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ESE – 2019 MAINS
OFFLINE TEST SERIES

MECHANICAL ENGINEERING

TEST – 11
SOLUTIONS

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01(a).

Sol: The wetted surface of the model is considered as an equivalent flat plate 3.0 m long and having a surface area of 3.50 m^2.

Reynolds Number,

$$Re_L = \frac{UL}{\nu} = \frac{3.0\times3.0}{1.0\times10^{-6}} = 9\times10^6$$

The boundary layer is turbulent and the drag coefficient appropriate to this $Re_L = 9\times10^6$ is

$$C_{Df} = \frac{0.074}{Re_L^{1/5}} - \frac{1700}{Re_L}$$

$$= \frac{0.074}{\left(9.00\times10^6\right)^{1/5}} - \frac{1700}{9.00\times10^6}$$

$$= 2.82\times10^{-3}$$

Drag force due to skin friction

$$F_{Df} = C_{Df}\times \text{area}\times\frac{\rho U^2}{2}$$

$$= 2.82\times10^{-3}\times3.5\times\frac{998(3)^2}{2} = 44.3 \text{ N}$$

Total measured drag

= skin friction drag + wave drag

$\therefore$ 70.0 = 44.3 + F_{DW}

Wave drag , F_{DW} = 70.0 – 44.3 = 25.7 N

01(b).

Sol: Comparison of Otto, Diesel and Dual cycles :

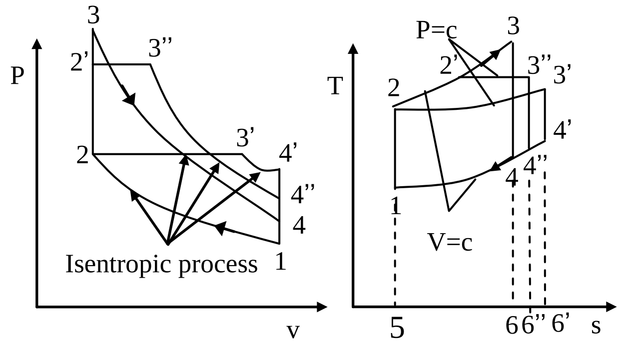
Generally Otto, Diesel and Dual cycles are compared.

For any comparison 'dual cycle' is always in between Otto and Diesel cycle.

The three cycles can be compared on the basis of either the same compression ratio or the same maximum pressure and temperature.

(i) For same compression ratio and heat addition :

The figures are shown in *P-V and T-s* diagrams for the same compression ratio and heat input.



$1\rightarrow2\rightarrow3\rightarrow4\rightarrow1$	Otto cycle
$1\rightarrow2\rightarrow3'\rightarrow4'\rightarrow1$	Diesel cycle
$1\rightarrow2\rightarrow2''\rightarrow3''\rightarrow4''\rightarrow1$	Dual cycle

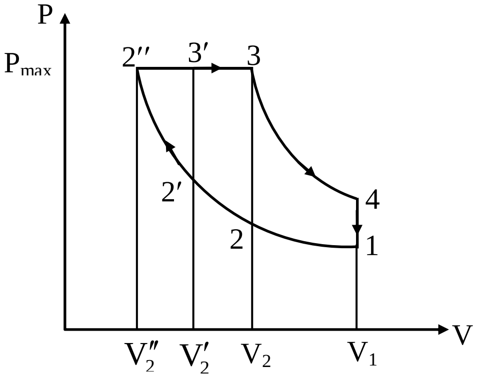
- From the *T-s* diagram, it can be seen that Area 5236 = Area 523′6′ = Area 522″3″6″ as this area represents the heat input which is the same for all cycles.
- All the cycles start from the same initial state point 1 and the air is compressed from state 1 to 2 as the compression ratio is same.
- It is seen from the *T-s* diagram for the same heat input, the heat rejection in Otto cycle (area 5146) is minimum and heat rejection in Diesel cycle (514′6′) is maximum. Consequently Otto cycle has the highest work output and Efficiency. Diesel cycle has the least efficiency and Dual cycle having the efficiency between the two.

From the above diagrams

- (Peak pressure)$_{Otto}$ > (Peak pressure)$_{Diesel}$
- (Peak temp.)$_{Otto}$ > (Peak temp)$_{Diesel}$
- (Expansion ratio)$_{Otto}$ > (Expansion ratio)$_{Diesel}$
- (Temp at beginning of heat rejection)$_{Otto}$ < (Temp at beginning of heat rejection)$_{Diesel}$
- (Heat rejected)$_{Otto}$ < (Heat rejected)$_{Diesel}$
- (Work)$_{Otto}$ > (Work)$_{Diesel}$
- $(\eta_{th})_{Otto} > (\eta_{th})_{Diesel}$

(ii) For the same peak pressure, peak temperature and heat rejection:

figure represents the p-V and T-s diagrams for the three cycles having the same peak pressure, peak temperature and heat rejection.



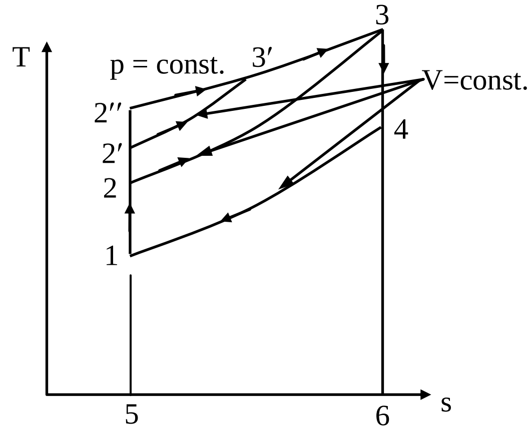


***Fig:** p-V and T-s diagram having the same peak pressure, peak temperature and heat rejection for the three cycles.*

Here,

1-2-3-4-1 represents the Otto cycle.

1-2′-3′-3-4-1 represents the Dual combustion cycle.

1-2″-3-4-1 represents the Diesel cycle.

In all the three cycles, the maximum pressure p_{max} is the same. The peak temperature is the temperature at point 3. i.e. T_3 is also the same for the three cycles. Heat rejection is during the process 4-1. This is also the same for the three cycles.

The compression ratio will now be different for the three cycles, such as:

$$r = \frac{V_1}{V_2} \text{ for Otto cycle}$$

$$r = \frac{V_1}{V_2'} \text{ for Dual combustion cycle}$$

$$r = \frac{V_1}{V_2''} \text{ for Diesel cycle}$$

$\therefore \quad r_{Diesel\ cycle} > r_{Dual\ cycle} > r_{Otto\ cycle}$

$Q_{1_{Diesel\ cycle}}$ = area 2″365 on the T-s diagram

$Q_{1_{Dual\ cycle}}$ = area 2′3′365 on the T-s diagram

$Q_{1_{Otto\ cycle}}$ = area 2365 on the T-s diagram

Now, area 2″365 > area 2′3′365 > area 2365

$$Q_{1_{Diesel\ cycle}} > Q_{1_{Dual\ cycle}} > Q_{1_{Otto\ cycle}}$$

$$\eta = 1 - \frac{Q_2}{Q_1}$$

For a given value of Q_2

$$\eta_{Diesel\ cycle} > \eta_{Dual\ cycle} > \eta_{Otto\ cycle}$$

01(c).

Sol: Given data :

D_p = 20 cm, L = 40 cm,

D_s = 10 cm, N = 90 rpm

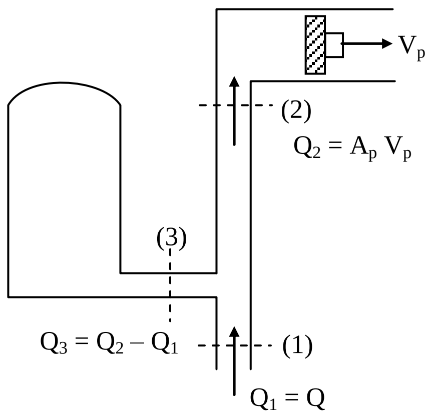
When slip is ignored theoretical discharge and actual discharge is same.

$$Q = Q_{th} = \frac{A_p L N}{60} = \frac{\pi}{4}\times D_p^2 \times L \times \frac{N}{60}$$

$$= \frac{\pi}{4}\times 0.2^2 \times 0.4 \times \frac{90}{60}$$

= 18.85 lit/s

Above discharge represents the average discharge. This discharge will be present in suction pipe upstream of air vessel. On downstream side of air vessel the discharge depends upon instantaneous piston speed. The difference between the discharges is stored or supplied by the air vessel. (Refer to the figure given below)



Q_1 = Q (average discharge)

= 18.85 lit/s

$Q_2 = A_p V_p$

$= A_p \times r \times \omega \times \sin\theta$

$$= \frac{\pi}{4}\times D_p^2 \times \left(\frac{L}{2}\right)\times\frac{2\pi N}{60}\times \sin\theta$$

$$= \frac{\pi}{4}\times 0.2^2 \times 0.2 \times \frac{2\pi \times 90}{60}\times \sin 30$$

= 29.61 lit/s

By continuity equation,

$Q_3 = Q_2 - Q_1$ = 29.61 – 18.85

= 10.76 lit/s (out of air vessel)

01(d).

Sol: Expression for energy available in the wind or power available in the wind.

The energy available in the wind can be derived by applying the concept of kinetics. The principle of wind machines is to convert kinetic energy of wind into mechanical energy. The energy available in the wind is in the form of kinetic energy. Generally, the power is expressed as energy per unit time and the kinetic energy is expressed as,

$$K.E = \frac{1}{2} mV^2 \text{ ------------(i)}$$

where, M = Mass of the particle and

V = Velocity of the particle

Consider that, the quantity of air passing in the unit time through an area 'A' with velocity 'V' is V×A. The mass of air is equivalent to its volume multiplied by its density of air 'ρ'.

∴ Mass = Volume of air × Density of air

$$m = V \times A \times \rho \text{ -------------(ii)}$$

On substituting equation (ii) in equation (i), we get,

$$K.E = \frac{1}{2}\times\rho\times A\times V\times V^2 \text{ Watts}$$

$$K.E = \frac{1}{2}\times\rho\times A\times V^3 \text{ Watts---------(iii)}$$

But, in case of horizontal axis wind turbine, area is considered as a circular section.

