



**ACE**  
Engineering Academy  
(Leading Institute for ESE/GATE/PSUs)

# **ESE – 2019 MAINS OFFLINE TEST SERIES**



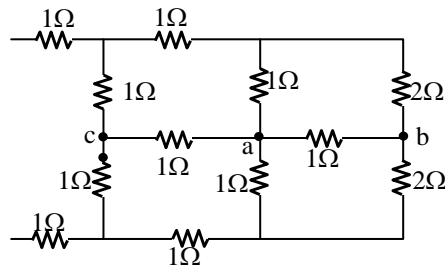
## **ELECTRICAL ENGINEERING TEST – 7 SOLUTIONS**

All Queries related to **ESE – 2019 MAINS Test Series** Solutions are to be sent to the following email address  
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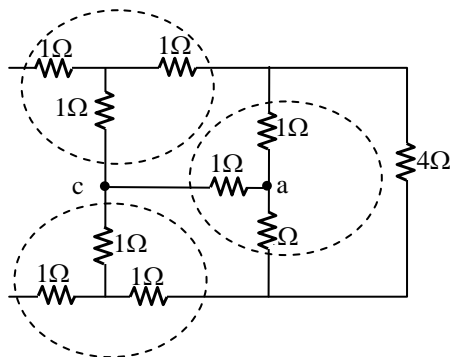


01.(a)

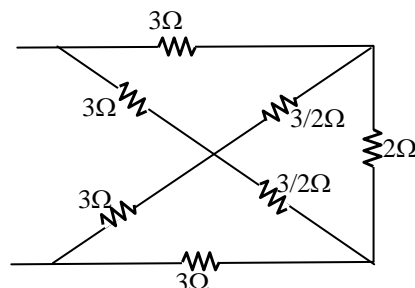
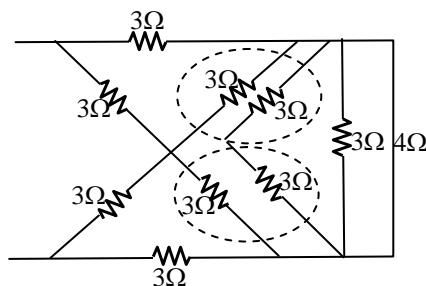
**Sol: (i)** The given circuit can be redrawn as

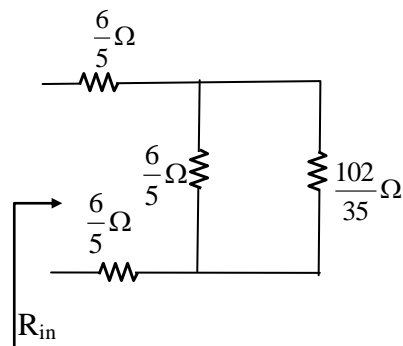
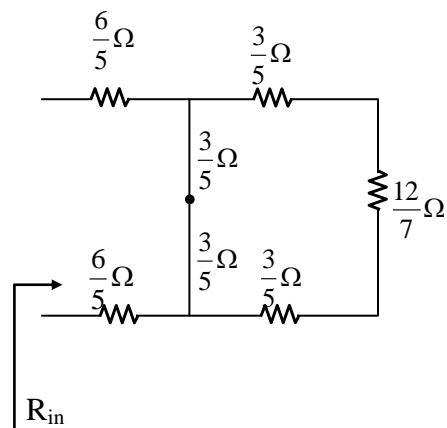


The  $1\Omega$  resistor between nodes 'a' and 'b' can be neglected by bridge balance.



By  $\Delta - Y$  transformation



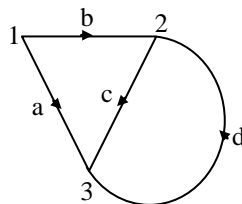


$$R_{in} = \frac{6}{5} + \left( \frac{6}{5} // \frac{102}{35} \right) + \frac{6}{5}$$

$$R_{in} = \frac{13}{4} \Omega$$

**(ii) Graph**

In the given circuit, there are three principle nodes and five branches. The graph of the circuit can be given as



$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ -1 & 0 & -1 & -1 \end{bmatrix}$$



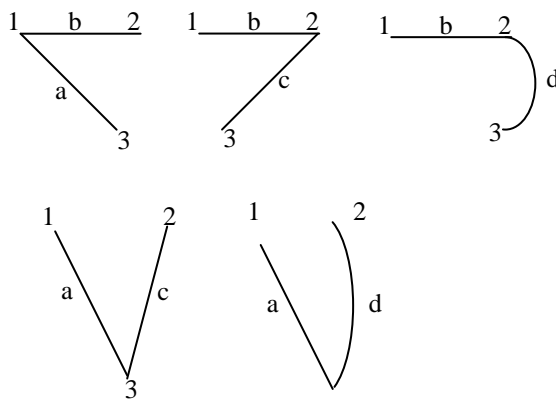
$$A_r = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} A_r^T = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A_r \cdot A_r^T = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

The number of trees =  $\det (A_r A_r^T)$

$$= 6 - 1 = 5$$

Five trees are possible for this graph and they are given by



01.(b)

**Sol:**

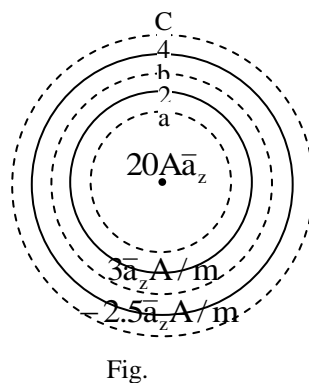


Fig.

