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ESE – 2019 MAINS
OFFLINE TEST SERIES

MECHANICAL ENGINEERING

TEST – 9
SOLUTIONS

All Queries related to **ESE – 2019 MAINS Test Series** Solutions are to be sent to the following email address
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01(a).

Sol: Biomass Resources :

Biomass resources are classified into five types. They are:

1. Forest
2. Agricultural wastes
3. Energy crops
4. Aquatic plants
5. Urban waste

1. Forest

Forests are the rich source of fuel, wood, charcoal and producer gas.

2. Agricultural Wastes

Crop residues such as straw, rice husk and waste wood are pressed to form lumps, known as fuel pellets and are used as solid fuel.

3. Energy Crops

Sugarcane, sweet sorghum, sugar beet, starch plants, cassava, oil producing plants are used as bio-resources.

4. Aquatic Plants

The water plants which provide raw materials for producing biogas or ethanol are water hyacinth, kelp, seaweed and algae.

5. Urban Waste

Urban wastes are of two types:

(i) Municipal Solid Waste (MSW)

(ii) Sewage (liquid waste)

Energy from MSW can be obtained from direct combustion (incineration) and from sewage, biogas can be obtained after some processing.

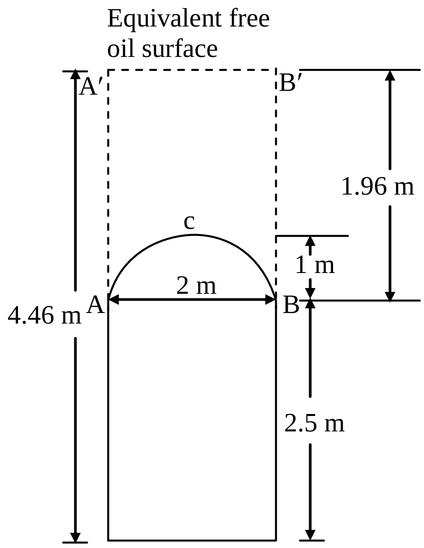
01(b).

Sol: Specific weight of oil = 0.80×9.81

$= 7.848 \text{ kN/m}^3$

Pressure head corresponding to a pressure of

$$35 \text{ kPa} = \frac{35}{7.848} = 4.46 \text{ m of oil}$$



The above figure shows the equivalent free oil surface at a height of 4.46 m above the base of the tank.

Upward force exerted by oil on the roof of the tank = weight of the block of oil A′B′BCA.

= 7.848×(rectangle A′B′BA–semi circle ACB)

$$= 7.848 \times 3\left[1.96 \times 2 - \frac{\pi \times 1^2}{2}\right] \text{kN}$$

= 55.31 kN

01(c).

Sol:

(i) **Fully developed flow:** It is used to describe the region in the flow where the axial point velocity profiles are not changing with axial distance.

It is the region in the flow where negligible changes in static pressure gradient with the flow direction are negligible and down stream from the point where the developing boundary layers meet.

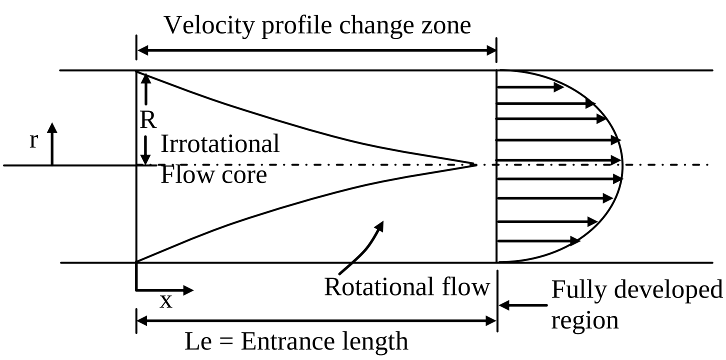
(ii) **Transition flow:**

This is in between laminar and turbulent flows, difficult to study analytically (or) experimentally, because of its unstable nature. The shift appeared to be dependent upon Reynolds number and size of flow. For analytical analysis, it is treated as turbulent flow. In this region both viscous and Reynolds stresses are of equal magnitude. The friction factor is not only function of Reynolds number, but also pipe roughness height.

(iii) **Entrance length of a pipe flow:**

It is concerned in fluid dynamics. It is the length (or) distance a flow travels after entering a pipe before the flow becomes fully developed.

The region from the pipe inlet to the point at which the boundary layer merges at the centreline is called entrance length (Le) beyond this flow become fully-developed flow.



$$Le_{\text{laminar pipe flow}} = 0.05Re.D$$
$$= 115\ D \quad (\text{for } Re = 2300)$$
$$Le_{\text{turbulent}} = 1.36\ Re^{1/4}.D$$

(iv) **Laminar sub-layer thickness:**

It is the region observed in turbulent flow, near the no-slip boundary and in which the flow is laminar. Viscous forces dominate over the inertia forces. It is experimentally given as

$$\delta' = \frac{11.6\nu}{V^*}$$

where, ν = Kinematic viscosity,

V^* = Shear or friction velocity

Approximate laminar sub-layer thickness

$$= \frac{5\nu}{V^*}$$

01(d).

Sol: Flow = Cylindrical free vortex, i.e., V = C/r

$$\text{Given } \frac{P_2 - P_1}{\gamma_{water}} = 0.05\,m$$

$$\text{So, } \frac{P_2}{\gamma_{air}} - \frac{P_1}{\gamma_{air}} = 0.05\left(\frac{9810}{12}\right)$$

$$= 40.875 \text{ m of air}$$

$$\text{But } \frac{P_2}{\gamma_{air}} - \frac{P_1}{\gamma_{air}} = \frac{C^2}{2g}\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) \text{ (For the bend)}$$

Thus, for the bend with

$r_1 = 1.8$ m and $r_2 = 3.0$ m:

$$40.875 = \frac{C^2}{2(9.81)}\left(\frac{1}{1.8^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \quad C = 63.72\ m^2/s$$

Thus noting that V = C/r and dA = B dr, where B is the width of the duct,

$$Q = \int V dA = BC\int_{r_1}^{r_2} dr/r = BC\ell n(r_2/r_1)$$

$$= 1.2(63.72)\ \ln(3.0/1.8)$$

$$= 39.06\ m^3/s = 2343.6\ m^3/min$$

01(e).

Sol: Impeller diameter, D = 0.76 m

Mass flow rate of air, $\dot{m} = 35\ \text{kg/sec}$

Compressor efficiency, $\eta_{comp} = 0.8$

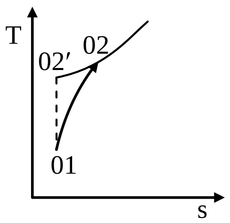
Pressure ratio, $\frac{P_{02}}{P_{01}} = 4.2$

Radial velocity at impeller tip,

$V_r = 120\,\text{m/sec}$

Inlet pressure, $P_{01} = 1\text{bar}$

Inlet temperature, $T_{01} = 273 + 47 = 320\,\text{K}$



$$T_{02'} = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 320(4.2)^{\frac{1.4-1}{1.4}} = 482.4\text{K}$$

$$T_{02} = T_{01} + \frac{T_{02'} - T_{01}}{\eta_{comp}}$$

$$= 320 + \frac{482.4 - 320}{0.8} = 523\,\text{K}$$

$$\rho_2 = \frac{P_{02}}{RT_{02}} = \frac{4.2\times10^5}{287\times523} = 2.8\,\text{kg/m}^3$$

$$\dot{m} = \rho_2 A_{tip} V_r$$

$$A_{tip} = \frac{\dot{m}}{\rho_2 V_r}$$

$$= \frac{35}{2.8\times120} = 0.1042\ \text{m}^2$$

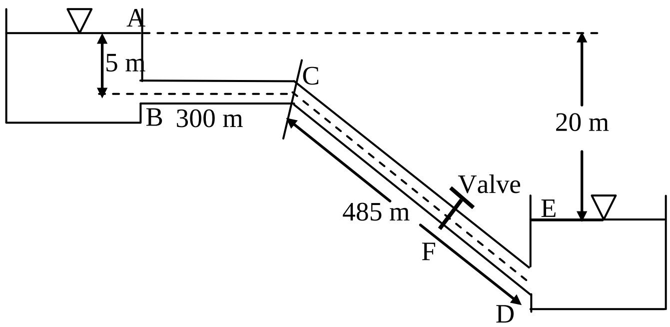
$$\pi D W = A_{tip}$$

$$W = \text{Axial width} = \frac{A_{tip}}{\pi D}$$

$$= \frac{0.1042}{\pi\times0.76} = 4.36\ \text{cm}$$

02(a).

Sol:



$$\frac{P_c}{\gamma_{water}} = \text{Pressure head at C} = \frac{3.77}{9.81}$$

$= 0.3843$ m of water

By Bernoulli's Equation,

Energy head at A = Energy head at C + loss at entry to pipe BC + friction loss in pipe BC

$$5 = \frac{P_c}{\gamma} + \frac{V_1^2}{2g} + 0.5\frac{V_1^2}{2g} + h_{f1}$$

where V_1 is the velocity of water flowing in pipe BC.

$$5 = \frac{p_c}{\gamma} + 1.5\frac{V_1^2}{2g} + h_{f1}$$

$$5 = 0.3843 + 1.5\frac{V_1^2}{2g} + \frac{f_1 L_1 V_1^2}{2gd_1}$$

$$= 0.3843 + \frac{V_1^2}{2g}\left[1.5 + \frac{f_1 L_1}{d_1}\right]$$

$$4.6157 = \frac{V_1^2}{2g}\left[1.5 + \frac{f_1\times300}{0.25}\right]$$

$$4.6157\times2\times9.81 = V_1^2[1.5 + 1200f_1]$$

$$90.56 = V_1^2[1.5 + 1200f_1] \;\ldots\ldots\; (1)$$

There are two unknowns, (V_1 & f_1).

To solve by trial and error method.

