ACE
Engineering Academy
(Leading institute for ESE/GATE/PSUs)

# ESE – 2019 MAINS
## OFFLINE TEST SERIES

# ELECTRICAL ENGINEERING

# TEST – 8
# SOLUTIONS

All Queries related to **ESE – 2019 MAINS Test Series** Solutions are to be sent to the following email address
**testseries@aceenggacademy.com** | **Contact Us : 040 – 48539866 / 040 – 40136222**

**1.(a)**

**Sol:** When H(S) = 1 + αs

Closed loop transfer function

$$M = \frac{G(s)}{1+G(s).H(s)}$$

$$= \frac{\frac{K}{s(s+p)}}{1+\frac{K}{s(s+p)}(1+\alpha s)}$$

$$= \frac{K}{s^2+ps+K+K\alpha s}$$

$$M = \frac{K}{s^2+s(p+K\alpha)+K}$$

**Sensitivity with respect to K:**

$$S_K^M = \frac{\partial M}{\partial K}\times\frac{K}{M}$$

$$= \frac{\left[s^2+s(p+K\alpha)+K\right]1-K.(s\alpha+1)}{\left[s^2+s(P+K\alpha)+K\right]^2}\times\frac{K}{\frac{K}{s^2+s(p+K\alpha)+K}}$$

$$= \frac{s^2+sp}{s^2+s(p+K\alpha)+K}$$

$$= \frac{s^2+3s}{s^2+s\left[3+(0.14\times 12)\right]+12}$$

$$S_K^M = \frac{s^2+3s}{s^2+4.68s+12}$$

**Sensitivity with respect to P:**

$$S_P^M = \frac{\partial M}{\partial P}.\frac{P}{M}$$

$$= \frac{-sK}{\left[s^2+s(p+K\alpha)+K\right]^2}\times\frac{p}{\frac{K}{s^2+s(p+K\alpha)+K}} = \frac{-sp}{s^2+s(p+K\alpha)+K} = \frac{-3s}{s^2+4.68s+12}$$

**Sensitivity with respect to α:**

$$S_{\alpha}^{M} = \frac{\partial M}{\partial \alpha}.\frac{\alpha}{M}$$

$$= \frac{-sK^2}{\left[s^2 + s(p+K\alpha)+K\right]^2}.\frac{\alpha}{\dfrac{K}{s^2+s(p+K\alpha)+K}} = \frac{-sK\alpha}{s^2+s(p+K\alpha)+K}$$

$$S_{\alpha}^{M} = \frac{-1.68s}{s^2+4.68s+12}$$

**1(b)**

**Sol:** Given, Input $x(t) = te^{-3t}\,u(t)$

$$X(s) = \frac{1}{(s+3)^2}$$

Impulse response $h(t) = 2e^{-4t}\,u(t)$

$$\therefore \quad H(s) = \frac{2}{s+4}$$

We know that, Output $y(t) = x(t) * h(t)$

$$\therefore \; Y(s) = X(s)\,H(s)$$

So, Output $y(t) = L^{-1}[X(s)\,H(s)]$

$$Y(s) = X(s)H(s) = \left[\frac{1}{(s+3)^2}\right]\left(\frac{2}{s+4}\right) = \frac{2}{(s+3)^2(s+4)}$$

Taking partial fractions, we have

$$Y(s) = \frac{2}{(s+3)^2(s+4)} = \frac{A}{(s+3)^2} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$Y(s) = \frac{2}{(s+3)^2} - \frac{2}{s+3} + \frac{2}{s+4}$$

Taking inverse Laplace transform on both sides, we have the output

$$y(t) = 2te^{-3t}\,u(t) - 2e^{-3t}\,u(t) + 2e^{-4t}\,u(t)$$

**1(c)**

**Sol: (i)** $\dfrac{d\delta}{dt} = \omega - \omega_s, \dfrac{d\omega}{dt} = \dfrac{P_a}{M}$

$$\omega = \frac{P_a}{M}t + \omega_s$$

$$\omega - \omega_s = \frac{P_a}{M}t$$

$$\Rightarrow \frac{d\delta}{dt} = \frac{P_a}{M}t$$

During fault, $P_a = 1$ p.u.

$$M = \frac{H}{\pi f}$$

$$= \frac{5}{\pi \times 50} = \frac{1}{10\pi} s^2/\text{elec.rad}$$

Now, $\dfrac{P_a}{M} = \dfrac{1}{\dfrac{1}{10\pi}} = 10\pi$ elec.rad/s$^2$

At the instant of clearing fault, $\delta_c = 50°$

$$t_c = \sqrt{\frac{2M(\delta_c - \delta_0)}{P_a}}$$

$$= \sqrt{\frac{2 \times \frac{1}{10\pi}(50° - 30°) \times \frac{\pi}{180}}{1}}$$

= 0.149 s

Now, $\left.\dfrac{d\delta}{dt}\right|_{t_c} = \dfrac{P_a}{M}t_c$

$$= 10\pi \times 0.149 \text{ elec.rad/s}$$

= 4.681 elec.rad/s

**(ii) Steady State Stability:**

The steady-state stability of a power system is defined as the ability of the system to bring itself back to its stable configuration following a small disturbance in the network (like normal load fluctuation or action of automatic voltage regulator). It can be considered only during a very gradual and infinitesimally small power change.

In case the power flow through the circuit exceeds the maximum power permissible, then there are chances that a particular machine or a group of machines will cease to operate in synchronism, and result in yet more disturbances. In such a situation, the steady-state limit of the system is said to have reached, or in other words, the steady state stability limit (SSSL) of a system refers to the maximum amount of power that is permissible through the system without loss of its steady state stability.

**Transient Stability:**

Transient stability of a power system refers to the ability of the system to reach a stable condition following a large disturbance in the network condition. In all cases related to large changes in the system like sudden application or removal of the load, switching operations, line faults or loss due to excitation the transient stability of the system comes into play. It in fact deals in the ability of the system to retain synchronism following a disturbance sustaining for a reasonably long period. And the maximum power that is permissible to flow through the network without loss of stability following a sustained period of disturbance is referred to as the transient stability limit (TSL) of the system. Going beyond that maximum permissible value for power flow, the system would temporarily be rendered as unstable.

**1(d)**

**Sol:** Given open loop transfer function

$$G(s) = \frac{K(s+4)}{s(s+1}$$

No. of root locus branches =2(P > Z)

No. of Asymptotes N = |P – Z|= 1

$$\text{Angle of Asymptotes} = \frac{(2\ell+1)180^0}{P-Z} \quad l = 0$$

$$= \frac{(2(0)+1)180^0}{1} = 180^0$$

Here, only one asymptote is present, therefore centroid is not required.

Break Point CE is $1 + KG_1(s)H_1(s) = 0$

$$K = \frac{-1}{G_1(s)H_1(s)}$$

$$G_1(s)H_1(s) = \frac{s+4}{s(s+1)}$$

$$\frac{dK}{ds} = \frac{d}{ds}\left(\frac{-1}{G_1(s)H_1(s)}\right) = 0$$

$$\frac{d}{ds}\left(-\frac{s(s+1)}{s+4}\right) = 0$$

$$\Rightarrow \frac{(s+4)(2s+1)-(s^2+s)(1)}{(s+4)^2} = 0$$

$$\Rightarrow \quad s^2+8s+4=0$$

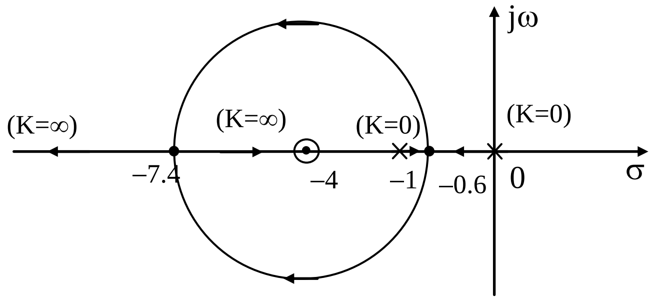


Fig. Root Locus diagram

$\therefore$ s = –0.6 and s = –7.4

The system is critically damped when s= –0.6 and s = –7.4 (roots are real and equal)

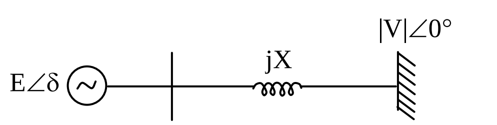
$$K = \frac{\text{Product of distances from poles}}{\text{Product of distances from zeros}}$$

$$K = \frac{(0.6)(0.4)}{3.4} = 0.07 \text{ (at } s = -0.6)$$

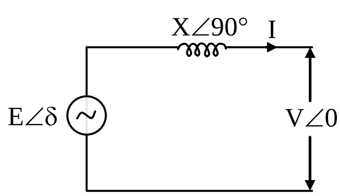
$$K = \frac{(7.4)(6.4)}{3.4} = 13.92 \text{(at } s = -7.4)$$

**1(e)**

**Sol:** A generator is connected to an infinite bus (SMIB) as shown in the figure. In this the reactance includes the reactance of the transmission line and the synchronous reactance (or) the transient reactance of the generator. The sending end voltage is then the internal emf of the generator.




Let the sending end and receiving end voltages be given by E∠δ and V∠0°



Apparent power (s) = VI*

$$= V\angle 0\left(\frac{E\angle\delta - V\angle 0}{X\angle 90^o}\right)$$

$$S= \frac{EV}{X}\angle(90-\delta)-\frac{V^2}{X}\angle 90^o$$

$$P = \frac{EV}{X}\cos(90-\delta)-\frac{V^2}{X}\cos 90^o$$

$$P_e = \frac{EV}{X}\sin\delta = \text{Real power output}$$

$$Q = \frac{EV}{X}\sin(90-\delta)-\frac{V^2}{X}\sin 90$$

$$Q = \frac{EV}{X}\cos\delta-\frac{V^2}{X} = \text{Reactive power output}$$

**2(a) (i)**

**Sol:** **Bode plot:** It has both magnitude and phase plots

Magnitude plot: |G(s)H(s)| in dB Vs frequency (ω).

