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ESE – 2019 MAINS
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MECHANICAL ENGINEERING

TEST – 14
SOLUTIONS

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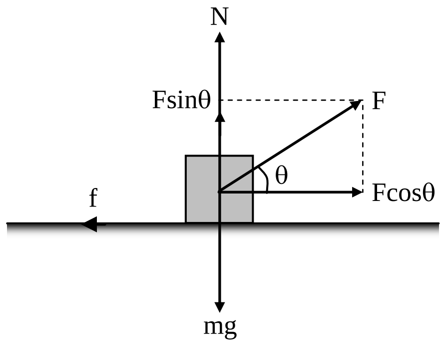
01(a).

Sol: Let the force F be applied at an angle θ to the horizontal plane. The forces acting on the block of mass m are its weight mg, normal reaction N, frictional force f and the applied force F.



Resolve $\vec{F}$ in the horizontal and the vertical directions. The net force in the vertical direction is zero (because there is no acceleration in the vertical direction) i.e.,

$$N + F\sin\theta = mg$$

Resolve $\vec{F}$ in the horizontal and the vertical directions. The net force in the vertical direction is zero (because there is no acceleration in the vertical direction) i.e.,

$$N + F\sin\theta = mg \qquad \text{---- (1)}$$

The block will just start moving when the horizontal component of the applied force is equal to the limiting value of the frictional force i.e.,

$$F\cos\theta = f_{max} = \mu N \qquad \text{---- (2)}$$

Eliminate N from equations (1) and (2) and simplify to get

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta} \qquad \text{---- (3)}$$

The applied force is minimum when $\frac{dF}{d\theta} = 0$

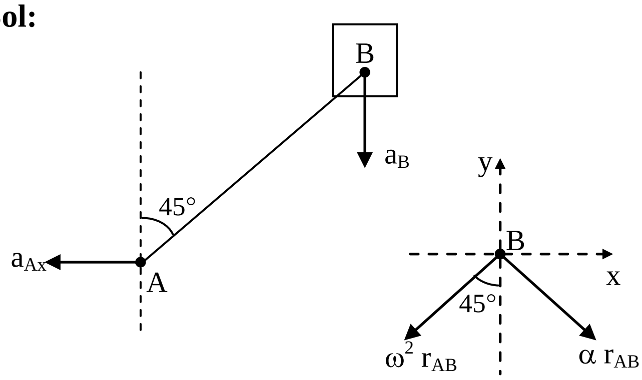
i.e., $\frac{dF}{d\theta} = -\frac{\mu mg}{(\cos\theta + \mu\sin\theta)}(-\sin\theta + \mu\cos\theta) = 0$

which gives $\theta = \tan^{-1}\mu$. Substitute θ in equation (3)

$$F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

01(b)(i).

Sol:



As the block sides in the guide acceleration of B along x-axis is zero.

$$\vec{a}_B = \vec{a}_{BA} + \vec{a}_A$$

where, $\vec{a}_{BA}$ = acceleration of B w.r.t A

$\vec{a}_A$ = acceleration of A

$\vec{a}_B$ = acceleration of B

Therefore, writing acceleration along x-axis.

$$a_{Bx} = a_{BAx} + a_{Ax}$$

$$a_{BX} = -\omega^2 r_{AB}\sin 45^\circ + \alpha\, r_{AB}\cos 45^\circ - 2 = 0$$

$$-(1)^2 \times (1) \times \frac{1}{\sqrt{2}} + \frac{\alpha(1)}{\sqrt{2}} - 2 = 0$$

$$-\frac{1}{\sqrt{2}} + \frac{\alpha}{\sqrt{2}} - 2 = 0$$

$$\Rightarrow \alpha = 2\sqrt{2} + 1 = 3.828 \text{ rad/s}^2$$

01(b)(ii).

Sol: $V_c = 100$ km/hr, $k_G = 300$ mm,

$R = 80$ m, $r = 400$ mm

$M = 16$ kg, $d = 1.3$ m

$I = 2\,M\,k_G^2 = 2\times16\times0.3^2 = 2.88$ kg.m^2

$$\omega_s = \frac{V_c}{r} = \frac{100\times\frac{5}{18}}{0.4} = 69.44 \text{ rad/s}$$

$$\omega_P = \frac{V_c}{R} = \frac{100\times\frac{5}{18}}{80} = 0.347 \text{ rad/s}$$

$G = I\,\omega_S\,\omega_P = 2.88\times69.44\times0.347 = 69.4$ Nm

$$\Delta F = \frac{G}{d} = \frac{69.4}{1.3}$$

$\Delta F = 53.4$ N

01(c).

Sol: The critical stresses occur on the surface of the torsion bar.

We have the torque,

$T = PL = 2600\,(0.36) = 936$ N.m

Moment, $M = Pa = 2600(0.12) = 312$ N.m act uniformly along this member.

Thus, maximum shear and bending stresses:

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16(936)}{\pi(0.034)^3} = 121.3\,\text{MPa}$$

$$\sigma_x = \frac{32\,M}{\pi d^3} = \frac{32(312)}{\pi(0.034)^3} = 80.86\,\text{MPa}$$

By maximum shear stress theory,

$$\left[(80.86)^2 + 4(121.3)^2\right]^{1/2} = \frac{580}{n}$$

$\Rightarrow$ $n = 2.27$

Maximum energy of distortion theory,

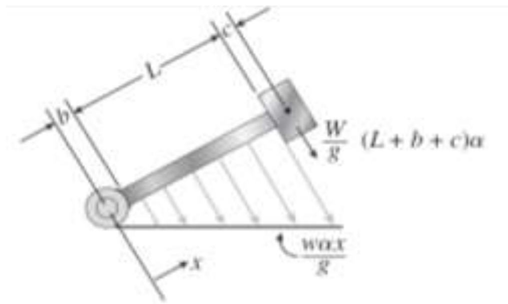
$$\left[(80.86)^2 + 3(121.3)^2\right]^{1/2} = \frac{580}{n}$$

Solving, we obtain $n = 2.58$

Comments: In as much as the maximum distortion energy criterion is more accurate, it makes sense for a higher factor of safety to be obtained by this theory.

01(d).

Sol: Rotating Centrifuge :



Tangential acceleration = $r\alpha$

Inertial force, $Mr\alpha = \dfrac{W}{g} r\alpha$

Maximum V and M occur at x = b

$$V_{max} = \frac{W}{g}(L+b+c)\alpha + \int_b^{L+b} \frac{w\alpha}{g} x\,dx$$

$$= \frac{W\alpha}{g}(L+b+c) + \frac{wL\alpha}{2g}(L+2b)$$

$$M_{max} = \frac{W\alpha}{g}(L+b+c)(L+c) + \int_b^{L+b} \frac{w\alpha}{g} x(x-b)dx$$

$$= \frac{W\alpha}{g}(L+b+c)(L+c) + \frac{wL^2\alpha}{6g}(2L+3b)$$

Substitute numerical data:

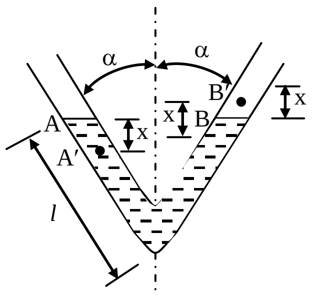
$W = 2.0\,wL$, $b = \dfrac{L}{9}$, $c = \dfrac{L}{10}$

$$V_{max} = \frac{91\,wL^2\,\alpha}{30g}$$

$$M_{max} = \frac{229\,wL^3\,\alpha}{75\,g}$$

01(e).

Sol: Let us the energy method to determine the equation of motion. If the free surface A′ is at a depth x below the original position A, then the PE can be evaluated as follows. We can imagine that the only change from the equilibrium configuration is that the column AA′ shits to BB′ when the CG of the former column goes up by x.



Cross-sectional area of the tube = A

$$V = \rho A \frac{x}{\cos\theta} gx = \rho A g \frac{x^2}{\cos\alpha} \quad \text{----(i)}$$

Where $\frac{\rho A x}{\cos\alpha}$ is the mass of AA′. Clearly, the magnitude of velocity everywhere in the liquid is $\frac{\dot{x}}{\cos\alpha}$.

Therefore, the KE of the system is

$$T = \frac{1}{2}\rho A\, 2\ell\left(\frac{\dot{x}}{\cos\alpha}\right)^2 = \rho A\ell\left(\frac{\dot{x}^2}{\cos^2\alpha}\right) \quad \text{----(ii)}$$

$$\frac{d}{dt}\left[\frac{\rho A\ell}{\cos^2 a}\times\dot{x}^2 + \frac{\rho A g}{\cos\alpha}\times x^2\right] = 0$$

$$\left[\frac{\rho A\ell}{\cos^2 a}\times\ddot{x} + \frac{\rho A g}{\cos\alpha}\times x^2\right] = 0$$

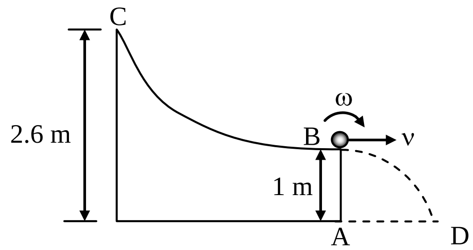
$$\ddot{x} + \frac{g\cos\alpha}{\ell}x = 0$$

So, the natural frequency of oscillations is

$$\omega_n = \left(\frac{g\cos\alpha}{\ell}\right)^{1/2}$$

02(a).