$\therefore$ Area of section, $A = \frac{\pi}{4}\times D^2$ ------------(iv)

where, D = Diameter

Now, substituting equation (iv) in equation (iii), we get,

$$K.E \text{ or } P = \frac{1}{2}\times\rho\times\frac{\pi}{4}\times D^2\times V^3$$

$$\therefore \quad K.E \text{ or } P = \frac{\pi}{8}\times\rho\times D^2\times V^3 \text{ Watts}$$

$\therefore$ The above equation represents the energy available in the wind or power available in the wind.

01(e).

Sol:

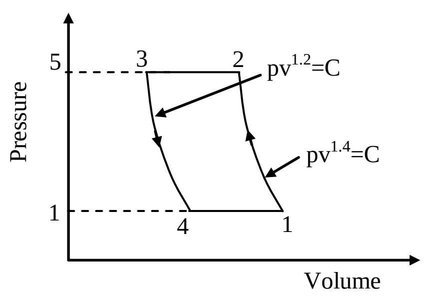
(i) Work done per kg of air

Let T_2 and T_4 = Temperatures at the end of compression and expansion respectively.

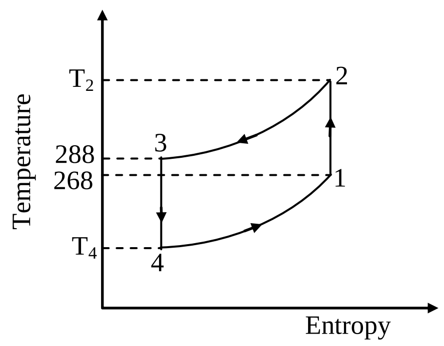
The compression process 1-2 is isentropic and follows the law $pv^{1.4}$ = constant.

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = 5^{0.286} = 1.585$$

Or, $T_2 = T_1\times1.585 = 268\times1.585 = 424.8$ K



(a) p-v diagram



(b) T-s diagram

The expansion process 3-4 follows the law $pv^{1.2}$ = constant.

$$\therefore \quad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.2-1}{1.2}} = 5^{0.167} = 1.31$$

$$\text{Or, } T_4 = \frac{T_3}{1.31} = \frac{288}{1.31} = 220 \text{ K}$$

we know that work done by the compressor during the isentropic process 1-2 per kg of air,

$$w_C = w_{1-2} = \frac{\gamma}{\gamma-1}\times R(T_2 - T_1)$$

$$= \frac{1.4}{1.4-1}\times0.287(424.8-268) = 159 \text{ kJ/kg}$$

(Taking R for air = 0.287 kJ/kg. K)

and workdone by the expander during the process 3-4 per kg of air,

$$w_E = w_{3-4} = \frac{n}{n-1}\times R(T_3 - T_4)$$

$$= \frac{1.2}{1.2-1}\times0.287(288-220) = 118.3 \text{ kJ/kg}$$

$\therefore$ Net work done per kg of air

$w = w_C - w_E$

$= 159 - 118.3 = 40.7$ kJ/kg

(ii). C.O.P of the plant

We know that heat absorbed during constant pressure process 4-1 per kg of air,

$$q_A = c_p(T_1 - T_4) = 1(268 - 220) = 48 \text{ kJ/kg}$$

$$\therefore \text{ C.O.P of the plant} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{w}$$

$$= \frac{48}{40.7} = 1.18$$

02(a).

Sol: Assumptions:

- *Steady operating conditions exit.*
- *The surfaces are opaque, diffuse, and gray.*
- *Convection heat transfer in between the furnace front plates is not considered.*
- *The ambient temperature is equal to the temperature of the surrounding surfaces,* $T_\infty = T_{surr}$

Properties:

The emissivity of all the surfaces is given as $\varepsilon = 0.1$

Analysis

The net radiation heat transfer between the parallel plates of the furnace front is

$$\dot{q}_{12} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(N+1)\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)}$$

The convection and radiation heat transfer between the outer furnace front surface and the surroundings are

$$\dot{q}_{conv} = h(T_2 - T_{surr}) \text{ and } \dot{q}_{rad} = \varepsilon\sigma\left(T_2^4 - T_{surr}^4\right)$$

Performing the energy balance on the outer surface, we have

$$\dot{q}_{12} = \dot{q}_{rad} + \dot{q}_{conv}$$

$$\frac{\sigma\left(T_1^4 - T_2^4\right)}{(N+1)\left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)} = h(T_2 - T_\infty) + \varepsilon\sigma\left(T_2^4 - T_{surr}^4\right)$$

Hence,

$$N = \left(\frac{1}{1/\varepsilon + 1/\varepsilon - 1}\right)\left[\frac{\sigma\left(T_1^4 - T_2^4\right)}{h(T_2 - T_\infty) + \varepsilon\sigma\left(T_2^4 - T_{surr}^4\right)}\right] - 1$$

$$= \left(\frac{1}{2/0.1 - 1}\right) \times \left[\frac{\left(5.67\times10^{-8}\right)\left(873^4 - 318^4\right)}{(10)(318 - 303) + (0.1)\left(5.67\times10^{-8}\right)\left(318^4 - 303^4\right)}\right] - 1$$

$$= 9.63 \approx 10$$

Thus, placing 10 radiation shields inside the furnace front will keep the outer surface temperature below 45°C to prevent thermal burn hazards.

02(b).

Sol: Velocity of steam leaving = V_1 = 900 m/sec

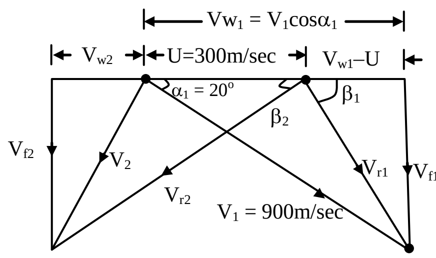
Nozzle angle = $\alpha_1 = 20^o$

Blade velocity = U = 300 m/sec

Blade velocity coefficient = $0.7 = \frac{V_{r_2}}{V_{r_1}}$

Symmetrical blading, $\beta_1 = \beta_2$

Mass flow rate = $\dot{m} = 1 \text{ kg/sec}$



Velocity of whirl at inlet = $V_{w1} = V_1 \cos\alpha_1$

$= 900 \cos 20°$

$= 845.73$ m/sec

$V_{w1} - U = 845.73 - 300 = 545.73$ m/sec

Velocity of flow at inlet = $V_{f1} = V_1 \sin\alpha_1$

$= 900 \sin 20°$

$= 307.83$ m/sec

$$\tan\beta_1 = \frac{V_{f1}}{V_{w1} - U} = \frac{307.82}{545.73}$$

$\beta_1 = 29.43^o = \beta_2$

$$\sin\beta_1 = \frac{V_{f1}}{V_{r_1}}$$

$$V_{r_1} = \frac{V_{f_1}}{\sin\beta_1} = \frac{307.82}{\sin 29.43^o} = 626.47 \text{ m/sec}$$

$$\frac{V_{r_2}}{V_{r_1}} = 0.7 \quad \text{(given)}$$

$V_{r2} = 0.7\, V_{r1} = 0.7 \times 626.47 = 438.53$ m/sec

Velocity of whirl at exit

$V_{w2} = V_{r2} \cos\beta_2 - U$

$V_{w2} = 438.53 \cos 29.43° - 300 = 81.56$ m/sec

Velocity of flow at exit = $V_{f2} = V_{r2} \sin\beta_2$

$= 438.53 \sin 29.43°$

$= 215.47$ m/sec

$$\text{Power on wheel} = \frac{\dot{m}.U\left(V_{w_1} + W_{w_2}\right)}{1000}$$

$$= \frac{1 \times 300(845.73 + 81.56)}{1000}$$

$= 278.19$ kW

$$\text{Force on wheel} = \frac{\dot{m}\left(V_{w_1} + V_{w_2}\right)}{1000}$$

$$= \frac{1(845.73 + 81.56)}{1000} = 0.9273 \text{ kN}$$

Axial thrust = $\dot{m}\left(V_{f_1} - V_{f_2}\right)$

$= 1(307.82 - 215.47)$

$= 92.35$ N

$$\text{Diagram efficiency} = \frac{\text{Power (kW)}}{\text{Energy input (kW)}}$$

$$= \frac{\text{Power (kW)}}{\frac{1}{2}\dot{m}\frac{V_1^2}{1000}(\text{kW})}$$

$$= \frac{278.19 \text{ kW}}{\frac{1}{2} \times 1 \times \frac{900^2}{1000} \text{ kW}} = 0.6868$$

02(c).

Sol: Gas required for lighting

$= 10 \times 0.227 \times 4 = 9.08$ m^3/day

Electrical energy required by six computers

$= 6 \times 250 \times 6 \times 60 \times 60 = 32.4$ MJ

Assuming the conversion efficiency of generator to be 80% and the thermal efficiency of the engine to be 25%, the thermal input to the engine to generate 54 MJ electrical energy = 32.4(0.25×0.80) = 162 MJ

Mechanical energy required for water pumping = 2×746×2×60×60 = 10.74 MJ

Assuming the thermal efficiency of engine to be 25%, the required thermal input

= 42.96 MJ

Total thermal input required by the engine

= 204.96 MJ

Assuming the heating value of biogas to be 23 MJ/m^3, the required volume of biogas for the engine = $\frac{204.96}{23}$ = 8.91 m^3/day

Therefore, total daily requirement of biogas

= 9.08 + 8.91 = 18 m^3

Let the cows required to feed the plant = n

Collectable cow dung per day = 7n kg/day

Weight of solid mass (18%) in the cow dung

$= 7n \times 0.18 = 1.26$ n kg/day

Assuming gas yield of 0.34 m^3/kg of dry mass, the gas produced per day

$= 1.26n \times 0.34 = 0.4284 \times n$ m^3/day

Therefore, $0.4284 \times n = 18$

$n = 42$

Thus, 42 cows are required to feed the plant.

