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ESE – 2019 MAINS
OFFLINE TEST SERIES

ELECTRONICS & TELECOMMUNICATION ENGINEERING (E&T)

TEST – 12
SOLUTIONS

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01. (a)

Sol:

(i) If f represents the system failure and f_H and f_L represent the failure of the upper and lower paths respectively in the system, then:

$P(f) = P(f_H f_L) = P(f_H)\,P(f_L) = [P(f_H)]^2$

$P(f_H) = 1 - P(\bar{f}_H) = 1 - (1 - 0.01)^{10} = 0.0956$

$P(f) = (0.0956)^2 = 0.009143$

$P(\bar{f}) = 1 - P(f) = 0.9908$

(ii) $P(\bar{f}) = 0.999$

$P(f) = 1 - 0.999 = 0.001$

$P(f_H) = \sqrt{0.001} = 0.0316$

$P(\bar{f}_H) = 1 - 0.0316$

$= 0.9684$

01. (b)

Sol:

Category	IP(v4)	IP(v6)
1. IP Address	• 32 bits IP Address • Classfull or Classless IP address	• 128 bits IP address • Only Classless IP
2. Range Problem	• Overcome by using private network • NAT table is used	• No any concept of private network • No range problem
3. Routing	• More processing overhead at intermediate router	• Less processing overhead • Routing is flexible and fast relatively
4. Communication	• Unicast, Multicast and Broadcast	• Unicast, Multicast and any cast • Broadcasting is not allowed
5. Security	• No any IP security support	• IP security (Authentication) provided

01. (c)

Sol: Properties of ROC

The properties of ROC are as follows:

1. The shape of the ROC is strips parallel to the imaginary axis in s-plane.
2. The ROC does not contain any poles.
3. If x(t) is a right-sided signal, the ROC of X(s) extends to the right of the right most pole and no pole is located inside the ROC
4. If x(t) is a left-sided signal, the ROC of X(s) extends to the left of the left most pole and no pole is located inside the ROC.
5. If x(t) is a two-sided signal, the ROC of X(s) is a strip in the s-plane bounded by poles and no pole is located inside the ROC.
6. Impulse function is the only function for which the ROC is the entire s-plane.
7. The ROC must be a connected region.
8. The ROC of an LTI stable system contains the imaginary axis of s-plane
9. The ROC of the sum of two or more signals is equal to the intersection of the ROCs of those signals.

01. (d)

Sol: 1 rotation time = $\frac{60}{7200}$sec

$$= \frac{60*1000\,\text{msec}}{7200}$$

= 8.33 msec

Average rotational latency in half of 1 rotation time.

Average rotational latency $= \frac{8.33}{2} = 4.16\text{msec}$

Transfer time for 8KB data $= \frac{8\text{KB}}{4\text{MB}}\text{sec} = 2\text{msec}$

Total time = seek time + average rotational latency + transfer time + controller's overhead

= (12 + 4.16 + 2 + 2) msec

= 20.16 msec

01. (e)

Sol: $V_{max} = V_i[1 + |\Gamma_L|]$, $V_{min} = V_i[1 - |\Gamma_L|]$

$$|\Gamma_L| = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{110 - j80 - 50}{110 - j80 + 50} = \frac{60 - j80}{160 - j80}$$

$$= \frac{3 - j4}{8 - j4} = 0.559\angle -26.52^\circ$$

$|\Gamma_L| = 0.559$

The power transmitted at the source will be same every where since there are no line losses. This power at any point is given by

$P = V_{max}\, I_{min} = V_{min}\, I_{max}$

Since P = 25 Watts

$$25 = \left[V_i\left[1 + |\Gamma_L|\right]\right]\left[\frac{V_i}{Z_0}\left[1 - |\Gamma_L|\right]\right]$$

$$\Rightarrow 25 = \frac{V_i^2}{Z_0}\left[1 - |\Gamma_L|^2\right]$$

$$\therefore V_i^2 = \frac{25 \times 50}{1 - (0.559)^2} = \frac{1250}{0.6875} = 1818.13\,\text{V}$$

$\therefore V_i = 42.64$V

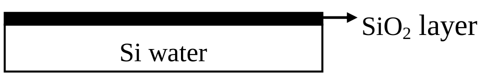
So, $V_{max} = 42.64[1 + 0.559] = 66.47$V

$V_{min} = 42.64[1 - 0.559] = 18.8$V

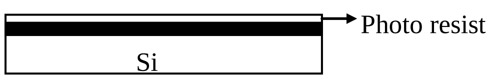
01. (f)

Sol: Photo lithography is the technology to create a pattern on the silicon water using an ultraviolet (UV) ray of light the steps involved in it are shown in below.

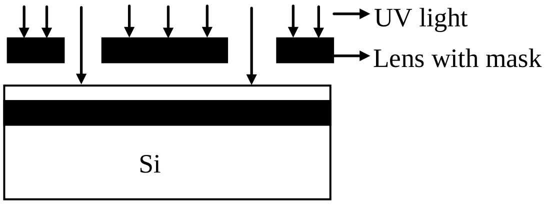
Step1



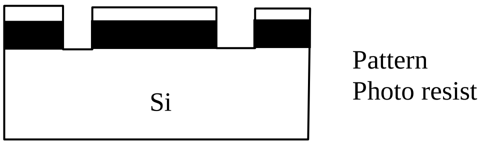
Step2



Step3



Step4



The components of photolithography process are as follows:
(1) Si water
(2) Photo resist a light sensitive material
(3) Lens
(4) Mask with the desired pattern
(5) UV light source
(6) Developer solution.

The water is first cleansed and SiO_2 layer is deposited on the surface of the water. The water is then coated with the photo resist on the top. Then the UV light is projected on the water through the mask and a lens. The mask has certain regions transparent and the other opaque. The transparent regions of the mask allow the UV light to pass through it and fall on the photo resist. Depending upon whether the photoresist is positive or negative it under go some chemical changes and becomes more soluble or less soluble in an etchant solution. A pattern is formed on the photo resist. For positive photoresist the pattern is same as the mask and for negative it is inverse of mask. Finally water is dipped into a developer solution. The soluble part of photo resist and the underneath part of the photoresist and the underneath SiO_2 layer are etched out. The photoresist is then stripped off and the replica of mask is formed on the SiO_2 layer.

02. (a)
Sol:
(i) Types of distortion is envelope detector output: There are two types of distortions:

(1) Diagonal clipping:

This type of distortion occurs when the RC time constant of load circuit is too long. Due to this, the RC circuit cannot follow the fast changes in the modulating envelope.

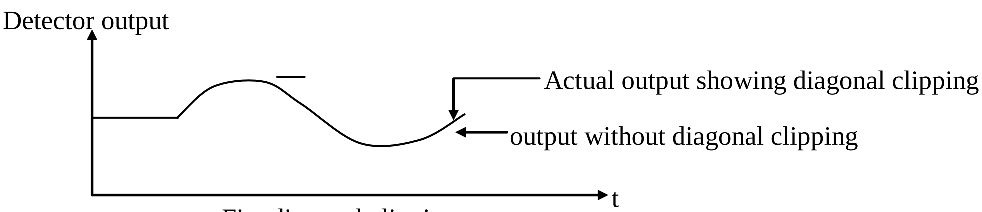


Fig: diagonal clipping

(2) Negative peak clipping:

This distortion occurs due to the fact that modulation index of output side of the detector is higher than that on its input side. Hence, at higher depths of modulation of the transmitted signed, the over modulation may take place at the output of detector negative peak clipping will take place as a result of this over modulation.

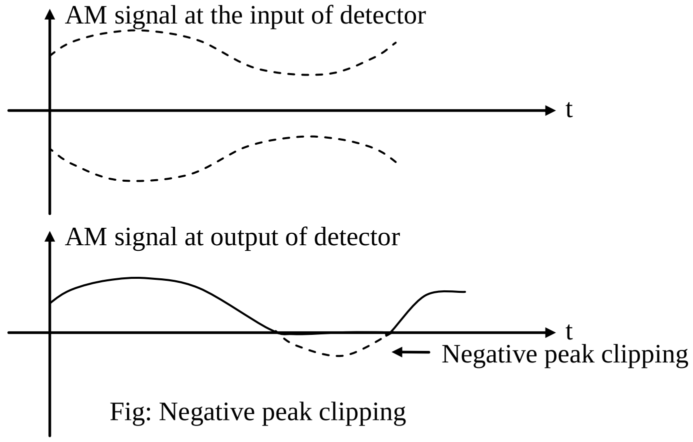


Fig: Negative peak clipping

The only way to reduce/eliminate the distortions is to choose RC time constants. The capacitor charges through D and R_s, when the diode is on and discharges through R when diode is off.

