



ACE
Engineering Academy
(Leading institute for ESE/GATE/PSUs)

ESE – 2019 MAINS OFFLINE TEST SERIES



ELECTRICAL ENGINEERING

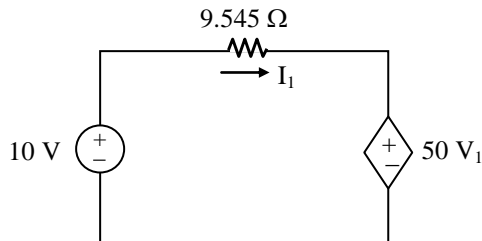
TEST – 11 SOLUTIONS



01(a)

Sol: From the circuit $I_1 = I_4 = I_2 + I_3$ and $V_1 = 3V_1$

The $4\ \Omega$ and $7\text{-}\Omega$ resistors are replaced by the equivalent



$$\text{Resistance of magnitude } \frac{4 \times 7}{4 + 7} = \frac{28}{11} = 2.545\Omega$$

$$\begin{aligned}\therefore \text{Total circuit resistance} &= 2.545 + 3 + 4 \\ &= 9.545\ \Omega\end{aligned}$$

The circuit reduces as shown in applying KVL to the loop we get

$$9.545I_1 + 50 V_1 = 10$$

$$V_1 = 3I_1$$

$$9.545I_1 + 50 (3I_1) = 10$$

$$9.545I_1 + 150 I_1 = 10$$

$$I_1 = \frac{10}{159.545} = 0.0626\ \text{A}$$

$$\text{We have } I_2 + I_3 = I_1 = 0.0626\ \text{A}$$

$$0 = 4I_2 - 7I_3$$

$$I_2 = 0.03986\ \text{A}; I_3 = 0.02279\ \text{A}$$

$$V_1 = 3I_1 = 3 \times 0.0626 = 0.188\ \text{V}$$

1(b).

Sol: Let $f(z) = (u + i v)$ be analytic function.

$$\text{Then } i f(z) = (i u - v)$$

Adding the above equations, we get

$$(1 + i) f(z) = (u - v) + i (u + v)$$



$F(z) = U + iV$ where $F(z) = (1+i) f(z)$,

$U = (u - v)$ and $V = (u + v)$.

$$U = (u - v) = (x - y) (x^2 + 4xy + y^2)$$

$$\frac{\partial U}{\partial x} = x^2 + 4xy + y^2 + (x - y) (2x + 4y) = 3x^2 + 6xy - 3y^2$$

$$\frac{\partial U}{\partial y} = -(x^2 + 4xy + y^2) + (x - y) (4x + 2y) = 3x^2 - 6xy - 3y^2$$

Put $x = z$ and $y = 0$ in above, we get

$$\frac{\partial U}{\partial x} = 3z^2 \text{ and } \frac{\partial U}{\partial y} = 3z^2$$

$$F'(z) = \frac{\partial U}{\partial x} - i \frac{\partial U}{\partial y}$$

$$F'(z) = 3z^2 dz - i3z^2 dz$$

$$F(z) = z^3 - iz^3 + ic$$

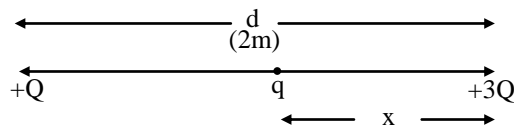
$$(1 + i) f(z) = (1 - i) z^3 + isc$$

$$f(z) = \frac{1-i}{1+i} z^3 + \frac{ic}{1+i}$$

$$\therefore f(z) = -iz^3 + \frac{(i-1)c}{2}$$

01(c)

Sol:



From the diagram the third charge 'q' is placed at a distance of 'x' from charge +3Q

For the system to be in equilibrium, q must be negative

$$\therefore F_{12} = F_{23} = F_{13}$$

$$\Rightarrow \frac{-Qq}{4\pi\epsilon_0(d-x)^2} = \frac{-3Qq}{4\pi\epsilon_0 x^2} = \frac{3Q^2}{4\pi\epsilon_0 d^2} \text{ ---- (1)}$$



That is, $3(d-x)^2 = x^2$

$$\Rightarrow 3d^2 - 6dx + 3x^2 = x^2$$

$$\Rightarrow 2x^2 - 6dx + 3d^2 = 0$$

$$x = \frac{6d \pm d\sqrt{12}}{4}$$

When $d = 2$, $x = 4.73\text{m}$ or 1.268m

As $x < d$, $x = 1.268\text{m}$ is considered.

Substitute $x = 1.268$ in equation (1)

$$\therefore \frac{-3Qq}{4\pi\epsilon_0 x^2} = \frac{3Q^2}{4\pi\epsilon_0 d^2}$$

$$Q = -Q(x^2/d^2)$$

$$\therefore q = -0.4Q$$

Location of third charge is 1.268m from $+3Q$ towards $+Q$ and the value of charge is $-0.4Q$.

1(d).

Sol: Superconductivity:

The resistivity of some materials abruptly becomes zero below a specific critical temperature. It is called Superconductivity. It was first observed in pure mercury at the critical temperature of 4.2°K . Different materials have different critical temperatures.

Effect of Magnetic Field:

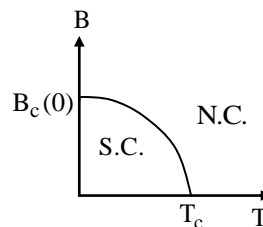
When the magnetic field is applied on a specimen in SC State ($T < T_c$) and it is increased gradually, at a specific field called critical magnetic field (B_c), it becomes a normal conductor. That is, magnetic field is capable of destroying superconductivity. This B_c depends on the temperature of the specimen below T_c as

$$B_c = B_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

At $T = T_c$, $B_c = 0$

and at $T = 0 \text{ K}$

$$B_c = B_c(0).$$





Effect of frequency:

- The superconducting metal offers zero resistance only for direct current of constant value.
- If the current is changing like in AC, an electric field is developed and some power is distributed.
- The superconductor contains two different types of electrons. Normal electrons and Super electrons which are responsible for superconductivity. Thus, the SC metal is like two parallel conductors one having a normal resistance and the other zero resistance.
- Super electrons, short circuit the normal electrons giving zero resistivity in DC current case.
- In AC current case, current is due to both super and normal electrons where some power dissipation is present.
- If the frequency of applied AC is high, super electrons behave like normal electrons and SC behave like normal metal.
- Electronic specific heat decreases exponentially

1(e)

Sol: The given is , $(D^2+2D+1)y = e^{2x} - \left(\frac{1+\cos 2x}{2}\right)$

$$\text{i.e., } (D+1)^2 y = e^{2x} - \frac{1}{2}(e^{0x} + \cos 2x)$$

A.E has roots $-1, -1$

$$\therefore \text{C.F: } y_c = (c_1 + c_2 x) e^{-x}$$

$$\text{PI: } y_p = y_{p_1} - \frac{1}{2}(y_{p_2} + y_{p_3})$$

Where

$$y_{p_1} = \frac{e^{2x}}{(D+1)^2} = \frac{e^{2x}}{(2+1)^2} = \frac{e^{2x}}{9}$$

$$y_{p_2} = \frac{e^{0x}}{(D+1)^2} = \frac{e^{0x}}{(0+1)^2} = \frac{1}{1} = 1$$

$$y_{p_3} = \frac{\cos 2x}{(D+1)^2} = \frac{\cos 2x}{(D^2 + 2D + 1)}$$



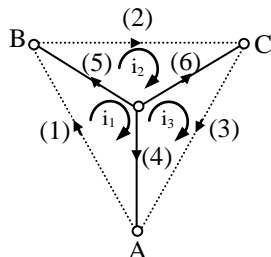
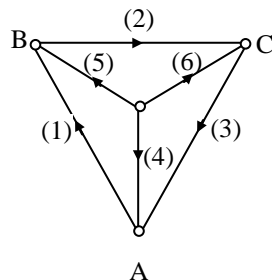
$$\begin{aligned}
 &= \frac{\cos 2x}{(-4 + 2D + 1)} = \frac{\cos 2x}{(2D - 3)} \\
 &= \frac{(2D + 3)}{(2D + 3)} \left(\frac{\cos 2x}{(2D - 3)} \right) = (2D + 3) \left(\frac{\cos 2x}{(4D^2 - 9)} \right) \\
 &= (2D + 3) \left(\frac{\cos 2x}{(4(-4) - 9)} \right) \\
 &= \frac{-1}{25} (-4 \sin 2x + 3 \cos 2x)
 \end{aligned}$$

∴ The required solution

$$y = y_c + y_p = (c_1 + c_2 x) e^{-x} + \left\{ \frac{e^{2x}}{9} - \frac{1}{2} \left(1 - \frac{1}{25} (-4 \sin 2x + 3 \cos 2x) \right) \right\}$$

2(a).

Sol: The directed graph of the above network is shown below



One of the possible tree of the graph.
The loops formed because of placing the links in the tree. The direction of loop currents is along the direction of links

From the above figure, various f circuits formed by placing links in the tree are

f-circuit (1) : {1,5,4}



f-circuit (2) : {2,6,5}

f-circuit (3) : {3,4,6}

Based on these f circuits, the f circuit matrix or tieset matrix will be

f – circuits	Branches
	1 2 3 4 5 6
1	$\begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$
B = 2	$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & -1 \end{bmatrix}$
3	$\begin{bmatrix} 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$

Since the given is resistive, the branch impedance matrix Z_b will be diagonal.

