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ESE – 2019 MAINS
OFFLINE TEST SERIES

ELECTRICAL ENGINEERING

TEST – 14
SOLUTIONS

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SECTION -A

1(a)

Sol: Common three-phase base power is 200 MVA

L-L base voltage on the generator side is 22 kV

For fault calculations, static load of 40 MW is neglected in the reactance diagram

The pu reactance on the common base MVA of 200 MVA

Generator G: $0.2 \times \frac{200}{200} = 0.20$ pu

Transformer T_1: $0.12 \times \frac{200}{100} = 0.24$ pu

Trans former T_2: $0.10 \times \frac{200}{100} = 0.20$ pu

Transformer T_3: $0.08 \times \frac{200}{100} = 0.16$ pu

Transformer T_4: $0.08 \times \frac{200}{100} = 0.16$ pu

Synchronous motor

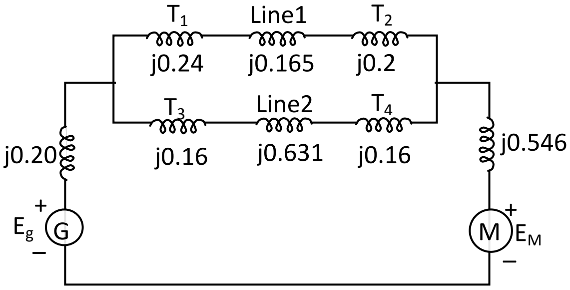
$$M = 0.18 \times \left(\frac{200}{60}\right) \times \left(\frac{10.5}{11}\right)^2$$

$= 0.546$ pu

Line 1: $\frac{40 \times 200}{(220)^2} = 0.165$ pu

Line 2: $\frac{55 \times 200}{(132)^2} = 0.631$ pu

The per unit reactance diagram of the system.



1(b)

Sol: $$\frac{X(z)}{z}=\frac{z^2-4z+5}{(z-1)(z-2)(z-3)}$$

$$\frac{X(z)}{z}=\frac{A}{z-1}+\frac{B}{z-2}+\frac{C}{z-3}$$

$$\frac{X(z)}{z}=\frac{1}{z-1}-\frac{1}{z-2}+\frac{1}{z-3}$$

$$X(z)=\frac{z}{z-1}-\frac{z}{z-2}+\frac{z}{z-3}$$

A. $2<|z|<3=(|z|>2)\cap(|z|<3)\cap(|z|>1)$

So, $x(n)=u(n)-(2)^n u(n)-(3)^n u(-n-1)$

B. $|z|>3=(|z|>1)\cap(|z|>2)\cap(|z|>3)$

So, $x(n)=u(n)-(2)^n u(n)+(3)^n u(n)$

C. $|z|<1=(|z|<1)\cap(|z|<2)\cap(|z|<3)$

So, $x(n)=-u(-n-1)+(2)^n u(-n-1)-(3)^n u(-n-1)$

1(c)

Sol: $$G(s)H(s)=\frac{K}{\left(1+\frac{1}{b}s\right)}\Rightarrow|G(j\omega)H(j\omega)|=\frac{K}{\sqrt{1+\left(\frac{1}{b}\omega\right)^2}}$$

at $\omega=5b\Rightarrow M=\dfrac{K}{\sqrt{1+\left[\frac{1}{b}(5b)\right]^2}}=\dfrac{K}{\sqrt{26}}$

$$M_{dB}=20\log\left(\frac{K}{\sqrt{26}}\right)=20\log K-20\log\sqrt{26}$$

$M_{dB}=20\log K-14.14\text{dB}$. ------------- (1)

From the Bode plot given at

$\omega=5b\Rightarrow$ slope = -20dB/dec.

at $\omega_1=b\Rightarrow M_1=20\log K$

$\omega_2=5b\Rightarrow M_2=?$

$$\text{slope} = \frac{M_2 - M_1}{\log \omega_2 - \log \omega_1}$$

$$-20 = \frac{M_2 - 20\log K}{\log(5b) - \log(b)} = \frac{M_2 - 20\log K}{\log\left(\frac{5b}{b}\right)}$$

$$M_2 - 20\log K = -20(\log 5) = -13.979$$

M_2 = 20logK – 13.979 --------------------- (2)

Equation (1) is exact analysis, equation (2) is approximate analysis.

Error = exact – approximate.

= (20logK – 14.14) – (20logK – 13.97)

= – 0.161 dB

Magnitude = 0.16 dB

1(d)

Sol: Electromagnetic torque, $T_e \propto E_2 I_2 \cos\theta_2$.

Since rotor circuit is purely resistive, $\cos\theta_2 = 1$.

$\therefore$ $T_e \propto E_2 I_2$

The standstill rotor voltage E_2 remains substantially constant; in view of this, $T_e \propto I_2$.

For torque balance, $T_e = T_L$

$\therefore$ Load torque $\propto I_2$

Here load torque $T_L \propto n_r$, But $T_e \propto I_2$

For torque balance, $T_e = T_L$

Or $I_2 \propto n_r$

$\propto n_s\,(1 - s)$,

At normal speed, $20 \propto n_s(1 - 0.03)$.

For slip s = 0.4, $I_2' \propto n_s(1 - 0.4)$

$$\therefore \quad \frac{I_2'}{20} = \frac{n_s(0.6)}{n_s(0.97)}$$

$$\text{or} \quad I_2' = \left(\frac{0.6}{0.97}\right)(20) = 12.88\,\text{A}$$

$$\therefore \quad \bar{E}_j + s\bar{E}_2 = \bar{I}_2' r_2$$

$$\text{Or} \quad \bar{E}_j + (0.4)(200) = (12.88)\,(0.3)$$

$$\therefore \quad \bar{E}_j = \mathbf{-76.136\ V}$$

$\therefore$ E_j should be in phase opposition to E_2.

For slip s = 0, $\dfrac{I_2'}{20} = \dfrac{n_s}{0.97 n_s}$

Or $I_2' = \dfrac{20}{0.97} = 20.62\text{ A}$

Now $\overline{E}_j + 0 = \overline{I}_2' r_2 = (20.62)(0.3)$

Or $\overline{E}_j$ **= 6.186 V**

$\therefore$ E_j should be in phase with E_2.

For slip, s = – 0.4, $\dfrac{I_2'}{20} = \dfrac{1.4 n_s}{0.97 n_s}$

Or $I_2' = \left(\dfrac{1.4}{0.97}\right)(20) = 28.88\text{A}$

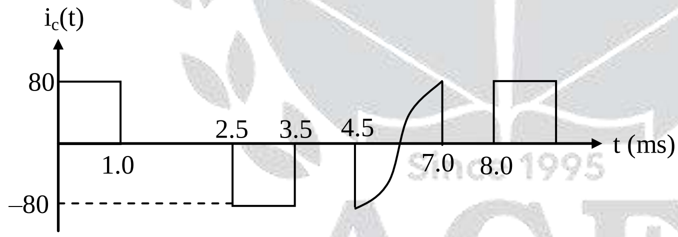
$\therefore$ E_j + (–0.4)(200) = (28.88) (0.3)

Or E_j = **88.664 V.**

$\therefore$ E_j should be in phase with E_2.

1(e)

Sol:



$$I_c = c.\left(\frac{dV}{dt}\right)$$

$$= 100\times10^{-6}\left(\frac{800-0}{1\times10^{-3}}\right) \qquad \text{for } 0 < t < 1\text{ms}$$

$$= 8\times10^{4}\times10^{-2}\,\text{m}^{-3} = 80\text{A}$$

For 4.5 < t < 7 ms

$$\frac{T}{2} = 2.5\,\text{ms}$$

$$\therefore\ \omega = \frac{2\pi}{T} = 400\,\pi$$

$$I_c = C\left(\frac{dV}{dt}\right)$$

$$= \left(100\times10^{-6}\right)\left(\frac{d}{dt}\right).\left(-637\sin\omega t\right)$$

$$= (100\times10^{-6})\ (-637)(\omega)(\cos\omega t)$$

$$= (10^{-4}).(-657),\ (400\pi).\ \omega t$$

$$X_c = -80\cos\omega t$$

$$\therefore\ P_{loss} = I_{rms}^2\times R$$

$$I_{rms}^2 = \frac{(80)^2\left(1\times10^{-3}\right)+(-80)^2(1\times10^{-3})+\left(\frac{80^2}{2}\right)\left(2.5\times10^{-3}\right)}{8\times10^{-3}}$$

$$= 10^2\times\left(\frac{3.25}{8}\right) = 2600$$

$$P_{loss} = I_{rms}^2\times R = 2600\times1.4\times10^{-3}$$

$$= 3.64\ \text{W}$$

2(a)

Sol: Time Scaling Property

The time scaling property states that, if

$$x(t)\xleftrightarrow{FS}C_n$$

$$\text{then}\qquad x(\alpha t)\xleftrightarrow{FS}C_n\ \text{with}\ \omega_0\to\alpha\omega_0$$

Proof: From the definition of Fourier series, we have

$$x(t) = \sum_{n=-\infty}^{\infty}C_n e^{jn\omega_0 t}$$

$$\therefore\ x(\alpha t) = \sum_{n=-\infty}^{\infty}C_n e^{jn\omega_0\alpha t} = \sum_{n=-\infty}^{\infty}C_n e^{jn(\alpha\omega_0)t} = FS^{-1}\left[C_n\right]$$

where $\omega_0\to\alpha\omega_0$

$\therefore\ x(\alpha t)\xleftrightarrow{FS}C_n$ with fundamental frequency of $\alpha\omega_0$ proved.

