ACE

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ESE – 2019 MAINS
OFFLINE TEST SERIES

ELECTRICAL ENGINEERING

TEST –13
SOLUTIONS

All Queries related to **ESE – 2019 MAINS Test Series** Solutions are to be sent to the following email address

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1(a)

Sol: $E_{RY} = 400 \angle 0°$, $E_{YB} = 400 \angle -120°$,

$E_{BR} = 400 \angle 120°$,

$$I_{RY} = \frac{400}{30 - j40} = \frac{400}{50\angle -53°} = 8\angle 53° = 4.8 + j6.4$$

$$I_{RB} = \frac{400\angle -60°}{40}$$

$\Rightarrow 10 \angle -60° = I_{ML}$,

[wattmeter current coil current]

For the wattmeter pressure coil circuit

$20\ I_{RB} + V_1 V_2 - (-j40\ I_{RY}) = 0$

i.e., $0 = 200\angle -60° + 40 \angle 90° \times 8\angle 53° + V_1V_2$

$\therefore - V_1V_2 = 100 - j173 + 320 \angle 143$

$= 100 - j173 - 255.6 + j\ 195.2$

$\therefore - V_1V_2 = -155.6 + j20$

$V_1V_2 = 155.6 - j20 = 156.9\angle -7.3°$

$\therefore$ the reading of wattmeter $= I_{ML}\ .\ V_1V_2$

$= 10\times 156.9 \times 10^{-3} \cos(52.7°)$ kW $= 0.94$ kW

1(b)

Sol: The operating point current and voltages in the circuit are:

$$I_{CQ} = I_E = \frac{|V_{EE}|}{R_E} = \frac{10V}{10\Omega} = 1A$$

And,

$V_{CEQ} = V_{CC} = 10V$

Therefore, maximum ac output power is,

$$P_{o(max)} = \frac{V_{CEQ}.I_{CQ}}{2} = \frac{10\times 1}{2} = 5W$$

To calculate the efficiency, η, the dc power drawn by collector-emitter circuit is,

$$P_{DC} = (|V_{CC}| + |V_{EE}|)I_{CQ}$$

$= (10 + 10)\times 1 = 20W$

Therefore efficiency,

$$\eta = \frac{P_{o(max)}}{P_{DC}} = \frac{5W}{20W}\times 100$$

Or $\eta = 25\%$

1(c)

Sol: (i) Process address space:

The process address space is the set of logical addresses that a process references in its code. For example, when 32-bit addressing is in use, addresses can range from 0 to 0x7fffffff; that is, 2^{31} possible numbers, for a total theoretical size of 2 gigabytes.

The operating system takes care of mapping the logical addresses to physical addresses at the time of memory allocation to the program. There are three types of addresses used in a program before and after memory is allocated.

Memory Addresses & Description

1. Symbolic addresses:

The addresses used in a source code. The variable names, constants and instruction labels are the basic elements of the symbolic address space.

2. Relative addresses:

At the time of compilation, a compiler converts symbolic addresses into relative addresses.

3. Physical addresses:

The loader generates these addresses at the time when a program is loaded into main memory. Virtual and physical addresses are the same in compile-time and load-time address-binding schemes. Virtual and physical addresses differ in execution-time address-binding scheme.

(ii) Static vs Dynamic Loading

The choice between Static or Dynamic Loading is to be made at the time of computer program being developed. If you have to load your program statically, then at the time of compilation, the complete programs will be compiled and linked without leaving any external program or module dependency. The linker combines the object program with other necessary object modules into an absolute program, which also includes logical addresses.

If you are writing a dynamically loaded program, then your compiler will compile the program and for all the modules which you want to include dynamically, only references will be provided and rest of the work will be done at the time of execution.

At the time of loading, with static loading, the absolute program (and data) is loaded into memory in order for execution to start.

If you are using dynamic loading, dynamic routines of the library are stored on a disk in relocatable form and are loaded into memory only when they are needed by the program.

(iii) Fragmentation in OS

As processes are loaded and removed from memory, the free memory space is broken into little pieces. It happens after sometimes that processes cannot be allocated to memory blocks considering their small size and memory blocks remains unused. This problem is known as Fragmentation.

Fragmentation is of two types:

1. External fragmentation:

Total memory space is enough to satisfy a request or to reside a process in it, but it is not contiguous, so it cannot be used.

2. Internal fragmentation:

Memory block assigned to process is bigger. Some portion of memory is left unused, as it cannot be used by another process.

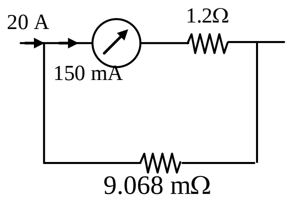
1(d)

Sol: Ammeter –X:

$R_m = 1.2\Omega$, $I_m = 150$ mA ; $I = 20$ A

$$R_{sh} = \frac{R_m}{m-1} = \frac{R_m}{\left(\frac{I}{I_m}-1\right)}$$

$$= \frac{1.2}{\frac{20}{150\times10^{-3}}-1} = 9.068 \times 10^{-3}\ \Omega$$



Ammeter – Y:

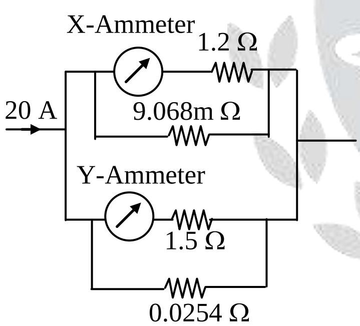
$R_m = 1.5\ \Omega$, $I_m = 250$ mA

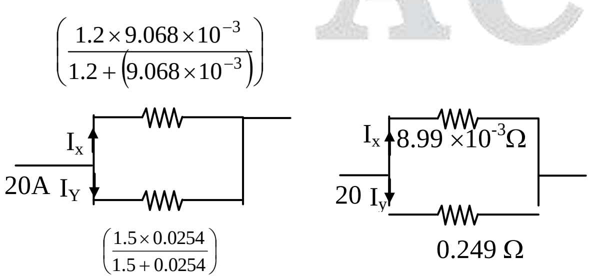
$I = 20$ A

$$R_{Sh} = \frac{R_m}{m-1} = \frac{R_m}{\left(\frac{I}{I_m} - 1\right)}$$

$$= \frac{1.5}{\frac{20}{250\times10^{-3}} - 1} = 0.0254\ \Omega$$

Two Ammeters are in parallel





$$I_x = 20\times\frac{0.249}{(0.249+8.99\times10^{-3})} = 19.303$$

$$I_y = 20\times\frac{8.99\times10^{-3}}{8.99\times10^{-3}+0.249} = 0.6969$$

1(e)

Sol: $I_E = I_B + I_L$

$I_L = I_C = \beta I_B$

$$I_B = \frac{I_L}{\beta} = .02\text{mA}$$

$$I_E = (2 + .02)\text{mA} = 2.02\text{mA}$$

For Q_2: KVL

$12 = V_{EB2} + V_Z + V_{R1}$

$V_{R_1} = 12 - V_{EB2} - V_Z$

$= 12 - .7 - 4 = 7.3\text{V}$

Hence,

$$R_1 = \frac{V_{R_1}}{I_Z + I_B} = \frac{7.3}{(0.02+5)\times 10^{-3}} = 1.45\,\text{k}\Omega$$

V_{R_2} = voltage drop across R_L

$= V_{EB_2} + V_Z - V_{EB_1}$

$= 4\text{V}$

$$R_2 = \frac{4}{2.02\text{mA}} = 1.98\text{k}\Omega$$

Voltage drop across Base-to-collector of Q_1 (from KVL)

$V_{BC} = 12 - V_{R2} - V_{EB_1} - I_L R_L$

$= 12 - 4 - .7 - I_L R_L$

$= 7.3 - (2R_L)10^{-3}$

For Q_1 to be in active Region V_{BC} should be +Ve means.