Phase plot: ∠G(s)H(s) Vs frequency (ω)

**Procedure to sketch the magnitude plot [|G(s) H(s)| in dB vs frequency (ω)]**

- Arrange the TF G(s)H(s) into the standard form.
- Find the corner frequencies and gain 'K"
- Prepare the slope/magnitude change table of G(s)H(s), in the increasing order of corner frequencies with differential (or) integral terms on top of the table.
- Use the above table to draw the magnitude plot.
- $|G(s)H(s)| = |Ks^{\pm n}|\Big|_{\omega\le\text{least corner frequency(LCF)}}$ = 20 log K ± n (20 log ω)

  Where n= no. of differential/integral terms.
- Starting point frequency is chosen in such a way that it is always less than the lowest corner frequency.
- Starting point (ω→0) or low frequency(less than the lowest corner frequency) asymptote slope of the Bode magnitude plot is

$\pm n(-20\text{dB/dec}) = \pm n(6\text{db/octave})$

where n = no. of integral/differential terms, + for differential term and – for integral term.

- High frequency($\omega \to \infty$) asymptote slope of Bode magnitude plot

  $= (P - Z)(-20\text{dB/dec})$

  $= (P - Z)(-6\text{dB/octave})$.

**Phase Plot:**

**Eg**: $G(s)H(s) = \dfrac{K(s+Z)}{s(s+P_1)(s+P_2)}$

Substitute s = j$\omega$ and write the phase as shown below.

$$\angle G(s)H(s) = \angle \tan^{-1}\frac{\omega}{Z} - \left(90 + \tan^{-1}\frac{\omega}{p_1} + \tan^{-1}\frac{\omega}{p_2}\right)$$

At different frequencies calculate the phase and draw the phase plot.

**2(a)(ii)**

**Sol:** $x(t) = \phi(t)x(0) + L^{-1}\left((sI-A)^{-1}BU(s)\right)$

$$sI-A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s+1 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{\text{Adj}(sI-A)}{|sI-A|}$$

$$\text{Adj}(sI-A) = \begin{bmatrix} s+2 & 1 \\ 0 & s+1 \end{bmatrix}$$

$$|sI-A| = (s+1)(s+2)$$

$$(sI-A)^{-1} = \frac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s+1 \end{bmatrix}$$

$$(sI-A)^{-1}BU(s) = \frac{1}{(s+1)(s+2)}\begin{bmatrix} s+2 & 1 \\ 0 & s+1 \end{bmatrix}_{2\times 2}\begin{bmatrix} 1 \\ -1 \end{bmatrix}_{2\times 1}\frac{1}{s}$$

$$= \frac{1}{s(s+1)(s+2)}\begin{bmatrix} s+1 \\ -(s+1) \end{bmatrix}_{2\times 1} = \begin{bmatrix} \dfrac{1}{s(s+2)} \\ \dfrac{-1}{s(s+2)} \end{bmatrix}_{2\times 1} = \begin{bmatrix} \dfrac{1}{2s} - \dfrac{1}{2(s+2)} \\ -\dfrac{1}{2s} + \dfrac{1}{2(s+2)} \end{bmatrix}_{2\times 1}$$

$$L^{-1}((sI-A)^{-1}BU(s))=\begin{bmatrix}\frac{1}{2}-\frac{1}{2}e^{-2t}\\ -\frac{1}{2}+\frac{1}{2}e^{-2t}\end{bmatrix}_{2\times 1}$$

$$\phi(t) = L^{-1}(sI-A)^{-1}$$

$$= L^{-1}\begin{bmatrix}s+2 & 1\\ 0 & s+1\end{bmatrix}.\frac{1}{(s+1)(s+2)}$$

$$= L^{-1}\begin{bmatrix}\frac{1}{s+1} & \frac{1}{(s+1)(s+2)}\\ 0 & \frac{1}{s+2}\end{bmatrix}$$

$$= L^{-1}\begin{bmatrix}\frac{1}{s+1} & \frac{1}{s+1}-\frac{1}{s+2}\\ 0 & \frac{1}{s+2}\end{bmatrix}$$

$$\phi(t) = \begin{bmatrix}e^{-t} & e^{-t}-e^{-2t}\\ 0 & e^{-2t}\end{bmatrix}$$

$$x(t)=\phi(t)x(0)+L^{-1}((sI-A)^{-1}BU(s))$$

Therefore X (t) is

$$x(t)=\begin{bmatrix}e^{-t} & e^{-t}-e^{-2t}\\ 0 & e^{-2t}\end{bmatrix}_{2\times 1}\begin{bmatrix}1\\ 1\end{bmatrix}_{2\times 1}+\begin{bmatrix}\frac{1}{2}-\frac{1}{2}e^{-2t}\\ -\frac{1}{2}+\frac{1}{2}e^{-2t}\end{bmatrix} \quad x(t)=\begin{bmatrix}e^{-t} & e^{-t}-e^{-2t}\\ 0 & e^{-2t}\end{bmatrix}_{2\times 2}\begin{bmatrix}1\\ 1\end{bmatrix}_{2\times 1}+\begin{bmatrix}\frac{1}{2}-\frac{1}{2}e^{-2t}\\ -\frac{1}{2}+\frac{1}{2}e^{-2t}\end{bmatrix}$$

$$=\begin{bmatrix}2e^{-t}-e^{-2t}\\ e^{-2t}\end{bmatrix}+\begin{bmatrix}\frac{1}{2}-\frac{1}{2}e^{-2t}\\ -\frac{1}{2}+\frac{1}{2}e^{-2t}\end{bmatrix}$$

$$\therefore x(t)=\begin{bmatrix}0.5+2e^{-t}-1.5e^{-2t}\\ -0.5+1.5e^{-2t}\end{bmatrix}$$

$$y(t) = [1 \quad 1]\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

$$y(t) = x_1(t)+x_2(t)$$

$= 0.5+2e^{-t}-1.5e^{-2t}-0.5+1.5e^{-2t}$

$y(t) = 2e^{-t}u(t)$

**2(b)**

**Sol: (i)**

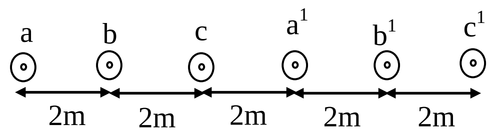


Fig: Bundle conductor arrangements

Self GMD of phase a = $\sqrt{r's}$

Where, $r' = 0.7788r$,

r = radius of the conductor in cm

s = spacing between phase a and $a'$

Self GMD of phase

a = $\sqrt{0.7788\times12\times10^{-3}\times6}$ = 0.237m.

Similarly, self GMD of phases b and c also same.

Self GMD of each phase

$= \sqrt[3]{Self\ GMD_a\,.self\ GMD_b\,.self\ GMD_c}$

$= \sqrt[3]{0.237\times0.237\times0.237} = 0.237$ m

Mutual GMD of phase a

$= \sqrt[4]{D_{ab}D_{ac}D_{ab'}D_{ac'}}$

$= \sqrt[4]{2\times4\times8\times10} = 5.029$ m

Mutual GMD of phase b

$= \sqrt[4]{D_{ba}D_{bc}D_{ba'}D_{bc'}}$

$= \sqrt[4]{2\times2\times4\times8} = 3.36$ m

Mutual GMD of phase c

$= \sqrt[4]{D_{ca}D_{cb}D_{ca'}D_{cb'}}$

$= \sqrt[4]{4\times2\times2\times4} = 2.83$ m

Mutual GMD of each phase

$$= \sqrt[3]{GMD_a.GMD_b.GMD_c}$$

$$= \sqrt[3]{GMD_a.GMD_b.GMD_c} = 3.62 \text{ m}$$

Inductance per phase

$$= 2 \times 10^{-7} \ln \frac{GMD}{GMR} \quad \text{H/m}$$

$$= 2 \times 10^{-7} \ln \frac{3.62}{0.237}$$

$$= 5.452 \times 10^{-7} \text{ H/m/phase}$$

$$= 0.545 \text{ mH/km/phase.}$$

(ii) $$\frac{2\pi\varepsilon_o\varepsilon_r}{\ln\left(\frac{D}{r}\right)} = 0.01\ \mu\text{F/km}$$

$$\frac{2\pi \times 8.85 \times 10^{-12} \times 1}{\ln\left(\frac{4}{r}\right)} = 0.01\ \mu\text{F/km}$$

$$\frac{2\pi \times 8.85 \times 10^{-12}}{0.01 \times 10^{-9}} = \ln\left(\frac{4}{r}\right)$$

$$\ell n\left(\frac{4}{r}\right) = \frac{2\pi \times 8.85 \times 10^{-12}}{0.01 \times 10^{-9}}$$

$$\ell n\left(\frac{4}{r}\right) = \frac{177}{100} \times \pi = 5.5606$$

$$\frac{4}{r} = e^{5.56} \Rightarrow \frac{4}{r} = 259.82$$

$$\Rightarrow \frac{4}{259.82} = r \Rightarrow 0.01539$$

In new configuration, $D_{eq} = \sqrt[3]{4 \times 4 \times 8}$

$$= 5.04$$

$$C_n = \frac{2\pi\,\varepsilon_0\,\varepsilon_r}{\ln\left(\frac{D_m}{D_r}\right)}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12}}{\ln\left(\frac{5.04}{0.015}\right)} = 0.0096\ \mu\text{F/km}$$

**2(c)(i)**

**Sol:** $x(n) \leftrightarrow X(k)$

$$x((n-m))_N \leftrightarrow e^{-j\frac{2\pi}{N}mk} X(k)$$

$$\text{DFT}\left[x((n-m))_N\right] = \sum_{n=0}^{N-1} x((n-m))_N e^{-j\frac{2\pi}{N}nk}$$

$$= \sum_{n=0}^{m-1} x((n-m))_N e^{-j\frac{2\pi}{N}nk} + \sum_{n=m}^{N-1} x((n-m))_N e^{-j\frac{2\pi}{N}nk}$$

Since, $x((n-m))_N = x(N-m+n)$

$$\sum_{n=0}^{m-1} x((n-m))_N e^{-j\frac{2\pi}{N}nk} = \sum_{n=0}^{m-1} x(N-m+n) e^{-j\frac{2\pi}{N}nk}$$

Let , $N-m+n = \ell$

$$\sum_{n=0}^{m-1} x((n-m))_N e^{-j\frac{2\pi}{N}nk} = \sum_{\ell=N-m}^{N-1} x(\ell) e^{-j\frac{2\pi}{N}(N-m+\ell)k}$$

$$= \sum_{\ell=N-m}^{N-1} x(\ell) e^{-j\frac{2\pi}{N}(\ell+m)k}$$

Similarly, $\displaystyle\sum_{n=m}^{N-1} x((n-m))_N e^{-j\frac{2\pi}{N}nk} = \sum_{\ell=0}^{N-1-m} x(\ell) e^{-j\frac{2\pi}{N}(m+\ell)k}$

