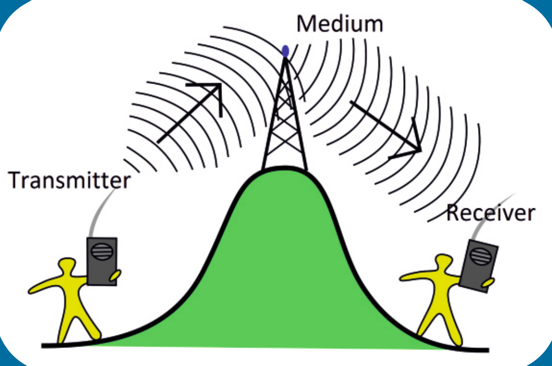
GATE | PSUs

ELECTRONICS & COMMUNICATION ENGINEERING

Communication Systems

Text Book : Theory with worked out Examples and Practice Questions



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Chapter 1 Introduction

(Solutions for Text Book Practice Questions)

01. Ans: (b)

Sol: We know that

$$e^{-at}u(t) \overset{F.T}{\leftrightarrow} \frac{1}{a+j\omega}$$

$$e^{at}u(-t) \overset{F.T}{\leftrightarrow} \frac{1}{a-j\omega}$$

$$e^{-at}u(t) - e^{at}u(-t) \overset{F.T}{\leftrightarrow} \frac{1}{a+j\omega} - \frac{1}{a-j\omega}$$

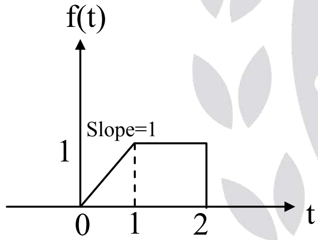
Put a = 0

$$u(t) - u(-t) \overset{F.T}{\leftrightarrow} \frac{1}{j\omega} - \frac{1}{-j\omega}$$

$$\text{sgn}(t) \overset{F.T}{\leftrightarrow} \frac{2}{j\omega}$$

02. Ans: (a)

Sol:



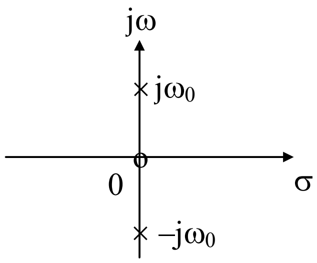
f(t) = r(t) – r(t – 1) – u(t – 2)

03. Ans: (a)

Sol: The convergence of Fourier transform is along the jω-axis in s-plane.

04. Ans: (a)

Sol:

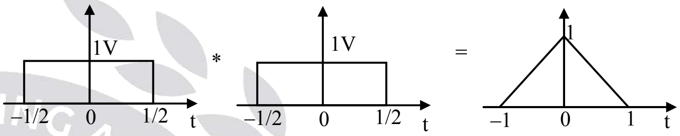


$$F(s) = \frac{s}{s^2+\omega_0^2} \overset{I.L.T}{\leftrightarrow} f(t) = \cos\omega_0 t$$

$$f(t) = \cos\omega_0 t \overset{I.L.T}{\leftrightarrow} F(\omega) = \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$$

05. Ans: (d)

Sol:



06. Ans: (c)

Sol: Given $x(t) = e^{-at^2}$

Fourier transform of x(t) is

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$= \int_{-\infty}^{\infty} e^{-at^2}e^{-j\omega t}dt$$

$$= \int_{-\infty}^{\infty} e^{-\left(at^2+j\omega t\right)}dt$$

$$= e^{-\frac{\omega^2}{4a}} \int_{-\infty}^{\infty} e^{-\left[\sqrt{a}t+\frac{j\omega}{2\sqrt{a}}\right]^2} dt$$

Let $p = \sqrt{a}t + \frac{j\omega}{2\sqrt{a}}$

$dp = \sqrt{a}dt$

$$X(\omega) = \frac{e^{-\frac{\omega^2}{4a}}}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-p^2}dp$$

$$\int_{-\infty}^{\infty} e^{-p^2}dp = \sqrt{\pi}$$

$$X(\omega)=\frac{e^{-\frac{\omega^2}{4a}}}{\sqrt{a}}\sqrt{\pi}$$

$$X(\omega)=\sqrt{\frac{\pi}{a}}e^{-\frac{\omega^2}{4a}}$$

07. Ans: (d)

Sol: The EFS expression of a periodic signal x(t)

is $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$

where, 'c_n' is EFS coefficient.

Apply F.T on both sides

$$X(\omega) = \sum_{n=-\infty}^{\infty} c_n \, FT\left[e^{jn\omega_0 t}\right]$$

$1 \leftrightarrow 2\pi\delta(\omega)$

$e^{jn\omega_0 t} \leftrightarrow 2\pi\delta(\omega - n\omega_0)$

$$X(\omega) = 2\pi\sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_0)$$

So, it is a train of impulse.

08. Ans: (a)

Sol: $V(j\omega) = e^{-j2\omega};\ |\omega| \le 1$

$$\text{Energy} = \frac{1}{2\pi}\int_{-\infty}^{\infty}|V(j\omega)|^2.d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1}\left|e^{-j2\omega}\right|^2.d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{1} 1 d\omega$$

$$= \frac{2}{2\pi}$$

$$= \frac{1}{\pi}$$

09. Ans: (b)

Sol: Parseval's theorem is used to find the energy of the signal in frequency domain.

$$\therefore \int_{-\infty}^{\infty}|f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|F(j\omega)|^2 d\omega$$

10. Ans: (a)

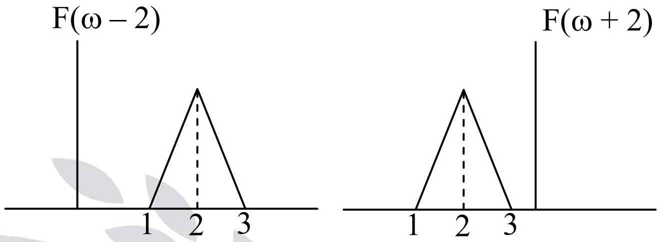
Sol: $f(t) = A.e^{-a|t|} \overset{F.T}{\leftrightarrow} F(j\omega) = \frac{2Aa}{a^2+\omega^2}$

11. Ans: (d)

Sol: m(t) = f(t) cos2t

Apply Fourier transform

$$M(f) = \frac{1}{2}[F(\omega-2)+F(\omega+2)]$$



12. Ans: (b)

Sol: For band limited signals,

$S(f) \ne 0;\ |f| < W$

$S(f) = 0;\ |f| > W$

13. Ans: (a)

Sol: In a communication system, antenna is used to convert voltage variations to field variation and vice-versa.

14. Ans: (d)

Sol: Hilbert transform of f(t) is

$$H.T\{f(t)\} = f(t) * \frac{1}{\pi t}$$

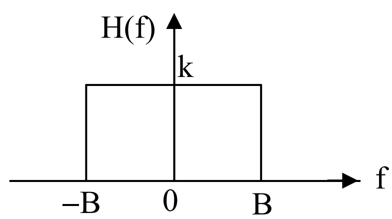
It is in the terms of 't'.

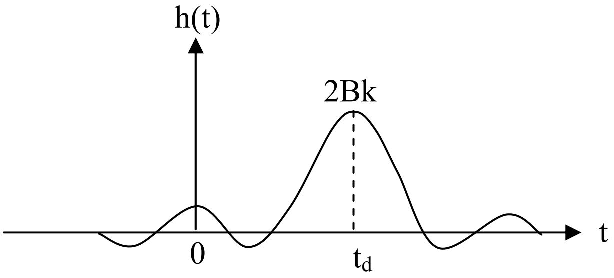
15. Ans: (a)

Sol: For an ideal LPF

$H(f) = k\ e^{-j\omega t_0}$ for $-B < f < B$

$h(t) = F^{-1}[H(f)] = 2Bk \text{ sinc } 2B(t - t_d)$





$h(t) \neq 0$ for $t < 0$

Output exists before input is applied i.e. non-causal, which is physically impossible.

16. Ans: (b)

Sol: $\delta(at) = \frac{1}{|a|}\delta(t)$

$\delta(2t) = \frac{1}{2}\delta(t)$

17. Ans: (a)

Sol: By modulation we are translating the low frequency spectrum into high frequency spectrum.

18. Ans: (a)

Sol: We know that

$P(dBm) = 10\log(P\times 10^3)$

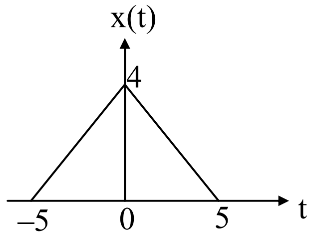
$-10 = 10\log(P\times 10^3)$

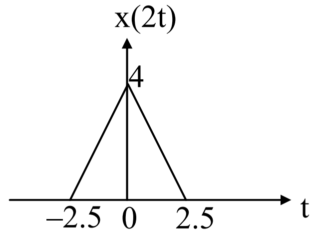
$P\times 10^3 = 10^{-1}$

$P = 10^{-4} = 100\ \mu W$

19. Ans: (a)

Sol: $x(2t)$ means signal time axis is compressed by 2





20. Ans: (b)

Sol: Audio frequency is between 20Hz to 20kHz

21. Ans: (d)

Sol: Telephone channel carries voice. Voice frequency is between 300 Hz to 3500 Hz. So bandwidth is 3200Hz. So we approximately consider 4kHz is the bandwidth requirement of a telephone channel.

22. Ans: (c)

Sol: From the signal spectrum f_H = 530 kHz, f_L = 50 kHz

Bandwidth = $f_H - f_L$ = 530 kHz – 50 kHz = 480 kHz

Chapter 2 Amplitude Modulation

01. Ans: (a)

Sol: $V(t) = A_c.\cos\omega_c t + 2\cos\omega_m t\,.\,\cos\omega_c t.$

Comparing this with the AM-DSB-SC signal

$A\cos\omega_c t + m(t).\cos\omega_c t$, it implies that

$m(t) = 2\cos\omega_m t \Rightarrow E_m = 2$

To implement Envelope detection,

$A_c \geq E_m$

$\therefore (A_c)_{min} = 2$

02. Ans: (d)

Sol: $m(t) = (A_c + A_m\cos\omega_m t)\cos\omega_c t.$

$$= A_c\left(1 + \frac{A_m}{A_c}\cos\omega_m t\right)\cos\omega_c t.$$

Given

$A_c = 2A_m$

$$= A_c\left(1 + \frac{1}{2}\cos\omega_m t\right)\cos\omega_c t.$$

$$P_T = \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right],\ P_s = \frac{A_c^2}{2}\left[\frac{\mu^2}{4}\right]$$

$$\frac{P_T}{P_S} = \frac{1 + \frac{\mu^2}{2}}{\frac{\mu^2}{4}} = \frac{1 + \frac{1}{8}}{\frac{1}{16}} = \frac{9}{8} \times 16$$

$P_T = 18\,P_s$

03. Ans: (a)

Sol: $m(t) = 2\cos 2\pi f_1 t + \cos 2\pi f_2 t$

$C(t) = A_c\cos 2\pi f_c t$

$S(t) = [A_c + m(t)]\cos 2\pi f_c t$

$$S(t) = A_c\left[1 + \frac{1}{A_C}m(t)\right]\cos 2\pi f_c t$$

$$K_a = \frac{1}{A_c}$$

$A_{m1} = 2,\ A_{m2} = 1$

$$\mu_1 = K_a A_{m1} = \frac{2}{A_C},\ \mu_2 = K_a A_{m2} = \frac{1}{A_C}$$

$$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

$$\Rightarrow 0.5 = \sqrt{\frac{4}{A_c^2} + \frac{1}{A_c^2}}$$

$$\Rightarrow A_C = \sqrt{20}$$

04. Ans: (c)

Sol: $m(t) = -0.2 + 0.6\sin\omega_1 t,\ k_a = 1,\ A_c = 100$

$S(t) = A_c[1 - 0.2 + 0.6\sin\omega_1 t]\cos\omega_c t$

$= 100[0.8 + 0.6\sin\omega_1 t]\cos\omega_c t$

$V_{max} = A_c[1 + \mu] = 100[0.8 + 0.6] = 140\text{ V}$

$V_{min} = A_c[1 - \mu] = 100[0.8 - 0.6] = 20\text{ V}$

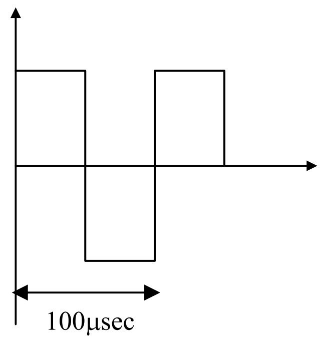
$= 20\text{V to }140\text{ V}$

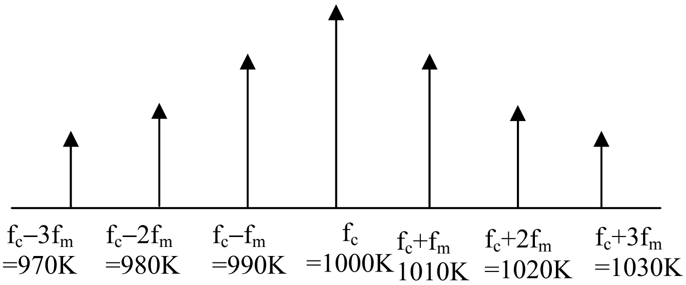
05. Ans: (c)

Sol: $f_C = 1\text{ MHz} = 1000\text{ kHz}$

The given m(t) is symmetrical square wave of period T = 100 μsec

$$f_m = \frac{1}{T_0} = 10\text{ kHz}$$



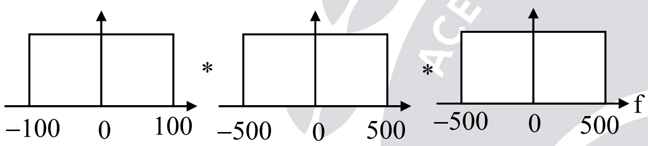


These frequencies 980k, 1020k are not present because the symmetrical square wave it consists of half wave symmetries only odd harmonics are present, even harmonics are dismissed

06. Ans: (d)

Sol: $m(t) = \text{sinc}(200t)\text{sinc}^2(1000t)$

$= \text{sinc}(200t)\text{sinc}(1000t)\text{sinc}(1000t)$

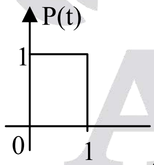


So, highest frequency component in the signal m(t) is 100 + 500 + 500 = 1100

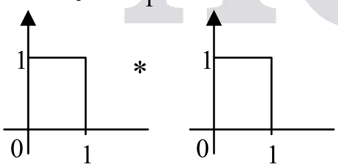
BW = 2 × 1100
BW = 2200 Hz

07. Ans: (a)

Sol: $P(t) = u(t) - u(t-1) \Rightarrow$



$g(t) = P(t) * P(t) =$



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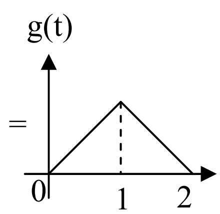


$x(t) = 100(P(t) + 0.5g(t))\cos\omega_c t$

$= 100(1 + 0.5t)\cos\omega_c t$

$= A_c(1 + K_a m(t))\cos\omega_c t$

$k_a = 0.5,\ m(t) = t$

$\mu = k_a[m(t)]_{max}$

$\mu = 0.5 \times 1 = 0.5$

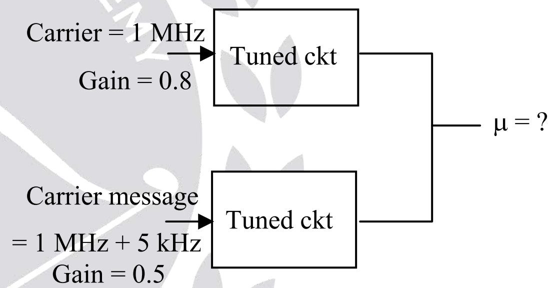
08. Ans: (d)

Sol: $R_L C \le \dfrac{\sqrt{1-\mu^2}}{2\pi f_m \mu}$

So it depends on depth of modulation and the highest modulation frequency.

