ACE Engineering Academy
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# ESE – 2019 MAINS
## OFFLINE TEST SERIES

## MECHANICAL ENGINEERING

# TEST – 8
# SOLUTIONS

All Queries related to **ESE – 2019 MAINS Test Series** Solutions are to be sent to the following email address
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**01(a).**

**Sol:** Let the given stresses be designated as:

$\sigma_x = 51$ MPa, $\sigma_y = 66$ MPa,

$\tau_{xy} = 18$ MPa

The principal stress magnitudes can be computed from equation.

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{51+66}{2} \pm \sqrt{\left(\frac{51-66}{2}\right)^2 + 18^2}$$

$$= 58.50 \pm 19.50$$

$\sigma_{p1} = 78.0$ MPa and $\sigma_{p2} = 39.0$ MPa

Since this is a cylindrical pressure vessel subjected to internal pressure only, we know that the principal stresses occur in the hoop and longitudinal directions. Thus, we can assert that:

$$\sigma_{p1} = \sigma_{hoop} = \frac{pd}{2t}$$

and $$\sigma_{p2} = \sigma_{long} = \frac{pd}{4t}$$

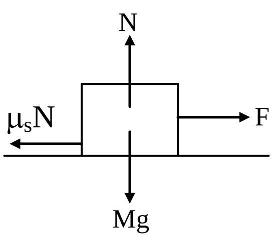
The internal pressure can be calcualted from either expression:

$$\frac{pd}{2t} = 78$$

$$\therefore p = \frac{2(10)(78)}{830} = 1.880 \text{ MPa}$$

**01(b).**

**Sol:** The forces acting on the block are shown in figure.




Here M = 2.5 kg and F = 15 N.

When F = 15 N is applied to the block, the block remains in limiting equilibrium. The force of friction is thus f = $u_s$ N. Applying Newton's first law,

$$f = u_s N \text{ and } N = mg$$

so that $F = u_s Mg$

or, $$\mu_s = \frac{F}{mg} = \frac{15\,N}{(2.5\,kg)(10\,m/s^2)} = 0.60$$

When the block is gently pushed to start the motion, kinetic friction acts between the block and the surface. Since the block takes 5 second to slide through the first 10 m, the acceleration a is given by

$$10\,m = \frac{1}{2}a(5s)^2$$

or, $$a = \frac{20}{25}\,m/s^2 = 0.8\,m/s^2$$

The frictional force is

$$f = u_k N = u_k Mg.$$

Applying Newton's second law

$$F - u_k Mg = Ma$$

$$\mu_k = \frac{F - Ma}{Mg}$$

$$= \frac{15N - (2.5kg)(0.8\,m/s^2)}{(2.5kg)(10\,m/s^2)} = 0.52$$

**01(c).**

**Sol:** $$A_1 = \frac{\pi}{4}(12)^2 = 113.0973\,mm^2$$

$$A_2 = \frac{\pi}{4}(16)^2 = 201.0619\,mm^2$$

The total static deformation is

$$\delta_{s1} = \frac{F_1 L_1}{A_1 E_1} + \frac{F_2 L_2}{A_2 F_2} = F_{s1}\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2}\right]$$

$$= F_{st}\left[\frac{800}{(113.0973)(70{,}000)} + \frac{1{,}300}{(201.0619)(105{,}000)}\right]$$

$$= \frac{F_{st}}{6{,}148.9798\ \text{N/mm}}$$

and the impact factor can be expressed as

$$n = \frac{\sigma_{max}}{\sigma_{st}} = \frac{200}{F_{st}/A}$$

$$= \frac{(200)(113.0973)}{F_{st}} = \frac{22{,}619.4600\ \text{N}}{F_{st}}$$

Substitute these two expressions into

$$n = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}}$$

and solve for $F_{st}$

$$\frac{22{,}619.4600\ \text{N}}{F_{st}} = 1 + \sqrt{1 + \frac{2(300)}{F_{st}/(6{,}148.9798\ \text{N/mm})}}$$

$$= 1 + \sqrt{1 + \frac{3{,}689{,}387.880}{F_{st}}}$$

$$\left(\frac{22{,}619.4600}{F_{st}} - 1\right)^2 = 1 + \frac{3{,}689{,}387.880}{F_{st}}$$

$$\frac{1}{F_{st}^2}(22{,}619.4600 - F_{st})^2 = \frac{F_{st} + 3{,}689{,}387.880}{F_{st}}$$

$$(22{,}619.4600 - F_{st})^2 = F_{st}(F_{st} + 3{,}689{,}387.880)$$

$$2(22{,}619.4600)F_{st} = (22{,}619.4600)^2 - 3{,}689{,}387.880$$

$$F_{st} = \frac{(22{,}619.4600)^2}{3{,}689{,}387.880 + 2(22{,}619.4600)}$$

$$= 136.9990\ \text{N}$$

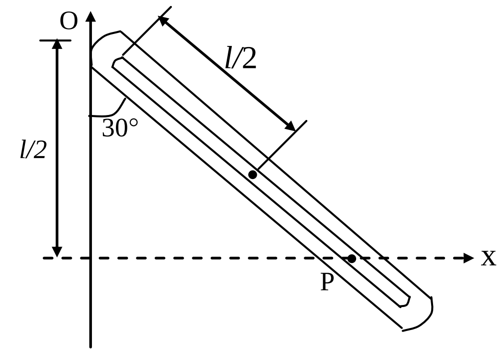
**Allowable mass:**

$$m = \frac{F_{st}}{g} = \frac{136.9990}{9.806650}$$

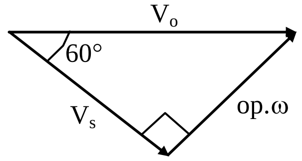
$$= 13.9700\ \text{kg} = 13.97\ \text{kg}$$

**01(d).**

**Sol:**




Given, $l$ = 50 cm, $V_0$ = 75 cm/s

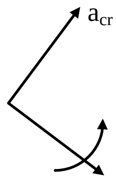



**(velocity diagram)**

$OP.\omega = V_o \sin 60$

$$\frac{\ell\omega}{2\times\cos 30} = V_o \sin 60$$

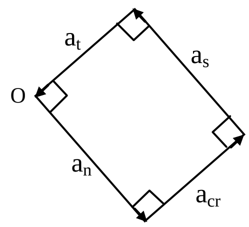
$\omega = 2.25\ \text{rad/s}$




$V_s = V_o \cos 60 = 37.5$ cm/s

Coriolis acceleration

$= 2V_s\omega = 168.75\ \text{cm/s}^2$




**(Acceleration diagram)**

As acceleration of point 'P' is zero. (As it is moving with constant velocity)

$a_t = a_{cr} = 168.75\ \text{cm/s}^2$

$$OP = \frac{25}{\cos 30} = 25.876\ \text{cm}$$

$OP.\alpha = 168.75$

$\alpha = 5.84\ \text{rad/s}^2$

**01(e).**

**Sol:**

- A governor with a range of speed zero is known as an isochronous governor. This means that for all positions of the sleeve or the balls, the governor has the same equilibrium speed. Any change of speed results in moving the balls and the sleeve to their extreme positions. However, an isochronous governor is not practical due to friction at the sleeve.
- For isochronism, $\omega_1 = \omega_2$ and thus $h_1 = h_2$
- For a porter governor, with all arms equal in length and intersection on the axis (neglecting friction),

  $h_1 = \frac{g}{\omega_1^2}\left(1+\frac{M}{m}\right)$ and $h_2 = \frac{g}{\omega_2^2}\left(1+\frac{M}{m}\right)$

  However, from the configuration of a Porter governor, it can be judged that it is impossible to have two positions of the balls at the same speed. Thus, a pendulum type of governor cannot possibly be isochronous.

  In the case of a Hartnell governor (neglecting friction)

  At $\omega_1$, $mr_1\omega_1^2 a = \frac{1}{2}(Mg + F_s)b$

  At $\omega_2$, $mr_2\omega_2^2 a = \frac{1}{2}(Mg + F_{a2})b$

  For isochronism, $\omega_1 = \omega_2$

  $\therefore \frac{Mg + F_{s1}}{Mg + F_{s2}} = \frac{r_1}{r_2}$

  Which is the required condition of isochronism.

**02(a)(i).**

**Sol:** Equation of Equilibrium: For entire post

$$+\uparrow\sum F_y = 0$$

$$F + 8.00 - 20 = 0$$

$$F = 12.0 \text{ kN}$$

**Internal Force FBD (b)**

$$+\uparrow\sum F_y = 0$$

$$-F(y) + 4y - 20 = 0$$

$$F(y) = (4y - 20) \text{ kN}$$

**Displacement:**

$$\delta_{A/B} = \int_0^L \frac{F(y)dy}{A(y)E} = \frac{1}{AE}\int_0^{2m}(4y-20)dy$$

$$= \frac{1}{AE}(2y^2 - 20y)\Big|_0^{2m}$$

$$= -\frac{32.0}{AE}$$

$$= -\frac{32.0(10^3)}{\frac{\pi}{4}(0.06^2)13.1(10^9)}$$

$$= -0.8639\times10^{-3} \text{ m}$$

$$= -0.864 \text{ mm}$$

Negative sign indicates that end A moves toward end B.