In this matrix all non-diagonal elements will be zero. The branch impedance matrix is shown below.

$$Z_b = \begin{bmatrix} 5 & & & & & 0 \\ & 5 & & & & \\ & & 5 & & & \\ & & & 10 & & \\ & & & & 5 & \\ 0 & & & & & 10 \end{bmatrix}$$

Since there are three loop currents I_1 , I_2 and I_3 , the loop current matrix I_L will be,

$$I_L = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

The network contains voltage source of 5 V in only branch 1. Hence the branch input voltage source matrix V_s will be as follows –



$$V_s = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Since there are no current sources in the network, the branch input current source matrix I_s will be zero i.e.

$$I_s = 0$$

The mesh or loop or KVL or tieset equilibrium equations are given by equation 7.6.1 as

$$BZB^T I_L = B (V_s - Z_b I_s)$$

Since $I_s = 0$, the above equation will be,

$$BZB^T I_L = B V_s$$

Putting the values for appropriate matrices in the above equation and on simplification we get,

$$\begin{bmatrix} 20 & -5 & -10 \\ -5 & 20 & -10 \\ -10 & -10 & 25 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}$$

This is the required equilibrium equation in matrix form on solving the above matrix equation

2(b)(i)

$$\text{Sol: } f(z) = \frac{1}{z^2 - 9} = \frac{1}{6(z-3)} - \frac{1}{6(z+3)}$$

Here the singular points are $z = 3, -3$ and the centre of the power series is $z_0 = 1$

\Rightarrow The radius of convergence is

$$r = |z_0 - \text{singular point}| = |1 - 3| = 2$$

\therefore The region of convergence is $|z - z_0| < r$

$$\Rightarrow |z - 1| < 2 \quad \text{or} \quad \left| \frac{z - 1}{2} \right| < 1$$

$$\text{Let } z - 1 = t \text{ then } z = 1 + t \text{ and } |t| < 2 \text{ (or) } \left| \frac{t}{2} \right| < 1$$



$$\begin{aligned}
 f(z) &= \frac{1}{6(t-2)} - \frac{1}{6(t+4)} \\
 &= \frac{-1}{12\left[1-\frac{t}{2}\right]} - \frac{1}{24\left[1+\frac{t}{4}\right]} \quad (\because |t| < 2 \Rightarrow |t| < 4) \\
 &= \frac{1}{-12}\left[1-\frac{t}{2}\right]^{-1} - \frac{1}{24}\left[1+\frac{t}{4}\right]^{-1} \\
 &= \left(\frac{-1}{12}\right)\left[1+\frac{t}{2}+\frac{t^2}{2^2}+\frac{t^3}{2^3}+\dots\right] - \frac{1}{24}\left[1-\frac{t}{4}+\frac{t^2}{4^2}+\dots\right] \\
 &= -\frac{1}{8} - \frac{1}{32}t - \frac{3}{128}t^2 - \frac{5}{512}t^3 - \dots
 \end{aligned}$$

$\therefore f(z) = -\frac{1}{8} - \frac{(z-1)}{32} - \frac{3(z-1)^2}{128} - \frac{5(z-1)^3}{512} - \dots$ is a Taylor's series expansion of $f(z)$ about $z = 1$ in the region $|z-1| < 2$.

2(b(ii)).

Sol: Given $AX = B$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & k \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$$[A | B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & k & 6 \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & k-1 & 3 \end{array} \right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & k-7 & 0 \end{array} \right]$$

If $k-7 \neq 0$ then the system will have unique solution.



∴ For $k = 7$ the system will not have unique solution.

2(c)

Sol: The experimental results state that under thermal equilibrium for any semiconductor, the product of the number of holes and the number of electrons is constant and is independent of the amount of donor and acceptor impurity doping. This relation is known as *mass-action law* and is given by $n.p = n_i^2$

- Where n is the number of free electrons per unit volume, p the number of holes per unit volume and n_i the intrinsic concentration.
- While considering the conductivity of the doped semiconductors, only the dominant majority charge carriers have to be considered.
- Charge densities in n-type and p-type semiconductors The law of mass-action has given the relationship between free electron concentration and hole concentration. These concentrations are further related by the law of Electrical Neutrality as explained below.
- Let N_D be the concentration of donor atoms in an n-type semiconductor. In order to maintain the electric neutrality of the crystal, we have

$$n_n = N_D + p_n \approx N_D$$

where n_N and p_N are electron and hole concentration in the n-type semiconductor. The value of p_n is obtained from the relations of mass-action law as

$$p_n = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_D}, \text{ which is } \ll n_n \text{ or } N_D$$

Similarly, in a p-type semiconductor we have

$$p_p = N_A + n_p \approx N_A$$

From mass-action law, $n_p = \frac{n_i^2}{p_p}$

Therefore, $n_p = \frac{n_i^2}{N_A}$, which is $\ll p_p$ or N_A

For a semiconductor material, conductivity is given as,

$$\sigma_{s.c} = (n\mu_n + p\mu_p)q$$



For p – type:

$$\sigma_p = (n\mu_n + p\mu_p)q \dots\dots\dots (1)$$

where n is minority carrier concentration,

$$n = \frac{n_i^2}{p} \dots\dots\dots (2)$$

Substitute (2) in (1)

$$\Rightarrow \sigma_p = \left(\frac{n_i^2}{p} \mu_n + p\mu_p \right) q$$

To get maximum conductivity $\frac{d\sigma_p}{dp} = 0$

$$\Rightarrow n_i^2 \left(\frac{-1}{p^2} \right) \mu_n + \mu_p = 0 \Rightarrow \frac{n_i^2}{p^2} \mu_n = \mu_p$$

$$\frac{n_i^2}{p^2} = \frac{\mu_p}{\mu_n} \Rightarrow n_i^2 = p^2 \frac{\mu_p}{\mu_n} \Rightarrow n_i = p \sqrt{\frac{\mu_p}{\mu_n}}$$

For n – type:

$$\sigma_p = (n\mu_n + p\mu_p)q \dots\dots\dots (3)$$

Where ‘p’ is minority carrier concentration given by,

$$p = \frac{n_i^2}{n} \dots\dots\dots (4)$$

Substitute (4) in (3)

$$\Rightarrow \sigma_n = \left(n\mu_n + \frac{n_i^2}{n} \mu_p \right) q$$

To get minimum conductivity $\frac{d\sigma_n}{dn} = 0$

$$\Rightarrow \mu_n + \left[\frac{-n_i^2}{(n)^2} \right] \mu_p = 0$$

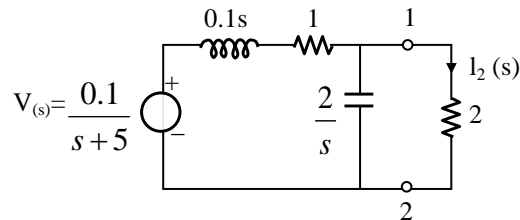
$$\mu_n = \frac{n_i^2}{n^2} \mu_p \Rightarrow \frac{\mu_n}{\mu_p} = \frac{n_i^2}{n^2} \Rightarrow n^2 = n_i^2 \frac{\mu_p}{\mu_n}$$



$$n = n_i \sqrt{\frac{\mu_p}{\mu_n}} \quad \text{Hence proved.}$$

3(a)

Sol: (i) The Laplace equivalent network of the above circuit is shown bellows



The supply voltage $v(t)$ to the network is

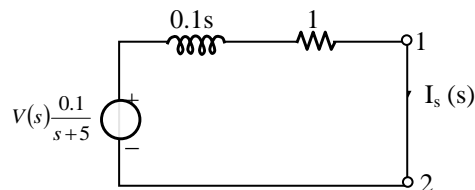
$$V(t) = 0.1 e^{-5t}$$

Taking Laplace transform of above equation we get,

$$V(s) = \frac{0.1}{s + 5}$$

The Norton equivalent current source is obtained by shorting terminals 1 and 2 and the evaluating this short circuit current.

The equivalent diagram is shown below in Figure



From above figure we can write

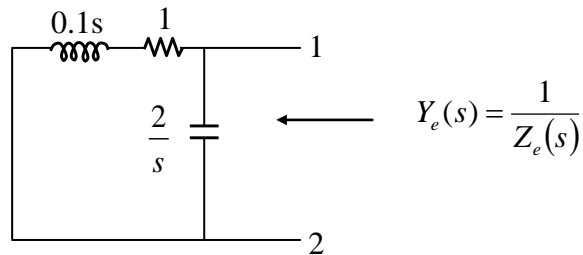
$$I_s(s) = \frac{V(s)}{0.1s + 1} = \frac{0.1}{(0.1s + 1)(s + 5)}$$

$$\therefore I_s(s) = \frac{1}{(s + 5)(s + 10)}$$

Now the equivalent admittance of the circuit viewed from points 1 and 2 is obtained by shorting voltage source.



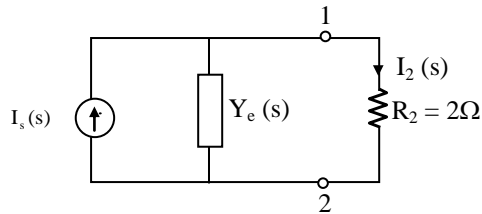
The equivalent network for this is shown below



From the equivalent network we can obtain $Y_e(s)$ as,

$$\begin{aligned} Y_e(s) &= \frac{1}{2/s} + \frac{1}{0.1s + 1} \\ &= \frac{0.5s^2 + 5s + 10}{s + 10} \end{aligned}$$

Thus the Norton equivalent network formed by $I_s(s)$, $Y_e(s)$ and R_2 is shown below



From the above figure it is clear that the voltage $V(s)$ across resistance R_2 will be,

$$V(s) = \frac{I_s(s)}{Y_e(s) + \frac{1}{R_2}}$$

∴ Current $I_2(s)$ through resistance R_2 will be,

$$\begin{aligned} I_2(s) &= \frac{V(s)}{R_2} = \frac{I_s(s)}{R_2 \left[Y_e(s) + \frac{1}{R_2} \right]} \\ &= \frac{I_s(s)}{R_2 Y_e(s) + 1} \end{aligned}$$

Putting values in above equation we get,



$$\begin{aligned}
 I_2(s) &= \frac{1}{2 \left[\frac{0.5s^2 + 5s + 10}{s + 10} \right] + 1} \\
 &= \frac{1}{(s + 5)(s^2 + 11s + 30)} = \frac{1}{(s + 5)[(s + 5)^2 + s + 5]} \\
 &= \frac{1}{(s + 5)^2(s + 6)}
 \end{aligned}$$

Let us rearrange the above equation as,

$$I_2(s) = \frac{(s + 6) - (s + 5)}{(s + 5)^2(s + 6)} = \frac{1}{(s + 5)^2} - \frac{1}{(s + 5)(s + 6)}$$

Again rearranging second term we get partial fraction expansion as follows

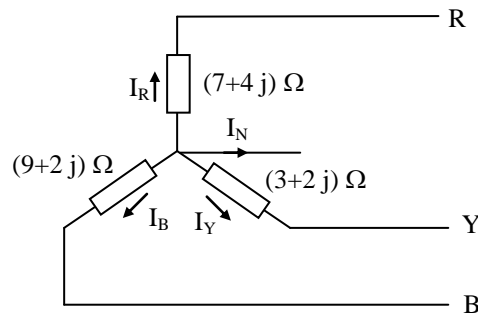
$$I_2(s) = \frac{1}{(s + 5)^2} - \frac{1}{s + 5} + \frac{1}{s + 6}$$