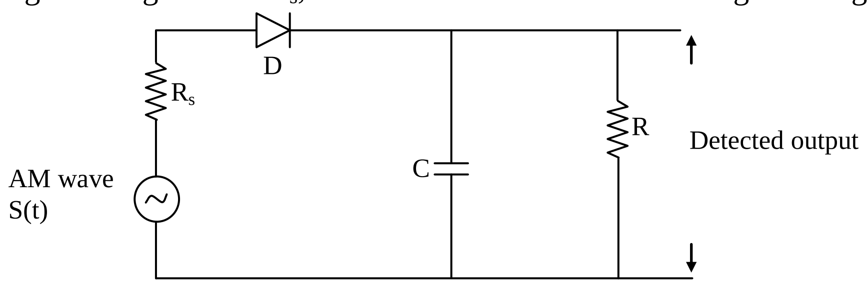


Fig: envelope detector for detection of AM wave.

The charging time constant R_sC should be short compared to the carrier period $1/f_c$ thus $R_sC << \frac{1}{f_c}$

On the other hand, discharging time constant RC should be long enough so that the capacitor discharges slowly through load resistance R. But this time constant shouldn't be too long which will not allow the capacitor voltage to discharge at the maximum rate of change of the envelope.

$$\therefore \frac{1}{fc} << RC << \frac{1}{W}$$

Where W = maximum modulating frequency.

(ii) **(A)** As f_m = 10 kHz

Given R = 50kΩ, C = 0.1 μF

For the detector output to follow envelope at all times & handle without distortion,

$$RC \le \frac{\sqrt{1-m^2}}{mw_m}$$

Where m = modulation index

$w_m = 2\pi f_m = 2 \times \pi \times 10\text{kHz}.$

$$m^2 w_m^2 (RC)^2 \le 1 - m^2$$

$$m \le \frac{1}{\sqrt{1+(w_m RC)^2}}$$

On substituting w_m, R. C. we get

$m \le 3.18 \times 10^{-3}$

$\therefore$ Maximum modulation index is 3.18×10^{-3}

(B) When f_m = 5 kHz

Similarly m ≤ 6.366 × 10^{-3}

∴ max modulation index = 6.366 × 10^{-3}

02. (b)

Sol: Put s = jω

$$G\,(j\omega) = \frac{K(j\omega)^3}{(j\omega+1)(j\omega+2)}$$

$$G\,(j\omega) = \frac{-jK\omega^3}{(2-\omega^2)+j3\omega}$$

$$G\,(j\omega) = \frac{-jK\omega^3[(2-\omega^3)-j3\omega]}{(2-\omega^2)^2+(3\omega)^2}$$

$$G(j\omega)\frac{-3K\omega^4}{(2-\omega^2)^2+(3\omega)^2}-\frac{jK\omega^3(2-\omega^2)}{(2-\omega^2)^2+(3\omega)^2}$$

The intersection of G (jω) plot with – ive real axis is obtained by equating imaginary part of G (jω) to zero and solving for ω. Therefore,

$$-\frac{K\omega^3(2-\omega^2)}{(2-\omega^2)^2+(3\omega)^2}=0$$

$$(2-\omega^2) = 0$$

The intersection with –ve real axis occurs at

ω = ± $\sqrt{2}$ rad/sec

The intersection is obtained by substituting ω = $\sqrt{2}$ in real part of G (jω), i.e.

$$G(j\sqrt{2} = \frac{-3K(\sqrt{2})^4}{(2-\sqrt{2^2})^2+(3\sqrt{2})^2} = \frac{-3K\times2\times2}{(2-2)^2+3\times3\times2}$$

$$G\,(j\sqrt{2}) = -\frac{2}{3}K$$

$$|G(j\omega)| = \frac{K\omega^3}{\sqrt{\omega^2+1}\sqrt{\omega^2+2^2}}$$

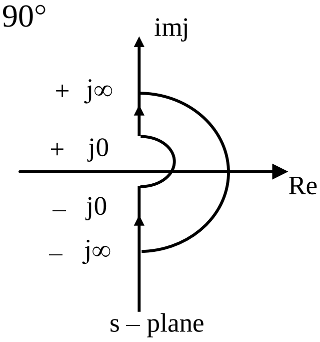
$$\angle G\,(j\omega) = 270° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$

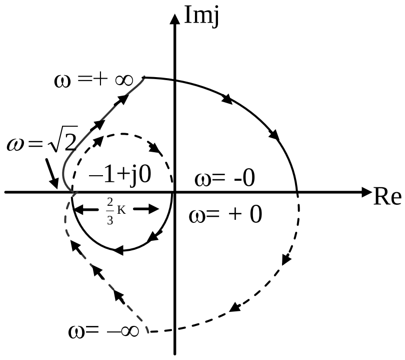
As $\omega \to 0 \Rightarrow |G(j\omega)| \to 0$

$\angle$ G (jω) → 270°

As $\omega \to \infty \Rightarrow |G\,(j\omega)| \to \infty$

$\angle$ G (jω)→ 90°





The completed Nyquist plot is shown in Figure

It is given that, the number of poles of G(s) having positive real part is nil i.e. $P_+ = 0$.

The encirclements of critical point (–1 + j0) are determined below.

(1) If $K < \frac{3}{2}$

The critical point (–1 + j0) lies outside the Nyquist plot, hence N =0

$\therefore\ N = P_+ - Z_+$

$0 = 0 - Z_+$

$\therefore\ Z_+ = \text{Nil}$

The system is stable

(2) If $K > \frac{3}{2}$

The critical point (–1 + j0) will be encircled twice in the clockwise direction by the Nyquist plot, hence

N = –2

$\therefore\ N = P_+ - Z_+$

$-2 = 0 - Z_+$

$\therefore\ Z_+ = 2$

The system is unstable.

For stability $K \le \frac{3}{2}$

02. (c)

Sol:

(i) Start

read a, b, c

if a = 0 and b = 0 then

write "illegal equation, cannot solve"

else

if a = 0 then

Set root to −c/b

Write "linear equation, the one root is", root

else

set discriminant to $b^2 - 4ac$

if discriminant < 0 then

Write "roots are complex, cannot solve"

else

set root -1 to $\frac{-b+\sqrt{\text{discriminant}}}{2a}$

set root -2 to $\frac{-b-\sqrt{discriminant}}{2a}$

write "answers are ", root-1, root-2

stop

END OF THE ALGORITHM

(ii) **function** power (x:real; n:integer):real;
{compute x raised to the power n recursively the function always returns 0 for x = 0}

```
begin
    if x = 0.0 then
    power: = 0.0
  else
    if n = 0 then
    power : = 1.0
  else
    if n < 0 then
    power : = power(x,n+1)/ x
  else
    power : = power(x,n–1)* x
  end; { of power}
```

03. (a)
Sol:
(i)

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} r^n \frac{\sin((n+1)\omega)}{\sin\omega} z^{-n}$$

$$= \sum_{n=0}^{\infty} \frac{r^n}{2j\sin\omega}\left[e^{j(n+1)\omega} - e^{-j(n+1)\omega}\right] z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(rz^{-1}e^{j\omega}\right)^n \frac{e^{j\omega}}{2j\sin\omega} - \sum_{n=0}^{\infty} \left(rz^{-1}e^{-j\omega}\right)^n \frac{e^{-j\omega}}{2j\sin\omega}$$

$$X(Z) = \frac{1}{2j\sin\omega}\left[\frac{e^{j\omega}}{1-rz^{-1}e^{j\omega}} - \frac{e^{-j\omega}}{1-rz^{-1}e^{-j\omega}}\right] \qquad |z| > |r|$$

$$= \frac{1}{\left(1-rz^{-1}e^{j\omega}\right)\left(1-rz^{-1}e^{-j\omega}\right)}$$

$$= \frac{1}{1-2rz^{-1}\cos\omega + r^2z^{-2}}$$

$$X(Z) = \frac{z^2}{z^2 - 2r\cos\omega z + r^2}$$

(ii) x(2n) = f(n) + g(n)
x(2n+1) = f(n) – g(n)

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk} = \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2Nk} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{(2n+1)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} [f(n)+g(n)] W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} [f(n)-g(n)] W_N^{(2n+1)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} f(n)\left[W_N^{2nk} + W_N^{(2n+1)k}\right] + \sum_{n=0}^{\frac{N}{2}-1} g(n)\left[W_N^{2nk} - W_N^{(2n+1)k}\right]$$

$$= \left(1+W_N^k\right)\sum_{n=0}^{\frac{N}{2}-1} f(n)W_N^{2nk} + \left(1-W_N^k\right)\sum_{n=0}^{\frac{N}{2}-1} g(n)W_N^{2nk}$$

$$= \left(1+W_N^k\right)\sum_{n=0}^{\frac{N}{2}-1} f(n)W_{\frac{N}{2}}^{nk} + \left(1-W_N^k\right)\sum_{n=0}^{\frac{N}{2}-1} g(n)W_{\frac{N}{2}}^{nk}$$

$$X(k) = \left(1+W_N^k\right)F(k) + \left(1-W_N^k\right)G(k)$$

03. (b)

