}$

(i) & (iii) are equivalent.

27. Ans: (c)

Sol: $L = (a+ba)^* b (a+b)^*$

strings of length ≤ 3 :

$b, ab, ba, bb, aab, aba, abb, baa, bab, bba, bbb$

Number of strings $= 11$

28. Ans: (b)

Sol: $r = (0^* + (10)^*)^* = (0+10)^*$
 $s = (0^*+10)^*$
 $\therefore L(r) = L(s)$

29. Ans: (d)

Sol: The following sets are countable sets.

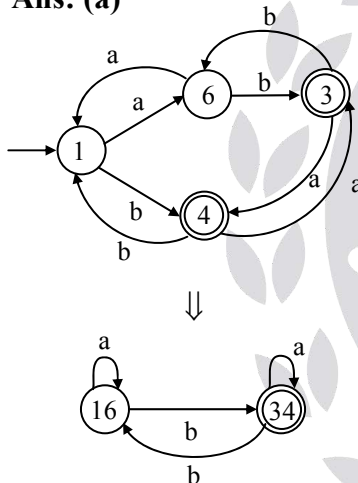
- 1) Set of regular sets
- 2) Set of CFLs
- 3) Set of Turing Machines

The set of real numbers is uncountable

The set of formal languages is uncountable.

30. Ans: (a)

Sol:



2 Equivalence classes.

31. Ans: (c)

Sol: $L = ((01)^* 0^*)^*$

$$\left. \begin{array}{l} h(a) = 0 \\ h(b) = 01 \end{array} \right\} \Rightarrow \begin{array}{l} h^{-1}(0) = a \\ h^{-1}(01) = b \end{array}$$

$$h^{-1}(L) = (b^* a^*)^* = (a+b)^*$$

32. Ans: (a)

Sol: $L_1 = a^*b$

$L_2 = ab^*$

$$L_1/L_2 = a^*b/ab^* = \{a^*b/ab, a^*b/a, \dots\}$$

$$= \{a^*, \phi, \dots\}$$

$$= a^*$$

33. Ans: (d)

Sol: (a) $L(r^*) \supset L(r^+)$

(b) $L((r+s)^*) \supset L(r^*+s^*)$

(c) $L((r+s)^*) \supset L((rs)^*)$

(d) $L(r^*) = L((r^+)^*)$

34. Ans: (b)

Sol: Arden's lemma cannot be applied to NFA with ϵ moves.

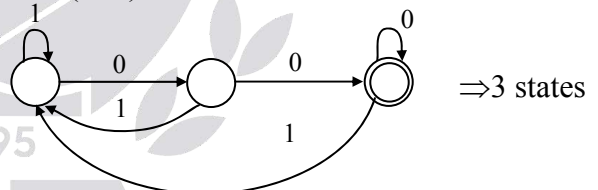
Arden's lemma applied to both DFA and NFA without ϵ moves.

35. Ans: (d)

Sol: Logic circuits, neural sets, toy's behavior can be modeled with regular sets.

36. Ans: (a)

Sol: $L = (0+1)^* 00$



$\Rightarrow 3$ states

37. Ans: (c)

Sol:

	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

= 5 states

38. Ans: (a)

Sol: 3rd symbol from ending is '1'

↓

DFA has 2^3 states.

39. Ans: (a)

Sol: $L = \{a^i b^j \mid i < 100, j \leq 10000\}$

$= \{\epsilon, a, b, \dots, a^{99} b^{10000}\}$

L is finite set

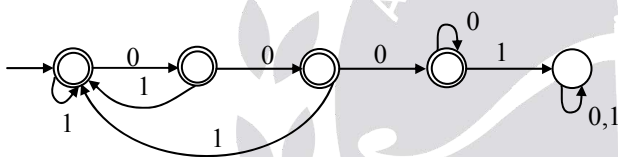
40. Ans: (a)

Sol: $L = (0+1)^* 0001 (0+1)^*$

DFA accepts L with 5 states

DFA that accepts complement of L also requires 5 states.

DFA that accepts complement of L.