Time Differentiation Property

The time differentiation property states that, if

$$x(t)\xleftrightarrow{FS}C_n$$

$$\text{then}\qquad \frac{dx(t)}{dt}\xleftrightarrow{FS}jn\omega_0 C_n$$

Proof: From the definition of Fourier series, we have

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

Differentiating both sides with respect to t, we get

$$\frac{dx(t)}{dt} = \sum_{n=-\infty}^{\infty} C_n \frac{d\left(e^{jn\omega_0 t}\right)}{dt} = \sum_{n=-\infty}^{\infty} C_n (jn\omega_0) e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} (jn\omega_0 C_n) e^{jn\omega_0 t} = FS^{-1}[jn\omega_0 C_n]$$

$$\frac{dx(t)}{dt} \xleftrightarrow{FS} jn\omega_0 C_n \text{ Proved.}$$

2(b)(i)

Sol: A. T.F = G(s) = C(s I – A)$^{-1}$ B

$$(sI - A)^{-1} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \frac{1}{s^2+3s+2}$$

$$(sI - A)^{-1} B = (sI - A)^{-1} \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 2s \end{bmatrix} \frac{1}{(s+1)\,(s+2)}$$

$$G(s) = [1 \;\; 1] \;\; (sI - A)^{-1} B$$

$$= \frac{2(s+1)}{(s+1)(s+2)} = \frac{2}{(s+2)}$$

B. $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, u(t) = unit step $\rightarrow \frac{1}{s}$

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = (sI - A)^{-1} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} + (sI - A)^{-1} B\, U(s)$$

$$= \begin{bmatrix} \frac{1}{(s+1)\,(s+2)} \\ \frac{s}{(s+1)\,(s+2)} \end{bmatrix} + \begin{bmatrix} \frac{2}{s(s+1)(s+2)} \\ \frac{2}{(s+1)(s+2)} \end{bmatrix}$$

$Y(s) = X_1(s) + X_2(s)$

$y(t) = x_1(t) + x_2(t)$

$$\frac{1}{(s+1)\,(s+2)} = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\frac{s}{(s+1)\,(s+2)} = \frac{-1}{s+1} + \frac{2}{s+2}$$

$$\frac{1}{s(s+1)\,(s+2)} = \frac{1}{2}\frac{1}{s} - \frac{1}{s+1} + \frac{1}{2}\frac{1}{s+2}$$

$$X_1(s) = \frac{1}{s+1} - \frac{1}{s+2} + \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2}$$

$$= \frac{1}{s} - \frac{1}{s+1}$$

$$X_2(s) = -\frac{1}{s+1} + \frac{2}{s+2} + \frac{2}{s+1} - \frac{2}{s+2}$$

$$= \frac{1}{s+1}$$

$x_1(t) = 1\,u(t) - e^{-t}\,u(t)$,

$x_2(t) = e^{-t}\,u(t)$

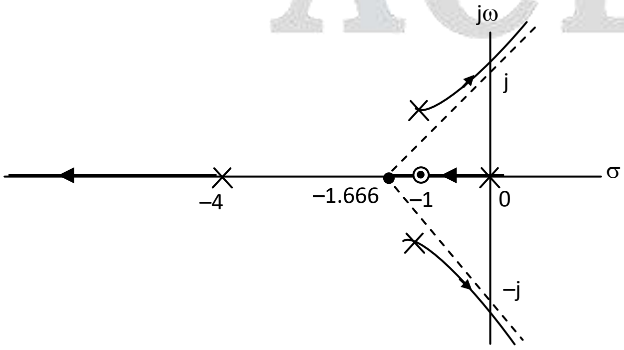
$y(t) = x_1(t) + x_2(t) = 1\,u(t)$

2(b)(ii)

Sol: The given characteristic equation

$s(s + 4)\,(s^2 + 2s + 2) + k\,(s + 1) = 0$

$$1 + \frac{k(s+1)}{s(s+4)(s^2+2s+2)} = 0$$



Angle of asymptotes

$$= \frac{(2\ell+1)}{P-Z} \times 180 \quad \ell = 0, 1, 2, ...$$

$= 60°, 180°, 300°$

Intercept of asymptotes means centroid

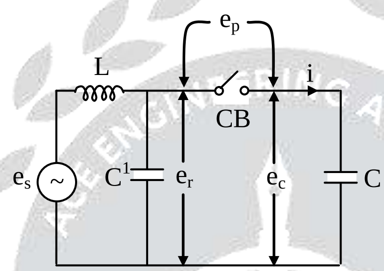
$$\text{Centroid} = \frac{\text{sum of poles} - \text{sum of zeros}}{P - Z}$$

$$= \frac{0-4-2+1}{3} = \frac{-5}{3} = -1.66$$

2(c)(i)

Sol: Switching of Capacitor Banks and Unloaded Lines:

Switching of capacitive loads (capacitor banks or unloaded lines) can lead to arc restriking in breakers, in which the voltage level across the capacitor builds up in steps to dangerous levels. This situation is illustrated in figure below from circuit point of view.



Capacitor switching

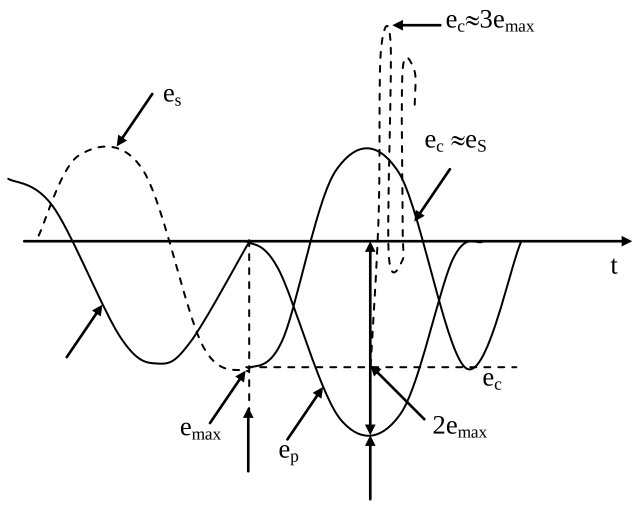
Normally C′ << C so that with the CB open,

$e_r \approx e_s$(system voltage)

Then the voltage across CB poles when the arc is interrupted is

$e_p = e_c - e_r$

The capacitive current (i) leads to voltage e_s by 90°, as shown in figure below. The arc extinguishes at i = 0 when the capacitor C is charged to $-e_{max}$, and stays charged beyond this time. The voltage e_p rises to $2e_{max}$ when the arc would be most likely to restrike. Arc restriking is equivalent to switching a capacitor charged to $-e_{max}$ to voltage source of value $+e_{max}$ through the inductance L. The amplitude of natural frequency oscillation would initially be $2e_{max}$, as indicated in the figure. The capacitor voltage would then rise to a peak value of $3e_{max}$, at which point the current I becomes zero and quenches once again. If this sequence of events repeats once again the capacitor voltage would rise to $5e_{max}$. It can lead to puncture of capacitor bank (or breakdown of line insulation). This phenomenon in capacitor switching is also avoided by resistance switching.



2(c)(ii)

Sol: A. The operating current of the relay = 1 × 0.50 = 0.5 A

The secondary voltage = $\frac{2VA}{0.5} = 4V$

The CT secondary voltage when current is

15 times, the relay setting = 15 × 4 = 60 V

The knee voltage must be slightly greater than 60 V.

B. $E = 4.44\, B_m A f N$

Where flux density, $B_m = 1.4$ Wb/m^2 Total turns, N = 50,

Frequency, f = 50 Hz

Area of cross section,

$$A = \frac{60}{4.44 \times 1.4 \times 50 \times 50}$$

$= 3.86 \times 10^{-3}$ sq.m = 38.6 sq.cm

3(a)(i)

Sol: Given data: $R = 20\Omega$, $V_S = 200V$

Chopper voltage drop = 1.5V, f = 2 kHz and

$\alpha = 0.5$, P_i, P_0, η = ?

Average output voltage $V_0 = \alpha\,(V_S - 1.5)$

$= 0.5\,(200 - 1.5)$

$= 99.25$ V

R.MS value of output voltage

$$V_{r.m.s} = \sqrt{\alpha}\left[V_S - 1.5\right] = \sqrt{0.5}\left[200 - 1.5\right] = 140.36 \text{ V}$$

Power delivered to the load,

$$P_0 = \frac{V_{rms}^2}{R} = \frac{(140.36)^2}{20} = 985.05 \text{ W}$$

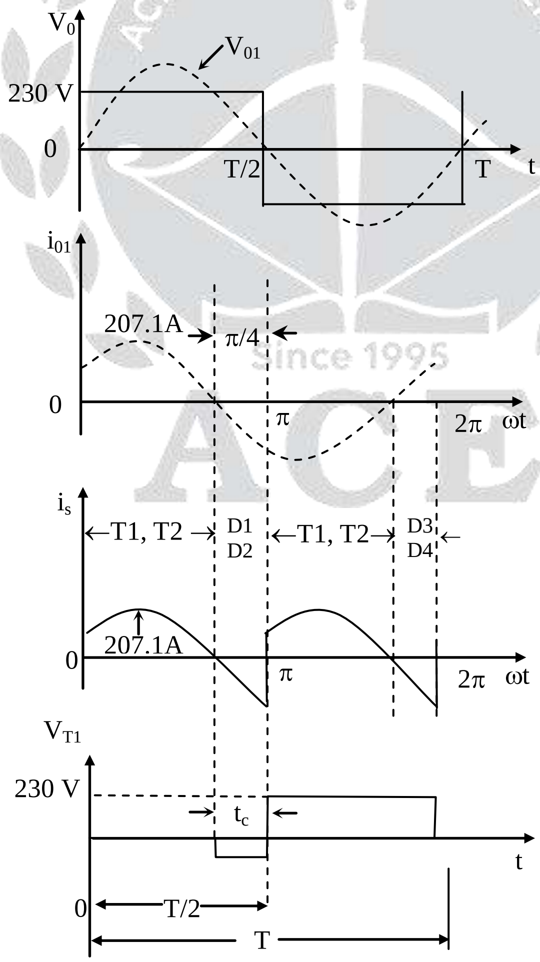
Input power $P_i = V_S \cdot I_0 = 200\left(\frac{99.25}{20}\right) = 992.5\,\text{W}$

$$\text{Chopper efficiency } \eta = \frac{\text{output power}}{\text{Input power}} \times 100$$

$$= \frac{985.05}{992.5} \times 100 = 99.24\%$$

3(a)(ii)

Sol: The load voltage waveform V_0 and its fundamental component V_{01} are shown in figure



Rms values of load voltage is

$$V_{01} = \frac{4Vs}{\pi\sqrt{2}} = \frac{4 \times 230}{\pi\sqrt{2}} = 207.1\text{V}$$

Rms value of current, $I_{01} = \frac{V_{01}}{Z_1}$

$$= \frac{V_{01}}{\left[R^2 + \left(\omega L - \frac{1}{\omega L}\right)^2\right]^{1/2}}$$

$$= \frac{207.1}{\left[1^2 + (-1)^2\right]^{1/2}} = \frac{207.1}{\sqrt{2}} = 146.46\text{A}$$

$$\phi = \tan^{-1}\frac{X_L - X_c}{R}$$

$$= \tan^{-1}(-1) = -45°$$

The fundamental component of current i_{01} as function of time is

$$i_{01} = \sqrt{2}\ I_{01}\sin(\omega t - \phi_1)$$

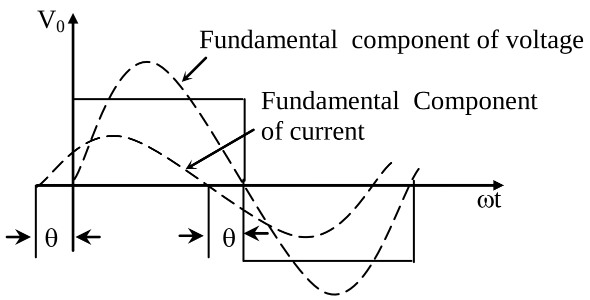
$$= \sqrt{2}\ \frac{207.1}{\sqrt{2}}.\ \text{Sin}(\omega t + 45°)$$

$$= 207.1\sin(\omega t + 45°)$$

Load current i_{01} and source current i_s are plotted in Fig. and the conducting components are also indicated

3(a)(iii)

Sol: The value of C should be such that RLC load is underdamped. Moreover, when load voltage passes through zero, the load current must pass through zero before the voltage wave, i.e. the load current must lead the load voltage by an angle θ as shown in fig.