Sol: Attenuation constant $\alpha = \underbrace{\alpha_d}_{\text{Due to dielectric losses}} + \underbrace{\alpha_c}_{\text{Due to conductor losses}}$

$$\alpha_d = \frac{\sigma_d \eta_d}{2\sqrt{1-\left(\frac{f_{c_{10}}}{f}\right)^2}} \qquad \text{where } \eta_d = \frac{120\pi}{\sqrt{2.6}}$$

$$\eta_d = 233.79\Omega$$

$$\therefore \alpha_d = \frac{10^{-15}\times 233.79}{2\sqrt{1-\left(\frac{2.21\times10^9}{9\times10^9}\right)^2}} \qquad f_{c_{10}} = \frac{c}{2a\sqrt{\epsilon_r}} = \frac{3\times10^{10}}{2\times4.2\times\sqrt{2.6}}$$

$= 1.206 \times 10^{-13}$ NP/m. $\qquad f_{c_{10}} = 2.21\text{GHz}.$

$$\alpha_c = \frac{R_s\left[\frac{2b}{a}\left(\frac{f_{c_{10}}}{f}\right)^2 + 1\right]}{b\eta_d\left[1-\left(\frac{f_{c_{10}}}{f}\right)^2\right]^{1/2}}$$

$$R_s = \sqrt{\frac{\pi f \mu}{\sigma_c}} = \sqrt{\frac{\pi\times9\times10^9\times4.\pi\times10^{-7}}{1.1\times10^7}} = 0.0568\Omega$$

$$\therefore \alpha_c = \frac{0.0568\left[\frac{2\times1.5\times10^{-2}}{4.2\times10^{-2}}\left[\left(\frac{2.21\times10^9}{9\times10^9}\right)^2\right]+1\right]}{\left(1.5\times10^{-2}\right)(233.79)\sqrt{1-\left(\frac{2.21\times10^9}{9\times10^9}\right)^2}}$$

$$= \frac{0.05924}{3.399} = 0.01742\,\text{NP/m}$$

$$\therefore \alpha_c = 1.742\times10^{-2}\,\text{NP/m}$$

$\therefore \alpha = \alpha_c + \alpha_d = 1.742 \times 10^{-2} + \underbrace{1.206 \times 10^{-13}}_{\text{Neglect this term}}$

$\therefore$ $\alpha = 1.742\times10^{-2}$ NP/m

Over 40cm, the attenuation (or) loss will be

$\alpha \times 40\text{cm} = 1.742 \times 10^{-2} \times 40 \times 10^{-2}$

$= 69.68 \times 10^{-4}$ NP.

$\therefore$ Attenuation (or) loss in decibels $= 69.68 \times 10^{-4} \times 8.68$

$= 604.8 \times 10^{-4}$

$= 60.48 \times 10^{-3}$ dB

03. (c)

Sol:

(i) Given $n_1 = 1.5$

$\Delta = 3\ \% = 0.03$

$\lambda = 0.82\ \mu m$

We know that

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$$n_1^2 - n_2^2 = 2.n_1^2\Delta$$

$$= 2\times(1.5)^2\times0.03$$

$$n_1^2 - n_2^2 = 0.135$$

The critical Radius of curvature for the multimode fiber

$$R_c = \frac{3n_1^2\lambda}{4\pi\left(n_1^2 - n_2^2\right)^{1/2}} = \frac{3\times(1.5)^2\times0.82\times10^{-6}}{4\pi\times(0.135)^{1/2}}$$

$R_c = 9\ \mu m$

(ii) Given that single mode fiber

$\lambda = 1.55\ \mu m$

$n_1 = 1.5$

$\Delta = 0.3\ \% = 0.03$

$$n_1^2 - n_2^2 = 2\Delta n_1^2$$

$$= 2\times0.003\times2.25$$

$$= 0.043$$

Cut off wavelength for a single - mode fiber is

$$\lambda_c = \frac{2\pi a n_1\sqrt{2\Delta}}{2.405}$$

$$\lambda_c = \frac{2\pi\times40^{-6}\times1.5\sqrt{(0.06)}}{2.405}$$

$\lambda_c = 1.214\mu m$

Critical radius of curvature for a single - mode fiber

$$R_{cs} = \frac{20\lambda}{(n_1^2 - n_2^2)^{1/2}}\left(2.748 - 0.996 \times \frac{\lambda}{\lambda c}\right)^{-3}$$

$$= \frac{20 \times 1.55 \times 10^{-6}}{(0.043)^{1/2}}\left(2.748 - \frac{0.996 \times 1.55 \times 10^{-6}}{1.214 \times 10^{-6}}\right)^{-3}$$

$R_{cs} = 34$mm

04. (a)

Sol:

(i) **(A)** The delay difference

$$\delta T_s = \frac{Ln_1\Delta}{c}$$

$$= \frac{6 \times 10^3 \times 1.5 \times 0.01}{2.998 \times 10^8}$$

$\delta T_s = 300$ns

(B) The RMS pulse broadening due to inter model dispersion may be obtained by

$$\sigma_s = \frac{Ln_1\Delta}{2\sqrt{3}C}$$

$$= \frac{1}{2\sqrt{3}} \times \frac{6 \times 10^3 \times 1.5 \times 0.01}{2.998 \times 10^8}$$

$\sigma_s = 86.7$ns

(C) Maximum bit rate

$$B_T = \frac{1}{2T}$$

$$= \frac{1}{2\delta T_s}$$

$$= \frac{1}{600 \times 10^{-9}}$$

$B_T = 1.7$Mbps

(or)

$$B_T = \frac{0.2}{\sigma_s} = \frac{0.2}{86.7 \times 10^{-9}}$$

$B_T = 2.3$Mbps

(D) The most accurate estimate of the maximum bit rate is

$B_T \times L = 2.3\text{M} \times 6\text{km}$

$= 13.8$MHz - km

(ii) For down link

$$\left(\frac{C}{N_o}\right) = \left(\frac{E_b}{N_o}\right) + 10\log M + 10\log R$$

Where M = margin

R = bit rate

$10\log_{10}M = 85 - 10 - 10\log(10^7)$

$10\log_{10}M = 5$ dB

We also know that

$$\left(\frac{C}{N_o}\right)_{dB} = EIRP + \frac{G_r}{T} - P_L - M - 10\log k$$

$P_L = 92.4 + 20\log_{10}(12.5) + 20\log_{10}(40,000)$

$P_L = -206$ dB

$10\log_{10}(k) = 10\log(1.38\times10^{-23})$

$= -228.6$ dB

$$\left(\frac{G_r}{T}\right)_{dB} = 85 - 57 + 206 - 228.6 + 5$$

$$\left(\frac{G_r}{T}\right)_{dB} = 10.4\text{dB}$$

$G_r = 10.4 + 10\log_{10}(310)$ $\quad (\because T = 310k)$

$G_r = 3396.25$

For dish antenna

$$G = \eta\pi^2\left(\frac{D}{\lambda}\right)^2$$

$$3396.25 = 0.55\pi^2\left(\frac{D}{\lambda}\right)^2 \quad (\because \lambda = \frac{3}{125})$$

$$\frac{D}{\lambda} = 25$$

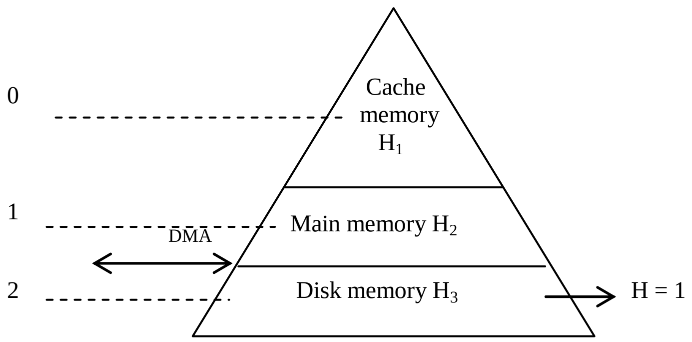
$$d = 25\times\frac{3}{125} = \frac{3}{5}$$

D = 0.6m

04. (b)

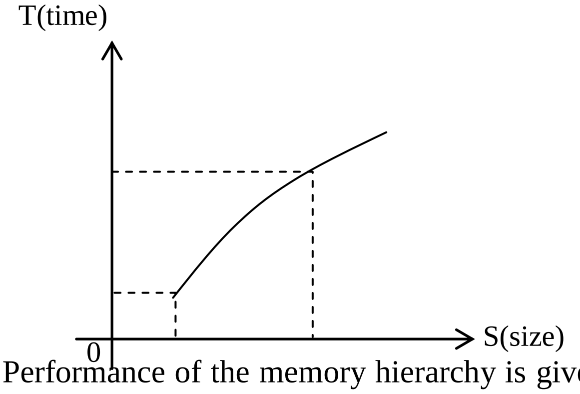
Sol: Objective: To reduce average access time by reducing the overall system cost

The structure of memory hierarchy has different memories of different capacities are organized such that i^{th} level memory is placed above $(i+1)^{th}$ level memory.