Sol: The sphere rolls without slipping from the point C to the point B.



The mechanical energy is conserved in rolling without slipping from C to B. At B, let ν be the velocity of the centre of mass and ω be the angular velocity of the sphere. The conservation of energy gives

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mg(h_C - h_B) \quad \text{-------(1)}$$

where $I = \frac{2}{5}mr^2$ is the moment of inertia of the sphere of radius r about its axis of rotation, h_C = 2.6 m is the height of C and h_B = 1 m is the height of B. In rolling without slipping, ν = ωT.

Substitute these values in equation (1) to get

$$v = \sqrt{\frac{g(h_C - h_B)10}{7}} = \sqrt{\frac{9.8(2.6-1.0)10}{7}}$$

$$= 4.73 \text{ m/s}$$

From B to D, the sphere will have projectile motion as if projected horizontally with a speed ν = 4.73 m/s.

The time taken by the sphere to travel a vertical distance h = 1 m is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 1}{9.8}} = 0.45 \text{ s}$$

The horizontal distance travelled by the projectile in time t is

AD = vt = 4.73 × 0.45 = 2.13 m

02(b).

Sol:

(i) Max. bending stress due to uniform load q

$$M_{max} = \frac{qL^2}{8} \qquad S = \frac{I}{\frac{h}{2}}$$

$$S = \frac{\frac{bh^3}{12}}{\frac{h}{2}} \qquad S = \frac{1}{6}bh^2$$

$$\sigma_{max} = \frac{M_{max}}{S} \qquad \sigma_{max} = \frac{\frac{qL^2}{8}}{\left(\frac{1}{6}bh^2\right)}$$

$$\sigma_{max} = \frac{3}{4}q\frac{L^2}{bh^2}$$

q = 5.8 kN/m, L = 4 m, b = 140 mm

h = 240 mm

$$M_{max} = \frac{qL^2}{8}$$

M_{max} = 11.6 kN.m

σ_{max} = 8.63 MPa

(ii) Maximum bending stress due to trapezoidal load q

$$R_A = \left[\frac{1}{2}\left(\frac{q}{2}\right)L + \frac{1}{3}\left(\frac{q}{2}\frac{1}{2}\right)L\right]$$

uniform load (q/2) and triangle load (q/2)

$$R_A = \frac{1}{3}qL$$

find x = location of zero shear

$$R_A - \frac{q}{2}x - \frac{1}{2}\left(\frac{x}{L}\frac{q}{2}\right)x = 0$$

$3x^2 + 6Lx - 4L^2 = 0$

$$x = \frac{-6L - \sqrt{(84L^2)}}{2\times 3}$$

$$\frac{x}{L} = \left(-1 + \frac{1}{6}\sqrt{84}\right)$$

X_{max} = 0.52753 L

$$M_{max} = R_A x_{max} - \frac{q}{2}\frac{x_{max}^2}{2} - \frac{1}{2}\left(\frac{x_{max}}{L}\frac{q}{2}\right)\frac{x_{max}^2}{3}$$

$M_{max} = 9.40376 \times 10^{-2}\ qL^2$

M_{max} = 8.727 kN.m

$$\sigma_{max} = \frac{M_{max}}{S}$$

$$\sigma_{max} = 6.493\times 10^3 \frac{N}{m^2}$$

σ_{max} = 6.49 MPa

02(c).

Sol:

(I). The three phases are:

(1) Crack initiation

(2) Crack propagation

(3) Final Fracture

The crack initiation phase may be modeled using the "local stress-strain" approach.

The crack propagation phase may be modeled using a fracture mechanics approach in which the crack propagation rate is empirically expressed as a function of the stress intensity factor range.

The final fracture phase may be modeled by using linear elastic fracture mechanics (LEFM) to establish the critical size that a growing crack should reach before propagating spontaneously to failure.

(II). *STATEMENTS OF ALL THEORIES OF FAILURES :*

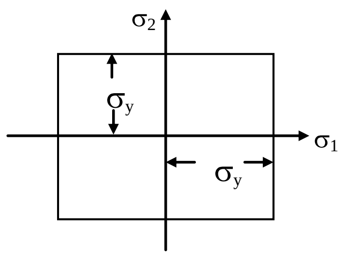
Maximum principal stress theory (Rankine's theory)

The theory state that the failure of the mechanical component subject to bi-axial or tri-axial stresses occurs when the maximum principal stress reaches the yield strength of the material.

$$|\sigma_1| = \sigma_y$$

Where, σ_y = tensile stress at yield point

σ_1 = maximum tensile stress



Rankine yield surface for 2D stress

For the biaxial stresses a yield surface is shown and yield occurs when the state of stress is at the boundary of the ***rectangle*** which is a ***yield locus.***

Maximum principal strain theory (St. Venant's theory)

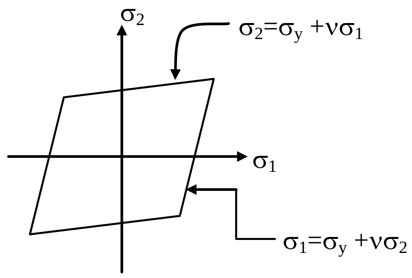
Yielding will occur when the maximum principal strain just exceeds the strain at the tensile yield point in either simple tension or compression.

$$\varepsilon_1 = \frac{\sigma_1 - \nu\sigma_2}{E} = \frac{\sigma_y}{E} \quad |\sigma_1| \geq |\sigma_2|$$

$$\varepsilon_2 = \frac{\sigma_2 - \nu\sigma_1}{E} = \frac{\sigma_y}{E} \quad |\sigma_2| \geq |\sigma_1|$$

Where, ν = poison's ratio,

σ_y = yield strength in simple tension



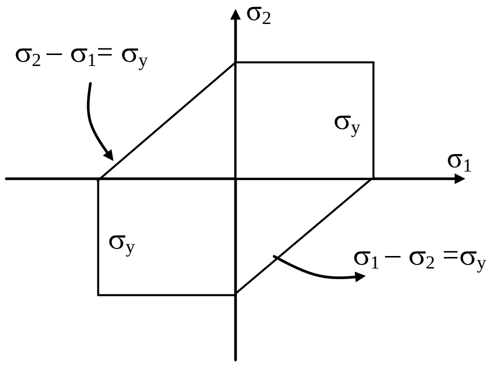
Yield surface for maximum principal strain

Here yield locus is ***parallelogram***

Maximum shear stress theory (Coulomb, Tresca and Guest's theory)

The theory state that the failure of a mechanical component subjected to bi-axial or tri-axial stresses occurs when the maximum shear stress at any point in the component becomes equal to the maximum shear stress in the standard specimen of the tension test, when yielding starts. In the tension test, the specimen is subjected to uni-axial stress (σ_1) and ($\sigma_2 = 0$). Thus maximum shear stress is $\frac{\sigma_y}{2}$

Share stresses in 3-D are $\left|\frac{\sigma_1 - \sigma_2}{2}\right|$, $\left|\frac{\sigma_2 - \sigma_3}{2}\right|$ and $\left|\frac{\sigma_3 - \sigma_1}{2}\right|$. Take maximum value ($\tau_{max}$) of these and equate to $\sigma_y/2$



Yield surface for maximum shear stress

Here yield locus is ***hexagon***

In a biaxial stress solution case $\sigma_3 = 0$ and this gives

$\sigma_y = \sigma_1 - \sigma_2$ if $\sigma_1 > 0, \sigma_2 < 0$

$\sigma_y = \sigma_2 - \sigma_1$ if $\sigma_1 < 0, \sigma_2 > 0$

$\sigma_y = \max(\sigma_1, \sigma_2)$ if $\sigma_1, \sigma_2 > 0$

$\sigma_y = \min(\sigma_1, \sigma_2)$ if $\sigma_1, \sigma_2 < 0$

Maximum strain energy theory (Beltrami's theory or Haigh's theory)

Failure would occur when the total strain energy absorbed at a point per unit volume exceeds the strain energy absorbed per unit volume at the tensile yield point

The strain energy per unit volume

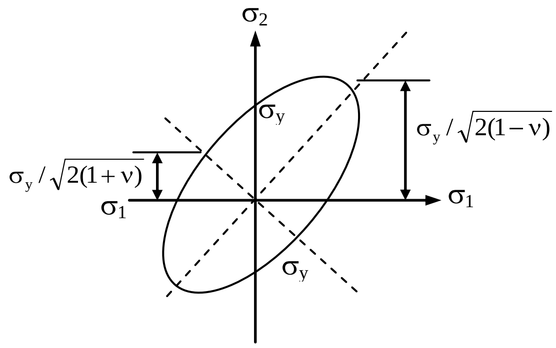
$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \nu(2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1)\right]$$

$$\sigma_1^{\ 2} + \sigma_2^{\ 2} + \sigma_3^{\ 2} - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) = \sigma_y^2$$

For biaxial this may be written as

$$\left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - 2\nu\left(\frac{\sigma_1\sigma_2}{\sigma_y^{\ 2}}\right) = 1$$

And corresponding figure depicting above *ellipse* is



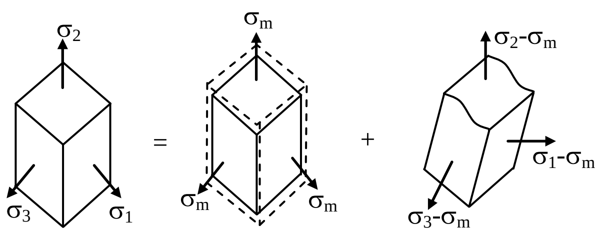
Yield surface for maximum strain

Distortion energy theory (von Mises and Hencky's theory)

According to this theory yielding would occur when total distortion energy absorbed per unit volume due to applied loads exceeds the distortion energy absorbed per unit volume at the tensile yield point.