From this phasor diagram,

$$\tan\theta = \frac{X_C - X_L}{R}$$

Here $X_C > X_L$ as the current is leading the voltage. Now (θ/ω) must be at lease equal to circuit turn–off time, i.e. 1.5 × 10 μsec

$$\therefore \quad \frac{\theta}{\omega} = 1.5 \times 10^{-6} \text{ sec}$$

$$\text{Now} \quad f = \frac{10^3}{0.1} = 10^4 \text{ Hz}$$

$$\therefore \theta = 2\pi \times 10^4 \times 15 \times 10^{-6} = 0.9424778 \text{ rad} = 54°$$

$$\therefore \quad \tan 54° = \frac{X_C - 10}{2}$$

or $$X_C = 12.752764 = \frac{1}{2\pi \times 10^4 \times C}$$

or $$C = 1.248\ \mu F$$

3(b)

Sol: A. Let us first convert the SC data to pu on 5 MVA base/phase.

For primary, $V_B = 6.35$ kV

$$I_B = \frac{5000}{6.35} = 787.4\text{A}$$

For secondary, $V_b = 1.91$ kV

$$I_B = \frac{5000}{1.91} = 2617.8\text{A}$$

Converting the given test data to pu yields:

Test No.	Winding involved	V	I
1	P and S	0.0787	0.5
2	P and T	0.1417	0.5
3	S and T	0.1212	0.5

From tests 1,2 and 3, respectively.

$$X_{12} = \frac{0.0787}{0.5} = 0.1574 \text{ pu}$$

$$X_{13} = \frac{0.1417}{0.5} = 0.2834 \text{ pu}$$

$$X_{23} = \frac{0.1212}{0.5} = 0.2424\,\text{pu}$$

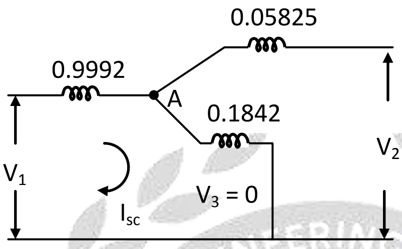
Therefore, X_1 = 0.5(0.1574 + 0.2834 –0.2424) = 0.0992 pu

X_2 = 0.5 (0.2424 + 0.1574 – 0.2834) = 0.05825 pu

X_3 = 0.5 (0.2834 + 0.2424 – 0.1574) = 0.1842 pu

B The base line-to-line voltage for the Y-connected primaries is $\sqrt{3}\times 6.35$ = 11kV, i.e. The bus voltage is 1pu. From figure for a short-circuit at the terminals of the tertiary, V_3 = 0.

Then



$$I_{sc} = \frac{V_1}{X_1 + X_3} = \frac{V_1}{X_{13}} = \frac{1.00}{0.2834} = 3.53\,\text{pu}$$

SC current primary side = 3.53 × 787.4 = 2779.5 A

SC current tertiary side = $3.53 \times \dfrac{5000 \times 1000}{400/\sqrt{3}} = 76424$ A (line current)

Neglecting the voltage drops due to the secondary load current, the secondary terminal voltage is the voltage at the junction point A , i.e

$V_A = I_{SC}X_3$ = 3.53 × 0.1842 = 0.6502 pu

$$V_A(\text{actual}) = 0.6502 \times 1.91\sqrt{3} = 2.15\,\text{kV(line-to-line)}$$

3(c)(i)

Sol: $H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n} = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3}$

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} = 1 + 3z^{-1} + 7z^{-2} + 10z^{-3} + 10z^{-4} + 7z^{-5} + 2z^{-6}$$

$$X(z) = \frac{Y(z)}{H(z)} = \frac{1 + 3z^{-1} + 7z^{-2} + 10z^{-3} + 10z^{-4} + 7z^{-5} + 2z^{-6}}{1 + 2z^{-1} + 3z^{-2} + 2z^{-3}}$$

$$\begin{array}{r|l} & 1+z^{-1}+2z^{-2}+z^{-3} \\ \hline 1+2z^{-1}+3z^{-2}+2z^{-3} & 1+3z^{-1}+7z^{-2}+10z^{-3}+10z^{-4}+7z^{-5}+2z^{-6} \\ & 1+2z^{-1}+3z^{-2}+2z^{-3} \\ \hline & z^{-1}+4z^{-2}+8z^{-3}+10z^{-4} \\ & z^{-1}+2z^{-2}+3z^{-3}+2z^{-4} \\ \hline & 2z^{-2}+5z^{-3}+8z^{-4}+7z^{-5} \\ & 2z^{-2}+4z^{-3}+6z^{-4}+4z^{-5} \\ \hline & z^{-3}+2z^{-4}+3z^{-5}+2z^{-6} \\ & z^{-3}+2z^{-4}+3z^{-5}+2z^{-6} \\ \hline & 0 \end{array}$$

$X(z) = 1+z^{-1}+2z^{-2}+z^{-3}$

$x(n) = \{1, 1, 2, 1\}$

3(c)(ii)

Sol: $h(t) = e^{-at}.\cos bt.u(t) \qquad h(n) = e^{-anT}.\cos(bnT)u(nT)$

$$H(z) = \sum_{n=0}^{\infty} h(n)z^{-n} = \sum_{n=0}^{\infty} e^{-anT}.\cos bnT.z^{-n}$$

$$= \sum_{n=0}^{\infty} e^{-anT}z^{-n}\left[\frac{e^{jnbT}+e^{-jnbT}}{2}\right]$$

$$= \frac{1}{2}\left[\sum_{n=0}^{\infty}\left(e^{-(a-jb)T}z^{-1}\right)^{n} + \left(e^{-(a+jb)T}z^{-1}\right)^{n}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1-e^{-(a-jb)T}z^{-1}} + \frac{1}{1-e^{-(a+jb)T}z^{-1}}\right]$$

$$H(z) = \frac{1}{2}\left[\frac{1-e^{-(a+jb)T}z^{-1}+1-e^{-(a-jb)T}z^{-1}}{\left(1-e^{-(a-jb)T}z^{-1}\right)\left(1-e^{-(a+jb)T}z^{-1}\right)}\right]$$

$$H(z) = \frac{1}{2}\left[\frac{1-e^{-aT}.e^{-jbT}z^{-1}+1-e^{-aT}z^{-1}.e^{jbT}}{1-e^{-(a+jb)T}z^{-1}-e^{-(a-jb)T}z^{-1}+e^{-(a+jb+a-jb)T}z^{-2}}\right]$$

$$H(z) = \frac{1}{2}\left[\frac{2-e^{-aT}z^{-1}\left[e^{-jbT}+e^{jbT}\right]}{1-e^{-aT}z^{-1}\left[e^{-jbT}+e^{jbT}\right]+e^{-2aT}z^{-2}}\right]$$

$$H(z) = \frac{1}{2}\left[\frac{2-2e^{-aT}\cos bTz^{-1}}{1-2e^{-aT}z^{-1}\cos(bT)+e^{-2aT}z^{-2}}\right]$$

$$H(z) = \frac{1-e^{-aT}\cos bTz^{-1}}{1-2e^{-aT}z^{-1}\cos(bT)+e^{-2aT}z^{-2}}$$

4(a)(i)

Sol: $\dfrac{\text{Per phase.h.v}}{\text{Per phase 1.v.}} = \dfrac{11{,}000}{433/\sqrt{3}} = \dfrac{11{,}000}{250} = 44$

$\therefore$ H.V. leakage impedance referred to L.V side

$$= (300 + j1500)\left(\frac{1}{44}\right)^2 = (0.155 + j0.775)\ \Omega \text{ per phase.}$$

Total per phase impedance between the transfer secondary and the load

$$= (0.155 + j0.775) + (0.2 + j1) + (0.5 + j1)$$

$$= 0.855 + j2.775\Omega = R + jX$$

Per phase load current $I_L = \dfrac{10{,}000}{\sqrt{3}(400)} = 14.43\text{A}$

Total impedance drop per phase

$$= I_L\,[R\cos\theta_2 + X\sin\theta_2]$$

$$= 14.43\,[0.855 \times 0.8 + 2.775 \times 0.6]$$

$$= 14.43\,(2.359)$$

$$= 34.05\text{ V}$$

Per phase voltage to be maintained at the load terminals $= \dfrac{400}{\sqrt{3}} = 231\text{ V}$

Per phase voltage that must be maintained at transformer L.V terminals

$$= 231 + 34.05 = 265.05\text{ V}$$

At no load, the transformers L.V terminal voltage $= \dfrac{433}{\sqrt{3}} = 250\text{ V}$

$\therefore$ The voltage boost that the tap-changer must provide = 265.05 – 250 = 15.05 V

$\therefore$ Tap setting = $\dfrac{15.05}{250} \times 100 = 6.02\%$

Tap down if the tapped coils are on the h.v. side or tap up if the tapped coils are on the 1.v. side.

4(a)(ii)

Sol: At no load, the counter emf is

$$E_{a1} = V_t - I_{a1}\, r_a$$

$$= 230 - 3.33 \times 0.3 = 229\text{ V.}$$

Field current, $I_f = \dfrac{230}{160} = 1.44\text{A}$

At full load, $I_{a2} = I_L - I_f = 40 - 1.44 = 38.56$ A.

$\therefore$ Counter e.m.f at full load is

$$E_{a2} = 230 - 38.56 \times 0.3 = 218.43 \text{ volts}$$

At full load, the field flux is

$$\phi_2 = 0.96\, \phi_1 \text{ (given)}$$

The counter e.m.f E_a is given by

$$E_a = K_a \phi\, \omega_m$$

$$\therefore \quad \frac{E_{a1}}{E_{a2}} = \frac{K_a\phi_1\omega_{m1}}{K_a\phi_2\omega_{m2}} = \frac{\phi_1 n_1}{\phi_2 n_2}$$

$$\therefore \quad \frac{229}{218.43} = \frac{1000 \times \phi_1}{n_2(0.96\phi_1)}$$

$\therefore$ Full load speed, $n_2 = 993.6$ r.p.m

At full load, $E_{a2} = K_a\, \phi_2\, \omega_m$

or $$K_a\phi_2 = \frac{218.43 \times 60}{2\pi \times 995}$$

$\therefore$ Electromagnetic, or developed, torque at full load, $T_e = K_a\phi_2 I_{a2}$

$$= \frac{218.43 \times 60}{2\pi \times 995} \times 38.56 = 80.95 \text{ Nm}$$

4(b)

Sol: (i) AC voltage controllers are semiconductor based circuits which convert fixed alternating voltage directly to variable alternating voltage without a change in the frequency.