Daily feeding of cow dung into the plant

$= 7 \times 42 = 294$ kg

This will be mixed with equal quantity of water to make the slurry. Thus the daily slurry produced = 294 + 294 = 588 kg

Assuming slurry density to be 1090 kg/m^3, the volume of slurry added per day

$$= \frac{588}{1090} = 0.5394 \text{ m}^3$$

For a 50 day retention time, the total volume of the slurry in the digester

$= 50 \times 0.5394 = 26.972$ m^3

As only 90% of the digester volume is occupied by the slurry, the net volume of the digester $= \dfrac{26.972}{0.9} = 29.97$ m^3

03(a),

Sol: Given data,

$H = 400$ m, $D = 20$ m,

$T_f = 8$ kN.m, $N = 350$ rpm,

$Q = 0.75$ m^3/s , $c_v = 0.98$

$180° - \beta_2 = 170°$

$\eta_m = 0.9$, $k = 0.9$

The overall efficiency is the product of all efficiencies

$\therefore \eta_o = \eta_n \times \eta_w \times \eta_m \times \eta_v$

Assuming volumetric efficiency (η_v) as 1

we get , $\eta_o = \eta_n \times \eta_w \times \eta_m$

The nozzle efficiency is given by

$\eta_n = c_v^2 = (0.98)^2 = 0.9604$

The wheel efficiency (η_w) is given by

$$\eta_w = \frac{2u(v-u)(1+k\cos\beta_2)}{V^2} \quad \text{--------(i)}$$

The jet velocity (V) is given by

$$V = C_v\sqrt{2gH}$$

$$= 0.9\sqrt{2\times9.81\times400} = 86.82 \text{ m/s}$$

The bucket velocity (u) is given by

$$u = \frac{\pi DN}{60} = \frac{\pi\times2.2\times350}{60} = 40.32 \text{ m/s}$$

Substituting in eq. (1) we get

$$\eta_w = \frac{2\times40.32(86.82-40.32)(1+0.9\cos10)}{86.82^2}$$

$= 0.938$

$\therefore \quad \eta_o = 0.9604 \times 0.938 \times 0.90 = 0.8108 = 81.08\%$

At normal running speed of 350 rpm the bucket speed is 40.32 m/s. Hence, the torque at normal speed is

$T = \rho aV (V – u) (1 + k \cos\beta_2) \times R$

$= \rho Q (V – u) (1 + k \cos\beta_2) \times R \quad [\because Q = aV]$

$= 1000 \times 0.75 (86.82 – 40.32) (1 + 0.9 \cos10) \times 1.1$

$= 72.36$ kN.m

At start, the bucket speed is zero

$T_{start} = \rho Q (V - 0)(1 + k\cos\beta_2) \times R$

$= 1000 \times 0.75 (86.82 - 0)(1 + 0.9\cos 10) \times 1.1$

$= 135.11$ kN.m

The runaway speed is the speed of runner corresponding to maximum possible bucket speed. Under no load condition, bucket can reach theoretical maximum speed equal to jet speed. This happens when frictional torque acting on the wheel is zero. At theoretical runaway speed ($N_{R,th}$) the bucket speed ($u_{R,th}$) is equal to jet velocity.

$$u_{R,th} = V$$

$$\frac{\pi D N_{R,th}}{60} = V$$

$$\frac{\pi \times 2.2 \times N_{R,th}}{60} = 86.82$$

$N_{R,th} = 753.7$ rpm

In reality actual runaway speed (N_R) will be slightly less than the theoretical runaway speed due to frictional loss. At actual runaway speed, torque developed by the runner is exactly equal to torque required to overcome frictional resistance.

$T_f = \rho Q (V - u_R)(1 + k\cos\beta_2) \times R$

$8 \times 10^3 = 1000 \times 0.75 (86.82 - u_R)(1 + 0.9\cos 10) \times 1.1$

$u_R = 81.68$ m/s

$$\frac{\pi D B N_R}{60} = 81.68$$

$$N_R = \frac{81.68 \times 60}{\pi \times 2.2} = 709 \text{ rpm}$$

03(b).

Sol: Final temperature of the body will be T_2

$$S_2 - S_1 = \int_{T_1}^{T_2} m c_v \left(\frac{dT}{T}\right) = m c_v \ell n\left(\frac{T_2}{T_1}\right)$$

[c_v = heat energy C_v]

$$\therefore (\Delta S)_{reservoir} = \frac{Q - W}{T_2}$$

$$\therefore (\Delta S)_{H.E} = 0$$

$$\therefore (\Delta S)_{univ} = (S_2 - S_1) + \frac{Q - W}{T_2} \ge 0$$

$\therefore T_2 (S_2 - S_1) + Q - W \ge 0$

$W \le Q + T_2 (S_2 - S_1)$

$W \le [Q - T_2 (S_1 - S_2)]$

$\therefore W_{max} = [Q - T_2(S_1 - S_2)]$

$$= Q + T_2\, C_v\, \ell n\left(\frac{T_2}{T_1}\right)$$

$$= C_v (T_1 - T_2) + T_2\, C_v\, \ell n\left(\frac{T_2}{T_1}\right)$$

$$= 8.4\left[373 - 303 + 303\, \ell n\left(\frac{303}{373}\right)\right]$$

$= 58.99$ kJ

03(c).

Sol:

(i) Theoretical power required:

First of all, let us find the temperature of superheated vapour at point 2(T_2).

We know that entropy at point 1,

$$s_1 = s_1' + 2.3 c_{pv} \log\left(\frac{T_1}{T_1'}\right)$$

$$= 0.7153 + 2.3 \times 0.615 \log\left(\frac{250}{245}\right) = 0.7277$$

-------- (1)

and entropy at point 2,

$$s_2 = s_2' + 2.3c_{pv}\log\left(\frac{T_2}{T_2'}\right)$$

$$= 0.6865 + 2.3\times 0.615\log\left(\frac{T_2}{299}\right)$$

$$= 0.6865 + 1.4145\log\left(\frac{T_2}{299}\right) \quad \text{--------(2)}$$

Since the entropy at point 1 is equal to entropy at point 2, therefore equating equations (1) and (2)

$$0.7277 = 0.6865 + 1.4145\log\left(\frac{T_2}{299}\right)$$

$$\log\left(\frac{T_2}{299}\right) = \frac{0.7277 - 0.6865}{1.4145} = 0.0291$$

$$\frac{T_2}{299} = 1.0693 \qquad \text{(Taking antilog of 0.0291)}$$

$\therefore$ $T_2 = 299\times1.0693 = 319.7$ K

We know that enthalpy at point 1,

$$h_1 = h_1' + c_{pv}(T_1 - T_1')$$

$$= 175.11 + 0.615(250 - 245) = 178.18 \text{ kJ/kg}$$

Enthalpy at point 2,

$$h_2 = h_2' + c_{pv}(T_2 - T_2')$$

$= 198.11+0.615(319.7 – 299)$

$= 210.84$ kJ/kg

Enthalpy of liquid refrigerant at point 3,

$$h_{f3} = h_{f3}' - c_{p\ell}(T_3' - T_3)$$

$= 60.67 – 0.963(299–295) = 64.52$ kJ/kg

We know that heat extracted or refrigerating effect per kg of the refrigerant,

$R_E = h_1 – h_{f3} = 178.18 – 64.52$

$= 113.66$ kJ/kg

Refrigerating capacity of the system,

$Q = 12$ TR $= 12\times210 = 2520$ kJ/min

$\therefore$ Mass flow of the refrigerant,

$$m_R = \frac{Q}{R_E} = \frac{2520}{113.66} = 22.17 \text{ kg/min}$$

Work done during compression of the refrigerant$= m_R(h_2 – h_1)$

$= 22.17(210.84 – 178.18)$

$= 724$ kJ/min

$\therefore$ Theoretical power required $= \frac{724}{60} = 12.07$ kW

(ii). C.O.P

we know that $\text{C.O.P} = \frac{h_1 - h_{f3}}{h_2 - h_1}$

$$= \frac{178.18 - 64.52}{210.84 - 178.18} = 3.48$$

(iii). Volumetric efficiency

Let ν_2 = specific volume at point 2, and

C = Clearance = 3% = 0.03 (Given)

First of all, let us find the specific volume at suction to the compressor, i.e. at point 1. Applying Charles law to points 1 and 1′,

$$\frac{\nu_1}{T_1} = \frac{\nu_1'}{T_1'}$$

Or, $\nu_1 = \nu_1' \times \frac{T_1}{T_1'}$

$$= 0.1475\times\frac{250}{245} = 0.1505 \text{ m}^3\text{/kg}$$

Again applying Charles' law to points 2 & 2′,

$$\frac{\nu_2}{T_2} = \frac{\nu_2'}{T_{2'}}$$

Or, $v_2 = v_{2'} \times \frac{T_2}{T_{2'}} = 0.0262 \times \frac{319.7}{299}$ = 0.028 m^3/kg

We know that volumetric efficiency,

$$\eta_v = 1 + C - C\left(\frac{v_1}{v_2}\right) = 1 + 0.03 - 0.03\left(\frac{0.1505}{0.028}\right)$$

= 0.87 or 87%

(iv). Bore and stroke of cylinder:

Let D = Bore cylinder,

L = Length of cylinder = 1.25 D and

N = Speed of compressor = 1000 r.p.m

We know that theoretical suction volume or piston displacement per minute.

$$= m_R \times v_1 \times \frac{1}{\eta_v}$$

$$= 22.17 \times 0.1505 \times \frac{1}{0.87} = 3.84 \text{ m}^3/\text{min}$$

Since the machine has six cylinders, single acting compressor, therefore, theoretical suction volume or piston displacement per cylinder per minute.

$$= \frac{3.84}{6} = 0.64 \text{ m}^3/\text{min}$$

We also know that suction volume or piston displacement per minute

= Piston area × Stroke × R.P.M.

$$= \frac{\pi}{4} \times D^2 \times L \times N = \frac{\pi}{4} \times D^2 \times 1.25D \times 1000$$

= 982 D^3 m^3/min

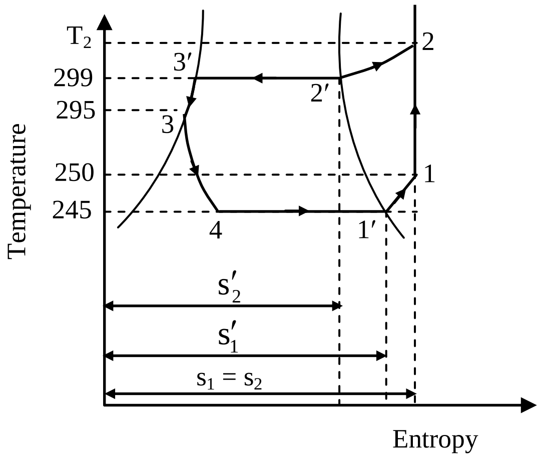
Equating equations (3) and (4)

D^3 = 0.64/982 = 0.000652

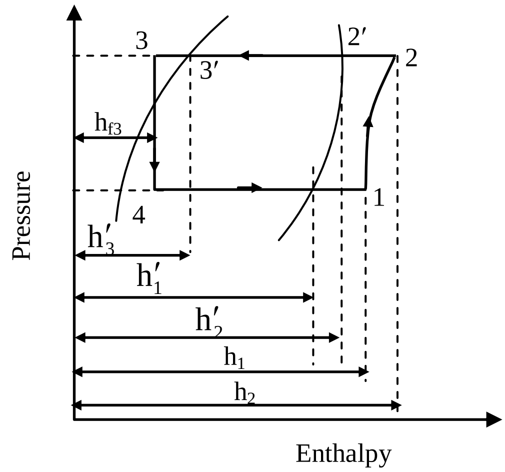
D = 0.0867 m = 86.7 mm

L = 1.25×86.7 = 108.4 mm

(v) T-s and P-h diagram



(a) T-s diagram



(b) p-h diagram

04(a).

