MECHANICAL ENGINEERING

Strength of Materials

Text Book: Theory with worked out Examples and Practice Questions
**Strength of Materials**

_Solutions for Text Book Practice Questions_

**Chapter 1**

Simple Stresses and Strains

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**Fundamental, Mechanical Properties of Materials, Stress Strain Diagram**

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**01. Ans: (b)**

**Sol:**

- **Ductility:** The property of materials to allow large deformations or large extensions without failure (large plastic zone) is termed as ductility.
- **Brittleness:** A brittle material is one which exhibits a relatively small extensions or deformations prior to fracture. Failure without warning (No plastic zone) i.e. no plastic deformation.
- **Tenacity:** High tensile strength.
- **Creep:** Creep is the gradual increase of plastic strain in a material with time at constant load.
- **Plasticity:** The property by which material undergoes permanent deformation even after removal of load.
- **Endurance limit:** The stress level below which a specimen can withstand cyclic stress indefinitely without failure.
- **Fatigue:** Decreased Resistance of material to repeated reversal of stresses.

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**02. Ans: (a)**

**Sol:**

- When the material is subjected to stresses, it undergoes to strains. After removal of stress, if the strain is not restored/recovered, then it is called inelastic material.
- For rigid plastic material:
  - Any material that can be subjected to large strains before it fractures is called a ductile material. Thus, it has large plastic zone.
  - Materials that exhibit little or no yielding before failure are referred as brittle materials. Thus, they have no plastic zone.

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**03. Ans: (a)**

**Sol:** Refer to the solution of Q. No. (01).

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**04. Ans: (b)**

**Sol:** The stress-strain diagram for ductile material is shown below.
A material is **homogeneous** if it has the same composition throughout the body. Hence, the elastic properties are the same at every point in the body in a given direction. However, the properties need not to be the same in all the directions for the material. Thus, both A and B are false.

**06. Ans: (a)**

**Sol:** Strain hardening increase in strength after plastic zone by rearrangement of molecules in material.

- **Visco-elastic material** exhibits a mixture of creep as well as elastic after effects at room temperature. Thus their behavior is time dependant.

**07. Ans: (a)**

**Sol:** Refer to the solution of Q. No. (01).

**08. Ans: (a)**

**Sol:** Modulus of elasticity (Young's modulus) of some common materials are as follow:

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Modulus (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Cast iron</td>
<td>100 GPa</td>
</tr>
<tr>
<td>Aluminum</td>
<td>60 to 70 GPa</td>
</tr>
<tr>
<td>Timber</td>
<td>10 GPa</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.01 to 0.1 GPa</td>
</tr>
</tbody>
</table>

**09. Ans: (a)**

**Sol:** Addition of carbon will increase strength, thereby ductility will decrease.
### Elastic Constants and Their Relationships

**01. Ans (c)**

Sol: We know that,

\[
Poisson's \ ratio = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{\Delta D/8}{\Delta L/L}
\]

\[\therefore \ \mu = \frac{\Delta D}{8} \frac{P}{AE} \]

\[\therefore \ \mu = \frac{\Delta D}{8} \frac{AE}{P} \]

\[\therefore \ 0.25 = \frac{\Delta D}{8} \frac{\pi^2 \times 10^6}{50000} \]

\[\Rightarrow \ \Delta D = 1.98 \times 10^{-3} \approx 0.002 \ cm\]

**02. Ans: (c)**

Sol: We know that,

\[
\text{Bulk modulus} = \frac{\delta P}{\delta V/V}
\]

\[\Rightarrow \ 2.5 \times 10^5 = \frac{200 \times 20}{\delta V} \]

\[\delta V = 0.016 \ m^3\]

### Linear and Volumetric Changes of Bodies

**01. Ans: (d)**

Sol:

\[
\varepsilon_z = 0
\]

\[
\varepsilon_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E}
\]

\[\therefore \ \ 0 = \frac{(-P)}{E} - \mu \frac{(-P)}{E} - \mu \frac{P_x}{E} \]

\[\Rightarrow \ P = \frac{\mu P_x}{(1 - \mu)}\]
02. Ans: (a)
Sol: Given that, $\sigma_c = 4\tau$