Advantages:

1. They have high efficiency, flexibility in control, compact size and less maintenance.
2. AC voltage controllers are also adaptable for closed-loop control systems.
3. No complex commutation circuitry is required in these controllers as ac voltage controllers are phase-controlled devices, thyristors and triacs are line commutated.

Disadvantages:

The main disadvantages of ac voltage controllers is the introduction of objectionable harmonics in the supply current and load voltage waveforms, particularly at reduced output voltage levels.

(ii) 1-Phase AC voltage Regulator/ controller with RL-Load:

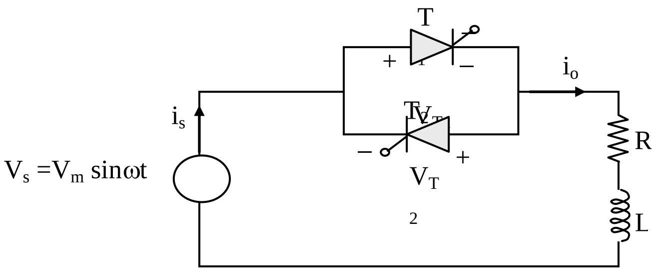


Fig.. (a)

Figure (a) shows 1-ϕ ACVR (or) ACVC with RL load.

Figure (b) shows waveforms of 1-ϕ ACVR with RL load for v_0 (output voltage), i_0 (output current), (Source current) i_s,

When T_1 is ON:

During zero to π, T_1 is forward biased

- AT $\omega t = (\alpha \geq 0)$ T_1 is triggred and $i_0 = i_{T1}$ starts building up through load.
- At $\omega t = \pi$ load and source voltages are zero but the current is not zero because of presence of inductance in load circuit.
- At $\beta > \pi$ load current reduces to zero.
- After π, T_1 is reverse biased but does not turn off because i_0 is not zero.
- At β only, when i_0 is zero, T_1 is turned off as it is already reverse biased.
- From β to $(\pi + \alpha)$, no current exists in the power circuit, therefore $V_0 = 0$, $V_{T1} = -V_s$ and $V_{T2} = V_s$

When T_2 is ON:

T_2 is turned on at $(\pi + \alpha) > \beta$

During T_2, i_0 is in opposite direction to i_s hence v_0 follows V_s in reverse direction.

$$R\, i_0 + L\frac{di_0}{dt} = V_m \sin\omega t$$

Where $\sqrt{R^2 + (\omega L)^2}$ = Load impedance

$\phi = \tan^{-1}\dfrac{\omega L}{R}$ = load p.f angle.

V_0 is same as V_s because no exponential term

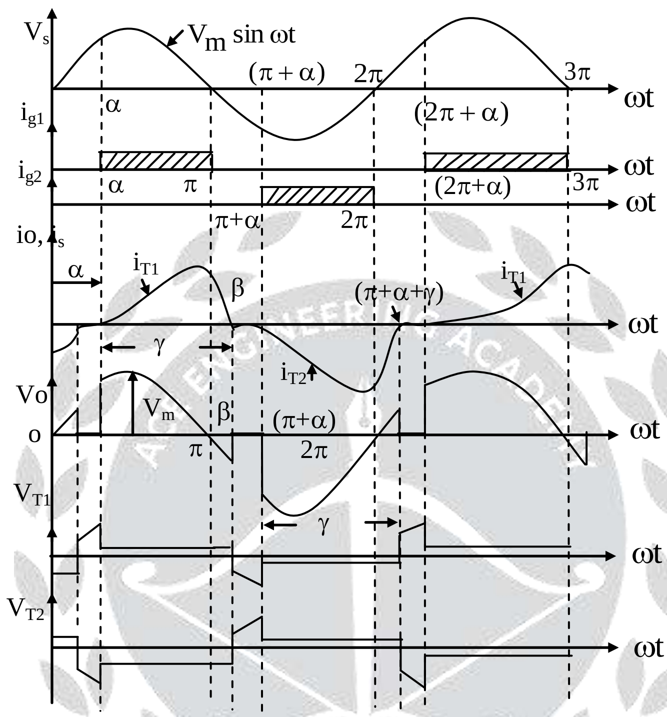
RMS output Voltage:

$$V_{0r}^2 = \frac{1}{\pi}\int_{\alpha}^{\beta} V_m^2 \sin^2\omega t\, d(\omega t)$$

$$= \frac{V_m^2}{2\pi}\int_{\alpha}^{\beta}(1-\cos 2\omega t)\,d\omega t$$

$$= \frac{V_m^2}{2\pi}\left[(\beta-\alpha)\frac{-\sin 2\beta-\sin 2\alpha}{2}\right]$$

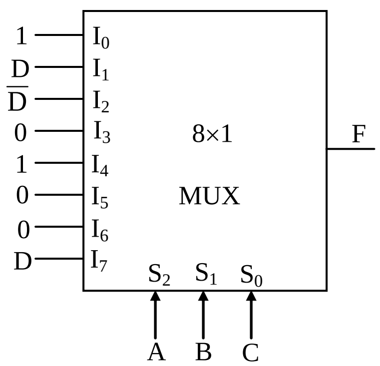
$$V_{or} = V_m\left[\frac{1}{2\pi}\left\{(\beta-\alpha)+\frac{\sin 2\alpha-\sin 2\beta}{2}\right\}\right]^{1/2}$$



4(c)(i)

Sol: Given F(A,B,C,D) = $\sum m(0,1,3,4,8,9,15)$

	$\bar A\bar B\bar C$	$\bar A\bar BC$	$\bar AB\bar C$	$\bar ABC$	$A\bar B\bar C$	$A\bar BC$	$AB\bar C$	ABC
$\bar D$	0	2	4	6	8	10	12	14
D	1	3	5	7	9	11	13	15
	$I_o = 1$	$I_1 = D$	$I_2 = \bar D$	$I_3 = 0$	$I_4 = 1$	$I_5 = 0$	$I_6 = 0$	$I_7 = 0$



4(c)(ii)

Sol: (A) $\frac{256k}{32k} = 8\text{chips}$

(B) 256k = 2^{18}

$\Rightarrow$ 18 address lines for memory

32k = 2^{15}

$\Rightarrow$ 15 address lines/chip

(C) For 256k, 18 address lines are required

For 32k, 15 address lines are required

$\Rightarrow$ (18 – 15) = 3 address lines.

So, a 3 × 8 decoder is used.

SECTION –B

5(a)

Sol: The turn-off process in a GTO

Before the initiation of turn-off process a GTO carries a steady current I_a. The total turn off time t_q is subdivided into three different periods. $t_q = t_s + t_f + t_t$

t_s = Storage period

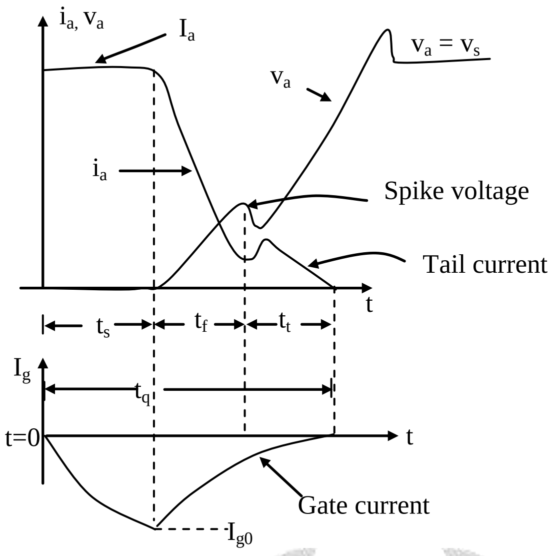
t_f = fall period

t_t = tail period

Initiation of turn-off process starts as soon as negative gate current begins to flow after t = 0. The rate of rise of this gate current depends upon the gate circuit inductance and the gate voltage applied. During the storage period. Anode current I_a and voltage V_a remains constant. Termination of the storage period is indicated by a fall in I_a and rise in V_a.

During storage time t_s, the negative gate current rises to a particular value and prepares the GTO for turning off by flushing out the stored carries.

The anode current falls to a certain valve, this interval is known as the fall time t_f.



After t_f, anode current i_a and anode voltage V_a keep moving towards their turn-off values for a time t_t called tail time. After t_t, anode current reaches zero value and V_a undergoes a transient overshoot and then stabilizes to its off-state value equal to the source voltage applied to the anode circuit. The turn-off process is complete when tail current reaches zero.

GTO has the following advantages over on SCR

(i) GTO has faster switching speed.

(ii) Its surge current capability is comparable with an SCR.

(iii) It has more $\frac{di}{dt}$ rating at turn-on

(iv) GTO circuit configuration has lower size and weight as compared to SCR circuit unit.

(v) GTO unit has higher efficiency because an increase in gate-drive power loss and on-state loss is more than compensated by the eliminated of forced commutation losses

(vi) GTO unit has reduced acoustical and electromagnetic noise due to elimination of commutation chokes.