09. Ans: (b)

Sol: $S(t) = 10\cos 2\pi 10^6 t + 8\cos 2\pi 5\times10^3 t\cos 2\pi 10^6 t$

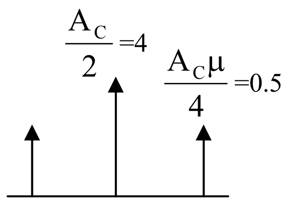


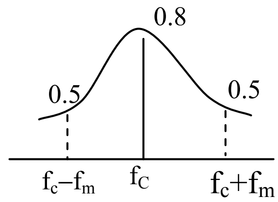
$S(t) = 0.8 \times 10\cos 2\pi 10^6 t$

$+ 0.5\times 8\cos 2\pi 5000t\cos 2\pi 10^6 t$

$= 8(1 + \frac{4}{8}\cos 2\pi 5000t)\cos 2\pi 10^6 t$

$\mu = \dfrac{4}{8} = \dfrac{1}{2} = 0.5$





10. Ans: (d)

Sol: $A_{max} = 10V$

$A_{min} = 5V$

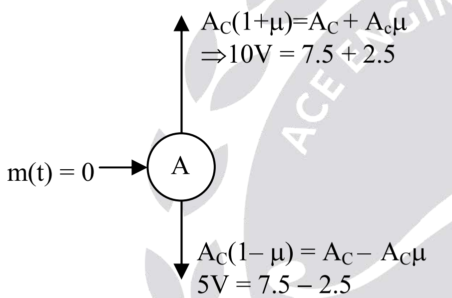
$\mu = 0.1$

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

$$= \frac{1}{3} = 0.33$$

$$A_C = \frac{A_{max} + A_{min}}{2}$$

$$= \frac{10+5}{2} = 7.5\text{ V}$$



Amplitude deviation $A_C\mu = 7.5\times\frac{1}{3} = 2.5$ V

$\mu_2 = 0.1 \Rightarrow A_{c2}\mu_2 = 2.5$

$A_{c2} = 25$ V

Which must be added to attain = 17.5

11. Ans: (d)

Sol: Modulation index

$\mu = k_a\,|m(t)|_{max}$

$$k_a = \frac{2b}{a} = \frac{2(\text{square term coefficient})}{\text{linear term coefficient}}$$

$|m(t)|_{max} = 1$

$$\mu = 2\left(\frac{b}{a}\right)$$

$$P_{SB} = \frac{1}{2}P_C \Rightarrow P_C\frac{\mu^2}{2} = \frac{1}{2}P_C$$

$$\mu^2 = 1 \Rightarrow \left(2\frac{b}{a}\right)^2 = 1$$

$$\Rightarrow 2\frac{b}{a} = 1 \Rightarrow \frac{a}{b} = 2$$

12. Ans: 0.125

Sol: $s(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2000\pi t)$

Here $4\cos(2400\pi t)$ is the carrier signal.

$\cos(2000\pi t)$ and $\cos(2000\pi t)$ are the sideband message signals.

$$P_c = \frac{4^2}{2} = 8\text{ W}$$

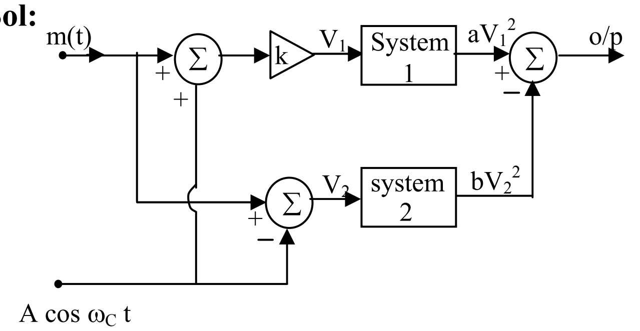
$$P_m = \frac{1}{2} + \frac{1}{2} = 1\text{ W}$$

$$\frac{P_m}{P_c} = \frac{1}{8} = 0.125$$

Chapter 3 Sideband Modulation Techniques

01. Ans: (c)

Sol:



$V_1 = k\,[m(t) + c(t)]$

$V_2 = [m(t) - c(t)]$

$V_0 = aV_1^2 - b\,V_2^2$

$= ak^2[m(t) + c(t)]^2 - b[m(t) - c(t)]^2$

$= ak^2\,[m^2(t) + c^2(t) + 2m(t)c(t)]$

$\quad - b[m^2(t) + c^2(t) - 2m(t)c(t)]$

$= [ak^2 - b]m^2(t) + [ak^2 - b]c^2(t)$

$\quad + 2[ak^2+b][m(t)c(t)]$

on verification if $k = \sqrt{\dfrac{b}{a}}$

$S(t) = 4bm(t)c(t) \rightarrow$ DSBSC Signal

02. Ans: (d)

Sol: Given A = 10

$m(t) = \cos 1000\pi t$

$b = 1$

B.W = ? and power = ?

$s(t) = 4b.A\cos 2\pi f_c t.\,\cos 2\pi\,(500)t$

$= 40.\cos 2\pi f_c t.\,\cos 2\pi\,(500)t$

$B.W = 2\,f_m$

$= 2\,(500)$

$= 1$ kHz

$$\text{Power} = \frac{A_c^2 A_m^2}{4}$$

$$= \frac{1600 \times 1}{4}$$

$$= 400\text{W}$$

03. Ans: (c)

Sol: Carrier = $\cos 2\pi\,(100 \times 10^6)t$

Modulating signal = $\cos(2\pi \times 10^6)t$

Output of Balanced modulator

$= 0.5[\cos 2\pi\,(101 \times 10^6)t + \cos 2\pi(99\times 10^6)t]$

The Output of HPF is $0.5\cos 2\pi(101 \times 10^6)t$

Output of the adder is

$= 0.5\cos 2\pi\,(101\times 10^6)\,t + \sin 2\pi\,(100\times 10^6)t$

$= 0.5\cos 2\pi[(100+1)10^6 t] + \sin 2\pi(100\times 10^6)t$

$= 0.5[\cos 2\pi\,(100 \times 10^6)t.\,\cos 2\pi\,(10^6)t$

$\quad - \sin 2\pi(100 \times 10^6)t.\,\sin 2\pi\,(10^6)t]$

$\quad + \sin 2\pi(100 \times 10^6)t]$

$= 0.5\cos 2\pi\,(100 \times 10^6)t.\,\cos 2\pi\,(10^6)t$

$\quad + \sin 2\pi(100\times 10^6)t\,[1 - 0.5\sin 2\pi\,(10^6)t]$

Let $0.5\cos 2\pi\,(10^6)t = r(t)\cos\theta(t)$

$1 - 0.5\sin 2\pi\,(10^6)t = r(t).\sin\theta(t)$

The envelope is

$r(t) = [\,0.25\cos^2 2\pi\,(10^6)t$

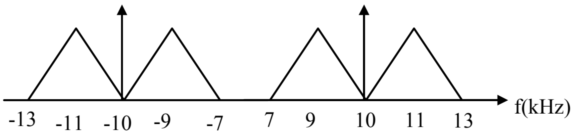
$\quad + \{1 - 0.5\sin 2\pi\,(10^6)t\}^2]^{1/2}$

$= [1.25 - \sin 2\pi(10^6)t]^{1/2}$

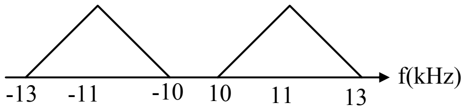
$= [\frac{5}{4} - \sin 2\pi\,(10^6)t]^{1/2}$

04. Ans: (b)

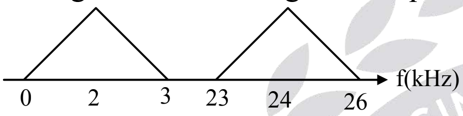
Sol: Output of 1st balanced modulator is



Output of HPF is



The Output of 2nd balanced modulator is consisting of the following +ve frequencies.



Thus, the spectral peaks occur at 2 kHz and 24 kHz

05. Ans: (c)

Sol: Given

$f_{m_1} = 100\text{Hz},\ f_{m_2} = 200\text{Hz},\ f_{m_3} = 400\text{Hz},$

$f_c = 100\text{KHz},\ f_{c_{L0}} = 100.02\text{KHz}$

$$S(t)/_{T_X} = \frac{A_c A_m}{2}[\cos(f_c + f_{m_1})t + \cos(f_c + f_{m_2})t + \cos(f_c + f_{m_3})t]$$

$$S(t)/R_x = [S(t)/T_x]A_c \cos 2\pi f_{c_{L0}} t$$

$$\Rightarrow \frac{A_c^2 A_m}{4}[\cos(f_c + f_{c_{Lo}} + f_{m_1}) + \cos(f_{m_1} - 20) + \cos(f_c + f_{c_{Lo}} + f_{m_2}) + \cos(f_{m_2} - 20) + \cos(f_c + f_{c_{Lo}} + f_{m_3}) + \cos(f_{m_3} - 20)]$$

Detector output frequencies:

80Hz, 180Hz, 380Hz

06. Ans: (b)

Sol: Given

SSB AM is used, LSB is transmitted

$f_{LO} = (f_c + 10)$

$$S(t)/T_X = \frac{A_c A_m}{2}\cos 2\pi[f_c - f_m]t$$

$$S(t)/R_X = \left[\frac{A_c A_m}{2}\cos 2\pi(f_c - f_m)t\right]\cos 2\pi(f_c + 10)t$$

$$\Rightarrow \frac{A_c A_m}{4}[\cos 2\pi(2f_c + 10 - f_m)t + \cos 2\pi(10 + f_m)t]$$

i.e., from 310 Hz to 1010 Hz

07. Ans: (b)

Sol: BW of Basic group = 12×4 = 48 kHz

BW of super group = 5×48 = 240 kHz

08. Ans: (d)

Sol: Given 11 voice signals

B.W. of each signals = 3 kHz

Guard Band Width = 1 kHz

Lowest f_c = 300 kHz

Highest f_c =

$\Rightarrow f_{c_H} + f_{m_{lost}} = 300\text{kHz} + 11(3\text{kHz}) + 10(1\text{kHz})$

$= 343$ kHz

$f_{c_H} = 343\,\text{kHz} - 3\,\text{kHz}$

$= 340\,\text{kHz}$

09. Ans: (b)

Sol: f_{m1} = 5 kHz → AM

f_{m2} = 10 kHz → DSB

f_{m3} = 10kHz → SSB

f_{m4} = 2kHz → SSB

f_{m5} = 5kHz → AM

f_g = 1kHz

BW = $(2f_{m_1} + 2f_{m2} + f_{m3} + f_{m4} + 2f_{m5} + 4f_g)$

= 2× 5 + 2×10 + 10 + 2 + 2×5 + 4×1

= 10 + 20 + 10 + 10 + 6

= 56 kHz

∴ BW = 56 kHz

Chapter 4 Angle Modulation

01. Ans: (a)

Sol: $s(t) = 10\cos(20\pi t + \pi t^2)$

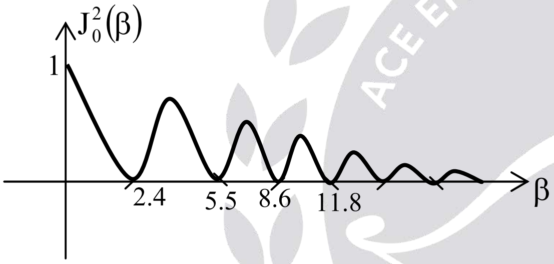
$$f_i = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

$$f_i = \frac{1}{2\pi}[20\pi + 2\pi t]$$

$$\frac{df_i}{dt} = \frac{1}{2\pi}\times 2\pi \times 1 = 1\,\text{Hz/sec}$$

02. Ans: (d)

Sol: $P_{fc} = \dfrac{A_c^{\,2} J_0^{\,2}(\beta)}{2}$



So, $J_0^2(\beta)$ is decreasing first, becoming zero and then increasing so power is also behave like $J_0^2(\beta)$.