Punching force = Shear resistance of plate
\[
\begin{align*}
\therefore \quad \sigma (\text{Cross section area}) &= \tau (\text{surface Area}) \\
\therefore \quad 4\times\tau\times\frac{\pi D^2}{4} &= \tau (\pi D t) \\
\therefore \quad D &= t = 10 \text{ mm}
\end{align*}
\]

03. Ans: (d)
Sol:
\[
\begin{align*}
\sigma_s &= 140 \text{ MPa} = \frac{P_s}{A_s} \\
\therefore \quad P_s &= \frac{140 \times 500}{3} \approx 23,300 \text{ N} \\
\sigma_{Al} &= 90 \text{ MPa} = \frac{P_{Al}}{A_{Al}} \\
\therefore \quad P_{Al} &= 90 \times 400 = 36,000 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\sigma_B &= 100 \text{ MPa} = \frac{P_B}{A_B} \\
\Rightarrow \quad P_B &= \frac{100 \times 200}{2} = 10,000 \text{ N}
\end{align*}
\]

Take minimum value from $P_s$, $A_{Al}$ and $P_B$.
\[
\Rightarrow \quad P = 10,000 \text{ N}
\]

04. Ans: (c)
Sol:
\[
\begin{align*}
\text{From similar triangle} \\
3a &= 2a \\
3\delta_B &= 2\delta_A \ldots \ldots (1)
\end{align*}
\]

Stiffness $K = \frac{W}{\delta}$
\[
\therefore \quad K_A = \frac{W_A}{\delta_A} \Rightarrow \delta_A = \frac{W_A}{2K} \\
\text{Similarly} \quad \delta_B = \frac{W_B}{K}
\]

From equation (1)
\[
3 \times \frac{W_B}{K} = 2 \times \frac{W_A}{2K}
\]
\[
\Rightarrow \quad \frac{W_A}{W_B} = 3
\]
1. **Ans:** (b)  
**Sol:** Free expansion = Expansion prevented  
\[ [\alpha t]_s + [\alpha t]_{Al} = \left[ \frac{P \ell}{AE} \right]_s + \left[ \frac{P \ell}{AE} \right]_{Al} \]  
\[ 11 \times 10^{-6} \times 20 + 24 \times 10^{-6} \times 20 \]  
\[ \frac{P}{100 \times 10^3 \times 200} + \frac{P}{200 \times 10^3 \times 70} \]  
\[ \Rightarrow P = 5.76 \text{ kN} \]  
\[ \sigma_s = \frac{P}{A_s} = \frac{5.76 \times 10^3}{100} = 57.65 \text{ MPa} \]  
\[ \sigma_{Al} = \frac{P}{A_{Al}} = \frac{5.76 \times 10^3}{200} = 28.82 \text{ MPa} \]  

2. **Ans:** (a)  
**Sol:**  
Strain in X-direction due to temperature,  
\[ \varepsilon_x = \alpha (\Delta T) \]  
Strain in X-direction due to volumetric stress,  
\[ \varepsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} \]  

3. **Ans:** (b)  
**Sol:**  
- Free expansion in x direction is \( \alpha t \).
- Free expansion in y direction is \( \alpha t \).
- Since there is restriction in y direction expansion doesn’t take place. So in lateral direction, increase in expansion due to restriction is \( \mu \alpha t \).

Thus, total expansion in x direction is,  
\[ = a \alpha t + \mu a \alpha t \]  
\[ = a \alpha t (1 + \mu) \]
Chapter 2: Complex Stresses and Strains

01. Ans: (b)
Sol: Maximum principal stress \( \sigma_1 = 18 \)
Minimum principal stress \( \sigma_2 = -8 \)
Maximum shear stress \( \frac{\sigma_1 - \sigma_2}{2} = 13 \)
Normal stress on Maximum shear stress plane
\[ \frac{\sigma_1 + \sigma_2}{2} = \frac{18 + (-8)}{2} = 5 \]