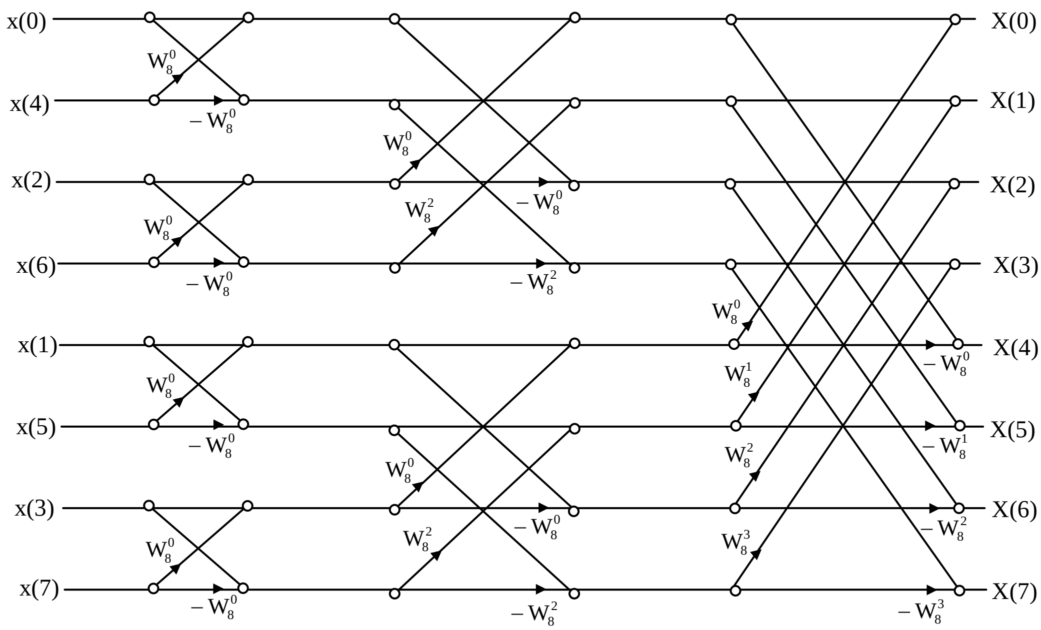
So, $\displaystyle\text{DFT}\left[x((n-m))_N\right] = \sum_{\ell=0}^{N-m-1} x(\ell) e^{-j\frac{2\pi}{N}(m+\ell)k} + \sum_{\ell=N-m}^{N-1} x(\ell) e^{-j\frac{2\pi}{N}(m+\ell)k}$

$$= e^{-j\frac{2\pi}{N}mk} \sum_{\ell=0}^{N-1} x(\ell) e^{-j\frac{2\pi}{N}\ell k}$$

$$= e^{-j\frac{2\pi}{N}mk} X(k)$$

**2(c)(ii)**

**Sol:** $W_8^0 = 1, W_8^1 = 0.707 - j0.707, W_8^2 = -j, W_8^3 = -0.707 - j0.707$




| Inputs | Stage 1 outputs | Stage 2 outputs | Stage 3 outputs |
|---|---|---|---|
| 1 | 1 + 4 = 5 | 5 + 5 = 10 | 10+10=20 |
| 4 | 1– 4 = –3 | –3 + (–j) 1 = –3–j | –3 –j + (0.707–j0.707)(–1–3j) =–5.828–j2.414 |
| 3 | 3 + 2 = 5 | 5 – 5 = 0 | 0 |
| 2 | 3 – 2 = 1 | –3 – (–j)1 = –3+j | –3 +j + (–0.707–j0.707)(–1+3j) =–0.172–j0.414 |
| 2 | 2 + 3 = 5 | 5 + 5 = 10 | 10–10=0 |
| 3 | 2 – 3 = –1 | –1 +(–j) 3 = – 1–j3 | –3 –j – (0.707–0.707)(–1–3j) =–0.172+j0.414 |
| 4 | 4 + 1 = 5 | 5 – 5 = 0 | 0 |
| 1 | 4 – 1 = 3 | –1 –(–j) 3 = – 1+j3 | (–3 +j) – (–0.707–0.707)(–1+3j) =–5.828+j2.414 |

X(k) = {20, –5.828–j2.414, 0, –0.172–j0.414, 0, –0.172+j0.414, 0, –5.828+j2.414}

**3(a)(i)**

**Sol: Features of wind wheel:** Based on the alignment of rotor axis wind wheel can be classified as follows:

1-Horizontal axis, 2-Vertical axis

Based up on the force utilized, they can be classified as follows:

| Lift Type | Drag Type |
|---|---|
| The force component perpendicular to the wind flow direction. | In line |
| High speed. | Low speed |
| Low torque | Rotor shaft torque is comparatively high |
| Aerofoil type blades are required to minimize the effect of drag forces. | Greater blade area is required |
| Blades have high thickness to chord ratio to produce high lift. | Blades are made of curved plates |

The wind wheel rotors are further divided as

1-Multiblade, 2-Prepeller type, 3-Savonious, 4-Davieus.

**1-Multiblade:**

**2-Propeller type:**

**3-Savonious type:**

**4-Darrieus type:**

Economic viability of wind power in comparison to conventional methods:

**Direct effect:**

- It compliments conventional methods by reducing their burden.
- Its running cost is very less.
- Can be integrated in to the conventional power grid.
- Better utilization of land area along with other utilities like farms, offshore plants etc.
- Below 100 kW if cost roughly \$3000 to \$8000 per kW.
- It has significant economy of scale.
- Smaller on residential mill costs more for per kW of electrical energy produced
- Most of windmill plants used are generally around 2 MW in size and roughly cost \$3 to \$4 million.
- With increasing technological development unit cost of production is expected to reduce significantly.
- Economic benefit on the basis of green energy. eg: Carbon credits as per Kyoto Protocol.

**Indirect Effect:**

- A promising green energy alternative.
- A positive impact towards eco-friendly energy.
- In a long term green economic benefits like increase in biodiversity and maintaining the concept of 'living with the nature'.
- It helps in up gradation of quality of life reducing pollution and resource depletion.

**3(a)(ii)**

**Sol:** $\lambda = (\text{Penalty factor}) \times \frac{dc}{dp}$

Let us assume

$\frac{dc_1}{dp_1} = \text{incremental fuel cost for the plant(1)}$

$\frac{dc_1}{dp_1} = 275/- \text{ per MWh}$

$\frac{dc_2}{dp_2} = \text{incremental fuel cost for the plant(2)}$

$\frac{dc_2}{dp_2} = 300/- \text{ per MWh}$

Coordination equation with losses is

$\lambda = L_1 \frac{dc_1}{dp_1} = L_2 \frac{dc_2}{dp_2}$

Since the system $\lambda$ should satisfy the above equation

$\lambda = (275)\, L_1 = (300)\, L_2$

Where $L_1$ = penalty factor for the plant(1)

$L_2$ = penalty factor for the plant(2)

Must be $L_1 > L_2$

Hence the penalty factor of the plant 1 is high.

Given that the cost per hour of increasing the load on the system $\lambda$=341/- per MWh.

From coordination equation,

$341 = 275\, L_1$

$L_1 = 1.24.$

**3(b)(i)**

**Sol:** The given characteristic equation

$s^3 + 4s^2 + 8s + 11 = 0$

| | | |
|---|---|---|
| $s^3$ | 1 | 8 |
| $s^2$ | 4 | 11 |
| $s^1$ | 21/4 | 0 |
| $s^0$ | 11 | |

No. of sign changes in the first column = 0

$\therefore$ Number of righ half of s-plane poles = 0

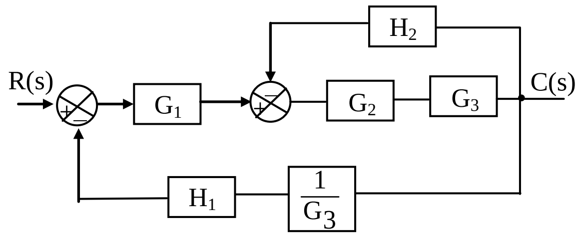
Number of $j\omega$ poles = 0

Number of left half of s-plane poles = 3

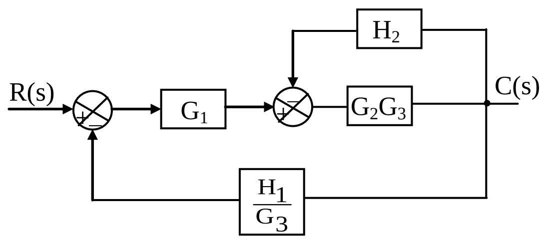
$\therefore$ System is stable.

**3(b)(ii)**

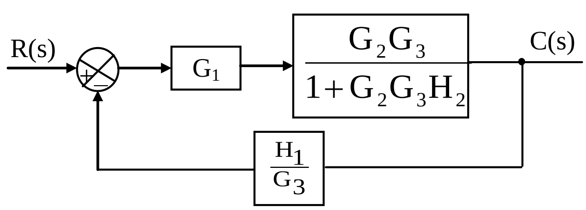
**Sol: Step (i):** Shifting the takeoff point before block $G_3$ to after block $G_3$.