Trial 1

Let $f_1 = 0.02$

$$90.56 = V_1^2(1.5 + 1200\times0.02)$$

$$= V_1^2\times25.5$$

$$V_1^2 = \frac{90.56}{25.5}$$

$V_1 = 1.8845$ m/s

$$Re_1 = \frac{\rho_1 V_1 d_1}{\mu} = \frac{10^3 \times 1.8845 \times 0.25}{1.002 \times 10^{-3}}$$

$= 4.702 \times 10^5$

Using Halland equation (given):

$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{Re} + \left(\frac{\varepsilon/d}{3.7}\right)^{1.11}\right]$$

where f = New value of f_1 after first trial and $Re = Re_1$

$$= -1.8\log\left[\frac{6.9}{4.702\times10^5} + \left(\frac{\frac{0.045}{250}}{3.7}\right)^{1.11}\right]$$

$= -1.8 \log [1.4675\times10^{-5} + 1.6317\times10^{-5}]$

$= 8.11574$

$\sqrt{f} = 0.1232$

$f = 0.0152$ which is different from f_1.

Trial 2

with new value of f = 0.0152

$$90.56 = V_1^2(1.5 + 1200 \times 0.0152)$$

$\Rightarrow V_1 = 2.142$ m/s

$$\Rightarrow Re = \frac{10^3 \times 2.142 \times 0.25}{1.002 \times 10^{-3}}$$

$= 5.344 \times 10^5$

Thus,

$$\frac{1}{\sqrt{f}} = -1.8\log\left[\frac{6.9}{5.334\times10^5} + \left(\frac{\frac{0.045}{250}}{3.7}\right)^{1.11}\right]$$

$= -1.8 \log [1.2912 \times 10^{-5} + 1.6317 \times 10^{-5}]$

$\Rightarrow$ f = 0.015 which is close to f_1 obtained after Trial 1.

Hence, $f_1 = 0.015$

therefore, $V_1^2 = \dfrac{90.56}{1.5 + 1200 \times 0.015} = 4.6441$

$V_1 = 2.155$ m/s

$$h_{f1} = \frac{f_1 L_1 V_1^2}{2gd_1} = \frac{0.015 \times 300 \times 2.155^2}{2\times9.81\times0.25} = 4.2606\text{ m}$$

For the flow from A to E, by energy equation,

$$20 = 0.5\frac{v_1^2}{2g} + h_{f1} + h_{f2} + h_L + h'_L$$

where, h_L = head lost at the valve = 6 m

and h'_L = loss at entrance to reservoir D

$$20 = 0.5\frac{V_1^2}{2g} + 4.2606 + h_{f2} + 6.05 + \frac{V_1^2}{2g}$$

$$= \frac{V_1^2}{2g}[0.5 + 1] + 4.2606 + h_{f2} + 6.05$$

$$= \frac{1.5\times2.155^2}{2\times9.81} + 10.3106 + h_{f2}$$

$\therefore$ $h_{f2} = 9.3344$

$$\therefore \quad = \frac{f_2 L_2 V^2}{2gd}$$

$$f_2 = \frac{9.3344\times2\times9.81\times0.25}{485\times2.155^2} = 0.02033$$

OR

Using Halland's equation for the pipe CD:

$$\frac{1}{\sqrt{f_2}} = -1.8\log\left[\frac{6.9}{Re_2} + \left(\frac{\varepsilon_2/d}{3.7}\right)^{1.11}\right]$$

where $\quad Re_2 = \dfrac{10^3 \times 2.155 \times 0.25}{1.002 \times 10^{-3}}$

$= 5.37675 \times 10^5$

$\varepsilon_2 = 0.26$ mm, $\quad$ d = 250 mm

Thus,

$$\frac{1}{\sqrt{f_2}} = -1.8\log\left[\frac{6.9}{5.37675\times10^5} + \left(\frac{0.26}{250\times3.7}\right)^{1.11}\right]$$

$= -1.8 \log [1.2833\times10^{-5} + 1.1434\times10^{-4}]$

$= -1.8 \log [1.27173\times10^{-4}] = 7.0121$

$\Rightarrow$ f = 0.0203

02(b).

Sol: Solar energy storage systems may be classified as below:

1. Thermal Energy Storage

- Thermal energy rise in can be stored in two ways that is sensible heat storage and latent heat storage.
- In sensible storage, heat is accumulated by the rise in temperature of the metal
- In the latent heat storage, the heat energy is stored, when the medium undergoes a phase transformation.

2. Electrical Storage

Large amount of electrical energy is stored in capacitors. Mostly batteries and capacitors are used to store energy. The expression for the total energy storage is given as:

$$H_{CAP} = \frac{1}{2} V \in E^2$$

where,

V– Volume of the dielectric

ε – Constant

E – Electric field strength

3. Chemical Storage

It is classified into two types. They are:

(i) Storage in the Form of Fuel

In this, the reactant in the battery is generated by a photochemical reaction promoted by solar radiation. This photochemically charged battery discharges electrical energy whenever needed. The photochemical reaction is shown below.

$$2NOC\ell + \text{Photons} \rightarrow 2NO + C\ell_2$$

$$AgC\ell(s) + \text{Photons} \rightarrow Ag(s) + \frac{1}{2}C\ell_2$$

$$NO_2 + \text{Photons} \rightarrow NO + \frac{1}{2}O_2$$

$$H_2C + \frac{1}{2}O_2 \xrightarrow{Z_nO} H_2O_2$$

(ii) Thermo-chemical Energy Storage

In this type of storage, some reversible chemical reactions are used for storage for high temperatures. In forward reaction heat is absorbed and stored in the form of products and heat is liberated in the reversible reaction.

$$A + B \rightleftharpoons AB$$

4. Mechanical Energy Storage

This is classified into three types:

(i) Pumped Hydroelectic Storage:

In this the pumped water at higher heads is allowed to flow through a hydraulic turbine, which drives an electric generator.

(ii) Compressed Air Storage

In this, air is pumped into a suitable pressurized, storage tank and used, when the wind is unavailable. This air drives an air turbine, which in turn drives a generator.

(iii) Flywheel Storage

In this, the rotation of the flywheel can be used to operate a generator to produce electricity, whenever required.

5. Electromagnetic Energy Storage

In this, super conducting .materials at lower temperatures are used, because they possess low resistance at lower temperatures and thus can carry strong electric currents with negligible loss. Due to the flow of electric current, an electromagnetic field is produced which can store energy.

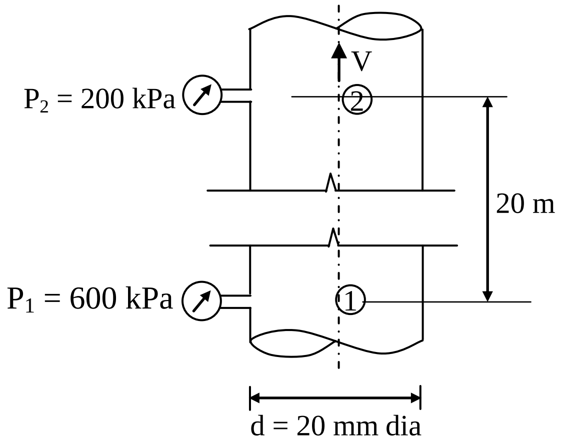
02(c).

Sol:

- Pressure gauges are provided to record pressure of main steam at the stop valve, in the steam chest, at the first stage and exhaust, the oil pressure to the bearings, the governing mechanism and pressure of steam or water to gland seals. For a condensing turbine, a vacuum gauge and a barometer are installed.
- Thermometers are provided to record steam temperature at stop valve, in the steam chest, at the first stage and at the gland. The oil temperatures entering and leaving the bearings are noted.
- A speed and cam shaft position recorder is required to record the turbine speed in rpm. During operation the turbine speed is obtained from the generator frequency recorder. The speed recorder is used to record cam shaft position, which determines opening of the valve and load on the turbine.
- An eccentricity recorder is provided to indicate and record the eccentricity of shaft at high pressure end of turbine.
- A vibration amplitude recorder is provided to record vibration of rotor.
- An expansion indicator is provided on turbine control board to show the axial expansion of turbine casing.
- A noise meter on control board is used to pick up and amplify the noise made by moving parts of the turbine.
- Flow metres are mounted on the turbine control board to indicate, record and integrate the mass flow rate of steam, steam bled at various points and flow to the condenser.
- Watt meters, voltmeters and ammeters are also provided on turbine control board, which along with flow meters are used to determine steam and heat rates of the unit.
- Hand wheels to operate various drain valves are located at turbine or on turbine board.
- Governor controls are located at turbine or on turbine control board for proper regulation of valves.
- A trip lock lever for testing the over speed trip is usually mounted on turbine control board.

03(a).

Sol: Assume initially basic flow i.e. laminar flow.



Apply energy equation:

$$\frac{P_1}{\rho g}+Z_1+\frac{V_1^2}{2g}-h_{loss}=\frac{P_2}{\rho g}+Z_2+\frac{V_2^2}{2g}$$

($Z_1 = 0$, $V_1 = V_2$, since pipe dia. constant.)

$$\frac{P_1-P_2}{\rho g} = Z_2+h_{loss\,(laminar)};$$

$$(h_{loss})_{laminar} = \frac{32\mu V\ell}{\rho g d^2} = \frac{128\mu.Q.\ell}{\pi d^4.\rho.g};$$

$$Z_1 - Z_1 = \ell = 20 \text{ m}$$

Q is the discharge of oil through the pipe.

$$\frac{600000-200000}{900\times 10} = 20+\left(\frac{128\mu.Q.\ell}{\pi d^4 \rho.g}\right)$$

$$44.44 = 20 + \frac{128 \times (150 \times 10^{-2} \times 0.1) Q \times 20}{\pi (0.02)^4 \times 900 \times 10}$$

$$44.44 = 20 + \frac{763.944 \times 10^6 Q}{9000}$$

$$44.44 = 20 + 84.883 \times 10^3 Q$$

$$\therefore \quad Q = \frac{44.44 - 20}{84.883 \times 10^3}$$

$Q = 2.88\times10^{-4}$ m^3/sec = 0.288 $l$ps

$\therefore$ Now Reynolds number = $\dfrac{\rho.V.d}{\mu} = \dfrac{4\rho Q}{\pi \mu d}$

$$= \frac{4 \times 900 \times 2.88 \times 10^{-4}}{\pi \times (150 \times 10^{-3}) \times 0.02}$$

$= 110 < 2000 \Rightarrow$ Laminar flow in the pipe

Hence, assumption is correct

$\therefore$ Mass flow rate $(\dot{m}) = \rho Q$

$= 900 \times 2.88\times10^{-4} \cong 0.26$ kg/sec

03(b).