02. Ans: (b)
Sol: Radius of Mohr’s circle, \( \tau_{\text{max}} = \frac{\sigma_1 - \sigma_2}{2} \)
\[ \therefore 20 = \frac{\sigma_1 - 10}{2} \]
\( \Rightarrow \sigma_1 = 50 \text{ N/mm}^2 \)

03. Ans: (b)
Sol: Given data,
\( \sigma_x = 150 \text{ MPa}, \ \sigma_y = -300 \text{ MPa}, \ \mu = 0.3 \)
Long dam \( \rightarrow \) plane strain member
\[ \varepsilon_z = 0 = \frac{\sigma_z - \mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} \]
\[ \therefore 0 = \sigma_z - 0.3 \times 150 + 0.3 \times 300 \]
\( \Rightarrow \sigma_z = 45 \text{ MPa} \)

04. Ans: (b)
Sol:
From the bove, we can say that Mohr’s circle is a point located at 175 MPa on normal stress axis.
Thus, \( \sigma_1 - \sigma_2 = 175 \text{ MPa} \)

05. Ans: (c)
Sol: Given that, \( \sigma_z = 0 \)
\[ \therefore \sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]
\[ \therefore \frac{\sigma_x + \sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]
\[ \therefore \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2 \]
\[ \therefore \tau_{xy}^2 = \sigma_x \sigma_y \]
\( \Rightarrow \tau_{xy} = \sqrt{\sigma_x \sigma_y} \)
01. Ans: (b)
Sol: Contra flexure is the point where BM is becoming zero.

Taking moment about A,

\[ \Sigma M_A = 0 \]
\[ 17.5 \times 4 - 20 \times 10 - R_B \times 8 = 0 \]
\[ \therefore R_B = 42.5 \text{ kN} \]

Now, \( M_x = -20x + R_B(x - 2) \)
For bending moment be zero \( M_x = 0, \)
\[ -20x + 42.5(x - 2) = 0 \]
\[ \Rightarrow x = 3.78 \text{ m from right i.e. from D.} \]

02. Ans: (b)
Sol:

Take \( \Sigma M_p = 0 \)
\[ \frac{1}{2} \times 25 \times 1.5 \times \left( \frac{1.5}{3} + 4 \right) - (R_Q \times 4) + 100 \times 2 + 25 = 0 \]
\[ \therefore R_Q = 77.34 \text{ kN} \]

Also, \( \Sigma V = 0 \)
\[ \therefore R_p + R_Q = 100 + \frac{1}{2} \times 25 \times 1.5 = 118.75 \text{ kN} \]
\[ \therefore R_p = 41.41 \text{ kN} \]
\[ \Rightarrow \text{Shear force at P} = 41.41 \text{ kN} \]

03. Ans: (c)
Sol: \( M_S = R_P (3) + 25 - (100 \times 1) = 49.2 \text{ kN-m} \)

04. Ans: (c)
Sol:

\[ 3 \text{ kN} \]
\[ 3 \text{ kN-m} \]
\[ 1 \text{ m} \]
\[ 1 \text{ m} \]
\[ 1 \text{ m} \]
\[ V_A \]
\[ V_B \]
\[ -V_B \times 3 + 3 = 0 \]
\[ \therefore V_C = 1 \text{ kN} \]
\[ \therefore \text{Bending moment at B,} \]
\[ \Rightarrow M_B = V_C \times 1 = 1 \text{ kN-m} \]
05. Ans: (a)
Sol:

![Diagram of a beam with reaction forces at supports]

Reaction at both the supports are 2 kN and in upward direction.

06. Ans: (c)
Sol:

![BMD Diagram]

Bending moment at \( \frac{l}{2} \) from left is \( \frac{PL}{4} \).

The given beam is statically determinate structure. Therefore equilibrium equations are sufficient to analyze the problem.

In statically determinate structure the BMD, SFD and Axial force are not affected by section (I), material (E), thermal changes.

07. Ans: (a)
Sol: As the given support is hinge, for different set of loads in different direction beam will experience only axial load.