41. Ans: (a)

Sol: $(00)^* + 0(00)^* + 00(000)^*$

$(00)^*$ = set of all even strings

$0(00)^*$ = set of all odd strings

$(00)^* + 0(00)^* = \text{set of all strings} = 0^*$

$\therefore (00)^* + 0(00)^* + 00(000)^* = 0^*$

42. Ans: (d)

Sol:

	0	1	2
→ q ₀	q ₀	q ₁	q ₂
→ q ₁	q ₃	q ₄	q ₅
→ q ₂	q ₀	q ₁	q ₂
→ q ₃	q ₃	q ₄	q ₅
→ q ₄	q ₀	q ₁	q ₂
→ q ₅	q ₃	q ₄	q ₅

same q₀, q₂, q₄ will be combined

same q₁, q₃ will be combined

Number of states = 3

$\{q_0, q_2, q_4\}, \{q_1, q_3\}, \{q_5\}$

43. Ans: (b)

Sol: i) $\{a^{2^n} \mid n \geq 1\}$ is not regular

ii) a^{prime} is not regular

iii) $\{0^i 1^j \mid i < j < 1000\}$ is finite. So regular

iv) Complement of L where

$L = (0+1)^* 000010101001010010(0+1)^*$

is also regular

\therefore (iii) & (iv) are regular sets.

44. Ans: (b)

Sol: i) n^{th} symbol from right end is '1' $\Rightarrow 2^n$ states

ii) n^{th} symbol from left end is '1' $\Rightarrow (n+2)$ states.

\therefore (i) has 64 states (ii) has 7 states.

45. Ans: (c)

Sol: $L = \{w \mid w \in (a+b+c)^*, n_a(w) = n_b(w) = n_c(w)\}$

L is not regular because symbols have dependency.

46. Ans: (a)

Sol: If $X = r + Xs$ and s has no ' ϵ ' then x has unique solution otherwise infinite solutions.

3. Context Free Languages (CFG, PDA)

01. Ans: (c)

Sol: CFLs are closed under:

- i) Finite union
- ii) Union
- iii) Concatenation
- iv) Kleene closure
- v) Reversal

CFLs are not closed under:

- i) Intersection
- ii) Complement
- iii) Infinite union

02. Ans: (a)

Sol: CFLs are closed under:

- i) Finite union
- ii) Homomorphism
- iii) Inverse Homomorphism
- iv) Substitution
- v) Reversal
- vi) Init
- vii) Quotient with regular set.

03. Ans: (d)

Sol: CFLs are not closed under:

- i) Intersection
- ii) Intersection with non CFL
- iii) Infinite union

04. Ans: (a)

Sol: Decidable problems for CFLs.

- i) Emptiness

ii) Finiteness

iii) Non emptiness

iv) Non finiteness (infiniteness)

v) Membership

Following problems are undecidable about CFLs:

- i) Equivalence
- ii) Containment
- iii) Totality

05. Ans: (a)

Sol: i) $\{0^n 1^n \mid n > 99\}$ is CFL

ii) $\{a^n b^n c^n \mid n < 990\}$ is finite, So CFL

iii) $\{a^n b^m c^l \mid m = l \text{ or } m = n\}$ is CFL

iv) $\{ww \mid w \in (a+b)^* \text{ and } |w| < 1000\}$ is finite, so CFL

All languages are CFLs

06. Ans: (a)

Sol: $L_1 = \{ww \mid w \in (0+1)^*\}$ is not CFL

$\bar{L}_1 = \Sigma^* - L_1$ is CFL

$L_2 = \{a^n b^n c^n \mid n > 1\}$ is not CFL

$\bar{L}_2 = \Sigma^* - L_2$ is CFL.

07. Ans: (b)

Sol: i) $\{ww^R \mid w \in (a+b)^*\}$ is CFL but not DCFL

ii) $\{w\$w^R \mid w \in (a+b)^*\}$ is DCFL but not regular

\therefore (ii) accepted by DPDA but (i) accepted by PDA.

08. Ans: (b)

Sol: i) $\{0^n 1^n \mid n > 1\}$ is DCFL

ii) $\{0^n 1^{2n} \mid n > 1\} \cup \{0^n 1^n \mid n > 10\}$ is CFL
 but not DCFL

\therefore (i) accepted by DPDA and (ii) accepted
 by PDA.

09. Ans: (c)

Sol: $S \rightarrow SS \mid a \mid \epsilon$

It is ambiguous CFG.

Every string generated by the grammar has
 more than one derivation tree.