03. Ans: (a)

Sol: In an FM signal, adjacent spectral components will get separated by $f_m = 5$ kHz

Since $BW = 2(\Delta f + f_m) = 1\text{MHz}$

$= 1000 \times 10^3$

$\Delta f + f_m = 500$ kHz

$\Delta f = 495$ kHz

The n^{th} order non-linearity makes the carrier frequency and frequency deviation increased by n-fold, with the base-band signal frequency (f_m) left unchanged since n = 3,

$\therefore (\Delta f)_{New} = 1485$ kHz &

$(f_c)_{New} = 300$ MHz

New BW $= 2(1485 + 5)\times 10^3$

$= 2.98$ MHz

$= 3$ MHz

04. Ans: (d)

Sol: $S(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos 2\pi(f_c + nf_m)t$

$\Delta f = 3(2f_m) = 12$ kHz

$\beta = \dfrac{\Delta f}{f_m} = 6$

$\therefore S(t) = \sum_{n=-\infty}^{\infty} 5.J_n(6)\cos 2\pi(f_c + nf_m)t$

$f_c = 1000$kHz, $f_m = 2$ kHz

$= \cos 2\pi(1008 \times 10^3)t$

$= \cos 2\pi(1000 + 4\times 2)\times 10^3 t$

i.e., n = 4

The required coefficient is $5.J_4(6)$

05. Ans: (c)

Sol: $2\pi f_m = 4\pi\, 10^3$

$\Rightarrow f_m = 2k$

$J_0(\beta) = 0$ at $\beta = 2.4$

$\beta = \dfrac{k_f\ A_m}{f_m} \Rightarrow 2.4 = \dfrac{k_f \times 2}{2k}$

$k_f = 2.4$ KHz /V

at $\beta = 5.5$

$$5.5 = \frac{2.4\,k \times 2}{f_m}$$

$\Rightarrow f_m = 872.72$ Hz

06. Ans: (c)

Sol: $\beta = 6$

$J_0(6) = 0.1506$; $J_3(6) = 0.1148$

$J_1(6) = 0.2767$; $J_4(6) = 0.3576$

$J_2(6) = 0.2429$;

$$\frac{P_{f_c \pm 4f_m}}{P_T} = ? \qquad P_T = \frac{A_c^{\,2}}{2R}$$

$$P_{f_c \pm 4f_m} = \frac{A_C^2}{R}\left[\frac{J_0^2(\beta)}{2} + J_1^2(\beta) + J_2^2(\beta) + J_3^2(\beta) + J_4^2(\beta)\right]$$

$$P_{f_c \pm 4f_m} = \frac{A_C^2}{R}\left[\frac{J_0^2(\beta)}{2} + J_1^2(\beta) + J_2^2(\beta) + J_4^2(\beta)\right]$$

$$\frac{P_{f_c \pm 4f_m}}{P_T} = \frac{0.2879}{1/2} = 0.5759 = 57.6\ \%$$

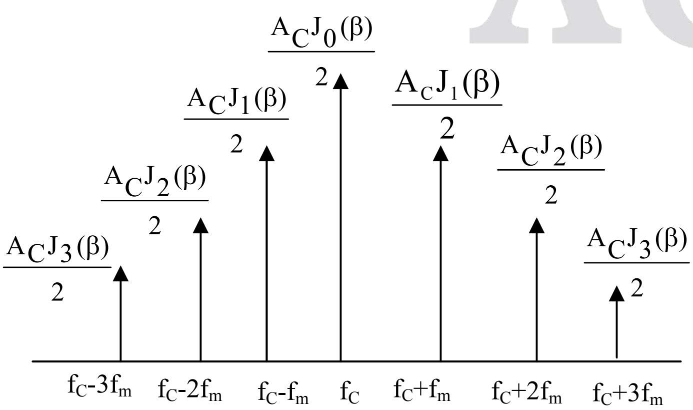
07. Ans: (c)

Sol: $m(t) = 10\cos 20\pi t$

$f_m = 10$ Hz

inserting correct signal and frequency

$$\beta = \frac{k_f\,A_m}{f_m} = \frac{5 \times 10}{10} = 5$$



From f_c to $f_c + 4f_m$ pass through ideal BPF

Powers in these frequency components

$$P = \frac{A_C^2}{2R}J_0^2(\beta) + 2\frac{A_C^2}{2R}J_1^2(\beta) + 2\frac{A_C^2}{2R}J_2^2(\beta)$$

$$+ 2\frac{A_C^2}{2R}J_3^2\beta + 2\frac{A_C^2}{1R}J_4^2(\beta)$$

$$= \frac{A_C^2}{2R}\left[\begin{array}{l}(-0.178)^2 + 2(-0.328)^2 + 2(0.049)^2 \\ + 2(0.365)^2 + 2(0.391)^2\end{array}\right]$$

$= 41.17$ Watts

08. Ans: (d)

Sol: $P_t = \dfrac{A_C^2}{2R}$ $(R = 1\Omega)$

$$= \frac{100}{2} = 50\ \text{W}$$

$$\%\ \text{Power} = \frac{\text{Power in components}}{\text{total power}} \times 100$$

$$= \frac{41.17}{50} \times 100$$

$= 82.35\%$

09. Ans: (d)

Sol: In frequency modulation the spectrum contains $f_c \pm nf_1 \pm mf_2$, where n & m = 0, 1, 2, 3………

10. Ans: (c)

Sol: Given $f_c = 1$MHz

$f_{max} = f_c + k_f\,A_m$

$k_p = 2\pi\,k_f$

$$k_f = \frac{k_p}{2\pi} = \frac{\pi}{2\pi}$$

$$= \frac{1}{2}$$

$= \left(10^6 + \frac{1}{2}\times 10^5\right) = \left(10^6 + 0.5\times 10^5\right)$

$= \left(10^6 + 5\times 10^4\right)$

$= \left(10^3 + 50\right)10^3$

$= (10^3 + 50)$ k

$= 1050$ kHz.

$f_{min} = f_c - k_f A_m$

$= \left(10^6 - \frac{1}{2}\times 10^5\right)$

$= \left(10^6 - 0.5\times 10^5\right)$

$= \left(10^6 - 5\times 10^4\right)$

$= \left(10^3 - 50\right)10^3$

$= (10^3 - 50)$ k

$= 950$ kHz

11. Ans: (d)

Sol: $\beta = \frac{\Delta f}{f_m}$

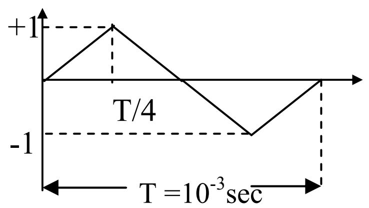
$\Delta\phi = \frac{\Delta f}{f_m}$

$\Delta f = \Delta\phi\, f_m$

$= k_p A_m f_m$

12. Ans: (c)

Sol: Given



$f_c = 100\times 10^3$ Hz

$k_f = 10\times 10^3$Hz

$m(t)|_{max} = +1$, $m(t)|_{min} = -1$

$f_i = f_c \pm \Delta f$

$= f_c \pm k_f A_m$

$= 100\times 10^3 \pm 10\times 10^3\,(m(t))$

$= 110$ kHz & 90 kHz

13. Ans: (c)

Sol: $S(t) = A_c \cos(\underbrace{2\pi f_c t + k_p m(t)}_{\theta_i(t)})$

$f_i = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t)$

$= \frac{1}{2\pi}\frac{d}{dt}(2\pi f_c t + k_p m(t))$

$= f_c + \frac{1}{2\pi}k_p\frac{d}{dt}m(t)$

$f_{max} = f_c + \frac{k_p}{2\pi}\frac{1}{\left(\frac{10^{-3}}{4}\right)} = f_c + \frac{k_p}{2\pi}\times 4\times 10^3$

$= 100\text{ kHz} + \frac{\pi}{2\pi}\times 4\times 10^3$

$= 102$ kHz

$f_{min} = f_c - k_p\frac{1}{\left(\frac{10^{-3}}{4}\right)}$

$= f_c - 2$ kHz

$f_{min} = 98$kHz

14. Ans: (c)

Sol: Given,

$S(t) = A_c \cos(\theta_i(t))$

$= A_c \cos(\omega_c t + \phi(t))$

$m(t) = \cos(\omega_m t)$

$f_i(t) = f_c + 2\pi k (f_m)^2 \cos\omega_m t$

$f_i = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$

$\theta_i(t) = \int 2\pi\, f_i(t)dt$

$$\theta_i(t) = \int 2\pi[f_c + 2\pi k(f_m)^2 \cos\omega_m t]dt$$

$$\theta_i(t) = 2\pi f_c t + (2\pi f_m)^2\, k\, \frac{\cos\omega_m t}{\omega_m t}$$

$$\theta_i(t) = \omega_c t + \omega_m k \sin\omega_m t$$

15. Ans: (b)

Sol: $\Delta f_{max} = K_f\, |m(t)|_{max}$

$$= \frac{100}{2\pi} \times [10]$$

$$\Delta f_{max} = \left(\frac{500}{\pi}\right)\text{Hz}$$

16. Ans: (b)

Sol: Given that

$s(t) = \cos[\omega_c t + 2\pi m(t)]$volts

$$f_i = \frac{1}{2\pi}\frac{d}{dt}[\omega_c t + 2\pi m(t)]$$

$$= \frac{1}{2\pi}\frac{d}{dt}[2\pi f_c t + 2\pi m(t)]$$

$$f_i = f_c + \frac{d}{dt}[m(t)]$$

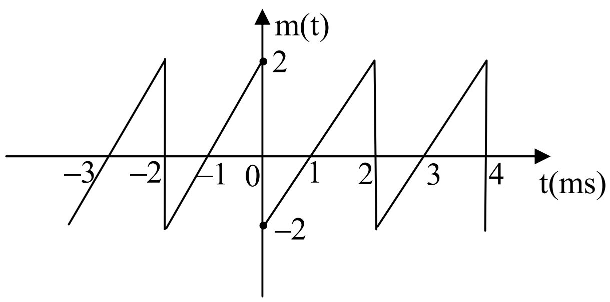
we know that $f_i = f_c + k_f\, m(t)$

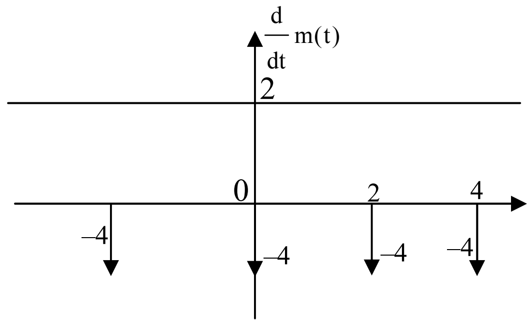
Here $k_f m(t) = \frac{d}{dt}[m(t)]$

$$\Delta f = \max\{k_f m(t)\}$$

$$\Delta f = \max\left[\frac{d}{dt} m(t)\right]$$

$$\Delta f = 2\text{kHz}$$





17. Ans: (a)

Sol: $\beta_p = k_p \max[|m(t)|] = 1.5 \times 2 = 3$

$$\beta_f = \frac{k_f \max[|m(t)|]}{f_m}$$

$$= \frac{3000 \times 2}{1000}$$

$$= 6$$

18. Ans: (a)

Sol: Using Carson's rule we obtain

$BW_{PM} = 2\,(\beta_p + 1)f_m = 8 \times 1000 = 8000\text{Hz}$

$BW_{FM} = 2\,(\beta_f + 1)f_m = 14 \times 1000 = 14000\text{Hz}$

19. Ans: 70 kHz

Sol: $s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$

$$f_i = f_c + \frac{k_p}{2\pi}\frac{d}{dt}x(t)$$

$$= 20k + \frac{5}{2\pi} \times 5\frac{d}{dt}\left(\sin 4\pi 10^3 t - 10\pi\cos 2\pi 10^3 t\right)$$

$$= 20k + \frac{25}{2\pi} \times \begin{bmatrix} \cos(4\pi 10^3 t - 10\pi\cos 2\pi 10^3 t) \\ \left(4\pi 10^3 + 10\pi \sin 2\pi 10^3 t \times 2\pi 10^3\right) \end{bmatrix}$$

$$f_{i(t=0.5ms)} = 20k + \frac{25}{2\pi} \times \cos(4\pi + 10\pi) \times 4\pi \times 10^3$$

$$= 20k + \frac{25}{2\pi} \times 4\pi \times 10^3$$

$$= 20k + 50k$$

$$f_{i(t=0.5ms)} = 70\text{kHz}$$

Chapter 5 Radio Receivers

01. Ans: (d)

Sol: The image channel selectivity of super heterodyne receiver depends upon Pre selector and RF amplifier only.

02. Ans: (b)

Sol: The image (second) channel selectivity of a super heterodyne communication receiver is determined by the pre selector and RF amplifier.

03. Ans: (d)

Sol: Given $f_s = 4$ to 10 MHz

IF = 1.8 MHz

$f_{si} = ?$

$f_{si} = f_s + 2\times IF$

= 7.6 MHz to 13.6 MHz

04. Ans: (a)

Sol: Image frequency $f_{si} = f_s + 2\times IF$

$= 700\times10^3 + 2(450\times10^3)$

= 1600 kHz

Local oscillator frequency, $f_l = f_s + IF$

$(f_l)_{max} = (f_s)_{max} + IF = 1650 + 450$

= 2100 kHz

$(f_l)_{min} = (f_s)_{min} + IF = 550 + 450$

= 1000 kHz

$$R = \frac{C_{max}}{C_{min}} = \left(\frac{f_{l\,max}}{f_{l\,min}}\right)^2 = \left(\frac{2100}{1000}\right)^2 = 4.41$$

05. Ans: (a)

Sol: f_s(range) = 88 - 108MHz

Given condition $f_{IF} < f_{LO}$, $f_{si} > 108$ MHz

$f_{si} = f_s + 2\times IF$

$f_{si} > 108$ MHz

$f_s + 2IF > 108$ MHz

88MHz + 2×IF > 108 MHz

IF > 10MHz

Among the given options IF = 10.7 MHz

06. Ans: (a)

Sol: Range of variation in local oscillator frequency is

$f_{Lmin} = f_{smin} + IF$

= 88 + 10.7

$f_{Lmin} = 98.7$ MHz

$f_{Lmax} = f_{smax} + IF$

=108 + 10.7

$f_{Lmax} = 118.7$ MHz

07. Ans: 5

Sol: $f_s = 58$ MHz – 68 MHz

When $f_s = 58$ MHz

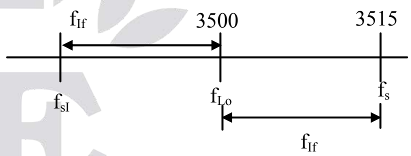
$f_{si} = f_s + 2IF > 68$ MHz

2IF > 10 MHz

IF ≥ 5 MHz

08. Ans: 3485 MHz

Sol:



$f_{If} = 15$ MHz

$f_{Lo} = 3500$ MHz

$f_s - f_{Lo} = f_{IF}$

$f_s = f_{Lo} + f_{IF} = 3515$ MHz

f_{si} = image frequency $= f_s - 2\,f_{IF}$

= 3515 – 2×15

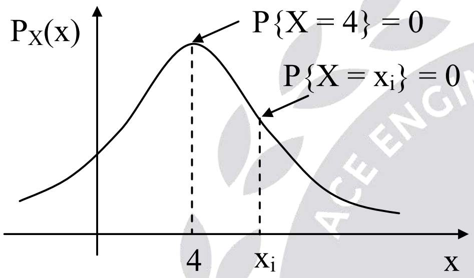
= 3485 MHz

Chapter 6 Random Variables & Noise

01. Ans: (c)

Sol: A continuous Random variable X takes every value in a certain range, the probability that X = x, is zero for every x in that range.