The energy of distortion can be obtained by subtracting the energy of volumetric change from the total energy.

∴ The total work done causes change in volume due to operation of direct stress and distortion due the shearing stress which does not affect the volumetric change.



Volume change due to normal stress

Distortion due to shear

$$\sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$\therefore U = U_v + U_s$

$\therefore$ Strain energy due to direct stress

$U_v = \frac{1}{2}(\text{average } \sigma)\ \Delta V$

$= \frac{1}{2}\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3}\right)\Delta V$

But $\Delta V = \frac{1}{E}\{\sigma_1 + \sigma_2 + \sigma_3 - 2\nu(\sigma_1+\sigma_2+\sigma_3)\}$

Per unit volume

$\therefore\ U_v = \frac{(\sigma_1+\sigma_2+\sigma_3)^2}{6E}(1-2\nu)$

Total strain energy

$U = \frac{1}{2}\times \text{stress} \times \text{strain} \times \text{unit volume}$

$= U_1 + U_2 + U_3$

Here $U_1 = \frac{1}{2}\sigma_1\varepsilon_1 = \frac{1}{2}\sigma_1\left[\frac{\sigma_1}{E} - \frac{\nu}{E}(\sigma_2+\sigma_3)\right]$

$= \frac{1}{2E}\left[\sigma_1^2 - \nu\sigma_1\sigma_2 - \nu\sigma_3\sigma_1\right]$

Similarly, $U_2 = \frac{1}{2E}\left[\sigma_2^2 - \nu\sigma_1\sigma_2 - \nu\sigma_2\sigma_3\right]$

$U_3 = \frac{1}{2E}\left[\sigma_3^2 - \nu\sigma_1\sigma_3 - \nu\sigma_3\sigma_2\right]$

$$U = \frac{1}{2E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2 - 2\nu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$$

$\therefore$ Distortion energy

$U_s = U - U_v$

$= \frac{1}{2E}[X - 2\nu Y] - \frac{(1-2\nu)}{6E}[X+2Y]$

Where, $X = \sigma_1^2+\sigma_2^2+\sigma_3^2$

$Y = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$

$U_S = \frac{1}{6E}[X(1+\nu) - Y(1+\nu)] = \frac{(1+\nu)}{6E}(X-Y)$

$$\therefore U_S = \frac{1+\nu}{3E}\left[\sigma_1^2+\sigma_2^2+\sigma_3^2 - (\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right]$$

$$U_s = \frac{1+\nu}{6E}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]$$

Under the action of uniaxial stress σ_y

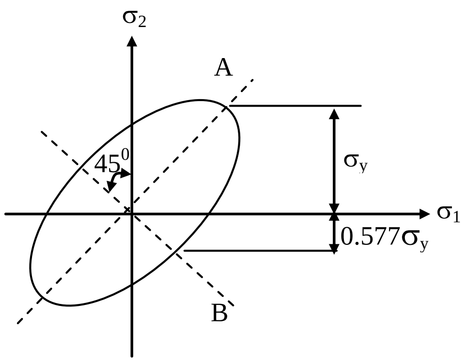
distortion energy $= \frac{1+\nu}{3E}\sigma_y^2$

$$\therefore\ \frac{1+\nu}{6E}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]$$

$$= \frac{1+\nu}{3E}\sigma_y^2$$

$$\Rightarrow (\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 = 2\sigma_y^2$$

In a 2-D situation if $\sigma_3 = 0$, then the criterion reduces to $\left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - \left(\frac{\sigma_1}{\sigma_y}\right)\left(\frac{\sigma_2}{\sigma_y}\right) = 1$



Yield surface for distortion energy

This is an equation of ***ellipse*** and the yield surface is as shown. Major and minor axes are along 45^0 lines. The stress values at points A, B could be solved by substituting $\sigma_1 = \sigma_2$, $\sigma_1 = -\sigma_2$ respectively; in terms of σ_y.

03(a).

Sol: Given data:

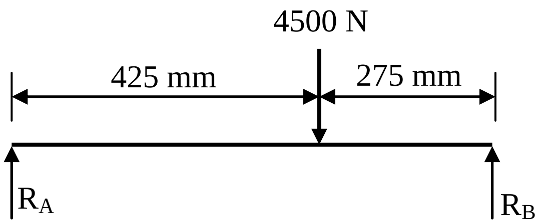
Load, $P = 4500$ N

$\sigma_y = 310\,\text{MN/m}^2 = 310\,\text{N/mm}^3$

$n = 3$

Drum radius, $r = \frac{200}{2} = 100$ mm

The given loading of the shaft can be shown as in figure.



Taking moments about A,

We have

$$R_F \times 700 - (4500 \times 425) = 0$$

$$R_a = 2732.14\,\text{N}$$

$$R_4 = 4500 - 2732.14$$

$$R_A = 1767.86\,\text{N}$$

$$\therefore\ BM_C = 2732.1 \times 275 = 751340.5\,\text{N} - \text{mm}$$

Torque M_t = Force×Radius of the drum

$$-4500 \times 100 = 45000\ \text{N} - \text{mm}$$

So, the shaft is under the action of

(i) Bending moment M_b = 751340.5 N-mm

(ii) Torque, M_t = 450000 N-mm

Shear stress due to

$$M_b\,\tau_{xy} = \frac{16.M_t}{\pi d^3} = \frac{16 \times 450000}{\pi d^3} - \frac{2.29 \times 10^6}{d^3}$$

Bending stress due to

$$M_b \sigma_z = \frac{32.M_t}{\pi d^3} - \frac{7.65 \times 10^6}{d^3}$$

Maximum shear stress,

$$\tau_{max} = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau xy^2}$$

$$= \sqrt{\left(\frac{7.65 \times 10^6}{d^3}\right)^2 + 4\left(\frac{2.29 \times 10^6}{d^3}\right)^2} - \frac{8.916 \times 10^6}{d^3}$$

According Tersca's theory designing

$$\frac{\sigma_x}{n} = \tau_{max}$$

$$\Rightarrow \frac{310}{3} = \frac{8.916 \times 10^6}{d^3}$$

$$\Rightarrow \quad d = 44.18\ \text{mm}$$

Substituting d = 35.07 in the expression for τ_{xy} and σ_z we have

$$\tau_{xy} = \frac{2.29 \times 10^6}{44.183} = 26.54\,\text{N/mm}^2$$

$$\sigma_x = \frac{7.65 \times 10^6}{44.18^2} = 88.712\,\text{N/mm}^2$$

So, maximum and minimum principal stresses are,

$$\sigma_1, \sigma_2 = \frac{1}{2}\left[(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau xy^2}\right]$$

$$= \frac{1}{2}\left[88.712 \pm \sqrt{88.712^2 + 4 \times 26.54^2}\right]$$

$$= 96\,\text{N/mm}^2 \,\&\, -7.144\,\text{N/mm}^2$$

i.e, $\sigma_1 = 96\,\text{N/mm}^2$

$\sigma_2 = -7.144\,\text{N/mm}^2\,(\text{comp})$

By von-Mises theory,

$$\left(\frac{\sigma_y}{n}\right)^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1$$

$$\therefore\ \left(\frac{\sigma_y}{n}\right)^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2$$

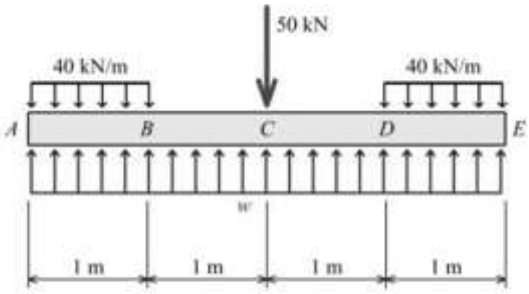
$$\text{i.e}\left(\frac{310}{n}\right)^2 = 96^2 + 7.144^2 - (96 - 7.144)$$

$$\Rightarrow \quad n = 3.101$$

Comment: Not much difference is found between the FOS values by these two theories.

03(b)(i).