Sol:

(i) Cut-in Speed

- Cut-in wind speed is the minimum wind speed at which the brakes are released and the prime mover starts rotating.
- The generator thus starts generation of power. It is the lower wind speed limit for the turbine to operate safely.

(ii) Cut - out Speed

- Cut-out wind speed is the maximum wind speed above which the turbine or the prime mover runs at a speed beyond their limits.
- Wind speed above the cut-out speed affects the mechanical capacity, which may be hazardous.
- It is the upper speed limit beyond which the turbine operation would be unsafe.

(iii) Yaw Control

- Yaw control is for the rotor according to the direction of wind, by controlling the nacelle about the vertical axis.
- In small wind turbines, the yaw control is achieved using a tail vane.
- In large turbines, a servo mechanism operated by a wind direction sensor is adopted.

(iv) Coefficient of Performance of a Wind Mill

Coefficient of performance of a wind mill is the ratio of rotor output power to the maximum wind power available.

Coefficient of performance,

$$C_p = \frac{\text{Rotor output power}\,(P)}{\text{Maximum wind power avaiable}\,(P_t)}$$

$$C_p = \frac{P}{\frac{1}{2}\rho A V^3}$$

where,

ρ - is the density of the air (~1.2 kg/m^3)

A - Swept area (m^2)

V - Velocity of the wind (m/s)

03(c).

Sol: Given data:

$P_{um} = 9.2\ \text{kW/m}^{3/2},\ H_m = 8\ \text{m},$

$Q_{um} = 1.25 \dfrac{\text{m}^3/\text{s}}{\sqrt{\text{m}}}, \quad N_{um} = 90 \dfrac{\text{rpm}}{\sqrt{\text{m}}}$

$\dfrac{D_m}{D_p} = \dfrac{1}{4}; \qquad H_p = 24\,\text{m}$

$$P_u = \frac{P}{H^{3/2}}, Q_u = \frac{Q}{\sqrt{H}}, N_u = \frac{N}{\sqrt{H}}$$

$$\Rightarrow P_m = P_{um} \times H_m^{3/2} = 9.2 \times 8^{3/2} = 208.2\,\text{kW}$$

$$Q_m = Q_{um} \times \sqrt{H_m} = 1.25 \times \sqrt{8} = 3.536\,\text{m}^3/\text{s}$$

$$N_m = N_{um} \times \sqrt{H_m} = 90 \times \sqrt{8} = 254.6\,\text{rpm}$$

$$\eta_m = \frac{P_m}{\rho g Q H} = \frac{208.2\times10^3}{9810\times3.536\times8} = 0.75$$

By similarity laws,

$$N_D \propto \sqrt{H}$$

$$\therefore \frac{N_m}{N_p}\times\frac{D_m}{D_p} = \sqrt{\frac{H_m}{H_p}}$$

$$\frac{254.63}{N_p}\times\frac{1}{4} = \sqrt{\frac{8}{24}}$$

$$N_p = 110.2$$

Also , $Q \propto D^2\sqrt{H}$

$$\therefore \frac{Q_m}{Q_p} = \left(\frac{D_m}{D_p}\right)^2 \times \sqrt{\frac{H_m}{H_p}}$$

$$\therefore \frac{3.536}{Q_p} = \left(\frac{1}{4}\right)^2 \times \sqrt{\frac{8}{24}}$$

$$\therefore Q_p = 98\,\text{m}^3/\text{s}$$

similarly, $P \alpha D^2 H^{3/2}$

$$\therefore \frac{P_m}{P_p} = \left(\frac{D_m}{D_p}\right)^2 \times \left(\frac{H_m}{H_p}\right)^{3/2}$$

$$\therefore \frac{208.2}{P_p} = \left(\frac{1}{4}\right)^2 \times \left(\frac{8}{24}\right)^{3/2}$$

$$\Rightarrow \quad P_p = 17309\,\text{kW}$$

03(d).

Sol: For pumps in series twice head is obtained for same discharge when compared with single pump. The efficiency of series combination will be equal to efficiency of each pump in the series.

H_s(m)	40	39	37	35	32.5	30	25
Q_s(m^3/s)	0	25	50	75	100	120	150
η_s(%)	0	35	60	80	85	87	82

Head for single pump (H), resistance head (H_R), head for series combination (H_S) and efficiency are plotted against discharge (Q) as shown in the graph.

Point of intersection of H – Q and H_R – Q curve give s the operating point for single pump.

i.e , at operating point for single,

H = 18.3 m, Q = 57.5 m^3/s, η = 69.5%

$$\therefore P = \frac{\rho g Q H}{\eta}$$

$$= \frac{9810\times57.5\times18.3}{0.695} = 14.85\,\text{MW}$$

Similarly, operating point for series combination is the point of intersection of $H_S - Q$ and $H_R - Q$ curve.

$\therefore$ At operating point of series combination

H = 31.5 m, Q = 111 m^3/s, η = 86%

$$P = \frac{\rho g Q H}{n}$$

$$= \frac{9810\times111\times31.5}{0.86} = 39.88\,\text{MW}$$

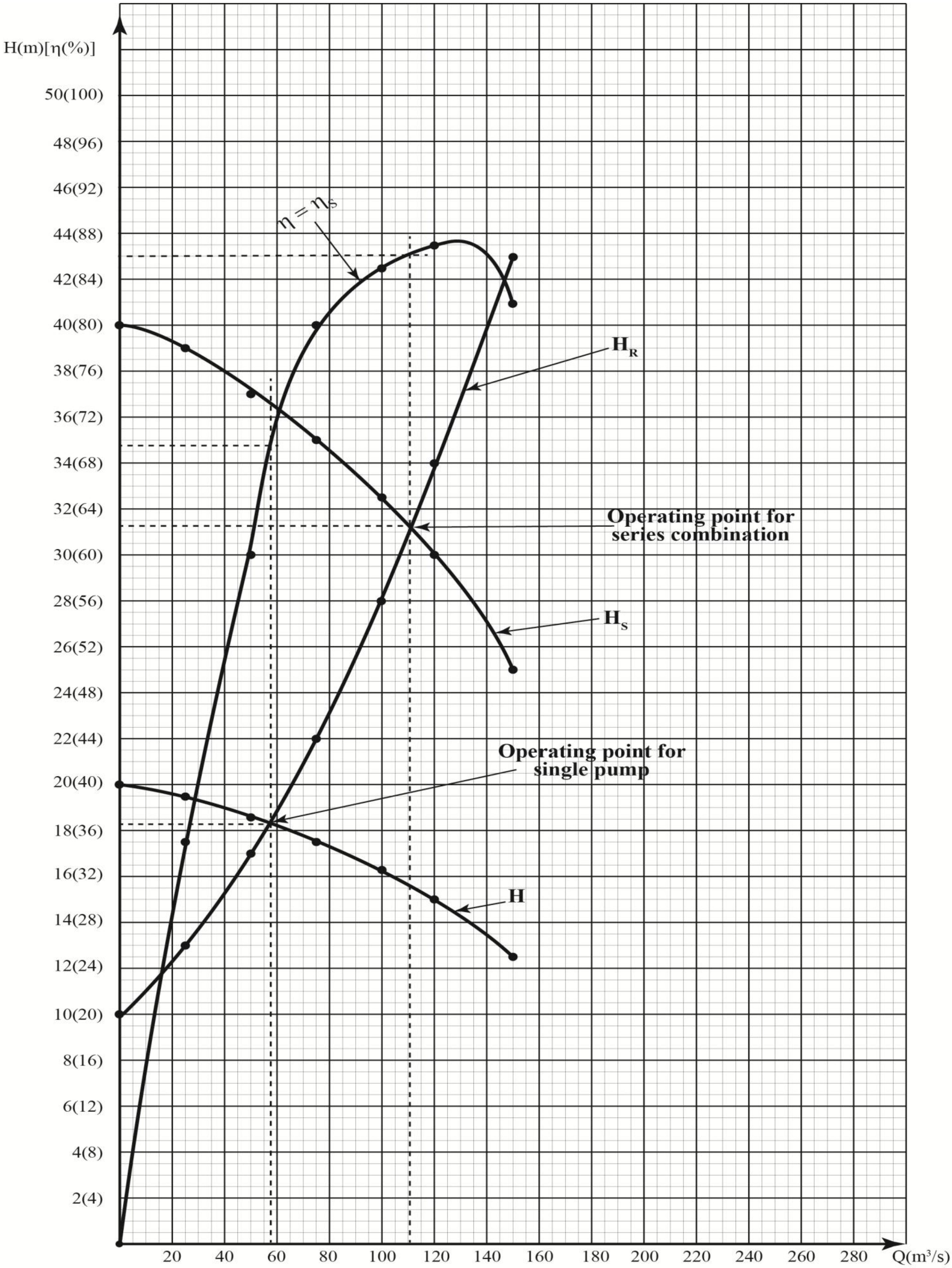
The pumping cost required to pump a given volume of liquid is directly proportional to power per unit discharge.

$$\left(\frac{P}{Q}\right)_{\text{sin gle}} = \frac{14.85\times10^3}{57.5} = 258.26\,\text{kJ/m}^3$$

$$\left(\frac{P}{Q}\right)_{\text{series}} = \frac{39.88\times10^3}{111} = 359.28\,\text{kJ/m}^3$$

As $\left(\frac{P}{Q}\right)_{\text{sin gle}} < \left(\frac{P}{Q}\right)_{\text{series}}$

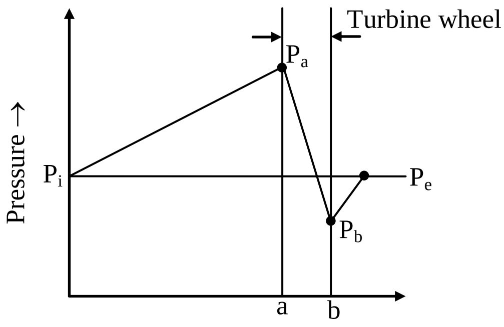
Therefore, single pump is more economic because it consumes less energy to pump given volume of liquid.