$7.3 \geq (2R_L)10^{-3}$

or $\quad 2R_L \leq 7.3$

or $\quad R_L \leq 3.65\ \text{k}\Omega$

hence

$0 \leq R_L \leq 3.65\ \text{k}\Omega$

2(a)(i)

Sol: $Q = K\theta_1 = K'd_1$

Where d_1 is linear deflection and K' is another constant

Now $Q = CV = 2.5\times10^{-6}\times25 = 62.5\times10^{-6}$ Coulomb

Hence ballistic sensitivity, $S_b = \dfrac{d_1}{Q}$

$$= \frac{25}{62.5\times10^{-6}}\,\text{cm/coulomb} = 4\ \text{mm}/\mu\text{C}$$

2(a)(ii)

Sol: Advantages:

The main function of a transducer is to respond only for the measurement under specified limits for which it is designed. It is, therefore, necessary to know the relationship between the input and output quantities and it should be fixed. Transducers should meet the following basic requirements.

1. Ruggedness: It should be capable of withstanding overload and some safety arrangement should be provided overload protection.

2. Linearity: It input-output characteristics should linear and it should produce these characteristics in symmetrical way.

3. Repeatability: It should reproduce same output signal when the same input signal is applied again and gain under fixed environmental conditions e.g., temperature, pressure humidity etc.

4. High output signal quality: The quality of output signal should be good i.e., the ratio of the signal to the noise should be high and the amplitude of the output signal should be enough.

5. High reliability and stability: it should give minimum error in measurement for temperature variations, vibrations and other various changes in surroundings.

6. Good dynamic response: Its output should be faithful to input when taken as a function of time. The effect is analyzed as the frequency response.

7. No hysteresis: It should not give any hysterisis during measurement while input signal is varied from its low value to high value and vice-versa.

8. Residual deformation: There should be no deformation on removal of input signal after period of application.

Applications:

Parameter	Applications
Resistance	
i. Potentiometer device	Pressure, displacement
ii. Resistance strain gauge	Force, torque, displacement.
iii. Resistance thermometer	Temperature, radiant heat
iv. Thermistor	Temperature, flow
v. Photoconductive cell	Photosensitive relay.
Capacitance	
i. Variable capacitance	Displacement, pressure.
ii. Capacitor microphone	Speech, music, noise.
iii. Dielectric gauge	Liquid level, thickness.
Inductance	
i. Magnetic circuit transducer	Pressure, force, displacement,
ii. Differential transformer.	Pressure, displacement, position.
iii.Eddy current gauge	Displacement, thickness.
Voltage and Current	
i. Hall effect pickup	Magnetic flux, current, power.
ii. Ionization chamber	Particle counting, radiation.
Self Generating Transducers	
i. Thermocouple and thermopile	Temperature, heat flow, radiation.
ii. Moving coil generator	Velocity, vibrations.
iii. Piezoelectric pickup	Sound, vibrations, acceleration, pressure changes.

2(a)(iii)

Sol: $R = 350\ \Omega$

Gauge factor = 2

$Y_{steel} = 2.1 \times 10^6\ kg/cm^2$

Stress =?

$R + \Delta R = 350.5\ \Omega$

$\Delta R = 0.5\ \Omega$

$$G_f = \frac{\Delta R / R}{\Delta \ell / \ell}$$

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$\text{stress} \times \frac{1}{Y} = \text{strain}$$

$$G_f \times \frac{1}{Y} \times \text{stress} = \frac{\Delta R}{R} \quad \left[\because \frac{\Delta \ell}{\ell} = \text{strain}\right]$$

$$2 \times \frac{1}{2.1 \times 10^6} \times \text{stress} = \frac{0.5}{350}$$

$$\text{Stress} = \frac{0.5}{350} \times 2.1 \times 10^6 \times \frac{1}{2}$$

$$= 0.0015 \times 10^6 \text{ kg/cm}^2$$

2(b)(i)

Sol:
```
#include<stdio.h>
void main ( )
{
        int n, a, b, c, i;
        a = 0;
        b = 1;
        printf("Enter the value of n:");
        scanf("%d", &n);
        if (n > = 0)
                printf("fibonacci(0) = 0 \n");
        if (n > = 1)
                printf("fibonacci(1) = 1 \n");
        if (n > = 2)
```

```
        {
                for(i = 2; i < = n; i++)
                {
                        c = a + b;
                        printf("fibonacci(%d) = %d \n", i, c);
                        a = b;
                        b = c;
                }
        }
}
```

2(b)(ii)

Sol: In direct mapped cache the main memory address is divided into 3 parts (fields) as follows:

$\longleftarrow$ Main memory address $\longrightarrow$

Tag	Cache block number	Byte offset

Block size = 64B = 2^6 B $\Rightarrow$ Byte offset = 6-bits

$$\text{Number of blocks in cache memory} = \frac{\text{cache size}}{\text{block size}}$$

$$= \frac{256\text{MB}}{64\text{B}}$$

$$= \frac{2^8 . 2^{20}}{2^6}$$

$$= 2^{22}$$

Hence cache block no. = 22-bits

Tag size = 32–(22+6) = 4-bits

$\longleftarrow$ 32 $\longrightarrow$

Tag	Cache block number	Byte offset
$\leftarrow$ 4 $\rightarrow$	$\leftarrow$ 22 $\rightarrow$	$\leftarrow$ 6 $\rightarrow$

Tag directory (Meta-data) size = no. of blocks in cache * 1 entry size

$= 2^{22} * (\text{Tag} + \text{extra bits})$

$= 2^{22} * (4+2)$ bits

$= 2^{22} * 6$ bits

$= 24 * 2^{20}$ bits

= 24 Mbits

= 0.3 Mbytes

2(c)

Sol: (i) Methods to reduce the ratio error of current transformer:

1. Leakage reactance increases ratio error.

   $\therefore$ Two windings primary and secondary should be close together to reduce the winding leakage reactance.

   Ex: use of ring shaped cores around which toroidal windings are uniformly distributed.
2. For a particular value of current and burden impedance, ratio error is corrected by reducing the secondary windings turns. This type of current transformer is called compensated current transformer.
3. For a particular value of current and burden, with the use of shunts across primary or secondary windings, the secondary winding current is reduced and ratio error is corrected.
4. Wilson compensation method
5. Two stage design. It utilises a second current transformer to correct the error in secondary current of first transformer.
6. By using materials of high permeability like Nickel iron cores, ratio errors are reduced.