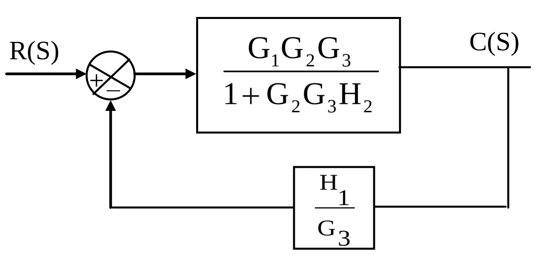

**Step (ii):** Cascading the blocks $G_2$ and $G_3$, $H_1$ and $1/G_3$.




**Step (iii):** Eliminate the feedback loop with $H_2$.




**Step (iv):** Cascade the forward path blocks.



**Step (v):** Eliminate the feedback loop with $H_1/G_3$.

$$\therefore \text{Transfer function, } \frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2}}{1+\dfrac{G_1G_2G_3\left(\dfrac{H_1}{G_3}\right)}{1+G_2G_3H_2}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1}$$

**3(b)(iii)**

**Sol:** A. Step input:

$$K_p = \underset{s\to 0}{\mathrm{Lt}}\, G(s) = \underset{s\to 0}{\mathrm{Lt}} \frac{s+1}{s^2(s+2)(s+4)} = \infty$$

Steady state error = $\dfrac{1}{1+K_p} = \dfrac{1}{1+\infty} = 0$

B. Ramp input:

$$K_v = \underset{s\to 0}{\mathrm{Lt}}\, sG(s) = \underset{s\to 0}{\mathrm{Lt}}\, s\frac{s+1}{s^2(s+2)(s+4)} = \infty$$

Steady state error = $\dfrac{1}{k_v} = \dfrac{1}{\infty} = 0$

C. Parabolic input:

$$K_a = \underset{s\to 0}{\mathrm{Lt}}\, s^2G(s) = \underset{s\to 0}{\mathrm{Lt}}\, s^2\frac{s+1}{s^2(s+2)(s+4)} = \frac{1}{8}$$

Steady state error = $\dfrac{1}{K_a} = 8$

**3(c)(i)**

**Sol:** The given second order system

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = F(t)$$

Taking Laplace transform on both sides

$$as^2\theta(s) + bs\theta(s) + c\theta(s) = F(s)$$

$$\Rightarrow \left(as^2+bs+c\right)\theta(s)=F(s)$$

Transfer function $=\dfrac{\theta(s)}{F(s)}=\dfrac{1}{as^2+bs+c}$

$\therefore$ Step response $\theta(S)=\left(\dfrac{1}{as^2+bs+c}\right)\dfrac{1}{s}$

$$=\left(\frac{1/a}{s^2+\frac{b}{a}s+\frac{c}{a}}\right)\left(\frac{1}{s}\right)$$

$$=\frac{1}{c}\left(\frac{\frac{c}{a}}{s^2+\frac{b}{a}s+\frac{c}{a}}\right)\frac{1}{s}$$

Compare with standard second system $\dfrac{\omega_n^2}{s\left(s^2+2\xi s\omega_n+\omega_n^2\right)}$, we get response as

$$\theta(t)=1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sin\omega_d t+\phi\right)$$

$$\omega_n=\sqrt{\frac{c}{a}}$$

$$2\xi\omega_n=\frac{b}{a}$$

$$\xi=\frac{1}{2}\frac{b}{\sqrt{ac}}$$

$$\omega_d=\omega_n\sqrt{1-\xi^2}$$

$$=\sqrt{\frac{c}{a}}\sqrt{1-\frac{1}{4}\frac{b^2}{ac}}$$

$$=\sqrt{\frac{c}{a}}\sqrt{\frac{4ac-b^2}{4ac}}$$

$$\omega_d=\frac{1}{2a}\sqrt{4ac-b^2}$$

Let assume $\xi=\dfrac{1}{2}\dfrac{b}{\sqrt{ac}}<1$

Then

$$\theta(t)=\frac{1}{c}\left[1-\frac{e^{\frac{-bt}{2a}}}{\sqrt{\frac{4ac-b^2}{4ac}}}\left(\sin\left[\frac{1}{2a}\sqrt{4ac-b^2}\right]t+\cos^{-1}\left(\frac{b}{2\sqrt{ac}}\right)\right)\right]$$

**3(c)(ii)**

**Sol:** The given transfer function

$$G(s)H(s)=\frac{K}{s(s+2)(s+4)}$$

Characteristic equation

$$1+G(s)H(s)=0$$

$s(s^2 + 6s +8) + K = 0$

$s^3 +6s^2 + 8s + K = 0$

Apply Routh Hurwitz criteria,

$$\begin{array}{c|cc} s^3 & 1 & 8 \\ s^2 & 6 & K \\ s^1 & \frac{48-K}{6} & 0 \\ s^0 & K & \end{array}$$

If there are any sign changes in the first column, the system is unstable. So to be stable $\frac{48-K}{6} > 0$, K

$> 0 \Rightarrow K < 48, K > 0$

$\therefore\ 0 < K < 48$

For the system to be stable.

For remaining values of K system is unstable. i.e., $K < 0, K > 48$.

At the point of intersection of root loci with imaginary axis system is critically stable.

$\therefore\ 48 - K = 0$

$K = 48$

Substitute K value in second row.

We will get auxiliary equation as

$6s^2 + K = 0$

$6s^2 + 48 = 0$

$s^2 + 8 = 0$

$s = \pm j2\sqrt{2}$

At $s = \pm j2\sqrt{2}$, root loci intersects with imaginary axis.

**4(a)(i)**

**Sol:** x(t) is expressed in terms of step functions

$x(t) = 1u(t) - 3u(t - T) + 4\,u(t - 2T)$

$- 4\,u(t - 4T) + 2\,u(t - 5T)$

Take the Laplace transform on both sides, and Use the pair $u(t) \rightarrow 1/s$ and time shifting property :

$$u(t - t_0) \rightarrow \frac{1}{s}\, e^{-st_0}$$

$$\therefore\ X(s) = \frac{1}{s}\,[1 - 3e^{-Ts} + 4e^{-2Ts} - 4e^{-4Ts} + 2e^{-5Ts}]$$

**4(a)(ii)**

**Sol:** The convolution property states that the convolution of two signals in time domain is equivalent to the multiplication of their spectra in frequency domain. This is called the time convolution theorem.

If $x_1(t) \xleftrightarrow{FT} X_1(\omega)$ and $x_2(t) \xleftrightarrow{FT} X_2(\omega)$

Then $x_1(t) * x_2(t) \xleftrightarrow{FT} X_1(\omega)X_2(\omega)$

**Proof:** We know that the convolution of two signals $x_1(t)$ and $x_2(t)$ is given by

$$x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t - \tau)d\tau$$

$$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} x_1(\tau)x_2(t - \tau)d\tau\right]e^{-j\omega t}dt$$

Interchanging the order of integration, we have

$$F[x_1(t) * x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau)\left[\int_{-\infty}^{\infty} x_2(t - \tau)e^{-j\omega t}dt\right]d\tau$$

Substituting $t - \tau = P$ in the second integration, we have

$$t = p + \tau \text{ and } dt = dp$$

$$F[x_1(t)*x_2(t)] = \int_{-\infty}^{\infty} x_1(\tau)\left[\int_{-\infty}^{\infty} x_2(p)e^{-j\omega(p+\tau)}dp\right]d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\left[\int_{-\infty}^{\infty} x_2(p)e^{-j\omega p}dp\right]e^{-j\omega\tau}d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)X_2(\omega)e^{-j\omega\tau}d\tau$$

$$= \left[\int_{-\infty}^{\infty} x_1(\tau)e^{-j\omega\tau}d\tau\right]X_2(\omega) = X_1(\omega)X_2(\omega)$$

$$x_1(t)*x_2(t) \xleftrightarrow{FT} X_1(\omega)X_2(\omega)$$

**4(a)(iii)**

**Sol:** let, $x(t) = L^{-1}[X(s)]$

$$= L^{-1}\left[\log\frac{s(s+1)}{s^2+1}\right]$$

$$L[x(t)] = \log\left[\frac{s(s+1)}{s^2+1}\right]$$

$$= \log s(s + 1) - \log (s^2 + 1)$$

$$= \log s + \log (s + 1) - \log (s^2 + 1)$$

$$L[tx(t)] = -\frac{d}{ds}\left[\log s + \log(s+1) - \log(s^2+1)\right]$$

$$= \frac{-1}{s} - \frac{1}{s+1} + \frac{2s}{s^2+1}$$

$$t\,x(t) = L^{-1}\left(\frac{-1}{s} - \frac{1}{s+1} + \frac{2s}{s^2+1}\right)$$

$$= -L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{1}{s+1}\right) + 2L^{-1}\left(\frac{s}{s^2+1}\right)$$

$$= [-1 - e^{-t} + 2\cos(t)]\,u(t)$$

$$\therefore \quad x(t) = \left[\frac{2\cos t - e^{-t} - 1}{t}\right]u(t)$$

**4(b)**

**Sol:** (i) From the data given

$$Z_n = \sqrt{\frac{\ell}{c}} = \sqrt{\frac{10^6}{11.1}} = 300\Omega$$

$$\beta = \omega\sqrt{\ell c} = 2\pi.50\sqrt{1\times 11.1\times 10^{-12}} \times \frac{180}{\pi}\text{ degrees}$$

$$= 0.06 \text{ deg/km}$$

$$\theta = \beta d = 0.06\times 600 = 36^0$$

$$P_{max} = \frac{P_n}{\sin\theta} = \frac{400\times 400}{300\sin(36^0)} = 907.4\text{MW}$$

$P_{max}$ occurs at $\delta = \delta_{max} = 90^0$. The midpoint voltage corresponding to this condition is

$$V_m = \frac{V\cos\left(\frac{\delta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{400\cos 45^0}{\cos 18^0} = 297.4\text{kV}$$

(ii) Since the power flow with a series capacitor is given by

$$P = \frac{P_n \sin\delta}{\sin\theta(1-k_{se})}$$

If power is to be doubled, $k_{se} = 0.5$. The expression for $k_{se}$ (when the series capacitor is connected at the midpoint) is given by

$$k_{se} = \frac{X_c}{2Z_n}\cot\left(\frac{\theta}{2}\right)$$

Substituting the values for $k_{se}$, and $Z_n$, we get

$$X_e = 2k_{se}Z_n \tan\left(\frac{\theta}{2}\right) = 300\times\tan 18^0 = 97.48\text{ohms}$$

(iii)The midpoint voltages, with a shunt capacitor connected there, is given by

$$V_m = \frac{V\cos\left(\frac{\delta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)(1-k_{sh})}, k_{sh} = \frac{B_c Z_n}{2}\tan\frac{\theta}{2}$$

Hence,

$$\cos\frac{\delta}{2} = 0.97\times\cos 18^0\left[1-\frac{300\tan 18^0}{450\times 2}\right]$$

$= 0.823$

$\delta = 69.31^0$

The power flow in the line is given by

$$P = \frac{VV_m \sin\frac{\delta}{2}}{Z_n \sin\frac{\theta}{2}} = \frac{1\times 0.97\times\sin 34.65^0}{\sin 18^0}\times P_n$$

$$= 1.785P_n = \frac{1.785\times 400\times 400}{300} = 952\,\text{MW}$$

**4(c)(i)**

**Sol:** Given that,

**A.** $A = 0.96\angle 1°$, $B = 100\angle 80°$

$|V_s|$ (L-L) = 110 kV, $|V_r|$ (L-L) = 110 kV

$\angle(V_s\,(ph) - V_r(ph)) = 30°$

Now, $V_s(ph) = \dfrac{110}{\sqrt{3}}\angle 30°$ kV

$V_r(ph) = \dfrac{110}{\sqrt{3}}\angle 0°$ kV (reference phasor)