Taking inverse Laplace transform we get.

$$i_2(t) = te^{-5t} - e^{-5t} + e^{-6t}$$

This is the current through resistance R_2

(ii) The unbalanced star connection described in the problem may be represented as shown



impedances on three phases are calculated as follows:

$$Z_R = (7 + j4) \Omega = 8.06 \angle 29.74^\circ \Omega$$

$$Z_Y = (3 + j4) \Omega = 5.0 \angle 56.3^\circ \Omega$$

$$Z_B = (9 + j2) \Omega = 9.22 \angle 12.53^\circ \Omega$$



$$V_L = 440V, V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03 \text{ V}$$

Phase or line currents can be calculated as given below.

$$I_R = \frac{V_{ph} \angle 0^\circ}{Z_1 \angle \phi_1^\circ} = \frac{254.03 \angle 0^\circ}{8.06 \angle 29.74^\circ} = (27.36 - j15.634)A = 31.5 \angle -29.74^\circ A$$

$$I_Y = \frac{V_{ph} \angle -120^\circ}{Z_1 \angle \phi_2^\circ} = \frac{254.03 \angle 120^\circ}{3.6 \angle 33.7^\circ} = -63.26 - j31.265 A = 70.56 \angle -153.7^\circ A$$

$$I_Z = \frac{V_{ph} \angle -240^\circ}{Z_3 \angle \phi_2^\circ} = \frac{254.03 \angle 240^\circ}{9.22 \angle 12.53^\circ} = (-8.27 + j26.28)A = 27.55 \angle 107.47^\circ A$$

Neutral current is given as $I_N = -(I_R + I_Y + I_B)$

$$= -(27.36 - j15.634 - 63.26 - j31.265 - 8.27 + j26.28)$$

$$= -(-44.17 - j20.619) A = (44.17 + j20.619) A$$

03(b)(i).

Sol: Given $\frac{dy}{dx} - y = x \dots\dots\dots (1)$

and $y(0) = 0 \dots\dots\dots (2)$

Also given $h = 0.1$

$y(0.3) = ?$

from (2), we have

$$x_0 = 0, y_0 = 0 \text{ and } f(x, y) = \frac{dy}{dx} = y + x$$

$$x_1 = x_0 + 1h = 0 + 0.1 = 0.1$$

$$x_2 = x_0 + 2h = 0 + 2(0.1) = 0.2$$

$$x_3 = x_0 + 3h = 0 + 3(0.1) = 0.3$$

Euler's method is given by

$$y_1 = y_0 + h f(x_0, y_0) = y_0 + h \left(\frac{dy}{dx} \right)_p$$

$$y_1 = 0 + (0.1) (x_0 + y_0) = 0.1 [0 + 0] = 0.0$$

$$y_2 = y_1 + h f(x_1, y_1)$$

$$= 0.0 + (0.1)[0.1 + 0.0] = 0.0 + 0.01 = 0.01$$



$$\begin{aligned} y_3 &= y_2 + h f(x_2, y_2) \\ &= 0.01 + (0.1) [0.2 + 0.01] \\ &= 0.01 + 0.021 \\ &= 0.031 \end{aligned}$$

03(b)(ii).

Sol: A. Since $\sum P(x) = 1$, $6K + 0.6 = 1$

$$\therefore k = \frac{1}{15}$$

\therefore The probability distribution becomes

X	-2	-1	0	1	2	3
P(x)	1/10	1/15	1/5	2/15	3/10	1/5

B $P(X < 2) = P(X = -2, -1, 0 \text{ or } 1)$

$$= P(X = -2) + P(X = -1) + P(X = 0) + P(X = 1)$$

[since the event $(X = -2), (X = -1)$ etc. are mutually exclusive.]

$$= \frac{1}{10} + \frac{1}{15} + \frac{1}{5} + \frac{2}{15} = \frac{1}{2}$$

$P(-2 < X < 2) = P(X = -1, 0 \text{ or } 1)$

$$= P(X = -1) + P(X = 0) + P(X = 1)$$

$$= \frac{1}{15} + \frac{1}{5} + \frac{2}{15} = \frac{2}{5}$$

C. the mean of X is defined as $E(X) = \sum xP(x)$

$$\begin{aligned} \therefore \text{Mean of X} &= \left(-2 \times \frac{1}{10}\right) + \left(-1 \times \frac{1}{15}\right) + \left(0 \times \frac{1}{5}\right) + \left(1 \times \frac{2}{15}\right) + \left(2 \times \frac{3}{10}\right) + \left(3 \times \frac{1}{5}\right) \\ &= -\frac{1}{5} - \frac{1}{15} + \frac{2}{15} + \frac{3}{5} + \frac{3}{5} = \frac{16}{15} \end{aligned}$$



03(c)(i).

Sol: $|\vec{E}| = \frac{k}{\rho}$

$$\begin{aligned} W_E &= \frac{\epsilon_0}{2} \int |\vec{E}|^2 dv \\ &= \frac{\epsilon_0}{2} \int_2^3 \int_0^{\pi/2} \int_0^7 \frac{k^2}{\rho^2} \times \rho d\rho d\phi dz \\ &= \frac{\epsilon_0}{2} \times k^2 \times [\ln \rho]_2^3 [\phi]_0^{\pi/2} [z]_0^7 \\ &= k^2 \times \frac{10^{-9}}{36\pi \times 2} \times \ln\left(\frac{3}{2}\right) \times \frac{\pi}{2} \times 2 \\ &= k^2 \times \frac{10^{-9}}{72} \times \ln\left(\frac{3}{2}\right) \end{aligned}$$

$$W_E = 1\mu J = 10^{-6}$$

$$\Rightarrow k^2 \times \frac{10^{-9}}{72} \times \ln\left(\frac{3}{2}\right) = 10^{-6}$$

$$\Rightarrow k^2 = \frac{72}{10^{-3} \times \ln\left(\frac{3}{2}\right)} = 177573.84$$

$$\Rightarrow k = 421.39$$

3(c)(ii)

Sol: $f_0 = 100 \text{ Hz}$

$$Z_1 = (10 + j8), Z_2 = (10 - jX_c) \Omega$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{10 + j8} = 0.06 - j0.04 \Omega$$

$$\begin{aligned} Y_z &= \frac{1}{Z_2} = \frac{1}{10 - jX_c} = \frac{10 + jX_c}{10^2 + (X_c)^2} \\ &= \frac{10}{100 + X_c^2} + \frac{jX_c}{100 + X_c^2} \end{aligned}$$

$$Y = Y_1 + Y_2$$



$$= 0.06 + \frac{10}{100 + X_c^2} + j \left[\frac{X_c}{100 + X_c^2} - 0.04 \right]$$

The condition for resonance is that the net susceptance should become equal to zero. That is,

$$\frac{X_c}{100 + X_c^2} - 0.04 = 0$$

$$\frac{X_c}{100 + X_c^2} = 0.04$$

$$X_c = 4 + 0.04 X_c^2$$

$$0.04X_c^2 - X_c + 4 = 0$$

$$X_c = \frac{1 \pm \sqrt{(1)^2 - 4(0.04)(4)}}{2 \times 0.04} = \frac{1 \pm \sqrt{1 - 0.4}}{0.08} = 20 \Omega$$

$$X_c = \frac{1}{2\pi f_c} \Rightarrow C = \frac{1}{2\pi f X_c} \Rightarrow \frac{1}{2\pi \times 100 \times 20} = 79.61 \mu F$$

$$C = 79.61 \mu F$$

4(a)(i)

Sol: Laser ablation: Laser ablation has been extensively used for the preparation of nanoparticles and particulate films. In this process a laser beam is used as the primary excitation source of ablation for generating clusters directly from a solid sample in a wide variety of applications. The small dimensions of the particles and the possibility to form thick films make this method quite an efficient tool for the production of ceramic particles and coatings and also an ablation source for analytical applications such as the coupling to induced coupled plasma emission spectrometry, ICP, the formation of the nanoparticles has been explained following a liquefaction process which generates an aerosol, followed by the cooling/solidification of the droplets which results in the formation of fog. The general dynamics of both the aerosol and the fog favours the aggregation process and micrometer-sized fractal-like particles are formed. The laser spark atomizer can be used to produce highly mesoporous thick films and the porosity can be modified by the carrier gas flow rate. ZrO_2 and SnO_2 nanoparticulate thick films were also synthesized successfully using this process with quite identical microstructure. Synthesis of other materials such as lithium manganate, silicon and carbon has also been carried out by this technique.



4(a)(ii)

Sol: The unusual properties of nano materials can be attributed to the following reasons: the number of atoms on the surface is comparable to the number of atoms at the lattice points. Therefore the properties are affected by the atoms at these locations. The other aspect is the Quantum Confinement effect; when the particles are of nano size, say, 100nm side cube, the presence of a few vacancies allows the crystal lattice to large relaxation. As a result, the optical, electronic and mechanical properties are affected significantly.

Some examples are given below:

- (i) They are more ductile at elevated temperatures as compared to the coarse-grained ceramics.
- (ii) Nanostructured semiconductors are known to show various non-linear optical properties.
- (iii) Semiconductor Q-particles also show quantum confinement effects which may lead to special properties, like the luminescence in silicon powders; silicon - germanium quantum dots as infrared optoelectronic devices.
- (iv) Cold welding properties combined with the ductility make them suitable for metal-metal bonding especially in the electronic industry.
- (v) Very small particles have special atomic structures with discrete electronic states, which give rise to special properties for high density information storage and magnetic refrigeration.
- (vi) They have large surface to volume ratio. Further, in a nano wire, electrons are confined to one dimensional ballistic motion which gives rise to special electrical properties.

04.(b)(i)

Sol: Let $f(z) = \frac{3z+4}{z(z-1)}$ Then the singular points of $f(z)$ are $z = 0$ & $z=1$. The given region is bounded by the

circle C: $|z| = 2$

Here the two singular points lie inside the circle 'C' and $z = 0$, $z = 1$ are poles of order one.

$$R_1 = \text{Res}\{f(z) : z = 0\}$$

$$= \lim_{z \rightarrow 0} \left[(z-0) \frac{3z+4}{z(z-1)} \right] = \frac{0+4}{0-1} = -4$$

$$R_2 = \text{Res}[f(z) : z = 1]$$

$$= \lim_{z \rightarrow 1} \left[(z-1) \frac{3z+4}{z(z-1)} \right] = \frac{3+4}{1} = 7$$



∴ By Cauchy's Residue Theorem, we have

$$\int_c \frac{3z+4}{z(z-1)} dz = 2\pi i (R_1 + R_2) = 2\pi i (-4 + 7) = 6\pi i$$

04.(b)(ii)

Sol: From figure, it is self-evident that $z = 4 - y$ is to be integrated over the circle $x^2 + y^2 = 4$ in the XY - plane.