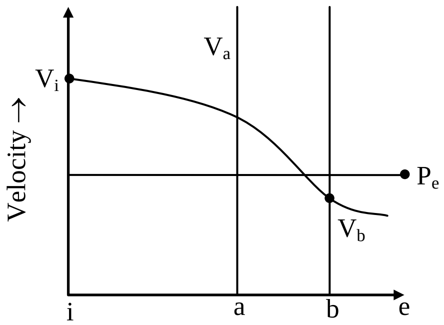

H(m)[η(%)]
50(100)
48(96)
46(92)
44(88)
42(84)
40(80)
38(76)
36(72)
34(68)
32(64)
30(60)
28(56)
26(52)
24(48)
22(44)
20(40)
18(36)
16(32)
14(28)
12(24)
10(20)
8(16)
6(12)
4(8)
2(4)
20
40
60
80
100
120
140
160
180
200
220
240
260
280
Q(m³/s)
η = ηs
H_R
Operating point for series combination
H_S
Operating point for single pump
H

04(a).

Sol: Power coefficient (C_p) for the wind mill is the ratio of maximum power to total power.

Consider a horizontal axis propeller type wind mill. Let, the wheel of it has thickness a–b, the incoming wind velocity and pressure at upstream section be V_i and P_i and the exit wind velocity and pressure at downstream section be V_e and P_e respectively.





Consider incoming air between 'i' and 'a' as thermodynamic system. The general energy equation is

$$\frac{P_i}{\rho}+\frac{V_i^2}{2}=\frac{P_a}{\rho}+\frac{V_a^2}{2}$$

$$P_i+\frac{\rho V_i^2}{2}=P_a+\frac{\rho V_a^2}{2} \quad \text{..............(1)}$$

The potential energy remains constant and there is no addition or removal of heat or work because of which potential energy terms, heat terms or work terms are not considered in energy equation.

Similarly, considering the air between b – e i.e in exit region, the general energy equation is,

$$\frac{P_e}{\rho}+\frac{V_e^2}{2}=\frac{P_b}{\rho}+\frac{V_b^2}{2}$$

$$P_e+\frac{\rho V_e^2}{2}=P_b+\frac{\rho V_b^2}{2} \quad \text{...........(2)}$$

$V_i > V_a$,

$V_b > V_e$

And , $P_a > P_i$ and $P_b < P_e$

Subtract equation (2) from equation (1),

$$\left[P_i+\frac{\rho V_i^2}{2}\right]-\left[P_e+\frac{\rho V_e^2}{2}\right]=\left[P_a+\frac{\rho V_a^2}{2}\right]-\left[P_b+\frac{\rho V_b^2}{2}\right] \quad \text{......(3)}$$

As wind pressure returns to ambient pressure,

$\therefore \quad P_e = P_i$ ....(4)

The velocity within the turbine does not change because the width of the blade a–b is thin compared with total distance.

$\therefore \quad V_a \approx V_b$ ....(5)

Substitute equations (4) & (5) in equation (3),

$$\frac{\rho V_i^2}{2}-\frac{\rho V_e^2}{2}=P_a-P_b$$

$$P_a-P_b=\frac{\rho}{2}\left(V_i^2-V_e^2\right)$$

Axial force on the turbine wheel area perpendicular to stream is,

$$F_x=(P_a-P_b)\times A=\frac{\rho}{2}\left(V_i^2-V_e^2\right)\times A$$

$$=\frac{\rho A}{2}\left(V_i^2-V_e^2\right) \quad \text{.................(6)}$$

Force = Rate of change of momentum of wind

$$F_x=\Delta(\dot{m}V)$$

where, $\dot{m}$ = Mass flow rate = $\rho A.V_t$

$$\therefore \; F_x=\rho A V_t(V_t-V_e) \quad (\because V=V_i-V_e) \text{.......(7)}$$

Equating (6) and (7),

$$\frac{\rho A}{2}\left(V_i^2 - V_e^2\right) = \rho A V_t \left(V_t - V_e\right)$$

$$V_t = \frac{V_i + V_e}{2}$$

Consider the total thermodynamic system between i and e. The change in potential energy, internal energy and flow energy is zero. Also no heat is added or removed.

So, the general energy equation for this system contains kinetic energy term and steady flow work W.

$$W = KE_i - KE_e$$

$$= \frac{1}{2}V_i^2 - \frac{1}{2}V_e^2$$

$$= \frac{1}{2}\left(V_i^2 - V_e^2\right)$$

Power 'P is the rate of work,

$$P = \frac{\dot{m}}{2}\left(V_i^2 - V_e^2\right)$$

$$= \frac{\rho A V_t}{2}\left(V_i^2 - V_e^2\right)$$

$$= \frac{\rho A}{2}\left(\frac{V_i + V_e}{2}\right)\left(V_i^2 - V_e^2\right)$$

$$\therefore P = \frac{\rho A}{4}\left(V_i + V_e\right)\left(V_i^2 - V_e^2\right)$$

For maximum power $\dfrac{dP}{dV_e} = 0$

$$\frac{d}{dV_e}\left[\frac{\rho A}{4}\left(V_i + V_e\right)\left(V_i^2 - V_e^2\right)\right] = 0$$

$$\frac{d}{dV_e}\left[\frac{\rho A}{4}\left(V_i^3 - V_i V_e^2 + V_e V_i^2 - V_e^3\right)\right] = 0$$

$$-2V_e V_i + V_i^2 - 3V_e^2 = 0$$

$$3V_e^2 + 2V_e V_i - V_i^2 = 0$$

Solving for V_e,

$$V_e = \frac{-2V_i \pm \sqrt{\left(-2V_i\right)^2 - 4\times 3\left(-V_i^2\right)}}{2\times 3}$$

$$= \frac{-2V_i \pm \sqrt{4V_i^2 + 12V_i^2}}{6}$$

$$= \frac{-2V_i \pm 4V_i}{6} = \frac{2V_i}{6}$$

$$\therefore V_e = \frac{V_i}{3}$$

$\therefore$ Maximum power is then,

$$P_{max} = \frac{\rho A}{4}\left[V_i + \frac{V_i}{3}\right]\left[V_i^2 - \left(\frac{V_i}{3}\right)^2\right]$$

$$= \frac{\rho A}{4}\times\frac{4V_i}{3}\times\frac{8V_i^2}{9}$$

$$\therefore P_{max} = \frac{8}{27}\rho A V_i^3$$

$$P_{Total} = \frac{1}{2}\rho A V_i^3$$

$$\text{Power Coefficient} = \frac{P_{max}}{P_{total}}$$

$$= \frac{\frac{8}{27}\rho A V_i^3}{\frac{1}{2}\rho A V_i^3}$$

$\therefore$ Power Coefficient = 0.593

04(b).

Sol: Discharge in pipe 2 = 0.605 m^3/s

$$h_{f2} = \frac{fL_2 Q_2^2}{12.1 d_2^5}$$

$$= \frac{0.02\times 450\times 0.605^2}{12.1(0.45)^5} = 14.75\ \text{m}$$

Energy head at the junction J

E_j = 60 + 14.75 = 74.75 m

For the flow from J to C

$E_j = E_c + h_{f3}$

$$\therefore\ 74.75 = 55 + 0 + \frac{V_3^2}{2g} + \frac{4\times 0.02\times 450}{0.3}\frac{V_3^2}{2g}$$

$$\therefore\ 19.75 = 31\frac{V_3^2}{2g}$$

$\therefore\ V_3 = 3.535$ m/s

$\therefore\ Q_3 = \frac{\pi}{4}(0.3)^2 3.535 = 0.250\ \ m^3/sec$

$\therefore\ Q_1 + Q_2 + Q_3 = 0.605 + 0.250$

$= 0.855\ m^3/sec$

$$h_{f1} = \frac{fl_1 Q_1^2}{12.1 d_1^5}$$

$$= \frac{0.02\times 450\times 0.855^2}{12.1\times(0.45)^5} = 29.460\ m$$

$\therefore$ Energy head to be supplied by the pump

$= E_j + h_{f_j} - E_a$

$= 74.75 + 29.46 - 50 = 54.21$ m

$\therefore$ Power to be supplied by the pump

$= 9.81 \times 0.855 \times 54.21$ kW

$= 454.7$ kW

04(c).

Sol: Given that:

S.P = 25 MW, N = 300 rpm, H = 400 m

Case-I: Double overhung turbine (two turbines with single jet)

$$V = C_v\sqrt{2gH}$$

$$= 0.98\times\sqrt{2\times 9.81\times 400} = 86.82\,m/s$$

$$u = k_u.V = 0.46\times 86.82 = 39.93\,m/s$$

$$u = \frac{\pi DN}{60}$$

$$\therefore\ D = \frac{39.93\times 60}{\pi\times 300} = 2.54\,m$$

Power per wheel per jet (P_j) is given by

$$P_j = \frac{S.P}{2} = \frac{25}{2} = 12.5\,MW$$

But $P_j = \eta_o \rho g Q H$

$$12.5\times 10^6 = 0.9\times 9810\times Q\times 400$$

$$\therefore\ Q = 3.539\,m^3/s$$

$$Q = \frac{\pi}{4}d^2V$$

$$\therefore d = \sqrt{\frac{4\times 3.539}{\pi\times 86.82}} = 0.228\,m$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300\sqrt{12.5\times 10^3}}{400^{1.25}}$$

$$N_s = 18.75$$

Case-II: Single wheel with single jet .

As net head (H), nozzle velocity coefficient (C_V) and speed ratio (k_u) are same, the jet speed (V), wheel speed (N) and runner diameter remain same for all cases.

