ACE
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ESE – 2019 MAINS
OFFLINE TEST SERIES

ELECTRICAL ENGINEERING

TEST – 9
SOLUTIONS

All Queries related to **ESE – 2019 MAINS Test Series** Solutions are to be sent to the following email address
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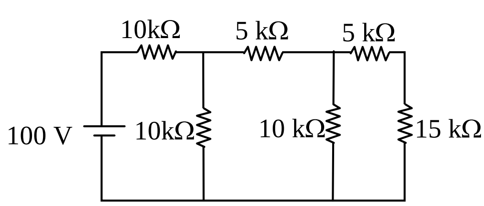
01(a)

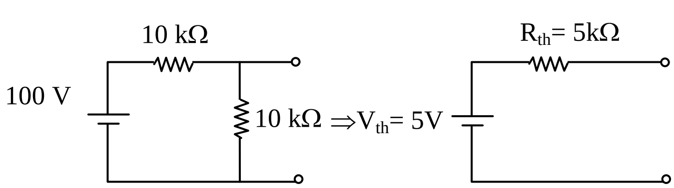
Sol:
```
int search (int * A, int size, int element)
{
    int i;
    for (i= 0; i < size; i+ +)
    if (A[i] = = element)
    return i; //position of element
    return –1; // error
}
void main (void)
{
        int A[30], element, index;
        index = search (A, 30, element);
        if(index = = –1)
        printf("element not found");
        else
        printf("element found");
}
```

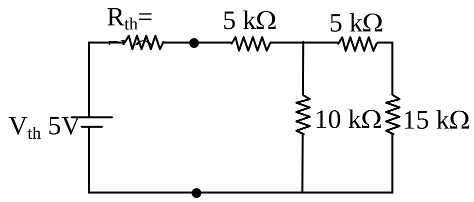
01(b)

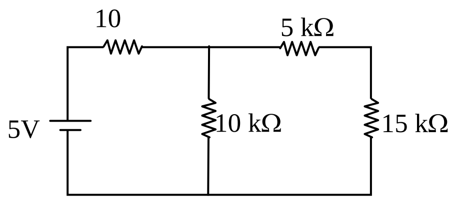
Sol:

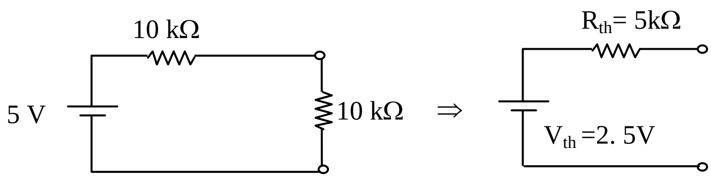


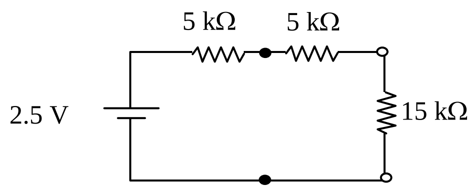
Using circuit minimizing techniques

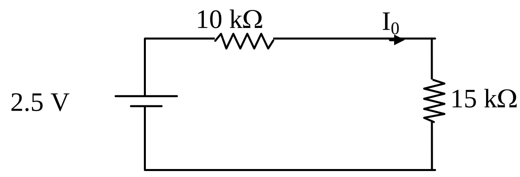






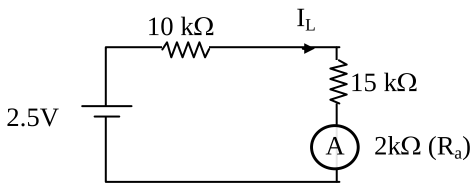






Value of current in 15kΩ (I_0) = $\dfrac{2.5V}{10k\Omega + 15k\Omega}$

I_0 = 100 μA



(A) reading (I_L)

$$= \frac{2.5\text{V}}{10\text{k}\Omega + 15\text{k}\Omega + 2\text{k}} = 92.6\mu\text{A}$$

I_L = 99% I_0 (from the question data)

$I_L = 0.99\ I_0,\ \ I_L = \dfrac{1}{1+\dfrac{R_a}{R_{ckt}}} I_0$

$$\frac{I_L}{I_0} = \frac{1}{1+\dfrac{R_a}{(10\text{k}+15\text{k}\Omega)}}$$

$$\Rightarrow 0.99 = \frac{1}{1+\dfrac{R_a}{25\text{k}\Omega}}$$

$R_a = 250\ \Omega$

01(c)

Sol: $\mathbf{A} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$

It's characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 1-\lambda & 2 \\ 2 & 2 & 1-\lambda \end{vmatrix} = 0$$

$R_1 + (R_2 + R_3)$:

$$\begin{vmatrix} 5-\lambda & 5-\lambda & 5-\lambda \\ 2 & 1-\lambda & 2 \\ 2 & 2 & 1-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1-\lambda & 2 \\ 2 & 2 & 1-\lambda \end{vmatrix} = 0$$

$(C_2 - C_1)$; $(C_3 - C_1)$:

$$(5-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 2 & -\lambda-1 & 0 \\ 2 & 0 & -\lambda-1 \end{vmatrix} = 0$$

$(5-\lambda)(-\lambda-1)(-\lambda-1) = 0$

$(5-\lambda)(1+\lambda)^2 = 0$

$(5-\lambda)(1+2\lambda+\lambda^2) = 0$

$(5+9\lambda+3\lambda^2-\lambda^3) = 0$

i.e $(\lambda^3-3\lambda^2-9\lambda-5) = 0$

From cayley – Hamilton theorem (1) is satisfied by 'A'

i.e $(A^3 - 3A^2 - 9A - 5I) = 0$ ------- (2)

$$A^2 - 4A - 5I\,\big)\; A^3 - 3A^2 - 9A - 5A \;\big(A + I$$
$$\underset{-}{A^3} \underset{+}{-4A^2} \underset{+}{-5A}$$
$$\overline{\quad A^2 - 4A - 5I \quad}$$
$$A^2 - 4A - 5I$$
$$\overline{\qquad\qquad}$$
$$0$$

$\therefore (A^3 - 3A^2 - 9A - 5I) = (A^2 - 4A - 5I)(A+I)$

i.e $(A^2 - 4A - 5I)(A+I) = 0$ (from (2))

But $(A+I) \neq 0 \Rightarrow (A^2 - 4A - 5I) = 0$

01(d)

Sol: $R = 120\Omega$; $G.F = \dfrac{\Delta R/R}{\varepsilon}$

$G.F = -12$, $\varepsilon = 0.01$ $\quad G.F \times \varepsilon = \dfrac{\Delta R}{R}$

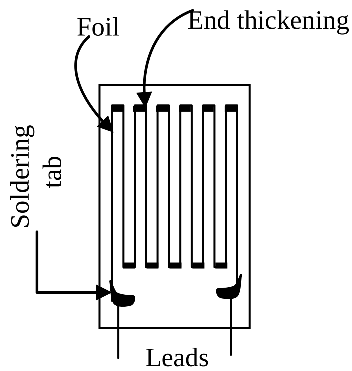
$12 \times 0.01 \times 120 = \Delta R$;

$R - \Delta R = 120\Omega - 14.4\ \Omega$

$= 105.6\ \Omega$

Metal foil gauge: The gauge is produced by printed circuit technique and consists of a foil on plastic backing. The desired grid pattern is first printed on a thin sheet of metal-alloy foil with an acid resistant ink and then the unprinted portion is etched away. This construction allows the use of varying sections throughout the grid length; larger area can be provided at the ends where lead connections are made.

