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ESE – 2019 MAINS
OFFLINE TEST SERIES

ELECTRICAL ENGINEERING

TEST – 12
SOLUTIONS

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1(a)

Sol: At 800 rpm, $E_a = (4.8I_a + 200)$ volts

At 600 rpm, $E_{a1} = (4.8I_{a1} + 200) \times \frac{600}{800} = 3.6\ I_{a1} + 150$

Power developed in armature = $E_{a1}.\ I_{a1} = (3.6I_a + 150)I_{a1}$ watts

Armature terminal power = $E_{a1}\ I_{a1} - I_{a1}^2 r_a$

Descending load torque $= 100 \times \frac{0.40}{2} \times 9.81\,\text{Nm}$ = Rheostatic braking torque, T_d

$\therefore\ T_d = 20 \times 9.81 = 192.2$ Nm

For torque balance, dynamic braking torque = Descending load torque

$\therefore\ \frac{(3.6I_{a1} + 150)I_{a1} - I_{a2}^2 \times 1}{2\pi \times 600/60} = 196.2$ Nm

Or $\quad 2.6\,I_{a1}^2 + 150I_{a1} = 12327.61$

Or $\quad I_{a1}^2 + 57.692I_{a1} - 4741.39 = 0$

Its solution gives, $I_{a1} = 45.81$ A.

Generated voltage at 600 rpm is $E_{a1} = 3.6 \times 45.81 + 150 = 314.92$ V.

If R_b is the braking resistance to be connected in series with armature, then

$$I_{a1} = \frac{E_{a1}}{R_b + r_a} = \frac{314.92}{R_b + 1} = 45.81$$

$$R_b = 5.874\ \Omega.$$

1(b)

Sol: $x(2n) = f(n) + g(n)$

$x(2n+1) = f(n) - g(n)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk} = \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2Nk} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{(2n+1)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} [f(n) + g(n)] W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} [f(n) - g(n)] W_N^{(2n+1)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} f(n)\left[W_N^{2nk} + W_N^{(2n+1)k}\right] + \sum_{n=0}^{\frac{N}{2}-1} g(n)\left[W_N^{2nk} - W_N^{(2n+1)k}\right]$$

$$= \left(1+W_N^k\right)\sum_{n=0}^{\frac{N}{2}-1} f(n)W_N^{2nk} + \left(1-W_N^k\right)\sum_{n=0}^{\frac{N}{2}-1} g(n)W_N^{2nk}$$

$$= \left(1+W_N^k\right)\sum_{n=0}^{\frac{N}{2}-1} f(n)W_{\frac{N}{2}}^{nk} + \left(1-W_N^k\right)\sum_{n=0}^{\frac{N}{2}-1} g(n)W_{\frac{N}{2}}^{nk}$$

$$X(k) = \left(1+W_N^k\right)F(k) + \left(1-W_N^k\right)G(k)$$

1(c)

Sol: $\%M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$

$$\therefore\ 26 = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

$$e^{+\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = \frac{100}{26}$$

$$\therefore\ \frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.34$$

$$1-\zeta^2 = \left(\frac{\zeta\pi}{\sqrt{1.34}}\right)^2$$

$$1-\zeta^2 = 5.49\,\zeta^2$$

$$\zeta^2 = \frac{1}{1+5.49}$$

$$\therefore\quad \zeta = \frac{1}{6.49} = 0.39$$

Resonant frequency:

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

$$8 = \omega_n\sqrt{1-2\zeta^2} = \omega_n\sqrt{1-2\times 0.39^2}$$

$= \omega_n \sqrt{0.699}$

$\omega_n = \dfrac{8}{\sqrt{0.699}}$ = 9.6 rad/sec

The overall transfer function is $M(s) = \dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)} = \dfrac{\dfrac{K}{s(ST+1)}}{1+\dfrac{K}{s(ST+1)}.1}$

$$= \frac{K}{s^2T+s+K} = \frac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}}$$

The characteristic equation is

$s^2 + \dfrac{1}{T}s + \dfrac{K}{T} = 0$

On comparing above equation with $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$ (characteristic equation of a second order system)

$2\zeta\omega_n = \dfrac{1}{T}$

$T = \dfrac{1}{2\zeta\omega_n} = \dfrac{1}{2\times 0.39\times 9.6} = 0.13$ and $\omega_n^2 = \dfrac{K}{T}$

$K = \omega_n^2 T = (9.6)^2 (0.13) = 11.9$

Resonant peak :

$M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}} = \dfrac{1}{2\times 0.39\sqrt{1-0.39^2}} = 1.39$

Gain crossover frequency:

$\omega_1 = \omega_n \sqrt{\left(4\zeta^4+1\right)^{1/2} - 2\zeta^2}$

$= 8.69 \sqrt{\left(4\times 0.39^4+1\right)^{1/2} - 2\times 0.39^2}$

$= 8.69 \sqrt{0.74} = 7.47$ rad/sec

Phase margin $\phi = \tan^{-1}\left[\dfrac{2\zeta}{\sqrt{\left(4\zeta^4+1\right)^{1/2} - 2\zeta^2}}\right]$

$$= \tan^{-1}\left[\frac{2\times0.39}{\sqrt{\left(4\times0.39^4+1\right)^{1/2}-2\times0.39^2}}\right]$$

$$= \tan^{-1}(1.054) = 46.5°$$

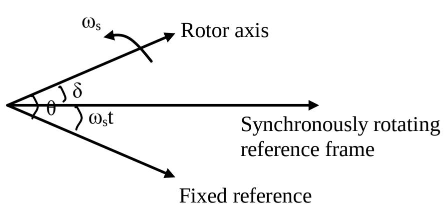
1(d)

Sol: **Swing equation:** The equation which gives the relation between accelerating power and angular acceleration is called as swing equation and also it describes the relative motion of the rotor with respect to the stator field as a function of time.

Derivation: The behavior of a synchronous machine during transients is described by the swing equation. Let θ be the angular position of the rotor at any instant 't'. However, θ is continuously changing with time. It is convenient to measure θ with respect to reference axis that is rotating at synchronous speed. If 'δ' is the angular displacement of the rotor in electrical degrees from the synchronously rotating reference axis and ω_s the synchronous speed in electrical radians, then θ can be expressed as the sum of

(i) Time varying angle ω_s t on the rotating reference axis,

(ii) The torque angle δ of the rotor w.r.t. the rotating reference axis.



In other words,

$\theta = \omega_s t + \delta$ electrical radians …… (1)

differentiating above equation w.r.t. 't', we get

$$\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt}$$

again differentiating above equation

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}$$

Angular acceleration of the rotor $\alpha = \frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}$ elec.rad/s^2 ……… (2)

If damping is neglected, the accelerating torque (T_a) in a synchronous generator is equal to the difference of input mechanical or shaft torque (T_s) and the output electromagnetic Torque (T_e).

$T_a = T_s - T_e$ ……..… (3)

Let ω_s = synchronous speed of the motor

J = moment of inertia of the rotor

N = angular moment of the rotor

P_s = mechanical power input

P_e = electrical power output

P_a = accelerating power

$M = J\omega_s$ ……….. (4)

Multiplying both sides of equation (3) by 'ω_s'

$\omega_s T_a = \omega_s T_s - \omega_s T_e$

$P_a = P_s - P_e$

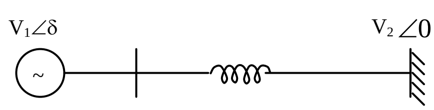
But $J\frac{d^2\theta}{dt^2} = T_a$; $J\frac{d^2\delta}{dt^2} = T_a$

$\omega_s J\frac{d^2\delta}{dt^2} = \omega_s T_a$

$M\frac{d^2\delta}{dt^2} = P_s - P_e = P_a$ ……..… (5)

Equation (5) gives the relation between the accelerating power and angular acceleration. It is called swing equation and it is a non-linear differential equation of the second order.

Application: The study of steady state stability of power system involves the study of dynamics of the system when the rate of application of load is quite slow as compared with the natural frequency of oscillation of the system. The dynamics of the system can be described by swing equation which is non-linear.



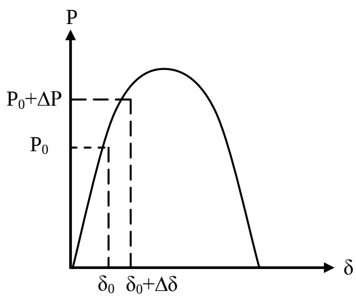
Consider a system consisting of a generator connected to an infinite bus bar through a lossless network with initial operating point on the power angle curve as (P_o, δ_o).

If the load is changed (increased) by ΔP, then the load angle changes by $\Delta\delta$ and is given as

$$M\frac{d^2\Delta\delta}{dt^2} = P_i - P_e$$

$$= P_i - (Po + \Delta P) = -\Delta P$$

$$= -\left(\frac{\partial P}{\partial \delta}\right)_{=\delta_0} .\Delta\delta \text{ …………..… (1)}$$



Let $\frac{d}{dt} = K$, then equation (1) reduces to

$$Mk^2 \cdot \Delta\delta + \left(\frac{\partial P}{\partial \delta}\right)_{=\delta_0} .\Delta\delta = 0$$

$$\left[Mk^2 + \left(\frac{\partial P}{\partial \delta}\right)_{=\delta_0}\right].\Delta\delta = 0 \text{ …………(2)}$$

Equation (2) is the characteristic equation with the two roots $k = \pm\left(\frac{\left(-\frac{\partial P}{\partial \delta}\right)_{\delta_0}}{M}\right)$

When $\frac{\partial P}{\partial \delta}$ is positive $\left(\delta < \frac{\pi}{2}\right)$, the two roots are pure imaginary and conjugate, then the rotor motion is oscillatory and damped around δo.

When $\frac{\partial P}{\partial \delta}$ is negative $\left(\delta > \frac{\pi}{2}\right)$, both the roots are real, one negative and other positive and hence the system is unstable.

When, $\frac{\partial P}{\partial \delta} = 0$, ($\delta$ = 90°), the system is critically stable. The frequency of oscillation is given by the roots of the characteristic equation.