01. Ans: (a)
Sol:

\[
\bar{y} = \frac{E_1 y_1 + E_2 y_2}{E_1 + E_2}
\]

\[
\bar{y} = \frac{2E_2 \left( h + \frac{h}{2} \right) + E_2 \times \frac{h}{2}}{2E_2 + E_2}
\]

\( \therefore E_1 = 2E_2 \)

\( \bar{y} = 1.167h \) from base

02. Ans: (b)
Sol:

\[
\bar{y} = \frac{A_1 E_1 Y_1 + A_2 E_2 Y_2}{A_1 E_1 + A_2 E_2}
\]

\[
= \frac{1.5a \times 3a^2 	imes E_1 + 1.5a \times 6a^2 \times 2E_1}{3a^2 E_1 + 6a^2 (2E_1)}
\]

\[
= \frac{22.5a^3 E_1}{15a^2 E_1} = 1.5a
\]

03. Ans: 13.875 bd³
Sol:

![Diagram of a rectangular section with CG]

\( y = \frac{5}{4}d \)
M.I about CG = \( I_{CG} = \frac{2b(3d)^3}{12} = \frac{9}{2}bd^3 \)

M.I about X-X at \( \frac{d}{4} \) distance = \( I_G + Ay^2 \)

\[
= \frac{9}{2}bd^3 + 6bd\left(\frac{5}{4}\right)d^2
\]

\[
= \frac{111}{8}bd^3 = 13.875bd^3
\]

04. Ans: \( 6.885 \times 10^6 \) mm\(^4\)

Sol:

\[
I_x = \frac{BD^3}{12} - \left( \frac{bd^3}{12} + Ah^2 \right)
\]

\[
= \frac{60 \times 120^3}{12} - 2\left( \frac{30 \times 30^3}{12} + (30 \times 30) \times 30^2 \right)
\]

\[
= 6.885 \times 10^6 \text{ mm}^4
\]

05. Ans: \( 152146 \) mm\(^4\)

Sol:

\[
I_x = \frac{30 \times 40^3}{12} - \frac{\pi \times 20^4}{64} = 152146 \text{ mm}^4
\]

\[
I_y = \frac{40 \times 30^3}{12} - \left( \frac{\pi \times 20^4}{64} + 2\left( \frac{\pi}{2} \times 10^3 \times \left(15 - \frac{4 \times 10}{3\pi}\right)^2 \right) \right)
\]

\[
= 45801.34 \text{ mm}^4
\]

02. Ans: (b)

Sol:

By using flexural formula, \( \sigma = \frac{M}{Z} \)

\[
\therefore \sigma = \frac{1}{Z} Z_B = \frac{6}{b \times \left(\frac{b}{2}\right)} = 2
\]

\[
\Rightarrow \sigma_A = 2\sigma_B
\]
\[ \sum M_A = 0 \]
\[ \therefore P \times 100 + 2P \times 200 + 3P \times 300 = R_B \times 400 \]
\[ \therefore R_B = 3.5P, R_A = 2.5P \]

Take moments about F and moment at F
\[ M_F = R_B \times 150 - 3P \times 50 = 375P \]
Also, \[ \frac{M_F}{I} = \frac{\sigma_b}{Y_F} \]
\[ \therefore 375P = \left( \frac{1.5 \times 10^{-6} \times 200 \times 10^3}{6} \right) \]
\[ \Rightarrow P = 0.29 \text{ N} \]

03. Ans: (b)
Sol: By using Flexural formula,
\[ \frac{E}{R} \frac{Y_{max}}{Y_{top}} = \frac{2 \times 10^6}{250} = \frac{\sigma_b}{(0.5/2)} \]
\[ \Rightarrow \sigma_b = 200 \text{ N/mm}^2 \]

04. Ans: (c)
Sol:

By using flexural formula,
\[ \frac{M}{I} \frac{f}{y} \]
\[ \therefore \frac{16 \times 10^6}{100 \times 150} = \frac{f}{25} \Rightarrow f = 14.22 \text{ MPa} \]
Now, Force on hatched area
\[ = \text{Average stress} \times \text{Hatched area} \]
\[ = \left( \frac{0 + 14.22}{2} \right) (25 \times 50) = 8.9 \text{ kN} \]