$\therefore\ D = 2.54$ m

$$P_j = 25\,MW$$

$$[\because \text{single jet is present on a single wheel}]$$

$$\therefore\ P_j = \eta_0 \rho g Q H$$

$$\therefore 25\times 10^6 = 0.9\times 9810\times Q\times 400$$

$\therefore\ Q = 7.08\ m^3/s$

$$Q = \frac{\pi}{4}d^2V$$

$$\therefore d = \sqrt{\frac{4\times 7.08}{\pi\times 86.82}} = 0.322\ m$$

$$N_s = \frac{N\sqrt{P}}{h^{5/4}} = \frac{300\sqrt{25\times 10^3}}{400^{1.25}} = 26.52$$

Case-III: Single wheel with four jets

$$V = 86.82\,m/s, D = 2.54m\ (\text{same as case I \& II})$$

$$P_j = \frac{S.P}{4} = \frac{25}{4} = 6.25\,MW$$

$$\therefore P_j = \eta_o \rho g Q H$$

$$6.25\times 10^6 = 0.9\times 9810\times Q\times 400$$

$$Q = 1.77m^3/s$$

$$Q = \frac{\pi}{4}d^2V$$

$$\therefore d = \sqrt{\frac{4\times 1.77}{\pi\times 86.82}} = 0.161\,m$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300\sqrt{6.25\times10^3}}{400^{1.25}} = 13.26$$

Note: To calculate specific speed of Pelton turbine, power per wheel per jet (in kW) is considered as per convention adopted.

05(a).

Sol: The different solar energy measuring equipments are:

- (1) Pyrhelimoter type
  - The Angstrom pyrheliometer
  - The Abbot silver disc pyrheliometer
  - Eppley pyrheliometer
- (2) Pyranometer type
  - Eppley pyranometer
  - Yellot solarimeter
  - Moll-Gorczy heski solarimeter
  - Bimetallic Actionographs of Rabitzsch type
  - Velochme pyranometer
  - Thermoelectric pyranometer

Differences between Pyrheliometer and Pyranometer

Pyrheliometer		**Pyranometer**	
1	A device used for measuring direct or beam radiations is called pyrheliometer.	1	A device used for measuring total solar radiation is known as pyranometer.
2.	A power supply is required for measuring the radiations.	2.	It does not require power supply.
3.	The sensor disc is placed at the base of the tube, whose axis is aligned with the direction of sun's rays. Thus, blocking the diffuse radiations from sensor surface.	3.	The beam radiation is prevented from falling on the sensor disc. Thus, measuring the diffuse radiations.

05(b).

Sol:

(i) For 2-D flow to be irrotational,

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0$$

For the given stream function, 0

$$\psi = x + 2x^2 - 2y^2$$

$$\frac{\partial\psi}{\partial x}=1.4x \text{ and } \frac{\partial\psi}{\partial y}=-4y$$

$$\frac{\partial^2\psi}{\partial x^2}=4 \text{ and } \frac{\partial^2\psi}{\partial y^2}=-4$$

Here, $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=4-4=0$

Thus, the given stream function does represent irrotational flow.

(ii) We know that the velocity components u and v are related to (velocity potential) and ψ (stream function) as:

$$\left.\begin{aligned} u &= -\frac{\partial\phi}{\partial x}=-\frac{\partial\psi}{\partial y}\\ \text{and } v &= -\frac{\partial\phi}{\partial y}=\frac{\partial\psi}{\partial x}\end{aligned}\right\}\text{..........(1)}$$

Thus, for the given stream function,

$$u=-\frac{\partial\phi}{\partial x}=-\frac{\partial}{\partial y}\left(x+2x^2-2y^2\right)=4y$$

or $\frac{\partial\phi}{\partial x}=-4y$

Integrating the above equation,

$$\phi = -4xy + f(y) \text{ ………….. (2)}$$

Differentiating the above equation w.r.t y,

$$\frac{\partial\phi}{\partial y}=-4x+f'(y)$$

But

$$\frac{\partial\phi}{\partial y}=-v=\frac{-\partial}{\partial x}\left(x+2x^2-2y^2\right)=-(1+4x)$$

Thus, $-4x + f'(y) = -1-4x$

or $f'(y) = -1$

and $f(y) = -y + C$

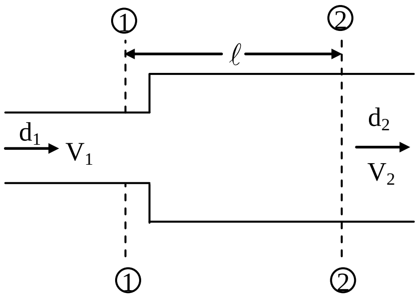
(C = Constant of integration)

Substituting f(y) in equation (2), we get

$$\phi = -4xy - y + C$$
$$= -y(4x+1) + C$$

05(c).

Sol:



$$\frac{d_2}{d_1}=\sqrt{2}$$

$$\frac{A_2}{A_1}=\frac{d_2^2}{d_1^2}=2$$

$A_2 = 2A=1$

∴ $V_1 = 2V_2$

Loss of head due to sudden enlargement

$$=\frac{(V_1-V_2)^2}{2g}=\frac{(2V_2-V_2)^2}{2g}=\frac{V_2^2}{2g}$$

Loss of head due to friction = $\frac{f\ell}{d}\cdot\frac{V_2^2}{2g}$

Applying Bernoulli's equation to section 1-1 and 2-2

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+\frac{f\ell}{d_2}\frac{V_2^2}{2g}+\frac{V_2^2}{2g}$$

Since $\frac{P_1}{\gamma}=\frac{P_2}{\gamma}$ and $V_1 = 2V_2$

$$4\frac{V_2^2}{2g}=2\frac{V_2^2}{2g}+\frac{f\ell}{d_2}\cdot\frac{V_2^2}{2g}$$

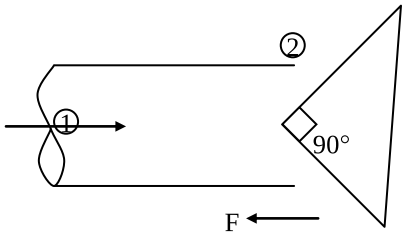
$$4=2+\frac{f\ell}{d_2}$$

$$\therefore \qquad \ell = \frac{2d_2}{f} = \frac{2\times d_2}{0.02} = 100d_2$$

Actual distance between the section 1-1 and 2-2 = 100 d_2 + additional length required for the expansion.

05(d).

Sol:



Assume flow at 2 entirely parallel to sides of the cone.

P_2 = Atmospheric pressure

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

$$= P_{atm} + \frac{1}{2}\rho V_2^2$$

or, $(P_1)_{gauge} = \frac{1}{2}\rho(V_2^2 - V_1^2) = \frac{1}{2}\times 1.22\times(60^2 - 15^2)$

= 2059 Pa (if velocities uniform)

Applying linear momentum equation:

$$2059\times\frac{\pi}{4}\times 0.6^2 - F = \rho Q(60\cos 45° - 15)$$

$$582.17 - F = 1.22\times\frac{\pi}{4}\times 0.6^2\times 15(60\times\cos 45° - 15)$$

= 141.91

$\Rightarrow$ F = 440.26 N

05(e).

Sol: Given data:

$D_P = 150\,mm,\ r = 125\,mm,\ N = 60\,rpm,$

$H_{st} = H_s + H_d = 30\,m,$

$h_f = 0.08Q^2$

$$Q_{th} = \frac{A_pLN}{60} = \frac{\pi}{4}\times 0.15^2\times 0.25^2\times\frac{60}{60}$$

$= 4.418\times 10^{-3}\,m^3/s$

$= 4.418\,lit/s$

$$\%S = \frac{Q_{th} - Q}{Q_{th}}\times 100$$

$$5 = \frac{4.418 - Q}{4.418}\times 100$$

$\therefore Q = 4.197\,lit/s$

$\therefore\ h_f = 0.08\times 4.197^2 = 1.41\,m$

The pumping power is given by

$$P = \frac{\rho g Q(H_s + H_d + h_f)}{\eta}$$

$$= \frac{8.5\times 10^3\times\times 4.197\times 10^{-3}(30 + 1.41)}{0.85}$$

= 1.318 kW

06(a).

Sol: Given data:

S.P = 18.6 MW,

H = 21 m, N = 135 rpm,

D_t = 5 m, D_h = 2.5,

η_0 = 89%, η_h = 92%,

$$S.P = \eta_0\,\rho g Q H$$

$$18.6\times 10^6 = 0.89\times 9810\times Q\times 21$$

$$Q = 92.13\,m^3/s$$

$$Q = \frac{\pi}{4}\times(D_t^2 - D_h^2)\times V_f$$

$$\therefore V_f = \frac{4\times 92.13}{\pi(5^2 - 2.5^2)}$$

i.e., $V_{f1} = V_{f2} = 6.256\,m/s$

Flow ratio, $k_f = \frac{V_{f1}}{\sqrt{2gH}} = \frac{6.256}{\sqrt{2\times 9.81\times 21}}$

= 0.308

$$u_t = \frac{\pi D_t N}{60} = \frac{\pi\times 5\times 135}{60}$$

i.e., $u_{t1} = u_{t2} = 35.34\,m/s$

$\therefore$ Speed ratio, $k_u = \dfrac{u_t}{\sqrt{2gH}}$

$$= \frac{35.34}{\sqrt{2\times 9.81\times 21}} = 1.74$$

At blade tip:

$u_{t1} = u_{t2} = 35.34\,m/s$

$V_{f1t} = V_{f2t} = 6.256\,m/s$

$\eta_h = 0.92$

$$\therefore \frac{V_{w1t}\times u_{t1}}{gH} = 0.92$$

$$\therefore V_{w1t} = \frac{0.92\times 9.81\times 21}{35.34} = 5.36 \text{ m/s}$$

$$\tan\beta_{1t} = \frac{V_{f1t}}{u_{t1}-V_{w1t}} = \frac{6.256}{35.34-5.36}$$

$\beta_{1t} = 11.8^\circ$

$$\tan\beta_{2t} = \frac{V_{f2t}}{u_{t2}} = \frac{6.256}{35.34}$$

$\beta_{2t} = 10^\circ$

$$\tan\alpha_{1t} = \frac{V_{f1t}}{V_{w1t}} = \frac{6.256}{5.36}$$

$\alpha_{1t} = 49.4^\circ$

At hub:

$V_{f1h} = V_{f2h} = 6.256\,m/s$

$$u_{h1} = u_{h2} = \frac{\pi D_h N}{60} = 17.67\,m/s$$

$$\eta_h = \frac{W_{w1h}\times u_{h1}}{gH}$$

$$\therefore V_{w1h} = \frac{0.92\times 9.81\times 21}{17.67} = 10.73\,m/s$$

$$\tan\beta_{1h} = \frac{V_{f1h}}{u_{h1}-V_{w1h}} = \frac{6.256}{17.67-10.73}$$

$\beta_{1h} = 42^\circ$

$$\tan\beta_{2h} = \frac{V_{f2h}}{u_1} = \frac{6.256}{17.67}$$

$\beta_{2h} = 19.5^\circ$

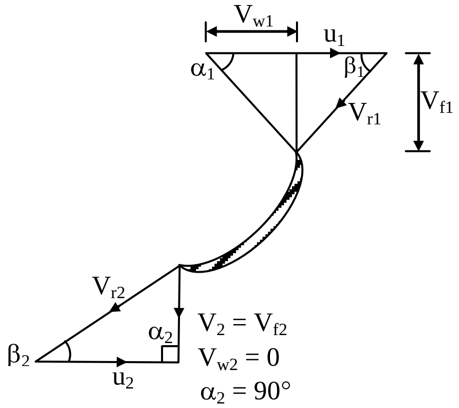
$$\tan\alpha_{1h} = \frac{V_{f1h}}{V_{w1h}} = \frac{6.256}{10.73}$$

$\beta_{2h} = 19.5^\circ$

$$\tan\alpha_{1h} = \frac{V_{f1h}}{V_{w1h}} = \frac{6.256}{10.73}$$

$\sigma_{1h} = 30.2$

Comparison of velocity triangles:



(a) Inlet velocity triangles :

$AC = u_{1t}$

$AE = u_{1h}$

$CB = v_{r1t}$

$ED = v_{r1t}$

$BF = v_{f1t}$

$DF = v_{f1h}$

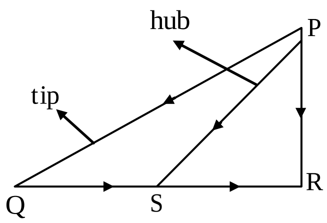
$\angle BAC = \alpha_{1t}$

$\angle DAE = \alpha_{1t}$

$\angle ACB = \beta_{1t}$

$\angle AED = \beta_{1h}$

(b) Exit velocity triangles



$QR = u_{2t}$

$SR = u_{2h}$

$PQ = v_{r1t}$

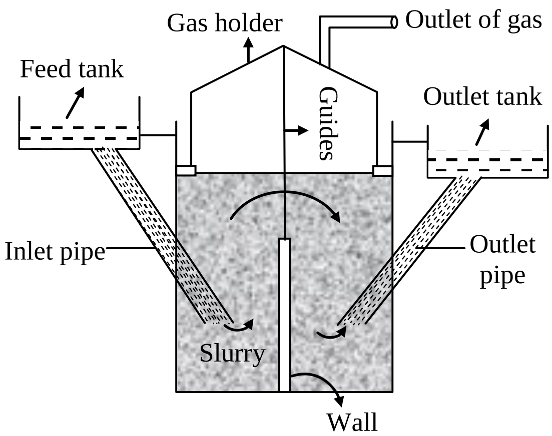
$PS = v_{r1t}$

$PR = v_{f2t} = v_{2t} = v_{f2h} = v_{2h}$

06(b).

Sol: Construction Details and Working of KVIC Digester

Digester is also called as fermentation tank. It is a pit made of masonry having diameter ranging from 1.3 m to 6 m and wall depth from 3 m to 6 m. It consists of biomass feed tank, gas holder, central guide pipe and outlet tank, as shown in figure.



A wall is constructed in the middle of the digester, which divides it into two compartments. This partition wall facilitates the circulation of the slurry. The partition wall is submerged in slurry, when the two compartments of the digester is full. The feed tank also known as inlet tank, is located at the surface level. The dung and water are mixed in the ratio 4:5 and this mixture is called slurry. This slurry is fed to the digester with the help of pipe connecting from feed tank to the digester known as inlet pipe.

The digester can hold the raw material for 60 days. Outlet tank is located slightly below the feed tank level. When the digester is full and if more slurry is added, then the equal amount of slurry flows out with the help of the pipe connecting from digester to outlet tank known as outlet pipe.

A mild steel drum is placed on the top of the digester and is known as gas holder. Its function is to collect the biogas produced from digester. A pipe is provided at the top of the holder for flow of gas. The gas holder maintains constant pressure inside the tank by moving up and down with the help of central guide pipe which is fitted to the frame at the bottom. The pressure inside the tank increases and gas holder moves up, so that the pressure is maintained constant. The pressure of gas in KVIC digester is about 7 cm to 9 cm of water column.

This pressure is sufficient to carry it upto a length of 20 m to 100 m depending on the size of the plant.

The cost of the gas holder is 40% of the total plant cost. It requires anti-corrosive paint coating from time to time, in order to improve the efficiency of the plant.

Applications of Biogas

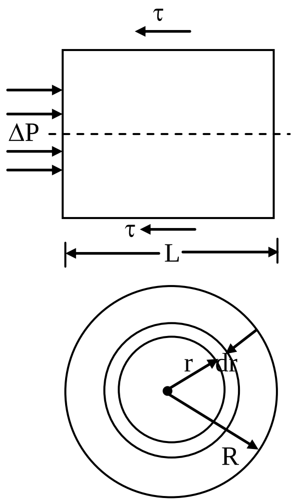
1. Cooking
2. Domestic lighting and heating
3. Fuel cells
4. I.C. engines
5. Petrol engines
6. Diesel engines.

06(c).

Sol: Given, $\tau = K\left(\frac{du}{dy}\right)^n$

Tube pressure loss, $\frac{\Delta P}{L} = 6400 \text{ Pa/m}$

(i) Take a cylindrical element of fluid as shown.



Shear force, $F_s = \tau \times$ surface area

$= \tau\,(2\pi rL)$

Pressure force = $\Delta P \times \pi r^2$

Total fore is zero,

$\Delta P \times \pi r^2 - \tau\,(2\pi rL) = 0$

or $\tau = \frac{\Delta P}{2L}.r$ …………. (1)

$\tau = -K\left(\frac{du}{dr}\right)^n$ ……..…… (2)

From equations (1) and (2)

$$-\frac{\Delta P}{2KL}.r = \left(\frac{du}{dr}\right)^n$$

$$du = -\left(\frac{\Delta P}{2KL}\right)^{1/n}.r^{1/n}dr$$

Integrating on both sides

$$\int_u^0 du = -\left(\frac{\Delta P}{2KL}\right)^{1/n}\int_r^R r^{1/n}dr$$

$$(0-u) = -\left(\frac{\Delta P}{2KL}\right)^{1/n}\left[\frac{r^{\frac{n+1}{n}}}{\left(\frac{n+1}{n}\right)}\right]_r^R$$

$$u = \frac{n}{n+1}\left(\frac{\Delta P}{2KL}\right)^{1/n} R^{\frac{n+1}{n}}\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]$$

at r = 0,

u = u_{max}

$$u_{max} = \frac{n}{n+1}\left(\frac{\Delta P}{2KL}\right)^{1/n} R^{\frac{n+1}{n}}$$

$$= \frac{0.8}{0.8+1}\times\left(\frac{6400}{2\times 0.05}\right)^{1/0.8}\times(0.003)^{\frac{0.8+1}{0.8}}$$

= 0.953 m/s

$$\therefore\ u = u_{max}\left(1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right) \quad …………….\ (3)$$

Discharge through the pipe,

$$Q = \int_0^R u.2\pi r dr$$

$$= 2\pi u_{max}\int_0^R r\left[1-\left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]dr$$

$$= 2\pi u_{max}\int\left(r - \frac{r^{\frac{2n+1}{n}}}{R^{\frac{n+1}{n}}}\right)dr$$

$$= 2\pi u_{max}\left[\frac{r^2}{2} - \frac{1}{R^{\frac{n+1}{n}}}\times r^{\frac{3n+1}{n}}\times\frac{n}{3n+1}\right]_0^R$$

$$= 2\pi u_{max}\left[\frac{R^2}{2} - \frac{n}{3n+1}\times R^{\frac{3n+1}{n}-\frac{n+1}{n}}\right]$$

$$= 2\pi u_{max}\times R^2\left[\frac{1}{2} - \frac{n}{3n+1}\right]$$

$$= 2\pi R^2 u_{max}.\frac{(n+1)}{2(3n+1)}$$

$$= \pi R^2 u_{max}.\left(\frac{n+1}{3n+1}\right)$$

Average velocity,

$$V = \frac{Q}{A} = \frac{\pi R^2 u_{max}\left(\frac{n+1}{3n+1}\right)}{\pi R^2}$$

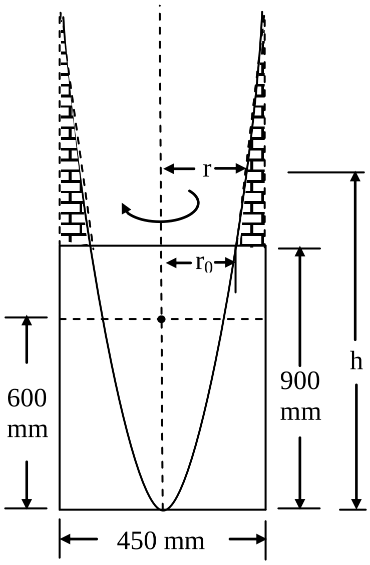
$$= \left(\frac{n+1}{3n+1}\right)u_{max}$$

$$= \left(\frac{0.8+1}{3\times 0.8+1}\right)\times 0.953$$

$$= 0.504 \text{ m/s}$$

07(a).