The gauge has been successfully employed to fillets and sharply curved shapes because of its fine and accurate construction.



Etched foil gauge

The significant advantages of the gauge over wire type are:

- Improved hysteresis,
- Easy soldering or welding of the leads,
- Better fatigue life,
- Very good lateral strain sensitivity,

Improved transmission of strain from the test surface to the strain sensitive grid, and Stability at high temperature.

01(c)

Sol: Let f (x, y, z) = $(x^2 + y^2 + z^2) + \lambda\,(x + y + z - 3a)$

$\frac{df}{dx} = 0 \Rightarrow (2x+\lambda) = 0$ ------(1)

$\frac{df}{dy} = 0 \Rightarrow (2y+\lambda) = 0$ ------(2)

$\frac{df}{dz} = 0 \Rightarrow (2z + \lambda) = 0$ ------(3)

From (1) ,(2) & (3)

2x = 2y = 2z

$\therefore$ x = y =z

We know that (x+y+z) = 3a

$\Rightarrow$ 3x = 3a

$$\left.\begin{array}{l} \therefore x = a \\ \Rightarrow y = a \\ \& \; z = a \end{array}\right\} \ldots\ldots (4)$$

$\therefore$ The required minimum value of $(x^2+y^2+z^2) = (a^2+a^2+a^2) = 3a^2$

2(a)

Sol: (i) D'Arsonval type Galvanometers, which are dc galvanometers, are made in three general types:

(1) Portable point type

(2) Laboratory rectifying type

(3) Box type

(ii) Given,

Height, l = 25 mm = 0.025 m

Width, b = 20 mm = 0.02 m

$B = 0.11$ Wb/m^2

Moment of inertia, $J = 0.28\times10^{-6}$ kg-m^2

Control spring constant,

$K = 3\times10^{-6}$ N-m/rad

Deflection, $\theta = 135°$

$$= 135 \times \frac{\pi}{180} = 2.356 \text{ rad}$$

Current, I = 0.11 mA

= 0.011 A

P. Number of turns (N):

For steady deflection, $GI = K\theta$

Displacement constant, $G = \dfrac{K\theta}{I} = \dfrac{3\times10^{-6}\times 2.356}{0.011} = 0.642\times10^{-3}$ N-m/A

$G = NB\ell b$

$N = \dfrac{G}{B\ell b} = \dfrac{0.642\times10^{-3}}{0.11\times0.025\times0.02}$

$= 11.67$

$N = 12$

Q. Resistance of coil for critical damping, R_c:

$$R_c = \frac{G^2}{2\sqrt{KJ}} = \frac{\left(0.642\times10^{-3}\right)^2}{2\sqrt{3\times10^{-6}\times0.28\times10^{-6}}} = 0.224\ \Omega$$

2(b)(i)

Sol: I) Initialization phase:

A (4 bit) (Multiplier) = 4 = 0100 (n bits)

B (5 bit) (Multiplicand) = –3 = 11101 (n+1) bits

R (8 bit) (Result) = $\underbrace{0\ 0\ 0\ 0\ 0}_{\text{RH (n+1) bits}}\ \underbrace{0\ 0\ 0}_{\text{RL (n–1) bits}}$ (2n bits)

AC (2 bits) = 00 SC (Sequence Count) = n = 4

A: AC = 0100:00

II) Calculation phase:

1. Perform right shift (one bit) on A: AC and arithmetic right shift (one bit) on R.

   A: AC = X 010: 00 R=00000 000

2. If (AC = = 01) ADD RH, B

   else if (AC = = 10) SUB RH, B

   else No operation; //AC = 00 or AC = 11

- AC is 00 so no operation

3. SC – – ;

SC = 3

If (SC > 0) go to step 1

1. A: AC = XX01 : 00 R= 00000 000
2. AC is 00 so no operation
3. SC = 2

1. A: AC = XXX 0 : 10   R= 00000 000
2. AC is 10 so SUB RH, B

$$\begin{array}{lll} & RH \rightarrow & 00000 \\ + & (-B) \rightarrow & \underline{00011} \\ & R_H \leftarrow & 00011 \end{array}$$

R = 000 11 000

3. SC =1

1. A: AC = XXXX : 01   R= 00001 100
2. AC is 01 so ADD R_H, B

$$\begin{array}{lll} & R_H \rightarrow & 00001 \\ + & B \rightarrow & \underline{11101} \\ & R_H \leftarrow & 11110 \end{array}$$

R = 11110 100

3. SC = 0

Result (R) = 11110100 = –12

2(b)(ii)

Sol: P = 0.02, t_m = 10 ns, S = 1000 ns

EAT = $t_m \times (1-P) + P \times S$

$= 10 \text{ ns} \times 0.98 + 0.02 \times 1000 \text{ ns}$

$= 9.8 \text{ ns} + 20 \text{ ns}$

$= 29.8 \text{ ns}$

2(c)

Sol: Given $f(x) = x^2$, $-\pi < x < \pi$

It is an even function

$\therefore$ The Fourier series is given by $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\alpha} a_n \cos nx$ ------- (1)

where $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x) dx$

$$= \frac{2}{\pi}\int_0^{\pi} x^2 dx$$

$$= \frac{2}{\pi}\left(\frac{x^3}{3}\right)_0^{\pi} = \frac{2\pi^2}{3} \text{ --------- (2)}$$

and $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\, dx$

$$= \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx dx$$

$$= \frac{2}{\pi}\left[x^2\left(\frac{\sin mx}{n}\right) - 2x\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{2\pi\cos n\pi}{n^2}\right] = \frac{4}{n^2}(-1)^n \text{ -------- (3)}$$

$\therefore$ from (2) & (3) in (1)

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\alpha}\frac{(-1)^n}{n^2} \cos nx$$

$$= \frac{\pi^2}{3} + 4\left(\frac{-\cos x}{1^2} + \frac{\cos 2x}{2^2} - \frac{\cos 3x}{3^2} + ----\right)$$

$$x^2 = \frac{\pi^2}{3} - 4\left(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} ----\right) \text{-------- } (4)$$

Put x = 0 (4)

$$0 = \frac{\pi^2}{3} - 4\left(\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ---\right)$$

$$\Rightarrow \left(\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ----\right) = \frac{\pi^2}{12} \quad \text{--------- } (5)$$

Put x = π in (4)

$$\pi^2 = \frac{\pi^2}{3} - 4\left(\frac{-1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} ----\right)$$

$$\frac{2\pi^2}{3} = 4\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2}\right) + ---$$

$$\therefore \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ---\right) = \frac{\pi^2}{6} \text{ ------- } (6)$$

$\therefore$ (5) + (6)

$$2\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + --\right) = \frac{\pi^2}{12} + \frac{\pi^2}{6}$$

$$= \frac{3\pi^2}{12}$$

$$= \frac{\pi^2}{4}$$

$$\Rightarrow \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ------\right) = \frac{\pi^2}{8}$$

03(a)