Given $P_X(x) = \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-4)^2}{18}}$ is a continuous Random variable therefore probability of the event {X = 4} is zero.



02. Ans: (b)

Sol: Given,

X & Y are two Random Variables

Y = cosπx

$f(x) = 1 \quad \frac{-1}{2} < x < \frac{1}{2}$

$\quad = 0$ else where

f(y) = ?

$$f(y) = f(x)\left|\frac{dx}{dy}\right|$$

$$x = \frac{1}{\pi}\cos^{-1}(y)$$

$$dx = \frac{1}{\pi} \times \frac{-1}{\sqrt{1-y^2}} dy$$

$$\Rightarrow \frac{dx}{dy} = \frac{-1}{\pi\sqrt{1-y^2}}$$

$$f(y) = \frac{1}{\pi\sqrt{1-y^2}}$$

$$\sigma_y^2 = E[y^2] - [E[y]]^2$$

03. Ans: (d)

Sol: The probability density function of the envelope of a sinusoidal plus narrrow band noise is Rician.

$$f_R(r) = \frac{r}{\sigma^2}\exp(-\frac{r^2 + A^2}{2\sigma^2})I_0(\frac{Ar}{\sigma^2})$$

04. Ans: (a)

Sol: Given,

Differential equation of a system is

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - x(t)$$

Applying Fourier transform,

$\Rightarrow Y(f)(1 + jf) = X(f)(jf - 1)$

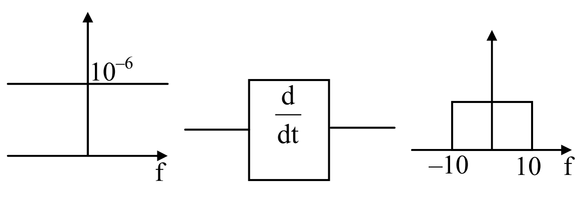
$$\frac{Y(f)}{X(f)} = \frac{-1 + jf}{1 + jf}$$

The transform function of system is a All pass filter

$\therefore S_y(f) = S_x(f)$

05. Ans: (a)

Sol:



$$S_{YY}(f) = |H(f)|^2 S_{XX}(f)$$

H(f) = j2πf

$$|H(f)|^2 = 4\pi^2 f^2$$

$$S_{YY}(f) = 4\pi^2 f^2 S_{XX}(f)$$

The Noise power at the output of the LPF is

$$N_o = \int_{-10}^{10} S_{YY}(f)df$$

$$N_o = \int_{-10}^{10} 4\pi^2 f^2 \times 10^{-6} df$$

$$= 2\times 4\pi^2 \times 10^{-6}\int_0^{10} f^2 df$$

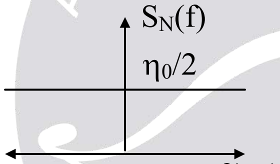
$$= 2\times 4\pi^2 \times 10^{-6} \times \frac{10^3}{3}$$

$$\boxed{\therefore\ N_o = 0.0263W}$$

06. Ans: (a)

Sol: Given,

PSD of Noise $= \frac{\eta_0}{2}$



$T = 27^\circ C \Rightarrow 300K$

$P_n = K.T.B$

$\eta_0 = KT$

$= 1.38\times10^{-23}\times300$

$PSD = \frac{\eta_0}{2}$

$= 1.38\times10^{23}\times150$

$= \frac{207}{10^{23}}$

07. Ans: (b)

Sol: $P_n = K.T.B$

$$= \left(\frac{1}{2}\times1.38\times10^{-23}\times300\right)\times2\times10^6\times2$$

$$= 8.28\times10^{-15}\,W$$

08. Ans: (b)

Sol: $E(X) = \int_{-1}^{3} x.p(x)dx = \frac{1}{4}\left[\frac{x^2}{2}\right]_{-1}^{3} = 1$

$E(X^2) = \int_{-1}^{3} x^2p(x)dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{3} = 7/3$

$Var(X) = E(X^2) - [E(X)]^2 = \frac{7}{3} - 1 = \frac{4}{3}$

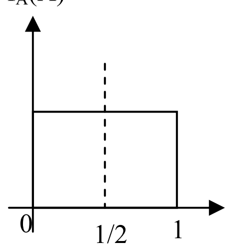
09. Ans: (d)

Sol: $R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)]$

$= E[A\cos\omega t_1 A\cos\omega t_2]$

$= \cos\omega t_1\cos\omega t_2\, E[A^2]$ $[\because E[A^2] = 1/3]$

$= \frac{1}{3}\cos\omega t_1\cos\omega t_2$



$\sigma^2 = \frac{(1)^2}{12} \to$ variance

$E[A^2] = \sigma^2 + [E[A]]^2$

$= \frac{1}{12} + \frac{1}{4}$

$E[A^2] = \frac{4}{12} = \frac{1}{3}$

10. Ans: (b)

Sol: $R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)]$

Let $t_2 - t_1 = \tau$

$E[(A\cos\omega t_1 + B\sin\omega t_1)(B\cos\omega t_2 - A\sin\omega t_2)]$

$\because E[AB] = E[A]\,E[B]$

$E[AB] = 0$

$E[BA] = 0$

$E[A^2] = \sigma^2$

$E[B^2] = \sigma^2$

$= \cos\omega t_1.\cos\omega t_2 E[AB] - \sin\omega t_1 \sin\omega t_2 E[BA]$

$- E[A^2] \cos\omega t_1 \sin\omega t_2$

$+ E[B^2] \sin\omega t_1 \cos\omega t_2]$

$= 0 - 0 - \sigma^2 \cos\omega t_1 \sin\omega t_2 + \sigma^2 \sin\omega t_1 \cos\omega t_2$

$= -\sigma^2(\cos\omega t_1 \sin\omega t_2 + \sin\omega t_1 \cos\omega t_2)$

$= -\sigma^2 \sin\omega(t_2 - t_1)\ (\because \tau = (t_2 - t_1))$

$= -\sigma^2 \sin\omega\tau$

11. Ans: (b)

Sol: X(t) = positive frequencies required

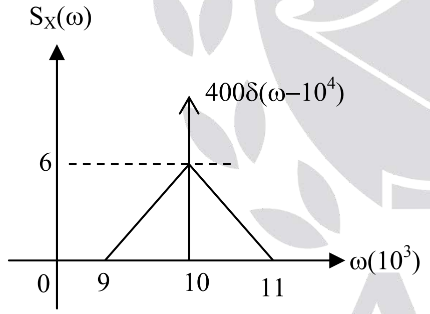
$E[X^2(t)]$ and $E[X(t)]$

$$E[X^2(t)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)d\omega$$

$$= \frac{1}{\pi}\left(400 + \frac{1}{2}(2000)\times 6\right) = \frac{6400}{\pi}$$

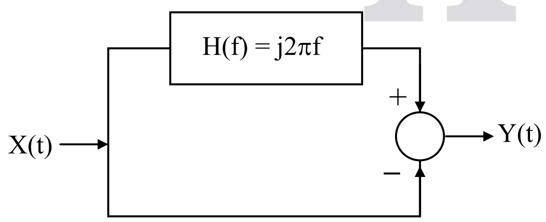
$E[X(t)] = 0$

[$\because$ The given function is periodic function]



12. Ans: (a)

Sol:



Overall $H(f) = j2\pi f - 1$

$R_X(\tau) = e^{-\pi\tau^2}$

$Y(t) = X(t)*h(t)$

$|H(f)|^2 = (4\pi^2 f^2 + 1)$

$R_{XX}(T) \xleftrightarrow{FT} S_{XX}(f)$

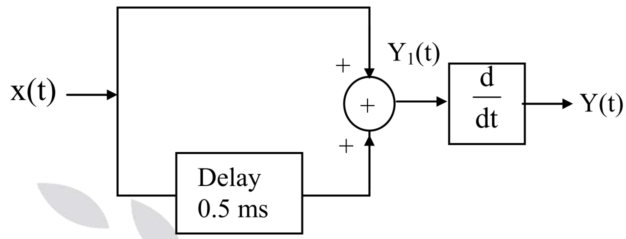
$e^{-\pi\tau^2} \xleftrightarrow{FT} e^{-\pi f^2}$

Normalized Gaussian function

$S_{YY}(f) = |H(f)|^2 S_{XX}(f) = (4\pi^2 f^2 + 1)\, e^{-\pi f^2}$

13. Ans: (d)

Sol:



$$Y(t) = \frac{d}{dt}\left(X(t) + X(t - t_d)\right)$$

$$Y(f) = j2\pi f\left(1 + e^{-j2\pi f t_d}\right)X(f)$$

$$H(f) = \frac{Y(f)}{X(f)}$$

$$= j2\pi f(1 + e^{-j2\pi f t_d})$$

$$|H(f)|^2 = 4\cos^2 \pi f t_d$$

$S_{YY}(f) = |H(f)|^2 S_{XX}(f)$

$= 4\pi^2 f^2 (2\cos(\pi f t_d))^2 S_{XX}(f)$

At $S_{YY}(f) = 0$

$$\pi f t_d = (2n+1)\frac{1}{2t_d}$$

$$f = (2n+1)\frac{1}{2\times 0.5\times 10^{-3}}$$

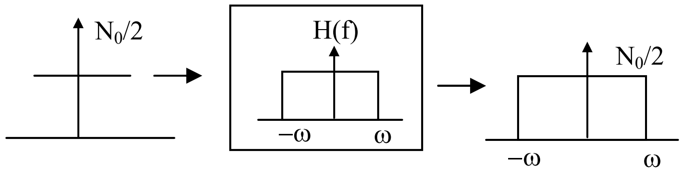
$f = (2n+1)10^3$

$f = (2n+1)f_0$

$f_0 = 1$ kHz

14. Ans: (b)

Sol:



Uncorrelated ⇒ cov(τ) ⇒ $R_{XX}(\tau) - \mu^2 \times (\tau)$

$\text{cov}(\tau) = R_{XX}(\tau) \Rightarrow R_{n_0}(\tau) = 0$

$\Rightarrow N\omega_0 \sin(2\omega\tau) = 0$, $\sin Cx = 0$; x is an integer

$2\omega\tau = m$

$\tau = \frac{m}{2\omega}$, integer m = 1, 2, 3 …….

15. Ans: (b)

Sol: We know that,

$$ACF \xleftrightarrow{F.T} S_x(f)$$

Taking Inverse Fourier Transform

$$F^{-1}[S_y(t)] = \int_{-\infty}^{\infty} S_y(t)\, e^{j2\pi f\tau} df$$

$$R_y(\tau) = \int_{-B_0}^{B_0} \frac{N_0}{2} e^{j2\pi f\tau} df = \frac{N_0}{2}\left[\frac{e^{j2\pi f\tau}}{j2\pi\tau}\right]_{-B_0}^{B_0}$$

$$= \frac{N_0}{2\pi\tau}\left[\frac{e^{j2\pi B_0\tau} - e^{-j2\pi B_0\tau}}{2j}\right]$$

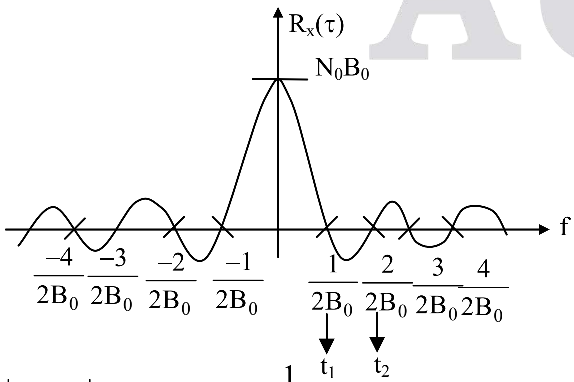
$$= \frac{N_0}{2\pi\tau}\sin(2\pi B_0\tau)$$

$$= N_0 B_0 \frac{\sin(2\pi B_0\tau)}{2\pi B_0\tau}$$

$$R_y(\tau) = N_0 B_0 \operatorname{sinc}(2B_0\tau)$$

16. Ans: (b)

Sol:



$|t_1 - t_2| = \text{multiple of } \frac{1}{2B}$

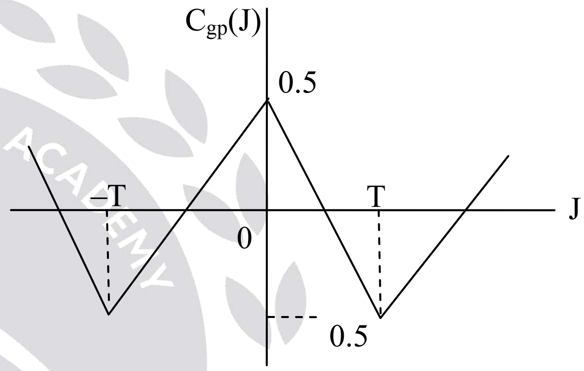
17.

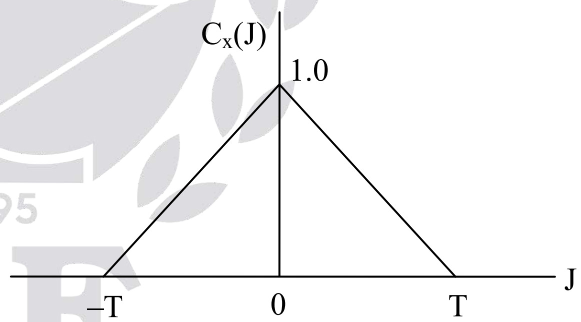
Sol: Since

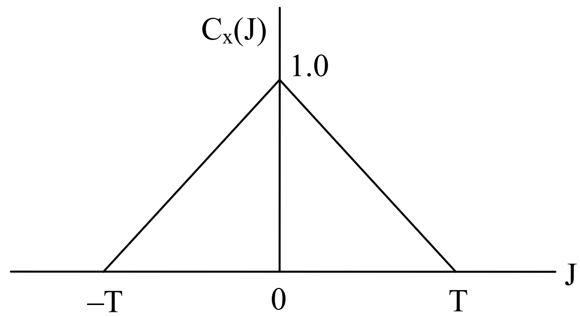
$y(t) = g_p(t) + X(t) + \sqrt{3/2}$

and $g_p(t)$ and X (t) are uncorrelated, then $C_Y(\tau) = C_{g_p}(\tau) + C_X(\tau)$.