05. Ans: (b)
Sol: By using flexural formula, \[ \frac{f_{\text{Tensile}}}{Y_{\text{top}}} = \frac{M}{I} \]
\[ \Rightarrow f_{\text{Tensile}} = \frac{0.3 \times 3 \times 10^6}{3 \times 10^6} \times 70 \]
(maximum bending stress will be at top fibre so \( y_1 = 70 \text{ mm} \))
\[ \Rightarrow f_{\text{Tensile}} = 21 \text{ N/mm}^2 = 21 \text{ MN/m}^2 \]

06. Ans: (c)
Sol: Given data:
\[ P = 200 \text{ N}, \quad M = 200 \text{ N.m} \]
\[ A = 0.1 \text{ m}^2, \quad I = 1.33 \times 10^{-3} \text{ m}^4 \]
\[ y = 20 \text{ mm} \]
Due to direct tensile force \( P \),
\[ \sigma_d = \frac{P}{A} = \frac{200}{0.1} = 2000 \text{ N/m}^2 \text{ (Tensile)} \]
Due to the moment \( M \),
\[ \sigma_b = \frac{M}{I} \times y \]
\[ = \frac{200}{1.33 \times 10^{-3}} \times 20 \times 10^{-3} \]
\[ = 3007.52 \text{ N/m}^2 \text{ (Compressive)} \]
\[ \sigma_{\text{net}} = \sigma_d - \sigma_b \]
\[ = 2000 - 3007.52 \]
\[ = -1007.52 \text{ N/m}^2 \]
Negative sign indicates compressive stress.
\[ \sigma_{\text{net}} = 1007.52 \text{ N/m}^2 \]
07. Ans: 80 MPa

Sol:

Maximum stress in timber = 8 MPa
Modular ratio, \( m = 20 \)
Stress in timber in steel level,

\[
100 \rightarrow \frac{8}{50} \rightarrow f_w
\]
\( \Rightarrow f_w = 4 \text{ MPa} \)

Maximum stress developed in steel is \( = m \cdot f_w \)

\[= 20 \times 4 = 80 \text{ MPa} \]

Convert whole structure as a steel structure by using modular ratio.

08. Ans: 2.43 mm

Sol: From figure \( A_1B_1 = l = 3 \text{ m} \) (given)

\[
AB = \left( R - \frac{h}{2} \right) \alpha = l - l\alpha_1 \quad \text{(1)}
\]

\[
A_2B_2 = \left( R + \frac{h}{2} \right) \alpha = l + l\alpha_2 \quad \text{(2)}
\]

Subtracting above two equations (2) – (1)

\[
h \ (\alpha) = l\alpha \ (t_2 - t_1)
\]

but \( A_1B_1 = l = R\alpha \)

\( \Rightarrow \alpha = \frac{l}{R} \)

\[\therefore h \left( \frac{l}{R} \right) = l\alpha (\Delta T) \]

\[
R = \frac{h}{\alpha(\Delta T)}
\]

\[= \frac{250}{(1.5 \times 10^{-5})(72 - 36)}
\]

\[R = 462.9 \text{ m} \]

From geometry of circles

\[
(2R - \delta)\delta = \frac{L}{2} \cdot \frac{L}{2} \quad \{\text{ref. figure in Q.No.02}\}
\]

\[
2R\delta - \delta^2 = \frac{L^2}{4} \quad \text{(neglect } \delta^2) \]

\[
\delta = \frac{L^2}{8R} = \frac{3^2}{8 \times 462.9} = 2.43 \text{ mm}
\]

\[\text{Shortcut:}\]

Deflection is due to differential temperature of bottom and top \( (\Delta T = 72^\circ - 36^\circ = 36^\circ) \).