**02(a)(ii).**

**Sol:** The thin walled cylindrical assumptions that must be satisfier are:

1. The wall must be thin enough to satisfy the assumption that the radial stress component ($\sigma_r$) at the wall is negligibly small compared to the tangential ($\sigma_t$) stress component.
2. The wall must be thin enough that $\sigma_t$ is uniform across it.

**02(b).**

**Sol:**

(a) For the outer spring $k_o = \dfrac{d_o^4 G}{64 R_o^3 N_o}$ and

For the inner spring $k_i = \dfrac{d_i^4 G}{64 R_i^3 N_i}$

$$R_o = \frac{1}{2}(D_i + d_i) = \frac{1}{2}(38 + 2.8) = 20.4\,\text{mm}$$

$d_o$ = 2.8 mm,

$N_o$ = 10

$$R_1 = \frac{1}{2}(D_o - d_o) = \frac{1}{2}(32 - 2.2) = 14.9\,\text{mm}$$

$d_i$ = 2.2 mm,

$N_i$ = 13

$$k_o = \frac{(0.0028)^4(79\times10^9)}{64(0.0204)^3(10)} \approx 894\ \text{N/m}$$

$$k_i = \frac{(0.0022)^4(79\times10^9)}{64(0.0149)^3(13)} \approx 672\ \text{N/m}$$

(b) Since the springs are in parallel

$k_{nest} = k_o + k_i$

= 894 + 672 = 1566 N/m.

Therefore

$F_{y=25} = k_{nest}(0.025) = 1566(0.025) = 39.15$ N

(c) For the inner and out springs,

$c_o = D_o/d_o = 40.4/2.8 = 14.6$,

$K_{w-o} = 1.097$,

$c_i = D_i/d_i = 29.8/2.2 = 13.6$

$K_{w-i} = 1.105$.

Therefore

$$(\tau_{max})_o = 1.097\left(\frac{16(39.15)(0.0202)}{\pi(0.0028)^3}\right) \approx 201\,\text{MPa}$$

$$(\tau_{max})_i = 1.105\left(\frac{16(39.15)(0.0149)}{\pi(0.0022)^3}\right) \approx 311\,\text{MPa}$$

The inner spring is the more highly stressed spring.

**02(c).**

**Sol:** Given that m = 100 kg, Δ = 8 mm

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{9.81}{8\times10^{-3}}} = 35.017\,\text{rad/s}$$

Let F be the vertical harmonic force at 80% of resonance frequency

$\therefore\ \omega = 0.8\,\omega_n = 0.8 \times 35.017$

= 28.0136 rad/sec

$$\Rightarrow \frac{\omega}{\omega_n} = 0.8$$

Let $A_1$ is amplitude without damping

$$A_1 = \frac{F/K}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]} = \frac{F/K}{(1-0.8^2)} \text{-----------(1)}$$

Let $A_2$ is the amplitude with damping at resonance, i.e., $\dfrac{\omega}{\omega_n} = 1$

$$\Rightarrow A_2 = \frac{F/K}{\sqrt{\left(2\zeta\frac{\varpi}{\varpi_n}\right)^2 + \left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}}$$

$$= \frac{F/K}{2\zeta} \text{ ----------- (2)}$$

$$\frac{\text{equ.(1)}}{\text{equ.(2)}} \Rightarrow \frac{A_1}{A_2} = \frac{2\zeta}{1-0.8^2}$$

$$\zeta = \frac{A_1}{2A_2}\times 1-0.8^2 = \frac{5}{2\times2}(1-0.8^2)$$

$\zeta$ = 0.45

(i) Damping coefficient

$$c = 2\,m\,\omega_n\,\zeta = 2 \times 100 \times 35.97 \times 0.45$$
$$= 3237.3 \text{ N/m/s}$$

(ii) Critical damping coefficient

$$c_c = \frac{c}{\zeta} = \frac{3237.3}{0.45} = 7194 \text{ N/m/s}$$

(iii) Damping ratio, $\zeta = 0.45$

(iv) Logarithmic decrement

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi \times 0.45}{\sqrt{1-0.45^2}} = 3.166$$

(v) Damping force = $c\dot{x} = cA_2\,\omega$

$$= 3237.3 \times \frac{2}{1000} \times 28.0136$$
$$= 181.3768 \text{ N}$$

**03(a).**

**Sol:** Since all shearing stress components are zero on the element shown in figure, it is a principal element and the principal stresses are $\sigma_1 = 290$ MPa, $\sigma_2 = 70$ MPa, $\sigma_3 = -35$ MPa

(a) Since e = 2%, the maximum normal stress theory is used $\sigma_{max} \geq \sigma_{fail} = S_u$.

Thus $\sigma_{max} = \sigma_1 = 290 \geq S_u = 248$

Failure is predicted by brittle fracture.

(b) Since e = 8%, the aluminum alloy is regarded as ductile, so both the distortional energy and maximum shearing stress theories will be used. From the distortional energy theory,

$$\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right] \geq \sigma_{fail}^2$$

Or,

$$\frac{1}{2}\left[(290-70)^2 + (70-(-35))^2 + (-35-290)^2\right] \geq (186)^2$$

$$8.25\times10^4 \geq 3.459\times10^4$$

Since the inequality is satisfied, failure is predicted (by yielding). From the maximum shearing stress theory,

$$|\tau_{max}| = \left|\frac{\sigma_{max}-\sigma_{min}}{2}\right| \geq |\tau_{fail}|_{max} = \frac{S_{yp}}{2}$$

$$|\sigma_1 - \sigma_3| \geq S_{yp} \Rightarrow 290 - (-35) \geq 186$$

$$\Rightarrow 325 \geq 186$$

Since the inequality is satisfied, failure is predicted (by yielding).

**03(b).**

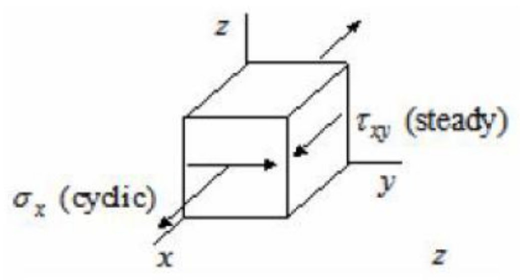
**Sol:** The torque applied to the shaft is

$$T = \frac{9549\,\text{kW}}{n} = \frac{9549 \times 150}{3600} = 398 \text{ N-m}$$

The bending moment, M = 280 N-m is completely reversed due to shaft rotation. Since the maximum shearing stress due to torsion is at the surface, and the cyclic bending stress is at the surface with each rotation. we have a state of stress as shown.

The shearing stress and flexural (bending) stress are given by

$$\tau_{xy} = \frac{Tr}{J} = \frac{16T}{\pi d^3} \text{ and } \sigma_x = \frac{Mr}{I} = \frac{32M}{\pi d^3}$$




This is relatively simple state of stress and the principal stress can be determined from Mohr's circle. Since it is a state of plane stress, we know that $\sigma_{eq} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$,

so $\quad \sigma_{eq-a} = \sqrt{\sigma_{x-a}^2 + 3\tau_{xy-a}^2}$

and $\quad \sigma_{eq-m} = \sqrt{\sigma_{x-m}^2 + 3\tau_{xy-m}^2}$

Noting that $T_{max} = T_{min} = T_m$ = 398 N-m.

$T_a$ = 0 with $M_{max}$ = +280 N-m

and $M_{min}$ = –280 N-m,

we determine

$M_m = 0$ and $M_a = 280$.

Therefore

$$\sigma_{x-a} = \frac{32\times 280}{\pi(0.032)^3} = 87 \text{ MPa and } \sigma_{x-m} = 0$$

$$\tau_{xy-a} = 0 \text{ and } \tau_{xy-m} = \frac{16\times(398)}{\pi(0.032)^3} = 61.9 \text{ MPa}$$

Therefore

$$\sigma_{eq-a} = \sqrt{\sigma_{x-a}^2 + 3\tau_{xy-a}^2} = \sqrt{(87)^2 + 3(0)^2}$$

$$= 87 \text{ MPa}$$

$$\sigma_{eq-m} = \sqrt{\sigma_{x-m}^2 + 3\tau_{xy-m}^2} = \sqrt{(0)^2 + 3(61.9)^2}$$

$$= 107 \text{ MPa}$$

$(S_u)_{540°C}$ = 821 MPa and

$(S_{yp})_{540°C} = 572\,\text{MPa}$.