Sol: Swept volume V_s = 2 litre = 0.002 m^3.

Volume of air induced for four-stroke engine per second $= \eta_v . V_s . \frac{N}{2 \times 60}$

where η_v is the volumetric efficiency and N is the engine rpm.

∴ Volume of air induced = $\frac{0.75 \times 0.002 \times 4500}{2 \times 60}$

= 0.05625 m^3/s

Each carburettor delivers an air flow of $\frac{0.05625}{2}$ = 0.028125 m^3/s

$\therefore\quad \dot{V}_1 = 0.028125\ m^3/s$

$$\dot{m}_a = \frac{p_1\dot{V}_1}{RT_1} = \frac{1.013\times10^5\times0.028125}{287\times288}$$

= 0.03447 kg/s

Velocity at throat,

$$C_2 = \sqrt{2c_pT_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{(\gamma-1)}{\gamma}}\right]}$$

$$\therefore\ \frac{p_2}{p_1} = \left[1-\frac{C_2^2}{2c_pT_1}\right]^{\frac{\gamma}{(\gamma-1)}}$$

$$= \left(1-\frac{100\times100}{2\times1005\times288}\right)^{1.5} = 0.9408$$

$\therefore$ p_2 = 1.013×0.9408 = 0.953 bar

Density of air,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1.013\times10^5}{287\times288} = 1.2256\ kg/m^3$$

$$\therefore\quad \rho_2 = \rho_1\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = 1.2256(0.9408)^{1/1.4}$$

= 1.1733 kg/m^3

Throat area,

$$A_2 = \frac{\dot{m}_a}{\rho_2C_2Cd_a} = \frac{0.03447}{1.1733\times100\times0.85}$$

= (3.456×10^{-4}) m^2

= 345.6 mm^2

$$\therefore\ \frac{\pi}{4}\left(D^2-d^2\right) = 345.6$$

d = 0.4D

Now,

$$\therefore\ \frac{\pi}{4}\left(D^2-0.16D^2\right) = 345.6$$

Or, $0.84\times\frac{\pi}{4}D^2 = 345.6$

Or, $D = \sqrt{\frac{345.6\times4}{\pi\times0.84}}$ = 22.89 mm

$$\dot{m}_f = Cd_fA_j\sqrt{2\rho_f(p_1-p_2-gZ\rho_f)}$$

$$\dot{m}_f = \frac{\dot{m}_a}{14} = \frac{0.03447}{14} = 0.002462\ kg/s$$

For the petrol, the pressure difference across the main jet is given by

$$p_1-p_2-gZ\rho_f = 1.013-0.953-\frac{9.81\times0.006\times750}{10^5}$$

= 1.013 – 0.953 – 0.00044

= 0.05956 bar = (0.05956×10^5) N/m^2

Area of the jet,

$$A_f = \frac{\dot{m}_f}{Cd_f\sqrt{2\rho_f(p_1-p_2-gZ\rho_f)}}$$

$$= \frac{0.002462}{0.66\sqrt{\left(2\times750\times0.05956\times10^5\right)}}$$

= 1.248 ×10^{-4} m^2 = 1.248 mm^2

$$\therefore\quad d_f = \sqrt{\left(\frac{4\times1.248}{\pi}\right)} = 1.26\ mm$$

04(b).

Sol: Given:

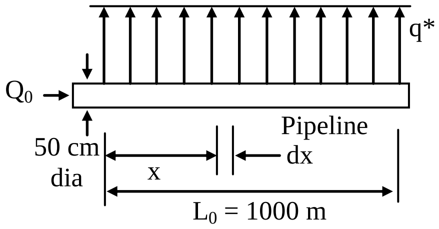
Diameter of pipeline, D = 50 cm = 0.5 m;

L = 4500 m ;

L_0 = 1000 m ;

f = 0.018.

First an expression for loss of head in a pipe having a uniform withdrawal of q* m^3/s per metre length is derived.



Consider a section at a distance x form the start of the uniform withdrawal at q^* per metre length.

Discharge, $Q_x = Q_0 - q^*x$

In a small distance dx,

$$h_f = \frac{fLV^2}{2gD}$$

$$dh_f = \frac{fV^2}{2gD} \times dx$$

$Q_x = Q_0 - q^* x$

$$V = \left(\frac{Q_0 - q^* x}{A}\right)$$

$$dh_f = \frac{f}{2g} \times \left(\frac{Q_0 - q^*x}{\frac{\pi}{4}D^2}\right)^2 \times \frac{1}{D} \times dx$$

$$= \frac{8f}{\pi^2 gD^5}\left(Q_0 - q^*x\right)^2 dx$$

$$h_f = \int_0^{L_0} dh_f = \frac{-8f}{3\pi^2 gD^5} \times \frac{1}{q^*} \times \left[\left(Q_0 - q^*x\right)^3\right]_0^{L_0}$$

$$h_f = \frac{8f}{3\pi^2 gD^5} \times \frac{1}{q^*} \times \left[Q_0^3 - \left(Q_0 - q^* L_0\right)^3\right]$$

here, $Q_0 = \dfrac{1000}{250} \times 0.075 = 0.3\,m^3/s$

$$q^* = \frac{0.075}{250} = 0.0003\,m^3/s$$

L_0 = 1000 m

H_L = Total head lost = [head lost in first (4500 – 1000) m with a discharge Q_d = 0.3 m^3/s] + [Head lost in 1000 m with a uniform withdrawal of q*] = $h_{f1} + h_{f2}$

$$h_{f1} = \frac{0.018 \times 3500 \times \left(0.3 / \left(\frac{\pi}{4} \times 0.5^2\right)\right)^2}{0.5 \times 2 \times 9.81} = 15\,m$$

$$h_{f2} = \frac{8f}{3\pi^2 gD^5} \times \frac{1}{q^*} \times \left[Q_0^3 - \left(Q_0 - q^* L_0\right)^3\right]$$

$$= \frac{8 \times 0.018}{3\pi^2 \times 9.81 \times (0.5)^5} \times \frac{1}{0.0003}\left[(0.3)^3 - (0.3 - 0.003 \times 1000)^3\right]$$

= 1.43 m

Total head loss = 15 + 1.43 = 16.43 m

Residual head at the dead end

= 145 – 16.43 = 128.57 m

04(c).

Sol: At 4000 kPa,

Saturation temperature, $T_{s,b} = 250.35^oC$

At 2.5 kPa,

Saturation temperature, $T_{s,c} = 21.08^oC$

For maximum increase in efficiency, feed water heater should be arranged in such a way that the difference in saturation temperature at each stage is constant.

$$\therefore T_{sat\ of\ FWH} = \frac{T_{sat.boiler} + T_{sat.condener}}{2}$$

$$= \frac{250.35 + 21.08}{2}$$

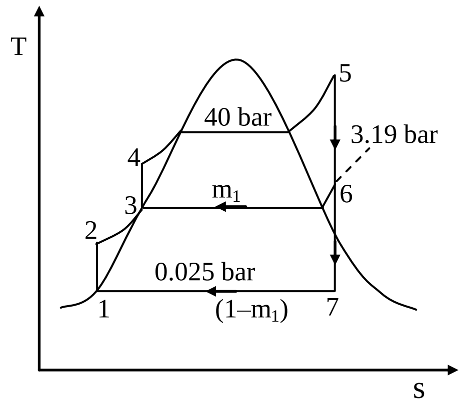
$T_{sat\ of\ FWH}$ = 135.715°C

By interpolation :

$$\frac{325 - 300}{136.27 - 133.52} = \frac{P_{FWH} - 300}{135.712 - 133.52}$$

P_{FWH} = 319.95 kPa

P_{FWH} = 3.19 bar



At State 1:

At 2.5 kPa, $h_{f1} = h_1 = 88.424$ kJ/kg

Work done by pump, $W = h_2 - h_1 = vdP$

$h_2 - h_1 = 0.001002\,(3.19 - 0.025)\times 100$

$h_2 = 88.741$ kJ/kg

At State 3 :

At 319 kPa, by interpolation

$h_{f3} = h_3 = 570.367$ kJ/kg

$v_3 = 0.001075$ m^3/kg

$h_4 - h_3 = v_3(P_4 - P_3)$

$= 0.001075 \times (40 - 3.19)\times 100$

$h_4 = 574.324$ kJ/kg

At state 5:

Superheated state :

At 40 bar, 500°C

$h_5 = 3446$ kJ/kg , $s_5 = 7.0922$ kJ/kg.K

At state 6:

$s_5 = s_6$ [$\because$ isentropic expansion]

s_g at 3.19 bar is < s_6

$\therefore$ Steam is superheated at state 6.

From superheated table :

$h_6 = 2780$ kJ/kg

At State 7:

$s_6 = 7.0922 = s_7$

$= s_f + x\, s_{fg}$

$7.0922 = 0.3118 + x\,(8.6421 - 0.3118)$

$\Rightarrow x_7 = 0.813$

$h_7 = h_{f1} + x_7 \times h_{fg}$

$= 88.424 + (0.813 \times 2450.976)$

$h_7 = 2081.06$ kJ/kg

Energy balance :

$m_1(h_6 - h_3) = (1 - m_1)\,(h_3 - h_2)$

$m_1(2780 - 570.367) = (1 - m_1)\,(570.367 - 88.741)$

$$\frac{m_1}{1-m_1} = 0.217$$

$\Rightarrow m_1(1.217) = 0.217$

Amount of steam bled in cycle,

$m_1 = 0.179$ kg/kg of steam.