1(e)

Sol: 20HP, 220V, 1000 rpm, D.C series motor controlled by a single phase half controlled converter

$R_f = 0.05\Omega$, $R_a = 0.12\ \Omega$

$V_s = 230$ V, $V_m = 230\sqrt{2}$ V,

50Hz frequency

K_b residual = 0.08 V /rad /sec, $K_T = 0.04$ Nm/A^2

$V_t = E_b + I_a (R_a + R_{se})$

For rated condition,

$220 = E_b + I_a(0.05+0.12)$

$E_b = (K_T)\ I_a \left(\frac{2\pi}{60} \times N\right)$

$E_b = (0.04\, I_a)\left(\frac{2\pi}{60} \times 1000\right)$

$E_b = 4.188\ I_a$

$220 = 4.188\ I_a + I_a\ (0.17)$

$I_a = 50.48$ Amp

$\therefore$ Rated current = 50.48A

$E_b = 4.188\ (50.48) = 211.41$V

$$\frac{V_m}{\pi}(1 + \cos\alpha) = E_b + I_a(R_a + R_{se})$$

$$\frac{230\sqrt{2}}{\pi}(1 + \cos 30) = E_b + 50.48(0.12 + 0.05)$$

$E_b = 184.61$ V

$E_b \propto \omega$

$$\frac{211.41}{184.61} = \frac{1000}{x}$$

x = 873.232rpm

$\therefore$ speed at which it is operating is 873.232rpm

Torque = $K_T\,(I_a)^2$

$= 0.04\,(50.48)^2 = 101.92$ Nm

2(a)(i)

Sol: For main winding

$$Z_m = \frac{100}{2} = 50\ \Omega$$

$$r_m = \frac{40}{(2)^2} = 10\ \Omega$$

$$\therefore \quad X_m = \sqrt{(50)^2 - (10)^2} = 48.99\ \Omega$$

For auxiliary winding, $Z_a = \dfrac{80}{1} = 80\ \Omega$

$$r_a = \frac{50}{I^2} = 50\ \Omega$$

$$X_a = \sqrt{(80)^2 - (50)^2} = 62.45\ \Omega$$

The value of x_c for obtaining the maximum starting torque is, $x_c = X_a + \dfrac{r_a \times r_m}{Z_m + X_m}$

$$x_c = 62.45 + \frac{50 \times 10}{50 + 48.99} = 67.50\ \Omega$$

$\therefore$ Capacitance C for maximum starting torque is

$$C = \frac{10^6}{2\pi \times 50 \times 67.50} = 47.16\ \mu F.$$

(b) If I_a leads V_1 by θ_a and I_m lags V_1 by θ_m, then

$$\theta_m + \theta_a = 90^o$$

Or $\theta_a = 90 - \theta_m$

Or $\tan\theta_a = \tan(90 - \theta_m)$

$= \cot\theta_m$

Since I_a leads V_1 by θ_a

$$\tan\theta_a = \frac{x_c - X_a}{r_a}$$

and $\tan\theta_m = \frac{X_m}{r_m}$

$\therefore \frac{x_c - X_a}{r_a} = \frac{r_m}{X_m}$

Or $x_c = X_a + \frac{r_a r_m}{X_m}$

$x_C = 62.45 + \frac{50\times10}{48.99} = 72.66\ \Omega$

Therefore, the starting capacitance C that would make I_a and I_m in time quadrature at starting, is given by

$$C = \frac{10^6}{2\pi\times50\times72.66} = 43.81\ \mu F$$

2(a)(ii)

Sol: Power in kVA for 8 Hr = $\frac{160}{0.8} = 200$ kVA

Power in kVA for 6 Hr = $\frac{100}{1} = 100$ kVA

Total copper losses for 24 hours

= copper losses for 8 hours + copper losses for 6 hours

Copper losses for 8 hours

= (% FL)^2.(FLCu loss)

$= \left(\frac{200}{2000}\right)^2 (3\times10^3)$ = 30 W

Copper losses for 6 hours

= (% FL)^2(FL Cu loss)

$= \left(\frac{100}{2000}\right)^2 (3\times10^3)$

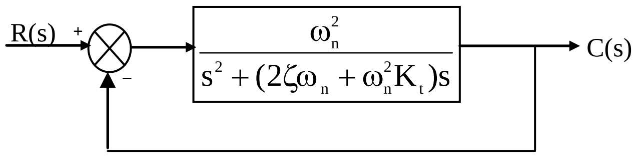
= 7.5 W

$\therefore$ Total copper loss = 8× 30 + 6 ×7.5

= 285 W

2(b)(i)

Sol: A.



From the block diagram figure the transfer function for a unity feedback second order – control system using derivate feedback control is determined below:

$$\frac{C(s)}{R(s)}=\frac{\frac{\omega_n^2}{s^2+(2\zeta\omega_n+\omega_n^2K_t)s}}{1+\frac{\omega_n^2}{s^2+\left(2\zeta\omega_n+\omega_n^2K_t\right)s}.1}$$

$$\frac{C(s)}{R(s)}=\frac{\omega_n^2}{s^2+(2\zeta\omega_n+\omega_n^2K_t)s+\omega_n^2}$$

The characteristic equation for the overall transfer function is

$s^2+(2\zeta\,\omega_n+\ \omega_n^2\ K_t)s+\ \omega_n^2\ =0$

The damping ratio for the above characteristic equation is

$$\zeta^{'}=\frac{2\zeta\omega_n+\omega_n^2K_t}{2\omega_n}$$

$$\zeta^{'}=\zeta+\frac{\omega_nK_t}{2}$$

The damping ratio is increased by using derivative feedback control and therefore, the maximum overshoot is decreased. However, the rise time is increased.

B. In the block diagram figure the forward path transfer function is

$$G(s)=\frac{\omega_n^2}{s^2+\left(2\zeta\omega_n+\omega_n^2K_t\right)s}$$

and the feedback path transfer function is H (s) = 1

The transfer function relating E(s) and R(s) is given by

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

Substituting

G(s) = $\dfrac{\omega_n^2}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s}$

and H (s) = 1 in $\dfrac{E(s)}{R(s)} = \dfrac{1}{1+G(s)H(s)}$ the following relation between the error and input signal for the derivative feedback control action is obtained

$$\frac{E(s)}{R(s)} = \frac{1}{1 + \dfrac{\omega_n^2}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s}.1}$$

$$\frac{E(s)}{R(s)} = \frac{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s + \omega_n^2}$$

For a unit ramp function

R (s) = $1/s^2$

$$\therefore E(s) = \frac{1}{s^2}.\frac{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s + \omega_n^2}$$

The steady state error is determined below:

$$e_{ss} = \lim_{s\to 0} s\,E(s)$$

$$= \lim_{s\to 0} s.\frac{1}{s^2}.\frac{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s + \omega_n^2}$$

$$e_{ss} = \frac{2\zeta}{\omega_n} + K_t$$

2(b)(ii)

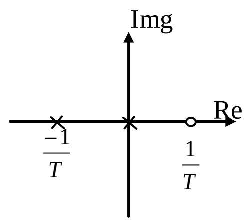
Sol: If the transfer function has symmetric pole and zero about the imaginary axis in s – plane then the transfer function is called all pass transfer function and given by

$$G(s) = \frac{1-sT}{1+sT}$$

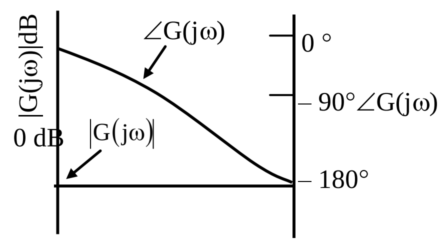
In sinusoidal form above transfer function is written as

$$G(j\omega) = \frac{1-j\omega T}{1+j\omega T}$$

The pole – zero configuration and Bode plot for all pass transfer function is shown in Figure



Pole –zero configuration



Bode plot for all pass transfer function

The magnitude plot lies on 0 db axis indicating that the actual gain for the frequencies is 1, thus the transfer function passes all frequencies.

The phase angle for all pass transfer function is given by

$$\angle\frac{1-j\omega T}{1+j\omega T} = \tan^{-1}(-\omega T) - \tan^{-1}(\omega T)$$

$$= -2\tan^{-1}(\omega T)$$

The phase angle for all pass transfer functions given by $-2\tan^{-1}(\omega T)$ and varies from 0° to –180° as the frequency is varied from $\omega = 0$ to $\omega = \infty$

2(c)

Sol: A. We have

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega = 2\pi x(t)$$

Substituting t = 0 in the above equation, we get

$$\int_{-\infty}^{\infty} X(\omega)d\omega = 2\pi x(0)$$

$$= 2\pi\ (1)$$

$$= 2\pi$$

B. From Parsavel's theorem, we have

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$\therefore \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 2\pi\int_{-\infty}^{\infty} |x(t)|^2 dt = 2\pi\left[\int_{-1}^{0}(t+1)^2 dt + \int_{0}^{2}(1+t)^2 dt + \int_{2}^{3}(t-3)^2 dt\right] = \frac{8\pi}{3}$$

C. We have $\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\, d\omega = 2\pi x(t)$

Substituting t = 2, we get

$$\int_{-\infty}^{\infty} X(\omega)e^{j2\omega}\, d\omega = 2\pi x(2)$$

$$= 2\pi\ (-1) = -2\pi$$

D. We have

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

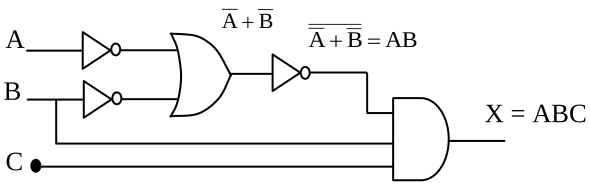
Substituting ω = 0, we get

$$X(0) = \int_{-\infty}^{\infty} x(t)dt$$

= 0 [∵ x(t) is a shifted odd signal.]

3(a)(i)

Sol:



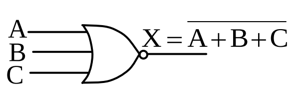
$$X = \left(\overline{\bar{A}+\bar{B}}\right)BC$$

$$X = \left(\overline{\overline{AB}}\right)BC = ABC$$

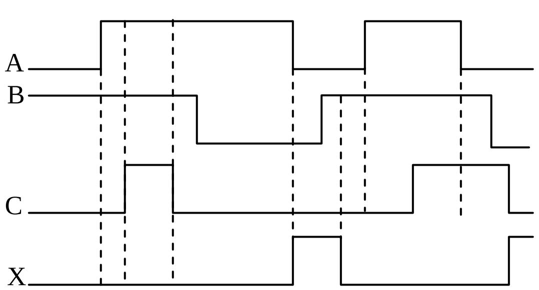
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

3(a)(ii)

Sol: A.