In addition. The maximum normal stress is

$$\sigma_{max} = \sigma_{eq-a} + \sigma_{eq-m}$$

$$= 87 + 107 = 194 \text{ MPa}$$

The equivalent completely reversed stress is

$$(\sigma_{eq-CR})_{540°C} = \frac{\sigma_a}{1-\frac{\sigma_m}{S_u}} = \frac{87}{1-\frac{107}{821}} = 100 \text{ MPa}$$

For the ultimate strength we are using

$S_f$ = 0.3(821) to 0.5(821) @ $10^8$ cycles,

$S_f$ = 246 to 411 MPa @ $10^8$ cycles

Comparing this to $(\sigma_{eq-CR})_{540°C} = 100\,\text{MPa}$ we conclude that infinite life is expected. A more accurate answer involves considering the strength influencing factors.

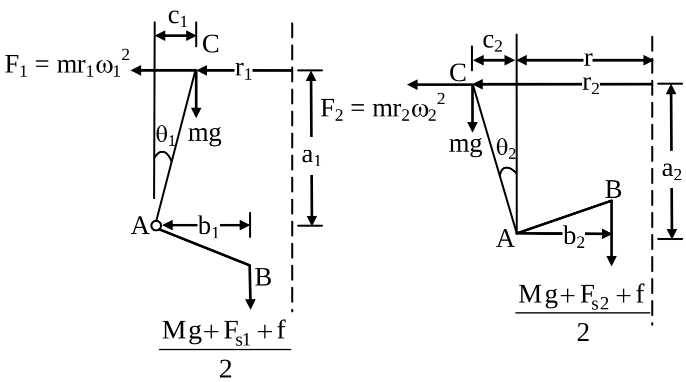
**03(c).**

**Sol:** Given:

m = 4 kg, $N_1$ = 200 rpm,

$h_1$ = 40 mm, $r_1$ = 90 mm, r = 115 mm,

a = 100 mm, b = 80 mm




Mean speed, $N = \frac{N_1 + N_2}{2}$

As, $\quad N = 16(N_2 - N_1)$

$$\therefore \quad \frac{N_1 + N_2}{2} = 16(N_2 - N_1)$$

$$\text{Or,} \quad \frac{200 + N_2}{2} = 16(N_2 - 200)$$

$N_2$ = 212.9 rpm

Angle turned by bell-crank lever between two extreme positions

$$= \frac{\text{Lift}(h_1)}{b} = \frac{c_1 + c_2}{a}$$

Or, $c_1 + c_2 = h_1 \frac{a}{b} = 40 \times \frac{100}{80}$ = 50 mm

But, $c_1 = r - r_1 = 115 - 90 = 25$ mm

$c_2 = 50 - 25 = 25$ mm

$r_2 = r + c_2 = 115 + 25 = 140$ mm

$b_1 = b_2 = \sqrt{b^2 - (h/2)^2}$

$= \sqrt{(80)^2 - (20)^2}$ = 77.46 mm

$a_1 = a_2 = \sqrt{(100)^2 - (25)^2}$ = 96.82 mm

$\omega_1 = \frac{2\pi \times 200}{600}$ = 20.94 rad/s

$\omega_2 = \frac{2\pi \times 212.9}{60}$ = 22.29 rad/s

In the extreme positions,

$mr_1\omega_1^2 a_1 = \frac{1}{2} F_{s1} b_1 + mgc_1$ (M = 0, f = 0)

$4 \times 0.09 \times (20.94)^2 \times 0.096\,82 =$

$\frac{1}{2} F_{s1} \times 0.07746 + 4 \times 9.81 \times 0.25$

$F_{s1}$ = 369.28 N

$mr_2\omega_2^2 a_2 = \frac{1}{2} F_{S2} b_2 + mgc_2$

$4 \times 0.14 \times (22.29)^2 \times 0.09682$

$= \frac{1}{2} F_{s2} \times 0.07746 - 4 \times 9.814 \times 0.025$

$F_{s2}$ = 720.86 N

$h_1 s = F_{s2} - F_{s1}$

$40 \times s = 720.86 - 369.28$

$\Rightarrow$ s = 8.79 N/mm

Initial compression = $\frac{F_{s1}}{s}$

$= \frac{369.28}{8.79}$ = 42.0 mm

$F_S$ at mid-position = $F_{s1}$ + 20s

= 369.28 + 8.79 × 20 = 545.3 N

Mean speed = $\frac{N_1 + N_2}{2}$

$= \frac{212.9 + 200}{2}$ = 206.45 rpm

At the mid-position, taking friction into account,

$mr\omega^2 a = \frac{1}{2}(F_s + f)b$

$4 \times 0.115 \times \omega_2^2 \times 0.1 = \frac{1}{2}(545.3 + 15) \times 0.08$

$\omega_1^2 = 487.2$,

$\omega_1 = \frac{2\pi N_1}{60}$ = 22.07

$N_1$ = 210.8 rpm

Also, $mr\omega_2^2 a = \frac{1}{2}(F_s - f)b$

$4 \times 0.115 \times \omega_2^2 \times 0.1 = \frac{1}{2}(545.3 - 15) \times 0.08$

$\omega_2^2 = 461.13$

$\omega_2 = \frac{2\pi N}{60}$ = 21.47

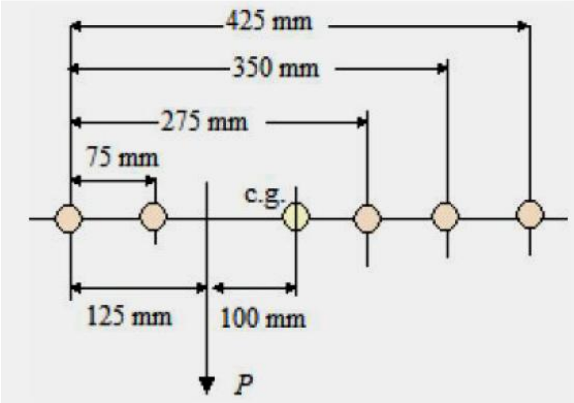
$N_2$ = 205.1 rpm

Alteration in speed = 210.8 – 205.1

= 5.7 rpm

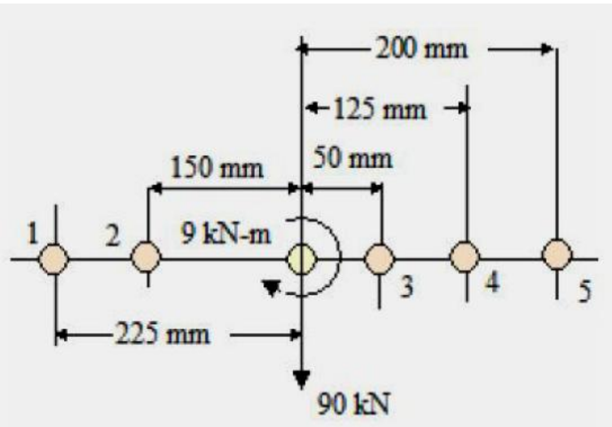
**04(a).**

**Sol:** We begin by locating the centroid of the rivet pattern and defining the loads that act there. All rivet diameters and therefore rivet cross-sectional areas are the same. The y coordinate of the centroid lies along the rivet centerline. The x coordinate (using rivet 1 as the origin) is




$$\bar{x} = \frac{(75+275+350+425)A_r}{5A_r} = 225 \text{ mm}$$

The loads acting at the centroid of the rivet pattern are as shown. The shear force supported by each rivet will consist of 2 components, one due to torsion (9 kN-m) and one due to direct shear. For defining an existing factor of safety we find the rivet supporting the largest stress. Rivets 1 and 5 are the furthest from the c.g and will have largest shear stress due to torsion component.




$$(\tau_1)_T = \frac{(90\times10^3)r_i}{J_j} = \frac{(90\times10^3)(0.225)}{J_j}$$

$$(\tau_5)_T = \frac{(90\times10^3)(0.20)}{J_j}$$

where

$$J_j = \sum A_i (r_i)^2$$

$$= A_r\left[(0.225)^2+(0.150)^2+(0.050)^2+(0.125)^2+(0.200)^2\right]$$

$$\frac{\pi(0.020)^2}{4}\left[(0.225)^2+(0.150)^2+(0.050)^2+(0.125)^2+(0.200)^2\right]$$

$$= 41.1\times10^{-6} \text{ m}^4$$

$$(\tau_1)_T = \frac{(90\times10^3)(0.225)}{41.1\times10^{-6}} = 493 \text{ MPa}\uparrow$$

$$(\tau_5)_T = \frac{(90\times10^3)(0.20)}{41.1\times10^{-6}} = 438 \text{ MPa}\downarrow$$

The shear stress at each rivet due to direct shear is

$$(\tau_i)_P = \frac{(90\times10^3)/5}{\pi(0.02)^2/4} = \frac{4(90\times10^3)}{5\pi(0.02)} = 57.3 \text{ MPa}\downarrow$$

Combining this with the shears due to torsion gives

$$\tau_1 = 493\uparrow + 57.3\downarrow = 435.7 \text{ MPa}\uparrow$$

$$\tau_5 = 438\downarrow + 57.3\downarrow = 495.3 \text{ MPa}$$

with the largest shear stress having been defined, we no assess failure modes.