Sol:



(i) Initial volume of air = $\pi R^2 \times 0.3$

Final volume of air = $\frac{1}{2}\pi r_0^2 \times 0.9$

Since volume of air remains constant,

$$\pi R^2 \times 0.3 = \frac{1}{2}\pi r_0^2 \times 0.9$$

$$r_0^2 = \frac{2\times 0.3}{0.9}\times R^2$$

$$= \frac{2}{3}R^2$$

$$r_0 = \sqrt{\frac{2}{3}}R$$

For the free liquid surface at the axis to touch the bottom of the cylinder we have,

$$0.9 = \frac{\omega^2 r_0^2}{2g} = \omega^2 \times \frac{2}{3}R^2 \times \frac{1}{2g}$$

$$\omega^2 = \frac{3\times 0.9\times 9.81}{0.225^2} = 523.2$$

$$\Rightarrow \omega = 22.87 \text{ rad/s}$$

(ii) The pressure intensity on the cover at any radius r,

$$P = \gamma(h-0.9)$$

$$F = \int_{r_0}^{R} P\,dA$$

$$= \int_{\sqrt{\frac{2}{3}}R}^{R} \rho g(h-0.9)2\pi r\,dr$$

where, $h = \frac{\omega^2 r^2}{2g}$

$$F = 2\pi\rho g\int_{\sqrt{\frac{2}{3}}R}^{R}\left(\frac{\omega^2 r^3}{2g} - 0.9r\right)dr$$

$$F = 2\pi\rho g\left[\frac{\omega^2 r^4}{8g} - \frac{0.9}{2}r^2\right]_{\sqrt{\frac{2}{3}}R}^{R}$$

$$= 2\pi\rho g\left[\frac{\omega^2}{8g}\left(R^4 - \frac{4}{9}R^4\right) - 0.45\left(R^2 - \frac{2}{3}R^2\right)\right]$$

$$= 2\pi\rho g\left[\frac{\omega^2}{8g}\times\frac{5}{9}R^4 - 0.45\times\frac{1}{3}R^2\right]$$

$$= 2\pi\times 850\times 9.81\left[\frac{22.87^2\times 5}{8\times 9.81\times 9}\times 0.225^4 - \frac{0.45\times 0.225^2}{3}\right]$$

$$= 52392.3 \times [9.489\times 10^{-3} - 7.594\times 10^{-3}]$$

$$= 99.28 \text{ N}$$

07(b).

Sol: Thermal Energy Storage:

- By heating melting or vaporization of a material, the energy can be stored and is available as heat by reversing the process.
- Thermal energy storage is essential for both domestic water and space heating applications and for the high temperature storage system needed for thermal power application.
- Thermal storage involves sensible heat storage, latent heat storage and thermo chemical storage system.

1. Sensible Heat Storage

It involves heating a liquid or a solid which does not undergo change in phase. The amount of energy stored is a function of the temperature change of the material and is given as,

$$E = m\int_{T_1}^{T_2} c_p dt$$

where 'm' is the mass, 'c_p' is the specific heat, T_1 and T_2 are the temperature limits of the storage system. The difference $(T_1 - T_2)$ is also referred as the temperature swing.

- Energy can also be stored in rocks or pebbles in an insulated vessel. This type of storage is used commonly for temperatures up to 100°C in integration with solar air heaters. It is simple in design and relatively inexpensive. The size of the rock used ranges from 1 cm to 5 cm and approximately 300 kg to 500 kg of rock per square meter of collector area is applied for space - heating application.
- Refractory materials like magnesium oxide (magnesia), aluminium oxide (alumina) and silicon oxide are suitable for high temperature sensible heat storage.

07(c).

Sol: Given:

Velocity profile a short distance downstream of the cylinder is given by

$$u = \begin{cases} 10\left[1 + 6\left(\dfrac{y}{0.1}\right)^2 - 4\left(\dfrac{|y|}{0.1}\right)^3\right], \text{if } |y| \le 0.1 \\ u_B \qquad\qquad\qquad \text{if } |y| \ge 0.1 \end{cases}$$

………….. (1)

(a) From the above velocity profile, it is evident that for $|y| \ge 0.1$, $u = u_B$ in equation (i)

$$u_B = 10\left[1 + 6\left(\frac{0.1}{0.1}\right)^2 - 4\left(\frac{0.1}{0.1}\right)^3\right]$$

$= 10\ (1 + 6 - 4)$

$= 30$ m/s.

(b) From the continuity equation:

$$u_A \times (h \times W) = 2\int_0^{0.1} u(dy \times W) + 2u_B \int_{0.1}^{0.15} W dy$$

where W is the width of the wind tunnel

$$= 2W\int_0^{0.1} 10\left[1 + 6\left(\frac{y}{0.1}\right)^2 - 4\left(\frac{|y|}{0.1}\right)^3\right]dy + 2Wu_B[y]_{0.1}^{0.15}$$

$$= 2W\left[10\left(y + \frac{6}{0.1^2}\times\frac{y^3}{3} - \frac{4}{0.1^3}\times\frac{y^4}{4}\right)\right]_0^{0.1} + 2W\times 30\times(0.05)$$

$$= 4W + 3W$$

$$u_A \times 0.3\ W = 7W$$

$$\Rightarrow \quad u_A = \frac{7}{0.3} = 23.33\ m/s$$

(c) Given that Bernoulli's equation is valid outside the wake of the cylinder, we can write

$$\frac{P_A}{\gamma}+\frac{V_A^2}{2g}+Z_A=\frac{P_B}{\gamma}+\frac{P_B^2}{2g}+Z_B$$

where, $Z_A = Z_B$

$V_A = u_A$ and $V_B = u_B$.

Thus, the pressure difference between upstream and downstream sections of the cylinder is

$$(P_A - P_B)=\rho\left(\frac{u_B^2-u_A^2}{2}\right)$$

$$= 1.2\left(\frac{30^2-23.33^2}{2}\right)$$

$$= 213.43 \text{ Pa}$$

07(d).

Sol: The major disadvantage of a turbo jet engine is relatively small power available for take off and climb compared to reciprocating engines. Following methods of thrust argumentation are used in turbo jet engines.

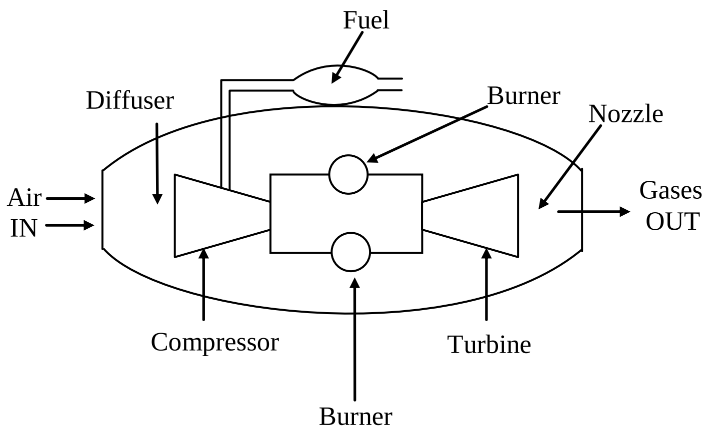
(a) Increasing mass flow rate of air

(b) Bleed burn cycle

(c) After burning or tailpipe burning.

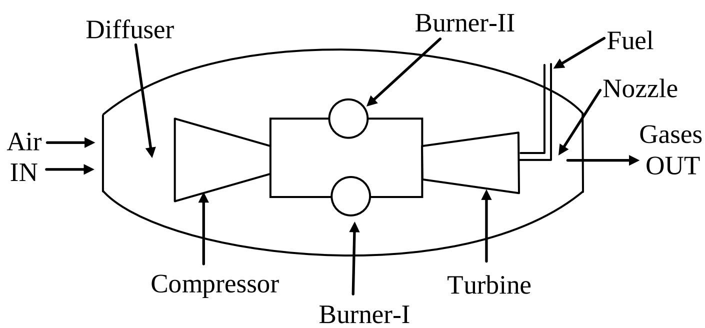
Increasing mass flow rate of air: The thrust produced is directly proportional to the mass flow rate and therefore the density of incoming air can be substantially increased by cooling. Water, alcohol, or a mixture of water and alcohol is sprayed over incoming air. The evaporation of liquid extracts heat from incoming air. The liquid in excess of saturation limit at compressor inlet is evaporated during compression process and air is further cooled. The cooling gives an increases in both pressure ratio and the air flows. This results in a thrust increase by about 20 percent but it depends on the coolant used. This method is used only during take off.

Bleed Burn Cycle.



A small fraction of compressed air is bled off from the compressor and enters an auxiliary combustion chamber. Additional fuel is injected inside, the auxiliary combustion chamber and hot gases coming out are allowed to expand through a secondary nozzle to produce additional thrust. In order to compensate for mass of air extracted from compressor, water is injected into main combustion chamber. This maintains usual mass flow rate of hot gases through the nozzle and the system is used to increase the thrust during take off period.

After burning or tail pipe burning



In this method additional fuel is injected between turbine outlet and nozzle inlet. This raises the enthalpy of gases entering the nozzle and results in an increased exit velocity of gases. A stable combustion of fuel in tail pipe would require sufficiently low velocities and to avoid excessive pressure loss a diffuser is provided between turbine outlet and tail pipe burner inlet. By this method the thrust can be increased by 20 to 25 percent but the specific fuel consumption increases. An aircraft will have to carry extra fuel. Therefore, this puts a limit during which augmentation can be achieved in flight.

08(a).

Sol: The advantage and disadvantages of fuel cells are as follows.

Advantages:

- Conversion efficiency is very high (approximately 70%)
- Transmission of power is not required because they can be installed near the usage.
- No noise pollution or poisonous effects
- Space requirement is less
- No cooling water is required on large scale
- They can start in less time
- There are no moving parts, therefore maintenance cost is less.
- Its weight and volume are considerably low compared to other energy sources.
- A wide variety of fuels can be used with the fuel cell.
- Part load efficiency remains constant.

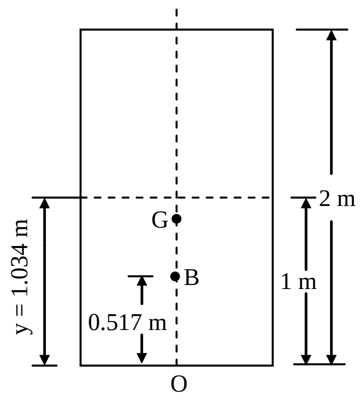
Disadvantages:

- Capital cost is very high
- Service life is considerably low.

08(b).