To cover the shaded half of this circle,

x varies from 0 to $\sqrt{4-y^2}$ and

y varies from -2 to 2

∴ Required volume

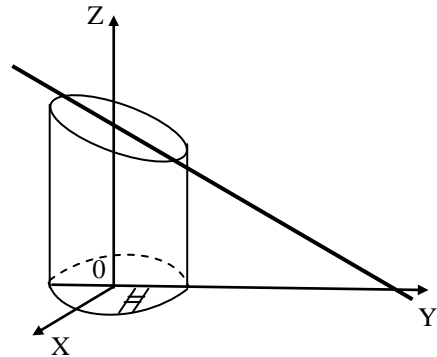
$$= 2 \int_{-2}^2 \int_0^{\sqrt{4-y^2}} z \, dx \, dy = 2 \int_{-2}^2 \int_0^{\sqrt{4-y^2}} (4-y) \, dx \, dy$$

$$= 2 \int_{-2}^2 (4-y) [x]_0^{\sqrt{4-y^2}} dy = 2 \int_{-2}^2 (4-y) \sqrt{4-y^2} dy$$

$$= 2 \int_{-2}^2 4\sqrt{4-y^2} dy - 2 \int_{-2}^2 y\sqrt{4-y^2} dy$$

$$= 8 \int_{-2}^2 \sqrt{4-y^2} dy \quad [\text{The second term vanishes as the integrand is an odd function}]$$

$$= 8 \left[\frac{y\sqrt{4-y^2}}{2} + \frac{4}{2} \sin^{-1} \frac{y}{2} \right]_{-2}^2 = 16\pi$$



4(c)

Sol: 1. Body centered cubic structure (B.C.C):

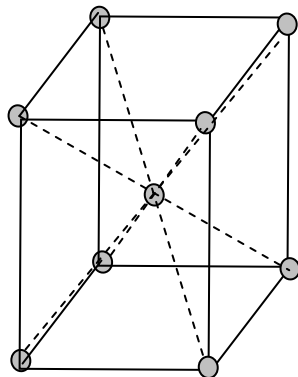




Figure shows the unit cell of B.C.C. structure. In this structure the eight corners of the cube are occupied by eight atoms and the centre of the cube is occupied by one atom. Metals that crystallize into B.C.C. structure are chromium, tungsten, iron, vanadium, molybdenum and sodium.

Number of atoms in the unit cell of B.C.C Structure:

In B.C.C. structure, the unit cell contains eight atoms at each corner of the cube and one atom in the centre of the cube. Since each corner atom is shared by eight surrounding cubes and the atom in the centre can not be shared by any other cube, the unit cell of the B.C.C. structure contains:

$$8 \text{ atoms at the corners} \times \frac{1}{8} = 1 \text{ atom}$$

$$1 \text{ centre atom} = 1 \text{ atom}$$

$$\therefore \text{Total} = 2 \text{ atoms}$$

Therefore the unit cell of B.C.C. structure contains two atoms.

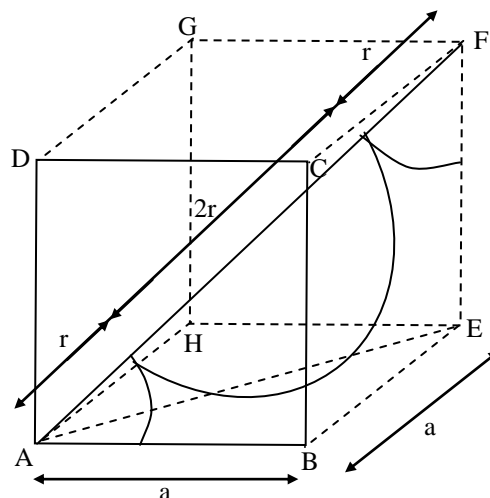
Atomic packing factor of B.C.C. structure:

From the figure

$$(AE)^2 = (AB)^2 + (BE)^2 = a^2 + a^2 = 2a^2$$

$$\text{and } (AF)^2 = (AE)^2 + (EF)^2 = 2a^2 + a^2 = 3a^2$$

$$\Rightarrow AF = a\sqrt{3}$$



We have from the figure

$$AF = 4r$$

$$\therefore 4r = a\sqrt{3}$$



$$\Rightarrow r = \frac{a\sqrt{3}}{4}$$

\therefore The radius of the atom (sphere) in the B.C.C. structure is $\frac{a\sqrt{3}}{4}$. And the number of atoms in the unit cell of B.C.C. structure are two.

$$\begin{aligned} \text{Volume of atoms in the unit cell} &= \frac{2 \times 4\pi r^3}{3} \\ &= \frac{8\pi \left(\frac{a\sqrt{3}}{4} \right)^3}{3} = \frac{\pi a^3 \sqrt{3}}{8} \end{aligned}$$

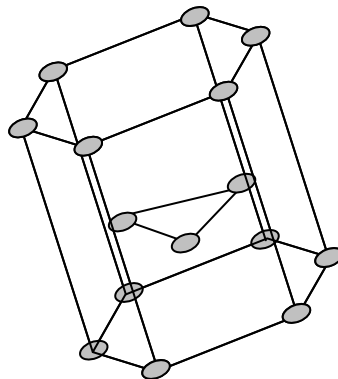
$$\text{Volume of unit cell} = a^3$$

Atomic packing factor

$$\begin{aligned} &= \frac{\text{Volume of atoms in the unit cell}}{\text{Volume of unit cell}} \\ &= \frac{\left(\frac{\pi a^3 \sqrt{3}}{8} \right)}{a^3} = \frac{\pi \sqrt{3}}{8} = 0.68 \end{aligned}$$

3. Hexagonal close packed structure (H.C.P.):

Figure shows the unit cell of H.C.P. structure. The H.C.P. structure contains i) One atom at each corner of the hexagon ii) One atom at the centre of the two hexagonal faces and iii) Three atoms in the form of a triangle midway between the two basal planes. Metals that crystallize into H.C.P. structure are zinc, cadmium, beryllium, magnesium, titanium, zirconium etc.





Number of atoms in the unit cell of H.C.P. structure:

Since each corner atom of the hexagon is shared by six surrounding hexagons, the centre atom of the hexagon face is shared by two surrounding hexagons and the three middle layer atoms can not be shared by any other hexagons, the unit cell of the H.C.P. structure contains.

$$12 \text{ atoms at the corners} \times \frac{1}{6} = 2 \text{ atoms}$$

$$2 \text{ face centered atoms} \times \frac{1}{2} = 1 \text{ atom}$$

$$3 \text{ middle layer atoms} = 3 \text{ atoms}$$

$$\therefore \text{Total} = 6 \text{ atoms}$$

Atomic packing factor of H.C.P. structure:

The volume of the unit cell can be found out by finding out the area of the basal plane and then multiplying this by its height.

The area of the basal plane is the area ABCDEFG. This area is six times the area of equilateral triangle ABC.

$$\text{Area of triangle ABC} = \frac{1}{2} (\text{base}) \times (\text{height}) = \frac{1}{2} \times a \times a \sin 60^\circ = \frac{1}{2} a^2 \sin 60^\circ$$

Total area of the basal plane

$$= 6 \times \frac{1}{2} \times a^2 \sin 60^\circ$$

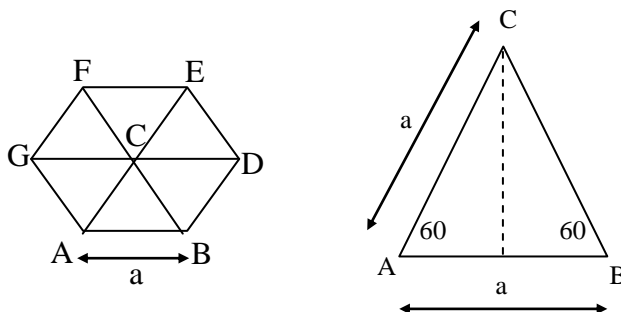
$$= 3 a^2 \sin 60^\circ$$

Volume of unit cell

$$= \text{Area of basal plane} \times \text{height}$$

$$= 3 a^2 \sin 60^\circ \times h$$

$$\text{For HCP structures, } a = 2r \Rightarrow r = \frac{a}{2}$$





Also we know that the number of atoms in the unit cell of HCP structure are six.

$$\text{Volume of atoms in the unit cell} = 6 \times \frac{4\pi r^3}{3}$$

Atomic packing factor

$$= \frac{\text{Volume of atoms in the unit cell}}{\text{Volume of unit cell}}$$

$$= \frac{6 \times \frac{4\pi r^3}{3}}{3a^2 \sin 60^\circ \times h}$$

$$= \frac{6 \times \frac{4\pi}{3} \left(\frac{a}{2}\right)^3}{3a^2 \sin 60^\circ \times h}$$

$$= \frac{\pi a}{3h \sin 60^\circ}$$

The h/a ratio for an ideal HCP crystal structure consisting of uniform spheres packed tightly together is 1.633.

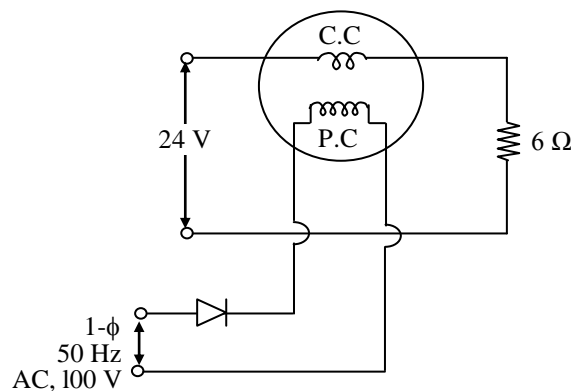
Therefore, substituting h/a = 1.633

We get,

$$\text{Atomic packing factor} = 0.74$$

05(a).