Bottom temperature being more, the beam deflects down.

\[
\delta = \frac{\alpha(\Delta T)l^2}{8h} = \frac{1.5 \times 10^{-5} \times 36 \times 3000^2}{8 \times 250}
\]

\[ = 2.43 \text{ mm (downward)} \]
01. Ans: (a)
Sol: \( \tau_{\text{max}} = \frac{3}{2} \times \tau_{\text{avg}} = \frac{3}{2} \times \frac{f}{b.d} \)
\[ 3 = \frac{3}{2} \times \frac{50 \times 10^3}{100 \times d} \]
\[ \therefore d = 250 \text{ mm} = 25 \text{ cm} \]

02. Ans: 37.3
Sol:

03. Ans: 61.43 MPa
Sol:
\[ I_{\text{NA}} = 13 \times 10^6 \text{ mm}^4 \]
\[ y_{CG} = 107 \text{ mm from base} \]
\[ \tau_{\text{max}} = \frac{F \bar{y}}{I_b} \]
\[ A \bar{y} = (120 \times 20 \times 43) + (33 \times 20 \times 16.5) \]
\[ = 114090 \text{ mm}^3 \]
\[ \tau_{\text{max}} = \frac{140 \times 10^3 \times 114090}{13 \times 10^6 \times 20} = 61.43 \text{ MPa} \]
01. Ans: (c)
Sol: Twisting moment = 2 × 0.5 + 1 × 0.5
    = 1.5 kN-m

02. Ans: (d)
Sol: \[
\frac{(\text{Strength})_{\text{solid}}}{(\text{Strength})_{\text{hollow}}} = \frac{1}{1-K^4}
\]
    = \frac{1}{1-(1/2)^4} = \frac{16}{15}

03. Ans: 43.27 MPa & 37.5 MPa
Sol: Given \(D_o = 30 \text{ mm} \), \( t = 2 \text{ mm}\)
    \( \therefore D_i = 30 - 4 = 26 \text{ mm}\)
    We know that \( \frac{\tau}{J} = \frac{q}{R} \)

\[
\frac{100 \times 10^3}{\pi(30^4 - 26^4)} = \frac{q_{\text{max}}}{32}\left(\frac{30}{2}\right)
\]
    \(q_{\text{max}} = 43.279 \text{ N/mm}^2\)

\[
\frac{100 \times 10^3}{\pi(30^4 - 26^4)} = \frac{q_{\text{min}}}{32}\left(\frac{26}{2}\right)
\]
    \(q_{\text{min}} = 37.5 \text{ N/mm}^2\)
03. Ans: (c)
Sol:
\[ \theta_{\text{max}} = \frac{wl^3}{6EI} = 0.02 \quad \text{-------(i)} \]
\[ y_{\text{max}} = \frac{wl^4}{8EI} \]
\[ \Rightarrow 0.018 = \left( \frac{wl^3}{6EI} \right) \times \frac{L \times 6}{8} \]
\[ \Rightarrow \quad 0.018 = \frac{0.02 \times L \times 6}{8} \quad \text{[': Equation (i)]} \]
\[ \Rightarrow \quad L = 1.2 \text{ m} \]

04. Ans: (a)
Sol:
\[ \downarrow y = \frac{wl^3}{48EI} \]
\[ \theta = \frac{wl^2}{16EI} \]
\[ \tan \theta = \frac{y}{(L - \ell)/2} \]
\[ \theta \text{ is small } \Rightarrow \tan \theta = \theta \]
\[ \therefore \theta = \frac{y}{(L - \ell)/2} \]

05. Ans: (c)
Sol: By using Maxwell’s law of reciprocals theorem
\[ \delta_{C/B} = \delta_{B/C} \]

Deflection at ‘C’ due to unit load at ‘B’
\[ = \text{Deflection at ‘B’ due to unit load at ‘C’} \]
As the load becomes half deflection becomes half.

06. Ans: (c)
Sol:
\[ y_A = y_B \Rightarrow \left( \frac{wL^3}{3EI} \right)_A = \left( \frac{wL^3}{48EI} \right)_B \]
\[ \therefore L_B = 400 \text{ mm} \]
07. Ans: 0.05

Sol:

\[ dydx = 0.004 \]

Integrating with respect to x,

\[ y = 0.004x^2 \]

At mid span, \( x = 5 \) m

\[ y = 0.05 \text{ m} \]

\[ \therefore \text{Curvature, } \frac{d^2y}{dx^2} = 0.004 \]