Disadvantages of GTO as compared to a Conventional thyristor:

(i) Magnitude of latching and holding current is more in a GTO

(ii) On state voltage drop and the associated loss is more in a GTO

(iii) Due to the multi cathode structure of GTO, triggering gate current is higher than that required for a conventional SCR

(iv) Gate drive circuit losses are more

(v) Its reverse-voltage blocking capability is less than it’s forward-voltage blocking

5(b)

Sol:

$$F(A,B,C,D,E) = A + D\overline{E}$$

$$\text{16 minterms}\left\{\begin{array}{ccccc} A & B & C & D & E \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ & . & & . & \\ 1 & 1 & 1 & 1 & 1 \end{array}\right. \qquad \overset{D\overline{E}}{\left.\begin{array}{ccccc} A & B & C & D & E \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ & . & & . & \\ & . & & . & \\ 1 & 1 & 1 & 1 & 0 \end{array}\right\}} \text{8 minterms.}$$

$$\left.\begin{array}{ccccc} A & B & C & D & E \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right\} \text{4 common minterms}$$

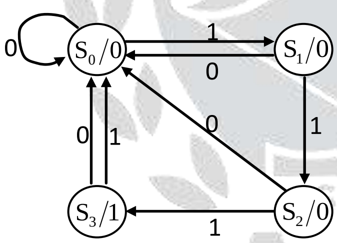
So, the number of non-redundant minterms = 16 + 8 – 4

= 20

5(c)

Sol:

The state diagram is shown below.



encoding:
S_o : 00
S_1 : 01
S_2 : 10
S_3 : 11

State table:

Present State input A B x	Next state A^+ B^+	Output Y
0 0 0	0 0	0
0 0 1	0 1	0
0 1 0	0 0	0
0 1 1	1 0	0
1 0 0	0 0	0
1 0 1	1 1	0
1 1 0	0 0	1
1 1 1	0 0	1

$$A^+ = \overline{A}Bx + A\overline{B}x = \left(\overline{A}B + A\overline{B}\right)x$$

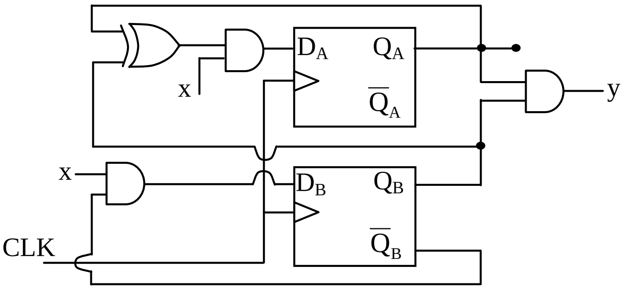
$$B^+ = \overline{A}\,\overline{B}x + A\overline{B}x = \overline{B}x$$

$$y = AB\overline{x} + ABx = AB$$

Implementation using D flip - flops:

$D_A = A^+ = (\overline{A}B + A\overline{B})x = (A \oplus B)x$

$D_B = B^+ = \overline{B}x;$ output, $y = AB$



5(d)

Sol: (i) $G(s)\Big|_{\tau_D = 0} = G_1(s) = \dfrac{1}{s(s+1)(s+2)}$

$$\left|G_1(s)\right|_{\omega = \omega_{gc} = 0.466} = 1$$

i.e., gain cross over frequency ω_{gc}= 0.466 rad/sec

$$PM = 180° + \angle \frac{1}{j\omega(j\omega+1)(j\omega+2)}$$

$$= 180° - \left(90° + \tan^{-1}\frac{\omega_{gc}}{1} + \tan^{-1}\frac{\omega_{gc}}{2}\right)$$

$$= 180° - (90° + 25° + 13°)$$

PM = 52°

(ii) If dead time increases the stability of the system decreases.

(iii) For just stability PM = 0°

GM = 0 dB

$$PM = 0° = 180° + \angle G(s)\Big|_{\omega = \omega_{gc}}$$

$$\Rightarrow \quad 0 = \pi + \angle \frac{e^{-j\omega\tau_D}}{j\omega(j\omega+1)(j\omega+2)}\Bigg|_{\omega = \omega_{gc}}$$

$$\Rightarrow \quad 0 = \pi - \left(\omega_{gc}\tau_D + \frac{\pi}{2} + \tan^{-1}\frac{\omega_{gc}}{1} + \tan^{-1}\frac{\omega_{gc}}{2}\right)\Bigg|_{\omega_{gc} = 0.466}$$

$\Rightarrow \quad \omega_{gc}\tau_D = 52°$

$\Rightarrow \quad \omega_{gc}\tau_D = 0.9075$

$\Rightarrow \quad \tau_D = \dfrac{0.9075}{0.466} = 1.947$

$\tau_D = 1.947$

For $\tau_D = 1.947$ system is just stable

$\tau_D < 1.947$ stable

$\tau_D > 1.947$ unstable

5(e)

Sol: (i) Full-load armature current per phase, $I_a = \dfrac{500}{\sqrt{3}\times 11} = 26.244$ A

Short-circuit load loss at half-full load

$$= 3\left(\frac{I_a}{2}\right)^2 \times r_a + \text{stray-load loss, which is zero here}$$

$$= 3\left(\frac{26.244}{2}\right)^2 \times 4 = 2066.24 \text{ W}$$

Total loss at half-full load = 1500+2500+2066.24+1000

= 7066.24 W

$$\text{Efficiency at half-full load} = \left[1 - \frac{7066.24}{500{,}000\times\frac{1}{2}\times 0.8 + 7066.24}\right]\times 100 = 96.587\%$$

(ii) For maximum efficiency,

Variable losses, $3I_{am}^2 r_a$ = rotational loss + field-circuit loss

$$3I_{am}^2 .4 = 1500 + 2500 + 1000 = 5000\,\text{W}$$

The current I_{am} at which maximum efficiency occurs is given by

$$I_{am} = \sqrt{\frac{5000}{12}} = 20.412 \text{ A}$$

Output maximum efficiency= $3V_t.\ I_{am}\ .\cos\theta$

$$= 3\times\frac{11000}{\sqrt{3}}\times 20.412\times 0.8 = 311{,}111.54 \text{ W}$$

Total losses at maximum efficiency = $2\times$ 5000 = 10,000 W

$$\therefore \text{ Maximum efficiency} = \left[1 - \frac{10{,}000}{311{,}111.54 + 10{,}000}\right] \times 100 = 96.886\%$$

6(a)(i)

Sol: (a) From the fig. we have,

$P(m_1) = 1 - p(m_0) = 1 - 0.5 = 0.5$

$P(r_0/m_0) = 1 - p(r_1/m_0) = 1 - p = 1 - 0.1 = 0.9$

$p(r_1/m_1) = 1 - p(r_0/m_1) = 1 - q = 1 - 0.2 = 0.8$

$\therefore p(r_0) = p(r_0/m_0)p(m_0) + p(r_0/m_1)p(m_1)$

$= 0.9\,(0.5) + 0.2(0.5)$

$p(r_0) = 0.55$

$p(r_1) = p(r_1/m_0)p(m_0) + p(r_1/m_1)p(m_1)$

$= 0.1(0.5) + 0.8(0.5)$

$\therefore p(r_1) = 0.45$

(b) using baye's rule, we have,

$$p(m_0/r_0) = \frac{p(m_0)p(r_0/m_0)}{p(r_0)} = \frac{(0.5)(0.9)}{0.55} = 0.818$$

(c) similarly,

$$p(m_1/r_1) = \frac{p(m_1)p(r_1/m_1)}{p(r_1)} = \frac{(0.5)(0.8)}{0.45} = 0.889$$

(d) $p_e = p(r_1/m_0)p(m_0) + p(r_0/m_1)p(m_1)$

$= 0.1(0.5) + 0.2(0.5)$

$\therefore \quad p_e = 0.15$

(e) The probability that the transmitted signal is correctly read at receiver is

$p_e = p(r_0/m_0)\, p(m_0) + p(r_1/m_1)\, p(m_1)$

$= 0.9(0.5) + 0.8(0.5)$

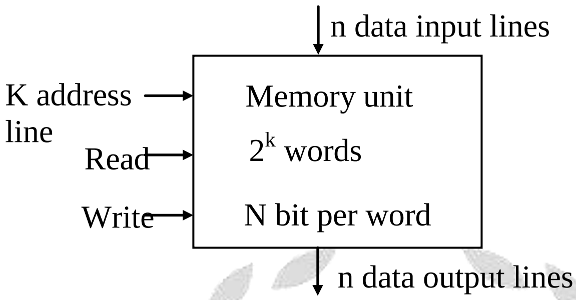
$p_e = 0.85$

(or) $p_e = 1 - p_e = 1 - 0.15 = 0.85$

6(a)(ii)

Sol: A memory unit is a collection of storage cells together with associated circuits needed to transfer information in and out of the device. The time it takes to transfer to or from any desired random location is always the same, hence the name Random-Access memory abbreviated as RAM. The Capacity of a memory unit is usually stated as the total number of bytes that it can store.

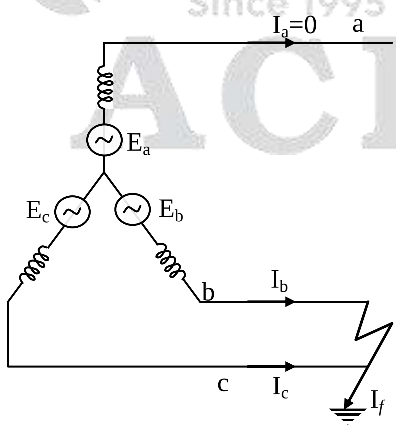
The communication between a memory and its environment is achieved through data input and output lines, address selection lines, and control lines that specify the direction of transfer. The block diagram of the memory unit is shown below.



The n data input lines provide the information to be stored in memory and the n data output lines supply the information coming out of memory. The K address lines specify the particular word chosen among the many available. The write input causes binary data to be transferred into the memory, and the read input causes binary data to be transferred out of memory.

6(b)(i)

Sol: **Double Line to Ground Fault (L-L-G)** Bolted fault means double line to ground fault at F of unloaded alternator with the fault impedance being $Z_f = 0$. The current and voltages at the fault conditions are expressed as



$I_a = 0 = I_{a0} + I_{a1} + I_{a2} = 0$ …. (1)

$$\because \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ I_c \end{bmatrix}$$

Hence, $I_{a0} = \frac{1}{3}(I_b + I_c)$

$(I_b+I_b) = 3I_{a0}$

$I_{a0} + a^2I_{a1} + aI_{a2} + I_{a0} + aI_{a1} + a^2I_{a2} = 3I_{a0}$

$I_{a1} + I_{a2} = I_{a0}$ ……….. (2)

$V_b = V_c = 0$

$V_{a0} + a^2V_{a1} + aV_{a2} = V_{a0} + aV_{a1} + a^2V_{a2}$

$V_{a1} = V_{a2}$ ……….. (3)

The symmetrical components of voltages are given by

$$\begin{bmatrix} V_{a1} \\ V_{a2} \\ V_{a3} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

or, $V_{a1} = V_{a2}$

Substituting equation (3) from equation (4),

$$V_{a0} - V_{a1} = \frac{1}{3}(2 - a - a^2)V_b$$

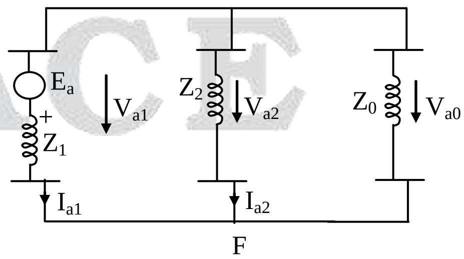
$\therefore\ V_{a0} = V_{a1}$ ....(4)

By observing equation (1),(2) (3) and (4), the Sequence network is drawn in below figure.

Here w.r.t. F, net circuit net impedance is $Z_1+(Z_2||Z_0)$.

$$\therefore\ I_{a1} = \frac{E_a}{Z_1 + (Z_2 || Z_0)}$$

$$= \frac{E_a}{Z_1 + \dfrac{Z_2Z_0}{Z_0 + Z_2}} \quad ....(5)$$



Again, $V_a = V_{a0}+V_{a1}+V_{a2}$

$= -I_{a0}Z_0 + (E_a - I_{a1}Z_1) - I_{a2}Z_2$

$$\therefore\ V_a = E_a\frac{Z_0Z_2}{D} + E_a - \left(E_aZ_1\frac{(Z_0+Z_2)}{D}\right) + \frac{E_aZ_2(Z_0)}{D} \quad ....(6)$$

[utilizing (6)]

Where $D = Z_1Z_2+Z_2Z_0+Z_0Z_1$ $\quad V_a = 3E_a\left\{\dfrac{Z_2Z_0}{Z_1Z_2 + Z_2Z_0 + Z_0Z_1}\right\}$

6(b)(ii)

Sol: Ratings of G_1 and G_2 are 50 MVA, 11 kV.