Plate tensile failure. No hole diameter was given. so we arbitrarily assume a diameter of $D_h$ = 22 mm

$$\sigma_t = \frac{F}{(b - N_r D_h)t}$$

$$= \frac{90000}{[0.075+0.20+0.075+0.075+2(1.5)(0.02)-5(0.022)](0.006)}$$

= 40 MPa

$$n_e = \frac{276}{40} = 6.9$$

**Rivet shear stress:**

The maximum rivet shear stress has been determined to be is

$\tau_{max} = \tau_5 = 495.3$ MPa↓

$$n_e = \frac{0.577(345)}{495.3} \approx 0.4$$

This is unacceptable and the joint must be redesigned.

Bearing failure between rivet and plate: The maximum rivet shear stress has been determined to be $\tau_{max} = \tau_5 = 495.3$ MPa↓. Since each rives experiences a different shear stress, the bearing stress at each will be different. The shear force supported by this rivet is therefore

$$F_{s-5} = A_r \tau_{max} = \frac{\pi(0.02)^2}{4}(495.3) \approx 156 \text{ kN}$$

$$\sigma_c = \frac{F_{s-5}}{tD_r} = \frac{156000}{0.006(0.02)} \approx 1300 \text{ MPa}$$

$$n_e = \frac{276}{1300} \approx 0.21$$

**04(b).**

**Sol:**

(a) Minimum diameter required for shaft BC. The torque applied to the shaft is

$T = Pb = (20,000)(210) = 4.200\times10^6$ N-mm

If the shear stress in the shaft is limited to 70 MPa, the minimum diameter required for the shaft is:

$$\frac{\pi}{16}d^3 \geq \frac{T}{\tau} = \frac{4.200\times10^6}{70} = 60,000 \text{ mm}^3$$

$\therefore$ $d \geq 67.356$ mm

If the vertical deflection of joint D is not to exceed 25 mm, then the rotation angle at C must not exceed

$$\sin\phi_c \leq \frac{25}{210} = 0.119048 \quad \phi_c \leq 0.1193306 \text{ rad}$$

The rotation angle at C is equal to the angle of twist in the shaft, therefore,

$$\phi_c = \frac{TL}{JG} \leq 0.1193306 \text{ rad}$$

The minimum polar moment of inertia required to satisfy this angular limit is

$$J \geq \frac{TL}{\phi G} = \frac{(4.200\times10^6)(1,200)}{(0.1193306)(80,000)} = 527,944.944 \text{mm}^4$$

Thus, the minimum required diameter to satisfy the deflection limit at D is

$$\frac{\pi}{32}d^4 \geq 527,944.944 \text{ mm}^4$$

$\therefore$ $d \geq 48.156$ mm

The minimum diameter required for the shaft BC is $d = 67.4$ mm

(b) Minimum diameter required for bolt A. Since the torque in the shaft is $4.200\times10^6$ N-mm, the force that acts on the bolt at A is

$$V_A = \frac{4.200\times10^6}{110} = 38,181.818 \text{ N}$$

The bolt acts in single shear. The area required to keep the average shear stress in the bolt to a value less than 100 MPa is

$$A_{bolt} \geq \frac{38,181.818}{100} = 381.818 \text{ mm}^2$$

Consequently, the bolt must have a minimum diameter of $d \geq 22.0$ mm

**04(c).**

**Sol:** Total mass to be balanced = $m_p$ + cm

$$= 280 + \frac{2}{3} \times 300 = 480\,\text{kg}$$

(i) Taking 1 as the reference plane and angle $\theta_2 = 0°$, writing the couple equations,

$m_2\, r_2\, l_2 \cos\theta_2 + m_3\, r_3\, l_3 \cos\theta_3 + m_4\, r_4\, l_4 \cos\theta_4 = 0$

$480 \times 300 \times 400 \cos 0° + 480 \times 300 \times 1000 \cos 90° + m_4 \times 620 \times 1400 \cos\theta_4 = 0$

$\Rightarrow m_4 \cos\theta_4 = -66.36$ --------(i)

$m_2\, r_2\, l_2 \sin\theta_2 + m_3\, r_3\, l_3 \sin\theta_3 + m_4\, r_4\, l_4 \sin\theta_4 = 0$

$480 \times 300 \times 400 \sin 0° + 480 \times 300 \times 1000 \sin 90° + m_4 \times 620 \times 1400 \sin\theta_4 = 0$

$\Rightarrow m_4 \sin\theta_4 = -165.9$ --------(ii)

Squaring and adding (i) and (ii), $m_4 = 178.7$ kg

Dividing (ii) by (i),

$$\tan\theta_4 = \frac{-165.9}{-66.36} = 2.5$$

$\Rightarrow \quad \theta_4 = 248.2°$

Taking 4 as the reference plane and writing the couple equations,

$m_2\, r_2\, l_2 \cos\theta_2 + m_3\, r_3\, l_3 \cos\theta_3 + m_1\, r_1\, l_1 \cos\theta_1 = 0$

$480 \times 300 \times 1000 \cos 0° + 480 \times 300 \times 400 \cos 90° + m_1 \times 620 \times 1400 \cos\theta_1 = 0$

$\Rightarrow \quad m_1 \cos\theta_1 = -165.9$ --------(iii)

similarly, $m_1 \sin\theta_1 = -66.36$ -------(iv)

From (iii) and (iv), $m_1 = 178.7\text{ kg} = m_4$

$$\tan\theta_1 = \frac{-66.36}{-165.9} = 0.4$$

$\Rightarrow \quad \theta = 201.8°$

The treatment shows that the magnitude of $m_1$ could have directly been written equal to $m_4$.

(ii) $$\omega = \frac{50 \times 1000 \times 1000}{60 \times 60} \times \frac{1}{\frac{1800}{2}} = 15.43\,\text{rad/s}$$

Swaying couple $= \pm \frac{1}{\sqrt{2}}(1-c)m\,r\,\omega^2\,I$

$$= \pm \frac{1}{\sqrt{2}}\left(1-\frac{2}{3}\right)\times 300\times 0.3\times (15.43)^2 \times 0.6 = 3030.3 \text{ N.m}$$

(iii) Variation in tractive force $= \pm\sqrt{2}(1-c)m\,r\,\omega^2$

$$= \pm\sqrt{2}\left(1-\frac{2}{3}\right)\times 300\times 0.3\times (15.43)^2 = 10100 \text{ N}$$

(iv) Balance mass for reciprocating parts only $= 178.7 \times \dfrac{\frac{2}{3}\times 300}{480}$ = 74.46 kg

Hammer-blow = m r $\omega^2$ = 74.46 × 0.62 × $(15.43)^2$ = 10991 N

Dead load = 3.5 × 1000 × 9.81 = 34335 N

Maximum pressure on rails = 34335 + 10991 = 45326 N

Minimum pressure on rails = 64335 – 10991 = 23344 N

(v) Maximum speed of the locomotive without lifting the wheels from the rails will be when the dead load becomes equal to the hammer-blow.

i.e., 74.46 × 0.62 × $\omega^2$ = 34335

⇒ $\omega$ = 27.27 rad/s

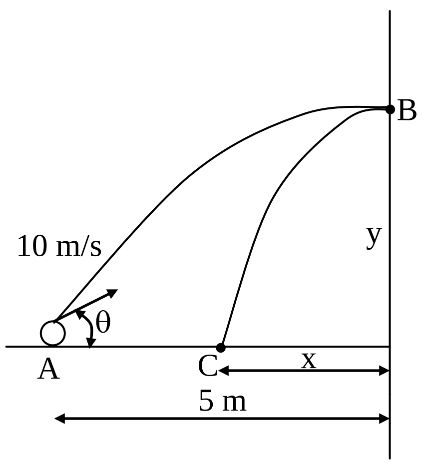
Velocity of wheels = $\omega$r = $\left(27.27\times\frac{1.80}{2}\right)$ m/s

$$= \left(27.27\times\frac{18}{2}\times\frac{60\times 60}{1000}\right) \text{ km/hr} = 88.36 \text{ km/hr}$$

**05(a).**

**Sol:** Let, B → Point on wall where ball strikes

C → Ball strikes ground after rebound.