Sol: Beam equilibrium:

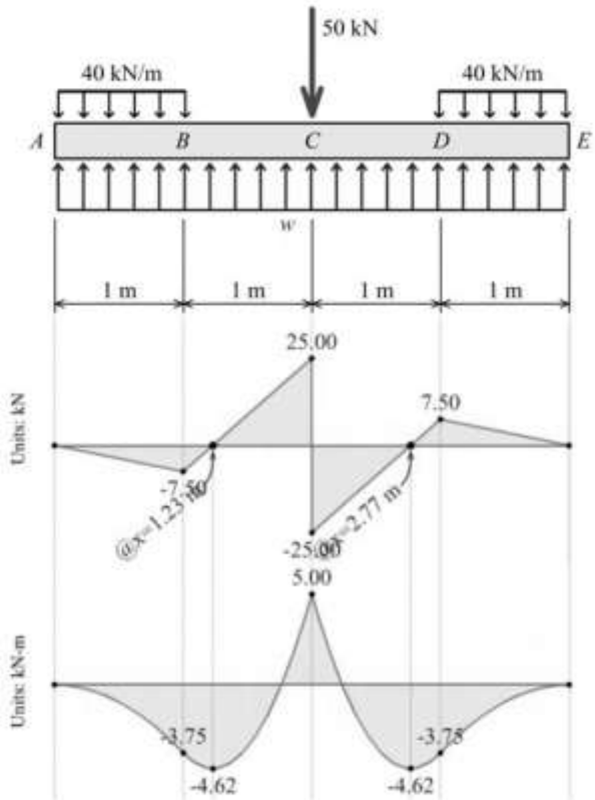


$\Sigma F_y = -40\times1 - 50$

$-40\times1 + w\times4 = 0$

$\therefore\ w = 32.5$ kN/m

Shear force and bending moment diagrams



(a) Maximum value of internal shear force:

V = ±25 kN @ x = 2 m

(b) Maximum value of internal bending moment:

M = –4.62 kN,m @ x = 1.23 m

M = –4.62 kN.m @ x = 2.77 m

M_{max} = 5.00 kN.m

03(b)(ii).

Sol: Assumptions:

(i) Plane cross sections before bending remain plane after bending (Bernoulli's Assumption).

(ii) Material is homogeneous, isotropic and obeys Hooke's Law and limits of eccentricity are not exceeded.

(iii) Every layer is free to expand or contract.

(iv) Modulus of elasticity has same value in tension and compression.

(v) The beam is subjected to pure bending and therefore *bends* in an arc of a circle.

(vi) Radius of curvature is large compared to the dimensions of the cross section

$$\frac{\mathbf{E}}{\mathbf{R}} = \frac{\mathbf{M}}{\mathbf{I}} = \frac{\mathbf{f}}{\mathbf{y}}$$

Where,

M = Bending moment at a cross section

I = Moment of inertia of entire cross section about neutral axis

f = Bending stress (tensile or compressive)

y = Linear distance from neutral axis

E = Modulus of elasticity

R = Radius of curvature

EI = Flexural rigidity

03(c).

Sol: The torsional stiffness of the shaft is

$$K = \frac{\pi G d^4}{32\ell} = \frac{\pi \times 8 \times 10^{10} \times 0.03^4}{32 \times 0.3}$$

$= 21206$ N-m/rad

The natural frequency of the system is

$\omega_n = (K/J)^{1/2}$

$= (21206 / 0.53)^{1/2} = 200$ rad/s

The frequency ratio is

$$r = \frac{\omega}{\omega_n} = \frac{314}{200} = 1.57$$

This value of r, we find the magnification factor is

$$M = \left|\frac{1}{1 - 1.57^2}\right| = 0.68$$

With a torque of 300 N-m, the static twist is

$$\theta_s = \frac{32T\ell}{\pi G d^4} = \frac{32 \times 300 \times 0.3}{\pi \times 8 \times 10^{10} \times 0.03^4} = 0.0141\,\text{rad}$$

The amplitude of the twist under the dynamic load is

$\theta = M\theta_s$

$= 0.68 \times 0.0141 = 0.0096$ rad

Hence, the maximum shear stress is

$$\tau = \frac{Gd\theta}{2\ell} = \frac{8 \times 10^{10} \times 0.03 \times 0.0096}{2 \times 0.3}$$

$= 0.384 \times 10^8$ N/m^2

$= 38.4$ MPa

When the diameter is increased to 35 mm, the foregoing calculations are repeated.

The values so obtained are

K = 39287 N-m/rad,

$\omega_n = 272$ rad/s,

r = 1.15,

M = 3.1,

$\theta_s = 0.0076$ rad,

$\theta = 0.0236$ rad,

$\tau = 110$ MPa

Note: The maximum shear stress drastically increases when the shaft diameter is increased. This is quite surprising at the first glance. Therefore, when designing a machine member to be subjected to dynamic loads, care has to be taken. The common thumb rule of increasing the dimensions to make it safer may not hold good when the loading is dynamic.

04(a).

Sol: The forces and couples are resolved in two mutually perpendicular planes. Summing up the last four columns of the table and equating each to zero,

we get,

$m_D x_D \cos\theta_D = -86.17$ ....(a)

$m_D x_D \sin\theta_D = -276.5$ ....(b)

$8m_D \cos\theta_D - 7.88m_A = -94.98$ ....(c)

$8m_D \sin\theta_D - 1.39m_A = -73.73$ ....(d)

Plane	Mass m (kg)	Eccentricity e (cm)	Distance from reference plane A x (cm)	Angle with reference line B θ (degrees)	Couple vector		Force vector	
					Vertical Component $mex\cos\theta$	Horizontal component $mex\sin\theta$	Vertical component $me\cos\theta$	Horizontal component $me\sin\theta$
A	m_A	8	0	190	0	0	$-7.88\ m_A$	$-1.39\ m_A$
B	18	6	10	0	1080	0	108	0
C	12.5	6	30	100	–390.6	2212	–13.02	73.73
D	m_D	8	x_D	θ_D	$8m_Dx_D \times\cos\theta_D$	$8m_Dx_D \times\sin\theta_D$	$8m_D \times\cos\theta_D$	$8m_D \times\sin\theta_D$

From (a) and (b), $\tan\theta_D = 3.21$. Since x_D is known to be positive, both $\sin\theta_D$ and $\cos\theta_D$ are negative. So, $\theta_D = 252.7^o$ (this is the angle between the masses at D and B). Thus, we get

$\cos\theta_D = -0.2975, \quad \sin\theta_D = -0.955$

Substituting these values in (c) and (d), we have

$-2.38m_D - 7.88m_A = -94.98$..................(e)

$-7.64m_D - 1.39\ m_A = -73.73$.................(f)

Solving (e) and (f) simultaneously, we get

$m_A = 9.67\,\text{kg}, \quad m_D = 7.89\,\text{kg}$

Then, using these values of m_D and $\cos\theta_D$ in (a), $x_{DF} = 36.57$ cm, and the distance of D from C is $(x_D - 30) = 6.57$cm

04(b).

Sol: Given data: M16 bolts

$l_1 = 20$,

$l_2 = 90$,

$l_3 = 160$,

$e_1 = 140$ mm,

$n_1 = n_2 = n_3 = 1$

Yield stress in shear,

$\sigma_y = 128$ MPa,

FOS = 1.6

$\tau_{max} = \frac{128}{1.6} = 80$ MPa

For the base plate bolts,

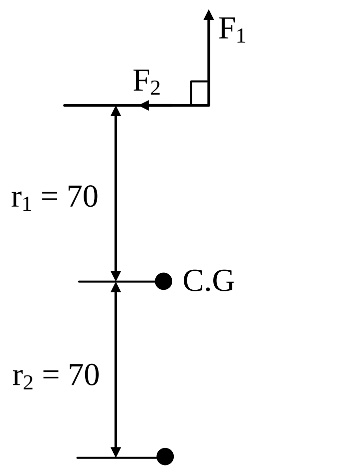
$n_3 = n_4 = n_5 = 2$

$l_3 = 25$

$l_4 = 25 + 175 = 200$ mm

$l_5 = 25 + 350 = 375$ mm,

$e_2 = 140 + 140 = 280$ mm



(i) Finding the maximum load for M16 bolts

The bolts are under two types of loads.

1. Direct shear load, F_1
2. Secondary shear load F_2

$$F_s = \frac{\text{Total load}}{\text{Number of bolts}}$$

i.e $F_1 = \frac{W}{2\times 3} = 0.1678$

(2 becomes the bolts are in double shear)

$$F_2 = \frac{W\times e_1\times r_1}{n_2 r_1^2 + n_2 r_2^2} - \frac{W\times 140\times 70}{70^2+70^2} = W$$

Resultant shear load,

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_1\cos\theta}$$

$$= \sqrt{(0.167W)^2 + W^2 + 2\times 0.167W\times W\times \cos 90}$$

$(\because \text{here}, \theta = 90^\circ)$

= 1.0138 W

The shear stress, $\tau = \frac{1.013}{A_c}$

Equating this to the gives τ_{max}, we have

$$\tau_{max} = 80 = \frac{1.013W}{A_c}$$

M16 bolts area, A_c = 157 mm^2

Substituting, we have W = 12.38 kN

∴ The maximum load for M16 bolts = 12.38 kN.

(ii) Bolt diameter for the base plate:

The same load of 12.38 kN is noting on those bolts

Considering only the top row of bolts

We have $\ell_5 = 375\,\text{mm}$

∴ Maximum load on each bolt due to tilting about 'O'

$$F_1 = \frac{W\times e_2\times \ell_1}{n_3\ell_3^2 + n_4\ell_4^2 + n_5\ell_5^2} = \frac{12.38\times 280\times 375}{2(25^2+200^2+375^2)}$$

F_1 = 3.58 kN

Direct shear load,

$$F_2 = \frac{\text{Total load}}{\text{No.of bolts}} = \frac{12.38}{6} = 2.06 \text{ kN}$$

Corresponding shears are

$$\sigma = \frac{3.58\times 10^2}{A_c}\,\text{N/mm}^3 \text{ and}$$

$$\tau = \frac{2.06\times 10^3}{A_s}\,\text{N/mm}^2$$

∴ Maximum shear stress

$$\tau_{max} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{\left(\frac{3.58\times 10^3}{A_c}\right)^2 + 4\left(\frac{2.06\times 10^3}{A_c}\right)^2}$$

$$= \frac{2.73\times 10^3}{A_c}\,\text{N/mm}^2$$

Equating this to the given τ_{max} we have

$$80 = \frac{2.73\times 10^3}{A_c}$$

$$A_c = 34.11\text{mm}^2$$

$$\frac{\pi}{4}d^2 = 34.11$$

$\Rightarrow d^2$ = 43.43

$\Rightarrow$ d = 6.59015 ≈ 8

So, M8 bolt is choosen.