Ratings of Transmission line are 50 MVA, 11 kV.

Sequence impedances of G_1 and G_2 are $Z_1 = Z_2 = j0.15$ and $Z_0 = j0.05$

Sequence impedances of transmission line are $Z_1 = Z_2 = j0.1$ and $Z_0 = j0.3$

Choose the base values as 11kV(LL), 50MVA(3-ϕ). All the apparatus impedances are given in p.u. on common base.

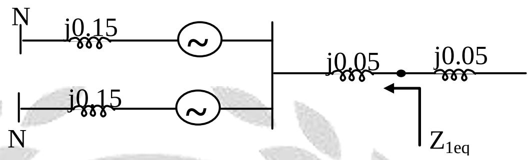
LG fault occur at mid point of transmission line:

For LG fault all three sequence networks will exist in fault analysis.

(i) Positive sequence network:

$Z_{1eq} = (j0.15 \parallel j0.15) + j0.05$

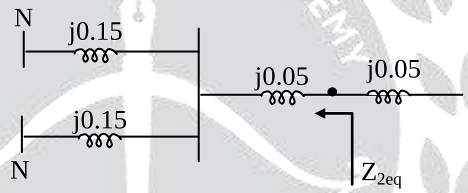
$= j0.125$ p.u.



(ii) Negative sequence network:

$Z_{2eq} = (j0.15 \parallel j0.15) + j0.05$

$= j0.125$ p.u.

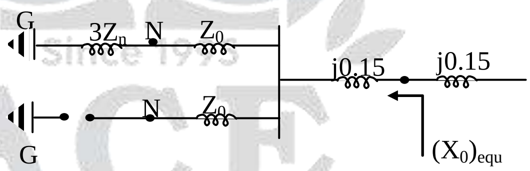


(iii) Zero sequence network:

$\Rightarrow (X_0)_{equ} = (3Z_n + Z_0) + (j0.15)$

$Z_{0eq} = (j0.15 + j0.05 + j0.15)$

$= j0.35$ p.u.



During LG fault, zero sequence current will be

$$I_{R0} = \frac{E_{R_1}}{(Z_1)_{epu} + (Z_2)_{epu} + (Z_0)_{epu}} = \frac{1}{j0.125 + j0.125 + j0.35}$$

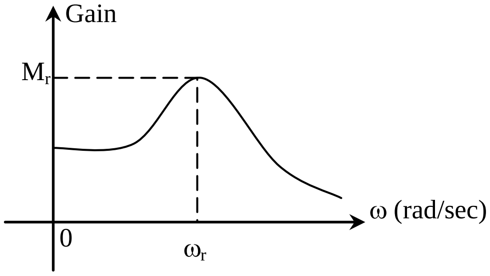
$$= \frac{1}{0.6} = -j1.667 \text{ p.u.}$$

$$I_{R0}(kA) = I_{R0}(p.u.) \times I_{base}(kA) = -j\ 1.667 \times \frac{50 \times 10^6}{\sqrt{3} \times 11 \times 10^3} \text{ A}$$

$$= -j\ 4.37 \text{ kA}$$

6(c)(i)

Sol:



Characteristics of gain Vs frequency

Resonant frequency (ω_r): The frequency at which maximum magnitude occurs.

Transfer function of a second order proto type system is, $\dfrac{C(s)}{R(s)} = \dfrac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

Sinusoidal transfer function is,

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

$$= \frac{1}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2 + j2\xi\left(\dfrac{\omega}{\omega_n}\right)}$$

Let $\dfrac{\omega}{\omega_n} = \mu = \dfrac{1}{(1-\mu^2) + j2\xi\mu}$

Magnitude,

$$M = \left|\frac{C(j\omega)}{R(j\omega)}\right| = \frac{1}{\sqrt{(1-\mu^2)^2 + (2\xi\mu)^2}}$$

To get maximum value of magnitude (M),

$$\frac{dM}{d\mu} = 0$$

$$\frac{-2((1-\mu^2)(-2\mu)) + (8\xi^2)\mu}{2\sqrt{(1-\xi^2)^2 + (2\xi\mu)^2}} = 0$$

$[2(1-\mu^2)(-2\mu)] = -8\xi^2\mu$

$-4\mu(1-\mu^2) + 8\mu\xi^2 = 0$

$-4\mu + 4\mu^3 + 8\mu\xi^2 = 0$

$8\xi^2 + 4\mu^2 - 4 = 0$

$2\xi^2 + \mu^2 - 1 = 0$

$$\mu = \sqrt{1 - 2\xi^2}$$

$$\frac{\omega}{\omega_n} = \sqrt{1-2\xi^2}\ ,\ \ \omega_r = \omega_n\sqrt{1-2\xi^2}$$

The resonant peak obtained by substituting ω_r in magnitude

$$\left|\frac{C(j\omega)}{R(j\omega)}\right| = \frac{1}{\sqrt{\left(1-\mu^2\right)^2 + 4\xi^2\mu^2}}$$

$$= \frac{1}{\sqrt{\left(1-1+2\xi^2\right)^2 + 4\xi^2\left(1-2\xi^2\right)}}$$

$$= \frac{1}{\sqrt{4\xi^4 + 4\xi^2 - 8\xi^4}}$$

$$\therefore\ M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

6(c)(ii)

Sol:

$+s^7$	1	1	1	1
$+s^6$	1	1	1	1
$+s^5$	0(3)	0(2)	0(1)	
$+s^4$	$\frac{1}{3}(1)$	$\frac{2}{3}(2)$	1(3)	
$-s^3$	–4(–1)	–8(–2)		
$+s^2$	$0(\varepsilon)$	3		
$+s^1$	$\frac{-2\varepsilon+3}{\varepsilon}$			
$+s^0$	3			

AE = $s^6 + s^4 + s^2 + 1 = 0$

$$\frac{dAE}{ds} = 6s^5 + 4s^3 + 2s$$

$$= 3s^5+2s^3+s$$

Number of sign changes = 2

Number of right hand side roots = 2

Number of $j\omega$ axis roots = 2

Number of left hand side roots = 3

7(a)

Sol: The characteristic equation is $s^2 + 1.6s + 16 = 0$. Comparing with the second order characteristic equation i.e.,

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$2\zeta\omega_n = 1.6$$

and $$\omega_n = \sqrt{16} = 4 \text{ rad/sec}$$

therefore, the damping ratio for the system without derivative feedback control is

$$\zeta = \frac{1.6}{2\omega_n} = \frac{1.6}{2\times 4} = 0.2$$

The damping ratio with derivative feedback control is given by

$$\zeta' = \zeta = + \frac{\omega_n K_t}{2}$$

As the damping ratio is to be made 0.8 and $\omega_n = 4$ rad/sec

$$0.8 = 0.2 + \frac{4K_t}{2}$$

$$K_t = 0.3$$

A. Without derivative feedback control:

Rise time $$t_r = \frac{\pi - \tan^{-1}\left(\frac{1-\zeta^2}{\zeta}\right)}{\omega_n\sqrt{1-\zeta^2}}$$

$$= \frac{\pi - \tan^{-1}\left(\frac{1-0.2^2}{0.2}\right)}{4\sqrt{1-0.2^2}} = \frac{\pi - \tan^{-1} 4.89}{4\times 0.98} = 0.45 \text{ sec}$$

Peak time $$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{4\sqrt{1-0.2^2}} = \frac{\pi}{4\times 0.98} = 0.8 \text{ sec}$$

% maximum overshoot

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\times 100} = e^{\frac{0.2\pi}{\sqrt{1-0.2^2}}} \times 100$$

$$= e^{-0.64} \times 100 = 52.6\%$$

Steady state error

$$e_{ss} = \frac{2\zeta}{\omega_n} = \frac{2\times 0.2}{4} = 0.1$$

B. With derivative feedback control

The overall transfer function of the system using derivative feedback control is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s + \omega_n^2}$$

Given that $\zeta = 0.2$, $\omega_n = 4$ and $K_t = 0.3$

$$\therefore \frac{C(s)}{R(s)} = \frac{4^2}{s^2 + (2\times 0.2\times 4\times + 4^2\times 0.3)s + 4^2}$$

Or $$\frac{C(s)}{R(s)} = \frac{16}{s^2 + 6.4s + 16}$$

$2\zeta'\omega_n = 6.4$ and $\omega_n = 4$

$$\zeta' = \frac{6.4}{2\omega_n} = \frac{6.4}{24} = 0.8$$

$$\text{Rise time } t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{-\zeta^2}}{\zeta'}\right)}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1-0.8^2}}{0.8}\right)}{4\sqrt{1-0.8^2}}$$

$$= \frac{\pi - \tan^{-1} 0.75}{2.4} = \frac{\pi - 0.64}{2.4} = 1.04 \text{ sec}$$

$$\text{Peak time } t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta'^2}} = \frac{\pi}{4\sqrt{1-0.8^2}} = 1.3 \text{ sec}$$

% Maximum overshoot

$$M_p = e^{-\frac{\zeta'\pi}{\sqrt{1-\zeta'^2}}\times 100} = e^{-\frac{0.8\pi}{\sqrt{1-0.8^2}}\times 100} = e^{-4\pi/3}\times 100 = 1.52\%$$

$$\text{Steady state error } e_{ss} = \frac{2\zeta}{\omega_n} + K_t = \frac{2\times 0.2}{4} + 0.3 = 0.4$$

	Without derivative feedback control	With derivative feedback control
Damping ratio ζ	0.2	0.8
Rise time t_r	0.45 sec	1.04 sec
Peak time t_p	0.8 sec	1.3 sec
% maximum overshoot M_p	52.6%	1.52%
Steady state error e_{ss}	0.1	0.4

7(b)(i)

Sol: Biased differential relay as shown in figure(a) and it is also called as percentage differential protection.

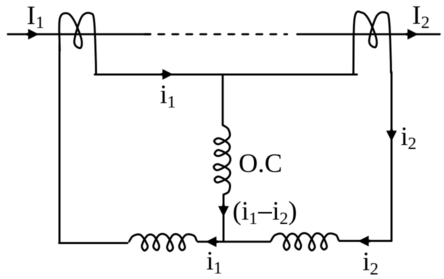


Figure: Biased differential relay

The relay consists of an operating coil and a restraining coil. The operating coil is connected to the mid point of the restraining coil. The operating current is variable quantity because of the restraining coil. Normally, no current flows through the operating coil under external fault condition. But due to the dissimilarities in C.T.S, the differential current through the operating coil is $|i_1 - i_2|$ and the equivalent current in restraining coil is $\frac{|i_1+i_2|}{2}$.