$W_T = (h_5 - h_6) + (1 - m)\,(h_6 - h_7)$

$= (3446 - 2780) + (1 - 0.179)\,(2780 - 2081.06)$

$= 666 + 0.821 \times 698.94$

$= 1239.83$ kJ/kg

$W_{pump} = (1 - m)\,(h_2 - h_1) + (h_4 - h_3)$

$= 0.821 \times 0.317 + 3.957$

$= 4.217$ kJ/kg

Heat supplied $= h_5 - h_4$

$= 3446 - 574.324$

$= 2871.676$ kJ/kg

$$\eta = \frac{W_T - W_{pump}}{\text{Heat supplied}}$$

$$= \frac{1239.83 - 4.217}{2871.676}$$

$\eta = 0.4303 = 43.03$ %

05(a).

Sol:

(i) Degree of saturation:

The ratio of the actual specific humidity ω to the specific humidity of saturated air(ω_s) at same temperature and pressure is termed as the degree of saturation denoted by the symbol (μ). Thus

$$\mu = \frac{\omega}{\omega_s} = \frac{P_v}{P_s}\left[\frac{1-\frac{P_s}{P_{atm}}}{1-\frac{P_v}{P_{atm}}}\right] = \phi\left[\frac{1-\frac{P_s}{P_{atm}}}{1-\frac{P_v}{P_{atm}}}\right]$$

Note:

$\phi = 0$, $\mu = 0$

$\phi = 50\%$ $\mu < 0.5$

$\phi = 100\%$ $\mu = 1$ i.e, $\omega = \omega_{sat}$

(ii) Enthalpy of moist air:

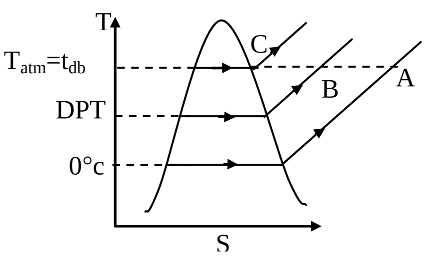
The enthalpy of moist air is the sum of the enthalpy of the dry air and the enthalpy of the water vapour.

$$h = h_a + \omega h_v \text{ kJ/kg of dry air.}$$

where h_a = enthalpy of dry air

h_v = enthalpy of vapour

Enthalpy of air calculated by taking account into different reference points.



(a) **at 0°C:** $h_v = (h_{fg})_{0^oC} + (c_p)_v(T_{atm} - 0)$

(b) **at DPT**

$$h_v = (c_p)_L(DPT-0) + (h_{fg})_{DPT} + (c_p)_v(T_{atm} - DPT)$$

Sensible heat = $(c_p)_L$ (DPT – 0)

Latent heat = $(h_{fg})_{DPT}$

Super heat = $(c_p)_v$ (T_{atm}–DPT)

(c) **at T_{atm}**

$$h_v = (c_p)_L(T_{atm} - 0) + (h_{fg})$$

Whatever may be the method out of the above 3 or any other method. Enthalpy of vapour h_v is same for all the cases .

$$\therefore (h_v)_A = (h_v)_B = (h_v)_C$$

- Only reference point is different but generally we take 0°C as the reference Point for 'h_v' measurement.

$h = h_a + \omega h_v$

$$= (c_p)_a(T_{atm} - 0) + \omega[(h_{fg})_{0^\circ c} + (c_p)_v(T_{atm} - 0)]$$

$$= [(c_p)_a + \omega(c_p)_v](T_{atm} - 0) + \omega(h_{fg})_{0^\circ c}$$

$$= (c_p)_{hs}T_{atm} + \omega[(h_{fg})_{0^\circ c} + (c_p)_v(T_{atm})]$$

$$h = 1.005(t_{db} - 0) + \omega[2500 + 1.88t_{db}]$$

where

t_{db} = dry bulb temperature

$(c_p)_{hs}$ = humid specific heat

$$(c_p)_{hs} = c_{pa} + \omega(c_p)_v$$

- Generally

  c_{pa} = 1.005 kJ/kgK , c_{pv} = 1.88 kJ/kgK

  $(c_p)_{hs}$ = 1.005 + 1.88 ω

- Since the second term 1.88 ω is very small compared to the first term 1.005, an approximated value of $(C_p)_{hs}$ is 1.0216 kJ/(kg dry air K) may be taken for all practical purposes in air-conditioning calculations.

(iii) Wet bulb depression (W.B.D):

It is the difference between dry bulb temperature and wet bulb temperature at any point. The wet bulb depression indicates relative humidity of the air.

(W.B.D) = DBT – WBT

- WBD is maximum when $\phi = 0$
- WBD = 0 when ϕ = 100% i.e. DBT = WBT

05(b).

Sol: The properties of steam at entry to the condenser

$h_1 = h_f + x \times h_{fg}$

= 191.83 + 0.8 × (2584.6 – 191.830

= 2225.8 kJ/kg

$s_1 = s_f + x\, s_{fg}$

= 0.6493 + 0.85 × (8.1501 – 0.6493)

= 7.025 kJ/kgK

The properties of condensate at exit

$h_2 = h_{f\ \text{at } 0.1\text{ bar}}$ = 191.83 kJ/kg

$s_2 = s_{f\ \text{at } 0.1\text{ bar}}$ = 0.6493 kJ/kg

Entropy change of condensate = $s_2 - s_1$

= 0.6493 – 7.025 = –6.38 kJ/kgK

Entropy change of cooling water = $\dfrac{h_1 - h_2}{27 + 273}$

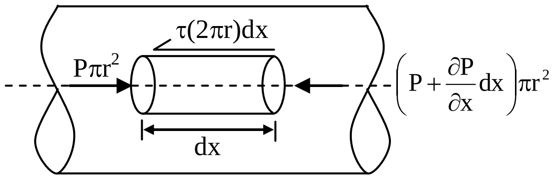
$$= \frac{2225.8 - 191.83}{300} = 6.78\,\text{kJ/kgK}$$

Net increase in entropy = 6.78 – 6.38

= 0.4 kJ/kgK

05(c).

Sol: Summation of forces along x-direction



$\Sigma F_x = 0$

$$P\pi r^2 - \left(P + \frac{\partial P}{\partial x}dx\right)\pi r^2 - 2\pi r dx\tau = 0$$

$$\frac{-\partial P}{\partial x}dx\pi r^2 = 2\pi r dx \tau$$

$$\tau = \left(\frac{-\partial P}{\partial x}\right)\frac{r}{2}$$

At the wall, $\tau = \tau_0$ at r = R

$$\tau_0 = \frac{-(P_2 - P_1)}{x_2 - x_1}\frac{D}{2} \times \frac{1}{2}$$

$$\tau_0 = \frac{D}{4L}(P_1 - P_2) \qquad \text{------ (i)}$$

We know that,

$$\frac{P_1 - P_2}{\rho g} = h_L$$

$$h_L = \frac{fLV^2}{D2g}$$

$\therefore P_1 - P_2 = \rho g h_L$

$$P_1 - P_2 = \frac{\rho g f L V^2}{2gD}$$

$$\tau_0 = \frac{D}{4L}\left(\frac{\rho f L V^2}{2D}\right) \qquad \text{------ (ii)}$$

$$\tau_0 = \frac{\rho f V^2}{8}$$

Friction factor, $f = \dfrac{64}{Re} = \dfrac{\mu 64}{\rho V D}$

$$\therefore \tau_0 = \frac{\rho V^2}{8} \times \frac{64\mu}{\rho VD}$$

$$\tau_0 = \frac{8\mu V}{D}$$

τ_0 will be maximum when V is maximum for given μ and D.

Reynolds number, $Re = \frac{\rho VD}{\mu}$

$Re_{max} = 2000$

$$2000 = \frac{\rho VD}{\mu}$$

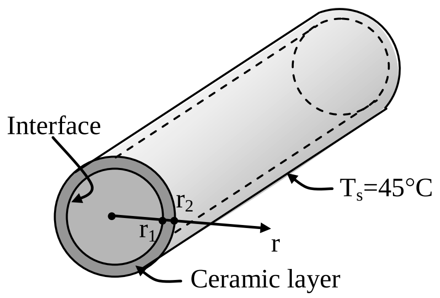
$$V = \frac{2000\mu}{\rho D}$$

$$\therefore \tau_0 = \frac{8\mu}{D} \times \frac{2000\mu}{\rho D}$$

$$\Rightarrow \tau_0 = \frac{16000\mu^2}{\rho D^2}$$

05(d).

Sol:



Assumptions:

- Heat transfer is steady since there is no change with time.
- Heat transfer is one-dimensional since this two-layer heat transfer problem possesses symmetry about the centerline and involves no change in the axial direction, and thus T = T (r).
- Thermal conductivities are constant.
- Heat generation in the wire is uniform.

Given data : $k_{wire} = 15\,W/mK$;

$K_{ceremic} = 1.2\,W/m-K$;

r_1 = 0.2 cm,

$r_2 = (r_1 + 0.5)$ cm = 0.7 cm

T_s = 45°C

Letting T_I denote the unknown interface temperature, the heat transfer problem in the wire can be formulated as:

$$\frac{1}{r}\frac{d}{dr}\left(r.\frac{dT_{wire}}{dr}\right) + \frac{\dot{e}_{gen}}{k} = 0 \quad \text{------(i)}$$

with boundary conditions as ;

$$T_{wire}(r_1) = T_I$$

$$\frac{dT_{wire}(0)}{dr} = 0$$

The solution of equation (i) is obtained as :

$$T_{wire}(r) = T_I + \frac{\dot{e}_{gen}}{4k_{wire}}\left(r_1^2 - r^2\right) \quad \text{-------(ii)}$$

Noting that the ceramic layer does not involve any heat generation and its outer surface temperature is specified, the heat conduction problem in that layer can be expressed as

$$\frac{d}{dr}\left(r\frac{dT_{ceramic}}{dr}\right) = 0 \quad \text{--------(iii)}$$

with boundary conditions as:

$$T_{ceramic}(r_1) = T_I$$

$$T_{ceramic}(r_2) = T_s = 45^{o}C$$

Its solution is determined to be

$$T_{ceramic}(r) = \frac{\ell n(r/r_1)}{\ell n(r_2/r_1)}(T_s - T_I) + T_1 \text{...........(iv)}$$

We have already utilized the first interface condition by setting the wire and ceramic layer temperatures equal to T_I at the interface $r = r_1$. The interface temperature T_I is determined from the second interface condition that the heat flux in the wire and the ceramic layer at $r = r_1$ must be the same:

$$-k_{wire}\frac{dT_{wire}(r_1)}{dr} = -k_{ceramic}\frac{dT_{ceramic}(r_1)}{dr}$$

$$\rightarrow \frac{\dot{e}_{gen}\, r_1}{2} = -k_{ceramic}\frac{T_s - T_1}{\ell n(r_2/r_1)}\left(\frac{1}{r_1}\right)$$

Solving for T_I and substituting the given values, the interface temperature is determined to be

$$T_1 = \frac{\dot{e}_{gen}\, r_1^2}{2k_{ceramic}}\ell n\left(\frac{r_2}{r_1}\right) + T_s$$

$$= \frac{(50\times10^6\,W/m^3)(0.002\,m)^2}{2(1.2\,W/m-K)}\ell n\frac{0.007\,m}{0.002\,m} + 45^\circ C = 149.4^\circ C$$

Knowing the interface temperature, the temperature at the centerline (r = 0) is obtained by substituting the known quantities into equation (i)

$$T_{wire}(0) = T_I + \frac{\dot{e}_{gen} r_1^2}{4k_{wire}}$$

$$= 149.4^\circ C + \frac{(50\times10^6\,W/m^3)(0.002\,m)^2}{4\times(15\,W/m-K)} = 152.7^\circ C$$

Thus the temperature of the centerline will be slightly above the interface temperature.