- $T_i < T_{i+1}$ (access time)
- $S_i < S_{i+1}$ (size)
- $C_i > C_{i+1}$ (cost per bit)
- $f_i > f_{i+1}$ (frequency of accessing)
- $I_i \subset I_{i+1}$ (information)
- $T\uparrow$, $S\uparrow$, $C\downarrow$, $f\downarrow$

Relation between size and access time



- Performance of the memory hierarchy is given by hit ratio (availability of referred information at referred palace is called as hit)
- The hit ratio of bottom most memory in the hierarchy is always one.

  $H_3 > H_2 > H_1$
- The hit ratio interm is directly proportional to the size and inversely proportional to average access time.

  $$H \propto S \;\&\; H \propto \frac{1}{T_{avg}}$$
- The side effect of memory hierarchy is the "data inconsistent" (same information is available differently at different places)
- The proper write operation reduces this problem.

Mathematical expressions for memory Hierarchy:-

Consider a 2 level memory with the following specifications

$T_1, S_1, C_1, H_1, \rightarrow$ level -1

$T_2, S_2, C_2, H_2, \rightarrow$ level -2

Total size $S_T = S_1 + S_2$

$$\text{Average cost } C_{avg} = \frac{\text{Total cost}}{\text{Total size}}$$

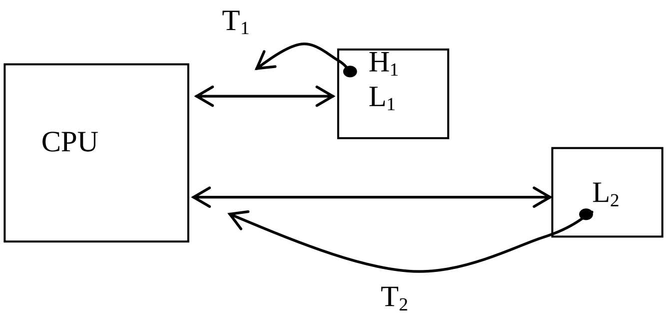
$$= \frac{S_1C_1 + S_2C_2}{S_T}$$

Average access time

$T_{avg} = H_1T_1 + (1 - H_1)T_2$

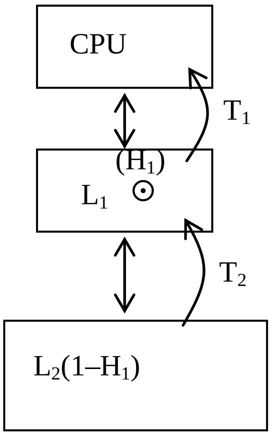
In three level mem ory system,

$T_{avg} = H_1C_1 + (1 - H_1)H_2C_2 + (1 - H_1)(1 - H_2)M$



CPU refers data if it is available in L_1 cache, otherwise goes to level 2.

*[level 2 size should be more than L_1]



$T_{avg} = H_1T_1 + (1 – H_1) (T_2 + T_1)$

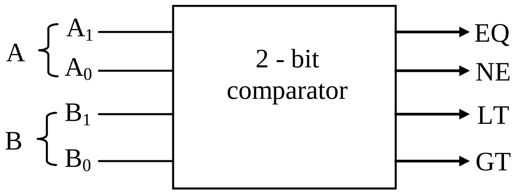
- If the present word is not available in level 1 then takes from level 2 as group of words.
- To reduce the future penalty due to the miss reference from level 1, block of words are moved from level2 to level1 for the current miss.

$T_{avg} = H_1T_1 + (1 – H_1) (T_B + T_1)$

$T_B = N * T_2$

04. (c)

Sol:



Truth table:

A_1	A_0	B_1	B_0	EQ	NE	LT(A<B)	GT(A>B)
0	0	0	0	1	0	0	0
0	0	0	1	0	1	1	0
0	0	1	0	0	1	1	0
0	0	1	1	0	1	1	0
0	1	0	0	0	1	0	1
0	1	0	1	1	0	0	0
0	1	1	0	0	1	1	0
0	1	1	1	0	1	1	0
1	0	0	0	0	1	0	1
1	0	0	1	0	1	0	1
1	0	1	0	1	0	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	1	0	1
1	1	0	1	0	1	0	1
1	1	1	0	0	1	0	1
1	1	1	1	1	0	0	0

K - map for EQ:

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00	1			
01		1		
11			1	
10				1

$$EQ = \overline{A}_1\overline{A}_0\overline{B}_1\overline{B}_0 + \overline{A}_1A_0\overline{B}_1B_0 + A_1A_0B_1B_0 + A_1\overline{A}_0B_1\overline{B}_0$$

K - map for NE:

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00		1	1	1
01	1		1	1
11	1	1		1
10	1	1	1	

$$NE = \overline{A}_0B_0 + A_0\overline{B}_0 + A_1\overline{B}_1 + \overline{A}_1B_1$$

K - map for GT:

$A_1A_0 \backslash B_1B_0$	00	01	11	10
00				
01	1			
11	1	1		1
10	1	1		

$$GT = \overline{A}_1\overline{B}_1 + A_0\overline{B}_1\overline{B}_0 + A_1A_0\overline{B}_0$$

K - map for LT

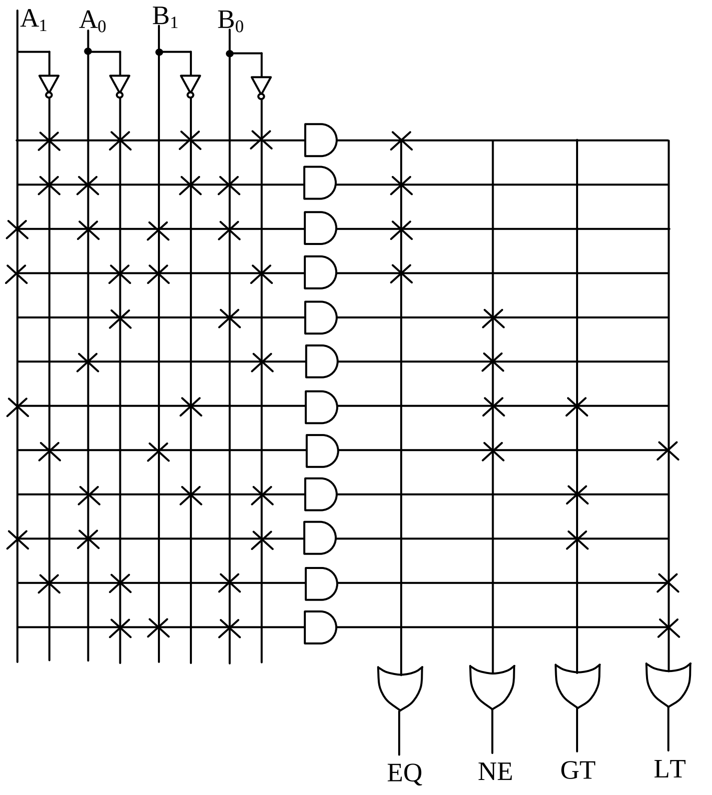
$A_1A_0 \backslash B_1B_0$	00	01	11	10
00		1	1	1
01			1	1
11				
10			1	

$$LT = \overline{A}_1B_1 + \overline{A}_1\overline{A}_0B_0 + \overline{A}_0B_1B_0$$

Common SOPs are $A_1\overline{B}_1, \overline{A}_1B_1$,

Logic diagram using PLA:

In PLA both AND and OR gates are programmable



05. (a)

Sol:

(i) Path loss exponent r = 6

First, let cluster size N = 7

Co-channel reuse ratio $(q) = \frac{D}{R} = \sqrt{3N} = 4.583$

$$\frac{C}{I} = \frac{1}{6(q)^{-r}}$$

$$= \frac{1}{6}(4.583)^6$$

$$\frac{C}{I} = 1544.35$$

$$\left(\frac{C}{I}\right)_{dB} = 10\log(1544.35)$$

$$\left(\frac{C}{I}\right)_{dB} = 31.88\,dB > 6dB$$

Since this is greater than minimum required $\frac{C}{I}$, N = 7 can be used.

(ii) Path loss exponent r = 2

Let N = 7

$$\frac{C}{I} = \frac{1}{6(q)^{-r}}$$

$$= \frac{1}{6}(4.583)^2$$

$$= 3.5$$

$$\left(\frac{C}{I}\right)_{dB} = 5.44\,dB < 6\,dB$$

Which is less than minimum required $\frac{C}{I}$, hence we need to use larger N.