Sol:





$$\text{Current through the current coil} = \frac{24}{6} = 4 \text{ A}$$

∴ Reading of wattmeter = Average power over a cycle

$$= \frac{1}{2\pi} \left[\int_0^{2\pi} v i \, d\theta \right]$$

$$\Rightarrow \frac{1}{2\pi} \left[\int_0^{\pi} \sqrt{2} \times 100 \sin \theta \times 4 \, d\theta \right]$$

$$= \frac{\sqrt{2} \times 100 \times 4}{2\pi} \left[\int_0^{\pi} \sin \theta \, d\theta \right]$$

$$= \frac{\sqrt{2} \times 100 \times 4}{2\pi} \left[-\cos \theta \Big|_0^{\pi} \right] = 180.06 \text{ W.}$$

5(b)

Sol: A. (i) The given oscillator is a Colpitts oscillator

(ii) For Colpitts oscillator, the expression for frequency of oscillation is given as

$$\omega_{\text{osc}} = \frac{1}{\sqrt{LC_{\text{eq}}}}$$

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} \Rightarrow C_{\text{eq}} = \frac{2 \times 2}{2 + 2} = 1 \text{ pF}$$

$$LC_{\text{eq}} = 100 \times 10^{-6} \times 1 \times 10^{-12}$$

$$\omega_{\text{osc}} = \frac{1}{\sqrt{LC_{\text{eq}}}} = \frac{1}{\sqrt{100 \times 10^{-18}}} = \frac{10^9}{\sqrt{100}}$$

$$= \frac{1000 \times 10^6}{\sqrt{100}} = 100 \text{ Mrad/s}$$

B. The output of integrator is

$$V_x = -\frac{1}{RC} \int V_0 \, dt \text{ and}$$

$$V_0 = -2V_x$$

$$\text{Then } V_x = -\frac{2}{RC} \int V_x \, dt$$



$$\frac{2}{RC} t V_x = V_x \Rightarrow t = \frac{RC}{2}$$

$$f = \frac{2}{RC}$$

$$f = \frac{2}{10^5 \times 10^{-9}} = 20 \text{ kHz}$$

5(c)

Sol: (i) Enum: Enumeration (or enum) is a user defined data type in C. It is mainly used to assign names to integral constants, the names make a program easy to read and maintain.

Following is an example of enum declaration:

enum flag{constant1, constant2, constant3, };

The name of enumeration is "flag" and the constant are the values of the flag. By default, the values of the constants are as follows:

constant1 = 0, constant2 = 1, constant3 = 2 and so on.

Variables of type enum can also be defined. They can be defined in two ways:

enum week{Mon, Tue, Wed};

enum week day;

Or

enum week{Mon, Tue, Wed}day;

In both of the above cases, "day" is defined as the variable of type week.

(ii) Typedef:

The C programming language provides a keyword called typedef, which you can use to give a type

a new name. Following is an example to define a term BYTE for one-byte numbers

typedef unsigned char BYTE;

After this type definition, the identifier BYTE can be used as an abbreviation for the type unsigned char, for example..

BYTE b1, b2;

By convention, uppercase letters are used for these definitions to remind the user that the type name

is really a symbolic abbreviation, but you can use lowercase, as follows

**typedef unsigned char byte;**

You can use typedef to give a name to your user defined data types as well. For example, you can use typedef with structure to define a new data type and then use that data type to define structure variables directly as follows

```
#include <stdio.h>
#include <string.h>

typedef struct Books {
char title[50];
char author[50];
char subject[100];
int book_id;
} Book;

int main( ) {
Book book;
strcpy( book.title, "C Programming");
strcpy( book.author, "Nuha Ali");
strcpy( book.subject, "C Programming Tutorial");
book.book_id = 6495407;
printf( "Book title : %s\n", book.title);
printf( "Book author : %s\n", book.author);
printf( "Book subject : %s\n", book.subject);
printf( "Book book_id : %d\n", book.book_id);
return 0;
}
```

When the above code is compiled and executed, it produces the following result

Book title : C Programming

Book author : Nuha Ali

Book subject : C Programming Tutorial

Book book_id : 6495407



5(d)

Sol: Loss of charge method: This method is specially suited for the measurement of a very high insulation resistance. The connections are shown in fig. 1. With the key K_1 closed and K_2 open, the capacitor is charged upto a suitable voltage. Then the capacitor is allowed to discharge through the unknown resistance X , by opening the key K_1 and closing K_2 . The terminal voltage is being observed for a long time (in hours).

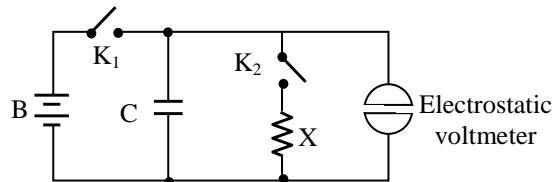


Fig. 1 Loss of charge method for measurement
Of insulation resistance

Let V be the terminal voltage and Q be the charge (in coulomb) at time t . Then the current through the resistance X is

$$i = \frac{dQ}{dt} = -C \frac{dV}{dt}$$

Where C is the capacitance of the capacitor in farad.

But, $i = \frac{V}{X}$

$$\therefore \frac{V}{X} = -C \frac{dV}{dt}$$

Or $C \frac{dV}{dt} + \frac{V}{X} = 0$

$$\frac{dV}{V} = -\frac{1}{CX} dt$$

Or $\log_e V = -\frac{t}{CX} + \log_e K$

Where K is a constant

Let $V = E$, when $t = 0$

Then , $K = E$



$$\therefore \log_e V = \log_e E - \frac{t}{CX}$$

$$\text{Or } \frac{t}{CX} = \log_e \frac{E}{V}$$

$$X = \frac{t}{C \log_e \frac{E}{V}} = \frac{0.4343t}{C \log_{10} \frac{E}{V}} \text{ ohm}$$

5(e)

Sol:(i) Applying KVL around the gate-source loop yields

$$V_G = V_{GSQ} + R_S I_{DQ} + V_{SS} \quad \text{----- (1)}$$

Solving (1) for I_{DQ} and equating the result to $I_{dss} \left(1 + \frac{V_{GSQ}}{V_{P0}}\right)^2$

$$\frac{V_G - V_{GSQ} - V_{SS}}{R_S} = I_{dss} \left(1 + \frac{V_{GSQ}}{V_{P0}}\right)^2 \quad \text{----- (2)}$$

Rearranging (2) leads to the following quadratic in V_{GSQ} :

$$V_{GSQ}^2 + V_{P0} \frac{V_{P0} + 2I_{dss}R_S}{I_{dss}R_S} V_{GSQ} + \frac{V_{P0}^2}{I_{dss}R_S} (I_{dss}R_S - V_G + V_{SS}) = 0 \quad \text{----- (3)}$$

Substituting known values into (3) and solving for V_{GSQ} with the quadratic formula lead to

$$V_{GSQ}^2 + 3 \frac{3 + (2)(5 \times 10^{-3})(8 \times 10^{-3})}{(5 \times 10^{-3})(8 \times 10^3)} V_{GSQ} + \frac{(3)^2}{(5 \times 10^{-3})(8 \times 10^3)} [(5 \times 10^{-3})(8 \times 10^3) - 0 - 8] = 0$$

So that $V_{GSQ}^2 + 6.225 V_{GSQ} + 7.2 = 0$ and $V_{GSQ} = -4.69 \text{ V}$ or -1.53 V . Since $V_{GSQ} = -4.69 \text{ V} < -V_{P0}$,

this value must be considered extraneous as it will result in $i_D = 0$. Hence, $V_{GSQ} = -1.53 \text{ V}$. Now,

$$\begin{aligned} I_{DQ} &= I_{dss} \left(1 + \frac{V_{GSQ}}{V_{P0}}\right)^2 \\ &= 5 \times 10^{-3} \left(1 + \frac{-1.53}{3}\right)^2 = 1.2 \text{ mA} \end{aligned}$$

and, by KVL,



$$V_0 = I_{DQ}R_S + V_{SS}$$

$$= (1.2 \times 10^{-3}) (8 \times 10^3) + (-8) = 1.6V$$

(ii) Substitution of known values into (3) leads to

$$V_{GSQ}^2 + 6.225 V_{GSQ} + 4.95 = 0$$

Which, after elimination of the extraneous root, results in $V_{GSQ} = -0.936 V$.

Then, as in part (a),

$$I_{DQ} = I_{DSS} \left(1 + \frac{V_{GSQ}}{V_{P0}} \right)^2$$

$$= 5 \times 10^{-3} \left(1 + \frac{-0.936}{4} \right)^2 = 2.37 \text{ mA and}$$

$$V_0 = I_{DQ}R_S + V_{SS}$$

$$= (2.37 \times 10^{-3}) (8 \times 10^3) + (-8)$$

$$= 10.96 V$$

6(a)(i).

Sol: $-5.5 = -101.101$

$$= -(1.01101) \times 2^{+2}$$

1	10000001	0110100....00
sign	Exponent	Mantissa
	(Excess127code)	(hidden bit)

1100 0000 1011 0100 0 ----- 0

[C0B40000]_H

06(a)(ii).

Sol: Control Unit is the part of the computer's central processing unit (CPU), which directs the operation of the processor.

It is the responsibility of the Control Unit to tell the computer's memory, arithmetic/logic unit and input and output devices how to respond to the instructions that have been sent to the processor. It



fetches internal instructions of the programs from the main memory to the processor instruction register, and based on this register contents, the control unit generates a control signal that supervises the execution of these instructions.

A control unit works by receiving input information to which it converts into control signals, which are then sent to the central processor. The computer's processor then tells the attached hardware what operations to perform. The functions that a control unit performs are dependent on the type of CPU because the architecture of CPU varies from manufacturer to manufacturer.

Functions of the Control Unit:

- It coordinates the sequence of data movements into, out of, and between a processor's many sub-units.
- It interprets instructions.
- It controls data flow inside the processor.
- It receives external instructions or commands to which it converts to sequence of control signals.
- It controls many execution units(i.e. ALU, data buffers and registers) contained within a CPU.
- It also handles multiple tasks, such as fetching, decoding, execution handling and storing results.

06(a)(iii).

Sol: Demand Paging:

- Virtual memory with paging
- Main Memory divided, into equal size blocks known as frames
- Virtual (user) program divided into 'pages', where page size is equal to frame size
- Whenever CPU generate read/write request for a word, that requested word belongs to one of the page of a program
- If demanded page is present in one of the frame of main memory then it is known as 'Page hit'
- If demanded page is not available in main memory then its condition is known as 'Page fault' and page fault interrupt invoked
- Page fault service routine load demanded page into main memory



6(b).