Where $C_{gp}(\tau)$ is the auto covariance of the periodic component and $C_x(\tau)$ is the auto covariance of the random component $C_Y(\tau)$ is the plot figure shifted down by 3/2, removing the DC component $C_{gp}(\tau)$ and $C_x(\tau)$ are plotted below







Both $g_p(t)$ and X(t) have zero mean, Average

(a) The power of the periodic component $g_p(t)$ is therefore,

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p^2(t)dt = C_{g_p}(0) = \frac{1}{2}$$

(b) The average power of the random component x(t) is

$E[X^2(t)] = C_x(0) = 1$

18.

Sol:

(a) The power spectral density consists of two components:

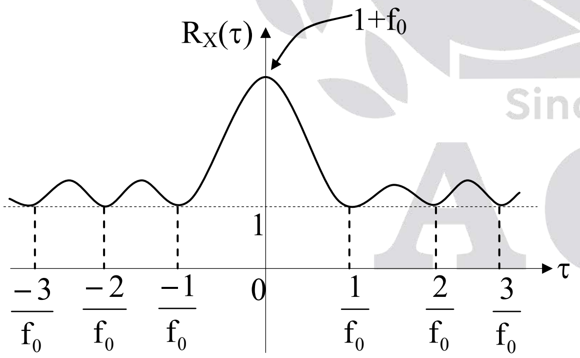
(1) A delta function δ(t) and the origin, whose inverse Fourier transform is one.

(2) A triangular component of unit amplitude and width $2f_0$, centered at the origin; the inverse Fourier transform of this component is $f_0 \, \text{sinc}^2(f_0\tau)$

Therefore, the autocorrelation function of X(t) is

$R_X(\tau) = 1 + f_0 \, \text{sinc}^2(f_0\tau)$

Which is sketched below:



(b) Since $R_X(\tau)$ contains a constant component of amplitude 1. It follows that the dc power contained in X(t) is 1.

(c) The mean-square value of X(t) is given by

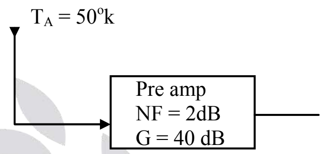
$$E[X^2(t)] = R_X(0)$$
$$= 1 + f_0$$

The ac power contained in X(f) is therefore equal to f_0.

(d) If the sampling rate is f_0/n, where n is an integer, the samples are uncorrelated. They are not, however, statistically independent. They would be statistically independent if X(t) were a Gaussian process.

19. Ans: (a)

Sol:



$10 \log_{10} NF = 2dB$

$\log_{10} NF = 0.2$

$NF = 10^{0.2}$

Noise temperature = $(F - 1) T_o$

$= (10^{0.2} - 1)\, 290o$

$= 169.36$ K

Noise power i/p = $k T_e B$

$= 1.38 \times 10^{-23} \times (169.36 + 50) \times 12 \times 10^6$

Noise power at o/p = $(3.632 \times 10^{-14}) \times 10^4$

$= 3.73 \times 10^{-10}$ watts

20. Ans: 100 W

Sol: $E[x^2(t)] = E[(3V(t) - 8)^2]$

$= E[(9V(t)^2 + 64 - 2\times 3V(t)\times 8]$

$= E[(9V^2(t) + 64 - 48V(t)]$

$= 9E[V^2(t)] + E[64] - 48E[V(t)]$

$[E[V(t)] = 0,\ E[V^2(t)] = MS = R(0) = 4e^{-5(0)} = 4,$

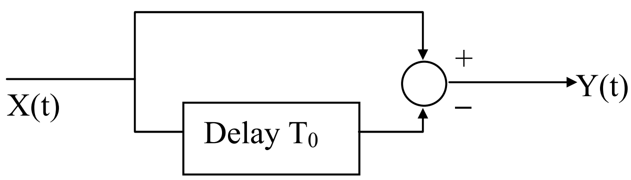
$E[\text{constant}] = \text{constant}]$

$E[x^2(t)] = 9\times 4 + 64 = 36 + 64$

$= 100$

21. Ans: (b)

Sol:



$Y(t) = X(t) - X(t - T_o)$

ACf of o/p $= R_y(\tau) = E[y(t)\, Y(t + \tau)]$

$R_y(\tau) = E[(X(t) - X(t - T_o)] [X(t + \tau) - X(t + \tau - T_o)]$

$R_y(\tau) = E[(X(t)X(t + \tau) - X(t)X(t+\tau - T_o) - X(t - T_o)X(t + \tau) + X(t - T_o)X(t + \tau - T_o)]$

$R_y(\tau) = [R_x(\tau) - R_x(\tau - T_o) - R_x(\tau + T_o) + R_x(\tau)]$

$R_y(\tau) = 2R_x(\tau) - R_x(\tau - T_o) - R_x(\tau + T_o)$

Chapter 7 Noise in Analog Communication

01. Ans: (d)

Sol: Output of the multiplier

$= m(t).\cos\omega_o t \cos(\omega_o t + \theta)$

$= \frac{m(t)}{2}[\cos(2\omega_o t+\theta)+\cos\theta]$

Output of LPF $V_0(t) = \frac{m(t)}{2}\cos\theta$

$= \frac{1}{2}\cos\theta\ m(t)$

Power of o/p signal $= \underset{T\to\infty}{Lt}\frac{1}{T}\int_{<T>} v_0^2(t)\,dt$

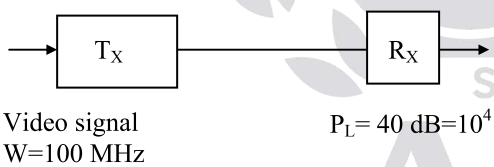
$= \underset{T\to\infty}{Lt}\frac{1}{T}\int_{<T>}\left(\frac{1}{2}\cos\theta\, m(t)\right)^2 dt$

$= \frac{1}{4}\cos^2\theta\left[\underset{T\to\infty}{Lt}\frac{1}{T}\int_{<T>} m^2(t)\,dt\right]$

$= \frac{1}{4}\cos^2\theta\ P_m$

02. Ans: (a)

Sol:



$n_i = n_0$

$n_i = n_0 \times W = 10^{-20} \times 100 \times 10^6$

$S_i = \frac{P_t}{P_L} = \frac{1mw}{10^4} = 1 \times 10^{-7}$

$n_i = 10^{-20} \times 100 \times 10^6$

$\frac{S_i}{n_i} = \frac{10^{-7}}{10^{-12}} = 10^5 = 50$ dB

$\frac{S_i}{n_i} = 50$ dB

03. Ans: (b)

Sol: $\Delta f = 75$ kHz

$f_m = 15$ kHz

$\left(\frac{S}{N}\right)_0 = 40dB = 10^4$

$FOM = \frac{3}{2}\beta^2;\ \beta = \frac{\Delta f}{f_m}$

$\frac{\left(\frac{S_0}{N_0}\right)}{\left(S_i/N_i\right)} = \frac{3}{2}\beta^2$

$\left(S/N\right)_i = \left(\frac{S}{N}\right)_0 \times \frac{2}{3} \times \frac{1}{\beta^2}$

$\left(S/N\right)_{i(dB)} = 24dB$

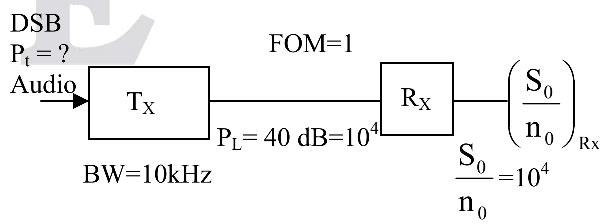
04. Ans: (c)

Sol: $\left(\frac{S}{N}\right)_i = 10$ dB ; FOM $= \frac{1}{3}$

$\left(\frac{S}{N}\right)_0 = \frac{1}{3}\times 10 = 3.33$

05. Ans: (a)

Sol:



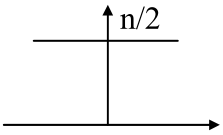
06. Ans: (a)

Sol: For SSB modulation

$\Rightarrow \frac{S_0}{N_0} = \frac{S_i}{N_i} = 10^4$

(Only SSB modulation in one sided n/2)

$P_t = ?$



$$\frac{S_i}{n_i} = \frac{S_0}{n_0} = 10^4$$

$S_i = 10^4 \times 10 \times 10^3 \times 2 \times 10^{-9}$ w/Hz

$S_i = 20 \times 10^{-2}$

$(S_i)_{dB} = (P_t)_{dB} - (P_t)_{dB}$

$(P_t)_{dB} = (S_i)_{dB} + (P_L)_{dB}$

$P_t = S_iP_L = 20 \times 10^{-2} \times 10^4$

$P_L = 2$ kW

07. Ans: (c)

Sol: For AM

$$\text{FOM} = \frac{1}{3} \text{ (if } \mu = 1)$$

$$\frac{S_0}{N_0} = \left(\frac{1}{3}\right)\frac{S_i}{N_i} \Rightarrow S_i = 3\left(\frac{S_0}{N_0}\right) \times N_i$$

$$= 3 \times 10^4 \times 2 \times 10^{-9} \times 10\text{kHz} = 0.6$$

$$\therefore P_t = S_i \times P_L = 0.6 \times 10^4 = 6\text{KW}$$

08. Ans: (b)

Sol: Noise figure $= \frac{(\text{SNR})_{I/P}}{(\text{SNR})_{O/P}}$

$Nf_{,dB} = SNR_{i,dB} - SNR_{o/p,dB}$

$SNR_{o/p,dB} = SNR_{I/P,dB} - Nf_{dB} = 37 - 3$

$= 34$ dB

Chapter 8 Baseband Data Transmission

01. Ans: (d)

Sol: $\Delta = \frac{V_{max} - V_{min}}{2^n}$

$\Delta \alpha \frac{1}{2^n}$; $\frac{\Delta_1}{\Delta_2} = \frac{2^{n_2}}{2^{n_1}}$

$\frac{0.1}{\Delta_2} = \frac{2^{n+3}}{2^n}$

$\Delta_2 = 0.1 \times \frac{1}{8}$

$= 0.0125$

02. Ans: (3)

Sol: $(BW)_{PCM} = \frac{n f_s}{2}$

Where 'n' is the number of bits to encode the signal and L = 2^n, where 'L' is the number of quantization levels.

$L_1 = 4 \Rightarrow n_1 = 2$

$L_2 = 64 \Rightarrow n_2 = 6$

$\frac{(BW)_2}{(BW)_1} = \frac{n_2}{n_1} = \frac{6}{2} = 3$

$(BW)_2 = 3\ (BW)_1$

03. Ans: (c)

Sol: Given,

Two signals are sampled with $f_s = 44100$s/sec and each sample contains '16' bits

Due to additional bits there is a 100% overhead.

Out put bit rate =?

$R_b = n^{|} f_s^{|}$

$f_s^{|} = 2f_{s|} = 2\ [44100]$

($\because$ two signals sampled simultaneously)

$n^{|} = 2n$

($\because$ due to overhead by additional bits)

$R_b = 4\ (nf_s) = 2.822$Mbps

04. Ans (c)

Sol: Number of bits recorded over an hour

$= R_b \times 3600 = 10.16$ G.bits

05. Ans: (c)

Sol: $p(t) = \frac{\sin(4\pi W t)}{4\pi W t\ (1-16W^2 t^2)}$

At $t = \frac{1}{4W}$; $P\left(\frac{1}{4W}\right) = \frac{0}{0}$

Use L-Hospital Rule

$\underset{t\to\frac{1}{4W}}{Lt}\ p(t) = \underset{t\to\frac{1}{4W}}{Lt}\ \frac{4\pi W\ \cos(4\pi W t)}{4\pi W - 64\pi W^3 (3t^2)}$

$= \frac{4\pi W(-1)}{4\pi W - 64\pi W^3\, 3\left(\frac{1}{16W^2}\right)}$

$= \frac{-4\pi W}{-8\pi W} = 0.5$

06. Ans: 35

Sol: Given bit rate R_b = 56 kbps, Roll of factor $\alpha = 0.25$

BW required for base band binary PAM system

$BW = \frac{R_b}{2}[1+\alpha] = \frac{56}{2}[1+0.25]\text{kHz} = 35\text{kHz}$

07. Ans: 16

Sol: $R_b = nf_s$ = 8bit/sample × 8kHz = 64 kbps

$(B_T)_{min} = \frac{R_b}{2\log_2 M}$

$= \frac{R_b}{2\log_2 4} = \frac{R_b}{2\times 2}$

$= \frac{R_b}{4} = \frac{64}{4}$

= 16kHz

08. Ans: (b)

Sol: Given $f_s = 1/T_s$ = 2k symbols/sec

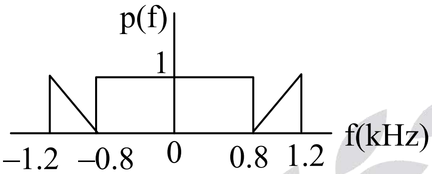
If $P(f) \overset{F.T}{\leftrightarrow} p(t)$,

Condition for zero ISI is given by

$$\frac{1}{T_s}\sum_{n=-\infty}^{\infty} P(f - n/T_s) = p(0)$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f - n/T_s) = p(0)T_s$$

p(0) = area under P(f)