Next possible value of N = 12 for i = j = 2

$$q = \sqrt{3N} = \sqrt{3\times 12} = 6$$

$$\left(\frac{C}{I}\right) = \frac{1}{6(q)^{-r}}$$

$$= \frac{1}{6}\times(6)^{+2}$$

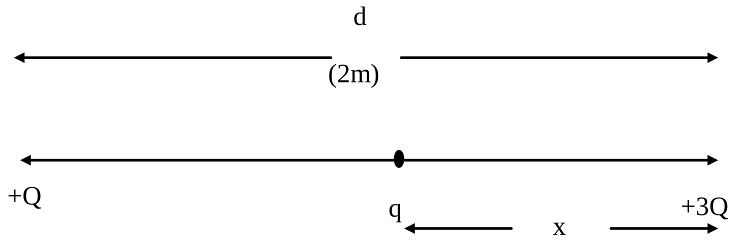
$$\frac{C}{I} = 6$$

$$\left(\frac{C}{I}\right)_{dB} = 7.78\,dB > 6\,dB$$

Since this is greater than minimum required $\frac{C}{I}$, N = 12 can be used.

05. (b)

Sol:



From the diagram the third charge 'q' is placed at a distance of 'x' from charge +3Q

For the system to be in equilibrium, q must be negative

$\therefore F_{12} = F_{23} = F_{13}$

$$\Rightarrow \frac{-Qq}{4\pi\varepsilon_o(d-x)^2} = \frac{-3Qq}{4\pi\varepsilon_o x^2} = \frac{3Q^2}{4\pi\varepsilon_o d^2} \text{ ——(1)}$$

That is, $3(d-x)^2 = x^2$

$\Rightarrow 3d^2 - 6dx + 3x^2 = x^2$

$\Rightarrow 2x^2 - 6dx + 3d^2 = 0$

$$x = \frac{6d \pm d\sqrt{12}}{4}$$

When d = 2, x = 4.73m or 1.268m

As $x<d$, x = 1.268m is considered.

Substitute x = 1.268 in equation (1)

$$\therefore \frac{-3Qq}{4\pi\varepsilon_o x^2} = \frac{3Q^2}{4\pi\varepsilon_o d^2}$$

$Q = -Q(x^2/d^2)$

$\therefore q = -0.4Q$

Location of third charge is 1.268m from +3Q towards +Q and the value of charge is – 0.4Q.

05. (c)

Sol:

(i) An address on a paging system is a logical page number and an offset. The physical page is found by searching a table based on the logical page number to produce a physical page number. Because the operating system controls the contents of this table, it can limit a process to accessing only those physical pages allocated to the process. There is no way for a process to refer to a page it does not own because the page will not be in the page table. To allow such access, an operating system simply needs to allow entries for non-process memory to be added to the process's page table. This is useful when two or more processes need to exchange data—they just read and write to the same physical addresses (which may be at varying logical addresses). This makes for very efficient inter process communication.

(ii) As such, C programming does not provide direct support for error handling but being a system programming language, it provides you access at lower level in the form of return values. Most of the C or even Unix function calls return –1 or NULL in case of any error and set an error code errno. It is set as a global variable and indicates an error occurred during any function call. You can find various error codes defined in <error.h> header file.

So a C programmer can check the returned values and can take appropriate action depending on the return value. It is a good practice, to set errno to 0 at the time of initializing a program. A value of 0 indicates that there is no error in the program.

The C programming language provides perror() and strerror() functions which can be used to display the text message associated with errno.

The perror() function displays the string you pass to it, followed by a colon, a space, and then the textual representation of the current errno value.

The strerror() function, which returns a pointer to the textual representation of the current errno value.

05. (d)

Sol: $\%M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$

$\therefore\ 26 = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$

$e^{+\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = \frac{100}{26}$

$\therefore\ \frac{\zeta\pi}{\sqrt{1-\zeta^2}} = 1.34$

$1-\zeta^2 = \left(\frac{\zeta\pi}{\sqrt{1.34}}\right)^2$

$1-\zeta^2 = 5.49\,\zeta^2$

$\zeta^2 = \frac{1}{1+5.49}$

$\therefore\ \zeta = \frac{1}{6.49} = 0.39$

Resonant frequency:

$\omega_r = \omega_n \sqrt{1-2\zeta^2}$

$8 = \omega_n \sqrt{1-2\zeta^2} = \omega_n \sqrt{1-2\times 0.39^2}$

$= \omega_n \sqrt{0.699}$

$\omega_n = \frac{8}{\sqrt{0.699}} = 9.6$ rad/sec

The overall transfer function is

$$M(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K}{s(ST+1)}}{1+\frac{K}{s(ST+1)}.1}$$

$$= \frac{K}{s^2T+s+K} = \frac{K/T}{s^2+\frac{1}{T}s+\frac{K}{T}}$$

The characteristic equation is

$s^2 + \frac{1}{T}s + \frac{K}{T} = 0$

On comparing above equation with $s^2 + 2\zeta\,\omega_n s + \omega_n^2 = 0$ (characteristic equation of a second order system)

$$2\zeta\,\omega_n = \frac{1}{T}$$

$$T = \frac{1}{2\zeta\omega_n} = \frac{1}{2\times 0.39\times 9.6} = 0.13$$

and $\omega_n^2 = \dfrac{K}{T}$

$$K = \omega_n^2 T = (9.6)^2\,(0.13) = 11.9$$

Resonant peak :

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$= \frac{1}{2\times 0.39\sqrt{1-0.39^2}} = 1.39$$

Gain crossover frequency:

$$\omega_1 = \omega_n \sqrt{\left(4\zeta^4+1\right)^{1/2} - 2\zeta^2}$$

$$= 8.69\sqrt{\left(4\times 0.39^4+1\right)^{1/2} - 2\times 0.39^2}$$

$$= 8.69\sqrt{0.74} = 7.47 \text{ rad/sec}$$

Phase margin $\phi = \tan^{-1}\left[\dfrac{2\zeta}{\sqrt{\left(4\zeta^4+1\right)^{1/2} - 2\zeta^2}}\right]$

$$= \tan^{-1}\left[\frac{2\times 0.39}{\sqrt{\left(4\times 0.39^4+1\right)^{1/2} - 2\times 0.39^2}}\right]$$

$$= \tan^{-1}(1.054)$$

$$= 46.5°$$

05. (e)

Sol: The bit duration is

$$T_b = \frac{1}{2.5\times 10^6\,\text{Hz}} = 0.4\mu s$$

The signal energy per bit is

$$E_b = \frac{1}{2}A_C^2 T_b$$

$$= \frac{1}{2}\left(10^{-6}\right)\times 0.4\times 10^{-6} = 2\times 10^{-13} \text{ joules}$$

(i) <u>Coherent Binary FSK</u>

The average probability of error is

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{E_b/2N_0}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\sqrt{2\times10^{-13}/4\times10^{-20}}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\sqrt{5\times10^{6}}\right)$$

Using the approximation

$$\text{erfc}(u) = \frac{\exp(-u^2)}{\sqrt{\pi}\,u`}$$

We obtain the result

$$P_e = \frac{1}{2}\frac{\exp(-5\times10^6)}{\sqrt{5\pi\times10^6}}$$

(ii) <u>MSK</u>

$$P_e = \text{er fc}\left(\sqrt{E_b/N_0}\right)$$

$$= \text{erfc}\left(\sqrt{10\times10^6}\right)$$

$$\approx \frac{\exp(-10\times10^6)}{\sqrt{10\pi\times10^6}}$$

(iii) <u>Non-coherent Binary FSK</u>

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$$

$$= \frac{1}{2}\exp(-5\times10^6)$$

06. (a)
Sol:

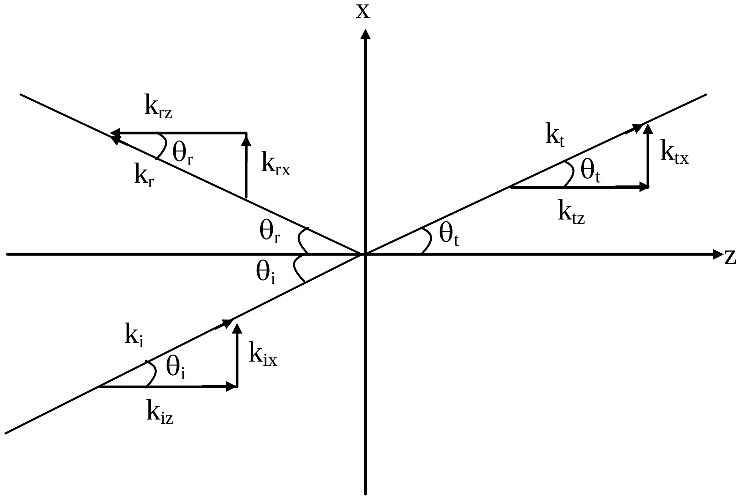
(i) From the incident $\vec{E}$ field, it is evident that the propagation vector is

$$\vec{k}_i = 4\hat{a}_x + 3\hat{a}_z \rightarrow k_i = 5 = \omega\sqrt{\mu_0\varepsilon_0} = \frac{\omega}{c}$$

Hence,
$\omega = 5c = 15 \times 10^8$ rad/s

A unit vector normal to the interface (z = 0) is $\hat{a}_z$. The plane containing $\vec{k}$ and $\hat{a}_z$ is y = constant which is xz-plane, the plane of incidence, since $\vec{E}_i$ is normal to this plane, so perpendicular polarization .