From A to B:

$s_x = u_x.t + ½ .a_x.t^2$

∴ 5 = 10.cos60 × t + 0

∴ t = 1 sec

$s_y = u_y.t + ½ . a_y.t^2$

$y = 10.\sin 60 \times 1 - \frac{9.81}{2}\times 1^2 = 3.76\,\text{m}$

v = u + at

$v_{bx} = u_x + at$ = 10 cos 60 + 0 = 5 m/s

$v_{by}$ = 10 sin 60 -9.81 × 1 = – 1.15 m/s

At B, $e = \frac{u'_{Bx}}{5}$

$\therefore \quad u'_{Bx} = 0.8 \times 5 = 4 m/s$

Velocity in x-direction after rebound = 4 m/s

From B to C

$$y = u_{y.}t + \frac{1}{2}.g.t^2$$

$$3.76 = 1.15 \times t + \frac{9.81}{2} \times t^2$$

$\therefore \quad t = 0.766$ sec

$$x = u_x \times t + \frac{1}{2} \times a_x.t^2,$$

$x = 4 \times 0.766 + 0$

$x = 3.064$ m

**05(b).**

**Sol:** In response to the 75°C temperature, the steel piece elongates by the amount

$\delta_{steel} = \alpha_{steel}\Delta TL_{steel}$

$= (11.9 \times 10^{-6})(75)(300) = 0.267750$ mm

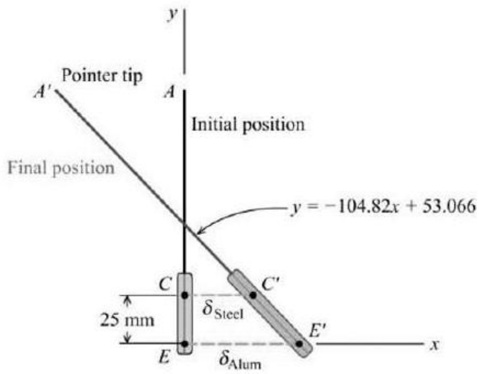
Thus, joint C moves to the right by 0.267750 mm

Next, calculate the deformation of the aluminum piece:

$\delta_{alum} = \alpha_{alum}\Delta TL_{Alum}$

$= (22.5 \times 10^{-6})(75)(300) = 0.50625$ mm

Joint E moves to the right by 0.50625 mm




Take the initial position of E as the origin. The coordinates of E in the deflected position are (0.50625 mm, 0) and the coordinates of C are (0.267750 mm, 25 mm). Use the deflected position coordinates of E and C to determine the slope of the pointer.

$$\text{slope} = \frac{y_C - y_E}{x_C - x_E} = \frac{25 - 0}{0.267750 - 0.50625}$$

$$= \frac{25}{-0.2385} = -104.821803$$

A general equation for the deflected pointer can be expressed as

$y = mx + b = -104.821803x + b$

Use the known coordinates of point E to determine b:

$0 = -104.821803(0.50625) + b$

$\therefore \quad b = 53.066038$ mm

Thus, the deflected pointer can be described by the line

$y = -104.821803x + 53.066038$ mm

or rearranging to solve for x

$$x = \frac{53.066038 - y}{104.821803}$$

At pointer tip A, y = 275 mm; therefore, the x coordinate of the pointer tip is

$$x = \frac{(53.066038 - 275)}{104.821803} = \frac{-221.933962}{104.821803}$$

$= -2.11725$ mm $= -2.12$ mm

The x coordinate is the same as the horizontal movement since we took the initial position of joint E as the origin.

**05(c).**

**Sol:** The given values are

$\varepsilon_a = \varepsilon_x = 875\ \mu$ m/m,

$\varepsilon_c = \varepsilon_y = 350\ \mu$ m/m,

$\varepsilon_b = \varepsilon_n = 700\ \mu$ m/m,

$$\theta_b = \tan^{-1}\left(\frac{4}{3}\right) = 53.130°$$

v = 0.30

$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta \cos\theta$

$\varepsilon_b = (875)\cos^2\theta_b + (350)\sin^2\theta_b + \gamma_{xy}\sin\theta_b \cos\theta_b = 700$

$\gamma_{xy} = 335.417\ \mu$ rad

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{335.417}{875-350}\right) = 16.287°,\ \ -73.713°$$

when $\theta_p = 16.287°$

$\varepsilon_n = (875)\cos^2\theta_p + (350)\sin^2\theta_p + (335.417)\sin\theta_p \cos\theta_p$

$= 924\ \mu \text{ m/m} = \varepsilon_{p1}$

$\varepsilon_{p2} = \varepsilon_x + \varepsilon_y - \varepsilon_{p1} = 301\ \mu$ m/m

$$\varepsilon_{p3} = \varepsilon_z = \frac{-v}{1-v}(\varepsilon_x + \varepsilon_y)$$

$$= \frac{-0.30}{1-0.30}(875+350)$$

$= -525\ \mu$ m/m

$\varepsilon_{p1} = +924\ \mu \text{ m/m} = 16.29°$

$\varepsilon_{p2} = +301\ \mu \text{ m/m} = 73.71°$

$\varepsilon_{p3} = -525\ \mu$ m/m

$\gamma_p = \varepsilon_{p1} - \varepsilon_{p2} = 623\ \mu$ rad

$\gamma_{max} = \varepsilon_{p1} - \varepsilon_{p3} = 1449\ \mu$ rad

$\varepsilon_n = (875)\cos^2(120°) + (350)\sin^2(120°) + (335.417)\sin(120°)\cos(120°)$

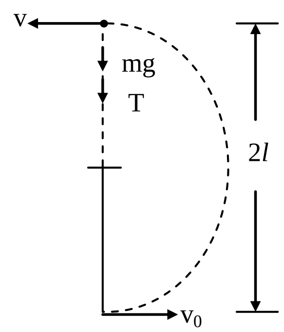
$\varepsilon_n = +336\ \mu$ m/m

**05(d).**

**Sol:** Suppose the bob is given a horizontal speed $v_0$ at the bottom and it describes a complete vertical circle. Let its speed at the highest point be v. Taking the gravitational potential energy to be zero at the bottom, the conservation of energy gives,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + 2mgl$$

or, $m^2 = mv_0^2 - 4mgl$ …………. (1)




The forces acting on the bob at the highest point+ are mg due to the gravity and T due to the tension in the string. The resultant force towards the centre is, therefore, mg + T. As the bob is moving in a circle, its acceleration towards the centre is $v^2/l$. Applying Newton's second law and using (i),

$$mg + T = m\frac{v^2}{\ell} = \frac{1}{\ell}(mv_0^2 - 4mg\ell)$$

or, $mv_0^2 = 5mgl + T\ell$.

Now, for $v_0$ to be minimum, T should be minimum. As the minimum value of T can be zero, for minimum speed,

$$mv_0^2 = 5mgl \text{ or, } v_0 = \sqrt{5g\ell}$$

**05(e).**

**Sol:** Given, N = 240 rpm,

$$\omega = \frac{2\pi \times 240}{60} = 25.14\,\text{rad/s}$$

Stroke = 300 mm = 0.3 m,

m = 50 kg, $m_1$ = 37 kg,

r = 150 mm = 0.15 m,

c = 2/3

(i) Balance mass required

Let,

B = balance mass required,

b = radius of rotation of the balance mass = 400 mm = 0.4 m

we know that,

$$B \times b = (m_1 + c.m) \times r$$

$$B \times 0.4 = \left(34 + \frac{2}{3} \times 50\right) \times 0.15 = 10.55$$

or B = 26.38 kg

(ii) Residual unbalanced force,

let,

θ = crank angle from top dead centre = 60°

we know that residual unbalanced force,

$$= m \times \omega^2 \times r\sqrt{(1-c)^2 \cos^2\theta + c^2 \sin^2\theta}$$

$$= 50 \times (25.14)^2 \times 0.15\sqrt{\left(1-\frac{2}{3}\right)^2 \cos^2 60^o + \left(\frac{2}{3}\right)^2 \sin^2 60^o}\ \text{N}$$

= 4740 × 0.601 = 2849 N

**06(a).**

**Sol:** Integrate the load distribution:

$$EI\frac{d^4v}{dx^4} = w_o \sin\frac{\pi x}{L}$$

$$EI\frac{d^3y}{dx^3} = -\frac{w_o L}{\pi}\cos\frac{\pi x}{L} + C_1$$

$$EI\frac{d^2y}{dx^2} = -\frac{w_o L^2}{\pi^2}\sin\frac{\pi x}{L} + C_1 x + C_2$$

$$EI\frac{dv}{dx} = \frac{w_o L^3}{\pi^3}\cos\frac{\pi x}{L} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EIv = -\frac{w_o L^4}{\pi^4}\sin\frac{\pi x}{L} + \frac{C_1 x^3}{6} + \frac{C_2 x^2}{2} + C_3 x + C_4$$

**Boundary conditions and evaluate constants:**

At x = 0, $\frac{dv}{dx} = 0$

$$\frac{w_o L^3}{\pi^3}\cos\frac{\pi(0)}{L} + \frac{C_1(0)^2}{2} + C_2(0) + C_3 = 0 \qquad \therefore C_3 = -\frac{w_o L^3}{\pi^2}$$