04(c).

Sol: Integrate the load distribution:

$$EI\frac{d^4v}{dx^4} = -w_o\cos\frac{\pi x}{2L}$$

$$EI\frac{d^3v}{dx^3} = -\frac{2w_oL}{\pi}\sin\frac{\pi x}{2L} + C_1$$

$$EI\frac{d^2v}{dx^2} = \frac{4w_oL^2}{\pi^2}\cos\frac{\pi x}{2L} + C_1x + C_2$$

$$EI\frac{dv}{dx} = \frac{8w_oL^3}{\pi^3}\sin\frac{\pi x}{2L} + \frac{C_1x^2}{2} + C_2x + C_3$$

$$EIv = -\frac{16w_oL^4}{\pi^4}\cos\frac{\pi x}{2L} + \frac{C_1x^2}{6} + \frac{C_2x^2}{2} + C_3x + C_4$$

Boundary conditions and evaluate constants:

At x = 0, $V = EI\frac{d^3v}{dx^3} = 0$

$\therefore$ $C_1 = 0$

At x = 0, $M = EI\frac{d^2v}{dx^2} = 0$

$$\frac{4w_oL^2}{\pi^2}\cos\frac{\pi(0)}{2L} + C_2 = 0$$

$$\therefore \quad C_2 = -\frac{4w_oL^2}{\pi^2}$$

At x = L, $\frac{dv}{dx} = 0$

$$\frac{8w_oL^3}{\pi^3}\sin\frac{\pi(L)}{2L} - \frac{4w_oL^2(L)}{\pi^2} + C_3 = 0$$

$$\therefore C_3 = -\frac{4w_oL^3}{\pi^3}(2-\pi)$$

At x = L, v = 0

$$-\frac{16w_oL^4}{\pi^4}\cos\frac{\pi(L)}{2L} - \frac{4w_oL^2(L)^2}{2\pi^2} - \frac{4w_oL^3(L)}{\pi^3}(2-\pi) + C_4 = 0$$

$$\therefore C_4 = \frac{2w_oL^4}{\pi^3}(4-\pi)$$

(i) Elastic curve equation:

$$EIv = -\frac{16w_oL^4}{\pi^4}\cos\frac{\pi x}{2L} - \frac{4w_oL^2x^2}{2\pi^2} - \frac{4w_oL^3}{\pi^3}(2-\pi) + \frac{2w_oL^4}{\pi^3}(4-\pi)$$

$$-\frac{w_o}{2\pi^4EI}\left[32L^4\cos\frac{\pi x}{2L} + 4\pi^2L^2x^2 + 8\pi L^2x(2-\pi) - 4\pi L^4(4-\pi)\right]$$

(ii) Deflection at left end of beam:

$$V_A = -\frac{W_o}{2\pi^4EI}\left[32L^4\cos\frac{\pi(0)}{2L} + 4\pi^2L^2(0)^2 + 8\pi L^3(0)(2-\pi) - 4\pi L^4(4-\pi)\right]$$

$$= -\frac{W_o}{2\pi^4EI}\left[32L^4 - 4\pi L^4(4-\pi)\right]$$

$$= -\frac{W_oL^4}{2\pi^4EI}\left[32 - 4\pi(4-\pi)\right]$$

$$= -0.1089\frac{W_oL^4}{EI}$$

(iii) Support reactions B_y and M_B:

$$V_B = EI\left.\frac{d^3v}{dx^3}\right|_{x=L} = -\frac{2w_oL}{\pi}\sin\frac{\pi(L)}{2L} = -\frac{2W_oL}{\pi}$$

$$\therefore \quad B_y = \frac{2W_oL}{\pi}\uparrow$$

$$M_B = EI\left.\frac{d^2v}{dx^2}\right|_{x=L}$$

$$= \frac{4w_oL^2}{\pi^2}\cos\frac{\pi(L)}{2L} - \frac{4w_oL^2}{\pi^2} = -\frac{4w_oL^2}{\pi^2}$$

$$\therefore M_B = \frac{4w_oL^2}{\pi^2}(cw)$$

05(a).

Sol: Roll (z-axis), pitch (y-axis), yaw (x-axis), rotations with fixed axes reference. so pre multiplication from given sequence

$\Rightarrow R(z, \pi), R(y, -\pi/2), R(x, -\pi/2)$

$$= \begin{bmatrix} c\pi & -s\pi & 0 \\ s\pi & c\pi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\left(\frac{-\pi}{2}\right) & 0 & s\left(\frac{-\pi}{2}\right) \\ 0 & 1 & 0 \\ -s\left(\frac{-\pi}{2}\right) & 0 & c\left(\frac{-\pi}{2}\right) \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} \text{ is resultant rotation matrix.}$$

05(b)(i) .

Sol: The Young's modulus (also known as 'modulus of elasticity' or 'elastic modulus) is influenced by the factors such as:

(i) Bonding character;

(ii) Temperature ;

(iii) Anisotropy

→ Strongly bonded solids with three-dimensional network possess high values of elastic modulus.

→ The effect of temperature is to lower the elastic modulus by 10% to 20% between 0 K to melting point.

→ Due to anisotropy, the materials show different modulus in different directions.

05(b)(ii) .

Ans: A material, whose stress-strain diagram has different slopes in tension and compression, is known as bi-modulus material. Such a material possesses unequal values of Young's moduli in tension and compression (Elastomer and wood may keep a bi-moduli character).

05(b)(iii) .

Ans:

- The behaviour of materials under fluctuating and reversing loads (or stresses) is known as fatigue.
- The main effects of fatigue on the properties of materials are: (i) Loss of ductility; (ii) Loss of strength; (iii) Enhanced uncertainty in strength and the service life of materials.

05(c).

Sol:

(i) Reliability (R) = e^{-t}

Failure rate $\lambda(t) = 0.003\left(\frac{t}{500}\right)^{0.5}$

Time t = 50hr, then

$\lambda(50) = 0.003\ (50/500)$^0.5

$= 0.000949$

Reliability = $e^{-0.0009490\times 50} = 0.953$

(ii) $R(t) = \exp\left(-\int_0^t \frac{0.5}{1000}\left(\frac{t}{1000}\right)^{-0.5} dt\right) = 0.9$

$$= \left(\frac{t}{1000}\right)^{0.5}\Bigg|_0^t = \exp\left(\left(\frac{t}{1000}\right)^{0.5}\right) = 0.9$$

$$\ell n 0.9 = \left(\frac{t}{1000}\right)^{0.5} \quad \rightarrow t = 11.1 \text{ years}$$

05(d).

Sol: Let x_1 and x_2 be the number of units of products A and B, respectively, to be purchased. The LPP may be stated as follows:

Minimize $Z = 20x_1 + 40x_2$ Total cost

Subject to

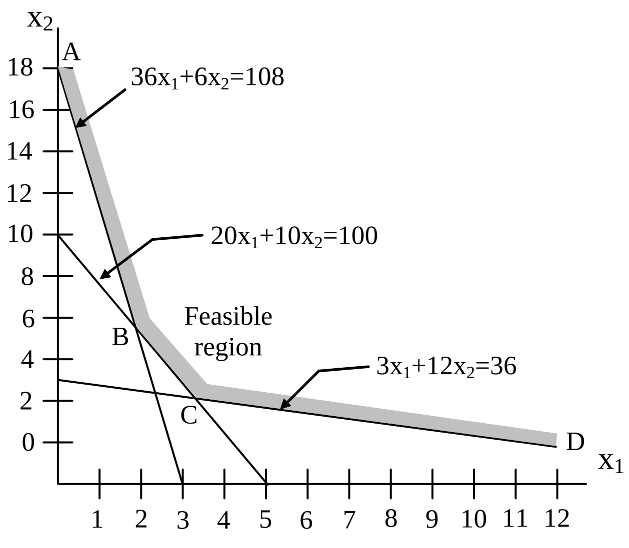
$36x_1 + 6x_2 \geq 108$ Nutrient 1

$3x_1 + 12x_2 \geq 36$ Nutrient 2

$20x_1 + 10x_2 \geq 100$ Nutrient 3

$x_1, x_2 \geq 0$

The feasible area has extremes A(0, 18), B(2, 6), C(4, 2), D(12, 0).



Accordingly, Z(A) = 720, Z(B) = 280, Z(C) = 160, and Z(D) = 240. Thus, optimal solution is $x_1 = 4$ and $x_2 = 2$.

05(e).

Sol: Forging defects:

Though forging process give generally prior quality product compared other manufacturing processes. There are some defects that are lightly to come a proper care is not taken in forging process design.

A brief description of such defects and their remedial method is given below.

(A) Unfilled Section:

In this some section of the die cavity are not completely filled by the flowing metal. The causes of this defect are improper design of the forging die or using forging techniques.