(ii) Reactance of pressure coil circuit $= 2\pi \times 50 \times 10 \times 10^{-3} = 3.14\Omega$

Resistance of pressure coil circuit $= 362\Omega$

Phase angle of pressure coil circuit, $\beta = \tan^{-1} 3.14/362 \approx 30'$

Phase angle of current transformer $\theta = +90'$

Phase angle of potential transformer $\delta = -45'$

Phase angle of load $= 50°$

Phase angle between pressure coil current I_p and a current I_s of wattmeter current coil is $\alpha = \phi - \theta - \beta - \delta = 50° - 90' - 30' - 45' = 47°15'$

Correction factor, $K = \dfrac{\cos\phi}{\cos\beta\cos\alpha} = \dfrac{\cos 50°}{\cos 30' \times \cos 47°15'} = 0.947$

$\therefore$ Percentage ratio error $= \dfrac{K_n - R}{R} \times 100$

$\therefore$ Actual ratio

$$R = \frac{K_n \times 100}{100 + \text{percentage error ratio}}$$

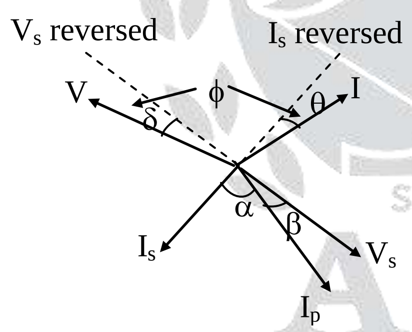
Actual ratio of C.T. $= \dfrac{20 \times 100}{100 - 0.2} = 20.04$

Actual ratio of P.T $= \dfrac{100 \times 100}{100 + 0.8} = 99.2$

Power = K × Actual ratio of P.T × Actual ratio of C.T × Wattmeter reading

$\therefore$ Power of load $= 0.947 \times 20.04 \times 99.2 \times 350 \times 10^{-3}$

$= 658.9\text{kW}$



Phasor diagram

3(a)(i)

Sol: $[C1B00000]_{16}$

$$\underset{\text{sign}}{\underline{1}}\ \underset{\text{exponent}}{\underline{10000011}}\ \underset{\text{Mantissa}}{\underline{01100......00}}$$

$(-1)^{\text{sign}} \times (1.\text{ Mantissa}) \times 2^{(\text{exponent} - \text{excess value})}$

$(-1)^1 \times (1.01100) \times 2^{(131-127)}$

$-1.01100 \times 2^{+4}$

–10110.0

$\boxed{-22.0}$ (in decimal)

3(a)(ii)

Sol: Round Robin process scheduling policy:

- Preemptive First come first served (FCFS)
- Every process gets its term for CPU execution for time quanta q units of time in round robin fashion as per ready queue order
- Pure implementation of multitasking operating system
- Additional hardware timer is required to generate time quanta expire interrupt
- Reduces average response time
- Every process in a system get fair chance of execution in early responsive way
- If time quanta decreases then number of context switches increases, but average response time decreases
- If time quanta increases then it may degenerate to FCFS.

3(a)(iii)

Sol: Direct Mapped cache organization:

- A main memory block is associated with particular cache line
- Associativity is one
- Mapping function

  (Cache line no.) ← (MM block no.) mod (no. of cache lines)
- More number of cache miss due to more number of collisions
- less expensive due to less hardware requirement in compare to other cache organization
- Fastest cache searching time in compare to other cache organization

3(b)

Sol: (a) when both switches are closed

$$I_B = \frac{9-0.7}{250k\Omega} = 33.2\mu A$$

$I_C = \beta I_B = 3.32$ mA

$$I_{C(sat)} = \frac{9-0.2}{1.5K\Omega} = 5.86mA$$

$\beta I_B < I_{c(sat)} \Rightarrow$ Active region

$V_{CE} = 9 - 1.5 \times 3.32 = 4.02$ volt

Operating point (4 volt, 3.3 mA) and transistor is in Active Region.

(b) when S_1 is closed, S_2 is open

$$I_B = \frac{9-.7}{(250+101\times1.2)\times10^3} \cong .0223 \text{ mA}$$

$\beta I_B = I_C \cong 2.23$ mA

$$I_{C(sat)} = \frac{9-0.2}{2.7K\Omega} = 3.26mA$$

$\beta I_B < I_{c(sat)} \Rightarrow$ Active region

$V_{CE} \cong 9 - 2.23 \times 2.7 = 2.962$ V

Operating point (2.962 V, 2.23 mA) and the transistor is in Active Region.

(c) when both S_1 and S_2 are open

$I_B = 0 = I_C$ Cut-off region.

3(c)(i)

Sol: If synchronization is not provided between co-operating or communicating processes then following problems may arise

(i) Loss of data

(ii) Inconsistency

(iii) Deadlock

Operating system is a resource allocator. There are many resources that can be allocated to only one process at a time, and we have seen several operating system features that allow this, such as mutexes, semaphores or file locks.

Sometimes a process has to reserve more than one resource. For example, a process which copies files from one tape to another generally requires two tape drives. A process which deals with databases may need to lock multiple records in a database.

In general, resources allocated to a process are not preemptable; this means that once a resource has been allocated to a process, there is no simple mechanism by which the system can take the resource back from the process unless the process voluntarily gives it up or the system administrator kills the process.

This can lead to a situation called *deadlock*. A set of processes or threads is deadlocked when each process or thread is waiting for a resource to be freed which is controlled by another process. Here is an example of a situation where deadlock can occur.

```
Mutex M1 ,M2;
/* Thread 1 */
while (1) {
   NonCriticalSection( )
   Mutex_lock(&M1);
   Mutex_lock(&M2);
   CriticalSection();
   Mutex_unlock(&M2);
   Mutex_unlock(&M1);
}
/* Thread 2 */
while (1) {
NonCriticalSection( )
Mutex_lock(&M2);
Mutex_lock(&M1);
CriticalSection();
Mutex_unlock(&M1);
Mutex_unlock(&M2);
}
```

Suppose thread 1 is running and locks M1, but before it can lock M2, it is interrupted. Thread 2 starts running; it locks M2, when it tries to obtain and lock M1, it is blocked because M1 is already locked(by thread 1). Eventually thread 1 starts running again, and it tries to obtain and lock M2, but it is blocked because M2 is already locked by thread 2. Both threads are blocked; each is waiting for an event will never occur.

3(c)(ii)

Sol:

	LAN	WAN
Distance coverage	Limited coverage, about up to 2 miles(or 2500 meters)	Unlimited (usually in 1000 km) range, uses repeater and other connectivity for range extension
Speed of operation	High, typically 10, 100 and 1000 Mbps	Slow, about 1.5 Mbps (May vary based on wireless technologies used)
Technologies used for medium	Locally installed, twisted pair, fiber optic cable, wireless (e.g. WLAN, Zigbee)	Locally installed and based on common carrier e.g. twisted pair wires, fiber, coaxial cable, wireless including wireless and cellular network based
Applications	Used mainly by fixed desktop computers and portable computers (e.g. laptops). Now-a-days it is used by smart phones due to emergence of WLAN network	Can be used by any devices, but desktop devices are mainly using this network type.

4(a)(i)

Sol: The distortion, D_{FB} in an amplifier with feedback and distortion, D without feedback in the amplifier are related as,

$$D_{FB} = \frac{D}{(1+AB)}$$

Where A is open loop gain of amplifier and B is the gain of feedback network.

Now, $D = 10\% = \frac{10}{100}$ (given)

and $D_{FB} = 1\% = \frac{1}{100}$ (given)

Therefore, using above equation,

$$\frac{1}{10} = \frac{1}{(1+AB)}$$

Or, (1 + AB) = 10

Now, $A_{FB} = \frac{A}{(1+AB)} = \frac{A}{10}$

Or, $A = 10 \times A_{FB} = 10 \times 120$ ($\because A_{FB} = 120$)

Or, A = 1200

4(a)(ii)

Sol: $V_{GS} = I_S R_S \cong I_D R_S$ ($\because I_G \cong 0$)

$I_D = I_{DSS} (1 - V_{GS}/V_P)^2$

Substitute the values in above equation,

$V_{GS} = 4R_S$

$4 = 12 (1 - 4 R_S/4)^2$

On solving we get

$R_S = 1.58\ k\Omega$ or $0.42\ k\Omega$

Using $R_S = 0.42\ k\Omega$, KVL for DS loop

$-12 - 6 + (R_D + R_S) I_D = 0$

With $R_S = 0.42\ k\Omega$, $R_D = 4.08\ k\Omega$

$\therefore R_D = 4.08\ k\Omega$

$R_S = 0.42\ k\Omega$

4(a)(iii)