Sol: $u = e^{-2xy} \sin(x^2 - y^2)$

$$\frac{\partial u}{\partial x} = 2x e^{-2xy} \cos(x^2 - y^2) - 2ye^{-2xy} \sin(x^2 - y^2) \quad \ldots\ldots\ldots\ldots\ldots.. (1)$$

$$\frac{\partial u}{\partial y} = -2ye^{-2xy}\cos(x^2 - y^2) - 2xe^{-2xy}\sin(x^2 - y^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{\partial^2 u}{\partial x^2} = 2e^{-2xy}\cos(x^2 - y^2) - 4xy.e^{-2xy}\cos(x^2 - y^2) - 4x^2e^{-2xy}\sin(x^2-y^2) - 2y(2xe^{-2xy}\cos(x^2 - y^2)$$

$$+2ye^{-2xy}\sin(x^2-y^2)$$

$$= 2e^{-2xy}\cos(x^2-y^2)(1-4xy) - 4e^{-2xy}(x^2 - y^2)\sin(x^2 - y^2) \quad \ldots\ldots\ldots.\ldots (3)$$

$$\frac{\partial^2 u}{\partial y^2} = -2e^{-2xy}\cos(x^2 - y^2) + 4xye^{-2xy}\cos(x^2 - y^2) - 4y^2e^{-2xy}\sin(x^2-y^2) - 2x[-2xe^{-2xy}\cos(x^2-y^2)$$

$$-2xe^{-2xy}\sin(x^2 - y^2)]$$

$$= 2e^{-2xy}\cos(x^2 - y^2)(4xy-1) - 4e^{-2xy}(y^2 - x^2)\sin(x^2 - y^2) \quad \ldots\ldots\ldots\ldots (4)$$

$$(3) + (4) \Rightarrow \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0$$

i.e., $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic.

Let $v(x, y) = 0$

$$\Rightarrow dv = \left(\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy\right) = 0$$

$\therefore\ dv = \left(-\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy\right) = 0$

From (1) & (2)

$dv = (2ye^{-2xy}\cos(x^2 - y^2) + 2xe^{-2xy}\sin(x^2 - y^2))dx + ((2xe^{-2xy}\cos(x^2 - y^2) - 2ye^{-2xy}\sin(x^2 - y^2))\,dy = 0$

$= e^{-2xy}\cos(x^2 - y^2)(2ydx + 2xdy) + e^{-2xy}\sin(x^2 - y^2)(2xdx - 2ydy) = 0$

$= 2e^{-2xy}\cos(x^2 - y^2)\,d(xy) + e^{-2xy}\sin(x^2 - y^2)\,d(x^2 - y^2) = 0$

$= -(d(e^{-2xy}\cos(x^2 - y^2)) = 0$

$V = -e^{-2xy}\cos(x^2 - y^2) + C$

We know that

$f'(z) = \left(\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right)$

$= \left(\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}\right)$

$= 2z\cos z^2 - i(-2z\sin z^2)$

$\Rightarrow f(z) = \int 2z\cos z^2 dz + i\int 2z\sin z^2 dz$

$= (\sin z^2 - i\cos z^2) + ic$

$= -ie^{iz^2} + ic$

03(b)

Sol: (i) $P_1 = 5000W$, $P_2 = -1000W$

Total power $(P_T) = P_1 + P_2 \Rightarrow$

$5000 - 1000 = 4000W$

Power Factor Angle (ϕ)

$$= \tan^{-1}\left[\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}\right]$$

$$\Rightarrow \tan^{-1}\left[\sqrt{3}\left[\frac{5000 - (-1000)}{5000 - 1000}\right]\right]$$

$$\Rightarrow \tan^{-1}\left[\sqrt{3}\left(\frac{6000}{4000}\right)\right] \Rightarrow \tan^{-1}\left[\frac{3\sqrt{3}}{2}\right]\phi$$

$= 68.94°$

$\therefore$ Power Factor $\cos\phi = \cos(68.94°)$

$\Rightarrow 0.3593$

≈ 0.36 Lag

(ii) Ans: Power consumed By each phase

$$= \frac{P_{Total}}{3} = \frac{4000}{3} = 1333.33\text{W}$$

In Δ connected system, voltage of each phase

$V_{ph} = V_L = 440\text{V}$

= supply voltage

$\rightarrow$ current in each phase

$$= \frac{1333.33}{440 \times 0.36} = 8.41748 \text{ Amp}$$

$\rightarrow$ Impedance of each phase

$$= \frac{440}{8.41748} = 52.27217\ \Omega$$

$\rightarrow$ Resistance of each phase

$$= \frac{1333.33}{(8.41748)^2} = 18.818\ \Omega$$

$\rightarrow$ Reactance (X) of each phase

$$= \sqrt{(52.27217)^2 - (18.818)^2} = 48.7674\Omega$$

$\rightarrow$ In order that one of the wattmeter's should read zero, the power factor should be 0.5

$\therefore \cos\phi = 0.5$, & $\tan\phi = 1.73$

$\therefore$ Reactance of circuit $\Rightarrow X = R\tan\phi$

$X = 18.818 \times 1.73 \Rightarrow X = 32.55514\Omega$

$\therefore$ Capacitive Reactance Required

$= 48.7674 - 32.55514$

$= 16.21226$

$$\text{Capacitance (C)} = \frac{1}{2\pi \times 50 \times 16.21226}$$

$C = 196.33\ \mu F$

03(c)(i)

Sol: Voltage across instrument for full scale deflection = 100mV.

Current in instrument for full scale deflection, $I = \frac{V}{R} = \frac{100\times10^{-3}}{20} = 5\times10^{-3}\ A$

Deflecting torque, $T_d = BINA = BIN(\ell \times d)$

$= 100 \times B \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 5 \times 10^{-3}$

$= B\times 375 \times 10^{-6}$

$\therefore$ Controlling torque for a deflection $\theta = 120°$

$T_c = K\theta = 0.375 \times 10^{-6} \times 120$

$= 45 \times 10^{-6}$ N-m

At final steady position, $T_d = T_c$

or $375 \times 10^{-6} \times B = 45\times 10^{-6}$

$\therefore$ Flux density in the air gap,

$B = \frac{45\times10^{-6}}{375\times10^{-6}} = 0.12\ Wb/m^2$

Resistance of coil winding, $R_c = 0.3 \times 20 = 6\ \Omega$

Length of mean turn

$l = 2\ (L+d) = 2\ (30+25) = 110$ mm

Let a be the area of cross-section of wire and P be the resistivity

Resistance of coil, $R_c = N\rho l/a$

$\therefore$ Area of cross-section of wire, $a = \frac{100\times1.7\times10^{-8}\times110\times10^{-3}}{6}\times10^{6}$

$= 31.37 \times 10^{-3}\ mm^2$

Diameter of wire, $d = \left[(4/\pi)(31.37\times10^{-3})\right]^{\frac{1}{2}} = 0.2$ mm

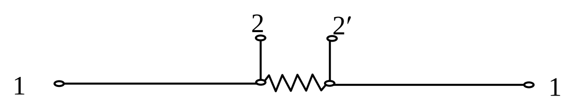
03(c)(ii)

Sol: Difficulties with measurement of low resistances: The methods used for measurement of medium resistances are unsuitable for measurement of low resistance. The reason is that resistance of leads and contacts, though small, are appreciable in comparison in the case of low resistance.

Ex: A Contact resistance of 0.002 Ω causes a negligible error where a resistance of 100Ω is being measured but the same contact resistance would cause an error of 10% if a low resistance of the value of 0.02 Ω is measured.

Hence special type of construction and techniques has to be used for the measurement low resistances in order to avoid serious errors occurring on account of the factors mentioned above.

Low resistance is constructed with four terminals as shown in below figure.



1 1′ - current terminals

2 2′ - voltage terminals

04(a)(i)

Sol: Here $\phi = y - \sin x$ and $\psi = \cos x$.