(ii) $\tan\theta_i = \dfrac{k_{ix}}{k_{iz}} = \dfrac{4}{3} \rightarrow \theta_i = 53.13^o$



Figure

(iii) Let

$$\vec{E}_r = E_{r0}\cos\left(\omega t - \vec{k}_r . \vec{r}\right)\hat{a}_y$$

where $\vec{k}_r = k_{rx}\hat{a}_x - k_{rz}\hat{a}_z$

$k_{rx} = k_r \sin\theta_r, \quad k_{rz} = k_r\cos\theta_r$

But $\theta_r = \theta_i$ and $k_r = k_i = 5$ because both k_r and k_i are in the same medium. Hence,

$\vec{k}_r = 4\hat{a}_x - 3\hat{a}_z$

From Snell's law

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i = \frac{c\sqrt{\mu_1\varepsilon_1}}{c\sqrt{\mu_2\varepsilon_2}}\sin\theta_i = \frac{\sin 53.13}{\sqrt{2.5}}$$

or

$\theta_t = 30.39^o$

$$\Gamma_\perp = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

where $\eta_1 = \eta_0 = 377\ \Omega$, $\eta_2 = \sqrt{\dfrac{\mu_0\mu_{r2}}{\varepsilon_0\varepsilon_{r2}}} = \dfrac{377}{\sqrt{2.5}} = 238.4\Omega$

$$\Gamma_\perp = \frac{238.4\cos 53.13^o - 377\cos 30.39^o}{238.4\cos 53.13^o + 377\cos 30.39^o} = -0.389$$

Hence, $E_{r0} = \Gamma_\perp E_{i0} = -0.389(8) = -3.112$

and $\vec{E}_r = -3.112\cos\left(15\times 10^8 t - 4x + 3z\right)\hat{a}_y\ V/m$

(iv) Similarly, let the transmitted electric field be

$$\vec{E}_t = E_{t0}\cos\left(\omega t - \vec{k}_t . \vec{r}\right)\hat{a}_y$$

where

$$k_t = \beta_2 = \omega\sqrt{\mu_2\varepsilon_2} = \frac{\omega}{c}\sqrt{\mu_{r_2}\varepsilon_{r_2}}$$

$$= \frac{15\times10^8}{3\times10^8}\sqrt{1\times2.5} = 7.906$$

From Figure

$$k_{tx} = k_t \sin\theta_t = 4$$

$$k_{tz} = k_t \cos\theta_t = 6.819$$

or

$$\vec{k}_t = 4\hat{a}_x + 6.819\hat{a}_z$$

$$\tau_\perp = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$= \frac{2\times238.4\cos53.13^o}{238.4\cos53.13^o + 377\cos30.39^o} = 0.611$$

Hence

$E_{t0} = \tau_\perp \; E_{i0} = 0.611 \times 8 = 4.888$

$\vec{E}_t = 4.888\cos(15\times10^8 t - 4x - 6.819z)\,\hat{a}_y \text{ V/m}$

$$\vec{H}_i = \frac{1}{\mu_2\omega}\vec{k}_t\times\vec{E}_t = \frac{\hat{a}_{k_t}\times\vec{E}_t}{\eta_2}$$

$$= \frac{4\hat{a}_x + 6.819\hat{a}_z}{7.906(238.4)}\times4.888\hat{a}_y\cos\left(\omega t - \vec{k}\,.\,\vec{r}\right)$$

$$\vec{H}_t = (-17.69\hat{a}_x + 10.37\hat{a}_z)\cos(15\times10^8 t - 4x - 6.819z)\text{mA/m}$$

06. (b)

Sol:

(i) For T = 0 and K =1 determine the gain cross over frequency ω_1

$$\therefore\; G(j\omega) = \frac{10}{j\omega(j\omega+1)(j\omega+7)}$$

At the phase crossover frequency

$\omega = \omega_1$; $|G(j\omega_1)| = 1$.

$$\left|\frac{10}{j\omega_1(j\omega_1+1)(j\omega_1+7)}\right| = 1$$

$$\frac{10}{\omega_1\sqrt{(\omega_1^2+1^2)}\sqrt{\omega_1^2+7^2}} = 1$$

$$\frac{10}{\omega_1\sqrt{\omega_1^2+1}\sqrt{\omega_1^2+49}} = 1$$

By inspection ω_1 = 1 rad/sec

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\frac{\omega}{1} - \tan^{-1} - \frac{\omega}{7}$$

$\angle G(j\omega_1) = -90° - \tan^{-1}\omega_1 - \tan^{-1}\dfrac{\omega_1}{7}$

$\omega_1 = 1$ rad/ sec

$G(j\omega_1) = -90° - \tan^{-1}1 - \tan^{-1}\dfrac{1}{7}$

$= -90° - 45° - 8.13° = -141.13°$

Incorporating time delay element $e^{-j\omega T,}$ the condition for marginal stability is given below:

$\angle G(j\omega_1) + \angle e^{-j\omega T} = -180°$

$\therefore -141.13 - \dfrac{\omega_1 T \times 180}{\pi} = -180$

$\because \omega_1 = 1$ rad/sec

$\therefore -141.13 - \dfrac{1 \times T \times 180}{\pi} = -180$

$\therefore T = \dfrac{(180 - 141.13)\pi}{1 \times 180} = 0.678 \text{ sec}$

(ii) T = 1 sec

$G(s) = \dfrac{10Ke^{-s.1}}{s(s+1)(s+7)}$

Put $s = j\omega$

$\therefore G(j\omega) = \dfrac{10Ke^{-j\omega}}{j\omega(j\omega+1)(j\omega+7)}$

$\angle G(j\omega) = -\dfrac{\omega \times 180}{\pi} - 90° - \tan^{-1}\dfrac{\omega}{1} - \tan^{-1}\dfrac{\omega}{7}$

$= -\dfrac{\omega \times 180}{\pi} - 90° - \tan^{-1}\dfrac{\omega + \frac{\omega}{7}}{1 - \omega.\frac{\omega}{7}}$

$= -\dfrac{\omega \times 180}{\pi} - 90° - \tan^{-1}\dfrac{8\omega}{7-\omega^2}$; $\qquad \omega < \sqrt{7}$

At the phase cross over frequency $\omega_{2,}$ $\angle G(j\omega_2) = -180°$

$-\dfrac{\omega_2 \times 180}{\pi} - 90° - \tan^{-1}\dfrac{8\omega_2}{7-\omega_2^2} = -180°$

$= -57.3\omega_2 - \tan^{-1}\dfrac{8\omega_2}{7-\omega_2^2} = -90°$

Solving by trial –error method: $\omega_2 = 0.79$ rad/sec

$\left|G(j\omega_2)\right| = \left|\dfrac{10K}{j\omega_2(j\omega_2+1)(j\omega_2+7)}\right|$

$= \dfrac{10K}{\omega_2\sqrt{\omega_2^2+1^2}\sqrt{\omega_2^2+7^2}}$

$\omega_2 = 0.79$ rad/sec

$$\therefore\ \left|G\left(j0.079_2\right)\right| = \frac{10K}{0.79\sqrt{(0.79)^2+1^2}\sqrt{(0.79)^2+7^2}} = \frac{K}{0.708}$$

For stability $\frac{K}{0.708}$<1; K < 0.708

For marginally stability: K = 0.708

06. (c)

Sol:

(i) We have

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega = 2\pi x(t)$$

Substituting t = 0 in the above equation, we get

$$\int_{-\infty}^{\infty} X(\omega)d\omega = 2\pi x(0) = 2\pi\ (1) = 2\pi$$

(ii) From Parsavel's theorem, we have

$$\int_{-\infty}^{\infty}|x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega$$

$$\therefore \int_{-\infty}^{\infty}|X(\omega)|^2 d\omega = 2\pi\int_{-\infty}^{\infty}|x(t)|^2 dt = 2\pi\left[\int_{-1}^{0}(t+1)^2 dt + \int_{0}^{2}(1-t)^2 dt + \int_{2}^{3}(t-3)^2 dt\right] = \frac{8\pi}{3}$$