Sol: $n_i = CT^{3/2} \exp\left(-\frac{E_g}{2kT}\right)$

Where $C = 2\left(\frac{2\pi mk}{h^2}\right)^{3/2}$

Conductivity $\sigma = n_i e(\mu_e + \mu_h)$

$$C = 2\left(\frac{2\times 22\times 9.109\times 10^{-31}\times 1.38\times 10^{-23}}{7x\left(6.626\times 10^{-34}\right)^2}\right)^{3/2}$$

$$= 2\left(\frac{554.098\times 10^{-54}}{307.327\times 10^{-68}}\right)^{3/2}$$

$= 2\ (1.7997 \times 10^{14})^{3/2}$

$= 2 \times 2.414 \times 10^{21}$

$C = 4.829 \times 10^{21}$

Taking T = 300K

$$2kT = \frac{2\times 1.38\times 10^{-23}\times 300}{1.6\times 10^{-19}}$$

$= 0.052eV$

$$n_i = CT^{3/2}\exp\left(-\frac{E_g}{2kT}\right)$$

$$= 4.829 \times 10^{21}\times 300^{3/2}\times \exp\left(\frac{-1.1}{0.052}\right)$$

$= 4.829 \times 10^{21}\times 300^{3/2}\times 6.501\times 10^{-10}$

$n_i = 163124.5 \times 10^{11}/m^3$

$\sigma_i = n_i e(\mu_e+\mu_h)$

$= 163124.5 \times 10^{11}\times 1.6\times 10^{-19}\times 0.493$

$= 128672.6\times 10^8$

$\sigma_i = 1.287 \times 10^{-3}\ \Omega^{-1} m^{-1}$

4(b)(i)

Sol: (i)A 20-V peak signal across a 16 - Ω load provides a peak load current of,

$$I_{L(P)} = \frac{V_{L(P)}}{R_L} = \frac{20V}{16\Omega} = 1.25A.$$

The DC value of the current drawn from the power supply is then

$$I_{dc} = \frac{2}{\pi}.I_{L(P)} = \frac{2}{\pi}(1.25A) = 0.796A.$$

And the input power delivered by the supply voltage is

$P_{i(dc)} = V_{cc}.I_{dc} = (30V)\,(0.796A) = 23.9$Watts.

The output power delivered to the load is

$$P_{o(ac)} = \frac{V^2{}_{L(P)}}{2R_L} = \frac{(20V)^2}{2(16\Omega)} = 12.5\text{Watts.}$$

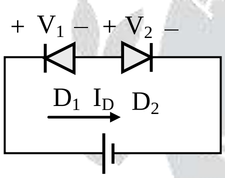
$$\therefore \text{ efficiency } (\eta) = \%\eta = \frac{P_{o(ac)}}{P_{o(dc)}} \times 100\%$$

$$= \frac{12.5W}{23.9W} \times 100\%$$

$= 52.3\%$

4(b)(ii)

Sol: (i) The applied voltage *V* makes D_1 – R.B & D_2 – F.B



$I_D = I_0$

$$\text{But } I_D = I_s\left[e^{\frac{V_2}{V_T}} - 1\right] \rightarrow \text{for } D_2$$

$$= I_s\left[1 - e^{\frac{-V_1}{V_T}}\right] \rightarrow \text{for } D_1$$

$$\therefore I_s\left[e^{\frac{V_2}{V_T}} - 1\right] = I_s\left[1 - e^{\frac{-V_1}{V_T}}\right]$$

$$e^{\frac{V_2}{V_T}} - 1 = 1 - e^{-\frac{V_1}{V_T}}$$

$$e^{\frac{V_2}{V_T}} + e^{-\frac{V_1}{V_T}} = 2$$

But $V_1 + V_2 = 50$ mV $\Rightarrow V_2 = 50 \times 10^{-3} - V_1$

$$e^{\frac{50\times10^{-3}-V_1}{V_T}} + e^{-\frac{V_1}{V_T}} = 2$$

$$e^{-\frac{V_1}{V_T}\left[e^2+1\right]} = 2$$

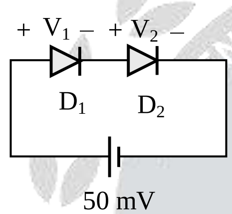
$$\Rightarrow \qquad e^{-\frac{V_1}{V_T}} = \frac{2}{e^2+1} = 0.2384$$

Then, $$I_D = I_S\left[1 - e^{-\frac{V_1}{V_T}}\right]$$

$$= I_S\,[1-0.2384]$$

$$\Rightarrow \qquad I_D = I_0 = 0.7614\ I_S$$

(ii) When diodes are F.B,



$V_1 + V_2 = 50$ mV

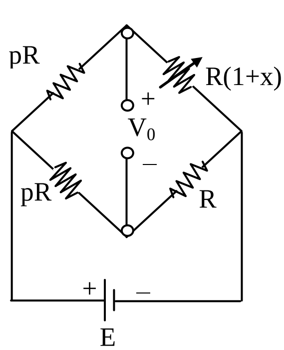
$\Rightarrow \quad V_1 = V_2 = 25$mV

$$I_D = I_S\left[e^{\frac{V_1}{\eta V_T}} - 1\right] = I_s\,[e^1 - 1]$$

$\Rightarrow I_D = 1.71828\ I_S$

4(c)(i)

Sol: As given Wheatstone bridge is shown below



The value of output voltage

$$V_0 = E\left[\frac{R(1+x)}{pR+R(1+x)} - \frac{R}{pR+R}\right]$$

$$= E\left[\frac{1+x}{p+(1+x)} - \frac{1}{1+p}\right]$$

For getting the max value of output voltage, $\frac{dV_0}{dp} = 0$

$$\Rightarrow \frac{dV_0}{dp} = E\left[\frac{-(1+x)}{[p+(1+x)]^2} - \frac{-1}{(1+p)^2}\right] = 0$$

$$\Rightarrow E\left[\frac{1}{(1+p)^2} - \frac{1+x}{[p+(1+x)]^2}\right] = 0$$

$$\Rightarrow \frac{1}{(1+p)^2} = \frac{1+x}{[p+(1+x)]^2}$$

$\Rightarrow [p + (1+x)]^2 = (1+x)(1+p)^2$

$\Rightarrow p^2+(1+x)^2 + 2p(1+x) = (1+x)\,[1+p^2 +2p]$

$= (1+x) + p^2\,(1+x) + 2p(1+x)$

$\Rightarrow \quad p^2[1+x-1] = (1+x)^2 - (1+x)$

$\Rightarrow \quad p^2x = 1+x^2+2x-1-x$

$\Rightarrow \quad p^2x = x^2+x$

$\Rightarrow \quad p^2 = 1+x$

$p = \sqrt{1+x}$

4(c)(ii)

Sol: Let $u = R^2 = 100^2 = 10{,}000$

Percentage error in $u = \frac{2\delta R}{R} = 2\times 5 = 10\%$

Let $v = \omega^2L^2 = (2\pi\times 50)^2 \times 2^2 = 394{,}784$

Percentage error in $v = \frac{2\delta L}{L} = 2\times 10 = 20\%$

Let $x = u + v = R^2 + \omega^2L^2$

$$=(100)^2+(2\pi\times 50)^2\times 2^2=404{,}784$$

Uncertainity in $x=\frac{u}{x}.\frac{\delta u}{u}+\frac{v}{x}.\frac{\delta v}{v}$

$$=\left[\frac{10{,}000}{404{,}784}\times 10+\frac{394{,}784}{404{,}784}\times 20\right]$$

= 0.24+19.5 = 19.74%

Now Z = $x^{1/2}$

So $\frac{\delta Z}{Z}=\pm\frac{1}{2}\frac{\delta x}{x}=\pm\frac{1}{2}\times 19.74$ **= 9.873%**