05(e).

Sol: Steam must be separated from mixture before it leaves the drum. Any moisture carried with steam to super heater tubes contains dissolved salts. In the superheater water is evaporated and salts remain deposited on the inner surface of the tube to form a scale which is difficult to remove. This scale reduces the rate of heat absorption ultimately leading to the failure of super heater tubes by over heating and rapture. The super heater tubes are exposed to highest steam pressure and temperature on inside and maximum gas temperature on the outside.

No vapour bubble should flow along with saturated water from the drum to the down comers. This will reduce the density difference and the pressure head for natural circulation. The bubbles tending to flow upward may also impedance the flow in down comer and thus affect natural circulation.

06(a).

Sol: Initial angular velocity of the flywheel

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi\times3000}{60} = 314.2 \text{ rad/s}$$

Initial available energy or exergy of the flywheel

$$= (K.E)_{initial} = \frac{1}{2}I\omega_1^2 = 0.54\times314.2^2$$

$= 2.66\times10^4$ Nm = 26.6 kJ

When this K.E is dissipated as frictional heat, if Δt is the temperature rise of the bearing, we have

Water equivalent of the bearings × Rise in temperature = 26.6 kJ

$$\therefore \quad \Delta t = \frac{26.6}{2\times 4.187} = 3.19^{\circ}C$$

∴ Final temperature of the bearings,

$t_f = 15 + 3.19 = 18.19°C$

The maximum amount of energy which may be returned to the flywheel as high grade energy is

$$A.E. = 2\times 4.187 \int_{288}^{291.19}\left(1-\frac{288}{T}\right)dT$$

$$= 2\times 4.187\left[(291.19-288)-288\ln\frac{291.19}{288}\right]$$

= 0.1459 kJ

The amount of energy rendered unavailable is

$U.E. = (A.E.)_{initial} - (A.E.)_{returnable\ as\ high\ grade\ energy}$

= 26.6 – 0.1459

= 26.4541 kJ

Since the amount of energy returnable to the flywheel is 0.146 kJ, if ω_2 is the final angular velocity and the flywheel is set in motion with this energy, then

$$0.146\times 10^3 = \frac{1}{2}\times 0.54\omega_2^2$$

$$\therefore \quad \omega_2^2 = \frac{146}{0.27} = 540.8$$

∴ ω_2 = 23.246 rad/s

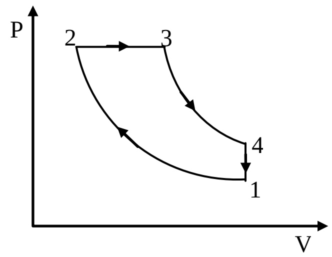
If N_2 is the final RPM of the flywheel

$$\omega_2 = 23.246 = \frac{2\pi N_2}{60}$$

$$N_2 = \frac{23.246\times 60}{2\times\pi} = 222 \text{ RPM}$$

06(b).

Sol:



$$r = 1+\frac{V_s}{V_c} = 1+8 = 9$$

Consider the process 1-2:

$$\frac{p_2}{p_1} = r^{\gamma} = 9^{1.4} = 21.67$$

$p_2 = 21.67\times 1.03\times 10^5 = 22.32\times 10^5$ N/m^2

= 22.32 bar

$$\frac{T_2}{T_1} = r^{(\gamma-1)} = 9^{0.4} = 2.408$$

$T_2 = 308 \times 2.408 = 741.6$ K = 468.6°C

Consider the process 2-3:

$p_3 = p_2 = 22.32 \times 10^5$ N/m^2 = 22.32 bar

$T_3 = 1773$ K = 1500°C

Consider the process 3-4:

$$\frac{T_3}{T_4} = r_e^{(\gamma-1)}$$

$$T_c = \frac{T_3}{T_2} = \frac{1773}{741.6} = 2.39$$

$$r_e = \frac{r}{r_c} = \frac{9}{2.391} = 3.764$$

$$\frac{T_3}{T_4} = 1.7$$

$$T_4 = \frac{T_3}{1.7} = \frac{1773}{1.7} = 1042.9 \text{ K} = 769.9°C$$

$$\frac{p_3}{p_4} = r_e^{\gamma} = 3.764^{1.4} = 6.396$$

$$p_4 = \frac{p_3}{6.396} = \frac{22.32\times10^5}{6.396}$$

$$= 3.49\times10^5 \text{ N/m}^2 = 3.49 \text{ bar}$$

$$\eta_{Cycle} = \frac{\text{Work output}}{\text{Heat added}} = 1 - \frac{\text{Heat rejected}}{\text{Heat added}}$$

$$= 1 - \frac{q_{4-1}}{q_{2-3}}$$

$q_{4-1} = c_v (T_4 - T_1)$

$= 0.717\times(1042.9 - 308) = 526.9$ kJ/kg

$q_{2-3} = c_p (T_3 - T_2)$

$= 1.004(1773 - 741.6)$

$= 1035.5$ kJ/kg

$$\eta_{Cycle} = 1 - \frac{526.9}{1035.5} = 0.4912 = 49.12\%$$

Work output $= q_{2-3} - q_{4-1}$

$= 1035.5 - 526.9$

$= 508.6$ kJ/kg

Power output = Work output × $\dot{m}_a$

$$\dot{m}_a = \frac{p_1V_1}{RT_1} \times \frac{N}{2}$$

$R = c_p - c_v = 0.287$ kJ/kg K

$$V_1 = V_s + V_c = \frac{9}{8}V_s$$

$$V_s = 6\times\frac{\pi}{4}d^2L = 6\times\frac{\pi}{4}\times10^2\times12$$

$= 5654.8$ sec $= 5.65\times10^{-3}$ m^3

$$V_1 = 5.65\times10^{-3}\times\frac{9}{8} = 6.36\times10^{-3} \text{ m}^3$$

$$\dot{m}_a = \frac{1.03\times10^5\times6.36\times10^{-3}\times30}{287\times308\times2}$$

$= 0.111$ kg/s

Power output $= 508.6 \times 0.111 = 56.45$ kW

06(c).

Sol: Given data,

$D_m = 55$ cm, $h = 9$ cm,

$N = 9000$ rpm, $\eta_T = 85\%$,

$T_{01} = 345$ K, $P_{01} = 1.7$ bar,

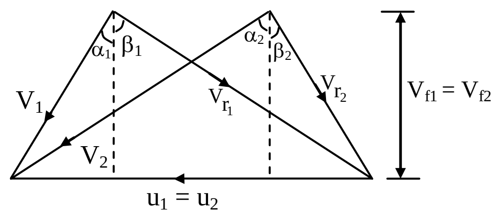
$\rho = 1.72$ kg/m^3, $\Delta\beta = 18°$

$$\dot{m} = \rho A V_f = \rho \times \pi \times D_m \times h \times V_f$$

$$V_f = \frac{45}{1.72\times\pi\times0.55\times0.09} = 168.2 \text{ m/s}$$

$$u_1 = u_2 = \frac{\pi D_m N}{60} = 259.2 \text{ m/s}$$

Velocity triangles for the axial compressor stage are shown below.



The work done per stage is given by

$W = c_p (T_{02} - T_{01})$

$= W_f\, u\, V_f (\tan\beta_1 - \tan\beta_2)$ ------(i)

where,

W_f = work done factor

V_f = velocity flow (axial velocity)

β_1 & β_2 = blade angles at inlet and exit

$$\tan\beta_1 = \frac{u_1 - V_{f1}\tan\alpha_1}{V_{f1}}$$

$$\tan\beta_1 = \frac{259.2 - 168.2\tan(28)}{168.2}$$

$\Rightarrow \beta_1 = 45.3°$

$\beta_1 - \beta_2 = 18°$

$\therefore \beta_2 = 45.3° - 18° = 27.3°$

From eq. (i)

$1.005 \times 10^3 \times (T_{02} - T_{01}) = 0.88 \times 259.2 \times 168.2 \times (\tan 45.3 - \tan 273)$

$T_{02} - T_{01} = 18.9°C$

Power $= \dot{m}W = \dot{m}c_p(T_{02} - T_{01})$

$= 45 \times 1.005 \times (18.9)$

$= 853.5$ kW

The isentropic compressor efficiency is given by

$$\eta_c = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}}$$

$T_{02'} = T_{01} + \eta_c (T_{02} - T_{01})$

$= 345 + 0.85 \times 18.9 = 361.1°C$

$$\frac{P_{02}}{P_{01}} = \left(\frac{T_{02'}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{361.1}{345}\right)^{\frac{1.4}{0.4}} = 1.173$$

07(a).