Sol: Exact Analysis:

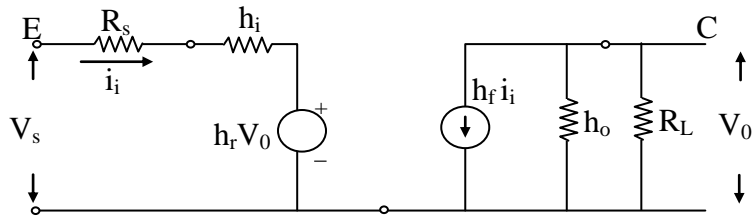


Figure: CB connection h parameter model

$$A_I = \frac{-h_{fb}}{1 + h_{ob}R_L} = \frac{-(-0.98)}{1 + (0.49 \times 10^{-6})(4 \times 10^3)}$$

$$\therefore A_I \cong 0.98$$

$$\begin{aligned} R_i &= h_{ib} + h_{rb} A_I R_L \\ &= 21.6 + (2.9 \times 10^{-4})(0.98)(4 \times 10^3) \end{aligned}$$

$$\therefore R_i = 22.7 \Omega$$

$$\begin{aligned} A_v &= A_I \frac{R_L}{R_i} \\ &= 0.98 \times \frac{4 \times 10^3}{22.7} \end{aligned}$$

$$\therefore A_v = 172.68$$

$$Y_0 = h_{ob} - \frac{h_{fb}h_{rb}}{h_{ib} + R_s}$$

$$Y_0 = 0.63 \times 10^{-6} \text{ S}$$

$$\therefore R_0 = 1/Y_0 = 1.58 \text{ M}\Omega$$

$$R'_0 = R_0 \parallel R_L = 3.98 \text{ K}\Omega$$

$$\text{Now, } A_{vs} = A_v \frac{R_i}{R_i + R_s} = 1.96$$



Approx, solution:

$$A_I = -h_{fb} = 0.98$$

$$R_i = h_{ib} = 21.6\Omega$$

$$A_v = -h_{fb} \frac{R_L}{h_{ib}} = 0.98 \times \frac{4K\Omega}{21.6}$$

$$A_v = 181.48$$

$$R_0 \rightarrow \infty \text{ and } R_i' = 4K\Omega$$

$$A_{vs} = 181.48 \times \frac{21.6}{21.6 + 2K\Omega}$$

$$A_{vs} = 1.94$$

6(c)(i)

Sol: HTTP:

- Hyper-text transfer protocol
- Application layer protocol which uses TCP as transport protocol
- Stateless protocol (Server never maintain state information of clients)
- Used to transfer resources between HTTP client and HTTP server
(resources can be HTML, XML or user files)

FTP:

- File Transfer Protocol
- Application protocol which uses TCP
- State full protocol (Server maintain state information of clients)
- Used to transfer user files between FTP client and server

SMTP:

- Simple mail transfer protocol
- Application protocol which uses TCP
- State full protocol
- Used to transfer electronic mail(e-mail) from mail client to mail server



6(c)(ii).

Sol: Mode of Data transfer in between I/O devices and CPU

- i) Programmed I/O
- ii) Interrupt driven I/O
- iii) DMA

1) Programmed I/O (Processor Control I/O)

- In this I/O operations are the result of I/O instructions written in the program
- For each data transfer one instruction is required to execute.
- The disadvantage of this is CPU stays in the program loop until the I/O unit is ready
- It is the time consuming process, since it keeps the processor busy needlessly

2) Interrupt Driven I/O (Device Control)

- The disadvantage in programmed I/O can be avoided by using an interrupt facility and special commands to inform the interface to issue an interrupt request signal when the data is available from the device.
- In mean time, the CPU can proceed to execute another program and the interface keeps monitoring the device.
- After receiving the ready signal from that device, CPU stops the task that it is processing and branches to service program request.

3) DMA:

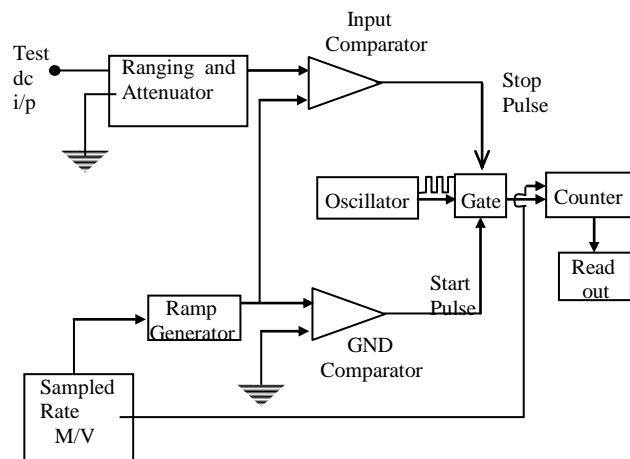
- It is mainly used for high speed data transfer in between main memory and auxiliary memory.
- When there is a miss in main memory, immediately DMA controller gets a signal known as DMA request for asking word transfer from auxiliary memory to main memory. Then the following operations performed
 - 1) DMAC sends a signals to Processor known as ***DMA Request*** for asking bus control.
 - 2) Immediately CPU completes its internal operation related to bus operation and then it hands over all the buses to DMAC by sending a special signal known as ***BUS Grant***
 - 3) DMAC gives acknowledgement for requested memory device known as ***DMA Grant***
 - 4) Then onwards, operations will be started to supervise the operation, DMAC uses one special register known as ***Terminal Count Register (TCR)***



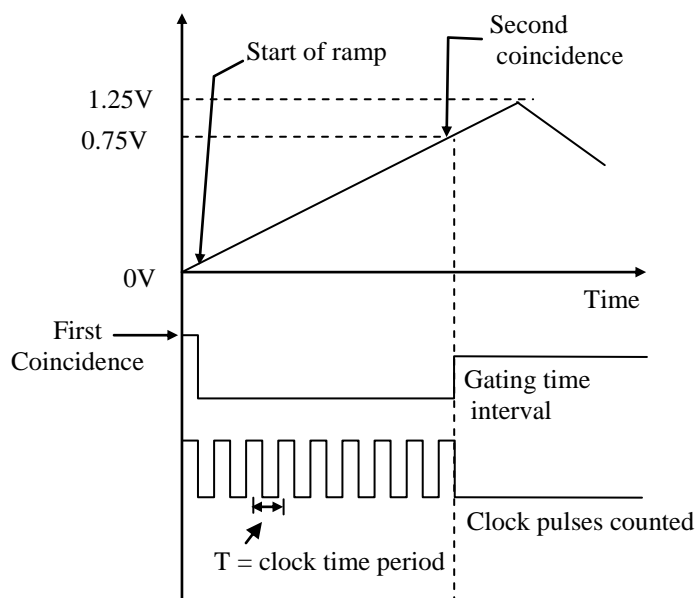
- 5) After each one word transfer, TCR will be tested and operation will be terminated, when TCR content becomes zero.

7(a)(i)

Sol: The operating principle is to measure the time that a linear ramp takes to change the input level to the ground level, or vice-versa. This time period is measured with an electronic time-interval counter and the count is displayed as a number of digits on an indicating tube or display. The operating principle and block diagram of a ramp type DVM are shown in fig.



The ramp may be positive or negative; in this case a negative ramp has been selected.





N1 = Number clock cycles in 1.25 V

$$= \frac{125 \text{ msec}}{1 \mu\text{sec}} = 125000$$

1.25 V \rightarrow 125000 counts

0.75 V \rightarrow x counts

x = 75000 counts counted into register for an input of 0.75 V

Method 2:

$$\text{Slope of ramp} = m = \frac{dV}{dt} = \frac{1.25 \text{ V}}{125 \text{ msec}} = 0.01 \left(\frac{\text{V}}{\text{msec}} \right)$$

\Rightarrow 0.75 V corresponding time required

$$= \frac{0.75}{0.01} \text{ msec}$$

$$= 75 \text{ msec}$$

$$\text{Number of clock pulses counted} = \frac{75 \text{ msec}}{T} = \frac{75 \text{ msec}}{1 \mu\text{sec}} = 75000$$

7(a)(ii)

Sol: True value = 20 mA

Measured value = 18 mA

(i) Absolute error = MV – TV = 18 – 20 = **-2 mA**

(ii) Percentage error = $\frac{-2}{20} \times 100 = -10\%$

(iii) Relative accuracy = $1 - \frac{(\text{Actual value} - \text{Measurement})}{\text{actual Value}}$

$$= 1 - \frac{(20 - 18)}{20} = 0.9$$

(iv) Percentage accuracy = $\frac{18}{20} \times 100 = 90\%$

(v) For 6th reading precision is $= 1 - \left| \frac{x_6 - \bar{x}}{\bar{x}} \right|$

$\therefore \bar{x}$ is the average value of the readings



$$= \frac{16 + 19 + 20 + 17 + 21 + 18 + 15 + 16 + 18 + 17}{10} = 17.7 \text{ mA}$$

$$\text{Precision} = 1 - \left| \frac{18 - 17.7}{17.7} \right| = 1 - 0.1695 = \mathbf{0.983}$$

7(b)(i)

Sol: A

Basis For Comparison	Program	Process
Basic	Program is a set of instructions.	When a program is executed, it is known as process.
Nature	Passive	Active
Lifespan	Longer	Limited
Required resources	Program is stored on disk in some file and does not require any other resources.	Process holds resources such as CPU, memory address, disk, I/O etc.

B. When the computer system starts CPU executes its first few instructions from ROM only because RAM is a volatile memory and RAM's content used to flushed out while system turn off.

CPU performs POST(Power On Self-Test) to check all hardware's and their working. After that CPU performs booting process which helps to bring OS programs from secondary memory to RAM. All these works are performed while executing the programs from ROM. That is why ROM is very important part of the system. And CPU can execute all these instructions from ROM because ROM is a non-volatile memory.

There are two main reasons that read-only memory is used for certain functions within the PC:

Permanence: The values stored in ROM are always there, whether the power is on or not. A ROM can be removed from the PC, stored for an indefinite period of time, and then replaced, and the data it contains will still be there. For this reason, it is called non-volatile storage. A hard disk is also non-volatile, for the same reason, but regular RAM is not.



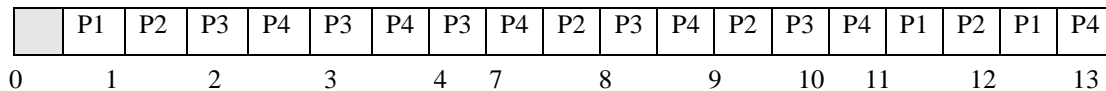
Security: The fact that ROM cannot easily be modified provides a measure of security against accidental (or malicious) changes to its contents. You are not going to find viruses infecting true ROMs, for example; it's just not possible. (It's technically possible with erasable EPROMs, though in practice never seen.)