5(a)

Sol: $V(t)=V(\infty)+[V(0)-V(\infty)]e^{-t/\tau}$

V(0) = 0V

$$V(\infty)=5\left[\frac{3k}{5k}\right]=3V$$

$$\tau=R_{eq}.C$$

$$R_{eq}=2k//3k=\frac{6}{5}k$$

$$T=\frac{6}{5}k\times 5nF=6\mu\sec$$

$$V(t)=3[1-e^{-t/6\mu}]$$

$$V(t=6\mu)$$

$$V(t)=3[1-e^{-1}]=3[1-0.367]=1.896\text{Volts}$$

5(b)

Sol: (i) Given equation in symbolic form is $(D^2+5D+6)\,x = 0$

It's A.E. is $D^2+5D+6=0$, i.e, $(D+2)(D+3)=0$

Whence D = – 2, –3

$\therefore$ C.S . is $x = c_1e^{-2t} + c_2e^{-3t}$ and $\frac{dx}{dt} = -2c_1e^{-2t} - 3c_2e^{-3t}$

When t = 0, x = 0

$\therefore$ $0 = c_1 + c_2$………..(1)

When t = 0 dx/dt = 15

$\therefore$ $15 = -2c_1 - 3c_2$……. (2)

Solving (1) and (2), $c_1 = 15$, $c_2 = -15$

Hence the required solution is

$x = 15\,(e^{-2t} - e^{-3t})$

(ii) $P.I = \frac{1}{D^3+1}\cos(2x-1)$ [put $D^2 = -2^2 = -4$]

$= \frac{1}{D(-4)+1}\cos(2x-1)$ [Multiply and divide by 1+4D]

$= \frac{(1+4D)}{(1-4D)(I+4D)}\cos(2x-1)$

$= (1+4D).\frac{1}{1-16D^2}\cos(2x-1)$ [put $D^2 = -2^2 = -4$]

$= (1+4D)\frac{1}{1-16(-4)}\cos(2x-1)$

$= \frac{1}{65}[\cos(2x-1) + 4D\cos(2x-1)]$

$= \frac{1}{65}[\cos(2x-1) - 8\sin(2x-1)]$

5(c)

Sol: Here Y-Parameter of the two-port network is parallel with 1Ω resistor, so the Y-parameters for the whole network is the summation of individual Y-parameters of network

$Y = Y_1 + Y_2$

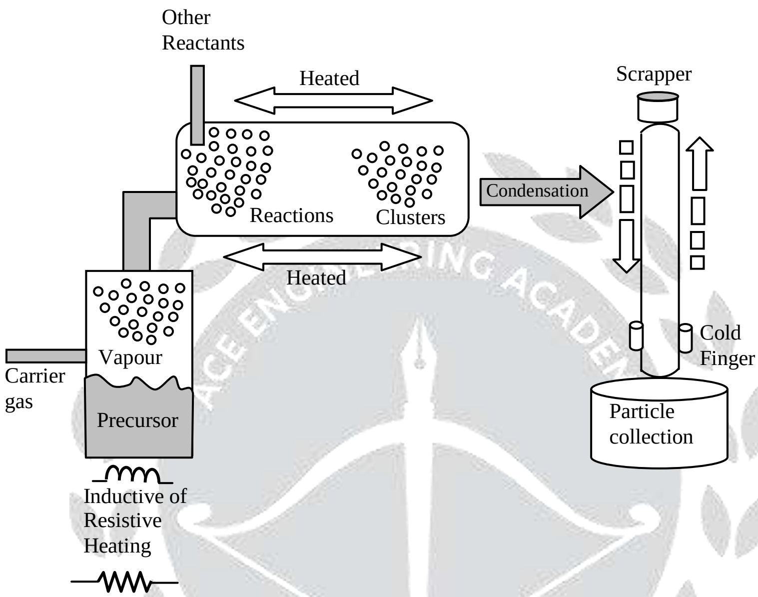
$Y_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ $\qquad Y_2 = \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$

$$Y = Y_1 + Y_2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ 1 & 3 \end{bmatrix}$$

$$Y = \begin{bmatrix} 6 & 1 \\ 0 & 4 \end{bmatrix}$$

5(d)

Sol:



Schematic of a typical chemical vapour deposition reactor

Chemical vapour deposition method nano particles are deposited from gas phase. Material is heated to form a gas and then allowed to deposit on a solid surface, usually under high vacuum. In deposition by chemical reaction new produce is formed. Nano powders of oxidises 2nd carbides of metals can be formed of vapours of carbon or oxygen are present with the metal.

It involves pyrolysis of vapours of metal organic precursors in a reduced precursors atmosphere in the simplest form shown in figure, a metal-organic precursor is introduced into the hot zone of the reactor using mass flow controller. The precursor is vaporized either by resistive or inductive heating. The carrier gas such as Ar or Ne carries the hot atoms to the reaction chamber. The hot atoms collide with cold atoms and undergo condensation through nucleation and form small clusters. In side reaction chamber other reactants are added to control the reaction rate. Then these clusters are allowed to condense on moving belt arrangement with scrapper to collect the nano-particles. The particle size could be controlled by rate of evaporation (energy input), rate of cluster formation (energy removal rate) and rate of condensation (cluster removal from the reaction chamber).

CVD method of synthesis of nano particles has many advantages.

1. The increased yield of nanoparticles
2. A wider range of ceramics including nitrides and carbides can be synthesised.
3. More complex oxides such as $BaTiO_3$ or composite structures can be formed
4. In addition to the formation of single phase nano particles by CVC of a single precursor the reactor allows the synthesis of
   (a) Mixtures of nano particles of two phases or doped nano particles by supplying two precursors at the front end of the reactor, and
   (b) Coated nano particles, i.e., n-ZrO_2 coated with n-Al_2O_3 or vice versa, by supplying a second precursor at a second stage of the reactor.

5(e)(i)

Sol: $I = \int_C \frac{z^2}{z^4-1}dz$

$$= \int \frac{z^2}{(z^2-1)(z^2+1)}\,dz$$

$$I = \frac{1}{2}\int\left(\frac{1}{(z^2-1)} + \frac{1}{(z^2+1)}\right)dz$$

$$I = \frac{1}{2}\oint_C \frac{1}{z^2-1}dz + \frac{1}{2}\oint_C \frac{1}{z^2+1}dz,\ C \text{ is } |z+1| = 1$$

$$= \frac{1}{2}\oint_C \frac{1}{(z-1)(z+1)}dz + \frac{1}{2}\oint_C \frac{1}{(z+i)(z-i)}dz$$

Only z = –1 lies inside the circle |z+1| = 1.