The torque developed by the operating coil is proportional to the ampere turns i.e., $T_0 \propto |i_1 - i_2|\, n_o$ where n_o is the number of turns in the operating coil. The torque due to restraining coil $T \alpha \frac{|i_1+i_2|}{2} n_r$ where n_r is the number of turns in the restraining coil. When the torque due to operating coil exceeds the torque due to the restraining coil, this relay operates.

$$|i_1 - i_2| n_0 \geq \frac{|i_1+i_2|}{2} n_r \qquad \frac{|i_1 - i_2|}{\frac{|i_1+i_2|}{2}} \geq \frac{n_r}{n_0}$$

The operating characteristics of percentage differential relay as shown in figure (b).

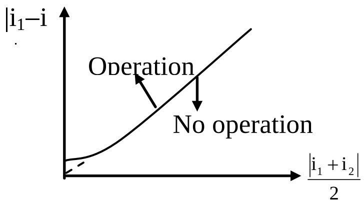


Fig.(b): operating characteristics of percentage differential relay

It is clear from the characteristic that except for the effect of the control spring at low currents, the ratio of the differential operating current to the average restraining current is a fixed percentage. This is why it is known as percentage differential relay.

The relay settings for transformer protection are kept higher than those for alternators. The typical value of alternator is 10% for operating coil and 5% for bias. The corresponding values for transformer may be 40% and 10% respectively. The reasons for a higher setting in the case of transformer protection are.

(i) A transformer is provided with on-load tap changing gear. The C.T. ratio cannot be changed with varying transformation ratio of the power transformer. The C.T. ratio is fixed and it is kept to suit the nominal ratio of the power transformer. Therefore, for taps other than nominal, an out of balance current flows through the operating coil of the relay during load and external fault conditions.

(ii) When a transformer is on no-load, there is no-load current in the relay therefore, its setting should be greater than no-load current.

7(b)(ii)

Sol: Given data:

$$P_s = P_{e1} = 1.0$$

$$P_{m1} = \frac{EV}{X_{1eq}} = EVy_{1\,eq} = 1.1 \times 1.0 \times 2.0 = 2.2 \text{ pu}$$

$$P_{m2} = \frac{EV}{X_{2eq}} = EVy_{2\,eq} = 1.1 \times 1.0 \times 0.5 = 0.55$$

$$P_{m3} = \frac{EV}{X_{3eq}} = EVy_{3eq} = 1.1 \times 1.0 \times 1.4 = 1.54$$

$$\delta_0 = \sin^{-1}\left(\frac{P_s}{P_{m1}}\right) = \sin^{-1}\left(\frac{1.0}{2.2}\right) \text{ elec. deg} = 27.03$$

$$\delta_{0\,(rad)} = \delta_0 \text{ elec deg} \times \frac{3.14}{180} = 0.4716$$

$$\delta_m = 180 - \sin^{-1}\left(\frac{P_s}{P_{m3}}\right) = 180 - \sin^{-1}\left(\frac{1.0}{1.54}\right) \text{ elec. deg} = 139.5$$

$$\delta_{m\,(rad)} = \delta_m \times \frac{3.14}{180} = 139.5 \times \frac{3.14}{180} = 2.435$$

$$\delta_c = \cos^{-1}\left[\frac{P_s(\delta_m - \delta_0) + P_{m3}\cos\delta_m - P_{m2}\cos\delta_0}{P_{m3} - P_{m2}}\right] \text{ elec. deg}$$

$$= \cos^{-1}\left[\frac{1.0(2.4345-0.4716)+1.54\times\cos(139.5)-0.55\times\cos(27.03)}{1.54-0.55}\right]$$

$$= \cos^{-1}(0.3055)$$

$\delta_c = 72.21°$

7(c)(i)

Sol: Per phase bus-bar voltage, $V_b = \frac{11000}{\sqrt{3}} = 6351\,V$

Alternator phasor diagram at unity pf gives

$$E_f = \sqrt{(V_b + I_a r_a)^2 + (I_a X_s)^2}$$

$$= \sqrt{(6351 + 200\times 0.5)^2 + (200\times 10)^2} = 6753.91\ V$$

Load angle, $\delta = \tan^{-1}\frac{2000}{6451} = 17.22°$

$$\alpha_z = \tan^{-1}\frac{r_a}{X_s} = \tan^{-1}\frac{0.5}{10} = 2.86°$$

$$Z_s = \sqrt{0.5^2 + 10^2} = 10.012\ \Omega$$

New values of excitation voltage, $E_{f_1} = 6753.91 \times 1.25 = 8442.4$ V

As the steam supply and alternator efficiency are unchanged power input to alternator with E_f = power input to alternator with 1.25 E_f

Or $\frac{E_f.V_b}{Z_s}\sin(\delta-\alpha_z)+\frac{E_f^2}{Z_s^2}r_a = \frac{1.25\,E_f}{Z_s}\sin(\delta'-\alpha_z)+\frac{(1.25\,E_f)^2}{Z_s^2}r_a$

Multiplying both sides of above equation by $\frac{Z_s}{E_f.V_b}$, we get

$$1.25\sin(\delta'-\alpha_z) + \frac{1.25^2 E_f}{V_b}.\frac{r_a}{Z_s} = \sin(\delta-\alpha_z)+\frac{E_f}{V_b}.\frac{r_a}{Z_s}$$

Or $1.25\sin(\delta' - 2.86) + \frac{1.25^2\times 6753.91}{6351}.\frac{0.5}{10.012} = \sin(17.22 - 2.86) + \frac{6753.91}{6351}.\frac{0.5}{10.012}$

Its simplification gives $\delta' = 12.91°$

With the excitation increased, let the armature current be I_{a_1}.

The alternator phasor diagram, shows that

$$I_{a_1}Z_s = \sqrt{(1.25E_f)^2 + V_b^2 - 2\times 1.25\,E_f\times V_b\cos\delta'}$$

Or $I_{a_1}(10.012) = \sqrt{8442.4^2 + 6351^2 - 2\times 8442.4\times 6351\times \cos 12.91^\circ}$

$\therefore$ $I_{a_1} = 265.85$ A

For the same steam input, $1.25\, E_f\, I_{a_1} \cos(\delta' + \theta) = E_f I_a \cos\delta$

Or $1.25\, I_{a_1} \cdot \cos(\delta' + \theta) = I_a \cos\delta$

$$\cos(12.91 + \theta) = \frac{200\cos 17.22}{265.85\times 1.25} \quad \text{or} \quad \theta = 42^\circ$$

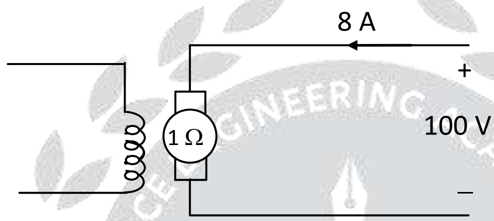
$\therefore$ New pf = cos 42° = 0.743 lag.

7(c)(ii)

Sol: $E_b = V - I_a R_a$

$E_b = 100 - 8 \times 1$

$E_b = 92$V



$$N \propto \frac{E_b}{\phi} \Rightarrow \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad (\because \phi \text{ constant})$$

$$E_{b_2} = 92 \times \frac{750}{500} = 138V$$

$T \propto \phi I_a$

$T \propto N^2$

$$\frac{T_2}{T_1} = \frac{N_2^2}{N_1^2} = \frac{I_2}{I_1}$$

$$\therefore I_2 = I_1 \left(\frac{N_2}{N_1}\right)^2$$

$$= 8 \times \left(\frac{750}{500}\right)^2 = 18A$$

$E_{b2} = V - I_2 R_a$

$V = E_{b2} + I_2 R_a$

$= 138 + 18\times 1$

$= 156$V

$\therefore$ The voltage to be applied to get a speed of 750rpm is 156V.

8(a)(i)

Sol: Given data, $\overline{Z}_2 = 5\angle 30°$

$$k = \frac{N_2}{N_1} = \frac{75}{150} = \frac{1}{2}$$

$$\overline{Z}_1 = \overline{Z}_2' = \frac{5\angle 30°}{k^2} = 20\angle 30°\Omega$$

$$V_2 = kV_1 = \frac{200}{2} = 100\text{V (Secondary terminal voltage)}$$

$$\bar{I}_2 = \frac{100\angle 0°}{5\angle 30°} = 20\angle -30°\text{ A.}$$

I_2 = 20 A and power factor cos 30°= 0.866 lag

$$\bar{I}_1 = \bar{I}_2' = k \times 20\angle -30^0 = 10\angle -30^0$$

I_1 = 10 A and power factor cos 30°= 0.866 lag

Secondary power output $P_2 = (20)^2 \times \text{Re } 5\angle 30°$

= 400 × 5 cos 30°

= 1.732 kW

As the transformer is loss less,

Primary power input (P_1) = P_2 = 1.732 kW

8(a)(ii)

Sol: Full load efficiency (η) at upf

$$= \frac{50kVA \times 1}{50kVA \times 1 + 500 + 600} \times 100$$

= 0.978 × 100 = 97.8 %

The load for maximum efficiency

$$= \sqrt{\frac{500}{600}} \times \text{rated load}$$

$$= \sqrt{\frac{5}{6}} \times 50\text{kVA} \times 1$$

= 45.64 kVA × 1

= 45.64 kW

At maximum efficiency, the variable copper losses equal the constant core losses, each of them being equal to 500 W.

8(a)(iii)

Sol: Effect of Armature reaction: The armature mmf produces two undesirable effects on the main field flux and these are.

(i) Net reduction in the main field flux per pole and

(ii) Distortion of the main field flux wave along the air gap periphery.

Armature reaction: When the armature of a d.c machine carries current effect of armature on main field flux is known as armature reaction. The machine air gap is now acted upon by the resultant mmf distribution caused by simultaneous action of the field, ampere-turns(AT_f) and armature ampere-turns(AT_a) As a result the air-gap flux density gets distorted as compared to the flat –topped (trapezoidal) wave with quarter wave symmetry when the armature did not carry any current.

- This reduction in the main field flux wave along the air-gap periphery.
- The cross magnetizing effect of armature mmf can be minimized at the design and construction stage of a D.C. Machine.

Various methods to reduce armature reaction:

(1) High-reluctance pole Tips

(2) Reduction in Armature flux

(3) Strong main-field flux

(4) Inter poles

(5) Compensating winding.

8(b)(i)

Sol: Total resistance of line 100×0.1 = 10 ohms.