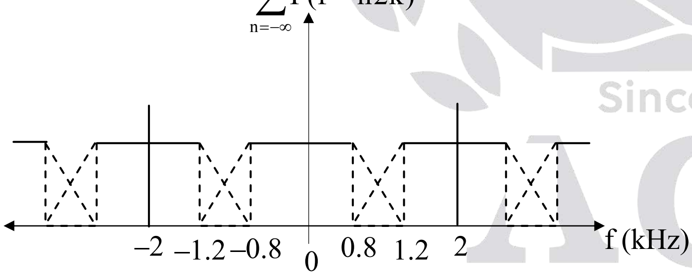
$$\text{Area} = 2\times\frac{1}{2}(1)(0.4)k + 2\times 0.8k = 2k$$

$$p(0)\ T_s = 2k \times \frac{1}{2k} = 1$$

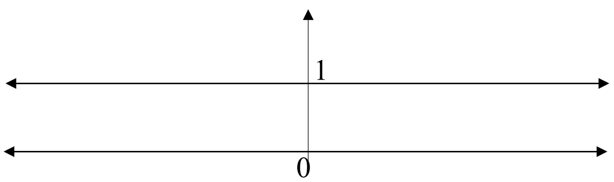
$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f - n/T_s) = 1$$

The above condition is satisfied by only option (b)

$$\sum_{n=-\infty}^{\infty} P(f - n2k)$$



⇓



$$\therefore \sum_{n=-\infty}^{\infty} P(f - n2k) = 1$$

Option (a) is correct if pulse duration is from –1 to + 1

Option (c) is correct if the transition is from 0.8 to 1.2, –0.8 to –1.2

Option (d) is correct if the triangular duration is from –2 to +2

09. Ans: 200

Sol: m(t) = sin 100πt + cos 100πt

$= \sqrt{2}\cos[100\pi t + \phi]$

$$\Delta = 0.75 = \frac{V_{max} - V_{min}}{L} = \frac{\sqrt{2} - (-\sqrt{2})}{L} = \frac{2\sqrt{2}}{L}$$

$$L = \frac{2\sqrt{2}}{0.75} \approx 4 = 2^n$$

So n = 2

f = 50 Hz so Nyquist rate = 100

So, the bit rate = 100 × 2 = 200 bps

10. Ans: (b)

Sol: Given

$f_{m_1} = 3.6\text{kHz} \Rightarrow f_{s_1} = 7.2\text{kHz}$

$f_{m_2} = f_{m_3} = 1.2\text{kHz} \Rightarrow f_{s_2} = f_{s_3} = 2.4\text{kHz}$

$f_s = f_{s_1} + f_{s_2} + f_3$

$= 12\text{kHz}$

No. of Levels used = 1024

⇒ n = 10bits

∴ Bit rate = nf_s

=10 × 12 kHz

=120 kbps

11. Ans: (a)

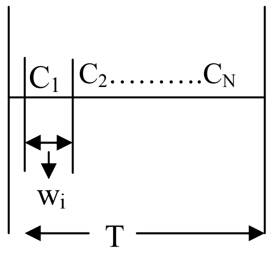
Sol: $(f_s)_{min} = (f_{s_1})_{min} + (f_{s_2})_{min} + (f_{s_3})_{min} + (f_{s_4})_{min}$

= 200 + 200 + 400 + 800

= 1600 Hz

12. Ans: (c)

Sol:



Minimum B.W of TDM is $\sum_{i=1}^{N} w_i$

13. Ans: (b)

Sol: Number of patients = 10

ECG signal B.W = 100Hz

$(Q_e)_{max} \le (0.25)\ \%V_{max}$

$$\frac{2V_{max}}{2\times 2^n} \le \frac{0.25}{100} V_{max}$$

$2^n \ge 400$

$n \ge 8.64$

$n = 9$

Bit rate of transmitted data = 10×9×200

= 18kbps

14. Ans: (a)

Sol: Peak amplitude → A_m

Peak to peak amplitude A_m

$$\frac{-\Delta}{2} \le Q_e \le \frac{\Delta}{2}$$

PCM maximum tolerable $\frac{\Delta}{2} = 0.2\%\ A_m$

$$\Delta = \frac{\text{Peak to peak}}{L} \Rightarrow \frac{2A/m}{2L} = \frac{0.2}{100} A_m$$

$$(\because \Delta = \frac{2A_m}{L})$$

$\Rightarrow L = 500$

$2^n = 500$

$n = 9$

$R_b = n(f_S)_{TDM} + 9$

$f_S = R_N + 20\%R_N = R_N + 0.2R_N$

$f_S = 1.2R_N = 1.2 \times 2\times\omega$

$f_S = 2.4$ K samples/sec

$(f_S)_{TDM} = 5(f_S)$

$= 5 \times 2.4$ K

$= 12$ K sample/sec

$R_b = (nf_S) + 0.5\%(nf_S)$

$$= (9 \times 12k) + \frac{0.5}{100}(9\times 12k)$$

= 108540 bps

15. Ans: (b)

Sol: To avoid slope over loading, rate of rise of the o/p of the Integrator and rate of rise of the Base band signal should be the same.

$\therefore \Delta f_s$ = slope of base band signal

$\Delta \times 32 \times 10^3 = 125$

$\Delta = 2^{-8}$ Volts.

16. Ans: (b)

Sol: $x(t) = E_m \sin 2\pi f_m(t)$

$\frac{\Delta}{T_S} < \left|\frac{dm(t)}{dt}\right|$ → slope overload distortion takes place

$\Delta f_S < E_m 2\pi f_m$

$$\Rightarrow \frac{\Delta f_S}{2\pi} < E_m f_m \qquad (\because \Delta = 0.628)$$

$$\Rightarrow \frac{0.628\times 40K}{2\pi} < E_m f_m$$

$f_S = 40$ kHz $\Rightarrow 4$ kHz $< E_m f_m$

Check for options

(a) $E_m \times f_m = 0.3 \times 8\text{ K} = 2.4\text{ kHz}$
$(4K \nless 2.4\text{ K})$

(b) $E_m \times f_m = 1.5 \times 4K = 6\text{ kHz}$
$(4K < 6\text{ K})$ correct

(c) $E_m \times f_m = 1.5 \times 2\text{ K} = 3\text{ kHz}$
$(4K \nless 3K)$

(d) $E_m \times f_m = 30 \times 1\text{ K} = 3\text{ kHz}$
$(4K \nless 3K)$

17. Ans: (a)

Sol: Given

$m(t) = 6\sin(2\pi\times10^3 t) + 4\sin(4\pi\times10^3 t)$

$\Delta = 0.314$ V

Maximum slope of $m(t) = \frac{d}{dt}(m(t)) / t = \frac{\pi}{2}$

$= 2\pi\times10^3(6) + 4\pi\times10^3[4] = 28\pi\times10^3$

18. Ans: (c)

Sol: Pulse rate which avoid distortion

$$\Delta f_s = \frac{d}{dt}m(t)$$

$$f_s = \frac{28\pi\times10^5}{0.314}$$

$f_s = 280\times10^3$ pulses/sec

Chapter 9 Bandpass Data Transmission

01. Ans: (c)

Sol: $(BW)_{BPSK} = 2f_b = 20$ kHz

$(BW)_{QPSK} = f_b = 10$ kHz

02. Ans: (b)

Sol: $f_H = 25$ kHz ; $f_L = 10$ kHz

$\therefore$ Center frequency

$= \left(\frac{25+10}{2}\right)$ kHz

$= 17.5$ kHz

$\therefore$ Frequency offset,

$\Omega = 2\pi\,(25 - 17.5) \times 10^3$

$= 2\pi\,(7.5) \times 10^3$

$= 15 \times 10^3\pi$ rad/sec.

The two possible FSK signals are orthogonal, if $2\Omega T = n\pi$

$\Rightarrow 2(15\pi) \times 10^3 \times T = n\pi$

$\Rightarrow 30 \times 10^3 \times T = n$ (integer)

This is satisfied for, T = 200µsec.

03. Ans: (a)

Sol: $r_b = 8$ kbps

Coherent detection

$\Delta f = \frac{nr_b}{2}$

Best possible n = 1

$\Delta f = \frac{8K}{2} = 4K$

To verify the options $\Delta f = 4k$

i.e. $f_{C2} - f_{C1} = 4K$

(a) 20 K – 16 K = 4 K

(b) 32 K – 20 K = 12 K

(c) 40 K – 20 K = 20 K

(d) 40 K – 32 K = 8 K

04. Ans: (a) & (c)

Sol: Non coherent detection of PSK is not possible. So to overcome that, DPSK is implemented. A coherent carrier is not required to be generated at the receiver.

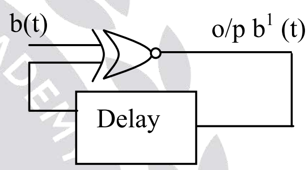
05. Ans: (c)

Sol: In QPSK baud rate $= \frac{\text{bit rate}}{2} = \frac{34}{2}$

$= 17$ Mbps

06. Ans: (d)

Sol:



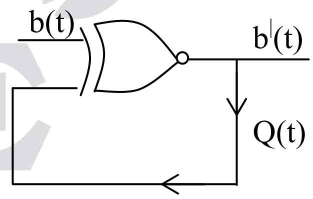
b(t)	0	1	0	0	1
$b^1(t)$ **(Ref.bit)**	0	0	1	0	0
Phase	π	π	0	π	π

07. Ans: (b)

Sol: Given

Bit stream 110 111001

Reference bit = 1



$b^{|}(t) = b(t) \odot Q(t)$

1 1 0 1 1 1 0 0 1

1

1 1 0 0 0 0 1 0 0

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

0 0 π π π π 0 π π

08. Ans: (d)

Sol: $r_b = 1.544 \times 10^6$

$\alpha = 0.2$

$$BW = \frac{r_b}{\log_2^M}(1+\alpha)$$

$$= \frac{1.544 \times 10^6}{2}(1+0.2) \quad (\because M = 4)$$

$BW = 926.4 \times 10^3$ Hz

09. Ans: 0.25

Sol: BW = 1500 Hz

BW required for M-ary PSK is

$$\frac{R_b[1+\alpha]}{\log_2 16} = 1500\text{Hz}$$

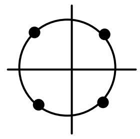
$\Rightarrow R_b [1 + \alpha] = 1500 \times 4 = 6000$

$$\Rightarrow (1+\alpha) = \frac{6000}{4800}$$

Roll off factor $\Rightarrow \alpha = \frac{6000}{4800} - 1 = 0.25$

10. Ans: (d)

Sol:



Here only phase is changing.
From options (d) is the optimum answer.

11. Ans: (b)

Sol: Here 16-points are available in constellation which are varying in both amplitude and phase. So, it 16QAM.

12. Ans: (d)

Sol: $BW = \frac{r_b}{\log_2 M}(1+\alpha)$

$$36 \times 10^6 = \frac{r_b}{2}(1+0.2)(\because M = 4, \text{QPSK})$$

$r_b = 60 \times 10^6$ bps

Chapter 10 Noise in Digital Communication

Noise Ratio

01. Ans: (b)

Sol: Signal to quantization noise ratio only depends on no. of quantization levels (L) and no. of bits per sample(n)

For sinusoidal input SQNR = 1.76+6n dB
= 1.76 + 6×12
= 73.76 dB

For uniform distributed signal = 6ndB
= 6×12
= 72 dB

02. Ans: (a)

Sol: For Bipolar pulses,

$$PSD = \frac{|P(\omega)|^2}{T_b}.\sin^2\left(\frac{\omega T_b}{2}\right)$$

The zero magnitude occurs for

$f = n/T_b$.

$\therefore$ The width of the major lobe = $1/T_b$
$= f_b$

$\therefore (B.W)_{min} = f_b$

Here, Data rate = nf_s
= 8(8 kHz) = 64 kbps

$\therefore (B.W)_{min}$ = 64 kHz

03. Ans: (c)

Sol: Since the signal is uniformly distributed,

$f(x) = \frac{1}{10}$ for $-5 \le x \le 5$
= 0 : else where.

$$\text{Signal Power} = \int_{-5}^{5} x^2 f(x)dx = \frac{25}{3}\text{volts}^2$$

$$\text{Step size} = \frac{V_{p-p}}{L} = \frac{10}{2^8} = 0.039\ V$$

$$N_q = \frac{\Delta^2}{12} = 0.126\ mW$$

Signal to noise ratio, SNR in dB is

$$SNR = 10\log\left(\frac{\text{signal power}}{\text{Noise power}}\right)$$

$$= 10\log\left(\frac{25/3}{0.126\times10^{-3}}\right)$$

$$= 48\ dB$$

04. Ans: (b)

Sol: For every one bit increase in data word length, quantization Noise Power becomes $\frac{1}{4}$th of the original. Hence, Data word length for n = 9 bits is,

$\therefore L = 2^n = 2^9 = 512$

05. Ans: (c)

Sol: V_{P-P} = −5V to 5V

20logL = 43.5

$L = 10^{2.175}$

= 149.6

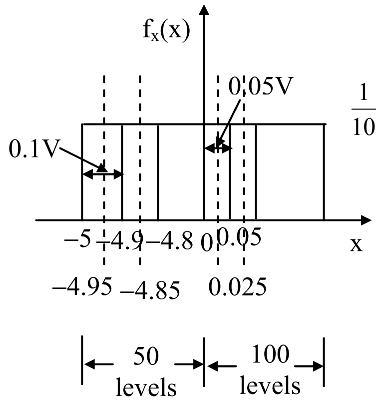
$$\Rightarrow \Delta = \frac{V_H - V_L}{L}$$

$$= \frac{5-(-5)}{10^{2.175}}$$

Δ = 0.06683

06. Ans: (c)

Sol:



Signal power $E[X^2] = \int_{-5}^{5} x^2\left(\frac{1}{10}\right)dx$

$$= \frac{1}{10}\left(\frac{x^3}{3}\right)_{-5}^{5} = \frac{1}{30}(250) = \frac{25}{3}\text{ W}$$

Quantization Noise power

$$= E[[X- Q(X)]^2]$$

$$= \int_{-5}^{5}[x - Q(x)]^2 \; f_X(x)\,dx$$

$$= \int_{-5}^{-4.9}[x-(-4.95)]^2\frac{1}{10}dx$$

$$+ \int_{-4.9}^{-4.8}[x-(-4.85)^2]\frac{1}{10}dx + ...(50\text{ times})$$

$$+ \int_{0}^{0.05}(x-0.025)^2\frac{1}{10}dx$$

$$+ \int_{0.05}^{0.1}[(x-0.075)^2]\frac{1}{10}dx + ...(100\text{ times})$$

$$= 50\int_{-5}^{-4.9}(x+4.95)^2\frac{1}{10}dx + 100\int_{0}^{0.05}(x-0.025)^2\frac{1}{10}dx$$

$$= 5\left[\frac{(x+4.95)^3}{3}\right]_{-5}^{-4.9} + 10\left[\frac{(x-0.025)^3}{3}\right]_{0}^{0.05}$$

$$= \frac{5}{3}[(0.05)^3 + (0.05)^3] + \frac{10}{3}[(0.025)^3 + (0.025)^3]$$

$$= \frac{5}{3}(125\times10^{-6} + 125\times10^{-6}) + \frac{10}{3}[(0.025)^3 + (0.025)^3]$$

$$= \frac{5}{3}(125\times10^{-6} + 125\times10^{-6}) + \frac{10}{3}(3.125\times10^{-5})$$