For NOR gate output is 1 only when all inputs are 0



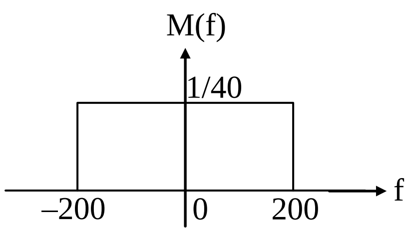
B. with C = 1, output X = 0 for all times, because

$$X = \overline{A+B+C} = \overline{A+B+1} = \overline{1} = 0$$

3(a)(iii)

Sol: m(t) = 10 sinc(400t)

Then M(f) represents a rectangular function as shown in the figure below.



$\therefore$ Then maximum frequency of message signal is 200(Hz)

A. $\beta = \frac{\Delta f_{max}}{f_{max}}$

$\therefore$ Δf_{max} = maximum frequency deviation = βf_{max}

$\therefore$ $\Delta f_{max} = 6 \times 200 = 1200$(Hz)

B. Since β = 6(>1) the modulated signal represents a WBFM signal.

Power of modulated signal = power of carrier signal

$$= \frac{A_C^2}{2}$$

$$\therefore \text{ Power } = \frac{(100)^2}{2} = 5(\text{kW})$$

3(b)(i)

Sol: Load MVA, $\overline{S}$ = 500 + j100

S = 510

MVA rating of each (single phase) transformer = 510/3 = 170

Voltage rating of each transformer = $\frac{345\sqrt{3}}{22}$ = 9.054

Let us choose voltage of star phase A as reference then

Star side: $\overline{V}_A = \overline{V}_{AN} = \dfrac{345}{\sqrt{3}}\angle 0^\circ = 199.2\angle{-240^o}$ kV

$\overline{V}_B = 199.2\angle{-120^o}$ kV

$\overline{V}_C = 199.2\angle{-240^o}$ kV

Note: Phase sequence is ABC

$\overline{V}_{AB} = \overline{V}_A - \overline{V}_B = 345\angle 30^o$ kV

$\overline{V}_{BC} = 345\angle{-90^o}$ kV

$\overline{V}_{CA} = 345\angle{-210^o}$ kV

Or $\overline{I}^* = \dfrac{500 + j100}{3\times 199.2} = 0.837 + j0.167$ kA; as $\overline{S} = VI^*$

$\overline{I}_A = 0.837 - j0.167 = 0.853\angle{-11.3^o}$ kA

$\overline{I}_B = 0.853\angle{-131.3^o}$ kA

$\overline{I}_C = 0.853\angle{-251.3^o}$ kA

Delta side: $\overline{V}_{ab} = \dfrac{\overline{V}_A}{a} = \dfrac{199.2}{9.054}\angle 0^\circ = 22\angle 0^o$ kV

$\overline{V}_{bc} = 22\angle{-120^o}$ kV

$\overline{V}_{ca} = 22\angle{-240^o}$ kV

$\overline{I}_{ab} = 9.054\times 0.853\angle{-11.3^\circ} = 7.723\angle{-11.3^o}$ kA

$\overline{I}_{bc} = 7.723\angle{-131.3^\circ}$ kA

$\overline{I}_{ca} = 7.723\angle{-251.3^\circ}$ kA

$\overline{I}_a = \overline{I}_{ab} - \overline{I}_{bc} = \sqrt{3}\times 7.723\angle(-11.3^\circ - 30^\circ) = 13.376\angle{-41.3^o}$

$\overline{I}_b = 13.376\angle(-120^o - 11.3^o) = 13.376\angle{-131.3^o}$ kA

$\overline{I}_c = 13.376\angle(-240^o - 11.3^o) = 13.376\angle{-251.3^o}$ kA

It is easily observed from above that line voltages and currents on the star side lead those on the delta side by 30^o.

3(b)(ii)

Sol:

- It increases the starting torque produced by Induction motor.
- It is also increases the rotor power factor at the time of starting.
- It limits the starting current drawn by Induction motor. Therefore no external starting methods are required to start this induction motor.
- If external resistance is varied on running condition then the speed of Induction motor will be controlled. This method of speed controlling is called resistance speed controlling.
- In running condition leakage flux is high and hence leakage reactance of rotor is high. Therefore power transfer from stator to rotor is less and the machine has less running torque.
- Slip ring induction motor has good starting performance but inferior running performance (high excitation current, low no-load and full load power factor, Low torque under running condition) when compared to squirrel cage induction machine.

3(c)(i)

Sol: Put s = jω

$$G(j\omega) = \frac{K(j\omega)^3}{(j\omega+1)(j\omega+2)}$$

$$G(j\omega) = \frac{-jK\omega^3}{(2-\omega^2)+j3\omega}$$

$$G(j\omega) = \frac{-jK\omega^3\left[(2-\omega^3)-j3\omega\right]}{(2-\omega^2)^2+(3\omega)^2}$$

$$G(j\omega)\frac{-3K\omega^4}{(2-\omega^2)^2+(3\omega)^2}-\frac{jK\omega^3(2-\omega^2)}{(2-\omega^2)^2+(3\omega)^2}$$

The intersection of G (jω) plot with – ive real axis is obtained by equating imaginary part of G (jω) to zero and solving for ω. Therefore,

$$-\frac{K\omega^3(2-\omega^2)}{(2-\omega^2)^2+(3\omega)^2}=0$$

$$(2-\omega^2)=0$$

The intersection with –ve real axis occurs at

$\omega = \pm\sqrt{2}$ rad/sec

The intersection is obtained by substituting $\omega = \sqrt{2}$ in real part of G (jω), i.e.

$$G(j\sqrt{2} = \frac{-3K\left(\sqrt{2}\right)^4}{\left(2-\sqrt{2^2}\right)^2+\left(3\sqrt{2}\right)^2} = \frac{-3K\times 2\times 2}{(2-2)^2+3\times 3\times 2}$$

$$G\left(j\sqrt{2}\right) = -\frac{2}{3}K$$

$$|G(j\omega)| = \frac{K\omega^3}{\sqrt{\omega^2+1}\sqrt{\omega^2+2^2}}$$

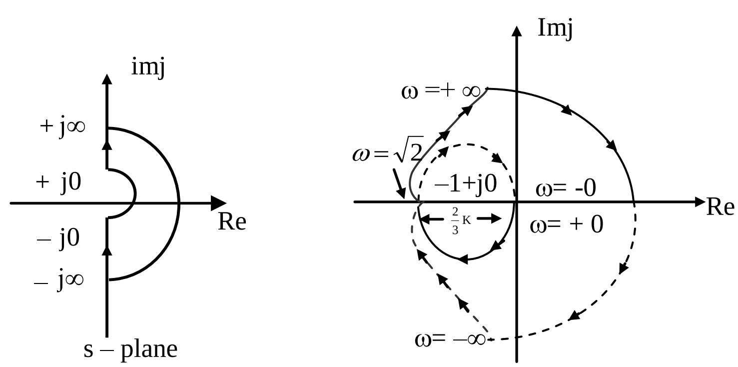
$$\angle G(j\omega) = 270° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

As $\omega \to 0 \Rightarrow |G(j\omega)| \to 0$

$\angle G(j\omega) \to 270°$

As $\omega \to \infty \Rightarrow |G(j\omega)| \to \infty$

$\angle G(j\omega) \to 90°$



The completed Nyquist plot is shown in Figure

It is given that, the number of poles of G(s) having positive real part is nil i.e. $P_+ = 0$.

The encirclements of critical point (–1 + j0) are determined below.

(1) If $K < \frac{3}{2}$

The critical point (–1+j0) lies outside the Nyquist plot, hence N =0

$\therefore\ N = P_+ - Z_+$

$0 = 0 - \; Z_+$

$\therefore\; Z_+ = \text{Nil}$

The system is stable

(2) If $K > \dfrac{3}{2}$

The critical point (–1 + j0) will be encircled twice in the clockwise direction by the Nyquist plot, hence

$N = -2$

$\therefore\; N = P_+ - Z_+$

$-2 = 0 - Z_+$

$\therefore\; Z_+ = 2$

The system is unstable. For stability $K \le \dfrac{3}{2}$

3(c)(ii)

Sol: $s^5 + s^4 + 2s^3 + 2s^2 + 11s + 10 = 0$

s^5	1	2	11
s^4	1	2	10
s^3	0	1	0
s^2	0.227	10	0

While forming the Routh array as above the third element in the first column is zero and thus the Routh criterion fails at this stage. The difficulty is solved if zero in the third row of the first column is replaced by a symbol ε and Routh array is formed as follows

+	s^5	1	2	11
+	s^4	1	2	10
+	S^3	ε	1	0
–	s^2	$\lim_{\varepsilon\to 0}\left(\frac{\varepsilon\times 2-1\times 1}{\varepsilon}\right)=-\infty$	10	0
+	s^1	$\lim_{\varepsilon\to 0}\left(1-\frac{10\varepsilon^2}{2\varepsilon-1}\right)=1$	0	0
+	s^0	10		

The limits of fourth and fifth element in the first column a $\varepsilon \to 0$ from positive side are $-\infty$ and +1 respectively, indicating two sign changes, therefore, the system is unstable and the number of roots with positive real part of the characteristic equation is 2.