At x = 0, v = 0

$$-\frac{w_oL^4}{\pi^4}\sin\frac{\pi(0)}{L}+\frac{C_1(0)^3}{6}+\frac{C_2(0)^2}{2}+C_3(0)+C_4=0 \qquad \therefore\ C_4=0$$

At x = L, $\frac{dy}{dx}=0$

$$\frac{w_oL^3}{\pi^3}\cos\frac{\pi(L)}{L}+\frac{C_1(L)^2}{2}+C_2(L)-\frac{w_oL^3}{\pi^3}=0 \qquad \therefore C_1L+2C_2=\frac{4w_oL^2}{\pi^3}\ \text{----------------(1)}$$

At x = L, v = 0

$$-\frac{w_oL^4}{\pi^4}\sin\frac{\pi(L)}{L}+\frac{C_1(L)^3}{6}+\frac{C_2(L)^2}{2}-\frac{w_oL^3}{\pi^3}(L)=0 \quad \therefore C_1L+3C_2=\frac{6w_oL^2}{\pi^3}\ \text{---------------(2)}$$

Solve eqs. (1) and (2) simultaneously to obtain:

$$C_2=\frac{6w_oL^2}{\pi^3}-\frac{4w_oL^2}{\pi^3} \qquad \therefore C_2=\frac{2w_oL^2}{\pi^3}$$

$$C_1L=\frac{4w_oL^2}{\pi^3}-2\left(\frac{2w_oL^2}{\pi^3}\right) \qquad \therefore C_1=0$$

**Reactions at supports A and B**

$$V_A=EI\left.\frac{d^3v}{dx^3}\right|_{x=0}=-\frac{w_oL}{\pi}\cos\frac{\pi(0)}{L}=-\frac{w_oL}{\pi}$$

$$\therefore\ A_y=\frac{w_oL}{\pi}\downarrow$$

$$V_B=EI\left.\frac{d^3v}{dx^3}\right|_{x=L}=-\frac{w_oL}{\pi}\cos\frac{\pi(L)}{L}=\frac{w_oL}{\pi}$$

$$\therefore B_y=\frac{w_oL}{\pi}\downarrow$$

$$M_A=EI\left.\frac{d^2v}{dx^2}\right|_{x=0}=-\frac{w_oL^2}{\pi^2}\sin\frac{\pi(0)}{L}+\frac{2w_oL^2}{\pi^3}=\frac{2w_oL^2}{\pi^3}$$

$$\therefore\ M_A=\frac{2w_oL^2}{\pi^3}(cw)$$

$$M_B=EI\left.\frac{d^2v}{dx^2}\right|_{x=L}=-\frac{w_oL^2}{\pi^2}\sin\frac{\pi(L)}{L}+\frac{2w_oL^2}{\pi^3}=\frac{2w_oL^2}{\pi^3}$$

$$\therefore\ M_B=\frac{2w_oL^2}{\pi^3}(ccw)$$

**06(b).**

**Sol:** $\phi = 270$ , b = 54 mm,

$\mu = 0.20$, T = 200 N, D = 210 mm

(i) Dimension $c_1$ will just prevent back motion :

When frictional pulley developed

$$\frac{P_1}{P_2} \le \exp\ f(\phi)$$

$$= \exp\left[0.2\left(\frac{3\pi}{2}\right)\right] = 2.566$$

To have the band tighten for CCW rotation.

Sum moment about rocket pivot,

$\sum M = 0$

$C_3\,\omega + C_1 P - C_2 P = 0$

$$\Rightarrow \ \omega = \frac{C_2P_2 - C_1P_1}{C_3}$$

The device is self locking for CCW rotation if $\omega$ is no longer needed, that is $\omega \le 0$. It follow from the equation above $\frac{P_1}{P_2} \ge \frac{C_2}{C_1}$

when friction is fully developed

$$2.566 = \frac{56}{C_1} \ \Rightarrow C_1 = \frac{56}{2.566} = 21.8238$$

(ii) When rocket designed with $C_1$ = 25 mm

$$\frac{P_1}{P_2} = \frac{C_2}{C_1} = \frac{56}{25} = 2.24$$

$$f = \frac{\ell n\left(\frac{P_1}{P_2}\right)}{\phi} = \frac{\ell n(2.24)}{\frac{3\pi}{2}} = 0.171$$

$$T = (P_1 - P_2)\frac{D}{2} = P_2\left(\frac{P_1}{P_2} - 1\right)\frac{D}{2}$$

$$P_2 = \frac{2\pi}{\left(\frac{P_1}{P_2} - 1\right)\times D}$$

$$= \frac{2(200)}{(2.24-1)\times 210} = 262.5$$

$P_1$ = 2.25 ($P_2$) = 2.24 ×(262.5) = 588

$$P = \frac{2P_1}{b\times D}$$

$$= \frac{2\times 588}{54\times 210} = \frac{1176}{11340} = 0.1037$$

(iii) The torque ratio is

$$P_2 = \frac{262.5}{18.18} = 14.438$$

$$P_1 = \frac{588}{18.18} = 32.34$$

**06(c).**

**Sol:** m = 3000 kg, r = 0.45,

R = 80 m, h = 1m, w = 1.4m

Moment of inertia of wheel,

$$I_w = \frac{32}{2} = 16\text{kg.m}^2$$

Moment of inertia of motor, $I_m = 16\,\text{kg.m}^2$

(i) Reaction due to weight

$$R_W = \frac{mg}{4} = \frac{3000\times 9.81}{4} = 7357.5\,\text{N(upwards)}$$

(ii) Reaction due to gyroscopic couple:

$$C_W = 4I_W\frac{v^2}{r.R} = 4\times 16\times\frac{v^2}{0.45\times 250} = 0.569v^2$$

$C_m = 2I_mG\omega_w\omega_p$ ( as there are two motors)

$$= 2\times 16\times 3\times\frac{v^2}{0.45\times 250} = 0.853v^2$$

$C_G = C_w - C_m$

(motors rotate in opposite direction)

$= 0.569v^2 - 0.853v^2 = -0.284v^2$

Reaction on each outer wheel,

$$R_{G0} = \frac{C_G}{2w} = \frac{0.284v^2}{2\times1.4} = 0.1014v^2$$

(downwards)

Reaction on each outer wheel,

$$R_{Gi} = 0.1014v^2 \text{ (upwards)}$$

(iii) Reaction due to centrifugal couple:

$$C_c = \frac{mv^2}{R}h = 3000\times\frac{v^2}{250}\times1 = 12v^2$$

$$R_{c0} = \frac{C_c}{2w} = \frac{12v^2}{2\times1.4} = 4.286v^2 \quad \text{(upwards)}$$

$$R_{ci} = \frac{C_c}{2w} = 4.286v^2 \quad \text{(downwards)}$$

Total reaction on outer wheel

$= 7357.5 - 0.1014v^2 + 4.286v^2$

$= 7357.5 + 4.1846v^2$

Total reaction on outer wheel

$= 7357.5 - 0.1014v^2 - 4.286v^2$

$= 7357.5 - 4.1846v^2$

Thus, the reaction on the outer wheel is always positive (upwards). There are chances that the inner wheels leave the rails.

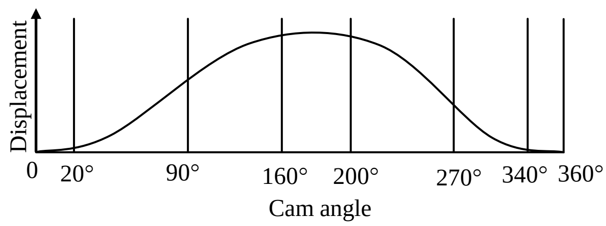
For maximum speed, $7357.5 - 4.1846v^2 = 0$

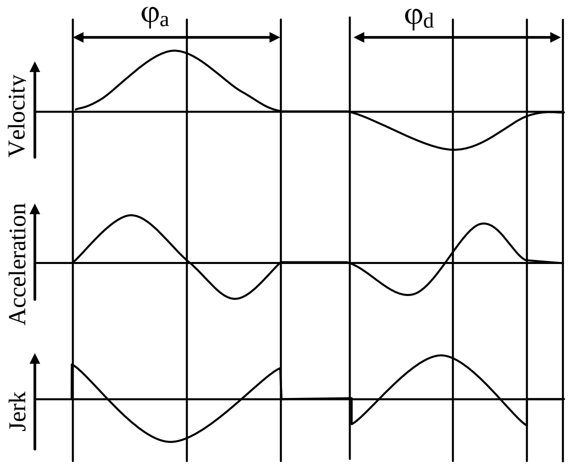
or $v^2 = 1758.2$, $v = 41.93$ m/s

or $$v = \frac{41.93\times3600}{1000} = 151\,\text{km/h}$$

**07(a)(i).**

**Sol:** From the plots of figure, it is observed that there are no abrupt changes in the velocity and the acceleration at any stage of the motion. The jerk therefore does not become infinite anywhere. This type of follower motion is, therefore, equally suitable at high speeds.