(iii) We have $\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega = 2\pi x(t)$

Substituting t = 2, we get

$$\int_{-\infty}^{\infty} X(\omega)e^{j2\omega}\,d\omega = 2\pi x(2) = 2\pi\ (-1) = -2\pi$$

(iv) We have

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Substituting ω = 0, we get

$$X(0) = \int_{-\infty}^{\infty} x(t)dt$$

$= 0$ [∵ x(t) is a shifted odd signal]

07. (a)

Sol:

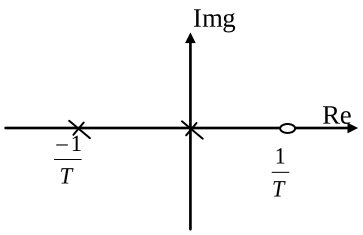
(i) If the transfer function has symmetric pole and zero about the imaginary axis in s - plane then the transfer function is called all pass transfer function and given by

$$G(s) = \frac{1-sT}{1+sT}$$

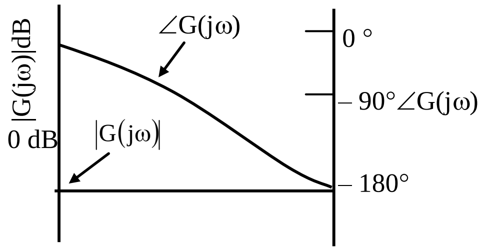
In sinusoidal form above transfer function is written as

$$G(j\omega) = \frac{1 - j\omega T}{1 + j\omega T}$$

The pole - zero configuration and Bode plot for all pass transfer function is shown in Figure



Pole –zero configuration



Bode plot for all pass transfer function

The magnitude plot lies on 0 db axis indicating that the actual gain for the frequencies is 1, thus the transfer function passes all frequencies.

The phase angle for all pass transfer function is given by

$$\angle\frac{1 - j\omega T}{1 + j\omega T} = \tan^{-1}(-\omega T) - \tan^{-1}(\omega T) = -2\tan^{-1}(\omega T)$$

The phase angle for all pass transfer functions given by $-2\tan^{-1}(\omega T)$ and

varies from 0° to –180° as the frequency is varied from $\omega = 0$ to $\omega = \infty$

(ii) $s^5 + s^4 + 2s^3 + 2s^2 + 11s + 10 = 0$

s^5	1	2	11
s^4	1	2	10
s^3	0	1	0
s^2	0.227	10	0

While forming the Routh array as above the third element in the first column is zero and thus the Routh criterion fails at this stage. The difficulty is solved if zero in the third row of the first column is replaced by a symbol ε and Routh array is formed as follows

+	s^5	1	2	11
+	s^4	1	2	10
+	S^3	ε	1	0
–	s^2	$\lim_{\varepsilon\to 0}\left(\frac{\varepsilon\times 2 - 1\times 1}{\varepsilon}\right) = -\infty$	10	0
+	s^1	$\lim_{\varepsilon\to 0}\left(1 - \frac{10\varepsilon^2}{2\varepsilon - 1}\right) = 1$	0	0
+	s^0	10		

The limits of fourth and fifth element in the first column a $\varepsilon \to 0$ from positive side are $-\infty$ and +1 respectively, indicating two sign changes, therefore, the system is unstable and the number of roots with positive real part of the characteristic equation is 2.

07. (b)

Sol:

(i) If the received signal is $Km(t)\cos\omega_c t$, the demodulator input is $[Km(t) + n_c(t)]\cos\omega_c t + n_s(t)\sin\omega_c t$. When this is multiplied by $2\cos\omega_c t$ and low-pass filtered the output is

$S_o(t) + n_o(t) = km(t) + n_c(t)$

Signal power $S_o = K^2\overline{m^2}$

Noise pwer $N_o = \overline{n_c^2}$

But the power of the received signal $km(t)\cos\omega_c t$ is $1\mu W$. Hence,

$$\frac{K^2\overline{m^2}}{2} = 10^{-6}$$

$$S_o = K^2\overline{m^2} = 2\times10^{-6}$$

To compute $\overline{n_c^2} = \overline{n^2}$

Where $\overline{n^2}$ is the power of the incoming bandpass noise of bandwidth 8 kHz centered at 500 kHz.

i.e., $$\overline{n^2} = \frac{2}{2\pi}\int_{2\pi(496000)}^{2\pi(504000)} \frac{1}{\omega^2 + a^2}d\omega$$

$$= \frac{1}{\pi}\frac{1}{a}\tan^{-1}\frac{\omega}{a}\Bigg|_{2\pi(496000)}^{2\pi(504000)}$$

$$= \frac{1}{\pi(10^6\pi)}\left[\tan^{-1}\frac{2\pi(504000)}{10^6\pi} - \tan^{-1}\frac{2\pi(496000)}{10^6\pi}\right]$$

$$= \frac{1}{10^6(3.14)^2}[0.788 - 0.780]$$

$= 8.113\times10^{-10}$

$= N_o$

$$\Rightarrow \frac{S_o}{N_o} = \frac{2\times10^{-6}}{8.113\times10^{-10}} = 2465.17$$

$= 33.83$ dB

(ii) Generator matrix $G = [I_k|P]$

Where I_k is $k\times k$ identity matrix

P is $k\times(n-k)$ matrix

Parity check matrix $H = [P^T|I_{n-k}]$

G is a $k\times n$ matrix

H is a $(n-k)\times n$ matrix.

$$GH^T = [I_K P]\begin{bmatrix} P \\ I_m \end{bmatrix} = P \oplus P = 0$$

07. (c)

Sol: Assume, $x(n) = x_1(n)$ Ⓝ $x_2(n)$

Apply DFT on both sides

$X(k) = X_1(k)\,X_2(k)$

$$X_1(k)=\sum_{n=0}^{N-1} x_1(n)e^{-j\frac{2\pi}{N}nk},\ k=0\ \text{to}\ N-1$$

N = 4

$$X_1(0)=\sum_{n=0}^{3} x_1(n)=5$$

$$X_1(1)=\sum_{n=0}^{3} x_1(n)e^{-j\frac{n\pi}{2}}=-1$$

$$X_1(2)=\sum_{n=0}^{3} x_1(n)e^{-jn\pi}=1$$

$$X_1(3)=\sum_{n=0}^{3} x_1(n)e^{-j\frac{3n\pi}{2}}=-1$$

$X_1(k) = \{5, -1, 1, -1\}$

$$X_2(k)=\sum_{n=0}^{N-1} x_2(n)e^{-j\frac{2\pi}{N}nk},\quad k=0\ \text{to}\ N-1$$

N = 4

$$X_2(0)=\sum_{n=0}^{3} x_2(n)=10$$

$$X_2(1)=\sum_{n=0}^{3} x_2(n)e^{-j\frac{n\pi}{2}}=-2+j2$$

$$X_2(2)=\sum_{n=0}^{3} x_2(n)e^{-jn\pi}=-2$$

$$X_2(3)=\sum_{n=0}^{3} x_2(n)e^{-j\frac{3n\pi}{2}}=-2-j2$$

$X_2(k) = \{10, -2 + j2, -2, -2 - j2\}$

$X_3(k) = X_1(k)\, X_2(k) = \{50, 2 - j2, -2, 2 + j2\}$

IDFT of $X_3(k)$ is $x_3(n)=\frac{1}{N}\sum_{k=0}^{N-1} X_3(k)e^{j\frac{2\pi}{N}nk},\quad k=0\ \text{to}\ N-1$

$$x_3(0)=\frac{1}{4}\sum_{k=0}^{3} X_3(k)=13$$

$$x_3(1)=\frac{1}{4}\sum_{k=0}^{3} X_3(k)e^{j\frac{\pi k}{2}}=14$$

$$x_3(2)=\frac{1}{4}\sum_{k=0}^{3} X_3(k)e^{j\pi k}=11$$

$$x_3(3)=\frac{1}{4}\sum_{k=0}^{3} X_3(k)e^{j\frac{3\pi k}{2}}=12$$

$x_3(n) = \{13, 14, 11, 12,\}$.

08. (a)

Sol:

(i) The waveform shown in Figure is periodic with a period $T = 2\pi$.