4(a)(i)

Sol: A. Given data: r = 1.25 cm

$\therefore$ External radius R = 1.25 + 0.5

= 1.75 cm

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{R}{r}\right)} = \frac{2\pi\times 8.85\times 10^{-12}\times 3.0}{\ln\frac{1.75}{1.25}}$$

[$\because$ $\varepsilon_0 = 8.85\times 10^{-12}$ F/mt]

$= 496\times 10^{-12}$ F/mt

For 4 km length,

$C = 496\times 10^{-12}\times 4\times 1000$

$= 1984\times 10^{-9}$ F/phase

B. charging current = $\frac{V_{ph}}{X_c}$

$$= \frac{33\times 10^3}{\sqrt{3}}\times 2\pi\times 50\times 1984\times 10^{-9}$$

= 11.87 A/ph

C. Total 3 phase charging kVAR

$$= \sqrt{3}\left(\sqrt{3}\,V_{ph}\right)I_{ph}$$

$$= 3\ V_{ph}\ I_c$$

$$= 3 \times \frac{33000}{\sqrt{3}} \times 11.87$$

$$= 678.48 \times 10^3\ \text{kVAR}$$

D. $\cos\phi = 0.02 \quad \therefore\ \phi = 88.85°$

$$\delta = \text{loss angle} = 90° - \phi$$

Dielectric loss/phase (P_e)

$$= VI\cos\phi = VI\cos(90° - \delta)$$

$$= VI\sin\delta = V^2\omega C\sin\delta$$

$$= \left(\frac{33\times10^3}{\sqrt{3}}\right)^2 \times 2\pi\times50\times1984\times10^{-9}\times\sin 1.146°$$

$$= 4.52\ \text{kW}$$

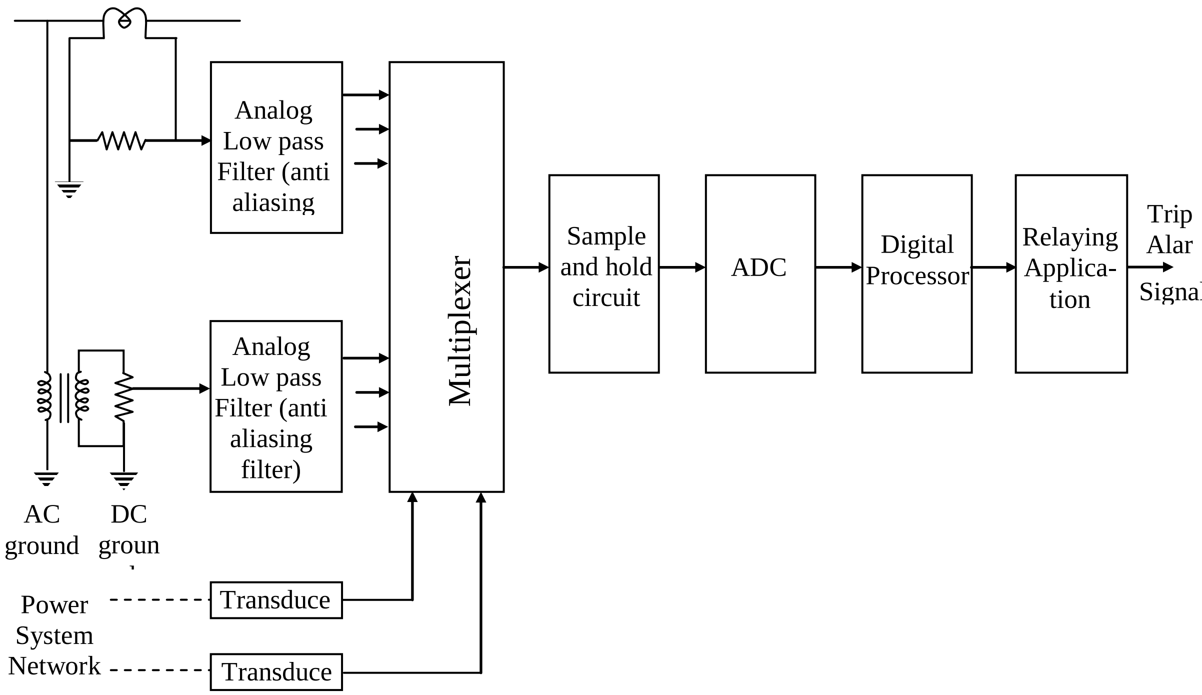
E. $$E_{max} = \frac{V}{r\ln\left(\frac{R}{r}\right)} = \frac{\left(\frac{33}{\sqrt{3}}\right)}{\left[1.25\ln\left(\frac{1.75}{1.25}\right)\right]}$$

$$= 45.3\ \text{kV/cm (rms)}.$$

4(a)(ii)

Sol: Digital signal processing:

The block diagram of digital signal processing is shown in figure various components and aspects of digital signal processing are explained below.



Data acquisition system:

The components of data acquisition system are shown in figure the current transformers and potential transformer are used for two purposes.

1. They are used to scale down the levels to become compatible with that of the digital sub system.
2. They provide the isolation between the power circuit and the measuring and protective hardware.

   The scaled-down analog signals must be converted to voltage signals suitable for conversion to digital form. The voltage developed across a resistor connected to the secondary of a CT injects the current juice to the numerical relay.

   Digital inputs to the numerical relay are usually the contact status, obtained from other relays (or) circuit breakers when the digital inputs are derived from contacts with in yard, it is necessary to apply surge filtering and optical isolation in order to isolate the numerical relay from the transient surge.

Sample and Hold (S/H) circuit:

The S/H circuit is an analog circuit which acts like a voltage memory device. The analog input voltage is acquired and stored on a high-quality capacitor with low leakage and low dielectric absorption characteristic.

In a relaying application, generally the three-phase voltages are to be acquired. So the magnitude as well as phase of each signal is significant, and hence all signals must be sampled at the same instant.

The multiplexer takes the different signals inside as per the sequence and the ADC interfacing rare coordinated by proper programming. Nowadays multichannel ADC, with simultaneous sampling are also available. So S/H and multiplexer functions are incorporated type ADC's having conversion time in the range of 15 to 30 μs are used for relay applying.

4(b)(i)

Sol: Types of distortion is envelope detector output: There are two types of distortions:

(a) Diagonal clipping:

This type of distortion occurs when the RC time constant of load circuit is too long. Due to this, the RC circuit cannot follow the fast changes in the modulating envelope.

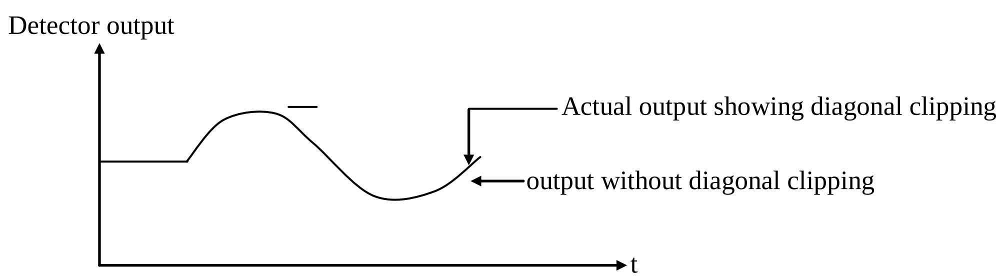


Fig: diagonal clipping

(b) Negative peak clipping

This distortion occurs due to the fact that modulation index of output side of the detector is higher than that on its input side. Hence, at higher depths of modulation of the transmitted signed, the over modulation may take place at the output of detector negative peak clipping will take place as a result of this overmodulation.

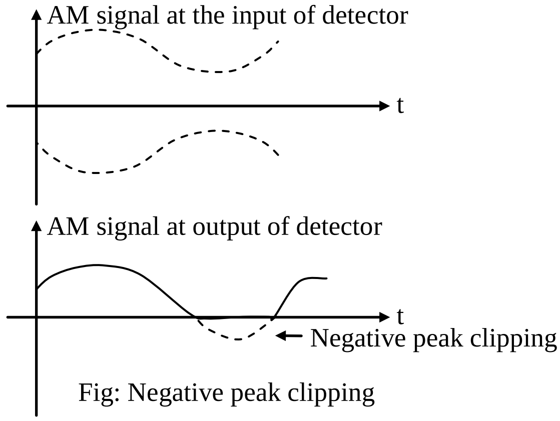


Fig: Negative peak clipping

The only way to reduce/eliminate the distortions is to choose RC time constants. The capacitor charges through D and R_s,when the diode is on and discharges through R when diode is off.

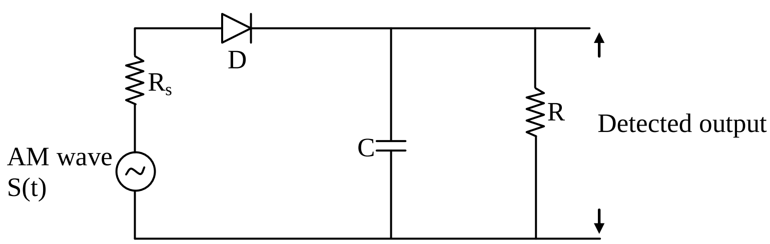


Fig: envelope detector for detection of AM wave.

The charging time constant R_sC should be short compared to the carrier period $1/f_c$ thus $R_sC << \frac{1}{f_c}$

On the otherhand, discharging time constant RC should be long enough so that the capacitor discharges slowly through load resistance R. But this time constant shouldn't be too long which will not allow the capacitor voltage to discharge at the maximum rate of change of the envelope.

$$\therefore \frac{1}{fc} << RC << \frac{1}{W}$$

Where W = maximum modulating frequency.

4(b)(ii)

Sol: Addressing is the method of specifying the location of data in an instruction. The different types of addressing modes in 8085 are,

(1) Direct addressing mode: The data is stored in memory and 16 bit address of data, in memory location is stored is specified in the instruction.

Eg: LDA 4500

LHLD 4200

(2) Immediate addressing mode: The required data for processing is given next to the opcode, in the instruction itself.

Eg: MVI A, 55

CPI 64

ADI 0A

(3) Register addressing mode: The data placed in a register and the register name is given in the instruction to access the data.

Eg: MOV A,B

ADD B

SUB C

(4) Register indirect addressing mode: The data is stored in memory and the 16–bit address of the data location in memory is placed in a register pair. This register pair holding the 16-bit address is given in the instruction to access the data.

Eg: LXI H, 4250

MOV A,M

(5) Implied addressing mode: The data location & the operation to be performed is given in the instruction itself.

Eg: CMA, RAR, XCHG

4(c)

Sol: Given data:

Source voltage $V_s = 230$ V

Average load current =10 A

$R = 0.4\ \Omega$

and $L = 2$ mH

(i) For E = 120 V, the full converter is operating as a controlled rectifier.

$$\therefore \frac{2V_m}{\pi}\cos\alpha = E + I_oR$$

Or $$\frac{2.\sqrt{2}.230}{\pi}\cos\alpha = 120 + 10\times 0.4 = 124\,V$$

Or $$\alpha = 53.208 \cong 53.21°$$

For α = 53.21°, power flows from ac source to dc load.

(ii) For E = – 120 V, the full-converter is operating as line commutated inverter.

$$\therefore \frac{2.\sqrt{2}.230}{\pi}\cos\alpha = -120 + 10\times 0.4$$

$$= -116\,V \text{ or } \alpha = 124.075$$

$$\cong 124.1°$$

For α = 124.1° , the power flows from dc source to ac load .