$\because\ V_s = AV_r + BI_r$

$$\Rightarrow \frac{110\times 10^3}{\sqrt{3}}\angle 30° = (0.96\angle 1°)\left(\frac{110}{\sqrt{3}}\angle 0°\right)$$

$$+ (100\angle 80)(I_r)$$

$\Rightarrow\quad I_r = 312.63\ \angle 20.98°$ A

Hence, receiving end current = $I_r$

$= 312.63\angle 20.98°$

And receiving end power factor

$= \cos(20.98)$ (leading)

$= 0.933$ (leading)

**B** If the load is suddenly thrown off then,

$$V_r = \frac{|V_s|}{|A|}(I_r = 0)$$

$$= \frac{\frac{110}{\sqrt{3}} \times 10^3}{0.96} = 66.154\,\text{kV}$$

Now, $V_r$(L-L) = 114.58 kV

**4(c)(ii)**

**Sol:** Since 85% to be protected against phase to ground fault remains 15% of winding unprotected

$$\text{Full load current} = \frac{100 \times 10^6}{\sqrt{3} \times 11 \times 10^3}$$

= 5248.6 A

Minimum fault current which will operates 20% full load current

$$= \frac{20}{100} \times 5248.6$$

= 1049.7 A

15% of winding

$$= \frac{15}{100} \times \frac{11 \times 10^3}{\sqrt{3}}$$

= 952.62 V

Fault current 15% winding will cause

$$= \frac{952.62}{r}$$

$$1049.7 = \frac{952.62}{r}$$

r = 0.907 Ω

**5(a)**

**Sol:** The energy of a signal x(t) is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-\infty}^{\infty} |e^{-4t}u(t)|^2\,dt$$

$$= \int_0^{\infty} \left|e^{-4t}\right|^2 dt = \int_0^{\infty} e^{-8t} dt = \left[\frac{e^{-8t}}{-8}\right]_0^{\infty} = \frac{1}{8} \text{ joule}$$

Now, according to Parseval's theorem, we have

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$X(\omega) = F[x(t)] = \int_{-\infty}^{\infty} e^{-4t} u(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(4+j\omega)t} dt = \left[\frac{e^{-(4+j\omega)t}}{-(4+j\omega)}\right]_0^{\infty} = \frac{1}{4+j\omega}$$

$$|X(\omega)| = \frac{1}{\sqrt{4^2+\omega^2}}$$

$$\therefore \qquad E = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{4^2+\omega^2} d\omega = \frac{1}{8\pi} \tan^{-1}\left[\frac{\omega}{4}\right]_{-\infty}^{\infty}$$

$$= \frac{1}{8\pi}\left[\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right] = \frac{1}{8} \text{ joule}$$

Thus, from the above equations, we see that energy is same in both the cases.

Hence Parseval's theorem is verified.

**5(b)**

**Sol: (i)** Causes of low power factor:

**1. Inductive loads**:

→ 90% of the industrial loads consists of induction machines (1-phase and 3-phase). They draw lagging reactive power from the system and work at low power factor

- For induction motors, the pf is usually extremely low (0.2 – 0.3) at light load conditions and (0.8 – 0.9) at full load.
- Other inductive loads are transformers, generators, arc lamps, electric furnaces etc.

**2. Variation of loads:**

When the system is lightly loaded the voltage increase and the current drawn by machines also increase, this results in low power factor.

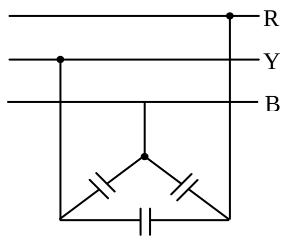
**3. Harmonic currents**:

- Harmonics currents reduce the power factor
- eg: power imbalance due to improper winding and electrical accidents result in low power factor.

**Measures of avoiding low power factor:**

**1. Capacitor bank**

- simplest method
- applied at areas where large inductive loads are present




**2. Synchronous condenser:**

- An over excited synchronous machine behaves as a synchronous condenser ($E \cos\delta > V$)
- Allows stepless power factor correction.

**3. Phase advancer:**

- Can be used only for induction motors required lagging current is supplied from an alternate source called phase advancer
- A phase advancer is basically an AC exciter. It is mounted on the same shaft as the main motor and connected in the rotor circuit.

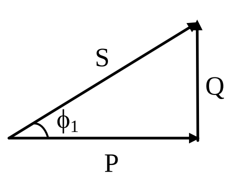
**4. Static compensation:**

- Static VAR compensators can be used for stepless and quick compensation of reactive power, thereby improving the power factor.

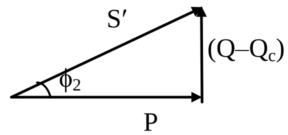
**(ii)** Initially, $Q = P \tan \phi_1$

With capacitor,

$Q - Q_c = P \tan \phi_2$



Then, $Q_c = P(\tan\phi_1 - \tan\phi_2)$




$= 20\times10^3(\tan(\cos^{-1}(0.8)) - \tan(\cos^{-1}(0.95))$

$Q_c = 8426.31$ VAR

$= 8.426$ kVAR

Hence, Rating of the capacitor

$= 8.426$ kVAR

**5(c)**

**Sol:** Arrange the given transfer function as,

$$\frac{Q(s)}{I(s)} = \frac{1}{J\left[s^2 + \frac{f}{J}s + \frac{K}{J}\right]} = \frac{\left[\frac{1}{J}\right]}{s^2 + \frac{f}{J}s + \frac{K}{J}}$$

Comparing denominator with $s^2+2\zeta\omega_n s + \omega_n^2$,

$$\omega_n^2 = \frac{K}{J} \quad \text{i.e. } \omega_n = \sqrt{\frac{K}{J}} \;\ldots..\;(1)$$

$$\text{and } 2\zeta\omega_n = \frac{f}{J} \quad \text{i.e., } \zeta = \frac{f}{2\sqrt{KJ}} \;\ldots..\;(2)$$

Now $M_p = 6\%$ i.e., 0.06

$$\therefore 0.06 = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

$$ln\,(0.06) = \frac{-\pi\zeta}{\sqrt{1-\zeta^2}}$$

solving for $\zeta$, $\zeta = 0.667$ ……. (3)

$$T_p = \frac{\pi}{\omega d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1 \text{ sec}$$

$\omega_n = \dfrac{\pi}{\sqrt{1-(0.667)^2}}$ = 4.2165 rad/sec. … (4)

The Laplace transform of output is Q(s).

Now input is step of 10 Nm hence corresponding Laplace transform is,

$I(s) = \dfrac{10}{s}$

$\therefore \dfrac{Q(s)}{\left(\dfrac{10}{s}\right)} = \dfrac{1}{Js^2 + fs + K}$

$\therefore Q(s) = \dfrac{1}{s(Js^2 + fs + K)}$

The steady state of output can be obtained by final value theorem.

Steady state output = $\lim\limits_{s\to 0} s\,Q(s)$

$0.5 = \lim\limits_{s\to 0} \dfrac{s.10}{s(Js^2 + fs + K)} = \dfrac{10}{k}$

$\therefore \quad K = 20$

Equating (1) and (4), $4.2165 = \sqrt{\dfrac{K}{J}}$

$4.2165 = \sqrt{\dfrac{20}{J}}$

$J = 1.1249$

From equation (2), $0.667 = \dfrac{f}{2\sqrt{KJ}}$

$0.067 = \dfrac{f}{2\sqrt{20 \times 1.1249}}$

$f = 6.3274$

**5(d)**

**Sol:** Given

Generator rating is

$V_R = 6.6$ kV

$(VA)_R = 20$ MVA

$X''_d = 10\% = 0.1$

$X'_d = 20\% = 0.2$

$X_d = 100\% = 1$

Generator is connected to transformer through a circuit breaker and a 3ϕ fault occurs between breaker and transformer.

(i) Sustained short circuit current through breaker ($I_s$) is the steady state fault current. During steady state,

Reactance = $X_d$ = 1 pu

$$\therefore I_s = \frac{V_{pu}}{X_{d\,pu}} = \frac{1}{1} = 1 \text{ pu}$$

$I_s$ = 1 pu

$$\text{Rated current } I_R = \frac{(VA)_R}{\sqrt{3}\,V_R}$$

$$= \frac{20\times10^6}{\sqrt{3}\times6.6\times10^3}$$

$\Rightarrow$ $I_R$ = 1749.5 A

$\therefore$ Sustained short circuit current

=1749.5A

(ii) Let initial symmetrical rms short circuit current = $I_1$

During initial times, reactance = $X''_d$

= 0.1 pu

$$\therefore \quad I_1 = \frac{V\,pu}{X''_d\,pu} = \frac{1}{0.1} = 10\,pu$$

$\therefore$ $I_{1\ actual} = I_{1\ pu} \times I_R$

= 10 × 1749.5 A

$I_{1\ actual}$ = 17495 A

$\therefore$ Initial symmetrical rms short circuit current = 17495 A.

**5(e)**

**Sol: (i)** $(a)^n u(n) \leftrightarrow \dfrac{1}{1-az^{-1}}$

$$-(a)^n u(-n-1) \leftrightarrow \frac{1}{1-az^{-1}}$$

Given, $x(n) = \left(-\dfrac{1}{5}\right)^n u(n) + 5\left(\dfrac{1}{2}\right)^{-n} u(-n-1)$

Apply z-transform

$$X(Z) = \frac{1}{1+\frac{1}{5}z^{-1}} - \frac{5}{1-2z^{-1}}$$

ROC = (|z|>0.2)$\cap$(|z|<2) = 0.2 < |z| < 2

**(ii)** $\delta(n) \leftrightarrow 1$

$\delta\,(n-n_0) \leftrightarrow z^{-n_0}$

So, $X(z) = \dfrac{1}{2} + z^{-1} - \dfrac{1}{3}z^{-2}$

ROC is entire z-plane except at z = 0.