(B) Cold Shut:

This appears as a small cracks at the corners of the forging. This is caused mainly by the improper design of die. Where in the corner and the fillet radie are small as a result of which metal does not flow properly into the corner and the ends up as a cold shut.

(C) Scale Pits:

This is seen as irregular depurations on the surface of the forging. This is primarily caused because of improper cleaning of the stock used for forging. The oxide and scale gets embedded into the finish forging surface. When the forging is cleaned by pickling, these are seen as depurations on the forging surface.

(D) Die Shift:

This is caused by the miss alignment of the die halve, making the two halve of the forging to be improper shape.

(E) Flakes:

These are basically internal ruptures caused by the improper cooling of the large forging. Rapid cooling causes the exterior to cool quickly causing internal fractures. This can be remedied by following proper cooling practices.

(F) Improper Grain Flow:

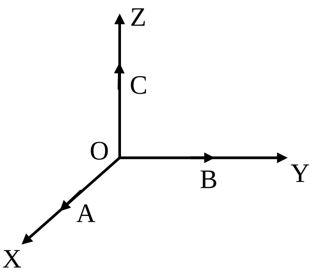
This is caused by the improper design of the die, which makes the flow of the metal not flowing the final interred direction.

06(a).

Sol: Frame 1(OXYZ) ⇒ Fixed frame,

Frame 2(OABC) ⇒ Movable frame,

$$\begin{bmatrix} \text{P} \\ \text{fixed} \\ \text{frame} \end{bmatrix} = \begin{bmatrix} \text{Equivalent} \\ \text{Rotation} \\ \text{Matrix} \end{bmatrix} \begin{bmatrix} \text{P} \\ \text{movable} \\ \text{frame} \end{bmatrix}$$



Set – A

Equivalent Rotation Matrix

R(90°, x) → T(5, –4, 6) → R(–90°, z)

All transformations with reference to fixed axis → Pre-multiplication.

⇒ R(–90°, z) × T(5, –4, 6) × R(90°, x)

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 & -4 \\ -1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & -1 & -4 \\ -1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \text{T.M}$$

(over all transformation matrix)

$$P_{xyz} = [\text{T.M}]^{P}_{abc}$$

$$\begin{bmatrix} -7 \\ -10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & -4 \\ -1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} . P_{ABC}$$

$$P_{ABC} = [\text{T.M}]^{-1} . P_{xyz}$$

$$[\text{T.M}]^{-1} = \begin{bmatrix} R^T & (-1)(R^T).D \\ 000 & 1 \end{bmatrix}$$

$$[\text{T.M}]^{-1} = \begin{bmatrix} 0 & -1 & 0 & -5 \\ 0 & 0 & 1 & -6 \\ -1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So,

$$P_{ABC} = \begin{bmatrix} 0 & -1 & 0 & -5 \\ 0 & 0 & 1 & -6 \\ -1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -7 \\ -10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 3 \\ 1 \end{bmatrix}$$

So point P with reference to movable frame (F_2) after set A translations are (5 4 3).

Set – B

R (–90°, C) → T (5, –4, 6) → R(–90°, A)

Translations are with reference to movable frame.

So Post multiplication.

$\Rightarrow$ R(–90°, C), T(5, –4, 6), R(–90°, A)

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 & -4 \\ -1 & 0 & 0 & -5 \\ 0 & -1 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$P_{xyz} = (T.M).P_{abc}$

$$\begin{bmatrix} -7 \\ -10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -4 \\ -1 & 0 & 0 & -5 \\ 0 & -1 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}.P_{abc}$$

$$P_{abc} = \begin{bmatrix} 0 & 0 & 1 & -4 \\ -1 & 0 & 0 & -5 \\ 0 & -1 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1}\begin{bmatrix} -7 \\ -10 \\ 10 \\ 1 \end{bmatrix}$$

$$[T.M]^{-1} = \begin{bmatrix} R^T & (-1)(R^T).D \\ 000 & 1 \end{bmatrix}$$

So, above matrix inverse is

$$[T.M]^{-1} = \begin{bmatrix} 0 & -1 & 0 & -5 \\ 0 & 0 & -1 & 6 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ So}$$

$$P_{abc} = \begin{bmatrix} 0 & -1 & 0 & -5 \\ 0 & 0 & -1 & 6 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -7 \\ -10 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ -3 \\ 1 \end{bmatrix}$$

So P_{abc} = (5, –4, –3)

So coordinates of point P with reference to movable frame 2(OABC) after Set A movements is (5, 4, 3)

Set B movements is (5, –4, –3).

06(b)(i).

Sol: Material removal rate (MRR) = V.f.d

= 70 (m/min) (0.4 mm/rev) (4mm) × 1000

= 112000 mm^3/min = 112 cm3/min

Machining power

= MRR × Sp. cutting energy

= 112 (cm^3/min) × 0.07 (kW/cm^3/min)

= 7.84 kW

RPM of spindle N

$$= \frac{V}{\frac{\pi(\text{outside dia D}) + (D - 2d)}{2}}$$

$$= \frac{2 \times 70(m/min) \times 1000}{\pi(50 + (50 - 2 \times 4))} = 485 \text{ rpm}$$

Power = 2πNT and torque T = Power / 2πN

$$= \frac{7840 \text{ Nm/sec} \times 60}{2\pi(485 \text{ rev/min})} = 155 \text{ Nm}$$

Time for machining = $\frac{Length}{fN}$

$$= \frac{(200) \times 60}{0.4 \times 45} = 63\text{sec}$$

Uncut chip thickness = f × cos (Cs)

= 0.4 × cos20° = 0.376 mm.

06(b)(ii).

Sol: *The different machine tools used for manufacture of spur gears are*

(i) *Gear hobbing:* In this the cutter will be rotating and reciprocating where as the work piece is only rotating, so that speed of cutter (hobbing tool) = Number of teeth to be produced on the blank × speed of work piece

(ii) *Gear broaching:* In this process the work piece is stationary but the tool is moving linearly.

(iii) *Shaping with single point tool:* In this single point cutting tool is reciprocating and work is stationary. But at the end of each teeth is completed, the work piece is indexed by using indexing mechanism.

(iv) *Gear Planning:* In this process, many number of single point tools are mounted around the work, i.e. the number of tools is equal to number of gear teeth to be made. In this process the tools are stationary but the work is reciprocating. No indexing is required.

(v) *Shaping with rack cutter :* In this process, work is stationary and rack shaped cutting tool is reciprocation in the vertical axis. But at the end of each teeth is completed, the work piece is indexed by using indexing mechanism.

(vi) *Shaping with pinion cutter : In* this process, work is stationary and pinion shaped cutting tool is reciprocation in the vertical axis. But at the end of each teeth is completed, the work piece is indexed by using indexing mechanism.

06(c)(i).

Sol: The sampling frequency is 1000 Hz.

Hence, the Nyquist frequency will be

$$1000/2 = 500 \text{ Hz.}$$

Hence, we can be able to measure frequency below 500 Hz accurately that means frequency 120, 200 and 460 Hz will be measured accurately.

However, frequency 700 Hz will appear as

$$1000 - 700 = 300 \text{ Hz}$$

Frequency 800 Hz will also appear as

$$1000 - 800 = 200 \text{ Hz}$$

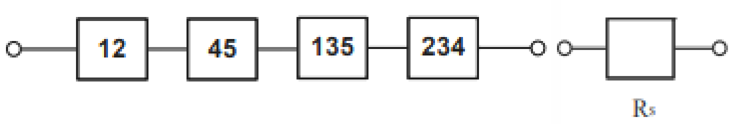
Frequency 900 Hz will appear as

$$1000 - 900 = 100 \text{ Hz signal}$$

So, the captured signal will contain erroneous high amplitude of 200 Hz signal with an additional frequency of 100 Hz which is actually not present at all in the actual signal

06(c)(ii).

Sol:



$R_{12} = 1 - P_{f1}\,P_{f2} = 1 - 0.2\times0.2 = 0.960$

$R_{45} = 1 - P_{f4}\,P_{f5} = 1 - 0.2\times0.2 = 0.960$

$R_{234} = 1 - P_{f2}\,P_{f3}\,P_{f4}$

$= 1 - 0.2\times0.2\times0.2 = 0.992$

$R_{135} = 1 - P_{f1}\,P_{f3}\,P_{f5}$

$= 1 - 0.2\times0.2\times0.2 = 0.992$

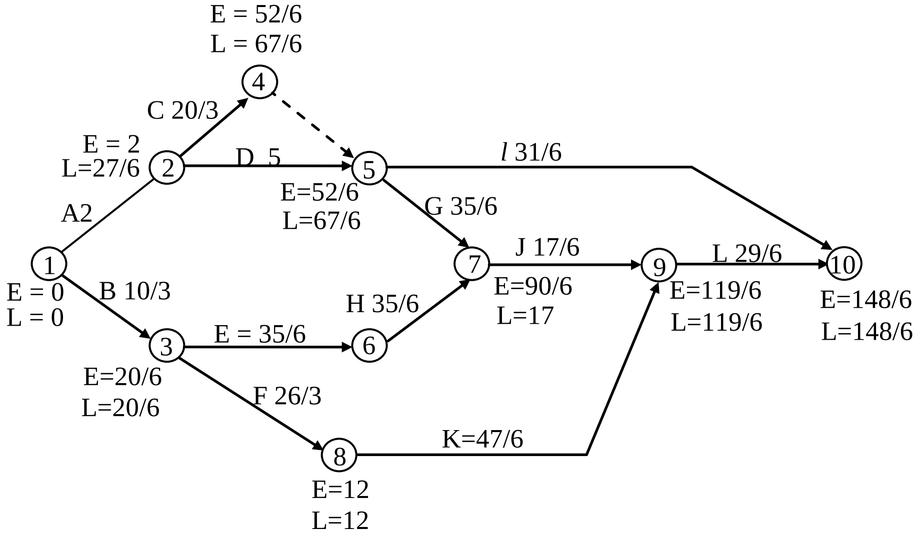
$R_s = R_{12} \times R_{45} \times R_{135} \times R_{234}$

$= 0.960 \times 0.960 \times 0.992 \times 0.992$

$= 0.907$

07(a).