10. Ans: (a), (b) and (c)

Sol: $S \rightarrow a \mid A$

$A \rightarrow a$

It is ambiguous CFG and has 2 parse trees
 for string 'a'

For string 'a', 2 parse trees, 2 LMD's and 2
 RMD's are there.

11. Ans: (d)

Sol: $L = \{a^l b^m c^n \mid l, m, n > 1\}$

$L = \{aa^+ bb^+ cc^+\}$

unambiguous CFG that generates L:

$S \rightarrow ABC$

$A \rightarrow aA \mid aa$

$B \rightarrow bB \mid bb$

$C \rightarrow cC \mid cc$

For given L, there exist unambiguous CFG,
 So L is called as Inherently unambiguous
 language.

12. Ans: (d)

Sol: i) $\{a^p \mid p \text{ is prime}\}$ is not regular

ii) $\{a^p \mid p \text{ is not prime}\}$ is not regular

iii) $\{a^{2^n} \mid n \geq 1\}$ is not regular

iv) $\{a^{n!} \mid n \geq 0\}$ is not regular

If language over 1 symbol is not regular then
 it is also not CFL. So all are not CFLs

13. Ans: (c) & (d)

Sol: i) $\{w \mid w \in (a+b)^*\} = (a+b)^*$ is regular

ii) $\{ww \mid w \in (a+b)^*\}$ is not CFL

iii) $\{www \mid w \in (a+b)^*\}$ is not CFL

iv) $\{ww^R w \mid w \in (a+b)^*\}$ is not CFL

Only (i) is regular and remaining are not
 regular.

So, only (i) is CFL and remaining are not
 CFLs.

14. Ans: (c)

Sol: Decidable problems about CFLs:

i) Emptiness

ii) Infiniteness

iii) Membership

15. Ans: (b)

Sol: Finiteness, Infiniteness, Membership are
 decidable for CFLs.

16. Ans: (c)

Sol: DCFLs are closed under:

- i) Complement
- ii) Inverse homomorphism
- iii) Intersection with regular set

17. Ans: (a)

Sol: DCFLs can be described by LR(k) grammars.

18. Ans: (a)

Sol: $L = \{1, 01, \dots, 110, 0110, \dots\}$
 It is neither regular nor CFL.

19. Ans: (a)

Sol: $L = 0^*10^*1$
 L is regular, so CFL.

20. Ans: (d)

Sol: In CNF, if length of string is n then derivation length is always $2n-1$.
 If Derivation length is k then string length is $(k+1)/2$

21. Ans: (a)

Sol: Top down parsing can use PDA.
 GNF CFG can be converted to PDA. Such PDA derives a string using LMD.

22. Ans: (a)

Sol: If PDA simulated by GNF CFG then the derivation of a string uses LMD.

23. Ans: (b)

Sol:

- i) $L = \{w \mid w \in (a+b)^*, n_a(w) \text{ is divisible by 3 and } n_b(w) \text{ is divisible by 5}\}$ is regular
 - ii) $L = \{w \mid w \in (a+b)^*, n_a(w) = n_b(w)\}$ is not regular but CFL
 - iii) $L = \{w \mid w \in (a+b)^*, n_a(w) = n_b(w), n_a(w) + n_b(w) \text{ is divisible by 3}\}$ is not regular but CFL
 - iv) $L = \{w \mid w \in (a+b)^*, n_a(w) \neq n_b(w)\}$ is not regular but CFLs.
- So, (i) is regular and remaining are CFLs.

24. Ans: (c)

Sol:

- i) $L = (a+b+c)^*$ is regular
- ii) $L = \{w \mid w \in (a+b+c)^*, n_a(w) = n_b(w) \text{ or } n_a(w) = n_c(w)\}$ is CFL.
- iii) $L = \{w \mid w \in (a+b+c)^*, n_a(w) = n_b(w) + n_c(w)\}$ is CFL.
- iv) $L = \{w \mid w \in (a+b+c)^*, n_a(w) = n_b(w), n_a(w) = 4n_c(w)\}$ is not CFL.

25. Ans: (a)

Sol: $L = \{w \mid w \in (a+b+c+d)^*, n_a(w) = n_b(w) = n_c(w) = n_d(w)\}$
 L is not CFL but \bar{L} is CFL
 $L_1 = \{ww \mid w \in (a+b)^*\}$
 L_1 is not CFL but \bar{L}_1 is CFL.