**(iii)** Given $x(n) = n\left(\dfrac{1}{3}\right)^n u(n) + 0.5\left(\dfrac{1}{3}\right)^n u(n)$.

$$\left(\frac{1}{3}\right)^n u(n) \leftrightarrow \frac{1}{1-\frac{1}{3}z^{-1}}$$

$$n\left(\frac{1}{3}\right)^n u(n) \leftrightarrow -z\frac{d}{dz}\left[\frac{1}{1-\frac{1}{3}z^{-1}}\right] = \frac{\frac{1}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)^2}$$

So, $X(Z) = \dfrac{\frac{1}{3}z^{-1}}{\left(1-\frac{1}{3}z^{-1}\right)^2} + \dfrac{0.5}{1-\frac{1}{3}z^{-1}}$

ROC is |z| > 1/3.

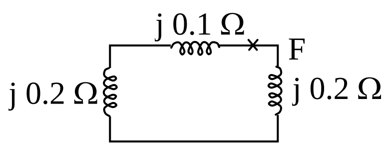
**6(a)(i)**

**Sol:** Taking base quantities as 50 MVA, 13.2 kV,

The base current = $\dfrac{50\times1000}{\sqrt{3}\times13.2}$ = 2187 amps

The pre fault voltage = $\dfrac{12.5}{13.2}$ = 0.9469 p.u.

Take this voltage as the reference.




The fault impedance = $\dfrac{j0.3\times j0.2}{j0.5}$ = j0.12 p.u.

$\therefore$ The fault current = $\dfrac{0.9469}{j0.12}$ = –j7.89 p.u.

The full load current before the fault takes place = $\dfrac{30\times1000}{\sqrt{3}\times12.5\times0.8}$ = 1732 amps

p.u. load current = $\dfrac{1732}{2187}$ = $0.792\angle36.8°$

= 0.634 +j0.474

The p.u. fault current supplied by the motor = –j7.89 × 3/5 = –j4.734

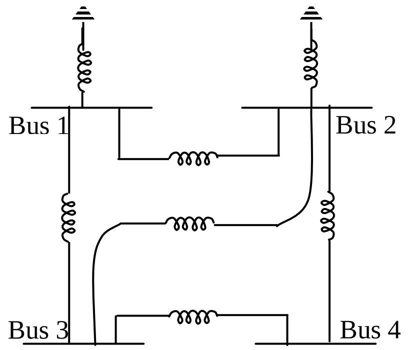
The net current supplied by the motor = –0.6344 –j0.4746 –j4.734

= (–0.6344–j5.2086)p.u.

= 5.247 p.u.

**6(a)(ii).**

**Sol:**



$Y_{11} = y_{11} + y_{12} + y_{13} + y_{14} = -j6$ ............... (i)

$Y_{22} = y_{21} + y_{22} + y_{23} + y_{24} = -j10$ ............. (ii)

$Y_{33} = y_{31} + y_{32} + y_{33} + y_{34} = -j9$ ............ (iii)

$Y_{44} = y_{41} + y_{42} + y_{43} + y_{44} = -j8$ ............ (iv)

From the given $Y_{BUS}$ matrix

$y_{12} = -j2, \quad y_{13} = -j2.5, \quad y_{14} = 0$ .............. (v)

$y_{21} = -j2, \quad y_{23} = -j2.5, \quad y_{24} = -j4$ ............(vi)

$y_{31} = -j2.5, \; y_{32} = -j2.5, \quad y_{34} = -j4$ ..........(vii)

$y_{41} = 0, \qquad y_{42} = -j4, \qquad y_{43} = -j4$ ..........(viii)

From (i) & (v)

$y_{11} - j2 - j2.5 + 0 = -j6$

$y_{11} = -j1.5 \;\Rightarrow X_{11} = j0.67$

From (2) & (6)

$-j2 + y_{22} - j2.5 - j4 = -j10$

$y_{22} = -j1.5 \;\Rightarrow X_{22} = j0.67$

From (iii) & (vii)

$-j2.5 - j2.5 + y_{33} - j4 = -j9$

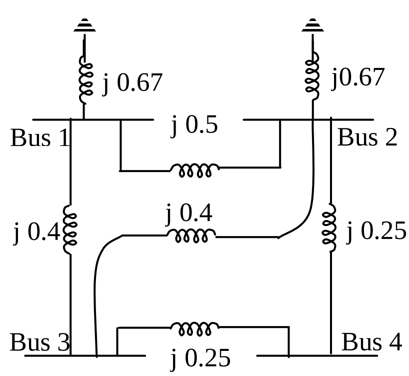
$y_{33} = 0 \;\Rightarrow X_{33} = \infty$

From (iv) & (viii)

$0 - j4 - j4 + y_{44} = -j8$

$y_{44} = 0 \;\Rightarrow X_{44} = \infty$

The Reactance diagram is



**6(b)**

**Sol:** Given $x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$

**(i) Autocorrelation:**

Autocorrelation of function x(t) is given by

$$R(\tau) = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{T}\int_{-T/2}^{T/2} x(t)x(t+\tau)dt$$

$$R(\tau) = \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{T}\int_{-T/2}^{T/2}\left[C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)\right]\left[C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + n\omega_0\tau + \theta_n)\right]$$

$$= \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{T}\int_{-T/2}^{T/2}\left[C_0^2 + C_0C_n\sum_{n=1}^{\infty}\cos(n\omega_0 t + n\omega_0\tau + \theta_n) + \sum_{n=1}^{\infty} C_0C_n\cos(n\omega_0 t + \theta_n)\right.$$

$$\left. + C_n^2\sum_{n=1}^{\infty}\cos(n\omega_0 t + \theta_n)\cos(n\omega_0 t + n\omega_0\tau + \theta_n)dt\right]$$

$$= \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{T}\int_{-T/2}^{T/2} C_0^2 dt + \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{T}\int_{-T/2}^{T/2} C_0C_n\sum_{n=1}^{\infty}\cos(n\omega_0 t + n\omega_0\tau + \theta_n)dt$$

$$+ \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{T}\int_{-T/2}^{T/2}\sum_{n=1}^{\infty} C_0C_n\cos(n\omega_0 t + \theta_n)dt$$

$$+ \underset{T\to\infty}{\mathrm{Lt}} \frac{1}{T}\int_{-T/2}^{T/2}\sum_{n=1}^{\infty} C_n^2\cos(n\omega_0 t + \theta_n)\cos(n\omega_0 t + n\omega_0\tau + \theta_n)dt$$

$$= C_0^2 + 0 + 0 + \sum_{n=1}^{\infty}\underset{T\to\infty}{\mathrm{Lt}} \frac{C_n^2}{2T}\int_{-T/2}^{T/2} 2\cos(n\omega_0 t + \theta_n)\cos(n\omega_0 t + n\omega_0\tau + \theta_n)dt$$

$$= C_0^2 + \sum_{n=1}^{\infty}\underset{T\to\infty}{\mathrm{Lt}} \frac{C_n^2}{2T}\int_{-T/2}^{T/2}\left[\cos(2n\omega_0 t + n\omega_0\tau + 2\theta_n) + \cos(n\omega_0\tau)\right]dt$$

$$= C_0^2 + \sum_{n=1}^{\infty}\underset{T\to\infty}{\mathrm{Lt}} \frac{C_n^2}{2T}\int_{-T/2}^{T/2}\cos n\omega_0\tau dt = C_0^2 + \sum_{n=1}^{\infty}\underset{T\to\infty}{\mathrm{Lt}} \frac{C_n^2}{2T}\cos(n\omega_0\tau)\left(\frac{T}{2} + \frac{T}{2}\right)$$

$$= C_0^2 + \sum_{n=1}^{\infty}\underset{T\to\infty}{\mathrm{Lt}} \frac{C_n^2}{2T}\cos n\omega_0\tau(T)$$

$$\therefore\ R(\tau) = C_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2\cos n\omega_0\tau$$

Power spectral density (PSD) and autocorrelation form a Fourier transform pair

$\therefore \quad PSD = F[R(\tau)]$

$$PSD = F\left[C_0^2 + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2 \cos n\omega_0\tau\right]$$

$$= C_0^2[2\pi\delta(\omega)] + \frac{1}{2}\sum_{n=1}^{\infty} C_n^2 \pi[\delta(\omega - n\omega_0) + \delta(\omega + n\omega_0)]$$

**6(c)**

**Sol: Hanning window:** $w_{Hn}(n) = 0.5 + 0.5\cos\left(\frac{2\pi n}{N-1}\right)$ for $-\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right)$

= 0 otherwise

N = 11

$w_{Hn}(0) = 1$

$w_{Hn}(1) = w_{Hn}(-1) = 0.9045$

$w_{Hn}(2) = w_{Hn}(-2) = 0.655$

$w_{Hn}(3) = w_{Hn}(-3) = 0.345$

$w_{Hn}(4) = w_{Hn}(-4) = 0.0945$

$w_{Hn}(5) = w_{Hn}(-5) = 0$

The filter coefficients are obtained as

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}).e^{j\omega n}d\omega = \frac{\sin \pi n - \sin\left(\frac{\pi}{4}n\right)}{\pi n}$$

$$h_d(0) = \underset{n\to 0}{\text{Lt}} \frac{\sin(n\pi) - \sin\left(\frac{n\pi}{4}\right)}{n\pi} = 1 - \frac{1}{4} = 0.75$$

$h_d(-1) = h_d(1) = -0.225$

$h_d(-2) = h_d(2) = -0.159$

$h_d(-3) = h_d(3) = -0.075$

$h_d(-4) = h_d(4) = 0$

$h_d(-5) = h_d(5) = 0.045$

The filter coefficients using hanning window are

$h(n) = h_d(n).w_{Hn}(n)$ for $-5 \le n \le 5$

$= 0$ otherwise

$h(0) = h_d(0)\ w_{Hn}(0) = 0.75$

$h(1) = h(-1) = h_d(1)\ w_{Hn}(1) = -0.204$

$h(2) = h(-2) = h_d(2)\ w_{Hn}(2) = -0.104$

$h(3) = h(-3) = h_d(3)\ w_{Hn}(3) = -0.026$

$h(4) = h(-4) = h_d(4)\ w_{Hn}(4) = 0$

$h(5) = h(-5) = h_d(5)\ w_{Hn}(5) = 0$

The transfer function of the filter is $H(z) = h(0) + \sum_{n=1}^{5} h(n)\left[z^n + z^{-n}\right]$

$$H(z) = 0.75 - 0.204\left(z + z^{-1}\right) - 0.104\left[z^2 + z^{-2}\right] - 0.026\left[z^3 + z^{-3}\right]$$

The transfer function of realizable filter is

$$H^1(z) = z^{-5}.H(z) = -0.026z^{-2} - 0.104z^{-3} - 0.204z^{-4} + 0.75z^{-5} - 0.204z^{-6} - 0.104z^{-7} - 0.026z^{-8}$$

The causal filter coefficients are $h(0) = h(1) = h(9) = h(10) = 0$

$h(2) = h(8) = -0.026$

$h(3) = h(7) = -0.104$

$h(4) = h(6) = -0.204$

$h(5) = 0.75$

**7(a)(i)**

**Sol:** A. Principle of Argument states that let F(s) be an analytic function and if an arbitrary closed clockwise contour is chosen in s-plane, so that F(s) is analytic at every point on the closed contour in s-plane then the corresponding F(s) plane contour mapped in the F(s) plane will encircle the origin, N times in anticlockwise direction, where N is the difference between the number of poles and number of zeros of F(s) that are encircled by the chosen closed contour in s-plane mathematically, it is expressed as, N = P – Z

**B.** The loop transfer function, G(s) H(s) is expressed as

G(s) H(s) = [1 + G(s) H(s)] –1 = F(s) – 1

From above equation, it can be concluded that contour of F(s) drawn with respect to origin of F(s) plane is same as contour of F(s) – 1 drawn with respect to –1 + j0 of F(s) plane. We know that F(s) = 1 + G(s) H(s) is the characteristic equation, origin (0, 0) is the critical point for F(s).