Sol:



Activity		a	m	b	t_e	σ^2	ES	EF	LS	LF	Total stock
A	1 – 2	2	2	2	2	0	0	2	15/6	27/6	15/6
B	1 – 3	1	3	7	20/6	1	0	20/6	0	20/6	0
C	2 – 4	4	7	8	40/6	4/9	2	52/6	27/6	67/6	15/6
D	2 – 5	3	5	7	5	4/9	2	7	37/6	67/6	25/6
E	3 – 6	2	6	9	35/6	49/36	20/6	55/6	32/6	67/6	2
F	3 – 8	5	9	11	52/6	1	20/6	72/6	20/6	12	0
G	5 – 7	3	6	8	35/6	25/36	52/6	87/6	67/6	17	15/6
H	6 – 7	2	6	9	35/6	49/36	55/6	90/6	67/6	17	2
I	5 – 10	3	5	8	31/6	25/36	52/6	83/6	117/6	148/6	65/6
J	7 – 9	1	3	4	17/6	9/36	15	107/6	17	119/6	2
K	8 – 9	4	8	11	47/6	49/36	12	119/6	12	119/6	0
L	9 – 10	2	5	7	29/6	25/36	119/6	148/6	119/6	148/6	0

Critical path : 1 - 3 - 8 - 9 – 10 (B - F - K - L)

Expected completion time = $\frac{148}{6}$ = 24.67 days

Variance, $\sigma = \sqrt{\frac{146}{36}}$ = 2.014 days

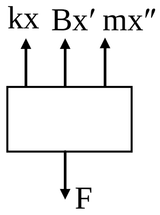
If X number of days give a 99% probability of completion, we have $2.33 = \frac{X - 24.67}{2.014}$

Thus, $X = 2.33 \times 2.014 + 24.67 = 29.36 \approx 30$ days

07(b).

Sol:

(i) To derive mathematical model free body diagram of mass (m).



As per the D Alemberts principle, algebraic sum of applied and opposition forces are zero.

So,

$F - mx'' - Bx' - kx = 0$ (or)

$$F = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx$$

$$= mx'' + Bx' + kx$$

So mathematical model of above system is

$$F = mx'' + Bx' + kx$$

(ii) To find state model equation state variable are

Let, displacement (x) = x_1

velocity (x′) = x_2

so,

$$F = mx'' + Bx' + kx$$

$\downarrow$

$$u = mx_2' + Bx_2 + kx_1$$

$$x_1' = x_2$$

Rearranged above equations as below

$$x_1' = 0.x_1 + 1.x_2 + 0.u$$

$$x_2' = \frac{-k}{m}x_1 + \left(\frac{-B}{m}\right)x_2 + \left(\frac{1}{m}\right)u$$

State model equation as

$x' = Ax + Bu$,

So. above equation are written as

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -B/m \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix}.u$$

is state model equation and y = x_1

$y = 1.x_1 + 0.x_2 + 0.u$, are output equation

$$\rightarrow\ y = [1o]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0.u$$

(iii) Based on Kalman's test for controllability

$x' = Ax + Bu$, A is 2×2 matrix

so, $Q_c = [BAB]$, so

B & AB calculated and written as 2×2 matrix

$$Q_c = \begin{bmatrix} 0 & 1/m \\ 1/m & -B/m^2 \end{bmatrix},$$

If determinant of $Q_c \Rightarrow |Q_c| \neq 0$, then above system is controllable, so

$$|Q_c| = 0 - \frac{1}{m^2} = \frac{-1}{m^2} \neq 0,$$

So, System is controllable

Kalman's test for observability

Output side y = x_1 (or)

$$y = 1.x_1 + 0.x_2 + 0.u(\text{or})$$

$$y = [1\ \ 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0.u$$

$$\rightarrow y = c\ \ x + 0.u$$

observability matrix

$$Q_o = [c^T A^T C^T]$$

$C^T \& A^T C^T$ calculated,

$Q_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and its determinant

$\left|Q_o\right| = 1 - 0 = 1 \neq 0$, so system is observable also.

So above given system is controllable as well as observable also.

07(c).

Sol:

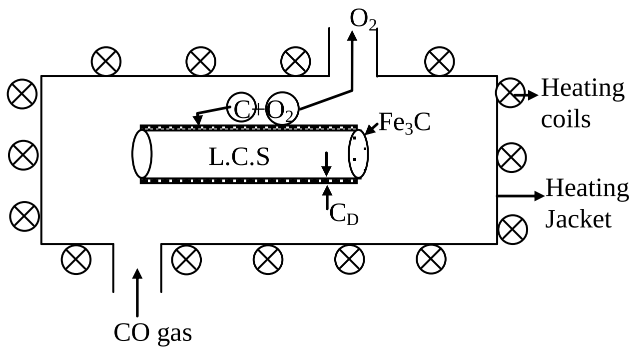
Material type	**Major alloying elements**	**Properties**	**Applications**
(i) Ferritic stainless steel	12%- 25% **Cr** , 0.1 - 0.35% **C**	Stronger than L.C.S Magnetic in nature Annealed condition strength is high strengthened by work hardening	Decorative trim, high pressure and high temp applications
(ii) Austenitic stainless steel	16- 26% **Cr,** 6 - 23% **Ni,** < 0.15% **C**	Shock resistant, difficult to machine without addition of sulphur, Highly anti corrosive, Non-magnetic	Domestic utensils, chemical processing equipment
(iii) Martensitic stainless steel	6- 18% **Cr**, up to 2% **Ni**, 0.1-1.5% **C**	High hard Cold workable, easily hardenable High creep and anti corrosive	Machine parts, knives
(iv) High speed steel	0.65-0.8**C,** 3.75-4 **Cr,** 17.25-18.75**W,** 0.9-1.3V, 0.1-0.4**Mn**, 0.2-0.4 **Si**	High hard with little ductility, wear resistant	Drills, milling cutters, tool bits, gear cutters, saw blades, punches, dies

08(a).

Sol:

(i) Carburising:

- Carburizing is a method of enriching the surface layer of low carbon steel with carbon in order to produce a hard case.
- This can be carried out by incorporating C atoms on to the envelope of the L.C.S component. It will be turned as hard by forming Fe_3C phase and is known as carburizing.



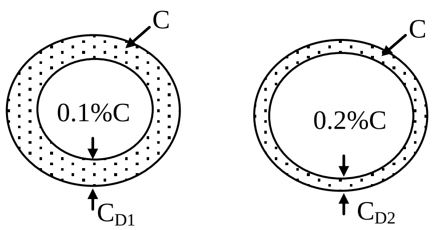
Case Depth = 0.5 mm/5 hour

- By heating the component to 850°C temperature CO gas is circulated, in the heating envelope.
- At that temp carbon monoxide decomposes into carbon and oxygen where carbon will penetrate into the component and oxygen goes out.
- Due to continuous penetration of carbon atom, the outer envelope will be produced with more iron carbide (Fe_3C) phase & turns as hard.
- The depth up to which form the surface of the component being hardened is known as case depth (C_D).

- Since the solubility of carbon is more in austenitic state than in ferritic state, fully austenitic state is required for carburizing.
- This can be achieved by heating the steel above the critical temperature. And diffusion of carbon is made by holding the heated steel in contact with carbonaceous material which may be a solid, a liquid or a gas.

Case 1:

Two L.C.S Component with different % of C has been carburized, then $CD_1 > CD_2$ because if the component contains low carbon content then penetration of external carbon atoms will be easy $\Rightarrow$ more case depth can be achieved.



$C_{D1} > C_{D2}$

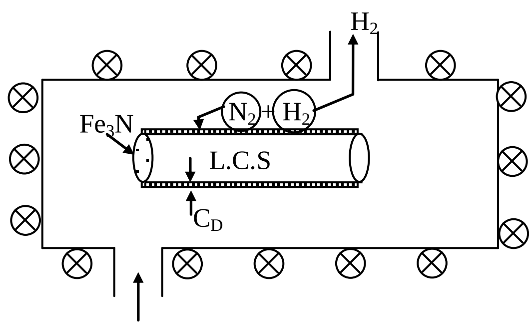
Note: In hardening process to achieve more D_h value, then carbon content should be high but in case of case hardening process to achieve to more case depth C_D, the carbon content should be low.