Let, $t_0 = 0$, $t_0 + T = 2\pi$

Then, Fundamental frequency $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

The waveform is described by

$$x(t) = \begin{cases} (A/\pi)t & \text{for } 0 \le t \le \pi \\ 0 & \text{for } \pi \le t \le 2\pi \end{cases}$$

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{2\pi}\int_0^{2\pi} x(t)dt$$

$$= \frac{1}{2\pi}\int_0^{\pi} \frac{A}{\pi} t\,dt = \frac{A}{2\pi^2}\left[\frac{t^2}{2}\right]_0^{\pi} = \frac{A}{4}$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{2\pi}\int_0^{\pi} \frac{A}{\pi} t\cos nt\,dt = \frac{A}{\pi^2}\int_0^{\pi} t\cos nt\,dt$$

$$= \frac{A}{\pi^2}\left[\left[\frac{t\sin nt}{n}\right]_0^{\pi} - \int_0^{\pi}\frac{\sin nt}{n}dt\right] = \frac{A}{\pi^2}\left[\frac{0-0}{n} + \left(\frac{\cos nt}{n^2}\right)_0^{\pi}\right]$$

$$= \frac{A}{\pi^2 n^2}(\cos n\pi - \cos 0)$$

$$a_n = \begin{cases} -(2A/\pi^2 n^2) & ;\ \text{for odd } n \\ 0 & ;\ \text{for even } n \end{cases}$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t\,dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} x(t)\sin nt\,dt = \frac{1}{\pi}\int_0^{\pi}\frac{A}{\pi} t\sin nt\,dt$$

$$= \frac{A}{\pi^2}\int_0^{\pi} t\sin nt\,dt = \frac{A}{\pi^2}\left[\left[\frac{t(-\cos nt)}{n}\right]_0^{\pi} - \int_0^{\pi}\frac{-\cos nt}{n}dt\right]$$

$$= \frac{A}{\pi^2}\left[-\pi\frac{\cos n\pi}{n} + \left(\frac{\sin nt}{n^2}\right)_0^{\pi}\right]$$

$$= \frac{-A}{n\pi}\cos n\pi = \frac{A}{n\pi}(-1)^{n+1}$$

$$b_n = \begin{cases} A/n\pi & ;\ \text{for odd } n \\ -(A/n\pi) & ;\ \text{for even } n \end{cases}$$

The trigonometric Fourier is:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$= \frac{A}{4} - \frac{2A}{\pi^2}\sum_{n=odd}^{\infty}\frac{\cos nt}{n^2} + \frac{A}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nt}{n}$$

$$= \frac{A}{4} - \frac{2A}{\pi^2}\left[\cos t + \frac{1}{3^2}\cos 3t + \frac{1}{5^2}\cos 5t + ...\right] + \frac{A}{\pi}\left[\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t + ...\right]$$

(ii) The signal shown in Figure is expressed as:

$$x(t) = \begin{cases} -A & ;\text{for } -T < t < 0 \\ A & ;\text{for } 0 < t < T \\ 0 & ;\text{ else where} \end{cases}$$

$$\therefore X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$= \int_{-T}^{0} -Ae^{-j\omega t}dt + \int_{0}^{T} Ae^{-j\omega t}dt = -\int_{0}^{T} Ae^{j\omega t}dt + \int_{0}^{T} Ae^{-j\omega t}dt$$

$$= -A\left[\frac{e^{j\omega t}}{j\omega}\right]_0^T + A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^T = -\frac{A}{j\omega}\left[\left(e^{j\omega T} - e^0\right) + \left(e^{-j\omega T} - e^0\right)\right]$$

$$= \frac{A}{j\omega}\left[2 - \left(e^{j\omega T} + e^{-j\omega T}\right)\right] = \frac{A}{j\omega}[2 - 2\cos\omega T] = \frac{2A}{j\omega}[1 - \cos\omega T]$$

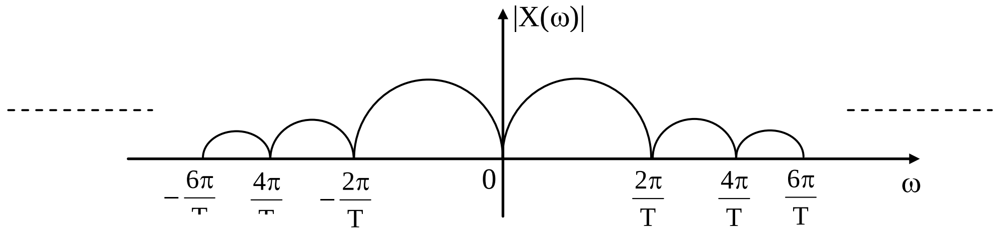
Therefore, the magnitude of $X(\omega)$ is:

$$|X(\omega)| = \frac{2A}{\omega}[1 - \cos\omega T]$$

$\therefore$ $|X(\omega)| = 0$, when $1 - \cos\omega T = 0$

i.e. $\omega T = 2n\pi$ or $\omega = \frac{2n\pi}{T}$

The magnitude spectrum is as shown in Figure.



08. (b)

Sol:

(i) The noise figure of the lossy cable is 3 dB = $10^{0.3}$ = 2.

The gain factor, of the cable is $\frac{1}{2} = 0.5$

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

Where $F_1 = 10^{0.6} = 3.98$

$G_1 = 100$
$F_2 = 2,\ G_2 = 0.5$
$F_3 = 10^{1.6} = 39.8,$

Substituting $F = 3.98 + \dfrac{2-1}{100} + \dfrac{39.8-1}{100\times 0.5}$
$= 3.98 + 0.01 + 0.776$
$= 4.766 + 10\log_{10}4.766 = 6.8\text{ dB}$

(ii) Upon removal of the pre-amplifier there are 2 blocks only the cable and the front end

Now $F_1 = 2$, $F_2 = 39.8$

$$F = F_1 + \frac{F_1 - 1}{G_1}$$

$$= 2 + \frac{39.8-1}{0.5} = 79.6 = 19\,dB$$

This shows the need of placing the pre-amplifier before the begin of the lossy line. The front end receiver gain has no contribution in reducing the noise figure.

(iii) When the amplifier is placed after the cable, is the first stage

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_2 - 1}{G_1 G_2}$$

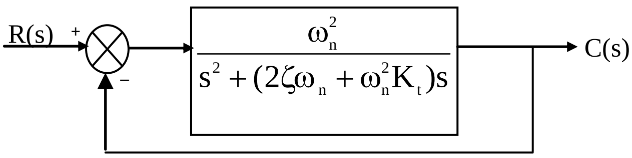
$$= 2 + \frac{3.98-1}{0.5} + \frac{39.8-1}{100\times 0.5}$$

$= 2 + 5.96 + 0.776$
$= 8.736$
$= 9.4$ dB

Thus the Noise figure changes by 2.6dB.

08. (c)

Sol: (i)



From the block diagram figure the transfer function for a unity feedback second order – control system using derivate feedback control is determined below:

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\omega_n^2}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s}}{1 + \dfrac{\omega_n^2}{s^2 + \left(2\zeta\omega_n + \omega_n^2 K_t\right)s}.1}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s + \omega_n^2}$$

The characteristic equation for the overall transfer function is

$s^2 + (2\zeta\,\omega_n + \omega_n^2\ K_t)s + \omega_n^2 = 0$

The damping ratio for the above characteristic equation is

2.16∠ 9.7°
3.04∠33.4°
Ω

$$\zeta' = \frac{2\zeta\omega_n + \omega_n^2 K_t}{2\omega_n}$$

$$\zeta' = \zeta + \frac{\omega_n K_t}{2}$$

The damping ratio is increased by using derivative feedback control and therefore, the maximum overshoot is decreased. However, the rise time is increased.

(ii) In the block diagram figure the forward path transfer function is

$$G(s) = \frac{\omega_n^2}{s^2 + \left(2\zeta\omega_n + \omega_n^2 K_t\right)s}$$

and the feedback path transfer function is H (s) = 1

The transfer function relating E(s) and R(s) is given by

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

Substituting

$$G(s) = \frac{\omega_n^2}{s^2 + \left(2\zeta\omega_n + \omega_n^2 K_t\right)s}$$

and H (s) = 1 in $\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$ the following relation between the error and input signal for the derivative feedback control action is obtained

$$\frac{E(s)}{R(s)} = \frac{1}{1 + \frac{\omega_n^2}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s}.1}$$

$$\frac{E(s)}{R(s)} = \frac{s^2 + \left(2\zeta\omega_n + \omega_n^2 K_t\right)s}{s^2 + \left(2\zeta\omega_n + \omega_n^2 K_t\right)s + \omega_n^2}$$

For a unit ramp function

R (s) = 1/s^2

$$\therefore\ E(s) = \frac{1}{s^2}.\frac{s^2 + \left(2\zeta\omega_n + \omega_n^2 K_t\right)s}{s^2 + (2\zeta\omega_n + \omega_n^2 K_t)s + \omega_n^2}$$

The steady state error is determined below:

$$e_{ss} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s.\frac{1}{s^2}.\frac{s^2 + \left(2\zeta\omega_n + \omega_n^2 K_t\right)s}{s^2 + \left(2\zeta\omega_n + \omega_n^2 K_t\right)s + \omega_n^2}$$

$$e_{ss} = \frac{2\zeta}{\omega_n} + K_t$$