By Green's theorem $\int_C [(y \sin x)dx + \cos x dy])$

$$= \iint_R \left(\frac{\partial\psi}{\partial x} - \frac{\partial\phi}{\partial y}\right) dx\, dy$$

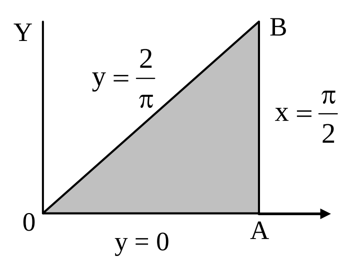


Fig.8.13

$$\int_{x=0}^{x=x/2}\int_{y=0}^{y=2x/x}(-\sin x - 1)dydx = -\int_0^{\pi/2}(\sin x + 1)/\Big|_0^{2x/\pi} dx$$

$$-\frac{2}{\pi}\int_0^{\pi/2} x(\sin x + 1)dx = -\frac{2}{\pi}\left\{x(-\cos x + x)\Big|_0^{\pi/2} - \int_0^{\pi/2} 1.(\cos x + x)dx\right\}$$

$$= -\frac{2}{\pi}\left\{\frac{\pi^2}{4} - \left|-\sin x + \frac{x^2}{2}\right|_0^{\pi/2}\right\} = \frac{\pi}{2} + \frac{2}{\pi}\left(-1 + \frac{\pi^2}{8}\right) = -\left(\frac{\pi}{4} + \frac{2}{\pi}\right)$$

04(a)(ii)

Sol: P A. Here $f(z) = z^2 - z + 1$ and $\alpha = 1$

Since f(z) is analytic within and on circle C:z| = 1 and $\alpha = 1$ lies on C

$\therefore$ by Cauchy's integral formula $\frac{1}{2\pi i}\int_C = \frac{f(z)}{z-\alpha} = f(\alpha) = 1 \text{ i.e.,} \int_C = \frac{z^2-z+1}{z-1}dz = 2\pi i$

B. In this case, $\alpha = 1$ lies outside the circle

C: |z| =1/2. so $(z^2 - z+1)/(z-1)$ is analytic everywhere within C.

$\therefore$ by Cauchy's theorem $\int_C \frac{z^2-z+1}{z-1}dz = 0$

Q. f(z) has simple poles at $z = 0, \pm \pi/2, \pm 3\pi/2,$— ….

Only the poles z = 0 and $z = \pm \pi/2$ lies inside

|z| =2

$$\therefore \text{Res } f(0) = \underset{x\to\pi/2}{\text{Lt}}\left[\left(z-\frac{\pi}{2}\right)f(z)\right] - \underset{x\to\pi/2}{\text{Lt}}\left\{\frac{(z-\pi/2\sin z)}{z\cos z}\right\}$$

$$= \underset{z\to\pi/2}{\text{Lt}}\frac{(z-\pi/2)\cos z + \sin z}{\cos z - z\sin z}$$

$$= \frac{1}{-\pi/2} = -\frac{2}{\pi} \qquad \left[\text{Being } \frac{0}{0} \text{ form}\right]$$

And

$$\text{Res } f(-\pi/2) = \underset{z\to-\pi/2}{\text{Lt}}\left\{\frac{(z+\pi/2)\sin z}{z\cos z}\right\} = \underset{x\to-\pi/2}{\text{Lt}}\frac{(z+\pi/2)\cos z + \sin z}{\cos z - z\sin z} = \frac{-1}{=\pi/2} = \frac{2}{\pi}$$

Hence sum of residues $0 - \frac{2}{\pi} + \frac{2}{\pi} = 0$

04(b)

Sol: Factors leading to in accuracies in measurements by A.C. bridges:

1) Mutual inductance effect due to magnetic coupling between various components of the bridge.
2) Stray capacitance effects, due to electrostatic fields between conductors at different potentials.
3) Stray conductance effect due to imperfect insulation

4) Residual in-components, the existence of small amount of series inductance or shunt capacitance in nominally non-reactive resistors.

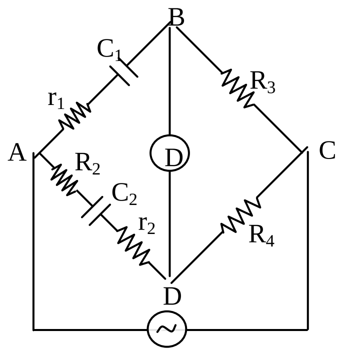
Problem:

Given Data:

r_1 =?, $r_2 = 0.4\ \Omega$, $R_3 = 2000\Omega$,

$R_4 = 2950\ \Omega$, $R_2 = 5\Omega$,

$C_2 = 0.5\ \mu F$, C_1 = ?



F = 450 Hz

At balance,

$$\left(r_1 + \frac{1}{j\omega C_1}\right)R_4 = \left(R_2 + r_2 + \frac{1}{j\omega C_2}\right)R_3$$

$$r_1R_4 + \frac{R_4}{j\omega C_1} = (R_2 + r_2)R_3 + \frac{R_3}{j\omega C_2}$$

By equating real and imaginary terms on both sides,

$$r_1R_4 = (R_2 + r_2)R_3 \quad \text{and} \quad \frac{R_4}{\omega C_1} = \frac{R_3}{\omega C_2}$$

$$\Rightarrow \frac{R_4}{R_3} = \frac{(R_2 + r_2)}{r_1} \quad \text{and} \quad \frac{R_4}{R_3} = \frac{C_1}{C_2}$$

$$\therefore\ C_1 = \frac{R_4}{R_3}C_2 = \frac{2950}{2000} \times 0.5 \times 10^{-6} = 0.74\mu F$$

$$r_1 = \frac{R_3}{R_4}(R_2 + r_2) = \frac{2000}{2950}(5 + 0.4) = 3.66\Omega$$

Dissipation factor of capacitor C_1:

$D = \tan\delta = \omega C_1 r_1$

$= 2\pi\times 450\times0.74\times10^{-6}\times3.66$

$= 0.00765$

04(c) (i)

Sol: MM size = 2^{30} Bytes

Physical Address width $\Rightarrow$ 30 bits

Cache size = 2^{21} Bytes

Cache line size = 2^{12} Bytes

Offset width $\Rightarrow$12 bits

$$\text{Number of cache lines} = \frac{\text{cache size}}{\text{cache line size}}$$

$= 2^9$ lines

Index width $\Rightarrow$ 9 bits

Tag entry width = physical address width – offset width – Index width

= (30–12–9) = 9 bits.