4. Recursive Enumerable Languages (REG, TM, REL, CSG, LBA, CSL, Undecidability)

01. Ans: (d)

Sol: Turing machine is equivalent to the following:

- TM with single tape
- TM start with blank tape
- TM with 2-way infinite tape
- TM with 2 symbols and blank

02. Ans: (a) & (c)

Sol: a. TM with one push down tape and read only is equivalent to push down automata
 b. TM with two push down tapes is equivalent to TM
 c. TM without alphabet is not equivalent to any machine.

03. Ans: (d)

Sol: a. TM with 4 counters is equivalent to TM
 b. TM with 3 counters is equivalent to TM
 c. TM with 2 counters is equivalent to TM

04. Ans: (d)

Sol: a. TM with multiple heads \cong TM
 b. Multi dimensional tape TM \cong TM
 c. n-dimensional tape TM \cong TM

05. Ans: (a)

Sol: a. TM that have no ink is equivalent to finite automata
 b. TM with 3 pebbles \cong TM
 c. 2-way infinite tape TM \cong TM
 d. 100000 tape TM \cong TM

06. Ans: (a) & (b)

Sol: (a) TM that cannot leave their input is equivalent to LBA
 (b) TM that cannot use more than $n!$ cells on 'n' length input is not equivalent to TM.
 (c) 3-tape TM is equivalent to TM
 (d) TM with single symbol alphabet is equivalent to TM

07. Ans: (d)

Sol: The set of partial recursive functions represent the sets computed by turing machines.

08. Ans: (a)

Sol: (a) Turing machines are equivalent to C programs.
 (b) TMs that always halt are equivalent to halting C programs.
 (c) Halting C programs not equivalent to turing machines
 (d) C++ programs are equivalent to turing machines.

09. Ans: (c)

Sol: Set of turing machines is logically equivalent to set of LISP programs.

10. Ans: (b)

Sol: Class of halting turing machines is equivalent to class of halting prolog programs
 \therefore The class of prolog programs describes a richer set of functions.

11. Ans: (a)

Sol: The class of an assembly programs is equivalent to class of all functions computed by turing machines.

12. Ans: (a)

Sol: Set of regular languages and set of recursive languages are closed under intersection and complement.

13. Ans: (c)

Sol:

- Non-deterministic TM is equivalent to deterministic TM
- Non-deterministic halting TM is equivalent to deterministic halting TM.

14. Ans: (d)

Sol: Universal TM is equivalent to TM.

15. Ans: (a)

Sol: $L = \text{Set of regular expressions}$

$$\bar{L} = \phi$$

L is REL and \bar{L} is also REL

So, L is recursive language.

16. Ans: (a)

Sol: Algorithms \cong Procedures \cong TMs

17. Ans: (a)

Sol: Hyper computer is equivalent to TM.

TM can accept non-regular.

18. Ans: (b)

Sol: TM head restricted to input accepts CSL

19. Ans: (b)

Sol: Type 0 grammar is equivalent to turing machine.

20. Ans: (c)

Sol: Type 1 grammar is equivalent to linear bounded automata.

21. Ans: (a & d)

Sol: $L = \{wwwww / w \in (a + b + c)^*\}$

L is CSL but not CFL

So, L is also recursive language

22. Ans: (a) & (d)

Sol: $L = \{a^n b^{n!} c^{(n!)^2} \mid n > 1\}$

L is CSL but not CFL

So, L is also recursive language

So a & d are false

23. Ans: (d)

Sol: $L = \{ww^R / w \in (a + b)^*\}$

L is CFL but not regular

24. Ans: (d)

Sol: $(0 + 1 + \dots + n + A + B + \dots + F)^* 1 (0 + 1 + \dots + 9 + A + B + C + D + E + F)^*$

It is regular language

25. Ans: (c)

Sol: $L = a^{47^n}$

L is CSL

26. Ans: (d)

Sol: Recursive languages are closed under union, intersection, complement, reversal and concatenation.

Recursive languages are not closed under substitution, homomorphism, quotient and subset.