(ii) Nitriding:

- Nitriding is the process of enriching the surface of steel with nitrogen by holding for a prolonged period at temperature of ammonia (NH_3).
- In this process the machined and heat treated (hardening by heating to 930°C and quenching in oil, then tempering at 650° to obtain the required properties in core) components are heated to a temperature of 500°C for between 40 to 100 hours (depending on case depth) in a gas tight chamber through which ammonia is allowed to circulate.
- By incorporating Nitrogen atom on the outer envelope of L.C.S component it will be termed as hard by forming iron Nitride phase
  Case Depth = 0.5 mm /hrs.
- Ammonia dissociates according to the following reaction.

$$2\,NH_3 \rightarrow 3H_2 + 2N$$

- The atomic nitrogen thus formed diffuses into iron, forms hard nitrides by combining with iron and certain alloying elements present in steel. The alloying elements having more affinity for nitrogen are aluminum, chromium and molybdenum.



Obtaining more case depth in Nitriding process is difficult because:

The size of the nitrogen atom is large and inert in nature ⇒ more ammonia should be consumed to obtain more case depth ⇒ expensive.

As it is condition low carbon steel contain low corrosion resistance but after Nitriding process they possess extreme corrosion resistance due to inert nature of non nitride phase on the surface.

Advantages :

- good surface finish
- less distortion and cracks
- good wear resistance
- used for mass production
- better than carburizing

Disadvantages :

- long operational times 100 hours for 0.038mm depth
- all alloys steels can not be used
- special equipment is needed
- More oxidation due to prolonged heating

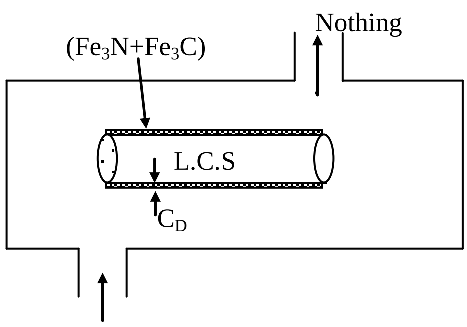
(iii) Cyaniding :

- During cyaniding the surface of steel is enriched with carbon and nitrogen by incorporating carbon & Nitrogen atoms simultaneously on to the outer envelope of the low carbon steels ⇒ it will be turned as hard by forming iron carbide & iron Nitride phases.
- In this process the components are immersed in a liquid bath of 30% NaCN, 40% Na_2CO_3 and 30% NaCl, maintained at a temperature of 800°C to 850°C.
- Then a measured amount of air is passed through the molten bath.
- Sodium cyanide reacts with oxygen of the air and is oxidized. The basic reactions in the bath are:

$$2\ NaCN + O_2 \rightarrow 2\ NaCNO$$

$$2\ NaCNO + O_2 \rightarrow Na_2\ Co_3 + CO + 2\ N$$

$$2\ CO \rightarrow CO_2 + C$$



- Carbon and nitrogen thus formed in atomic form diffuse into steel surface.
  Case Depth = 0.5 mm/10 hrs
- This process usually requires 30 to 90 minutes for completion.
- After cyaniding, the components are taken out and quenched in water or oil followed by low temperature tempering.

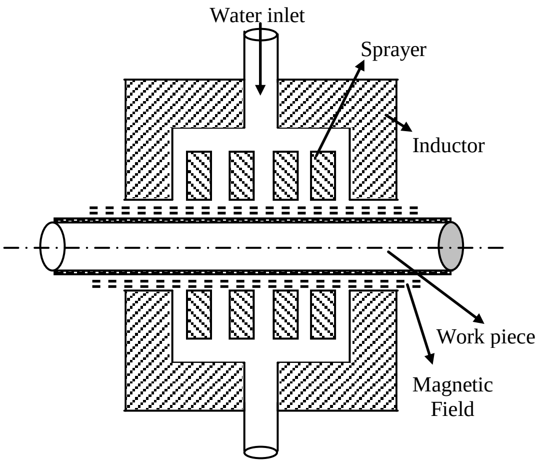
Advantages :

- can be applied to Low carbon and medium carbon steels
- bright finish in parts can be obtained
- cracks and distortions are minimized
- Most suitable for parts subjected to high loads.

Disadvantages :

- risk of splitting of poisonous salts
- unhealthy fumes are formed

(iv) Induction Hardening:



The disadvantage of flame hardening i.e., over-heating may be avoided by inducing heat electrically in the surface of steel.

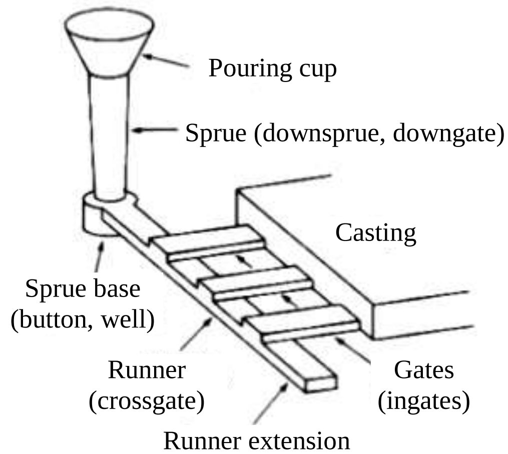
- In Induction hardening the heating time is only a few seconds.
- Heat generated in the work piece by induction is mostly confined to outer surface which is to be hardened.
- The depth to which heat penetrates is inversely proportional to the square root of frequency of the current. Hence, the hardened depth decreases with increase in frequency of the current.
- Similar to flame hardening, the induction hardened work piece is also subjected to low temperature tempering to relieve stresses.

08(b)(i).

Sol: The various elements of gating system are

Sprue: It is a circular cross-section minimizing turbulence and heat loss and its area is quantified from choke area and gating ratio. Ideally it should be large at top and small at bottom.

Sprue base : It is designed to restrict the free fall of molten metal by directing it in a right angle towards the runner. It aids in reducing turbulence and air aspiration. Ideally it should be shaped cylindrically having diameter twice as that of sprue exit and depth twice of runner.



Runner: Mainly slows down the molten metal that speeds during the free fall from sprue to the ingate. The cross section are of a runner should be greater than the sprue exit. It should also be able to fill completely before allowing the metal to enter the ingates. In systems where more than one ingate is present, it is recommended that the runner cross section area must be lowered after each ingate connection to ensure smooth flow.

Ingate : It directs the molten metal from the gating system to the mold cavity. It is recommended that ingate should be designed to reduce the metal velocity; they must be easy to fettle, must not lead to a hot spot and the flow of molten metal from the ingate should be proportional to the volume of casting region.

08(b)(ii).

Sol: Welding, soldering and brazing are the metal joining process. Each type of joining process has its own significance. Type of joining process to be applied for joining two parts depends on many factors. Below tabular comparison tells us the differences between the joining processes welding, soldering and brazing in aspects like strength comparison, temperature requirement, change in properties after joining, cost involved, heat treatment, preheating, etc.

S.No	Welding	Soldering	Brazing
1	Welding joints are strongest joints used to bear the load. Strength of the welded portion of joint is usually more than the strength of base metal.	Soldering joints are weakest joints out of three. Not meant to bear the load. Use to make electrical contacts generally.	Brazing joints are weaker than welding joints but stronger than soldering joints. This can be used to bear the load up to some extent.
2	Temperature required is 3800°C in welding joints.	Temperature requirement is up to 450°C in soldering joints.	Temperature may go to 600°C in brazing joints.
3	To join work pieces need to be heated till their melting point.	Heating of the work pieces is not required.	Work pieces are heated but below their melting point.
4	Mechanical properties of base metal may change at the joint due to heating and cooling.	No change in mechanical properties after joining.	May change in mechanical properties of joint but it is almost negligible.
5	Heat cost is involved and high skill level is required.	Cost involved and skill requirements are very low.	Cost involved and sill required are in between other two.
6	Heat treatment is generally required to eliminate undesirable effects of welding.	No heat treatment is required.	No heat treatment is required after brazing.
7	No preheating of workpiece is required before welding as it is carried out at high temperature.	Preheating of workpieces before soldering is good for making good quality joint.	Preheating is desirable to make strong joint as brazing is carried out at relatively low temperature.

08(c)(i) .

Sol: Drive systems commonly used are three types to actuate robotic joints.

They are

(i) Electric drives

(ii) Hydraulic

(iii) Pneumatic drives

Electric motors, such as servo motor, stepper motors are widely used in robotics. Due to advancement of electric motor technology, these are preferred in commercial robotic applications. These are easily compatible to computing systems.

Hydraulic and Pneumatic actuators such as piston-cylinder and rotary vane actuators are used to joint motions to accomplish linear (or) rotary movements.

Pneumatic drives used for smaller simple, robotic applications due to its limited load carrying and slow movement due to inertia.

Hydraulic drives used where large speed and heavy duty / load carrying applications. But these are not flexible as electric drives.

In general electric and hydraulic drives are preferred in sophisticated industrial robotics (or) Mechatronic systems.

08(c)(ii) .

Sol:

(i) Economic run length,

$$ERL = \sqrt{\frac{2C_oD}{C_c(1-(d/p))}}$$

where,

C_o = setup cost = Rs. 7.50

D = annual demand = 72000 bars/year

C_C = carrying cost = Rs. 1.50 /bar-year

d = daily demand rate

$= \frac{72000}{360}$ = 200 bars/day

p = daily production rate = 400 bars/day

$$ERL = \sqrt{\frac{2\times7.50\times72000}{1.50\left(1-\left(\frac{200}{400}\right)\right)}}$$

= 1200 bars/run

(ii) Optimal number of days of the run is

$$\text{Number of days} = \frac{12000\ \text{bars}}{400\ \text{bars/day}}$$

= 3 days