Size of Tag memory = (Tag entry width + extra bits) * Number of cache lines

= (9+2) bits * 2^9 = 11 * 2^9 bits = 11 * 2^6 Bytes

04(c)(ii)

Sol:

Category	Array	Liked List
1) Data structure	* Static Data structure * Index based data structure, each element associated with an index	* Dynamic Data structure * Reference based data structure, each node contains reference of next node
2) Memory Allocation	* Continuous Allocation	* Non-continuous allocation
3) Space overhead	* No extra space overhead	* Needed extra space to store pointers

4) Size	* Fixed in size, doesn't vary at runtime	* It may grow and shrink as per requirement at runtime
5) Insertion and Deletion of element	* Relatively slow due to shifting	* Easier and faster
6) Searching	* Binary and linear search	* Only linear search
7) Memory utilization	* Ineffective	* Efficient due to dynamic behavior

05(a)

Sol: Number of cache lines = N

Number of cache sets = S

Main Memory block Number = i

Cache set Number = j

Mapping function for K-way set associative cache

Cache set Number = (MM block No.) mod (No. of cache sets)

$$\boxed{j = (i) \bmod (s)}$$

The line numbers belongs to cache set number j are jk to (j+1) k–1

$\text{Set}_0 \rightarrow \text{line}_0, \text{line}_1,$ ------------ $\text{line}_{(k-1)}$

$\text{Set}_1 \rightarrow \text{line}_k,$ ------------------ $\text{line}_{(2k-1)}$

$\text{Set}_2 \rightarrow \text{line}_{2k}$ ------------------ $\text{line}_{(3k-1)}$

$\vdots$

$\text{Set}_j \rightarrow \text{line}_{jk}$ ----------------- $\text{line}_{(j+1)k-1}$

The line numbers $\Rightarrow$ jk to (j+1) k – 1

$\Rightarrow$ jk to jk + (k–1)

$$\boxed{\Rightarrow (i \bmod s)\, k \text{ to } (i \bmod s)\, k + (k-1).}$$

05(b)

Sol: Direct Memory Access (DMA):

- It allows certain hardware subsystem to access main memory, independent of CPU
- Whenever I/O request comes, CPU initializes the DMA for I/O
- DMA perform I/O instead of CPU
- DMA perform faster I/O than CPU

There are 3 methods in DMA known as

1) Burst DMA (Continuous DMA)

2) Cycle Stealing Method

3) Interleaved DMA

1) Burst DMA:

In Burst DMA until completion of the target file transfer, CPU does not get the control on buses i.e. valuable CPU time is wasted and this method is specially used for transferring large block of data only.

2) Cycle Stealing Method:

In cycle stealing mode, DMAC starts buses for one clock cycle time when these are free from CPU. This method is used for transferring smallest size of data only and in this method, there is no wastage time for CPU

3) Interleaved DMA:

In interleaved DMA, both DMAC and CPU share the bus control time, DMAC uses bus for first clock cycle time and CPU uses bus for next clock cycle time and so on.

05(c)

Sol: Let $f(z) = \dfrac{z\cos z}{\left(z - \dfrac{\pi}{2}\right)^3}$

$z = \dfrac{\pi}{2}$ lies inside 'c' and is a pole of order '3' of f(z).

$\therefore \left[\text{Residue of } f(z)\right]_{\text{at } z=\frac{\pi}{2}}$

$$= \frac{1}{2!} \underset{z \to \frac{\pi}{2}}{\mathrm{Lt}} \frac{d^2}{dz^2}\left[\left(z-\frac{\pi}{2}\right)^3 f(z)\right]$$

$$= \frac{1}{2} \underset{z \to \frac{\pi}{2}}{\mathrm{Lt}} \frac{d^2}{dz^2}[z\cos z]$$

$$= \frac{1}{2} \underset{z \to \frac{\pi}{2}}{\mathrm{Lt}} \frac{d}{dz}[z(-\sin z)+\cos z]$$

$$= \frac{1}{2} \underset{z \to \frac{\pi}{2}}{\mathrm{Lt}}[-\sin z-(z\cos z+\sin z)]$$

$$= \frac{1}{2}[-1-(0+1)]$$

$$= -1$$

From Cauchy's residue theorem

$$\oint_c \frac{z\cos z}{\left(z-\frac{\pi}{2}\right)^3} dz = 2\pi i(-1) = -2\pi i$$

05(d)

Sol: Differential output taken from an inductive transducer: Normally, the change in self inductance ΔL is adequate for detection for subsequent changes of instrumentation system. However, if the succeeding instrumentation responds to ΔL rather than to L+ ΔL the sensitivity and accuracy will be much higher. The transducer can be designed to provide 2 outputs, one of which is an increase of self inductance. The succeeding stages of instrumentation system measures the difference between there outputs i.e., 2 ΔL. This is known as differential output.

The advantages of differential output are:

1. The sensitivity and accuracy are increased.
2. The output is less affected by external magnetic field
3. The effective variations due to temperature changes are reduced.
4. The effects of changes in supply voltage and frequency are reduced.

The differential arrangement consists of a coil which is divided into two parts. In response to a physical signal, which is normally a displacement, the inductance of one part increases from L to

$L+\Delta L$ while that of other part decreases from L to L–ΔL. The change is measured as the difference between the two, resulting in an output as the difference between of the two resulting in an output of $2\Delta L$ instead of ΔL when only a single winding is used.

Example of Transducers working on Inductance:

1. LVDT 2. RVDT 3. Synchros 4. Resolver

05(e)

Sol: Power calculated by freshman

$P = I^2 R = (30.4)^2 \times 0.0105$

$= 9.70368$ W

True value of current (I_T) =

$30.4A + \left(\frac{1.2}{100} \times 30.4A\right) = 30.77$ A

True value of resistance (R_T) =

$0.0105\Omega + \left(\frac{0.3}{100} \times 0.0105\Omega\right) = 0.010532\Omega$

True power (P_T) = $(30.77)^2 \times 0.010532\Omega$

$= 9.971$W

$$\frac{\text{True value of power}}{\text{power calculated by freshman}} \times 100$$

$$= \frac{9.971\text{W}}{9.70368} \times 100$$

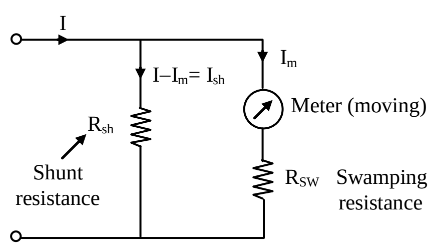
$= 102.75\%$

06(a)(i)

Sol: Given data:

$R_m = 5\ \Omega$, $R_{SW} = 4\ \Omega$

$I_m = 5$ mA $I = 1$ A



So, voltage across shunt coil

$= I_m R_m + I_m R_{sh}$

$= (0.005)\ [4+5]$

$= 45\times10^{-3}$V

The shunt should carry I_{sh}

$= I - I_m$

$= 1 - 0.005$

$= 0.995$A

$$\therefore \text{The shunt resistance} = \frac{45\times10^{-3}}{0.995}$$

$= 0.045\Omega$

Compensation of Temperature Error:

The temperature error can be eliminated when the shunt and the moving-coil are made of the same material and kept at the same temperature. This method, however, is not satisfactory in practice as the temperature of the two parts are not likely to change at the same rate. Disadvantage of using copper shunt is that they are likely to be bulky as the resistivity of copper is small. Copper shunts are only occasionally used in instruments with built – in shunts.

In this case, a "swamping resistance" of manganin (which has a negligible temperature co-efficient) having a resistance 20 to 30 times the coil resistance is connected in series with the coil and a shunt of manganin is connected across this combination.

06(a)ii)

Sol: P. Dynamic error: It is the difference between the true value of the quantity (under measurement) changing with time and the value indicated by the measurement system if no static error is assumed. It is also called Measurement Error.