Output voltage and load current waveforms for

α = 53.1°and α = 124.1° are shown in below fig .(a) and fig .(b) respectively.

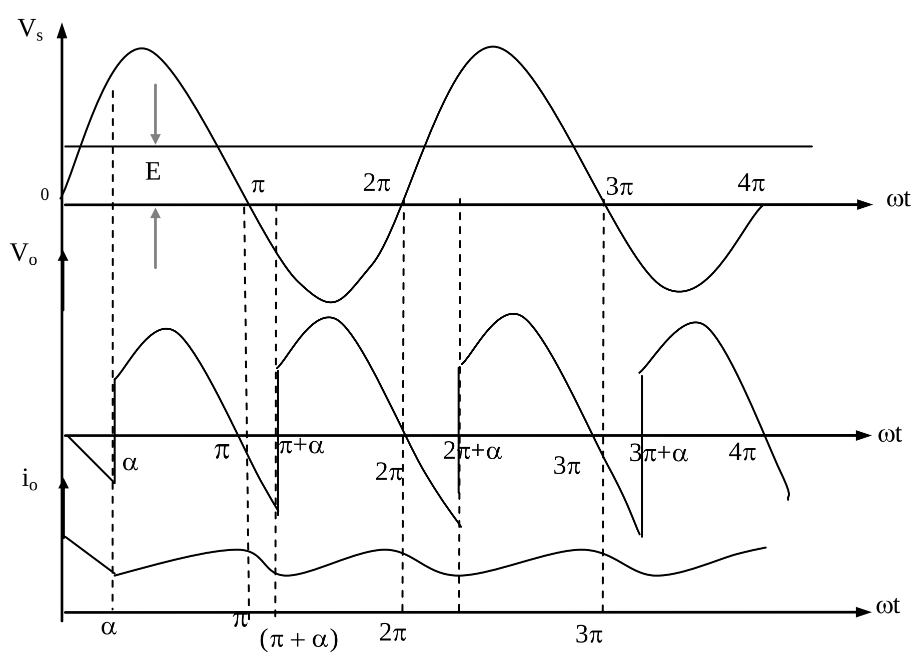


Fig. (a) Voltage and current waveforms for single-phase full converter for α < 90°

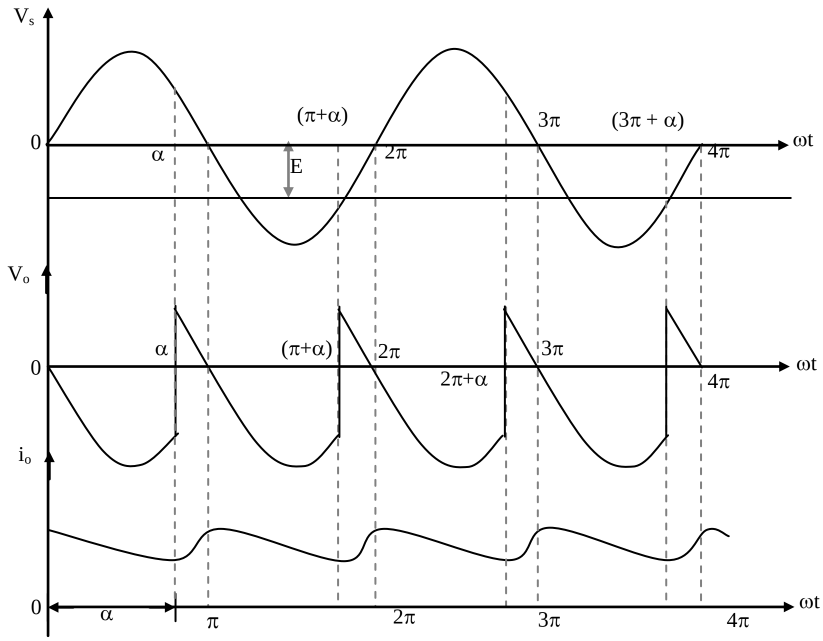


Fig. (b) Voltage and current waveforms for single-phase full converter for α > 90°

(iii) For constant load current, rms value of load current I_{or} is

$$I_{or} = I_o = 10\,A$$

$$\therefore \quad V_s . I_{or} \cos\phi = EI_o + I_{or}^2 R$$

For α=53.21° , $\cos\phi = \dfrac{120 \times 10 + 10^2 \times 0.4}{230 \times 10} = 0.5391\,\text{lag}$

For α = 124.1°, $\cos\phi = \dfrac{120 \times 10 - 40}{230 \times 10} = 0.5043\,\text{lag.}$

5(a)

Sol: as f_m = 10 kHz

Given R = 50kΩ, C = 0.1 μF

For the detector output to follow envelope at all times & handle without distortion,

$$RC \le \frac{\sqrt{1 - m^2}}{m w_m}$$

Where m = modulation index

$w_m = 2\pi f_m = 2\times\pi\times 10\text{kHz}.$

$$m^2 w_m^2 (RC)^2 \le 1 - m^2$$

$$m \le \frac{1}{\sqrt{1+(w_m RC)^2}}$$

On substituting w_m, R. C. we get

$m \le 3.18\times10^{-3}$

$\therefore$ Maximum modulation index is 3.18×10^{-3}

(b) when $f_m = 5$ kHz

Similarly $m \le 6.366\times10^{-3}$

$\therefore$ max modulation index = 6.366×10^{-3}

5(b)

Sol: (i) $\frac{V_0}{V_g} = \frac{N_2}{N_1}\frac{D}{1-D}$

So, $\frac{200}{13} = \frac{120}{6}\frac{D}{1-D}$

And D = 0.435

(ii) $L_M = \frac{n_1^2}{R} = \frac{n_1^2\mu_o A_c}{\ell_g}$

$$= \frac{(6^2)(4\pi\times10^{-7})(1.7\times10^{-4})}{0.4\times10^{-3}} = 19\,\mu H$$

(iii) Turns ratio $K = \frac{N_2}{N_1} = \frac{120}{6} = 20$

$$R = \frac{V_0^2}{P} = \frac{200^2}{50} = 800\,\Omega$$

$$\frac{I_M}{K} = \frac{V_o}{R(1-D)}$$

$$\Rightarrow I_M = (20)\frac{200}{800(1-0.435)}$$

$= 8.85$ A

$$\Delta i_M = \frac{V_g DT_s}{2L_M} = \frac{(13)(0.435)(10\mu)}{2(19\mu)} = 1.49\text{ A}\quad i_{m,peak} = I_M + \Delta i_M = 8.85 + 1.49 = 10.34\text{ A}$$

(iv) $N_1 i_{m,peak} = B_{peak} A_c R_g$

$$\Rightarrow B_{peak} = \frac{N_1 i_{m,peak}}{A_c\left(\frac{\ell_c}{\mu_o A_c}\right)} = \frac{\mu_o N_1 i_{m,peak}}{\ell_c}$$

$$\therefore B_{peak} = \frac{(4\pi\times 10^{-7}\,H/m)(13)(10.34)}{0.4\times 10^{-3}\,m}$$

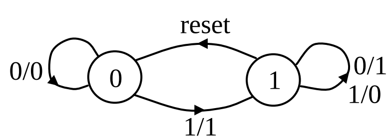
$= 0.195$T

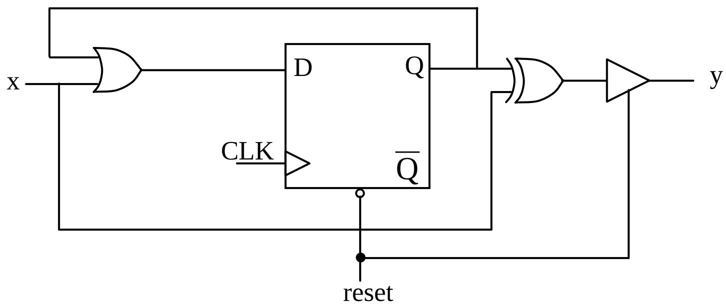
As the B_{peak} is less than the typical saturation flux density of Ferrite cores(0.3-0.5T), there is no Saturation of the given fly back transformer.

5(c)

Sol: The output is 0 for all 0 inputs until the first 1 occurs, at that time the output is 1. Thereafter, the output is the complement of the input. The state diagram has two states. In state 0, output = input; in state 1, output = $\overline{\text{input}}$

Present state	X input	Next State	Output Y
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	0



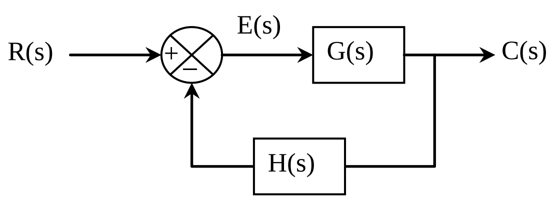


D = Q + x

$$y = Q\overline{x} + x\overline{Q} = Q \oplus x$$

5(d)

Sol: Closed –Loop –control system. For the closed– loop control system shown in Figure the overall transfer function is



$$M(s) = \frac{G(s)}{1+G(s)H(s)}$$

Differentiating w.r.t G(s)

$$\frac{\partial M(s)}{\partial G(s)} = \frac{[1+G(s)H(s)]-[G(s)H(s)]}{[1+G(s)H(s)]^2}$$

$$\frac{\partial M(s)}{\partial G(s)} = \frac{1}{[1+G(s)H(s)]^2}$$

The sensitivity of M(s) w.r.t G(s) for a closed – Loop control system is obtained as

$$S_G^M = \frac{G(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial G(s)}$$

$$= \frac{\frac{G(s)}{G(s)}}{1+G(s)H(s)} \frac{1}{[1+G(s)H(s)]^2}$$

$$S_G^M = \frac{1}{1+G(s)H(s)}$$

The sensitivity function for the overall transfer function M(s) with respect to variations in the feedback path transfer function H(s) is written as

$$S_H^M = \frac{H(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial H(s)}$$

The overall transfer function M(s) for the closed – Loop control system is

$$M(s) = \frac{G(s)}{1+G(s)H(s)}$$

Differentiating w.r.t. H(s)

$$\frac{\partial M(s)}{\partial H(s)} = -\frac{[G(s)]^2}{[1+G(s)H(s)]^2}$$

Therefore sensitivity of M(s) w.r.t. H(s) for a closed-loop control system is obtained as

$$S_H^M = -\frac{G(s)H(s)}{1+G(s)H(s)}$$

Comparing sensitivity functions it is concluded that a closed-loop control system is more sensitive to variations in feedback path parameters than the variations in forward path parameters therefore, the specifications of feedback elements in a closed-loop control system should be more rigid as compared to that forward path elements.