Sol: **Case (i):**

When the anchor is not provided (as shown in the below figure)



Weight of the buoy = 12 kN = 12000 N

∴ Volume of water displaced

$$= \frac{12000}{10055} = 1.19\,\text{cum}$$

∴ Depth of immersion

$$y = \frac{1.19}{\frac{\pi}{4}(1.2)^2} = 1.052\,\text{m}$$

$$\therefore \quad OB = \frac{1.052}{2} = 0.526\,\text{m}$$

$$BM = \frac{I}{V} = \frac{\frac{\pi}{64}(1.2)^2}{1.19} = 0.086\,\text{m}$$

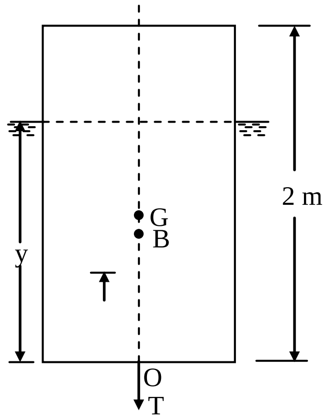
∴ OM = OB + BM

= 0.526 + 0.086 = 0.612 m

But OG = 1 m

Since OM < OG the metacentre is below G. Hence the equilibrium is not stable.

Case (ii): When the anchor chain is provided.



Let the tension in the anchor chain be T kN

Total downward force = (W + T) N

$$W + T = \gamma_{SW} A d$$

where d is the depth of immersion.

$$d = \frac{W+T}{\gamma_{SW} A} = \frac{(12+T)\times 4}{10.055\times\pi\times 1.2^2}$$

$$= 0.088\,(12 + T)$$

$$d = 1.055 + 0.088\,T$$

$$\overline{OB} = \frac{d}{2} = 0.5275 + 0.044T$$

$$\overline{BM} = \frac{I}{V} = \frac{\pi}{64}\times\frac{1.2^4\times 4}{\pi\times 1.2^2\times(1.055+0.088T)}$$

$$\overline{BM} = \frac{0.09}{1.055+0.088T}$$

$$\overline{GM} = \overline{BM} - \left(\overline{OG} - \overline{OB}\right)$$

$$= \frac{0.09}{1.055+0.088T} - (1.0 - 0.05275 - 0.044T)$$

$$= \frac{0.09}{1.055+0.088T} - 0.4725 + 0.044T$$

$$= \frac{0.09 - 0.49895 - 0.04167 + 0.0464T + 3.872\times10^{-3}T^2}{1.055+0.088T}$$

$$\overline{GM} = \frac{-0.4085 + 4.8\times10^{-3}T + 3.872\times10^{-3}T^2}{1.055+0.088T}$$

For stable condition; the numerator should be zero.

$$3.872\times10^{-3}\,T^2 + 4.8\times10^{-3}T - 0.4085 = 0$$

$$T^2 + 1.24\,T - 105.5 = 0$$

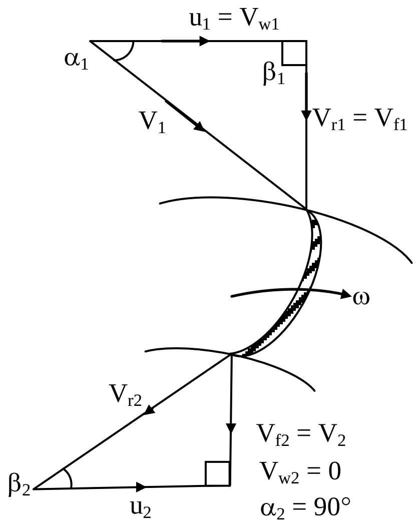
$$T = \frac{-1.24 \pm \sqrt{1.24^2 + 4\times 105.5}}{2}$$

$$= \frac{-1.24 \pm 20.58}{2}$$

$$\Rightarrow T = 9.67 \text{ kN}$$

08(c).

Sol:



As relative velocity is radial at inlet β_1 is 90°

As there is no whirl at exit

$$V_{w2} = 0$$

$$V_{f2} = V_2 \qquad \text{or} \qquad \alpha_2 = 90^\circ$$

The hydraulic efficiency is given by,

$$\eta_h = \frac{v_{w1}u_1}{gH}$$

$$\eta_h = \frac{u_1^2}{gH}\text{.....}(1)$$

$$[\because v_{w1} = u_1 \text{ from inlet velocity triangle}]$$

By energy equation

Net head = Head developed + Head lost

$$H = \frac{v_{W1}u_1}{g} + \frac{v_2^2}{2g}$$

$$= \frac{u_1^2}{g} + \frac{(kv_{f1})^2}{2g}$$

$$= \frac{u_1^2}{g} + \frac{(ku_1 \tan\alpha_1)^2}{2g}$$

$$H = \frac{u_1^2}{g}\left(1 + \frac{k^2 \tan^2\alpha_1}{2}\right) \text{...........(2)}$$

From (1) and (2)

$$\eta_h = \frac{u_1^2}{g \times \frac{u_1^2}{g}\left(1 + \frac{k^2 \tan^2\alpha_1}{2}\right)}$$

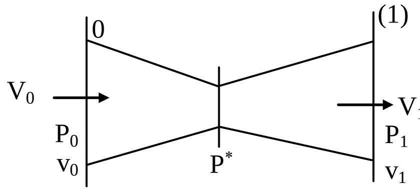
$$\eta_h = \frac{2}{2 + k^2 \tan\alpha_1}$$

08(d).

Sol: Flow is in a Convergent - Divergent nozzle

- Flow is adiabatic
- No work interaction
- Potential energy change is neglected

The expansion follows the law $Pv^k = c$



For isotropic flow in nozzle,

$T_{ds} = dh - vdP$

ds = 0

$\therefore$ dh = v dP.....(1)

Applying SFEE, we have

$$h_0 + \frac{V_0^2}{2} = h_1 + \frac{V_1^2}{2}$$

$$h_0 - h_1 = \frac{V_1^2 - V_0^2}{2} \text{.....(2)}$$

$$\int_0^1 dh = \int_0^1 vdP$$

$$h_1 - h_0 = \int_0^1 vdP$$

$$\Rightarrow h_0 - h_1 = -\int_0^1 vdP = \frac{V_1^2 - V_0^2}{2}$$

The law governing expansion process is $Pv^K = C$ and after neglecting inlet velocity we have,

$$\frac{V_1^2}{2} = -\int_0^1 vdP = -\int_0^1 C^{\frac{1}{k}} P^{-\frac{1}{k}} dP$$

$$= -C^{1/k}\left[\frac{P^{\frac{-1}{k}+1}}{\frac{-1}{k}+1}\right]_0^1$$

$$= \frac{-k}{(k-)}\left[C^{\frac{1}{k}} P_1^{\frac{k-1}{k}} - C^{\frac{1}{k}} P^{\frac{k-1}{k}}\right]. \quad P_0 v_0^k = P_1 v_1^k = C$$

$$= -\left(\frac{k}{k-1}\right)\left[(P_1 v_1^k)^{\frac{1}{k}} P_1^{\frac{k-1}{k}} - (P_0 v_0^k)^{\frac{1}{k}} P_0^{\frac{k-1}{k}}\right]$$

$$= \frac{k}{k-1}[P_0 v_0 - P_1 v_1]$$

$$\frac{V_1^2}{2} = \frac{k}{k-1} P_0 v_0 \left[1 - \frac{P_0 v_1}{P_0 v_0}\right]$$

$$P_1 v_1^k = P_0 v_0^k$$

$$\left(\frac{v_1}{v_0}\right)^k = \frac{P_0}{P_1}$$

$$\frac{v_1}{v_0} = \left(\frac{P_0}{P_1}\right)^{1/k} = \left(\frac{P_1}{P_0}\right)^{-1/k}$$

$$= \frac{k}{k-1} P_0 v_0 \left[1 - \left(\frac{P_1}{P_0}\right)\left(\frac{P_1}{P_0}\right)^{-1/k}\right]$$

$$= \frac{k}{k-1} P_0 v_0 \left[1 - \left(\frac{P_1}{P_0}\right)^{\frac{k-1}{k}}\right]$$

$$V_1 = \sqrt{\frac{2k}{k-1} P_0 v_0 \left[1 - \left(\frac{P_1}{P_0}\right)^{\frac{k-1}{k}}\right]}$$

Above is the expression for velocity of any medium leaving the nozzle.

For critical velocity we should have maximum mass flow rate condition. For discharge through nozzle, optimum pressure ratio at which discharge is maximum is given by

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

Critical velocity ,

$$V_1^* = \sqrt{\frac{2k}{k-1}P_0v_0\left[1-\left(\frac{P^*}{P_0}\right)^{\frac{k-1}{k}}\right]}$$

Substituting the above relationship in critical velocity we have

$$V_1^* = \sqrt{\frac{2k}{k-1}P_0v_0\left[1-\left[\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}\right]^{\frac{k-1}{k}}\right]}$$

$$V_1^* = \sqrt{\frac{2k}{k-1}P_0v_0\left[1-\frac{2}{k+1}\right]}$$

$$= \sqrt{\frac{2k}{k-1}P_0v_0\left[\frac{k+1-2}{k+1}\right]}$$

$$= \sqrt{\frac{2k}{k-1}P_0v_0\left[\frac{k-1}{k+1}\right]}$$

$$= \sqrt{\frac{2k}{k+1}P_0v_0}$$

For dry saturated steam k = 1.135

$$V_1^* = \sqrt{\frac{2\times1.135}{1.135+1}P_0v_o} = 1.03\sqrt{P_0v_0}$$

For superheated steam k = 1.3

$$V_1^* = \sqrt{\frac{2\times1.3}{1.3+1}P_0v_o} = 1.06\sqrt{P_0v_0}$$

For Air k = 1.4

$$V_1^* = \sqrt{\frac{2\times1.4}{1.4+1}P_0v_o} = 1.08\sqrt{P_0v_0}$$

08(e).

Sol: $V_1 = \dfrac{4Q}{\pi d_1^2}$

$$= \frac{4\times0.00265}{\pi(0.075)^2} = 0.60\,\text{m/s}$$

$$V_2 = \left(\frac{d_1}{d_2}\right)^2\times V_1 = \left(\frac{75}{25}\right)^2 V_1$$

= 9 × 0.6 = 5.4 m/s.

Applying Bernoulli's theorem to sections 1-1 and 2-2.

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$0 + \frac{P_1}{\gamma} + \frac{(0.60)^2}{2\times9.81} = 3 + 0 + \frac{(5.4)^2}{2\times9.81}$$

$$\frac{P_1}{\gamma} + 0.0183 = 3 + 1.4862$$

$$\therefore \quad \frac{P_1}{\gamma} = 4.4679\ \text{m}$$

Now considering the manometer,

13.6 h = 4.4679 + 0.6

∴ h = 0.373 m