By Cauchy's Integral Formula and Cauchy's Integral Theorem, we have

$$I = \frac{1}{2}\,2\pi i\left[\frac{1}{z-1}\right]_{Z=-1} + \frac{1}{2}\,2\pi i\ (0)$$

$$\therefore\ I = -\frac{\pi i}{2}$$

5(e)(ii)

Sol: Let f(x) = $\frac{(\pi - x)^2}{4}$

The fourier series of f(x) in (0, 2π) is

$$f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} \{a_n \cos(nx) + b_n \sin(nx)\}$$

Now, $a_o = \frac{1}{\pi}\int_0^{2\pi} f(x)\ dx$

$$= \frac{1}{\pi}\int_0^{2\pi} \frac{(\pi - x)^2}{4}\ dx$$

$$= \frac{1}{\pi}\left[\frac{(\pi - x)^3}{-12}\right]_0^{2\pi}$$

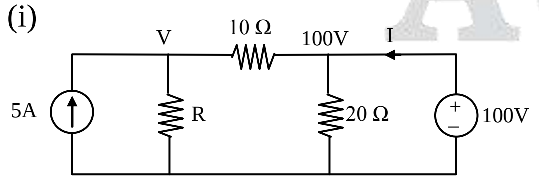
$$= \frac{-1}{12\pi}\left[-\pi^3 - \pi^3\right]$$

$$= \frac{2\pi^3}{12\pi} = \frac{\pi^2}{6}$$

∴ The constant term = $\frac{a_0}{2} = \frac{\pi^2}{12}$

6(a)

Sol: (i)



$$-5 + \frac{V}{R} + \frac{V - 100}{10} = 0 ----(1)$$

$$\frac{100 - V}{10} + \frac{100}{20} - I = 0 ----(2)$$

5V = 100 I ----(3)

Solve these three equations

We can get

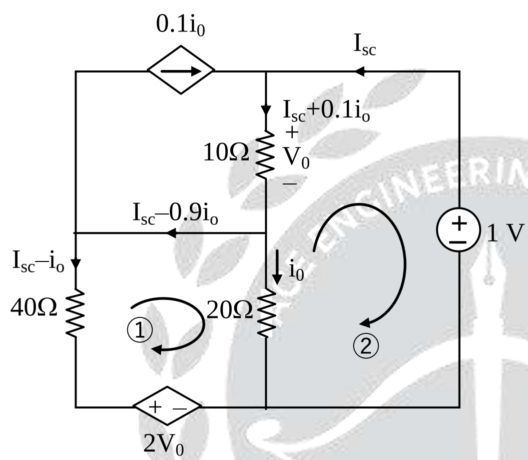
V = 100, I = 5A

Sub eq (1)

$$-5+\frac{100}{R}+\frac{100-100}{10}=0$$

R = 20 Ω

(ii)



KVL to Loop ①

$20i_0 - 2V_0 - 40\,(I_{sc} - i_c) = 0$

$20i_0 - 2V_0 - 40I_{sc} + 40\,i_0 = 0$

$40I_{sc} = 60i_0 - 2V_0$

$I_{sc} = \frac{6}{4}i_0 - \frac{1}{20}V_0$ .... (1)

$V_0 + 20i_0 = 1$ .... (2)

$V_0 = 10\,(I_{sc} + 0.1i_0)$

$V_0 = 10I_{sc} + i_0$ ....(3)

Solving (1) ,(2) and (3) we get

$I_{sc} = 0.03152$ A

$i_0 = 0.0326$ A

$V_0 = 0.3478$ V

$$R_{th} = \frac{1}{I_{sc}}$$

$$R_{th} = 31.72\ \Omega$$

6(b)(i)

Sol: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be the required matrix.

The given eigen vectors of a matrix $A_{2\times2}$ corresponding to eigen values $\lambda_1 = -2$ and $\lambda_2 = 5$ and $X_1 = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$ and $X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ respectively.

Consider $Ax_1 = \lambda_1 x_1$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -4 \\ 3 \end{bmatrix} = -2\begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$-4a + 3b = 8$…….(1)

$-4c + 3d = -6$……(2)

Consider $Ax_2 = \lambda_2 x_2$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$a + b = 5$…….(3)

$c + d = 5$…….(4)

Solving (1) and (3), we get

$a = 1$ & $b = 4$.

Again Solving (2) and (4), we get

$c = 3$ & $d = 2$

$\therefore$ The required matrix is $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$

6(b)(ii)

Sol: Given $\tan y \sec y \dfrac{dy}{dx} + \tan x \sec y = \cos^2 x$

Let secy = z

Then $\sec y \tan y \dfrac{dy}{dx} = \dfrac{dz}{dx}$

$\therefore \dfrac{dz}{dx} + (\tan x)\, z = \cos^2 x$

I.F = $e^{\int \tan x\, dx} = e^{\log(\sec x)} = \sec x$

The solution of given D.E is

z.secx = $\int \cos^2 x . \sec x\, dx = \int \cos x\, dx$

= sinx + c

$\therefore$ secy. secx = sinx + c ($\because$ z = secy)

Bernoulli's Equation:

The equation $\left(\dfrac{dy}{dx}\right) + Py = Qy^n$ …….. (1)

Where P, Q are functions of 'x' is reducible to the Leibnitz's linear equation and is usually called the Bernoulli's equation.

To solve (1), divide both sides by y^n, so that

$y^{-n}\left(\dfrac{dy}{dx}\right) + Py^{1-n} = Q$ …… (2)

Put $y^{1-n} = z$

Then (2) becomes

$\left(\dfrac{1}{(1-n)}\right)\dfrac{dz}{dx} + Pz = Q$ (or)

$\left(\dfrac{dz}{dx}\right) + P(1-n)z = Q(1-n)$

which is linear in 'z'.

6(c)(i)

Sol: All crystalline materials contain defects in some form or other. The nature and effects of such defects are very important in understanding the properties of materials. Technically important properties such as mechanical strength, ductility, electrical conduction in semiconductors are influenced by the defects. such effects of crystal imperfections are as given.

(a) Electrical Properties:

1. When a pure semiconductor is doped with pentavalent or trivalent impurities it results in extrinsic semiconductors. In such semiconductors electrical conductivity increases with doping concentration.
2. In the case of metals, the presence of impurity decreases the electrical conductivity.
3. Imperfections account for dielectric strength of a material. Even a micro void present reduces the dielectric strength drastically.

(b) Optical properties

1. The presence of impurity atoms in the crystal lattice results in characteristic colours to the crystals. These are called colour centres.
2. When aluminium oxide is doped with chromium atoms, we get ruby which is a very active laser medium. Many solid state lasers are prepared by proper doping only.

(c) Mechanical properties

The mechanical properties of metals are usually controlled by imperfections.

1. Plastic deformation is mainly due to motion of edge dislocation. By reducing the mobility of dislocations, the mechanical strength may be enhanced. Restricting or hindering dislocation motion renders a material harder and stronger.
2. Copper added to gold increases the ductility of gold so that it can be drawn into wires.
3. The presence of carbon atoms as interstitial impurity in iron lattice increases the strength of iron
4. Tin as substitutional impurity in copper lattice increases the bearing properties of copper.
5. Further imperfections account for creep yield strength, fracture strength, oxidation and corrosion characteristics.

6(c)(ii)

Sol: $a = 2.9\text{A}° = 2.9\times10^{-8}$ cm

Atomic weight (A) = 55.85

Avogadro's number (N_A) = 6.023×10^{23} g/mole

Density (ρ) = 7.87 g/cc

$$\rho = \frac{nA}{AN\times V_{vc}}$$

$$n = \frac{\rho\times AN\times V_{vc}}{A}$$

$$= \frac{7.87\times6.023\times10^{23}\times(2.9\times10^{-8})^3}{55.85}$$

= 2.069

6(c)(iii)

Sol: Given data;

$\lambda = 0.58\text{A}°$

$\theta = 9.5°$

$$d = \frac{a}{\sqrt{h^2+k^2+\ell^2}}$$

$$d_{200} = \frac{a}{\sqrt{2^2+0^2+0^2}} = \frac{a}{2} = 0.5a$$

From Bragg's law

$2d\sin\theta = n\lambda$

$2d_{200}\sin(9.5) = 1\times0.58$

$2\times0.5\,a\times0.165 = 0.58$

$a = 0.52\ \text{A}°$

7(a)

Sol: (i) Let $f(x) = x^4 - x - 10$

So that $f(1) = -10 = -\text{ve}$, f(2)

= 16 – 2 – 10 = 4 = + ve

$\therefore$ a root of f(x) = 0 lies between 1 and 2. Let us take $x_0 = 2$

$f'(x) = 4x^3 - 1$

Newton-Raphson's formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \text{ -------(i)}$$

Putting n = 0, the first approximation x_1 is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)}$$

$$= 2 - \frac{4}{4\times 2^3 - 1} = 2 - \frac{4}{31} = 1.871$$

Putting n = 1 in (i), the second approximation is

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.871 - \frac{f(1.871)}{f'(1.871)}$$

$$= 1.871 - \frac{(1.871)^4 - (1.871) - 10}{4(1.871)^3 - 1}$$

$$= 1.871 - \frac{0.3835}{25.199} = 1.856$$

putting n = 2 in (i), the third approximation is

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.856 - \frac{(1.856)^4 - (1.856) - 10}{4(1.856)^3 - 1} = 1.856 - \frac{0.010}{24.574} = 1.856$$

Here $x_2 = x_3$.