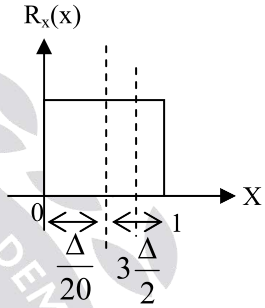
$$= \frac{1250}{3}\times10^{-6} + \frac{312.5}{3}\times10^{-6}$$

$$= 520.83333 \times 10^{-6}$$

$$(SNR)_{dB} = 10\log\left(\frac{25}{3\times5.2\times10^{-4}}\right)$$

$$= 42.04\text{ dB}$$

$$\approx 42\text{ dB}$$



07. Ans: (b)

Sol: $E[X - Q(x)]^2$

$$= \int_{0}^{0.3}(X-0)^2(1)dx \int_{0.3}^{1}(X-(0.7))^2(1)dx$$

$$= \left[\frac{x^3}{3}\right]_{0}^{0.3} + \left[\frac{(x-0.7)^3}{3}\right]_{0.3}^{1}$$

$$= \frac{(0.3)^3}{3} + \frac{(0.3)^3}{3} + \frac{(0.4)^3}{3}$$

$$= 0.198$$

08. Ans: (b)

Sol: Since, all the quantization levels are equiprobable,

$$\int_{-a}^{a}\frac{1}{4}dx = \frac{1}{3} \quad \Rightarrow a = \frac{2}{3}$$

09. Ans: (a)

Sol: $\int_{-2/3}^{2/3} x^2 f(x).dx = \frac{1}{4}\int_{-2/3}^{2/3} x^2.dx = \frac{4}{81}$

Matched Filter

01. Ans: (d)

Sol: The time domain representation of the o/p of a Matched filter is proportional to Auto correlation function of the i/p signal, except for a time delay

$$R_{ss}(\tau) = \int_0^{10^{-4}} S(t).S(t+\tau)dt$$

$$= \int_0^{10^{-4}} 10\sin(2\pi\times10^6 t).10\sin(2\pi\times10^6(t+\tau))]dt$$

$$= 50\int_0^{10^{-4}}\left[\cos(2\pi\times10^6\tau)-\cos(4\pi\times10^6 t+2\pi\times10^6\tau)\right]dt$$

$$= 50\times10^{-4}\cos(2\pi\times10^6)\,\tau$$

$\therefore$ The Peak is 5mV

02. Ans: (b)

Sol: The matched filter has maximum value of output at t = T is energy of the signal

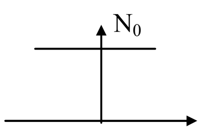
$$\Rightarrow E_s = \int_0^1 A^2 dt + \int_2^3 A^2(1)\,dt$$

$$= A^2 + A^2 = 2A^2$$

03. Ans: (d)

Sol: $(SNR)_0 = \dfrac{E_S}{N_0} = \dfrac{\frac{B^2}{2}.T}{N}$

$$= \frac{B^2T}{2N}$$



04. Ans: (b)

Sol: Given,

$$\frac{S_{02}(t)}{N} = \frac{S_{01}(t)}{N} \Rightarrow \frac{2E_{s_1}}{N} = \frac{2E_2}{N}$$

$$A^2T = \frac{B^2}{2}T \;\Rightarrow\; A = \frac{B}{\sqrt{2}}$$

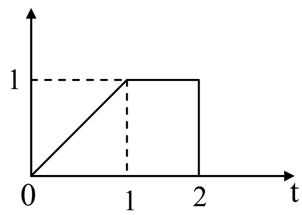
05. Ans: (d)

Sol: Output of the matched filter is maximum which is equal to the energy in the signal

$$E = \int_0^1 1.t^2 dt + \int_1^2 (1)dt$$

$$= \left[\frac{t^3}{3}\right]_0^1 + 1[t]_1^2$$

$$= \frac{1}{3} + 1 = 4/3$$



The time instant which occurs the maximum value is its time period T = 2

06. Ans: (c)

Sol: Given,

$$H(f) = \frac{1-e^{-j\omega t}}{j\omega}$$

$$H(f) = \frac{1}{j\omega} - \frac{e^{-j\omega t}}{j\omega}$$

Applying I.F.T

$h(t) = 0.5(\text{sgn}(t) - \text{sgn}(t - T_0))$

$$\left(\because F(\text{sgn}(t)) = \frac{2}{j\omega}\right)$$

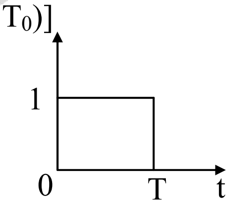
$= 0.5[2u(t) - 1 - [2u(t-T_0) - 1]]$

$= [u(t) - u(t - T_0)]$

We know that

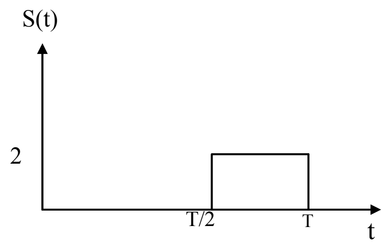
$h(t) = s^*(t - T)$

$\therefore S_i(t)$



07. Ans: (d)

Sol: The maximum value in the output is energy inside the signal



$$\Rightarrow S_0(t)\big|_{max} = \int_{T/2}^{T} 2^2.dt$$

$$= 4\int_{T/2}^{T} 1.dt$$

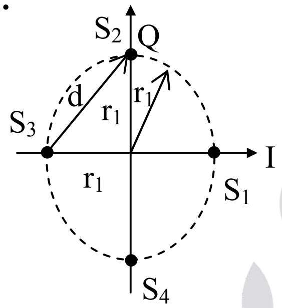
$$= 4[T - T/2]$$

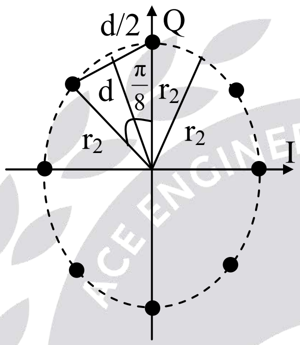
$$= 2T$$

Probability of Error

01. Ans: (d)

Sol:





$d = \sqrt{2}\, r_1$ $\qquad \sin\frac{\pi}{8} = \frac{(d/2)}{r_2}$

$$\Rightarrow r_1 = \frac{d}{\sqrt{2}} = 0.707\,d$$

$$\Rightarrow r_2 = \frac{d}{2\sin\frac{\pi}{8}} = 1.307\,d$$

02. Ans: (d)

Sol: 4-PSK, 8-PSK both have same error probability when both signals have same minimum distance between pairs of signal points.

$$P_e = Q\left(\frac{\sqrt{d_{min}^2}}{2N_0}\right)$$

$$P_e = 2Q\left(\sqrt{\frac{2E_s}{N_0}\sin^2\left(\frac{\pi}{M}\right)}\right)$$

Where E_s is the average symbol energy

Given both constellation d_{min} is same i.e.,'d'

Average Symbol Energy:

$$(E_s)_{4PSK} = \frac{E_{s_1} + E_{s_2} + E_{s_3} + E_{s_4}}{4}$$

Where E_{s_k} is the symbol 'S_k' Energy

= (distance from the origin to the symbol 'S_k')^2

$$(E_s)_{4PSK} = \frac{r_1^2 + r_1^2 + r_1^2 + r_1^2}{4} = r_1^2$$

Similarly, For 8 PSK

$$(E_s)_{8PSK} = r_2^2$$

$$\frac{(E_s)_{8PSK}}{(E_s)_{4PSK}} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{1.307d}{0.707d}\right)^2$$

In dB,

$$(E_s)_{8PSK(dB)} - (E_s)_{4PSK(dB)} = 10\log\left(\frac{1.307}{0.707}\right)^2$$

$$= 5.33 \text{ dB}$$

$$(E_s)_{8PSK} = (E_s)_{4PSK} + 5.33 \text{ dB}$$

8 PSK required additional 5.33 dB

03. Ans: (b)

Sol: Constellation 1:

$s_1(t) = 0$;

$s_2(t) = -\sqrt{2}\ a\,\phi_1 + \sqrt{2}\ a\,\phi_2$

$s_3(t) = -2\sqrt{2}\ a.\phi_1$;

$s_4(t) = -\sqrt{2}\ a\,\phi_1 - \sqrt{2}\ a\,\phi_2$

Energy of $S_1(t) = E_{S1} = 0$; $E_{S2} = 4a^2$; $E_{S3} = 8a^2$; $E_{S4} = 4a^2$

Average Energy of constellation 1

$$= \frac{E_{S1} + E_{S2} + E_{S3} + E_{S4}}{4} = 4a^2$$

Constellation 2:

$s_1(t) = a\phi_1 \quad \Rightarrow \quad E_{S1} = a^2$

$s_2(t) = a.\phi_2 \quad \Rightarrow \quad E_{S2} = a^2$

$s_3(t) = -a.\phi_1 \Rightarrow E_{S3} = a^2$

$s_4(t) = -a.\phi_2 \Rightarrow E_{S4} = a^2$

Average Energy of constellation 2

$$= \frac{E_{S1}+E_{S2}+E_{S3}+E_{S4}}{4} = a^2$$

The required Ratio is 4

04. Ans: (a)

Sol: The distance between the two closest points in constellation 1 is $d_1 = 2a$.

The same in constellation 2,

$d_2 = \sqrt{2}\,a$

Since $d_1 > d_2$, Probability of symbol error for constellation 1 is lower

05. Ans: (a)

Sol: $$S(t) = \sqrt{\frac{2E}{T_b}}\left[\cos\left(\omega_c t + \frac{2\pi}{m}(i-1)\right)\right]$$

$$= \sqrt{\frac{2E}{T_b}}\left[\cos\omega_c t..\cos\left(\frac{2T}{m}(i-1)\right) - \sin\omega_c t.\sin\frac{2\pi}{m}(i-1)\right]$$

$$= \sqrt{\frac{2}{T_b}}\cos\omega_c t\sqrt{E}\cos\left(\frac{2\pi}{m}(i-1)\right) - \sqrt{\frac{2}{T_b}}\sin\omega_c t\sqrt{E}\sin\frac{2\pi}{m}(i-1)$$

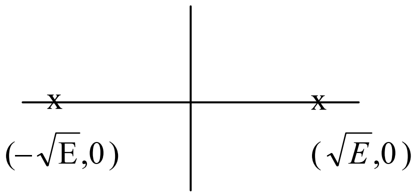
Given binary digital communication m = 2

$$\sqrt{\frac{2}{T_b}}\cos\omega_c t\sqrt{E}\cos\pi$$

$\because$ basic function $= 2\cos\omega_c t$

$\Rightarrow T_b = 1/2$

$$2\cos\omega_c t\left(\sqrt{E}\cos\pi(f-1)\right) - [2\sin\omega_c t]\sqrt{E}\sin\pi(i-1)$$



Distance between two points is:

$$\sqrt{(\sqrt{E}+\sqrt{E})^2 + 0}$$

$$\sqrt{4E} = 2\sqrt{E}$$

Energy of the signal:

$$\int_0^{T_b}(A\cos\omega_c t)^2 = \frac{A^2 T}{2}$$

$$\Rightarrow d = 2\sqrt{\frac{A^2 T_b}{2}} = 2\sqrt{\frac{A^2 \times T_b}{2}} = A$$

$(\because T_b = 1/2)$ $\quad\therefore\ d = A$

06. Ans: (c)

Sol: $$P_e = Q\left[\sqrt{\frac{E_b}{N_o}}\right]$$

$$E_b = \frac{\alpha^2 T_b}{2} = \frac{\alpha^2}{2R_b}$$

$\alpha = 4$mV, $R_b = 500$ kbps,

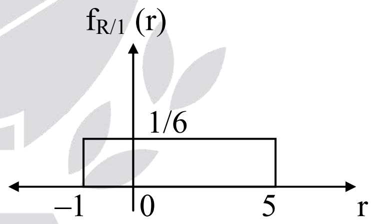
$N_o = 10^{-12}$W/Hz.

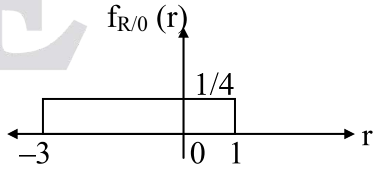
$$\frac{E_b}{N_o} = \frac{16\times10^{-6}}{2\times500\times10^3\times10^{-12}} = 16$$

$$P_e = Q\left[\sqrt{16}\right] = Q[4]$$

07. Ans: (d)

Sol:



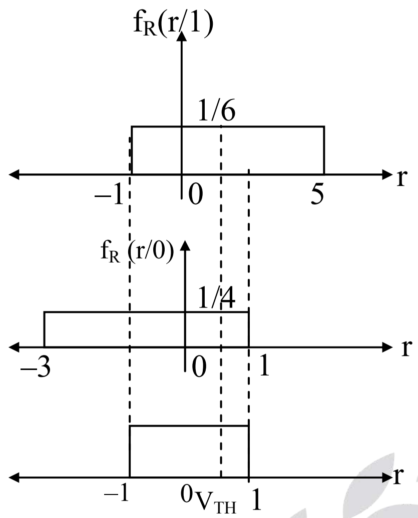


P(0) = 1/3; P(1) = 2/3

The probability of error of the symbols 0 & 1 are not the same.

$\therefore$ The intersection point of the two pdf's is not the threshold of detection.