01. Ans: (b)

Sol: \( \tau_{max} = \sigma_l = \frac{\sigma_h - 0}{2} = \frac{PD}{4t} \)

\[ \therefore \tau_{max} = \frac{1.6\times900}{4\times12} = 30 \text{ MPa} \]

02. Ans: 2.5 MPa & 2.5 MPa

Sol: Given data:

\( R = 0.5 \text{ m}, \quad D = 1 \text{ m}, \quad t = 1 \text{ mm}, \quad H = 1 \text{ m}, \quad \gamma = 10 \text{ kN/m}^3, \quad h = 0.5 \text{ m} \)

At mid-depth of cylindrical wall (\( h = 0.5 \text{ m} \)):

Circumferential (hoop) stress,

\[ \sigma_c = \frac{P_{at h=0.5m} \times D}{4t} = \frac{\gamma h \times D}{4t} \]

\[ = \frac{10\times10^3 \times (2 \times 0.5)}{4 \times 1 \times 10^{-3}} \]

\[ = 2.5 \times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa} \]

Longitudinal stress at mid-height,

\[ \sigma_l = \frac{\gamma \times \text{Volume}}{\pi D \times t} \]

\[ = \frac{\gamma \times \frac{\pi}{4} D^2 L}{\pi D \times t} = \frac{\gamma \times DL}{4t} \]

\[ = \frac{10\times10^3 \times 1 \times 1}{4 \times 1 \times 10^{-3}} \]

\[ = 2.5 \times 10^6 \text{ N/m}^2 = 2.5 \text{ MPa} \]
Chapter 10

Columns

01. Ans: (c)
   Sol: By using Euler’s formula, \( P_e = \frac{\pi^2 \times EI}{l_e^2} \)

   For a given system, \( l_e = \frac{l}{2} \)

   \[ P_e = \frac{4\pi^2 \times EI}{l_e^2} \]

02. Ans: (b)
   Sol: We know that, \( P_e = \frac{\pi^2 EI}{l_e^2} \)

   \[ \therefore \frac{P_1}{P_2} = \frac{l^2}{2l^2} \]

   \[ \therefore \frac{P_1}{P_2} = \frac{1}{2} \implies P_1 : P_2 = 1 : 4 \]

03. Ans: 4
   Sol: Euler’s crippling load,

   \[ P = \frac{\pi^2}{l^2} EI \]

   \[ \therefore P \propto I \]

   \[ \therefore \frac{P}{P_e} = \frac{I_{\text{bonded}}}{I_{\text{loose}}} = \frac{[\frac{b(2t)^3}{12}]}{2[\frac{bt^3}{12}]} = 4 \]

04. Ans: (c)
   Sol: Euler’s theory is applicable for axially loaded columns.

   Force in member AB, \( P_{AB} = \frac{F}{\cos 45^\circ} = \sqrt{2}F \)

   \[ P_{AB} = \frac{\pi^2 EI}{l_e^2} \]

   \[ \therefore \sqrt{2} F = \frac{\pi^2 EI}{l_e^2} \]

   \[ \implies F = \frac{\pi^2 EI}{2L^2} \]

05. Ans: (a)
   Sol: Given data:

   \( L_e = L = 3 \text{ m} \),

   \( \alpha = 12 \times 10^{-6} /\text{C} \),

   \( d = 50 \text{ mm} = 0.05 \text{ m} \)

   Buckling load, \( P_e = \frac{\pi^2 EI}{L_c^2} \)

   \[ \therefore P_e \frac{L}{AE} = L \alpha \Delta T \]

   \[ \implies \frac{\pi^2 EI \times L}{L^2 \times AE} = L \alpha \Delta T \]

   \[ \implies \frac{\pi^2 \times E \times \frac{\pi}{64} \times d^4 \times L}{L^2 \times \frac{\pi}{4} \times d^2 \times E} = L \alpha \Delta T \]

   \[ \therefore \Delta T = \frac{\pi^2 \times d^2}{16 \times L^2 \times \alpha} = \frac{\pi^2 \times (0.05)^2}{16 \times 3^2 \times 12 \times 10^{-6}} \]

   \[ \implies \Delta T = 14.3^\circ \text{C} \]
01. Ans: (*)
Sol:
- Slope of the stress-strain curve in the elastic region is called modulus of elasticity.
  For the given curves,
  \[(\text{Modulus of elasticity})_A > (\text{Modulus of elasticity})_B\]
  \[\therefore E_A > E_B\]
- The material for which plastic region is more is stress-strain curve is possessed high ductility. Thus, \(D_B > D_A\).