**07(a)(ii).**

**Sol:** Given, $\phi = 20°$, $t = 20$,

$$G = \frac{T}{t} = 2,$$

m = 5 mm,

v = 1.2 m/s,

Addendum = 1 module = 5 mm

(i). Angle turned through by pinion when one pair of teeth is in mesh

we know that pitch circle radius of pinion,

$$r = \frac{m.t}{2} = \frac{5\times20}{2} = 50\,\text{mm}$$

and pitch circle radius of wheel,

$$R = \frac{m \times T}{2} = \frac{m.G.t}{2}$$

$$= \frac{2 \times 20 \times 5}{2} = 100 \text{ mm} \quad (\because T = Gt)$$

∴ Radius of addendum circle of pinion,

$r_A$ = r + Addendum = 50 + 5 = 55 mm

Radius of addendum circle of wheel,

$R_A$ = R + Addendum = 100 + 5 = 105 mm

we know that length of the path of approach (i.e., the path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(105)^2 - (100)^2\cos^2 20^o} - 100\sin 20^o$$

= 46.85 – 34.2 = 12.65 mm

The length of path of recess (i.e., the path of contact when disengagement occurs).

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(55)^2 - (50)^2\cos^2 20^o} - 50\sin 20^o$$

= 28.6 – 17.1 = 11.5 mm

∴ Length of the path of contact,

KL = KP + PL = 12.65 + 11.5 = 24.15 mm

Length of the arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

$$= \frac{24.15}{\cos 20^o} = 25.7 \text{ mm}$$

We know that angle turned through by pinion

$$= \frac{\text{Length of arc of contact} \times 360^o}{\text{Circumference of pinion}}$$

$$= \frac{25.7 \times 360^o}{2\pi \times 50} = 29.45^o$$

(ii) Maximum velocity of sliding,

Let $\omega_1$ = angular speed of pinion, and

$\omega_2$ = angular speed of wheel,

We know that pitch line speed,

$v = \omega_1 r = \omega_2 R$

$$\omega_1 = \frac{v}{r} = \frac{120}{5} = 24 \text{ rad/s}$$

$$\omega_2 = \frac{v}{R} = \frac{120}{10} = 12 \text{ rad/s}$$

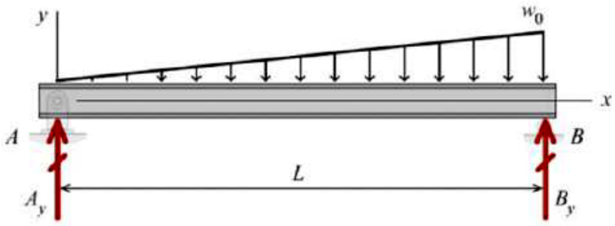
∴ Maximum velocity of sliding,

$$v_s = (\omega_1 + \omega_2)KP$$

= (24 + 12) × 12.65 = 455.4 mm/s

**07(b).**

**Sol:**



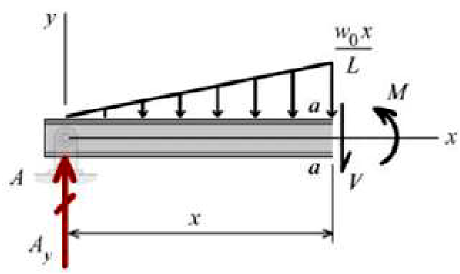

**Beam equilibrium:**

$$\sum M_B = -A_y L + \frac{w_o L}{2}\left(\frac{L}{3}\right) = 0$$

$$\therefore A_y = \frac{w_o L}{6}$$

$$\sum M_A = +B_Y L - \frac{w_o L}{2}\left(\frac{2L}{3}\right) = 0$$

$$\therefore B_y = \frac{w_o L}{3}$$



**Section a-a:**

$$\sum F_y = A_y - \frac{w_o x}{L}\left(\frac{x}{2}\right) - V$$

$$= \frac{w_o L}{6} - \frac{w_o x^2}{2L} - V = 0$$

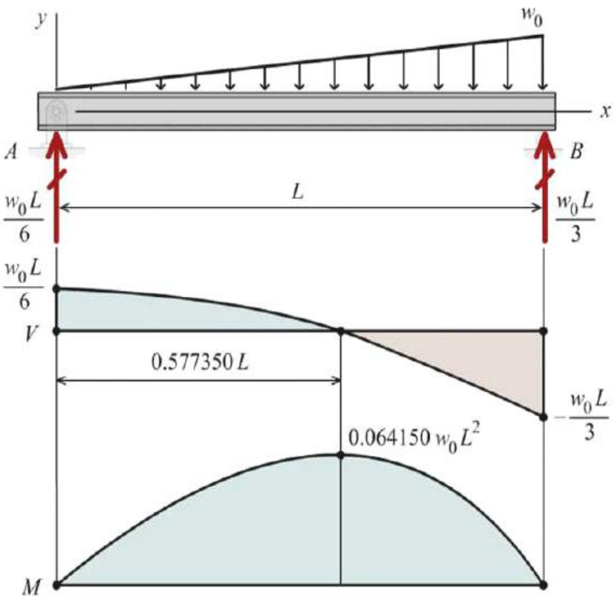
$$\therefore\ V = \frac{w_o L}{6} - \frac{w_o x^2}{2L}$$

$$\sum M_{a-a} = -A_y x + \frac{w_o x}{L}\left(\frac{x}{2}\right)\left(\frac{x}{3}\right) + M$$

$$= -\frac{w_o L x}{6} + \frac{w_o x^3}{6L} + M = 0$$

$$\therefore\ M = -\frac{w_o x^3}{6L} + \frac{w_o L x}{6}$$

**(b) Shear-force and bending moment diagrams**




**07(c)(i).**

**Sol:**

**(A)** Centre of mass and centroid

- Centre of mass is the centre of gravity and it is a point where entire mass of body is considered to be concentrated.
- Centroid is the geometric centre of the body.
- If the object has uniform density, then both will be at same point.
- In general, centre of mass is applied to 3–D bodies, i.e bodies with a mass, while centroid is applied to plane areas

**(B)** Mass M.I and Area M.I

- Mass M.I represents distribution of mass, and area moment of inertia represent distribution of area.
- Mass M.I represents inertia of body to angular rotation and area moment of inertia represents property which is useful against bending loading, deflection.

**07(c)(ii).**

**Sol:** The portion of the strings between the ceiling and the cylinder is at rest. Hence the points of the cylinder where the strings leave it are at rest. The cylinder is thus rolling without slipping on the strings. Suppose the centre of the cylinder falls with an acceleration a. The angular acceleration of the cylinder about its axis is $\alpha = a/R$, as the cylinder does not slip over the strings.

The equation of motion for the centre of mass of the cylinder is

$$mg - 2T = ma \text{ ………. (i)}$$

and for the motion about the centre of mass, it is

$$2Tr = \left(\frac{1}{2}mr^2\alpha\right) = \frac{1}{2}mra$$

or, $$2T = \frac{1}{2}ma. \text{ ……….. (ii)}$$

From (i) and (ii)

$$a = \frac{2}{3}g \text{ and } T = \frac{mg}{6}$$

As the centre of the cylinder starts moving from rest, the velocity after it has fallen through a distance h is given by

$$v^2 = 2\left(\frac{2}{3}g\right)h$$

or, $$v = \sqrt{\frac{4gh}{3}}$$

**08(a).**

**Sol: Section properties:**

$A = 120 \times 160 = 19{,}200 \text{ mm}^2$

$$I_x = \frac{120 \times (160)^3}{12} = 40.960 \times 10^6 \text{ mm}^4$$

$$I_z = \frac{160 \times (120)^3}{12} = 23.040 \times 10^6 \text{ mm}^4$$

**Equivalent forces K:**

F = 210 kN = 210,000 N

$V_x$ = –65 kN = –65,000 N

$V_z$ = –95 kN = –95,000 N

$M_x$ = –(95)(150) – (210)(50)

= –24750 kN.mm

= $-24.750 \times 10^6$ N-mm

$M_x = -65 \times 150 = 9{,}750 = 9.750 \times 10^6$ N-mm

Axial stress at K due to F:

$$\sigma_{axial} = \frac{210{,}000}{19{,}200} = 10.938 \text{ MPa(C)}$$

Shear stress at K due to $V_x$:

$Q_k = 0 \text{ mm}^3$

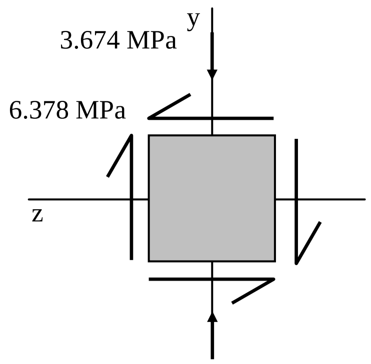
$\therefore\ \tau_k = 0$ MPa

Shear stress at K due to $V_z$:

$Q_k = 120 \times 50 \times 55 = 330{,}00 \text{ mm}^3$

$$\tau_k = \frac{95{,}000 \times 330{,}000}{(40{,}960 \times 10^6)(120)} = 6.378 \text{ MPa}$$

Bending stress at K due to $M_x$:




$$\sigma_{bendx} = \frac{M_x z}{I_x} = \frac{(24.750 \times 10^6)(30)}{40.960 \times 10^6} = 18.127 \text{ MPa(C)}$$

Bending stress at K due to $M_z$:

$$\sigma_{bendz} = \frac{M_z x}{I_z} = \frac{(9.750 \times 10^6)(60)}{23.040 \times 10^6} = 25.391 \text{ MPa(T)}$$

Summary of stresses at K:

$\sigma_z = 0$ MPa

$\sigma_y = -10.938 - 18.127 + 25.391$

$= -3.674$ MPa

$\tau_{yz} = -6.378$ MPa

**Principal stress calculations:**

The principal stress magnitudes can be computed from equation. For use in this equation, the negative z axis will be taken as the x axis, which causes the shear stress value to change sign.

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{0 + (-3.674)}{2} \pm \sqrt{\left(\frac{0 - (-3.674)}{2}\right)^2 + (6.378)^2}$$

$= -1.837 \pm 6.637$

$\sigma_{p1} = 4.80$

and $\sigma_{p2} = -8.48$ MPa

$\tau_{max}$ = 6.64 MPa (maximum in-plane shear stress)

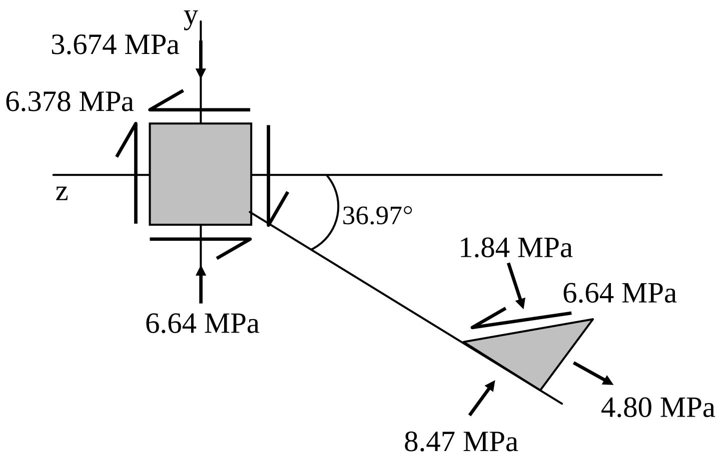
$\sigma_{avg}$ = 1.837 MPa (C) (normal stress on planes of maximum in-plane shear stress)

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$= \frac{6.378}{[0 - (-3.674)]/2}$$

$$= \frac{6.378}{1.837} = 3.471761$$

$\therefore$ $\theta_p$ = 36.97° (counter clockwise from the z axis to the direction of $\sigma_{p1}$)




**08(b)(i).**

**Sol:** The internal pressure induces circumfrential stress on the plate.

The pressure varies from 0 to 6 N/mm$^2$.

Circumfrential stress = $\sigma_c = \frac{P.D}{4t}$

$(\sigma_C)_{max}$ = Max. circumfrential stress

$$= \frac{P_{max}.D}{4t}$$

$$= \frac{6 \times 500}{4 \times t} = \frac{750}{t}$$

$(\sigma_C)_{min}$ = Min. circumfrential stress

$$= \frac{P_{min}.D}{4t} = 0$$

Since $(\sigma_C)_{min}$ = 0, the type of stress is repeated.

$\sigma_y$ = yield stress = 242 N/mm$^2$

Let $K_f = 1$, i.e., $K_t = 1$

$$\sigma_{mean} = \sigma_{amp} = \frac{(\sigma_C)_{max}}{2} = \frac{750}{t} \times \frac{1}{2} = \left(\frac{375}{t}\right) N/mm^2$$

$$\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + K_f \frac{\sigma_a}{\sigma_{-1}} \qquad [\ \sigma_{-1} = 0.5\sigma_u]$$

$$\frac{1}{3.5} = \frac{375}{t \times 242} + \left(\frac{375}{t \times 220}\right)$$

$$\frac{1}{3.5} = \frac{1.549}{t} + \frac{1.704}{t}$$

(It is repeated comp)

t = 11.38 mm

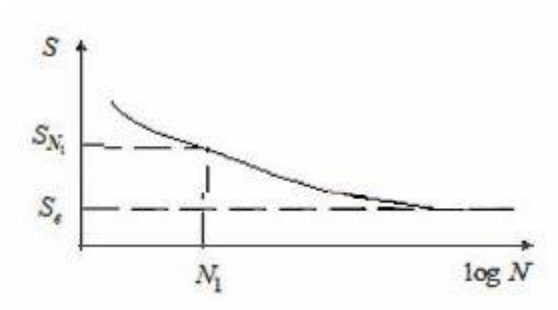
To find endurance strength for finite life.

$$\sigma_f = \sigma_{-1}\left(\frac{10^6}{N}\right)^{0.09}$$

N = the required life in cycles.

**08(b)(ii).**

**Sol:** A typical S-N curve has the appearance shown.




***Defining terms:***

$S_{N_1} = S_{N=N_1}$ = fatigue strength corresponding to $N_1$ cycles of life.

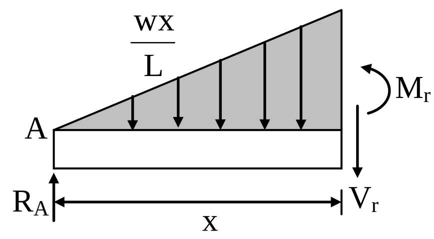
$S_e$ = $S_{N=\infty}$ = fatigue endurance limit; corresponding to strength asymptote (if one exists) to the S-N curve.

A designer might use an S-N curve as follows:

- Select an appropriate design life, say $N_d = N_1$
- Read up from $N_1$ and left to $S_{N_1}$, which is the fatigue strength corresponding to the selected design life.
- Determine the design stress as $\sigma_d = S_{N_1}/n_d$, where $n_d$ is the design factor of safety.
- Configure the part so that stress at the most critical location in the part does not exceed the design stress $\sigma_d$.

**08(c)(i).**

**Sol:**




From overall equilibrium,

$$R_A = \left(\frac{M_B}{L} + \frac{wL}{6}\right)\uparrow$$

Then

$$M_r = R_A x - \frac{wx^3}{6L} = \frac{M_B x}{L} + \frac{wLx}{6} - \frac{wx^3}{6L}$$

$$\left(\frac{\partial M_r}{\partial M_B} = \frac{x}{L}\right)$$

$$\theta_B = \frac{1}{EI}\int_0^L M_r \frac{\partial M_r}{\partial M_B} dx$$

$$= \frac{1}{EI}\int_0^L \left(\frac{M_B x^2}{L^2} + \frac{wx^2}{6} - \frac{wx^2}{6L^2}\right) dx$$

$$= \frac{1}{EI}\left[\frac{M_B x^3}{3L^2} + \frac{wx^3}{18} - \frac{wx^5}{30L^2}\right]_0^L = \frac{-wL^3}{24EI}$$

$$M_B = -\frac{23wL^2}{120} = \frac{23wL^2}{120}(CW)$$

$$R_A = \frac{M_B}{L} + \frac{wL}{6} = \frac{-23wL}{120} + \frac{wL}{6}$$

$$= \frac{-wL}{40} = \frac{wL}{40}\downarrow$$

**08(c)(ii).**

**Sol:** The four main categories are:

1. Hydrodynamic lubrication
2. Boundary Lubrication
3. Hydrostatic lubrication
4. Solid film lubrication

*Hydrodynamic lubrication* is characterized by a rotating shaft in an annular journal bearing so configured that a viscous lubricant may be "pumped" into the wedge shaped clearance space by the shaft rotation to maintain a stable thick fluid film through which asperities of the rotating shaft cannot contact surface asperities of the journal.

*Boundary lubrication* may be characterized by a shaft and journal bearing configuration in which the surface area is too small or too rough, or if the relative velocity is too low, or if temperatures increase too much (so the velocity is lowered too much), or if loads become too high, asperity contacts may be induced through the (thin) oil film.

*Hydrostatic lubrication* may be characterized by a pair of sliding surfaces in which a thick lubricant film is developed to separate the surfaces by an external source of pressurized lubricant.

*Solid film lubrication* may is characterized by bearing for which dry lubricants, such as graphite or molybdenum disulfide or self lubricating polymers such as Teflon or nylon are used.