$\therefore$ (–1, 0) is the critical point for F(s) –1 = G(s)H(s)

$\therefore$ The critical point in using the Nyquist criterion is (–1,j0) in G(s)H(s) plane and not the origin (0,j0).

C. For a minimum phase transfer function, no right hand poles for G(s)H(s) should be present.

For stability, the polar plot of a minimum phase system should not enclose (-1,j0) critical point

$\therefore$ Polar plot is sufficient to determine the stability of a system

**7(a)(ii)**

**Sol:** State variable representation is in controllable phase variable form hence system is controllable. Now it is possible to design SVFB gain controller 'K'.

From the given matrix 'A' Characteristic equation with SVFB gain 'K' is $\left|sI - [A - BK]\right| = 0$

$$A - BK = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$$

$$A - BK = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_1 & k_2 & k_3 \end{bmatrix}$$

$$A - BK = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(6+k_1) & -(11+k_2) & -(6+k_3) \end{bmatrix}$$

$$sI - [A - Bk] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(6+k_1) & -(11+k_2) & -(6+k_3) \end{bmatrix} \qquad sI - [A - Bk] = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ (6+k_1) & (11+k_2) & s+(6+k_3) \end{bmatrix}$$

CE= $\left|sI - (A - Bk)\right| = 0$

$s^{3+}(6+k_3)s^2 + (11+k_2)s+(6+k_1) = 0$--------(1)

Desired pole locations are -3, -4 and -5, hence CE is (s+3)(s+4)(s+5)=0

Desired CE= $s^3+12s^2+47s+60=0$ -------(2)

Comparing eqs. (1) and (2)

$6+k_3=12 \Rightarrow k_3=6$

$11+k_2=47 \Rightarrow k_2=36$

$6+k_1=60 \Rightarrow k_1=54$

Hence SVFB gain controller K=[54 36 6]

**7(b)(i)**

**Sol:** In a power system, there are mainly two types of buses: load and generator buses. For these buses we have specified the real power 'P' injections.

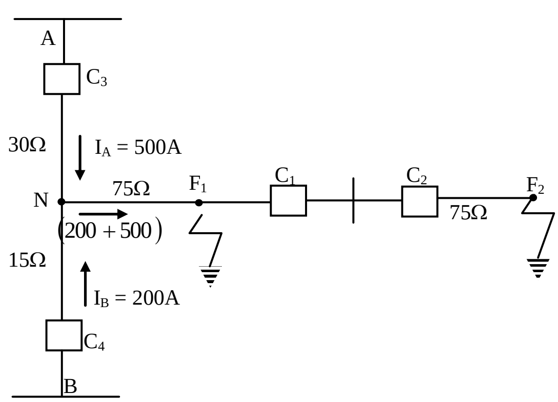
Now $\sum_{i=1}^{n} P_i$ real power loss where $P_i$ is the power injection at the buses, which is taken as positive for generator buses and is negative for load buses. The losses remain unknown until the load flow solution is complete. For this reason that generally one of the generator buses is made to take additional real and reactive power to supply transmission losses. That is why this type of bus is known as "Slack or Swing or Reference Bus". At this bus, the voltage magnitude V and phase angle $\delta$ are specified where as real and reactive powers $P_G$ and $Q_G$ are obtained.

The phase angle of the voltage at the slack bus is usually taken as the reference. In the following analysis the real and reactive components of voltage at a bus are taken as the independent variables for the load flow equations i.e.,

$V_i \angle \delta_i = e_i + jf_i$

**7(b)(ii)**

**Sol:**



Let us assume, the common point between A&B distance relays is 'N'

The zone 1 setting at A and B is 150Ω (assume)

$Z(1) = 150\ \Omega$

A. Fault at $F_1$, the voltage drop from A to $F_1$

$$= I_{AN} Z_{AN} + I_{NF_1} Z_{NF1}$$

$$I_{NF1} = 500 + 200 = 700\,A$$

$$Z_{NF_1} = 75\Omega,\ I_{AN} = 500 = I_A$$

$$Z_{AN} = 30\Omega$$

Voltage drop = $500 \times 30 + 700 \times 75$

$= 67500$ V

Impedance seen by the relay at $A = \frac{V}{I}$

$$= \frac{\text{voltage drop}}{I_A} = \frac{67500}{500} = 135\Omega$$

Zone1, setting of the distance relay at

A = 150Ω

∴ The relay at A will see the fault at $F_1$ and trip before the circuit breaker at B has tripped.

B. Voltage drop from B to $F_1$ $F_1 = I_{BN} Z_{BN} + I_{NF1} Z_{NF1}$

$$= I_B Z_{BN} + I_{NF_1} Z_{NF_1}$$

$$= 200 \times 15 + 700 \times 75 = 55500\ \text{V}$$

The impedance measured by the relay at B

$$= \frac{\text{Voltage drop}}{I_B} = \frac{55500}{200} = 277.5\ \Omega$$

∴ the relay at B will not trip before the circuit breaker at A has tripped.

when the circuit breaker at A has tripped, the relay at B will measure the impedance

= 15 + 75 = 90 Ω and trip the circuit.

C. Impedance measured by the relay at A

$$= \frac{V}{Z} = \frac{\text{Voltage drop upto } F_2}{Z_A}$$

Voltage drop up to $F_2 = V_{AN} + V_{NF_2}$

$$= I_A Z_{AN} + (I_A + I_B)(Z_{NF_1} + Z_{F_1F_2})$$

$$= 500 \times 30 + (500 + 200)(75 + 75)$$

$$= 120000 \text{ V}$$

Impedance $= \dfrac{120000}{500} = 240\Omega > 150\Omega$

Impedance measured by the relay at B

$$= \frac{\text{Voltage drop upto } F_2}{Z_B}$$

$$V = V_{BN} + V_{NF_2}$$

$$= I_B Z_{BN} + (I_A + I_B) Z_N F_2$$

$$= 200 \times 15 + 700 \times 150 = 108000 \text{ V}$$

Impedance $= \dfrac{108000}{200} = 540\,\Omega > 150\,\Omega$

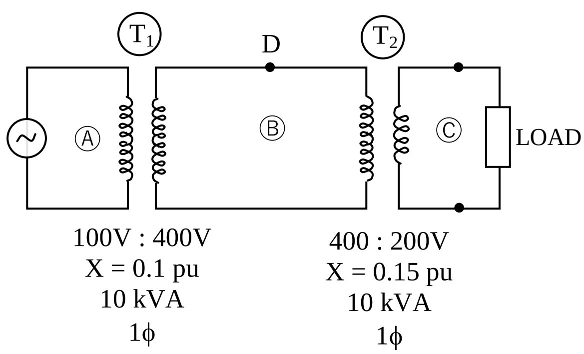
The relay at A will not operate

The relay at B also will not operate

The fault at $F_2$ will be cleared by back-up protection.

**7(c)**

**Sol:** Given system is,




Given, load voltage $V_L = 200$ V

Load kVA $(VA)_L = 10$ kVA

And load is purely resistive

$\therefore$ Load resistance $R_L = \dfrac{V_C^2}{(VA)_L} = \dfrac{(200)^2}{10 \times 10^3} = 4\Omega$

Let base values in region Ⓒ be

$\left.\begin{array}{l} V_C = 200 \text{ V} \\ (VA)_C = 10 \text{ kVA} \end{array}\right\}$ Base values in region Ⓒ

$\therefore$ Corresponding base values in Ⓑ are

$(VA)_B = 10$ kVA

$V_B = 400$ V

Corresponding base values in region Ⓐ are

$V_A = 100$ V

$(VA)_A = 10$ kVA

Base impedance in Ⓐ $Z_A = \dfrac{V_A^2}{(VA)_A}$

$$= \frac{(100)^2}{10\times10^3} = 1\ \Omega$$

Base impedance in Ⓑ $Z_B = \dfrac{V_B^2}{(VA)_B}$

$$= \frac{(400)^2}{10\times10^3} = 16\ \Omega$$

Base impedance in Ⓒ $Z_C = \dfrac{V_C^2}{(VA)_C}$

$$= \frac{(200)^2}{10\times10^3} = 4\ \Omega$$

Load impedance $R_L$ seen from the secondary of $T_2$ in Ⓒ is

$Z_{LC} = R_L = 4\ \Omega$

Load impedance $R_L$ seen from the primary of $T_2$ in Ⓑ is, $Z_{LB}$

$\dfrac{Z_{LB}}{Z_{LC}} = \left(\dfrac{N_1}{N_2}\right)_{T_2}^2$ where $\left(\dfrac{N_1}{N_2}\right)_{T_2}$ turns ratio of transformer $T_2$