5(e)

Sol: Given data:

$$P_S = P_{e1} = 0.5$$

Before fault $P_{m1} = \frac{EV}{X} \Rightarrow \frac{1.5\times 1.0}{1.5} = 1.0$

During fault P_{m2}= 0, After the fault P_{m3}=1.0

$$\delta = \sin^{-1}\left(\frac{P_S}{P_{m1}}\right) \Rightarrow \sin^{-1}\left(\frac{0.5}{1.0}\right) = 30^{\circ}$$

$$\delta_0(\text{radians}) = \frac{30\times\pi}{180} = 0.52 \text{ rad}$$

$$\delta_{max} = 180 - Sin^{-1}\left(\frac{P_S}{P_{m3}}\right)$$

$$= 180 - Sin^{-1}\left(\frac{0.5}{1}\right) = 150^o$$

$$\delta_{max}(\text{radians}) = \frac{150 \times \pi}{180} = 2.618 \text{ rad}$$

Critical clearing angle

$$\delta_C = \cos^{-1}\left[\frac{P_S(\delta_{max} - \delta_o) + P_{m3}Cos\delta_{max}}{P_{m3}}\right] = \cos^{-1}\left[\frac{0.5(2.618 - 0.52) + 1.0\,Cos\,150^0}{1.0}\right]$$

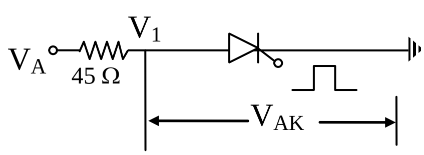
$$= 79.45^o$$

6(a)(i)

Sol: $I_H = 2$ mA

SCR will turn off when $I_a \leq$ Holding current

$$I_a \leq 2mA$$



$$2mA = \frac{V_A - V_1}{45}$$

Given voltage across SCR = 1V

$$2 \times 10^{-3} = \frac{V_A - 1}{45}$$

$$V_A = (90 \times 10^{-3}) + 1$$

$$= 0.09 + 1 \Rightarrow V_A = 1.09 \text{ V}$$

6(a)(ii)

Sol: Energy loss during turn-on

$$= \int_0^{t_{on}} i_C . v_{CE}\, dt$$

$$= \int_0^{t_{on}} \left(\frac{I_{CS}}{60} \times 10^6 t\right) \times \left(V_{CC} - \frac{V_{CC}}{50} \times 10^6 t\right) dt$$

$$= \int_0^{t_{on}} \left(\frac{240}{60} \times 10^6 t \right) \left(300 - 6 \times 10^6 t \right) dt$$

$$= \int_0^{t_{on}} \left(4 \times 10^6 t \right) \left(300 - 6 \times 10^6 t \right) dt$$

= 0.5 watt-sec

Energy loss during turn-off

$$= \int_0^{t_{off}} \left(240 - \frac{240}{60} \times 10^6 t \right) \left(\frac{300}{75} \times 10^6 t \right) dt$$

$$= \int_0^{t_{off}} \left(240 - 4 \times 10^6 t \right) \left(4 \times 10^6 t \right) dt$$

= 1.44 watt-sec

Total energy loss in one cycle

= 0.5 + 1.44 = 1.94 W-sec

Available switching frequency = $\frac{500}{1.94}$

f = 257.73 Hz

6(a)(iii)

Sol: Given data:

Initial voltage across capacitor is V_s = 230 V

Load current = 300 A

C = 20 μ F and L= 5 μ H .

Peak value of resonant current,

$$I_p = V_s \sqrt{\frac{C}{L}}$$

$$= 230 \sqrt{\frac{20}{5}} = 460 \text{ A}$$

Resonant frequency, $\omega_o = \frac{1}{\sqrt{LC}}$

$$= \frac{10^6}{\sqrt{100}} = 0.1 \times 10^6 \text{ rad/s}$$

A. Conduction time for auxiliary thyristor

$$= \frac{\pi}{\omega_o} = \frac{\pi}{0.1\times10^6} = 31.416\,\mu s$$

B. $\omega t_3 = \sin^{-1}\left(\frac{I_0}{I_P}\right)$

$$\omega t_3 = \sin^{-1}\left(\frac{300}{460}\right) = 40.706° \text{ or } 0.71045 \text{ rad.}$$

Voltage across main thyristor, when it gets turned-off is

$\therefore$ $V_{ab} = V_s \cos \omega t_3$

$= 230 \cos (40.706°)$

$= 174.355$ V

C. Circuit turn-off time for main thyristor is

$$t_c = C\frac{V_{ab}}{I_0}$$

$$= 20\times10^{-6}\frac{174.355}{300}$$

$$= 11.624\,\mu s.$$

6(b)

Sol: Given Data

$R = 25.3\ \Omega$, $X = 66.5\ \Omega$; $Z = (25.3 + j66.5)\ \Omega$

$Y = j0.442 \times 10^{-3}$ mho

$\gamma l = \sqrt{zy}\, l = \sqrt{zl.yl} = \sqrt{ZY}$

$\sqrt{ZY} = \sqrt{(25.3 + j66.5)(j0.442\times10^{-3})}$

$= (0.0327 + j0.174)$

$A = D = \cosh (\gamma l) = \cosh\left(\sqrt{ZY}\right)$

$= \cosh (0.0327 + j0.174)$

$= 0.986 \angle 0.32°$

$B = Z_c \sinh(\gamma l) = \left(\sqrt{\dfrac{Z}{Y}}\right) \text{Sin h} \left(\sqrt{ZY}\right)$

$\sqrt{\dfrac{Z}{Y}} = (393 - j72.3)$

$B = 70.3\angle 69.2°$

$C = \dfrac{1}{Z_c} \text{ Sin h } (\gamma l)$

$= \left(\sqrt{\dfrac{Y}{Z}}\right) \text{sin h} \left(\sqrt{ZY}\right)$

$= 4.44 \times 10^{-4} \angle 90°$

ii) Load at 60 MVA at 124 kV (L – L)

Load current $I_R = \dfrac{60\times10^6}{\sqrt{3}\times124\times10^3}$

$= 279.36$ A

Power factor is 0.8 (lagging)

$I_R = 279.36 \angle -36.87°$ A

$V_R = \dfrac{124}{\sqrt{3}} = 71.6$ kV (ph.Voltage)

$V_s = AV_R + BI_R$

$V_s = 0.986\angle 0.32° \times 71.6\angle 0° + \dfrac{(70.3\angle 69.2 \times 279.36\angle -36.87)}{1000}$

$V_s = 87.84 \angle 7.12$ kV

$V_{s,\,L\text{-}L} = \sqrt{3} \times 87.84\angle 7.12 = 152.14 \angle 7.12$ kV

$I_s = CV_R + DI_R$

$I_s = j4.44 \times 10^{-4} \times 71.6\angle 0 \times 1000 + 0.986 \angle 0.32 \times 279.36 \angle -36.87$

$I_s = 221.28 - j132.24$

$I_s = 257.78 \angle -30.86$ Amp

Power factor angle at the sending and

$= 7.12 - (-30.86°) = 37.98°$

Sending end power factor = cos (37.98)

= 0.788

(iii)Voltage regulations = $\dfrac{\dfrac{|V_s|}{|A|}-|V_R|}{|V_R|}\times 100$

$= \dfrac{\left(\dfrac{152.14}{0.986}-124\right)}{124}\times 100$

= 24.43

(iv) Sending end power

$P_s = \sqrt{3}\times 152.14 \times 257.18 \times \cos(37.98)$ kW

= 53520 kW = 53.52 MW

Receiving end power

$P_R = 60 \times 0.80 = 48$ MW

Efficiency $\eta = \dfrac{48}{53.52} = 89.68\%$

= 89.68%

6(c)(i)

Sol: Properties of ROC

The properties of ROC are as follows:

1. The shape of the ROC is strips parallel to the imaginary axis in s-plane.
2. The ROC does not contain any poles.
3. If x(t) is a right-sided signal, the ROC of X(s) extends to the right of the right most pole and no pole is located inside the ROC
4. If x(t) is a left-sided signal, the ROC of X(s) extends to the left of the left most pole and no pole is located inside the ROC.
5. If x(t) is a two-sided signal, the ROC of X(s) is a strip in the s-plane bounded by poles and no pole is located inside the ROC.
6. Impulse function is the only function for which the ROC is the entire s-plane.
7. The ROC must be a connected region.

8. The ROC of an LTI stable system contains the imaginary axis of s-plane
9. The ROC of the sum of two or more signals is equal to the intersection of the ROCs of those signals.

6(c)(ii)

Sol:

$$X(Z)=\sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$=\sum_{n=0}^{\infty} r^n \frac{\sin((n+1)\omega)}{\sin\omega} z^{-n}$$

$$=\sum_{n=0}^{\infty} \frac{r^n}{2j\sin\omega}\left[e^{j(n+1)\omega}-e^{-j(n+1)\omega}\right]z^{-n}$$

$$=\sum_{n=0}^{\infty}\left(rz^{-1}e^{j\omega}\right)^n \frac{e^{j\omega}}{2j\sin\omega}-\sum_{n=0}^{\infty}\left(rz^{-1}e^{-j\omega}\right)^n \frac{e^{-j\omega}}{2j\sin\omega}$$

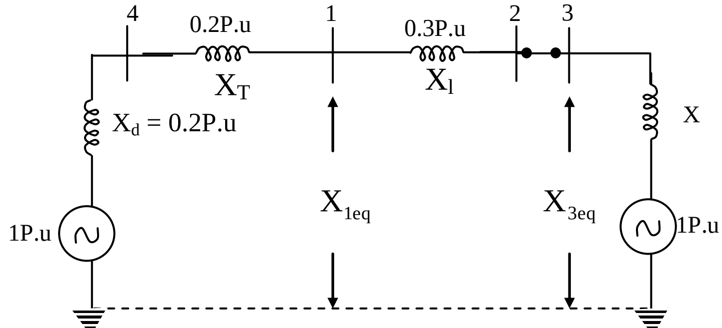
$$X(Z)=\frac{1}{2j\sin\omega}\left[\frac{e^{j\omega}}{1-rz^{-1}e^{j\omega}}-\frac{e^{-j\omega}}{1-rz^{-1}e^{-j\omega}}\right] \qquad |z|>|r|$$

$$=\frac{1}{\left(1-rz^{-1}e^{j\omega}\right)\left(1-rz^{-1}e^{-j\omega}\right)} =\frac{1}{1-2rz^{-1}\cos\omega+r^2z^{-2}}$$

$$X(Z)=\frac{z^2}{z^2-2r\cos\omega z+r^2}$$

7(a)(i)

Sol: Per unit positive sequence reactance diagram of the given system when the breaker closed is shown in fig.