Resolution: The smallest voltage that can be measured in lowest voltage range is known as resolution of an instrument.

$$\text{Scale resolution } = \frac{1}{10^N}$$

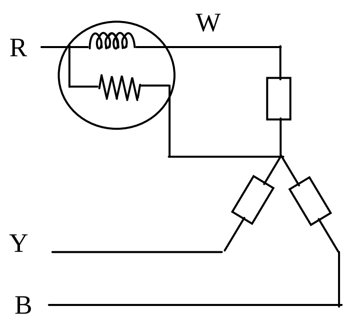
Where N = no. of full digits

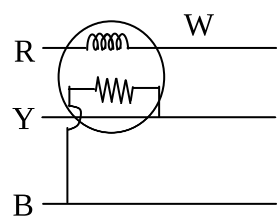
Resolution in a selected voltage range is given as $= \dfrac{\text{Selected voltage range}}{\text{Total counts}}$

(or)

$$= \frac{\text{Full scale reading in that range}}{\text{Maximum count}}$$

Q.





W = 400 watt.

$W = V_{ph}\, I_{ph} \cos\phi$

$V_{ph}\, I_{ph} = 400/0.8$

This type of connection gives reactive power, $W = \sqrt{3}\ V_p\, I_p \sin\phi$

$$= \sqrt{3} \times {}^{400}\!/_{0.8} \times 0.6$$

$$= 519.6 \text{ VAR}$$

06(b)

Sol: Cathode Ray Oscilloscope (CRO):

CRO is a common laboratory instrument that provides accurate time and amplitude measurements of voltage signals over a wide range of frequencies. Its reliability, stability and ease of operation make it suitable as a general purpose instrument. Below fig shows the basic block diagram of a general purpose CRO.

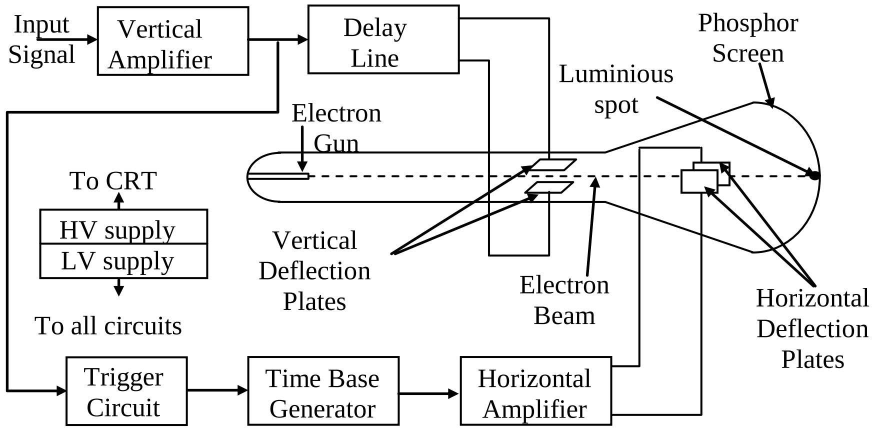


Fig: Block diagram of Cathode Ray Oscilloscope

A general purpose oscilloscope consists of the following parts

1. Cathode ray tube
2. Vertical amplifier
3. Delay line
4. Time base generator
5. Horizontal amplifier
6. Trigger circuit
7. Power supply.

Working of CRO:

1. The cathode ray is a beam of electrons which are emitted by the heated cathode (negative electrode) and accelerated towards the fluorescent screen. The assembly of the cathode, intensity grid, focus grid, and accelerating anode (positive electrode) is called an electrode gun. The function of electron gun is to control the intensity and focus.
2. There are two deflection plates between electron gun and fluorescent screen called horizontal deflection plate and vertical deflection plate which provide horizontal and vertical deflection of the beam and combination of these two allows the beam to reach any point of the screen. Whenever the electron beam hits the screen, the phosphor is excited and light is emitted from the point.

3. The linear deflection or sweep of the beam horizontally is accomplished by use of a sweep generator that is incorporated in the oscilloscope circuit. Usually the signal to be analyzed is first amplified and then applied to the vertical deflection plate to deflect the beam vertically and at the same time a voltage that increases linearly with time is applied to the horizontal deflection plate thus causing the beam to be deflected horizontally at a uniform rate.
4. The signal applied to the vertical plates is thus displayed on the screen as a function of time. The horizontal axis serves as a uniform time scale.

06(c)(i)

Sol: Gantt Chart

P_1	P_2	P_3	P_4	P_2	P_1

0 2 5 7 15 24 36 54

Ready Queue → $\cancel{P_1}\ \cancel{P_2}\ P_1\ \cancel{P_3}\ \cancel{P_2}\ \cancel{P_4}$

TAT = FT-AT	**WT = TAT – BT**
$P_1 \rightarrow 54 - 0 = 54$	$P_1 \rightarrow 54 - 20 = 34$
$P_2 \rightarrow 36 - 2 = 34$	$P_2 \rightarrow 34 - 15 = 19$
$P_3 \rightarrow 15 - 5 = 10$	$P_3 \rightarrow 10 - 10 = 0$
$P_4 \rightarrow 24 - 7 = 17$	$P_4 \rightarrow 17 - 9 = 8$

$$\text{Avg. W.T.} = \frac{34+19+0+8}{4} = \frac{61}{4} = 15.25$$

06(c)(ii)

Sol:

Category	**IP(v4)**	**IP(v6)**
1. IP Address	• 32 bits IP Address • Classfull or Classless IP address	• 128 bits IP address • Only Classless IP
2. Range Problem	• Overcome by using private network • NAT table is used	• No any concept of private network • No range problem
3. Routing	• More processing overhead at intermediate router	• Less processing overhead • Routing is flexible and fast relatively
4. Communication	• Unicast, Multicast and Broadcast	• Unicast, Multicast and any cast • Broadcasting is not allowed
5. Security	• No any IP security support	• IP security (Authentication) provided

06(c)(iii)

Sol: $t_c = 2$ ns

$h_c = 0.8$

$t_m = 10$ ns

$T_{avg} = h_c * t_c + (1 - h_c) * t_m$

$= 0.8 * 2 \text{ ns} + 0.2 * 10 \text{ ns}$

$= 3.6$ ns

07(a)(i)

Sol: The given equations can be represented in the form AX = 0 as follows

$$\underset{A}{\begin{bmatrix}1 & 1 & -2 & 3\\ 1 & -2 & 1 & -1\\ 4 & 1 & -5 & 8\\ 5 & -7 & 2 & -1\end{bmatrix}}\underset{X}{\begin{bmatrix}x\\ y\\ z\\ w\end{bmatrix}} = \underset{O}{\begin{bmatrix}0\\ 0\\ 0\\ 0\end{bmatrix}}$$

$$A = \begin{bmatrix}1 & 1 & -2 & 3\\ 1 & -2 & 1 & -1\\ 4 & 1 & -5 & 8\\ 5 & -7 & 2 & -1\end{bmatrix}$$

$R_4 - (R_2 + R_3)$; $(R_3 - 4R_1)$; $(R_2 - R_1)$

$$\sim\begin{bmatrix}1 & 1 & -2 & 3\\ 0 & -3 & 3 & -4\\ 0 & -3 & 3 & -4\\ 0 & -6 & 6 & -9\end{bmatrix}$$

$(R_4 - 2R_3)$; $(R_3 - R_2)$

$$\sim\begin{bmatrix}1 & 1 & -2 & 3\\ 0 & -3 & 3 & -4\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -1\end{bmatrix}$$

$R_3 \leftrightarrow R_4$

$$\sim\begin{bmatrix}1 & 1 & -2 & 3\\ 0 & -3 & 3 & -4\\ 0 & 0 & 0 & -1\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$\rho(A) = 3 <$ number of variables '4'