Hence the desired value is 1.856 correct to three decimal places.

(ii) We have $\frac{\partial z}{\partial x} = f'(x+ct).\frac{\partial}{\partial x}(x+ct) + \phi'(x-ct)\frac{\partial}{\partial x}(x-ct) = f'(x+ct) + \phi'(x-ct)$ and

$$\frac{\partial^2 z}{\partial x^2} = f''(x+ct) + \phi''(x-ct) \text{ -----------(1)}$$

Again $\frac{\partial z}{\partial t} = f'(x+ct)\frac{\partial}{\partial t}(x+ct) + \phi'(x-ct)\frac{\partial}{\partial t}(x-ct)$

$$= cf'(x+ct) - c\phi'(x-ct)$$

And $\frac{\partial^2 z}{\partial t^2} = c^2 f''(x+ct) + c^2\phi''(x-ct) = c^2[f''(x+ct) + \phi''(x-ct)$ ---------(2)

from equ. (1) and (2), it follows that $\frac{\partial^2 z}{\partial t^2} = c^2\frac{\partial^2 z}{\partial x^2}$

(iii) Operating $R_1 \to R_1 - R_2 - R_4$, $R_2 \to R_2 - 3R_3$, $R_3 \to R_3 - 2R_4$, the given determinant becomes

$$\Delta = \begin{bmatrix} -9 & -12 & 0 & -2 \\ 6 & -2 & 0 & 1 \\ -6 & -6 & 0 & -1 \\ 6 & 7 & 1 & 2 \end{bmatrix}$$

Expand by C_3,

$$= -\begin{vmatrix} -9 & -12 & -2 \\ 6 & -2 & 1 \\ -6 & -6 & -1 \end{vmatrix} = -24$$

7(b)

Sol: (i) $|\overline{E}| = \frac{k}{\rho}$

$$W_E = \frac{\varepsilon_0}{2}\int |\overline{E}|^2 \, dv$$

$$= \frac{\varepsilon_0}{2}\int_2^3\int_0^{\pi/2}\int_5^7 \frac{k^2}{\rho^2} \times \rho \, d\rho \, d\phi \, dz$$

$$= \frac{\varepsilon_0}{2} \times k^2 \times [\ln\rho]_2^3 [\phi]_0^{\pi/2} [z]_5^7$$

$$= k^2 \times \frac{10^{-9}}{36\pi \times 2} \times \ln\left(\frac{3}{2}\right) \times \frac{\pi}{2} \times 2$$

$$= k^2 \times \frac{10^{-9}}{72} \times \ln\left(\frac{3}{2}\right)$$

$W_E = 1\mu J = 10^{-6}$

$$\Rightarrow k^2 \times \frac{10^{-9}}{72} \times \ln\left(\frac{3}{2}\right) = 10^{-6}$$

$$\Rightarrow k^2 = \frac{72}{10^{-3} \times \ln\left(\frac{3}{2}\right)} = 177573.84$$

$\Rightarrow$ k = 421.39

(ii) point P is above conducting plane z=2 . If we drop a perpendicular from point P on the plane z=2, the coordinate of the foot of the perpendicular will be (2,–3,2). Hence the distance of point P from the z=2 plane is

$$\sqrt{(2-2)^2 + (-3+3)^2 + (5-2)^2} = 3.$$

Consider a point P′ which is mirror image of point P.The distance of point P′ from the plane z=2 will be 3.Hence the co ordinate of point p′ be (2,–3,-1).If a perpendicular is dropped from P′ on plane z=2, the co ordinates of foot of perpendicular will be (2,–3,2). At this point P′, the charge of –25nC(which is image of 25nC) is located.

V at (3,2,4) is = V due to 25nC+V due to –25nC

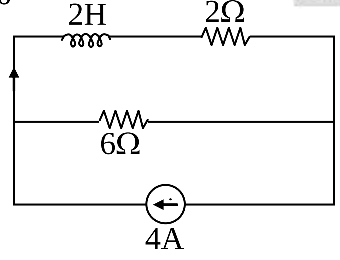
$$= \frac{25\times10^{-9}}{4\pi\varepsilon_0\sqrt{(3-2)^2+(2+3)^2+(4-5)^2}} +$$

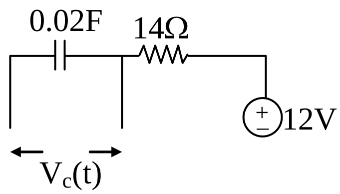
$$\frac{-25\times10^{-9}}{4\pi\varepsilon_0\sqrt{(3-2)^2+(2+3)^2+(4+1)^2}}$$

=11.7789V

7(c)

Sol: At $t = 0^-$



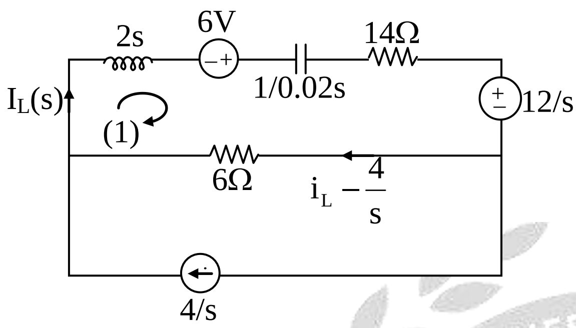


Inductor is short circuited

$I_L(0^-)$ at steady state $= 4\times\frac{6}{8} = 3A$

$V_c(0^-) = 0V$

At $t = 0^+$



Apply KVL in loop (1)

$$6-\left(2s+\frac{1}{0.02s}+14\right)i_L-\frac{12}{s}-(6)\left(i_L-\frac{4}{s}\right)=0$$

$$6-\frac{12}{s}+\frac{24}{s}=i_L\left(14+2s+\frac{50}{s}+6\right)$$

$$\frac{6s+12}{s}=i_L\left(20s+2s^2+50\right)$$

$$I_L(s)=\frac{6(s+2)}{2s^2+20s+50}=\frac{3(s+2)}{s^2+10s+25}$$

$$=\frac{3(s+2)}{(s+5)^2}$$

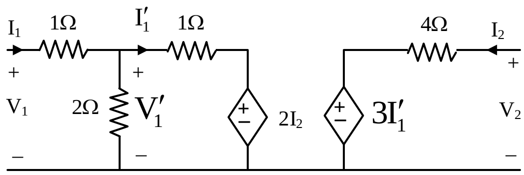
$$i_L=\frac{3(s+5-3)}{(s+5)^2}$$

$$=\frac{3}{(s+5)}-\frac{9}{(s+5)^2}$$

$i_L(t) = 3e^{-5t} - 9te^{-5t}$

$= (3 - 9t)e^{-5t}A$

(ii)



$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$V_1 = V_1' = -2I_1' = 1\times I_1' + 2I_2$$

$$\Rightarrow -3I_1' = 2I_2$$

$$I_1' = \frac{-2}{3}I_2 \qquad V_1 = -2\times\frac{-2}{3}I_2 = \frac{4}{3}I_2$$

$V_1 = Z_{11}\ I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

$$\therefore Z_{12} = \frac{4}{3}\Omega$$

8(a)(i)

Sol: Properties and requirements of good insulating materials:

The requirement of good insulating materials can be classified as electrical, mechanical, thermal and chemical.