Fig

The equivalent reactance with respect to point “1” is [short circuit 1P.u sources]

$$X_{1eq} = (X_T + X_d) // (X_1 + X)$$

$$= \frac{0.4 \times (0.3 + X)}{0.4 + 0.3 + X} = \frac{0.12 + 0.4X}{0.7 + X}$$

Given prefault voltage(V_{th}) = 1P.u.

$$\text{fault current}(I_f) = \frac{V_{th}}{X_{1eq}}$$

$$= \frac{1}{\left(\frac{0.12 + 0.4X}{0.7 + X}\right)} = 5P.u$$

0.7 + X = 5(0.12 + 0.4X)

$\therefore$ X = 0.1P.u

To find fault level at bus ‘3’:

The equivalent reactance w.r.t. point ‘3’ in reactance diagram is

$$X_{3eq} = (X_d + X_T + X_1) // X$$

= (0.2+0.2+0.3) // 0.1

$$= \frac{0.7 \times 0.1}{0.8} = 0.0875\text{p.u}$$

$$\therefore \text{fault current}(I_{f_3}) = \frac{V_{th}}{X_{3eq}} = \frac{1.0}{0.0875}$$

= 11.43 p.u

7(a)(ii)

Sol: Smart grid definition: A "smart grid" is an electrical grid which includes a variety of operational and energy measures including smart meters, smart appliances, renewable energy resources, and energy efficient resources. Electronic power conditioning and control of the production and distribution of electricity are important aspects of the smart grid. The ultimate aim is increasing user-interface for real time system optimization.

Difference of smart grid from conventional grid:

(i) Encouraging distributed generation sources like solar, wind, fuel cells, biomass plants etc.

(ii) Two way flow of electricity and communication between centralized generation and distribution systems.

(iii) Using the dynamic pricing for electricity.

(iv) Using smart meters in order to make the electricity bill clearly visible for consumers.

(v) Improvisation in self healing property of grid.

(vi) Capable to operate in island mode efficiently.

7(b)(i)

Sol: (a) Air gap flux (ϕ) $\propto \dfrac{\text{voltage}}{\text{frquency}}$

Given, $\phi = K\left(\dfrac{V}{f}\right)$ = constant

$\therefore \quad \dfrac{V_2}{V_1} = \dfrac{f_2}{f_1}$

$\Rightarrow \quad V_2 = 400\left(\dfrac{40}{50}\right) = 320$ V

(b) For 4-pole, 50 Hz machine, N_s = 1500 rpm

Slip $s_1 = \dfrac{N_s - N}{N_s} = \dfrac{1500-1470}{1500} = 0.02$

Electromagnetic torque, $\tau_{em} = \dfrac{180}{2\pi N_s}\dfrac{sE_2^2R_2}{R_2^2+(sx_2)^2}$

For small values of slip, $sx_2 <<< R_2$

Therefore, approximate torque equation, $\tau_{em} = \dfrac{180}{2\pi N_s}\dfrac{sE_2^2}{R_2}$

For same full load torque, $\dfrac{sE_2^2}{N_s}$ = constant

$$\Rightarrow \quad \frac{s_1}{s_2} = \frac{f_1}{f_2} \times \left(\frac{E_{22}}{E_{21}}\right)^2$$

$$\Rightarrow \qquad s_2 = \left(\frac{f_2}{f_1}\right)\left(\frac{E_{21}}{E_{22}}\right)^2 \times s_1$$

$$= \left(\frac{40}{50}\right)\left(\frac{400}{320}\right)^2 \times 0.02$$

$$= 0.025$$

Now $N_s = \frac{120f}{P} = \frac{120 \times 40}{4} = 1200$ rpm

Now the motor speed $N_r = N_s(1 - s)$

$$N_r = 1200(1 - 0.025)$$

$$= 1170 \text{ rpm}$$

7(b)(ii)

Sol: Here energy stored= $\frac{1}{2}J\omega^2 = 5000 \times 150$ Nm or Joules

$$\omega = \frac{2\pi \times 750}{60} = 25\,\pi \text{ rad/sec}$$

$$\therefore\ J = \frac{5000 \times 150 \times 2}{(25\pi)^2} = 243.17 \text{ kgm}^2$$

For dc motor, the dynamic equation is,

$$T_e = J\frac{d\omega}{dt} + T_L$$

Or $\qquad dt = \frac{J}{T_e - T_L}d\omega$

Or $\qquad t = \frac{J}{T_e - T_L}\,\omega$

It is given that load torque, T_L = motor full-load torque, $T_{e.fl}$

Where $\quad T_{e\,fl} = \frac{150 \times 746}{25\pi}$ Nm

As current during starting is limited to 1.5 times the full-load current, the starting torque,

$T_{e\,st} = 1.5T_{e.fl}$

$\therefore\ t = \dfrac{J}{T_{est} - T_{efl}}\ \omega$

$= \dfrac{243.17 \times 25\pi}{0.5 \times 150 \times 746} \times 25\pi\ = 26.8096$ sec

7(c)

Sol: (i) For T = 0 and K =1 determine the gain cross over frequency ω_1

$\therefore\ G(j\omega) = \dfrac{10}{j\omega(j\omega+1)(j\omega+7)}$

At the phase crossover frequency

$\omega = \omega_1$; $|G(j\omega_1)| = 1$.

$$\left|\frac{10}{j\omega_1(j\omega_1+1)(j\omega_1+7)}\right| = 1$$

$$\frac{10}{\omega_1\sqrt{(\omega_1^2+1^2)}\sqrt{\omega_1^2+7^2}} = 1$$

$$\frac{10}{\omega_1\sqrt{\omega_1^2+1}\sqrt{\omega_1^2+49}} = 1$$

By inspection $\omega_1 = 1$ rad/sec

$$\angle G(j\omega) = -90° - \tan^{-1}\frac{\omega}{1} - \tan^{-1}\frac{\omega}{7}$$

$$\angle G(j\omega_1) = -90° - \tan^{-1}\omega_1 - \tan^{-1}\frac{\omega_1}{7}$$

$\omega_1 = 1$ rad/ sec

$$G(j\omega_1) = -90° - \tan^{-1}1 - \tan^{-1}\frac{1}{7}$$

$$= -90° - 45° - 8.13° = -141.13°$$

Incorporating time delay element $e^{-j\omega T,}$

the condition for marginal stability is given below:

$$\angle G(j\omega_1) + \angle e^{-j\omega T} = -180°$$

$$\therefore -141.13 - \frac{\omega_1 T \times 180}{\pi} = -180$$

$\because \quad \omega_1 = 1$ rad/sec

$$\therefore -141.13 - \frac{1\times T\times 180}{\pi} = -180$$

$$\therefore T = \frac{(180-141.13)\pi}{1\times 180} = 0.678\,\text{sec}$$

(ii) T = 1 sec

$$G(s) = \frac{10Ke^{-s.1}}{s(s+1)(s+7)}$$

Put s = jω

$$\therefore \quad G(j\omega) = \frac{10Ke^{-j\omega}}{j\omega(j\omega+1)(j\omega+7)}$$

$$\angle G(j\omega) = -\frac{\omega\times 180}{\pi} - 90° - \tan^{-1}\frac{\omega}{1} - \tan^{-1}\frac{\omega}{7}$$

$$= -\frac{\omega\times 180}{\pi} - 90° - \tan^{-1}\frac{\omega+\frac{\omega}{7}}{1-\omega.\frac{\omega}{7}}$$

$$= -\frac{\omega\times 180}{\pi} - 90° - \tan^{-1}\frac{8\omega}{7-\omega^2}; \qquad \omega < \sqrt{7}$$

At the phase cross over frequency ω_2, $\angle G(j\omega_2) = -180°$

$$-\frac{\omega_2\times 180}{\pi} - 90° - \tan^{-1}\frac{8\omega_2}{7-\omega_2^2} = -180°$$

$$= -57.3\omega_2 - \tan^{-1}\frac{8\omega_2}{7-\omega_2^2} = -90°$$

Solving by trial –error method: $\omega_2 = 0.79$ rad/sec

$$|G(j\omega_2)| = \left|\frac{10K}{j\omega_2(j\omega_2+1)(j\omega_2+7)}\right|$$

$$= \frac{10K}{\omega_2\sqrt{\omega_2^2+1^2}\sqrt{\omega_2^2+7^2}}$$

$\omega_2 = 0.79$ rad/sec

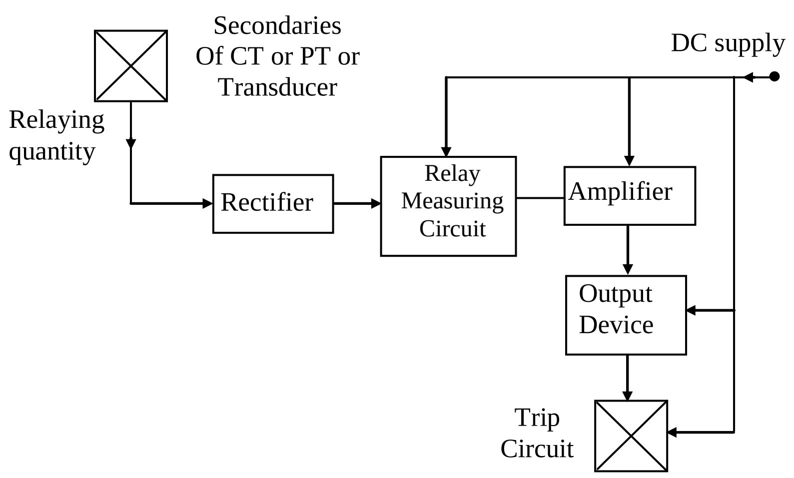
$$\therefore \; |G(j0.079_2)| = \frac{10K}{0.79\sqrt{(0.79)^2+1^2}\sqrt{(0.79)^2+7^2}} = \frac{K}{0.708}$$

For stability $\frac{K}{0.708} < 1$; $K < 0.708$

For marginally stability: $K = 0.708$

8(a)(i)

Sol: Static relay: Static relay uses solid state devices, it has no moving part. The components used are transistors, diodes, resistors, capacitors etc. The measurement of the actuating quantity is performed by electronic, magnetic, optical or another component without mechanical motion. The static component of a static relay are shown in fig(i).