$\therefore$ The given system has infinite number of solutions as shown below

$$w = 0$$

$$-3y + 3z - 4w = 0$$

$$\Rightarrow \quad 3y = 3z$$

$$y = z = k \text{ (say)}$$

$$x + y - 2z + 3w = 0$$

$$\therefore x = 2z - y = 2k - k = k$$

$$\therefore\ X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} k \\ k \\ k \\ 0 \end{bmatrix} \text{ is the general solution.}$$

07(a)(ii)

Sol: Let $\frac{dy}{dx} = (3x^2 + 1) = f(x, y)$

$y(0) = 2 \Rightarrow x_0 = 0 \;\&\; y_0 = 2$

$y(0.5) = y(0 + 2(0.25) = y(x_0 + 2h) = y_2 = ?$

We know that from Euler's method

$y_1 = y_0 + hf(x_0, y_0)$

$= 2 + (0.25)(3(0)^2 + 1)$

$= 2 + 0.25 = 2.25$

$y_2 = y_1 + hf(x_1, y_1)$

$= 2.25 + (0.25)\, f(0.25, 2.25)$

$= 2.25 + (0.25)(3(0.25)^2 + 1)$

$= 2.25 + (0.25)[1.1875]$

$= 2.5469$

07(b)(i)

Sol:
```
#include <stdio.h>
void main( )
{
    int n;
    printf("Enter a number : ");
    scanf("%d", & n);
    if (((n% 3) = = 0) && ((n % 7) = = 0))
    {
  printf("number %d is divisible by both 3 and 7", n);
     }
```

```
    else
    {
        printf("Number %d is not divisible by both 3 and 7", n);
    }
}
```

Output:

Test Case 1:

Enter a number : 35

Number 35 is not divisible by both 3 and 7

Test Case 2:

Enter a number : 63

Number 63 is divisible by both 3 and 7

07(b)(ii)

Sol:

Category	**Cache memory**	**Main Memory**
1. Type	* static-RAM (SRAM)	* Dynamic-RAM (DRAM)
2. Access time	* Relatively faster memory access	* Relatively a slower memory access
3. Size	* Relatively smaller in size	* Relatively larger in size
4. Cost	* Relatively more expensive	* Relatively less expensive
5. Organization	* Different types of cache organization on the basis of associativity	* Main memory is managed by memory management module of operating system
6. Hardware	* Formed by flip-flops	* Formed by capacitor

07(c)

Sol: The given equation is Euler cauchy's homogeneous linear equation

$\therefore$ Let $x = e^z \Rightarrow z = \log x$

& $D = \dfrac{d}{dz}$

Hence the given equation reduces to

$D(D-1)\,y + Dy + y = z \sin z$

$(D^2 + 1)\,y = z \sin z$

AE: $(m^2 + 1) = 0$

$m = \pm i$

CF: $y_c = C_1 \cos z + C_2 \sin z$

$= C_1 \cos(\log x) + C_2 \sin(\log x)$

PI: $y_p = \dfrac{z \sin z}{(D^2+1)}$

$$= z\frac{\sin z}{(D^2+1)} - \frac{2D}{(D^2+1)^2}(\sin z)$$

$$= z\left(\frac{-z}{2}\cos z\right) - \frac{2D}{(D^2+1)}\left(\frac{-z}{2}\cos z\right)$$

$$y_p = \frac{-z^2}{2}\cos z + \frac{(-z\sin z + \cos z)}{(D^2+1)}$$

$$= \frac{-z^2}{2}\cos z - \frac{z\sin z}{(D^2+1)} + \frac{\cos z}{(D^2+1)}$$

$$= \frac{-z^2}{2}\cos z - y_p + \frac{z}{2}\sin z$$

$$2y_p = \frac{-z^2}{2}\cos z + \frac{z}{2}\sin z$$

$$\therefore y_p = \frac{-z^2}{4}\cos z + \frac{z}{4}\sin z$$

$$= \frac{-1}{4}(\log x)^2\cos(\log x) + \frac{1}{4}\log x \sin(\log x)$$

$\therefore$ $y = (y_c + y_p)$

$$y = C_1 \cos(\log x) + C_2 \sin(\log x) - \frac{1}{4}(\log x)^2 \cos(\log x) + \frac{1}{4}\log x \sin(\log x)$$

8(a)

Sol: Network Topology:

- Arrangement of hosts in a communication network
- Network Topology are of two types

  i) Point to point topology

  ii) Point to Multipoint topology

i) Point to Point topology:

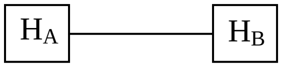
- Direct link, no any channel access protocol required
- Point to Point Protocol (PPP) is used for communication
- Types of point to point topology

a) Point to Point Link

b) Mesh topology

c) Star topology

a) Point to Point link:



Single communication link between two end communication point for serial communication

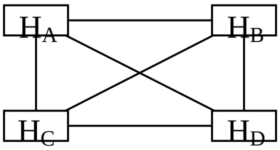
b) Mesh topology: Mesh topologies are of 2 types:

I. Fully mesh topology

II. Partial mesh topology

I. Fully mesh topology:

- Separate communication link between every pair of hosts
- If N number of hosts then $\frac{N \times (N-1)}{2}$ links are required.



Advantages:

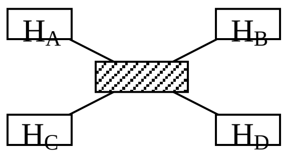
1. Fastest communication among all topologies
2. Secure communication
3. Parallel communication

Disadvantages:

1. Installation cost is very high due to too many links
2. Poor link utilization

c) Star topology:

- Multiple hosts are connected to centralized server
- Centralized server can be Hub, Switch or Router



Advantages:

1. Insertion and Removal of hosts are too easy
2. Star topology can be extended in daisy chain manner

Disadvantages:

1. If centralized server fail then topology will not work
2. More processing overhead at centralized server

ii) Point to multipoint topology:

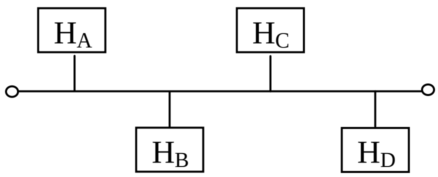
- Multiple hosts are connected to common broadcast media
- One sender and all are receiver
- Channel access mechanism required

Types of point to multipoint topology:

a) Bus topology

b) Ring topology

a) Bus topology:



- Multiple hosts are connected to common coaxial cable (backbone media)
- Different channel access protocols are ALOHA, CSMA and Token Bus

Advantages:

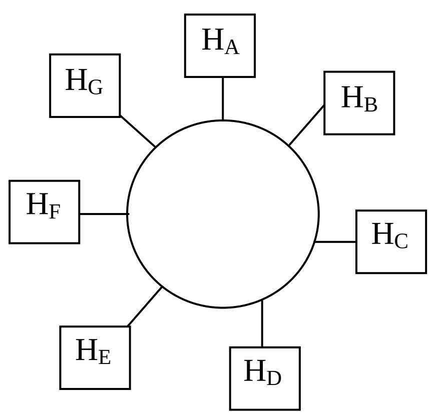
1. Installation cost is very less among all topologies
2. Preferred for large area networks

Disadvantages:

1. If common channel fail then topology will not work
2. Limited number of hosts per segment
3. Chance of collision

b) Ring Topology:

- Multiple hosts are connected to circular ring
- Channel access mechanism over token ring
- Data is moving only in one direction



Advantages:

1. Gives better performance over bus topology
2. No chance of collision

Disadvantages:

1. If ring fail then topology will not work
2. Need network monitor for network maintenance

8(b)

Sol: A multimeter is a multi purpose, multi – range instrument that can be used to measure voltage (DC/AC), current (DC/AC) and resistance.