Assume the threshold value to be V_{TH}



For minimum error the V_{TH} should lie in the area of intersection of the 2 pdf's.

$$P_{e_1} = \int_{-1}^{V_{TH}} \left(\frac{1}{6}\right) dr = \frac{1}{6}\left(V_{TH}+1\right)$$

$$P_{e_0} = \int_{V_{TH}}^{1} \left(\frac{1}{4}\right) dr = \frac{1}{4}\left(1-V_{TH}\right)$$

Decision error probability

$$= P_{e_0}P(0) + P_{e_1}P(1)$$

$$= \frac{1}{4}(1-V_{TH})\left(\frac{1}{3}\right) + \frac{1}{6}(1+V_{TH})\left(\frac{2}{3}\right)$$

$$P_e = \frac{1-V_{TH}}{12} + \frac{2(1+V_{TH})}{18}$$

For minimum decision error probability,

$$-1 \le V_{TH} \le 1$$

For $V_{TH} = -1$

$$BER = \frac{1-(-1)}{12} = \frac{1}{6} \text{ (min value)}$$

$\therefore$ Decision error probability = 1/6

08. Ans: (c)

Sol: The optimum threshold value is

$$\hat{x} = \frac{\sigma^2}{x_1 - x_2}\left[\ell n \frac{P(x_2)}{P(x_1)} + \frac{x_1^2 - x_2^2}{2\sigma^2}\right]$$

$x_1 = 1$, $x_2 = -1$

$P(x_1) = 0.75$, $P(x_2) = 0.25$

$$\hat{x} = \frac{\sigma^2}{2}\left[\ell n \frac{0.25}{0.75}\right] = -\frac{\sigma^2}{2}$$

So $\hat{x}$ should be strictly negative.

09. Ans: (c)

Sol: $Y = X + Z$

Z is Gaussian RV with mean βx

$x \in \{-a, +a\}$

when $\beta = 0$ $E[y] = E[x] + E[z]$

$E[y] = E[x] = +a$

$= a$

$BER = Q(a) = 1 \times 10^{-8}$

$$Q(v) = \frac{1}{\sqrt{2\pi}}\int_{v}^{\infty} e^{\frac{-v^2}{2}} du \cong e^{\frac{-v^2}{2}}$$

$$Q(a) = 1 \times 10^{-8} \approx e^{\frac{-a^2}{2}}$$

$a = 6$

when $\beta = -0.3$ mean $= 6 \times -0.3 = -1.8$

so $E(y) = E(x) + E(z)$

$= 6 - 1.8 = 4.2$

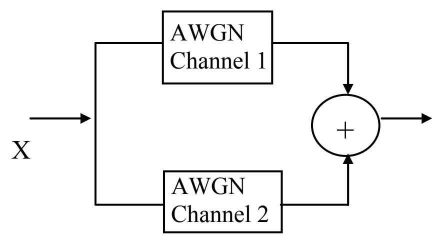
so $BER = Q(4.2) \cong e^{\frac{-(4.2)^2}{2}}$

$\cong 0.0001$

$\cong 10^{-4}$

10. Ans: 1.414

Sol: When the signal is transmitted through a channel BER = $Q[\sqrt{r}]$.



At the input of the receiver signal amplitude is doubled. But when two independent Gaussian Random Variables are added, the resultant random variables is also a Gaussian random. The pdf is the convolution of individual pdf's.

The variance indicates the noise power

But the variance is doubled.

Signal power increased by a factor of 4(mean is doubled).

But the noise increases by a factor of 2

So the signal to noise increases by a factor of 2

So b = $\sqrt{2}$ =1.414

BER = $Q[\sqrt{2r}] = Q[\sqrt{2}\sqrt{r}] = Q[1.414\sqrt{r}]$

So b = 1.414

11. Ans: (a)

Sol: Probability of error for an AWGN channel for binary transmission is given as

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

Where $E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt$

Given $s_1(t) = g(t)$

$s_2(t) = -g(t)$

$$E_d = \int_0^T [g(t) - (-g(t))]^2 dt$$

$$= 4\int_0^T g^2(t)dt$$

$$E_{d,a} = 4\int_0^1 (1)^2 dt = 4$$

$$E_{d,b} = 4\left[\int_0^{1/2} (2t)^2 dt + \int_{1/2}^1 (-2t+2)^2\right] dt$$

$$= \frac{4}{6} + \frac{4}{6} = \frac{4}{3}$$

$$E_{d,c} = 4\int_0^1 (1-t)^2 dt = \frac{4}{3}$$

$$E_{d,d} = 4\int_0^1 (t)^2 dt = \frac{4}{3}$$

P_e is minimum when E_d is maximum

E_d of signal (a) is more when compared to E_d of other signals.

$\therefore$ Probability of error is minimum for signal (a).

12. Ans: (b)

Sol: o/p Noise Power = o/p PSD × B.W

$= 10^{-20} \times 2 \times 10^6$

$= 2 \times 10^{-14}$ W

Since mean square value = Power

$$\frac{2}{\alpha^2} = 2 \times 10^{-14} \Rightarrow \alpha = 10^7$$

13. Ans: (d)

Sol: When a 1 is transmitted:

$Y_k = a + N_k$

Threshold $Z = \frac{a}{2} = 10^{-6}$

$\Rightarrow a = 2\times10^{-6}$

For error to occur, $Y_k < 10^{-6}$

$2\times10^{-6} + N_k < 10^{-6}$

$N_k < -10^{-6}$

$$\therefore P(0/1) = \int_{-\infty}^{-10^{-6}} P(n)\,dn:$$

$$= \int_{-\infty}^{-10^{-6}} (0.5)\alpha.e^{-\alpha n}.dn, \text{with } \alpha = 10^7$$

$= 0.5 \times e^{-10}$

When a '0' is Transmitted:

$Y_k = N_k$

For error to occur, $Y_k > 10^{-6}$

$$\therefore P(1/0) = \int_{10^{-6}}^{\infty} P(n)\,dn = 0.5 \times e^{-10}$$

Since, both bits are equiprobable, the Probability of bit error

$= \frac{1}{2}$ [P(0/1) + P(1/0)]

$= 0.5 \times e^{-10}$

14. Ans: (a)

Sol: $P(0/1) = P(1/0) = p$

$\Rightarrow P(1/1) = P(0/0) = 1 - p$.

Reception with error means getting at most one 1.

$\therefore$ P(reception with error)

$= P(X = 0) + P(X = 1)$

$= 3_{C_0}(1-p)^0 p^3 + 3_{C_1}(1-p)^1 p^2$

$= p^3 + 3p^2(1-p)$

15. Ans: (d)

Sol: p = probability of a bit being in error $= 10^{-3}$

q = probability of the bit not being in error

$= 1 - p = 1 - 10^{-3}$

$= 0.999$

(1) Total number of bits = 10;

P_e = probability of error

$= 1 - P(X = 0)$

$P(X = 0)$ = Probability of no error

$\therefore P_e = 1 - [{}^{10}C_0(10^{-3})^0(1-10^{-3})^{10}] = 0.00995$

(2) Total number of bits = 100

$P_e = 1 - [{}^{100}C_0(10^{-3})^0(1-10^{-3})^{100}]$

$= 0.0952$

(3) Total number of bits = 1000

$P_e = 1 - [{}^{1000}C_0(10^{-3})^0(0.999^{1000})]$

$P_e = 0.632$

(4) If total number of bits = 10, 000

$= 1 - [({}^{10,000}C_0)(1-10^{-3})^0(0.999)^{10,000}]$

$= 0.9999$

Conclusion: As the number of bits increases, the probability of error increases and it approaches unity.

16. Ans: (a)

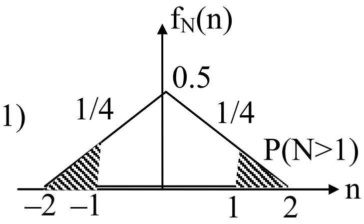
Sol: Higher modulation techniques requires more power i.e., to achieve same probability of error, bit energy has to be increased. So, power also increased.

17. Ans: (a)

Sol: Higher modulation techniques requires more power i.e., to achieve same probability of error, bit energy has to be increased. So, power also increased.

18. Ans: 0.125

Sol:



$$P(E) = P(x=-1)P\left(\frac{R}{x=-1} > 0\right) + P(x=1)P\left(\frac{R}{x=+1} < 0\right)$$

$= 0.5P(x+N>0) + 0.5\,P(x+N<0)$

$= 0.5\,P(-1+N>0) + 0.5P(1+N<0)$

$= 0.5\,P(N>1) + 0.5P(N<-1)$

$$= 0.5\left[\frac{1}{2}\frac{1}{4}(1)\right] + 0.5\left[\frac{1}{2}\frac{1}{4}\right]$$

$$= \frac{1}{8} = 0.125$$

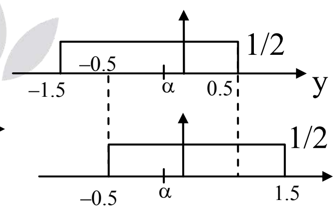
19. Ans: –0.5

Sol: $x = \{-0.5, 0.5\}$

$P(x = -0.5) = ¼,\ P(x = 0.5) = 3/4$

$f\left(\frac{y}{x=-0.5}\right) \rightarrow$

$f\left(\frac{y}{x=+0.5}\right) \rightarrow$



P_e in the overlap region $-0.5 < \alpha < 0.5$

$$P_e = \frac{1}{4}\frac{1}{2}(0.5-\alpha) + \frac{3}{4}\left(\frac{1}{2}\right)(\alpha + 0.5)$$

$$= \frac{0.5}{8} + \frac{1.5}{8} + \left(\frac{3}{8} - \frac{1}{8}\right)\alpha$$

$$= \frac{2}{8} + \frac{2}{8}\alpha$$

$\therefore P_e$ is minimum for $\alpha = -0.5$

Chapter 11 Information Theory & Coding

01. Ans: (b)

Sol: Huffman encoder is the most efficient source encoder

0.5	1	0.5	0
0.25	00	0.5	1
0.25	01		

$\overline{L} = 1\times0.5+2\times0.25+2\times0.25$

$= 1.5$ bits/symbol

Average bit rate $= 3000 \times 1.5$

$= 4500$ bps

02. Ans: (c)

Sol: Assuming all the 64 levels are equiprobable, $H = \log_2 64 = 6$ bits/pixel

Total No. of pixels $= 625 \times 400 \times 400$

$= 100$ M pixels /sec

Data rate $= 6$ bits/pixel$\times 100\times10^6$ pixel/sec

$= 600$ Mbps

03. Ans: (b)

Sol: $C = B \log (1+\frac{S}{N})$

Since $\frac{S}{N} >>1.\ 1+\frac{S}{N} \cong \frac{S}{N}$

$\therefore\ C_1 = B \log \frac{S}{N}$

$C_2 = B \log (2.\frac{S}{N})$

$= B \log 2 + B \log (\frac{S}{N}) = C_1 + B$

04. Ans: (b)

Sol: Given

B. W = 3 kHz

SNR = 10dB

$\Rightarrow 10 \log_{10} (\text{SNR}) = 10$

$\text{SNR} = 10^1 = 10$

Number of characters = 128

Channel capacity $= B \log_2 \left(1+\frac{S}{N}\right)$

$= 3 \times 10^3 \log_2(1 + 10)$

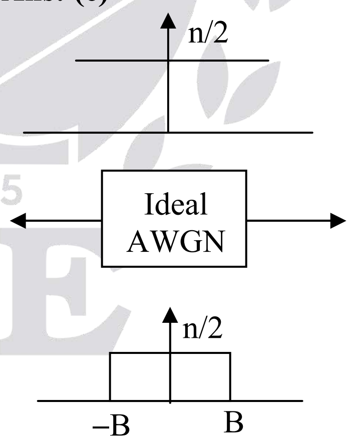
$= 10378$bps

05. Ans: (b)

Sol: Number of characteristics can be sent without any error $= \frac{c}{\log_2 M} = \frac{c}{7} = 1482$.cps

06. Ans: (c)

Sol:



$C = B\log_2(1+\frac{S}{N})$

$\lim\limits_{B\to\infty} C = \lim\limits_{B\to\infty} \frac{S}{n}\times\frac{n}{S} B \log_2\left(1+\frac{S}{nB}\right)$

$\lim\limits_{B\to\infty} C = \frac{S}{n}\log_2 e$

$(\because \lim_{n\to\infty} x\log\left(1+\frac{1}{Q}\right) = \log e)$

$$\lim_{B\to\infty} C = 1.44\frac{S}{n}$$

07. Ans: (b)

Sol: Max. entropy = $512\times512\times\log_2 8$

= 786432 bits

08. Ans: (d)

Sol: Maximum entropy of a binary source:

$$H(x)/_{max} = \log_2 M$$

$$H(x)/_{max} = \log_2 2 = 1 \text{ bit/symbol}$$

09. Ans: 0.4

Sol: $P\left(\frac{x=1}{y=0}\right) = \frac{P(x=1, y=0)}{P(y=0)}$

$$= \frac{P(x=1)P\left(\frac{y=0}{x=1}\right)}{P(x=1)P\left(\frac{y=0}{x=1}\right)+P(x=0)P\left(\frac{y=0}{x=0}\right)}$$

$$= \frac{0.8\times\frac{1}{7}}{0.8\times\frac{1}{7}+0.2\times\frac{6}{7}} = 0.4$$

Chapter 13 Optical Fiber Communication

01. Ans: (d)

Sol: NA = 0.25

$$n_2 = \frac{C}{V}$$

$$n_2 = \frac{C}{\frac{C}{\sqrt{\varepsilon_r}}}$$

$$n_2 = \sqrt{\varepsilon_r} = \sqrt{2.4375}$$

$n_2 = 1.56$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$(NA)^2 = n_1^2 - n_2^2$$

$$n_1 = \sqrt{NA^2 + n_2^2}$$

$$= \sqrt{(0.25)^2 + 2.4375}$$

$$= \sqrt{\frac{10}{4}} = \sqrt{2.5}$$

02. Ans: (d)

Sol: Number of modes $M = \frac{V^2}{2}\frac{\alpha}{\alpha+2}$

$$= \frac{1}{2}\left(\frac{2\pi a}{\lambda}(NA)^2\right)\frac{\alpha}{\alpha+2}$$

Here a = core radius

λ = wavelength

α = refractive index profile.

03. Ans: (b)

Sol: Power loss = 0.25dB/km

For 100km, the power load = 100 × 0.25

= 25 dB

The optical power at 100km

$= 10 \log 0.1\times10^{-3} - 25$

= –65 dB

In dBm

→ –65 dB + 30 = –35 dBm.

04. Ans: (c)

Sol: Numerical Aperture is used to describes light gathering (or) light collecting ability of an optical fiber.

05. Ans: (c)

Sol: The refractive index of the cladding material should be less than that of the core.

06. Ans: (d)

Sol: Fibers with higher numerical aperture exhibit greater losses and lower bandwidth.