Sol:

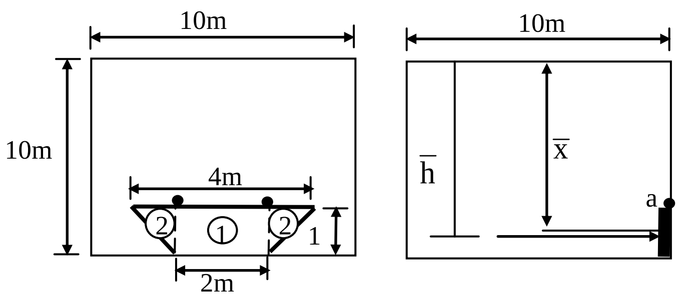


Fig: side view   Fig: front view

Let total pressure be P;

For rectangle, $A_1 = 2 \times 1 = 2\ m^2$

$\bar{x}_1 = (9 + 0.5) = 9.5$ m

$P_1 = \rho g A_1 \bar{x}_1 = 1000 \times 9.81 \times 2 \times 9.5$

$= 186390$ N

$= 186.39$ kN

Let P_1 acts at $\bar{h}_1$,

$$\bar{h}_1 = \bar{x} + \frac{I_G}{A\bar{x}} = 9.5 + \frac{\frac{2 \times 1^3}{12}}{2 \times 1 \times 9.5}$$

$\bar{h}_1 = 9.51$ m

For 2 triangles, $A_2 = \frac{1}{2} \times 2 \times 1 = 1\ m^2$

$\bar{x}_2 = 9 + \frac{1}{3} = 9.33$ m

$P_2 = \rho g A_2 \bar{x}_2 = 1000 \times 9.81 \times 1 \times 9.33$

$= 91.527$ kN

Let P_2 be acting at $\bar{h}_2$,

$$\bar{h}_2 = \bar{x}_2 + \frac{I_G}{A_2 \bar{x}_2} = 9.33 + \frac{\frac{2 \times 1^3}{36}}{1 \times 9.33}$$

$\bar{h}_2 = 9.336$ m

Total pressure $P = P_1 + P_2 = 186.39 + 91.527$

$P = 277.92$ kN

$\therefore$ Centre of pressure $\bar{h} = \frac{P_1\bar{h}_1 + P_2\bar{h}_2}{P_1 + P_2}$

$= \frac{186.39 \times 9.51 + 91.527 \times 9.336}{186.39 + 91.527}$

$= 9.45$ m

Now take the moments about the hinge (a)

$P \times 0.45 = F \times 1$

$277.92 \times 0.45 = F$

$\Rightarrow F = 125.064$ kN

where F = Force to be applied at the bottom of the gate.

If the tank is filled half only:

Let total pressure be P;

For rectangle, $A_1 = 2 \times 1 = 2m^2$

$\bar{x}_1 = (4 + 0.5) = 4.5$ m

$P_1 = \rho g A_1 \bar{x}_1 = 1000 \times 9.81 \times 2 \times 4.5$

= 88.29 kN

Let P_1 acts at $\bar{h}_1$,

$$\bar{h}_1 = \bar{x} + \frac{I_G}{A\bar{x}} = 4.5 + \frac{\frac{2\times 1^3}{12}}{2\times 1\times 4.5}$$

$$\bar{h}_1 = 4.518 \text{ m}$$

For 2 triangles, $A_2 = \frac{1}{2}\times 2\times 1 = 1 \text{ m}^2$

$$\bar{x}_2 = 4 + \frac{1}{3} = 4.33 \text{ m}$$

$P_2 = \rho g A_2 \bar{x}_2 = 1000\times 9.81\times 1\times 4.33$

= 42.51 kN

Let P_2 be acting at $\bar{h}_2$,

$$\bar{h}_2 = \bar{x}_2 + \frac{I_G}{A_2\bar{x}_2} = 4.33 + \frac{\frac{2\times 1^3}{36}}{1\times 4.33}$$

$\bar{h}_2$ = 4.34 m

Total pressure P = $P_1 + P_2$ = 88.29 + 42.51

P = 130.8 kN

∴ Centre of pressure $\bar{h} = \frac{P_1\bar{h}_1 + P_2\bar{h}_2}{P_1 + P_2}$

$$= \frac{88.2\times 4.518 + 42.51\times 4.34}{130.8}$$

= 4.46 m

Now take the moments about the hinge (a)

$P\times 0.46 = F\times 1$

$130.8\times 0.46 = F$

⇒ F = 60.168 kN

where F = Force to be applied at the bottom of the gate.

Hence, if the water is filled half of the tank, force applied at the bottom of the tank is reduced nearly to half.

07(b).

Sol: Given data:

I_{d4} = 27°C, t_{w4} = 21°C,

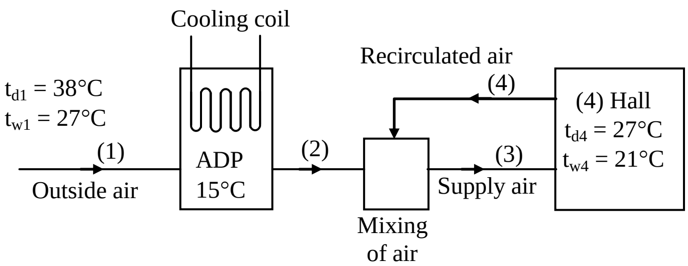
Q_{s4} = 46.5 kW, Q_{L4} = 17.5 kW

t_{d1} = 38°C, t_{w1} = 27°C,

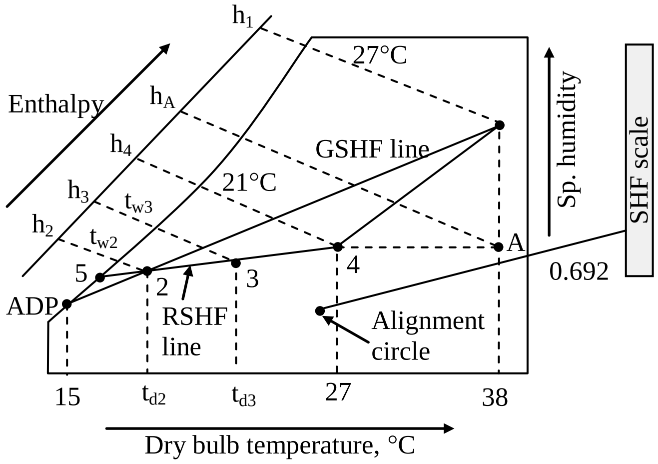
ν_1 = 25 m^3/min, ADP = 15°C

The line diagram for the processes involved in the air conditioning of a hall is shown in figure.

These processes are shown on the psychrometric chart as discussed below:



First of all, mark the condition of outside air i.e. at 38°C dry bulb temperature and 27°C wet bulb temperature on the psychrometric chart as point 1, as shown in figure. Now mark the condition of air in the hall, i.e. at 27°C dry bulb temperature and 21°C wet bulb temperature, at point4. Mark point A by drawing vertical and horizontal lines from points 1 and 4 respectively. Since 25 m^3/min of outside air at t_{d1} = 38°C and t_{w1} = 27°C is supplied directly into the room through ventilation and infiltration, therefore the sensible heat and latent heat of 25 m^3/min infiltrated air are added to the hall in addition to the sensible heat load of 46.5 kW and latent heat load of 17.5 kW.



From the psychrometric chart, we find that enthalpy of air at point 1,

h_1 = 85 kJ/kg of dry air

Enthalpy of air at point 4,

h_4 = 61 kJ/kg of dry air

Enthalpy of air at point A,

h_A = 72.8 kJ/kg of dry air

Also specific volume of air at point 1,

v_{s1} = 0.907 m^3/kg of dry air

∴ mass of air infiltrated into the hall,

$$m_a = \frac{\nu}{\nu_{s1}} = \frac{25}{0.907} = 27.56\,\text{kg/min}$$

Sensible heat load due to the infiltrated air,

$Q_{s1} = m_a(h_A - h_4)$

$= 27.56(72.8 - 61) = 325.21$ kJ/min

$= \frac{325.21}{60} = 5.42$ kW

and latent heat load due to the infiltrated air,

$Q_{L1} = m_a(h_1 - h_A)$

$= 27.56(85 - 72.8) = 336.23$ kJ/min

∴ Total room sensible heat load,

RSH = $Q_{s4} + Q_{L1}$ = 46.5 + 5.42 = 51.92 kW

and total room latent heat load

RLH = $Q_{L4} + Q_{L1}$ = 17.5 + 5.6 = 23.1 kW

We know that room sensible heat factor,

$$\text{RSHF} = \frac{\text{RSH}}{\text{RSH} + \text{RLH}} = \frac{51.92}{51.92 + 23.1} = 0.692$$

Now mark this calculated value of RSHF on the sensible heat factor scale and join with the alignment circle (i.e. 26°C DBT and 50% RH) as shown in figure. From point 4, draw a line 4-5 (known as RSHF line) parallel to this line. Since the outside air marked at point 1 is passed through the cooling coil whose ADP = 15°C, therefore join point 1 with ADP = 15°C on the saturation curve. This line is the GSHF line and intersects the RSHF line at point 2, which represents the condition of air leaving the cooling coil. Also 60% of the air from the hall is recalculated and mixed with the conditioned air after the cooling coil. The mixing condition of air is shown at point 3 such that

$$\frac{\text{Length } 2-3}{\text{Length } 2-4} = 0.6$$

(i). Condition of air after the coil and before the recirculated air mixes with it

The condition of air after the coil and before the recirculated air mixes with it is shown by point 2 on the psychrometric chart, as shown in figure. At point 2, we find that

Dry bulb temperature, t_{d2} = 19°C

Wet bulb temperature, t_{w2} = 17.5°C

(ii). Condition of air entering the hall, i.e. after mixing with recirculated air

The condition of air entering the hall, i.e. after mixing with recirculated air, is shown by point 3 on the psychrometric chart, as shown in figure. At point 3, we find that

Dry bulb temperature, $t_{d3} = 24°C$

Wet bulb temperature, $t_{w3} = 19.8°C$

(iii). Mass of fresh air entering the cooler

The mass of fresh air passing through the cooling coil to take up the sensible and latent heat of the hall is given by

$$m_F = \frac{\text{Total heat removed}}{h_4 - h_2} = \frac{RSH + RLH}{h_4 - h_2}$$

$$= \frac{51.92 + 23.1}{61 - 49}$$

$= 6.25$ kg/s $= 6.25\times60 = 375$ kg/min

(From psychrometric chart, $h_2 = 49$ kJ/kg of dry air)

(iv). By pass factor of the cooling coil

We know that by-pass factor of the cooling coil,

$$BPF = \frac{t_{d2} - ADP}{t_{d1} - ADP} = \frac{19 - 15}{38 - 15} = 0.174$$

(v). Refrigerating load on the cooling coil:

We know that the refrigerating load on the cooling coil

$= m_F(h_1 - h_2)$

$= 375(85 - 49) = 13500$ kJ/min

$$= \frac{13500}{210} = 64.3 \text{ TR}$$

07(c).