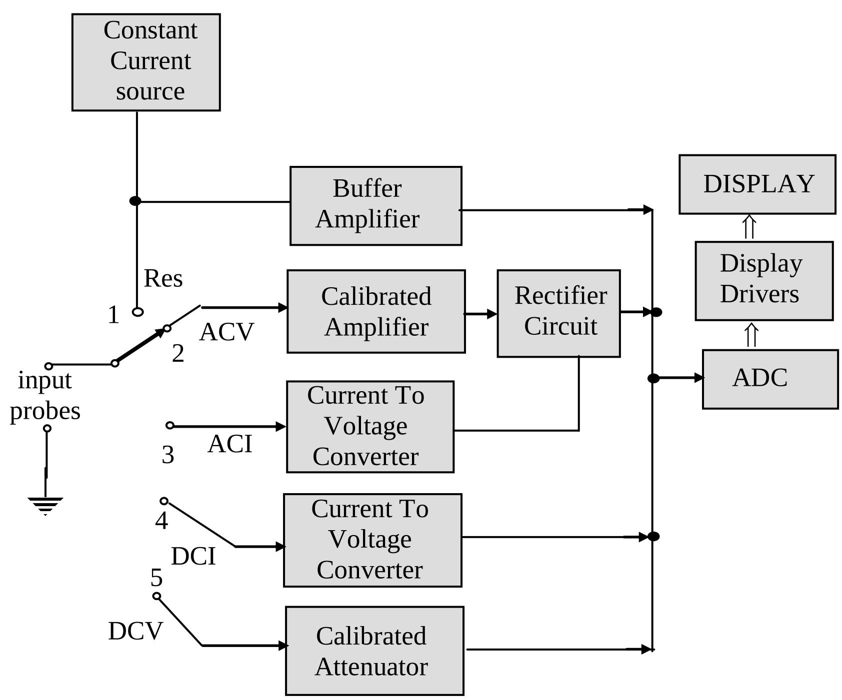
There are two types of multimeters

1. Analog multimeter (AMM)
2. Digital multimeter (DMM)

Analog multimeters are further categorised as ordinary electromechanical multimeter (VOM) and solid state electronic multimeter (EMM)

Ordinary electromechancial multimeter is a combination of voltmeter, ohmmeter & ammeter, whereas solid state electronic multimeter consists of electronic elements (like amplifier) at primary stage. EMM offers high & constant input impedance, high sensitivity, less loading effect, better accuracy than ordinary multimeter.

Digital multimeter schematic



DMM offers the following advantages compared to AMM

(a) High input impedance

(b) Less loading effect

(c) Better accuracy

(d) Superior resolution

(e) No observational errors

(f) No frictional errors

(g) Facilities like auto ranging & auto polarity

For DC voltage measurement, switch is thrown to position (5). The voltage is attenuated (if it is above the selected range) and then fed to A to D converter, whose output is displayed on readout in volts

For DC current measurement, switch is kept in position (4). The current is converted proportionally into voltage with the help of I –V converter. Now, this DC voltage is fed to A to D converter, whose output is displayed on readout in amperes. This is indirect measurement.

For AC current measurement, switch is kept in position (3). The current is converted proportionally into voltage with the help of I – V converter and then rectified. Now this DC voltage is fed to A to D converter, whose output is displayed on readout in amperes. This is indirect measurement.

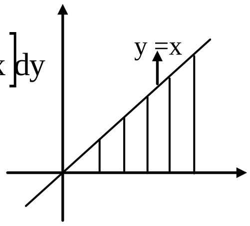
For AC voltage measurement, switch is kept in position (2). The voltage is attenuated (if it is above the selected range) and then rectified to convert it into proportional DC voltage. Now, this DC voltage is fed to A to D converter, whose output is displayed on readout in volts. This is indirect measurement.

For resistance measurement, switch is kept in position (1) and unknown resistor is placed across its input probes. Constant current source pumps a fixed current through the unknown resistor. The voltage developed across the unknown resistor is proportional to the resistance. This voltage is buffered by the buffer amplifier and than fed to A to D converter whose output is displayed on readout in ohms.

8(c)

Sol: (i). $\int_0^{\alpha}\int_0^{x} xe^{-x^2/y}\ dydx$

Becomes $\int_0^{\alpha}\left[xe^{-x^2/y}\ dx\right]dy$



$\int_0^{\infty}\left[\int_{y^2}^{\infty}\left(e^{-t/y}.\frac{dt}{2}\right)_{y^2}^{\infty}\right]dy$

(let x^2 = t $\Rightarrow$2xdx=dt & as x: y $\rightarrow\infty$

t: $y^2\rightarrow\infty$)

$$=\frac{1}{2}\int_0^{\infty}\left[\frac{e^{-1/y(t)}}{\left(\frac{-1}{y}\right)}\right]_{y^2}^{\infty}\ dy$$

$$=-\frac{1}{2}\int_0^{\infty}\left[y\,e^{-t/y}\right]_{y^2}^{\infty}\ dy$$

$$=-\frac{1}{2}\int_0^{\infty}\left[0-ye^{-y}\right]\ dy$$

$$=\frac{-1}{2}\left[e^{-y}\left(y+1\right)_0^{\infty}\right]=\frac{-1}{2}(0-1)=\frac{1}{2}$$

(ii) Let two curves intersect at points whose abscissa are given by $4x-x^2= x$

$x^2-3x=0$ i.e. x= 0,3.

Using vertical strips, the required area lies between

x=0, x=3 and y=x, $y=4x-x^2$.

$\therefore$ Required area $\int_0^3\int_x^{4x-x^2} dy\,dx$

$$=\int_0^3[y]_x^{4x-x^2}dx$$

$$=\int_0^3(4x-x^2-x)\,dx=\int_0^3(3x-x^2)\ dx$$

$$= \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3 = \frac{27}{2} - 9 = 4.5$$

(iii) Given surface is f(x,y,z)= xy–z^2……(1)

Let $\bar{n}_1$ and $\bar{n}_2$ be the normals to this surface at (4, 1, 2) and (3, 3, –3) respectively.

Differentiating partially, we get

$$\frac{\partial f}{\partial x} = y, \frac{\partial f}{\partial y} = x, \frac{\partial f}{\partial z} = -2z$$

$$\therefore \text{grad}\, f = y\bar{i} + x\bar{j} - 2z\,\bar{k}$$

$$\bar{n}_1 = (\text{grad}\, f)\, \text{at}(4,1,2) = \bar{i} + 4\bar{j} - 4\bar{k}$$

$$\bar{n}_2 = (\text{grad}\, f)\, \text{at}(3,3,-3) = 3\bar{i} + 3\bar{j} - 6\bar{k}$$

Let θ be the angle between the two normals.

$$\therefore \cos\theta = \frac{\bar{n}_1.\bar{n}_2}{|\bar{n}_1||\bar{n}_2|} = \frac{(i+4j-4k).(3i+3j+6k)}{\sqrt{1+16+16}\sqrt{9+9+36}} = \frac{3+12-24}{\sqrt{33}\sqrt{54}} = \frac{-9}{\sqrt{33}\sqrt{54}}$$

$$\theta = \cos^{-1}\left(\frac{-1}{\sqrt{22}}\right)$$