1. **Rectifier:** It rectifier the output of CT or PT of a transducer.
2. **Relay measuring circuit:** It consist of comparators, level detectors and logic circuits. The output is actuated when the dynamic input i.e., the relaying quantity attains the threshold value.
3. **Amplifier:** It amplifies the output of relay measuring circuit and fed the amplified quantity to the output device.
4. **Output device:** It activates the trip coil when the relay operates.

8(a)(ii)

Sol: Open circuit voltage $V_{oc} = 0.24$ V

Short circuit current $I_{sc} = -9$ mA

Maximum load voltage $(V_{max}) = 0.14$ V

Maximum load current $(I_{max}) = -6$ mA

Area (A) = 4 cm^2

Maximum power output $(P_o) = V_{max} \times I_{max}$

$= 0.14\text{V} \times (-6 \text{ mA})$

$= 0.14 \times (-6) \times 10^{-3}$

$= -0.84$ mW

∴ Maximum power = –0.84 mW

Total power available P_{in} = Power intensity × area

$= 200 \text{ W/m}^2 \times 4 \times 10^{-4}$

= 80 mW

$$\therefore \text{ Efficiency} = \frac{P_{out}}{P_{in}} = \frac{0.84\,\text{mW}}{80\,\text{mW}}$$

$\eta = 1.05\%$

∴ Efficiency of solar cell = 1.05%

8(a)(iii)

Sol: Distributed generation refers to diversification of generation of electricity. It includes renewable sources of energy like solar, wind etc. along with conventional sources like coal, gas, hydel.

The main idea is to meet the local demand locally and reduce burden on the rest of the system. It also finds application in case of strategic loads for better security and optimization.

Role in contemporary developments in electricity industry:

- Mitigation of the complexity of the system on the premise of economic operation.
- Aims at developing micro grids consisting of number of distributed generation centres to meet some strategic and specific loads.
- Helps in reducing the dependence on the conventional sources of energy.
- Distributed generation helps in achieving the goal of electricity for all.

8(b)(i)

Sol: Given 44MVA,

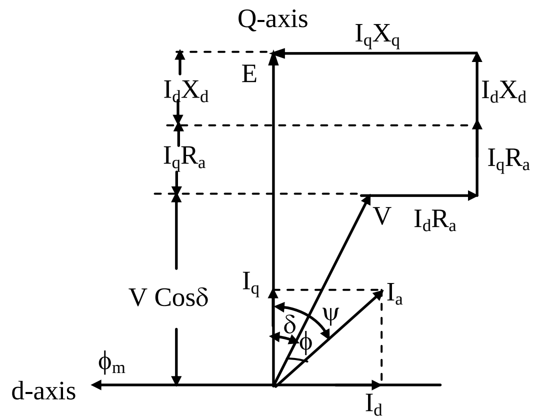
Y connected 3-ϕ salient pole synchronous generator and delivers a rated load at 0.8 PF lag.

V= 10.5 kV

f = 50 Hz

$X_d = 1.83\ \Omega$

$X_q = 1.21\ \Omega$



As it is Y connected

$$\tan\psi = \frac{V\sin\phi \pm I_a x_q}{V\cos\phi + I_a R_a}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{10.5\times10^3}{\sqrt{3}} = 6062.17$$

$$I_a = \frac{P}{\sqrt{3}V_L} = \frac{44\times10^6}{\sqrt{3}\times10.5\times10^3}$$

$= 2419.3\angle{-36.86}$

$$\tan\psi = \frac{(6062.17\times0.6)\pm(2419.3\times1.21)}{(6062.17\times0.8)+(2419.3\times0)}$$

$\psi = 53.546$

$\psi = \delta \pm \phi$; '+' for lag power factor

'−' for lead power factor

$\delta = 53.546 - 36.86 = 16.864$

$I_d = I_a\sin\psi = 2419.3 \times \sin(53.546) = 1945.925$

$I_q = I_a\cos\psi = 2419.3 \times \cos(53.546) = 1437.49$

$E = V\cos\delta + I_qR_a \pm I_dX_d$

$= 6062.17 \times \cos(16.6841) + (1437.492\times0) + (1945.925\times1.83)$

$E = 9368.0117$ V

$$\%\text{ Regulation} = \frac{E-V}{V}\times100$$

$$= \frac{9368.0117 - 6062.17}{6062.17}\times100$$

$= 54.53\%$

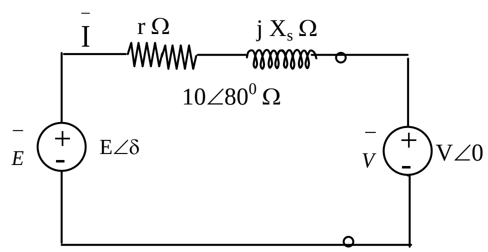
$P_{gen} = 3E_aI_a\cos\psi$

$= 3\times 9368.0117\times 2419.3 \times \cos(53.546)$

$= 40.399$ MW

8(b)(ii)

Sol: (i) Equivalent circuit per phase is shown in figure.



Since generator operation is being considered, $\bar{E}$ is shown as leading $\bar{V}$ (assuming δ is +ve). (For motor operation, $\bar{E}$ would lag $\bar{V}$). Reference direction for $\bar{I}$ is chosen to represent generator operation.

$$\bar{I} = \frac{E\angle\delta - V\angle 0}{10\angle 80^0} = \frac{E}{10}\angle(\delta - 80^0) - \frac{V}{10}\angle -80^0$$

$$\bar{I}^* = \frac{E}{10}\angle(80^0 - \delta) - \frac{V}{10}\angle 80^0$$

Power received by the bus $= \text{Re}\,\bar{V} I^*$

$$= \frac{EV}{10}\cos\left(80^0 - \delta\right) - \frac{V^2}{10}\cos 80^0$$

This will remain constant as E and δ change if E cos (80^0 - δ) remains constant (rest of the terms in the above expression are constants).

8(c)(i)

Sol: The time period of of oscillation is given by

$$\frac{\pi}{\sqrt{1/LC-(R/2L)^2}}=\frac{\pi}{\left[\frac{10^3\times10^6}{6\times1.2}-\frac{100^2\times10^6}{4\times36}\right]^{1/2}}s$$

$$=\frac{\pi}{1000(8.333)}=0.377m$$

Then output frequency, $f=\frac{10^3}{(0.377\times2)+(2\times0.2)}=866.55Hz$

When R = 40Ω, output frequency

$$=\frac{1}{\frac{2\pi}{\left[\frac{10^3\times10^6}{6\times1.2}-\frac{1600\times10^6}{4\times36}\right]}+0.4\times10^{-3}}=1046.2Hz$$

When R = 140 Ω, output frequency

$$=\frac{1}{\frac{2\pi}{\left[\frac{10^3\times10^6}{6\times1.2}-\frac{140^2\times10^6}{4\times36}\right]^{1/2}}+0.4\times10^{-3}}=239.8Hz$$

∴ Range of output frequency = 239.8Hz to 1046.2Hz.

8(c)(ii)

Sol: PWM: In this method, a fixed dc input voltage is given to the inverter and a controlled ac output voltage is obtained by adjusting the on and off periods of the inverter components. This is the most popular method of controlling the output voltage and is termed as Pulse Width Modulation (PWM) control.

Sinusoidal pulse width modulation:

In Sinusoidal pulse width modulation, the pulse width is a sinusoidal function of the angular position of the pulse in a cycle.

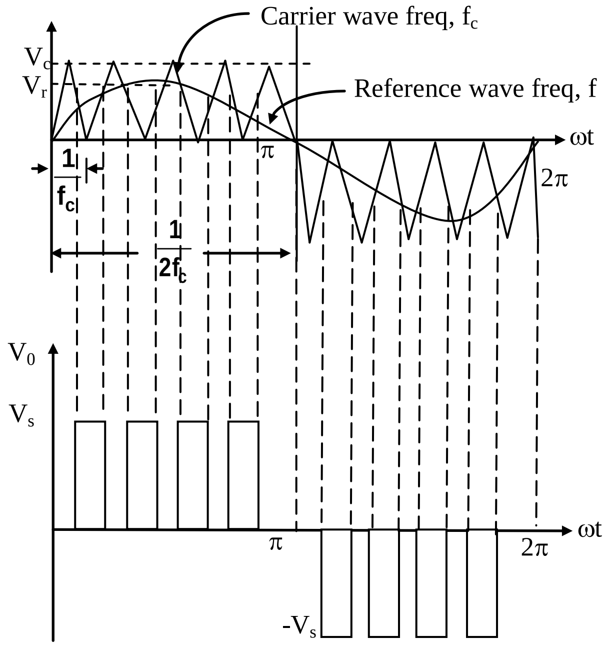


Fig. Output voltage waveforms with sinusoidal pulse modulation

For realizing Sinusoidal PWM, a high-frequency triangular carrier wave V_C is compared with a sinusoidal reference wave V_r of the desired frequency. The intersection of V_C and V_r waves determines the switching instants and commutation of the modulated pulse. The carrier and reference waves are mixed in a comparator.

When triangular carrier wave has it's peak coincident with zero of the reference sinusoid, there are $N = \frac{fc}{2f}$ pulses per half cycle. In case zero of the triangular wave coincides with zero of the reference sinusoid, there are (N-1) pulses per half cycle i.e. $\left(\frac{fc}{2f} - 1\right)$.

Important features of sinusoidal pulse modulation:

(i) For MI (modulation index = $\frac{V_r}{V_c}$ less than one, largest harmonic amplitudes in the output voltage are associated with harmonics of order fc/f ±1 (or) 2N±1. where N is the no. of pulses per half cycle. By increasing the value of N, the order of dominant harmonic frequency can be raised, which can then be filtered out easily and filtering requirements are accordingly minimized and at the same time switching losses becomes more, therefore an impaired inverter efficiency.

(ii) For MI greater than one, lower order harmonics appear, since for MI > 1, pulse width is no longer a sinusoidal function of the angular position of the pulse.