$Z_{LB} = Z_{LC}\left(\dfrac{400}{200}\right)^2 = 4\times(2)^2 = 16\ \Omega$

$Z_{LB} = 16\ \Omega$

Load impedance seen by primary of $T_1$ is in region Ⓐ is $Z_{LA}$

$$\frac{Z_{LA}}{R_L} = \left(\frac{N_1}{N_2}\right)_{\pi}^2\left(\frac{N_1}{N_2}\right)_{T_2}^2 = \left(\frac{100}{400}\right)^2\left(\frac{400}{200}\right)^2$$

$$= \frac{1}{16} \times 4 = \frac{1}{4}$$

$$Z_{LA} = R_L.\frac{1}{4} = 4.\frac{1}{4} = 1\ \Omega$$

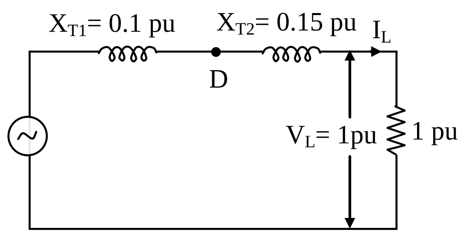
pu load impedance in region Ⓑ is

$$Z_{LB\ pu} = \frac{Z_{LB}}{Z_B} = \frac{16}{16} = 1\ \text{pu}$$

Pu load impedance in region Ⓐ is

$$Z_{LA\,pu} = \frac{Z_{LA}}{Z_A} = \frac{1\Omega}{1\Omega} = 1\text{pu}$$

Single line diagram of the circuit is given by




By taking current as reference

$$I_{L_{pu}} = \frac{V_{L_{pu}}}{R_{L_{pu}}} = \frac{1}{1} = 1\text{pu}$$

$$\therefore \overline{V}_D = \left(R_{L_{pu}} + jX_{T2_{pu}}\right)\overline{I}_{L_{pu}}$$

$$= (1 + j0.15)\ (1)$$

$$\overline{V}_{D_{pu}} = 1 + j0.15$$

$$\left|\overline{V}_D\right|_{pu} = \sqrt{1^2 + (0.15)^2}$$

$$= 1.011187\ \text{pu}$$

$$\therefore V_{D_{pu}} = 1.011187\ \text{pu}$$

Point D is in region Ⓑ

$\therefore$ Actual voltage at D, $V_D$ = pu voltage at D × base voltage at D

$\therefore\ V_D = V_{Dpu} \times V_B$

$= 1.011187 \times 400$

$V_D = 404.47$ V

$\therefore$ Voltage at point D = 404.47 V

**8(a)(i)**

**Sol:** The state equation representation is

$$\dot{X} = AX$$

If $X(0) = \begin{bmatrix} 1 \\ -3/2 \end{bmatrix}$, the response $X(t) = \begin{bmatrix} e^{-2t} \\ \frac{-3}{2} e^{-2t} \end{bmatrix}$

$$\therefore \dot{X} = A.X$$

$$\dot{X}(0) = A.X(0)$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ assume

$$\dot{X}(t) = \frac{d}{dt}\left(X(t)\right) = \begin{bmatrix} -2e^{-2t} \\ 3e^{-2t} \end{bmatrix}$$

$$\dot{X}(0) = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ -3/2 \end{bmatrix}$$

$$a - \frac{3}{2}b = -2 \qquad \text{............ (1)}$$

$$c - \frac{3}{2}d = 3 \qquad \text{............ (2)}$$

And $X(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, the response is

$$X(t) = \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$$

$$\dot{X}(t) = \begin{bmatrix} -e^{-t} \\ e^{-t} \end{bmatrix}$$

$$\dot{X}(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$a - b = -1 \qquad \ldots\ldots\ldots\ldots (3)$$

$$c - d = 1 \qquad \ldots\ldots\ldots\ldots (4)$$

From equations (1) and (3),

$$\frac{-1}{2}b = -1, \; b = 2$$

$$a - \frac{3}{2}(2) = -2, \; a = 1$$

From equations (2) and (4),

$$\frac{-1}{2}d = 2, \; d = -4$$

$$c - \frac{3}{2}(-4) = 3, \; c = -3$$

$$\therefore \text{ System matrix } = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ -3 & -4 \end{bmatrix}.$$

**8(a)(ii)**

**Sol:** $$\text{CLTF} = \frac{\dfrac{2K}{(1+0.02s)(s)(s+2)}}{1 + \dfrac{2K}{(1+0.02s)(s)(s+2)}}$$

$$= \frac{2K}{s(1+0.02s)(s+2) + 2K}$$

Characteristic equation

$s(1+0.02s)(s+2)+2K = 0$

$s(50+s)(s+2)+100K = 0$

$\Rightarrow s\,(s^2 + 52s + 100) + 100\,K = 0$

$\Rightarrow s^3 + 52s^2 + 100\,s + 100\,K = 0$

$$\begin{array}{c|cc} s^3 & 1 & 100 \\ s^2 & 52 & 100k \\ s^1 & \dfrac{5200-100k}{52} & 0 \\ s^o & 100k & 0 \end{array}$$

For system to be stable,

$100\,K > 0 \Rightarrow K > 0$

$5200 - 100\,K > 0$

$\Rightarrow 52 > K$

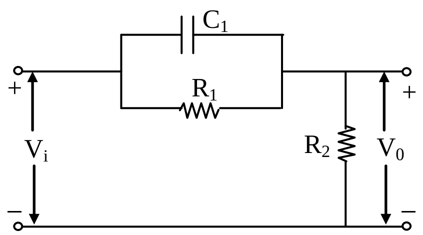
System is stable for $0 < K < 52$

$\therefore$ Maximum value of K = 52

**8(b) (i)**

**Sol:** **Lead Compensator:** The output of the lead compensator always leads with respect to the input. In other words, lead compensator always produces the positive phase.

The lead compensator can be realized by the following RC network.




**Effects of lead Compensator:**

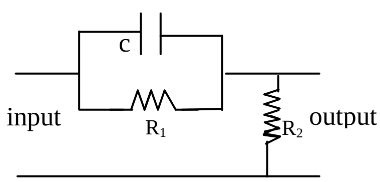
1. The lead compensator adds a zero to the right of the pole, which causes increased damping.
2. The increase in damping means less overshoot, less rise time and less delay time. Due to this, the transient performance is increased
3. It improves the gain margin and phase margin of system hence, the relative stability is improved.

4. It increases the bandwidth of the system more and gives quick response. Steady state error is not affected.

**Limitations:**

1. This lead compensator requires additional increase in gain (a) to offset (nullify) the attenuator.
2. To obtain large gain, more number of amplifiers are to be used, which increases the space, weight and cost.
3. Sometimes, more bandwidth may not be required, which makes the system more noisy.
4. From a single lead compensator, the maximum phase lead obtained is nearly $40^0$ to $60^0$. If required phase is more than $60^0$, multiple stages are used.

**Lead controller:**

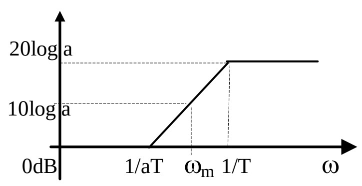



$$TF = \left(\frac{1+aTs}{1+Ts}\right)$$

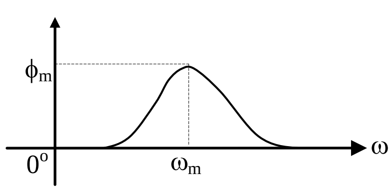
Where $T = \frac{R_1R_2}{R_1+R_2}C$

$$a = \frac{R_1+R_2}{R_2}$$

Magnitude plot




Phase plot



**8(b)(ii)**

**Sol:** O.L.T.F $G(s)H(s) = \dfrac{K}{(s+2)^2(s+3)}$

$$G.M = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$$

For $\omega_{pc}$, $\angle G(j\omega)H(j\omega) = -180°$

$$-2\tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right) = -180°$$

$$\tan^{-1}\left(\frac{\omega}{1-\frac{\omega^2}{4}}\right) + \tan^{-1}\left(\frac{\omega}{3}\right) = 180°$$

$$\left(\frac{\omega}{1-\frac{\omega^2}{4}}\right) + \left(\frac{\omega}{3}\right) = 0$$

$$\frac{\omega^2}{4} - 1 = 3$$

$$\omega^2 = 16$$

Phase crossover frequency, $\omega_{pc} = 4$ rad/sec

Given G.M $\geq 3$

$$\frac{(\omega^2+4)\sqrt{\omega^2+9}}{K} \geq 3$$

$$\frac{(16+4)(5)}{3} \geq K$$

$$K \leq \frac{100}{3}$$

**8(c)**

**Sol:** The periodic waveform shown in Figure, with a period $T = 2\pi$ can be expressed as:

$$x(t) = \begin{Bmatrix} A & ; 0 \leq t \leq \pi \\ -A & ; \pi \leq t \leq 2\pi \end{Bmatrix}$$

Let, $t_0 = 0$

$\therefore\ t_0 + T = 2\pi$ and

Fundamental frequency $\omega_0 = \dfrac{2\pi}{T} = \dfrac{2\pi}{2\pi} = 1$

Exponential Fourier series

$$C_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{2\pi}\left(\int_0^{\pi} Adt + \int_{\pi}^{2\pi} -Adt\right) = \frac{A}{2\pi}\left[(t)_0^{\pi} - (t)_{\pi}^{2\pi}\right] = 0$$

$$C_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt$$

$$= \frac{1}{2\pi}\left(\int_0^{\pi} Ae^{-jnt}dt + \int_{\pi}^{2\pi} -Ae^{-jnt}dt\right) = \frac{A}{2\pi}\left[\left(\frac{e^{-jnt}}{-jn}\right)_0^{\pi} - \left(\frac{e^{-jnt}}{-jn}\right)_{\pi}^{2\pi}\right]$$

$$= -\frac{A}{j2n\pi}\left[\left(e^{-jn\pi} - e^{0}\right) - \left(e^{-j2n\pi} - e^{-jn\pi}\right)\right]$$

$$= -\frac{A}{j2n\pi}\left[\left[(-1)^n - 1\right] - \left[1 - (-1)^n\right]\right] = -\frac{A}{2jn\pi}\left[-2 + 2(-1)^n\right]$$

$$= \frac{A}{jn\pi}\left[1 - (-1)^n\right]$$

$$\therefore\ C_n = \begin{cases} -j\dfrac{2A}{n\pi} & ;\text{for odd } n \\ 0 & ;\text{for even } n \end{cases}$$

$$\therefore\ x(t) = C_0 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} C_n e^{jn\omega_0 t} = \sum_{n=-\infty}^{\infty} -j\frac{2A}{n\pi}e^{jnt} \qquad \text{for odd } n$$

$$\therefore\ C_0 = 0, C_1 = C_{-1} = \frac{2A}{\pi}, C_3 = C_{-3} = \frac{2A}{3\pi}, C_5 = C_{-5} = \frac{2A}{5\pi}$$

The frequency spectrum is shown in Figure