1. Electrical properties:

- Electrically, the insulating material should have high resistivity to reduce the leakage current and high dielectric strength to enable it to withstand higher voltage without being punctures or broken down.
- Further, the insulator should have small dielectric loss.

2. Mechanical properties:

- Since the insulators are used on the basis of volume and not weight, a low density is preferred.
- The insulators should also have small thermal expansion to prevent mechanical damage.

- Further, it should be non-ignitable, or if ignitable, it should be self extinguishable.

3. Thermal properties:

- A uniform viscosity for liquid insulators ensures uniform thermal and electrical properties.
- Liquid and gaseous insulators are used also used as coolants. For example, transformer oil, hydrogen and helium are used both for insulation and cooling purposes. For such materials, good thermal conductivity is a desirable property.

4. Chemical properties:

- Chemically, the insulators should be resistant to oil, liquids, gas fumes, acids and alkalies.
- It should not deteriorate by the action of chemicals in soils or by contact with other metals.
- The insulator should not absorb water particles, since water lowers the insulation resistance and the dielectric strength.
- Insulating materials should have certain mechanical properties depending on the use to which they are put. Thus when used for electric machine insulation, the insulator should have sufficient mechanical strength to withstand vibration. Good heat conducting property is also desirable in such cases.
- Materials with large electronic and ionic polarizabilities and therefore large permittivity are used for making dielectric capacitors. Titanium oxide which has a permittivity of about 100 is a good example of such a material.
- The use of molecules with a permanent dipole moment is not desirable because of the possibility of large dielectric losses at high frequencies.

8(a)(ii)

Sol: Given data:

$Q = 2\times10^{-10}$ C

$d = 4\text{mm} = 4\times10^{-3}$ m

$\varepsilon_r = 3.5$

$A = 650\ \text{mm}^2 = 650\times10^{-6}\ \text{m}^2$

$$V = \frac{Qd}{\varepsilon_0\varepsilon_r A}$$

$$= \frac{2\times10^{-10}\times4\times10^{-3}}{8.85\times10^{-12}\times3.5\times650\times10^{-6}}$$

$= 3.973 \times10^{-4} \times10^{5}$

$= 39.73$ V

8(b)(i)

Sol: Given $f(x) = 3x^3 - 7x^2 + 5x + 6$ in $[0, 2]$

$\Rightarrow f'(x) = 9x^2 - 14x + 5$ & $f''(x) = 18x - 14$

Consider $f'(x) = 0$ for stationary points

$\Rightarrow 9x^2 - 14x + 5 = 0$

$\Rightarrow (x - 1)(9x - 5) = 0$

$\therefore x = 1, \frac{5}{9}$ are stationary points

At $x = 1$, $f''(1) = 18 - 14 = 4 > 0$

$\Rightarrow$ Local minimum exists at $x = 1$

At $x = \frac{5}{9}$, $f''\left(\frac{5}{9}\right) = 18\left(\frac{5}{9}\right) - 14 = -4 < 0$

$\Rightarrow$ Local maximum exists at $x = \frac{5}{9}$

Now, $f(0) = 6$, $f(2) = 12$ and $f\left(\frac{5}{9}\right) = 7.13$

$\therefore$ The maximum value of $f(x)$ in $[0, 2] = \max\left\{f(0), f(2), f\left(\frac{5}{9}\right)\right\}$

$= \max\{6, 12, 7.13\} = 12$

8(b)(ii)

Sol: Given $\overline{F} = (2xy + z^3)\overline{i} + x^2\,\overline{j} + 3xz^2\,\overline{k}$

$$\text{Curl } \overline{F} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy + z^3 & x^2 & 3xz^2 \end{vmatrix}$$

$= \overline{i}[0-0] - \overline{j}[3z^2 - 3z^2] + \overline{k}[2x - 2x] = \overline{0}$

$\Rightarrow$ $\overline{F}$ is irrotational

$\Rightarrow$ Work done by $\overline{F}$ is independent of path of curve

$\Rightarrow \overline{F} = \nabla\phi$

Where $\phi(x, y, z)$ is scalar potential

$\Rightarrow (2xy + z^3)\overline{i} + x^2\,\overline{j} + 3xz^2\,\overline{k} = \frac{\partial\phi}{\partial x}\overline{i} + \frac{\partial\phi}{\partial y}\overline{j} + \frac{\partial\phi}{\partial z}\overline{k}$

$\Rightarrow d\phi = (2xy + z^3)\,dx + x^2\,dy + 3xz^2\,dz$

$\Rightarrow \int d\phi = \int (2xy + z^3)dx + x^2\,dy + 3xz^2\,dz$

$\Rightarrow \int d\phi = \int d(x^2y + xz^3)$

$\Rightarrow \phi(x, y, z) = x^2y + xz^3$

$\therefore$ Work done $= \int_C \overline{F}.d\overline{r} = \phi(3,1,4) - \phi(1,-2,1)$

$= [9(1)+3(64)] - [1(-2) + 1(1)]$

$= 202$

8(c)

Sol: (i). Factors affecting resistivity:

i. **Temperature:** The electrical resistance of most metals increases with increase of temperature while those of semiconductors and electrolytes decreases with increase of temperature.

ii. **Alloying:** A solid solution has a less regular structure than a pure metal. Consequently, the electrical conductivity of a solid solution alloy drops off rapidly with increased alloy content.

In other words, the addition of small amounts of impurities leads to a considerable increase in resistivity.

- The temperature coefficient of metals is very small. Therefore, increase in resistivity due to the addition of impurities is temperature independent. This suggests the existence of alloys, whose resistance varies little with temperature.
- The resistivity may be said to be consists of two parts; one part is resistance at absolute zero of temperature, and another part, which arises from crystal imperfections and this part would be zero only in undistorted crystals.
- The residual resistance of alloys is obtained by extrapolating the temperature resistance curves to absolute zero, therefore, is quite appreciable. Further, different atoms dissolved in a given solvent metal, affect the resistivity in different ways, which is largely dependent upon the balance of the solvent and solute atoms.

(iii)Cold Work: Mechanical distortion of the crystal structure decreases the conductivity of a metal because the localized strains interfere with electron movement. Thus, hard drawn copper wire has a lower conductivity than annealed copper. Subsequent annealing restores the electrical conductivity by establishing greater regularity in the crystal lattice.

iv. Age Hardening: Age hardening increases the resistivity of an alloy.

(ii) Given data;

$L = 1\text{ cm} = 1\times10^{-2}\text{m}$

$A = 10^{-3}\times10^{-3} = 10^{-6}\text{ m}^2$

$n_i = 2.5\times10^{19}/\text{m}^3$

$\mu_e = 0.39\text{m}^2\text{V}^{-1}\text{s}^{-1}$

$\mu_p = 0.19\text{m}^2\text{V}^{-1}\text{s}^{-1}$

Resistance $R = \dfrac{1}{\sigma A} = \dfrac{1}{n_i e(\mu_p+\mu_e)A}$

$$= \frac{10^{-2}}{2.5\times10^{19}\times1.6\times10^{-19}(0.39+0.19)}$$

$$= \frac{10^{-2}\times10^{6}}{2.5\times0.58\times1.6} = 4.31\times10^{3}\ \Omega$$