Read-only memory is most commonly used to store system-level programs that we want to have available to the PC at all times. The most common example is the system BIOS program, which is stored in a ROM called (amazingly enough) the system BIOS ROM. Having this in a permanent ROM means it is available when the power is turned on so that the PC can use it to boot up the system. Remember that when you first turn on the PC the system memory is empty, so there has to be something for the PC to use when it starts up.

07(b)(ii).

Sol:

Gantt Chart



Process	Arrival time (AT)	Burst time (BT)	Completion time (CT)	Turn Around Time (TAT= CT – AT)	Waiting Time (WT = TAT – BT)
P1	1	2	18	17	15
P2	2	4	12	17	13
P3	3	6	20	17	11
P4	4	8	21	17	9

$$\text{Average Turn around time} = \frac{17+17+17+17}{4} = 17 \text{ msec}$$

$$\text{Average waiting time} = \frac{15+13+11+9}{4} = 12 \text{ msec}$$



07.(c)

Sol: (i). Operating Principle of Dynamometer type moving coil instrument.

The operating torque is produced by the reaction between the magnetic field of the fixed coils and the current through the moving coil. The torque is always positive regardless of the direction of the current, as with change in direction of the current in the moving coil the field of the fixed coils also changes its direction. To derive the expression for the torque we will consider the energy stored in the magnetic circuits. The total energy stored in the magnetic circuits is

$$W = \frac{i_1^2 L_1}{2} + \frac{i_2^2 L_2}{2} + i_1 i_2 M \dots\dots\dots (1)$$

If the moving system rotates through a small angle $d\theta$ and the corresponding change in energy is dW then the work done in moving the system must be equal to the change in energy dW , i.e.,

$$dW = T_d d\theta$$

Where T_d is the deflecting torque.

$$T_d = \frac{dW}{d\theta} \dots\dots\dots (2)$$

From Eq. (1)

$$\frac{dW}{d\theta} = i_1 i_2 \frac{dM}{d\theta}$$

Since L_1 and L_2 are independent of θ

$$\therefore T_d = I_1 I_2 \frac{dM}{d\theta} \dots\dots\dots (3)$$

In case of direct current

$$T_d = I_1 I_2 \frac{dM}{d\theta} \dots\dots\dots (4)$$

In case of ammeters and voltmeters

$$I_1 = I_2 = I$$

$$T_d = I^2 \frac{dM}{d\theta} \dots\dots\dots (5)$$

In case of steady-steady sinusoidal currents, if

$$i_1 = I_{m1} \sin \omega t$$

$$\text{and } i_2 = I_{m2} \sin(\omega t - \phi)$$



Then the average torque,

$$T_d = \frac{1}{\tau} \int_0^{\tau} i_1 i_2 \frac{dM}{d\theta} dt \quad \left(\because \tau = \frac{2\pi}{\omega} \right)$$

$$\begin{aligned} \text{(or)} \quad T_d &= \frac{dM}{d\theta} \frac{1}{\tau} \int_0^{\tau} I_{m1} I_{m2} \sin \omega t \sin(\omega t - \phi) dt \\ &= \frac{I_{m1} I_{m2}}{2} \frac{dM}{d\theta} \cos \phi \\ &= I_1 I_2 \frac{dM}{d\theta} \cos \phi \quad \dots\dots\dots (6) \end{aligned}$$

Where I_1 and I_2 are the rms values of currents.

In case of ammeters and voltmeters,

$$I_1 = I_2 = I \text{ and } \phi = 0$$

$$\therefore T_d = I^2 \frac{dM}{d\theta} \quad \dots\dots\dots (7)$$

By suitable design of coil sections and radii, it is practicable to obtain constancy of $\frac{dM}{d\theta}$ over the usual working range.

(ii) **Dynamometer voltmeters**

The dynamometer voltmeter is the most accurate form of voltmeter for measuring alternating voltages of moderate magnitude (about 50 – 500 V) at power frequency. The current through the moving coil is led by the ordinary control springs, since the current does not exceed 75 mA. The fixed and moving coils are connected in series with a high non-inductive resistance as shown in fig.

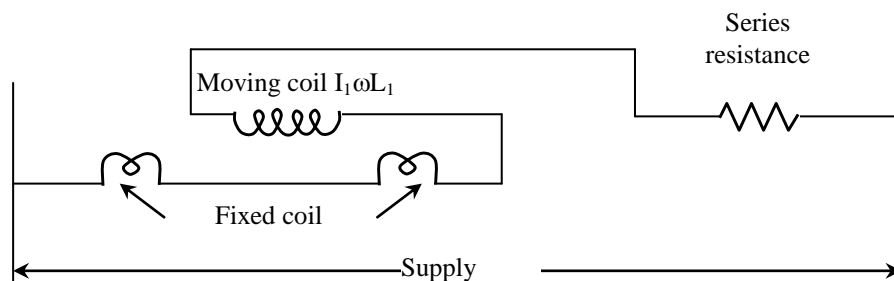


Fig. 1: Dynamometer voltmeter



The deflecting torque, $T_d = I^2 \frac{dM}{d\theta}$

But $\frac{dM}{d\theta}$ is constant over the usual working range, hence

$$T_d \propto K_1 I^2$$

If Z is the total impedance of the circuit shown in fig.

$$T_d = K_1 \frac{V^2}{Z^2} \quad \dots\dots\dots (8)$$

Where V is the voltage to be measured.

As the control is by springs,

$$S\theta = K_1 \frac{V^2}{Z^2}$$

$$\theta = \frac{K_1 V^2}{SZ^2}$$

$$\theta = K_2 \frac{V^2}{Z^2} \quad \left(K_2 = \frac{K_1}{S} \right) \quad \dots\dots\dots (9)$$

$$= KV^2 \quad \dots\dots\dots (10)$$

Dynamometer Ammeters

For low range ammeters (about 0.2 A) the moving and fixed coils are connected in series as shown in fig. 2. The current carrying capacity of the control springs limits the use of ammeters of range higher than 0.2 A without a shunt. The arrangement of a higher range ammeter is shown in fig. 3.

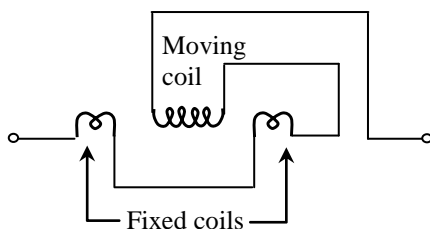


Fig. 2: Low range dynamometer ammeter

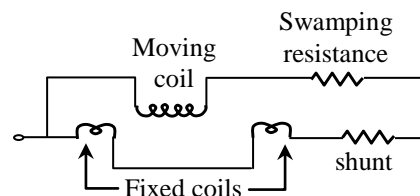


Fig. 3: High range dynamometer ammeter



In first case, as shown in fig. 2, the current I_1 and I_2 are same both I magnitude and phase. For the second arrangement (fig. 3) to satisfy the condition $\phi = 0$, it is essential that the time constants L/R of two parallel paths are equal,

For first case $I_1 = I_2 = I$ and $\phi = 0$

$$\therefore T_d = I^2 \frac{dM}{d\theta}$$

$$\text{And} \quad \theta = K^2 I^2 \quad \dots\dots\dots (11)$$

$$\text{Where,} \quad K' = \frac{dM/d\theta}{S}$$

08(a)(i)

$$\text{Sol: } L_p = \sqrt{D_p \tau_p} = \sqrt{100 \times 3 \times 10^{-7}} = 5.477 \times 10^{-3} \text{ cm} = 54.77 \mu\text{m}$$

Therefore, the common emitter current gain is

$$\beta_0 = \frac{2L_p^2}{W^2} = \frac{2(54.77 \times 10^{-4})^2}{(2 \times 10^{-4})^2} = 1500$$

08(a)(ii)

$$\text{Sol: } \tau_p = \tau_n = 10^{-6}, D_n = 21 \text{ cm}^2/\text{sec and } D_p = 10 \text{ cm}^2/\text{sec}$$

The saturation current calculation

$$\text{We know, } L_p = \sqrt{D_p \tau_p}$$

$$\begin{aligned} J_s &= \frac{qD_p P_{n0}}{L_p} + \frac{qD_n n_{p0}}{L_n} = qn_i^2 \left(\frac{1}{N_D} \sqrt{\frac{D_p}{\tau_{p0}}} + \frac{1}{N_A} \sqrt{\frac{D_n}{\tau_{n0}}} \right) \\ &= 1.6 \times 10^{-19} \times (9.65 \times 10^9)^2 \left(\frac{1}{10^{18}} \sqrt{\frac{10}{10^{-6}}} + \frac{1}{10^{16}} \sqrt{\frac{21}{10^{-6}}} \right) \\ &= 6.87 \times 10^{-12} \text{ A/cm}^2 \end{aligned}$$

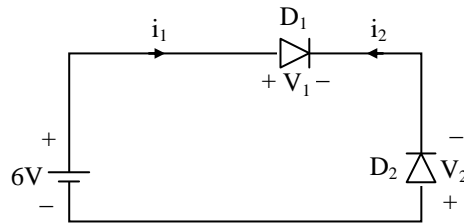
Given that the cross-sectional area $A = 1.2 \times 10^{-5} \text{ cm}^2$, we obtain

$$I_s = A \times J_s = 1.2 \times 10^{-5} \times 6.87 \times 10^{-12} = 8.244 \times 10^{-17} \text{ A}$$



08(a)(iii).

Sol:



Analysis:

By looking at the circuit, we can simply say that diode 'D₁' is forward bias while 'D₂' is in reverse – biased.