27. Ans: (d)

Sol:

- Regular sets are closed under finite union, intersection, complement, homomorphism, inverse homomorphism and reversal.
- Containment, equivalence, emptiness, totality problems are decidable for regular sets.

28. Ans: (d)

Sol: The following problems are undecidable for CFL's

1. Equivalence
2. Totality
3. Containment

29. Ans: (c)

Sol: The following problems are undecidable for CSL's

1. Finiteness
2. Emptiness
3. Totality (Σ^*)
4. Equivalence
5. Containment

30. Ans: (d)

Sol: Undecidable problems for recursive sets:

1. Emptiness
2. Infiniteness
3. Regularity
4. Equivalence
5. Containment

Membership problem is decidable for recursive sets

31. Ans: (d)

Sol: Given TM accepts only 2 strings of length one $L = \{0, 1\}$

5. Theory of Complexity

01. Ans: (a)

Sol: $L = \{a^n b^n c^n \mid n \geq 1\}$ is CSL but P-Problem can be accepted by TM in $O(n^2)$ moves. It is P-Problem.

02. Ans: (d)

Sol: (a) If L is accepted by DTM in polynomial time then L is P-Problem.

(b) If L is accepted by NTM in polynomial time then L is NP-Problem.

(c) If L is verified by DTM in polynomial time then L is NP-Problem.

03. Ans: (d)

Sol: $L = \{a^n b^n \mid n \geq 0\}$ is P-Problem

$L = \{a^n b^n c^n \mid n \geq 1\}$ is P-Problem

$L = \{www \mid w \in \Sigma^*\}$ is P-Problem

$L = \{ \langle r \rangle \mid r \text{ is regular expression and } L(r) = \emptyset \}$ is NPH-Problem.

04. Ans: (a)

Sol: Regular language can be accepted by DTM in $O(n)$ time.

It can be take $O(1)$ space to accept.

05. Ans: (c)

Sol: $L = \{ 0^n 1^n \mid n \geq 1 \}$ takes $O(n^2)$ time and $O(n)$ space (i) & (iii) are correct.

06. Ans: (d)

Sol: Complement of NP-Problem need not be NP-Problem.

07. Ans: (b)

Sol: P and NP class is closed under homomorphism.

08. Ans: (c)

Sol: If NTM takes $t(n)$ time to decide any problem then DTM can take $2^{O(t(n))}$ time to decide the same problem.

09. Ans: (c)

Sol: If multitape NTM decides a language L in $t(n)$ time then single tape NTM requires $(t(n))^2$ time.

10. Ans: (d)

Sol: $P \subseteq NP \subseteq PSPACE \subseteq EXP$

11. Ans: (c)

Sol: All the problems take exp time.

12. Ans: (d)

Sol: Conversion from NFA to DFA takes $O(2^n)$ time.

13. Ans: (c)

Sol: $f(n) = \max(n^2, n+1, 30) = O(n^2)$

14. Ans: (b)

Sol: SAT is NP-Problem

1-SAT and 2-SAT are P-Problems (So NP-Problems)

3-SAT and n-SAT are NP-Problems.

15. Ans: (b)

Sol: $NP = Co-NP$ iff L and \bar{L} are in NP.

16. Ans: (b)

Sol: L is in NP iff L is polynomial time verifiable

L is in P iff L is decidable in polynomial time

If L is in P then \bar{L} is in P

If L is in NP then \bar{L} need not be in NP.

17. Ans: (c)

Sol: L is in NPC iff L is in both NP and NPH.

18. Ans: (a)

Sol: If $L \in P$ and $P = NP$ then $NPC = P$.

So, $L \in NPC$.

19. Ans: (d)

Sol:

(i), (ii) & (iii) are true

(i) $\text{NPH-Problem} \leq L_1 \Rightarrow L_1$ is NPH-Problem

(ii) If $\text{NPC} \leq L_1$ and L_1 is in NP $\Rightarrow L_1$ is in NPC

(iii) If L is in NPC and $L \in P$ then $P = NP = \text{NPC}$

20. Ans: (a) & (c)

Sol: (a) Integer Linear Programming is NPC problem

(b) Primarily is NP-Problem

(c) 3-CNF is NPC problem

21. Ans: (b)

Sol: CYK algorithm is membership algorithm uses dynamic programming.

It takes $O(n^3)$ time.