Sol: Given data,

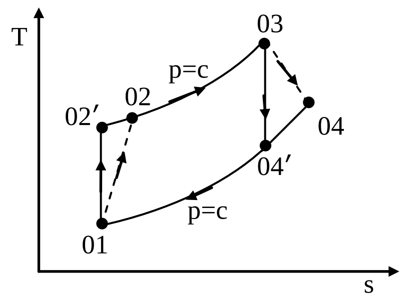
$r_p = 11.3$,

$T_{01} = 300$ K,

$W_{ent} = 0$

$Q_s = 476.4$ kJ/kg

The practical gas turbine cycle by considering turbine and compressor efficiencies is shown below on T-s diagram.



$$\frac{T_{02'}}{T_{01}} = \frac{T_{03}}{T_{04'}} = r_p^{\frac{\gamma-1}{\gamma}}$$

$= 11.3^{0.286} = 2$ ----- (1)

$\therefore\ T_{02'} = 2\times T_{01} = 2\times300 = 600$ K

$W_{net} = 0$ i.e., $W_C = W_T$

$c_p (T_{02} - T_{01}) = c_p (T_{03} - T_{04})$

Rearranging we get

$c_p (T_{03} - T_{02}) = c_p (T_{04} - T_{01})$ ------(2)

but $Q_s = c_p (T_{03} - T_{02})$ ------ (3)

From eq. (2) & (3)

$\therefore\ Q_s = c_p (T_{04} - T_{01})$

$476.4 = 1.005 (T_{04} - 300)$

$T_{04} = 774$ K

Isentropic efficiency of the turbine is given by

$$\eta_T = \frac{1 - T_{04}/T_{03}}{1 - T_{04'}/T_{03}} = \frac{1 - T_{04}/T_{03}}{\left[1 - \left(\frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}\right)\right]}$$

i.e., $0.71 = \dfrac{1-\dfrac{T_{04}}{774}}{1-\dfrac{1}{2}}$

$$\frac{T_{03}}{774} = 1 - \frac{0.71}{2}$$

$T_{03} = 1200$ K

From equation (1).

$T_{02} - T_{01} = T_{03} - T_{04}$

$T_{02} - 300 = 1200 - 774$

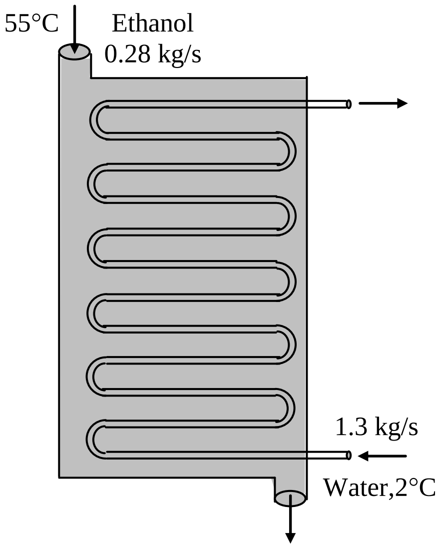
$T_{02} = 726$ K

The isentropic compression efficiency is given by

$$\eta_c = \frac{T_{02'} - T_{01}}{T_{02} - T_{01}} = \frac{600-300}{726-300} = 0.704$$

08(a).

Sol:



Assumptions:

- Steady operating conditions exist.
- The heat exchanger is well insulated so that heat loss to the surroundings is negligible.
- Changes in the kinetic and potential energies of fluid streams are negligible.
- Fluid properties are constant.
- Thermal resistance of tube wall is negligible.

The specific heat of ethanol is given to be $c_{ph} = 2630\,\text{J/kg.K}$.

The specific heat of water at 5°C is $c_{pc} = 4205\,\text{J/kg} - \text{K}$.

In the effectiveness - NTU method, we first determine the heat capacity rates of the hot and cold fluids and identify the smaller one:

$$C_h = \dot{m}_h c_{ph} = (0.28\,\text{kg/s})(2630\,\text{J/kg} - \text{K})$$
$$= 736.4\,\text{W/K}$$

$$C_c = \dot{m}_c c_{pc} = (1.3\,\text{kg/s})(4205\,\text{J/kg} - \text{K})$$
$$= 5466.5\,\text{W/K}$$

Therefore,

$$C_{min} = C_h = 736.4\,\text{W/K}$$

And

$$c = C_{min}/C_{max} = 736.4/5466.5 = 0.1347$$

Then the maximum heat transfer rate is determined from

$$\dot{Q}_{max} = C_{min}(T_{h.in} - T_{c,in})$$
$$= (736.4\,\text{W/K})(55-2)\text{K} = 39{,}029\,\text{W}$$

The actual rate of heat transfer is

$$\dot{Q} = C_h(T_{h,in} - T_{h,out})$$
$$= (736.4\,\text{W/K})(55-15)\text{K} = 29{,}456\,\text{W}$$

Thus, the effectiveness of the heat exchanger is

$$\varepsilon_1 = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{29{,}456}{39{,}029} = 0.7547$$

For this one - shell pass heat exchanger, knowing the effectiveness (ε), the NTU of this heat exchanger can be determined from the given appropriate relation.

$$NTU_1 = -\frac{1}{\sqrt{1+c^2}}\ell n\left(\frac{2/\varepsilon_1 - 1 - c - \sqrt{1+c^2}}{2/\varepsilon_1 - 1 - c + \sqrt{1+c^2}}\right)$$

$$= -\frac{1}{\sqrt{1+0.1347^2}}\ell n\left(\frac{2/0.7547 - 1 - 0.1347 - \sqrt{1+0.1347^2}}{2/0.7547 - 1 - 0.1347 + \sqrt{1+0.1347^2}}\right)$$

= 1.592

Note that for this one-shell pass heat exchanger $\varepsilon_1 = \varepsilon$ and $NTU_1 = NTU$.

Then the number of tube passes can be determined using

$$NTU = \frac{UA_s}{C_{min}} = \frac{U(\pi DL)n}{C_{min}} = 1.592$$

Thus,

$$n = \frac{C_{min}NTU}{U\pi DL} = \frac{(736.4\,W/K)(1.592)}{(700\,W/m^2.K)\pi(0.015\,m)(3m)}$$
$$= 11.85 \approx 12 - \text{tube passes}$$

08(b).

Sol: Let at any time 't' the pressure of gas in the vessel be 'p' and mass be 'm'

Then $PV_o = mRT$ -------(i)

Let after a time interval of 'dt' the pressure of the gas in the vessel be 'P – dP' and mass be ' m – dm'.

Applying equation of state to the gas inside the cylinder:

$(P - dP)\,V_o = (m - dm)\,RT$

$PV_o - dP \times V_o = mRT - dmRT$

$\Rightarrow dP \times V_o = dmRT$ ----(ii) $[\because PV_o = mRT]$

Applying equation of state to the gas taken out in the time interval 'dt'

$\Rightarrow (P - dP)\,rdt = dmRT$ ------(iii)

[∵ In time interval of 'dt' volume taken out is $r \times dt$ $\left(\because r = \frac{dV}{dt}\right)$]

Equation (iii) can be written as

$Pr\,dt = dmRT$ --------(iv)

[$\because r \times dP \times dt$ is very small]

Equating (ii) and (iv)

$$dP \times V_o = P\,r\,dt$$

$$\frac{dP}{P} = -\frac{r}{V_o}dt$$

(negative sign indicates that pressure decreases)

Integrating we get

$$[\ell n\,P]_{P_o}^{P} = -\frac{r}{V_o}[t]_o^t$$

$$\ell n\left(\frac{P}{P_o}\right) = -\frac{r}{V_o}(t)$$

$$P = P_o\,e^{-\frac{r}{V_o}(t)}$$

(ii) If half the gas is pumped out then the pressure inside the vessel becomes $\frac{P_o}{2}$

Now, $P = P_o\,e^{-\frac{r}{V_o}(t)}$

If $P = \frac{P_o}{2}$ then

$$\frac{P_o}{2} = P_o\,e^{-\frac{r}{V_o}(t)}$$

$$e^{-\frac{r}{V_o}(t)} = \frac{1}{2}$$

$$e^{\frac{r}{V_o}(t)} = 2$$

$$\frac{r\,t}{V_o} = \ell n(2)$$

$$\Rightarrow\ t = \frac{V_o}{r}\ell n(2)$$

08(c)(i).

Sol: *The main difficulties in the development of tidal energy are as follows.*

- Fluctuation in the output power due to variations in the tidal ranges is its major drawback.
- The range of tidal energy is restricted to a small value.
- The equipments used for tidal energy may get corroded due to high corrosiveness of sea water.
- The turbines have to work on a wide range of head vibrations which reduces the efficiency of the plant.
- The power cannot be produced continuously due to difference in the level of sea inland basin.
- The planning of daily load sharing in a grid is difficult as the power cycle occurrence is not constant.
- The installation of the plant is quite difficult in sea or in estuary.
- It obstructs the natural habitat of fishing.
- It also effects the navigation.
- It is comparatively not as much economical as all sources of energy.

08(c)(ii).

Sol: Given data:

Total number of power generators = 24

Maximum head (H′) = 13.5 M

Total time (t) = 2×6 hours per day

= 12 hours = 12×3600 sec

Efficiency (η) = 93% = 0.93

Total power of 24 generators

= 24 × 10 = 240 MW

$P = 2.4 \times 10^8$ W ($\because$ 1 MW = 10^6 W)

Average discharge,

$$Q = \frac{V}{t}$$

$$Q = \frac{V}{12\times 3600}$$

Power at any instant, $P = \rho g Q H' \times \eta$

$$2.4\times 10^8 = \frac{v\times 1025\times 9.81\times 13.5}{12\times 3600}\times 0.93$$

$v = 8.213\times 10^7\ m^3$

Basin capacity = $8.213\times 10^7\ m^3$

Annual energy production

= P × t × η × No. of days in a year

= $2.4 \times 10^8 \times 12 \times 3600 \times 365 \times 0.93$

= 3.52×10^{15} J