The inductance of the line

$$= 2\times10^{-7}\times100\times1000\times\ln\frac{200}{0.75\times0.7788}\text{H}$$

$$= 11.17\times10^{-2}\text{ H}$$

$\therefore$ Inductive reactance

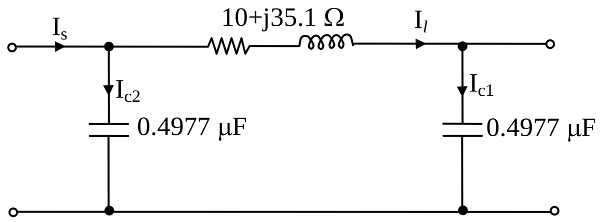
$$= 314\times11.17\times10^{-2} = 35.1\text{ohm}$$

The capacitance/phase

$$= \frac{2\pi\times8.854\times10^{-12}}{\ln\frac{200}{0.75}}\times100\times1000$$

$$= 9.954\times10^{-7} = 0.9954\,\mu\text{F}.$$

Nominal -π method: The nominal - π circuit for the problem is as follows:



For nominal $-\pi$ it is preferable to take receiving end voltage as the reference phasor. The current $I_r = 218.68(0.8 - j0.6)$.

Current $I_{c1} = j\omega CV_r = j314 \times 0.4977 \times 10^{-6} \times 38104$

$$= j5.95\,\text{amp}$$

$$\therefore I_\ell = I_r + I_{c1}$$

$$= 174.94 - j131.20 + j5.95 = 174.94 - j125.25$$

$$\therefore \; V_s = V_r + I_\ell Z$$

$$= 38104 + (174.94 - j125.25)\,(10 + j35.1)$$

$$= 38104 + 1749.4 - j1252.5 + j6140 + 4396$$

$$= 44249 + j4886 \text{ volts}$$

$$\therefore \; |V_s| = 44518 \text{ Volts}$$

The no load receiving end voltage will be

$$\frac{44518(-j6398)}{10 + j35.1 - j6398} = \frac{44518(-j6398)}{10 - j6363} = 44762 \text{ volts}$$

$$\therefore \quad \%\,\text{regulation} = \frac{44762 - 38104}{38104} \times 100$$

$$= 17.47\,\%$$

The line current $I_\ell = 215.15$ Amp

$$\therefore \text{Loss} = 3 \times 215.15^2 \times 10 = 1.388\,\text{MW}$$

$$\therefore \; \%\eta = \frac{20 \times 100}{21.388} = 93.5\%$$

8(b)(ii)

Sol: The voltage boost due to a shunt capacitor is distributed over the transmission line where as the change in voltages between the two ends of the series capacitor where it is connected, is sudden.

Let Q_c^1 be the reactive power of the shunt capacitor, V be the receiving end voltage and X be the reactance of the line. The current through the capacitor will be $\left(\frac{Q_c^1}{V}\right)$ and the drop due to this current in hv line will be $\left(\frac{Q_c^1}{V}\right)$ X.

Similarly let Q_c be the reactive power of the series capacitor, I be the line current and $\sin\phi$ be the sine of the power factor angle of the load. The voltage drop across the series capacitor will be $\left(\frac{Q_c}{I}\right)\sin\phi$, since the magnitude of the voltage across the capacitor is $\left(\frac{Q_c}{I}\right)$.

For a typical load with p.f 0.8 lag,

$\sin\phi_r = 0.6$ and assume $\frac{IX}{V} = 0.1$

For equality of voltage boost with both shunt and series capacitors.

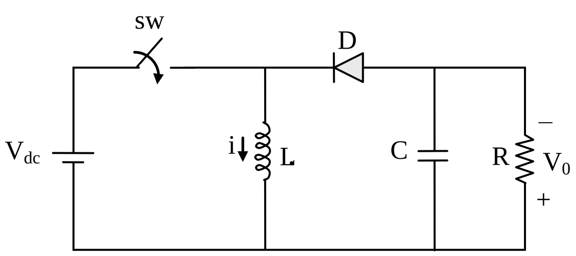
$$\left(\frac{Q_c^1}{V}\right)X = \left(\frac{Q_c}{I}\right)\sin\phi_r$$

$$\frac{Q_c^1}{Q_c} = \frac{\sin\phi}{\left(\frac{IX}{V}\right)} = \frac{0.6}{0.1} = 6.$$

It is evident that for same voltage boost, reactive power capacity of a shunt capacitor is greater than that of a series capacitor. Therefore, the shunt capacitor improves the p.f. of the load where as the series capacitor has little effect on the p.f.

8(c)(i)

Sol:



Given data:

$V_{dc} = 6V$, $V_0 = 15V$, $I_0 = 0.5A$,

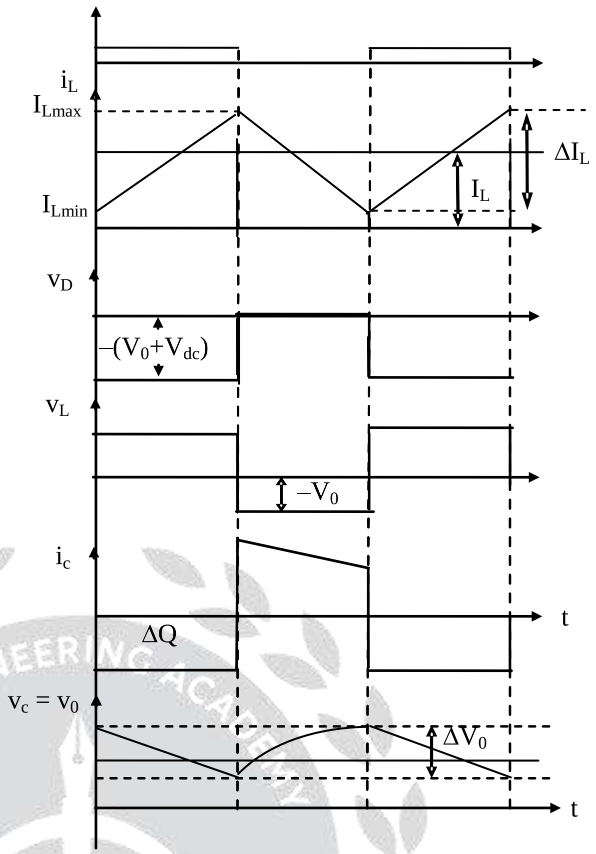
$F = 20$ kHz ,$L = 250\mu H$, $C = 440\mu F$

We know that $V_0 = \frac{D}{1-D}V_{dc}$

$$15 = \frac{D}{1-D}.6$$

$$15 = 21D$$

$$D = \frac{15}{21} = \frac{5}{7}$$



Ripple current in inductor (ΔI_L):

Since current is varying in a linear fashion and having a slope of $\frac{V_{dc}}{L}$

$$\Delta I_L = \text{slope} \times \text{time}$$

$$= \frac{V_{dc}}{L} \times T_{ON}$$

$$\Delta I_L = \frac{V_{dc}}{L}.DT$$

$$= \frac{6}{250 \times 10^{-6}} \times \frac{5}{7} \times \frac{1}{20 \times 10^3} = 0.857\,A$$

Ripple voltage of capacitor (ΔV_0):

$$\Delta V_0 = \frac{\Delta Q}{C}$$

Where ΔQ = charge either accumulated or discharge across capacitor.

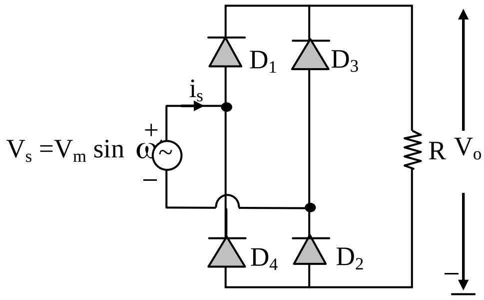
$$\Delta V_0 = \frac{I_0.DT}{C}$$

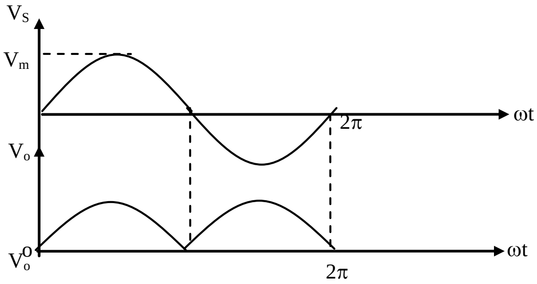
$$= \frac{0.5}{440 \times 10^{-6}} \times \frac{5}{7} \times \frac{1}{20 \times 10^3}$$

$$= 0.0405V$$

8(c)(ii)

Sol: Circuit diagram of a single phase bridge rectifier:





When D_1, D_2 are conducting, $V_0 = V_m \sin \omega t$

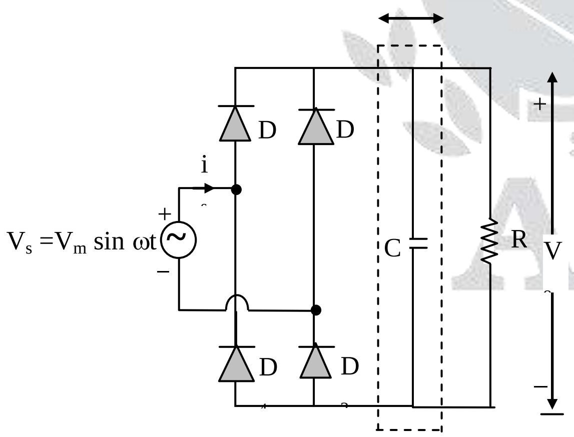
D_3,D_4 are conducting, $V_0 = -V_m \sin \omega t$

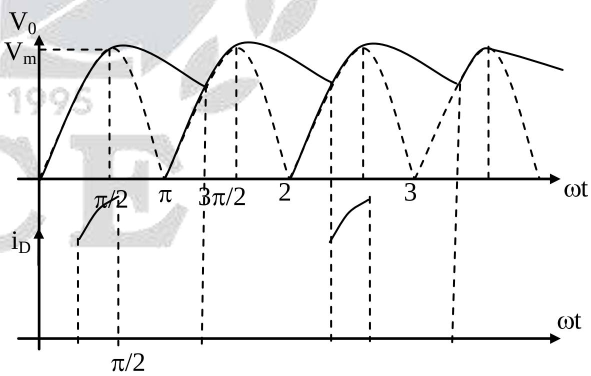
Average output voltage $= \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, d\omega t$

$$= \frac{V_m}{\pi}(-\cos\theta)_0^{\pi} = \frac{2V_m}{\pi}$$

$$\therefore \ V_0 = \frac{2V_m}{\pi}$$

If capacitor filter is used at output,





Current through diode $D_1 = \frac{V_o}{R}$ when it is conducting

Current through diode $D_1 = 0$ when it is not conducting