Current through a forward biased diode $I_{D1} = I_{S1} (e^{V_1/\eta V_T} - 1)$

Current through a reverse biased diode, $I_{D2} = I_{S2}$

So, 20nA (which is saturation current of D₂) will flow through the circuit

$$\Rightarrow 20\text{nA} = I_{D1} = i_1$$

$$\Rightarrow -20\text{nA} = I_{D2} = i_2$$

From this ,

$$i_1 = I_{D1} = I_{S1} (e^{V_1/\eta V_T} - 1) = 20\text{nA}$$

$$\Rightarrow 20\text{nA} = 1\text{nA} (e^{V_1/2V_T} - 1)$$

$$\Rightarrow V_1 = 2V_T \ln (21)$$

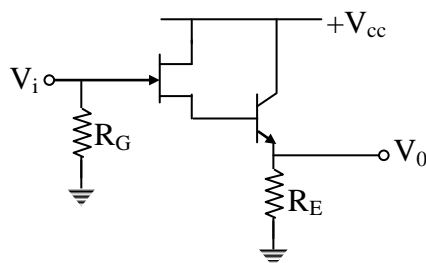
$$= 2 \times 0.025 \times \ln (21) = 0.152\text{V}$$

$$\Rightarrow 6 + V_2 = V_1$$

$$V_2 = -5.848 \text{ V}$$

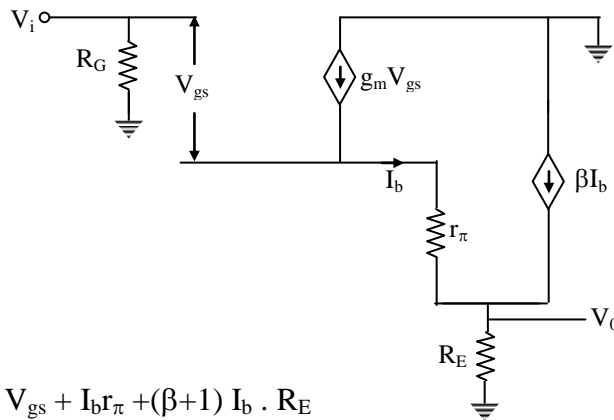
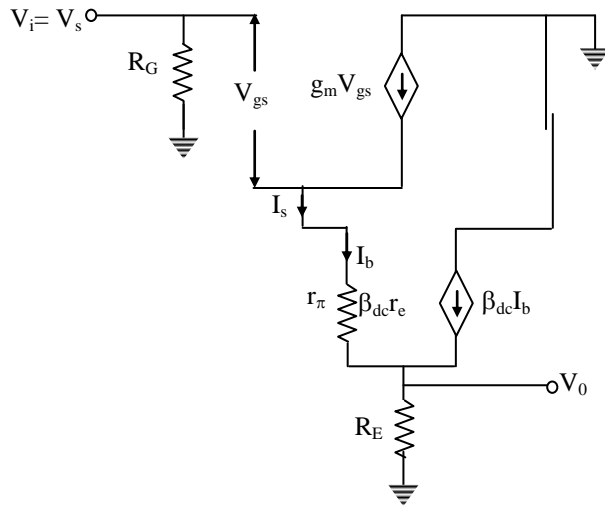
8(b)(i).

Sol:





It is multistage cascading amplifier with source follower to emitter follower.



$$V_i = V_{gs} + I_b r_{\pi} + (\beta + 1) I_b \cdot R_E$$

$$V_i = V_{gs} + g_m V_{gs} r_{\pi} + (\beta + 1) g_m V_{gs} R_E$$

$$[\because I_b = g_m V_{gs}]$$

$$V_i = V_{gs} [1 + g_m r_{\pi} + (\beta + 1) g_m R_E] \dots\dots\dots (1)$$

$$V_o = (\beta + 1) I_b \cdot R_E$$

$$= (\beta + 1) g_m V_{gs} \cdot R_E$$

$$\frac{V_o}{V_i} = \frac{(\beta + 1) g_m V_{gs} \cdot R_E}{V_{gs} [1 + g_m r_{\pi} + (\beta + 1) g_m R_E]}$$

$$A_v = \frac{(\beta + 1) g_m R_E}{[1 + g_m r_{\pi} + (\beta + 1) g_m R_E]}$$

$$\text{Output resistance } R_o = R_E \parallel \left[\frac{V_o}{I_o} \right]$$



[$\therefore V_i$ is short circuited] [$\therefore I_b = g_m V_{gs}$]

$$I_0 = (\beta + 1)I_b = (\beta + 1)g_m V_{gs}$$

$$V_0 = (1 + g_m r_\pi)V_{gs}$$

$$\frac{V_0}{I_0} = \frac{1 + g_m r_\pi}{(\beta + 1)g_m}$$

$$\begin{aligned} \therefore \text{output resistance } R_0 &= R_E \parallel \left(\frac{V_0}{I_0} \right) \\ &= R_E \parallel \left[\frac{1 + g_m r_\pi}{(\beta + 1)g_m} \right] \end{aligned}$$

08(b)(ii).

Sol: When $I_L = 0$ A, I_z is maximum

$$= \frac{V_{in} - V_z}{R} = 25.5 \text{ mA}$$

If R_L is removed from the circuit, then $I_L = 0$ A since, $I_{z\max} < I_{zm}$, therefore, $I_L = 0$ A is acceptable for minimum. i.e $I_{\min} = 0$ Amp

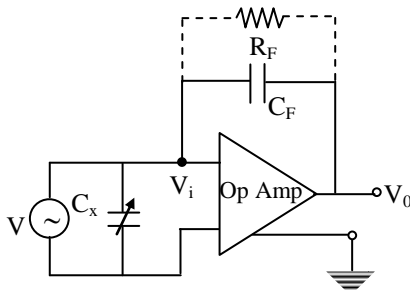
I_L maximum occurs for $I_z = I_{zk}$ and given as $I_{L(\max)} = I - I_{zk} = 25.5 - 1 = 24.5$ mA

$$R_{(\min)} \text{ for regulation} = \frac{12V}{24.5 \text{ mA}} = 490 \Omega$$

8.(c)(i)

Sol: Charge Amplifier

In order to improve the low frequency response of the piezoelectric transducer we require a charged amplifier.





We know that

$$\frac{V_0}{V_i} = -\frac{C_x}{C_F} ; V_0 = \left(\frac{-C_x}{C_F} \right) \cdot V_i \text{ -----(1)}$$

$$\text{We know that } V_i = \frac{q}{C_x} \text{ -----(2)}$$

By putting the value of equation-2 in equation-1

$$\text{we get } V_0 = -\frac{C_x}{C_F} \times \frac{q}{C_x} = \frac{-q}{C_F} \text{ -----(3)}$$

$$V_0 = -\frac{q}{C_f} = \frac{K_q \cdot X_i}{C_F}$$

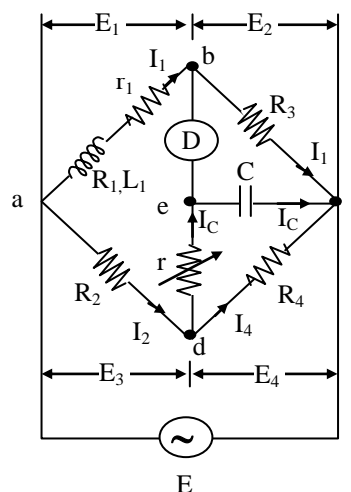
Equation-3 indicates output voltage V_0 is linearly related to input displacement x_i . It also indicates that the output changes instantaneously with input without loss in the steady state response. But in practice, it is not so, this is because op-Amp doesn't have an ' ∞ ' input resistance though very high. The input resistance and the leakage resistance of CF, exhibit a steady changing of CF by the leakage current till the amplifier is saturated. In order to overcome this problem, a resistance R_F is connected across capacitor C_F in the feedback path. This presents a small leakage current to charge the capacitor heavily. The connection of R_F is shown as dotted line.

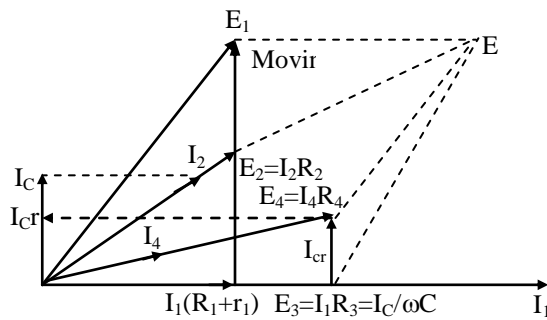
8(c)(ii)

Sol: This bridge, in fact, is a modification of the Maxwell's inductance capacitance bridge. In this method, the self inductance is measured in terms of a standard capacitor.

This method is applicable for precise measurement of self inductance over a very wide range of values.

Figure below shows the connections and the phasor diagram of the bridge for balanced conditions.





L_1 = self inductance to be measured

R_1 = resistance of self inductor,

r_1 = resistance connected in series with self inductor

$$r, R_2, R_3, R_4 = \text{known non inductance resistances,}$$

and C = fixed standard capacitor.

At balance, $I_1 = I_3$ and $I_2 = I_c + I_4$

$$\text{Now, } I_1 R_3 = I_C \times \frac{1}{j\omega C}$$

$$\therefore I_C = I_1 j\omega C R_3.$$

Writing the other balance equations

$$I_1(r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_C r$$

$$\text{And } I_C \left(r + \frac{1}{j\omega C} \right) = (I_2 - I_C) R_4$$

Substituting the value of I_C in the above equations, we have

$$I_1(r_1 + R_1 + j\omega I_1) = I_2 R_2 + I_1 j\omega C R_3 r$$

Or $I_1(r + R_1 + j\omega L_1 - j\omega CR_3r) = I_2R_2$

$$\text{And } j\omega CR_3 I_1 \left(r + \frac{1}{j\omega C} \right) = (I_2 - I_1 j\omega CR_3) R_4$$

$$\text{Or } I_1(j\omega CR_3r + j\omega CR_3R_4 + R_3) = I_2R_4$$



From equations (i) and (ii) we obtain

$$I_1(r_1 + R_1 + j\omega L_1 - j\omega CR_3r) \\ = I_1 \left(\frac{R_2 R_3}{R_4} + \frac{j\omega CR_2 R_3 r}{R_4} + j\omega CR_3 R_2 \right)$$

Equating the real and the imaginary parts,

$$R_1 = \frac{R_2 R_3}{R_4} - r_1 \text{ and } L_1 = C \frac{R_3}{R_4} [r(R_4 + R_2) + R_2 R_4]$$

An examination of balance equations reveals that to obtain easy convergence of balance alternate adjustments of r_1 and r should be done as they appear in only one of the two balance equations.

Advantages:

1. In case adjustments are carried out by manipulating control over r_1 and r , they become independent of each other. This is a marked superiority over sliding balance conditions met with low Q coils when measuring with Maxwell's bridge. A study of convergence conditions would reveal that it is much easier to obtain balance in the case of Anderson's bridge than in Maxwell's bridge for low Q-coils.
2. A fixed capacitor can be used instead of a variable capacitor as in the case of Maxwell's bridge
3. This bridge may be used for accurate determination of capacitance in terms of inductance.