02. Ans: (b)
Sol:

\[\frac{U_B}{U_A} = \frac{(V_1 + V_2)_B}{(V_1 + V_2)_A}\]
\[\Rightarrow \frac{U_B}{U_A} = \frac{\frac{\sigma_1^2}{2E} \times V_1 + \frac{\sigma_2^2}{2E} \times V_2}{\frac{\sigma_1^2}{2E} \times V_1 + \frac{\sigma_2^2}{2E} \times V_2}_B\]
\[= \frac{\frac{p^2}{A_1} \times L_1 + \frac{p^2}{A_2} \times L_2}{\frac{p^2}{A_1} \times L_1 + \frac{p^2}{A_2} \times L_2}_A\]
\[= \frac{\frac{L_1}{A_1} + \frac{L_2}{A_2}}{\frac{L_1}{A_1} + \frac{L_2}{A_2}}_A\]
\[= \frac{7.165}{4.77} = \frac{3}{2}\]

03. Ans: (a)
Sol:

\[\frac{U_B}{U_A} = \frac{(V_1 + V_2)_B}{(V_1 + V_2)_A}\]
\[\Rightarrow \frac{U_B}{U_A} = \frac{\frac{\sigma_1^2}{2E} \times V_1 + \frac{\sigma_2^2}{2E} \times V_2}{\frac{\sigma_1^2}{2E} \times V_1 + \frac{\sigma_2^2}{2E} \times V_2}_B\]
\[= \frac{\frac{p^2}{A_1} \times L_1 + \frac{p^2}{A_2} \times L_2}{\frac{p^2}{A_1} \times L_1 + \frac{p^2}{A_2} \times L_2}_A\]
\[= \frac{\frac{L_1}{A_1} + \frac{L_2}{A_2}}{\frac{L_1}{A_1} + \frac{L_2}{A_2}}_A\]
\[= \frac{7.165}{4.77} = \frac{3}{2}\]

04. Ans: (c)
Sol:
\[A_1 = \text{Modulus of resilience}\]
\[A_1 + A_2 = \text{Modulus of toughness}\]
\[A_1 = \frac{1}{2} \times 0.004 \times 70 \times 10^6 = 14 \times 10^4\]
\[A_2 = \frac{1}{2} \times \left(0.008 \times 50 \times 10^6\right) + \left(0.008 \times 70 \times 10^6\right)\]
\[= 76 \times 10^4\]
\[A_1 + A_2 = (14 + 76) \times 10^4 = 90 \times 10^4\]
05. Ans: (d)

Sol: Strain energy, \( U = \frac{P^2}{2A^2E}V \)

\[ \therefore \ U \propto P^2 \]

Due to the application of \( P_1 \) and \( P_2 \) one after the other,

\[ (U_1 + U_2) \propto P_1^2 + P_2^2 \ \cdots \ (1) \]

Due to the application of \( P_1 \) and \( P_2 \) together at the same time,

\[ U \propto (P_1 + P_2)^2 \ \cdots \ \cdots \ (2) \]

It is obvious that,

\[ (P_1^2 + P_2^2) < (P_1 + P_2)^2 \]

\[ \Rightarrow \ (U_1 + U_2) < U \]

06. Ans: 1.5

Sol: Given data:

\[ L = 100 \text{ mm} \]

\[ G = 80 \times 10^3 \text{ N/mm}^2 \]

\[ J_1 = \frac{\pi}{32} (50)^4; J_2 = \frac{\pi}{32} (26)^4 \]

\[ U = U_1 + U_2 = \frac{T^2 L}{2GJ_1} + \frac{T^2 L}{2GJ_2} \]

\[ \Rightarrow \ U = 1.5 \text{ N-mm} \]