Consider a circle of radius  $a$  which encloses the current of  $20A$  only. Apply Ampere's circuital law to

$$\oint \vec{H} \cdot d\vec{L} = I_{\text{encl}}$$



$$\text{LHS} = \oint \vec{H} \cdot d\vec{L}$$

As the current is directed in  $\hat{a}_z$  direction,  $H_\phi$  component of  $\vec{H}$  is produced.

$$\therefore \text{LHS} = \int_0^{2\pi} (H_\phi \hat{a}_\phi) (a d\phi \hat{a}_\phi) = \int_0^{2\pi} a H_\phi d\phi = 2\pi a H_\phi$$

$$\text{RHS} = I_{\text{encl}} = 20$$

$$\therefore 2\pi a H_\phi = 20 \text{ or } H_\phi = \frac{10}{\pi a}$$

$$\therefore \vec{H} = \frac{10}{\pi a} \hat{a}_\phi$$

Replacing  $a$  by  $\rho$ ,  $\vec{H} = \left( \frac{10}{\pi \rho} \right) \hat{a}_\phi \text{ A/m for } \rho < 2\text{m}.$

Consider a circle of radius  $b$  as shown in Fig. which encloses  $20\text{A } \hat{a}_z$  and  $3\hat{a}_z \text{ A/m}$ . Apply ampere's circuital law.

$$\oint \vec{H} \cdot d\vec{L} = I_{\text{encl}}$$

$$\begin{aligned} \therefore \text{LHS} &= \oint \vec{H} \cdot d\vec{L} = \int_0^{2\pi} (H_\phi \hat{a}_\phi) (b d\phi \hat{a}_\phi) \\ &= \int_0^{2\pi} b H_\phi d\phi = 2\pi b H_\phi \end{aligned}$$

$$\text{RHS} = I_{\text{encl}} = 20 + 3 \times \text{circumference of } \rho = 2\text{m circle.}$$

$$= 20 + 3 \times 2\pi \times 2 = 57.669 \text{ A}$$

$$\therefore 2\pi b H_\phi = 57.669,$$

$$\therefore H_\phi = \frac{9.178}{b} \text{ or } \vec{H} = \frac{9.178}{b} \hat{a}_\phi$$

Replacing  $b$  by  $\rho$ ,  $\vec{H} = \frac{9.178}{\rho} \hat{a}_\phi \text{ A/m for } 2 < \rho < 4$

Consider a circle of radius  $c$  as shown in fig. which encloses  $20 \text{ A } \hat{a}_z$ ,

$3\hat{a}_z \text{ A/m}$  and  $-2.5\hat{a}_z \text{ A/m}$ . Apply ampere's circuital law to this circular path,

$$\oint \vec{H} \cdot d\vec{L} = I_{\text{encl}}$$

$$\text{LHS} = \int_0^{2\pi} (H_\phi \hat{a}_\phi) (c d\phi \hat{a}_\phi) = \int_0^{2\pi} c H_\phi d\phi = 2\pi c H_\phi$$

$$\text{RHS} = I_{\text{end}} = 20 + 3 \times 2\pi \times 2 - 2.5 \times 2\pi \times 4 = -5.13274 \text{ A}$$



$$\therefore 2\pi c H_{\phi} = -5.13274$$

$$\text{or } \bar{H} = -\frac{0.8169}{c} \hat{a}_{\phi} \text{ A/m}$$

$$\text{Replacing } c \text{ by } \rho, \bar{H} = -\frac{0.8169}{\rho} \hat{a}_{\phi} \text{ A/m for } \rho > 4\text{m.}$$

Hence

$$\bar{H} = \begin{cases} \frac{10}{\pi\rho} \hat{a}_{\phi} & \rho < 2 \\ \frac{9.183}{\rho} \hat{a}_{\phi} & 2 < \rho < 4 \\ -\frac{0.8169}{\rho} \hat{a}_{\phi} & \rho > 4 \end{cases}$$

**01.(c)**

**Sol: (i) Dielectric constant ( $\epsilon_r$ )**

The dielectric characteristic of a material are determined by the dielectric constant or relative permittivity  $\epsilon_r$  of that material. Dielectric constant is the ratio between the permittivity of the medium and the permittivity of free space.

$$\text{i.e., } \epsilon_r = \frac{\epsilon}{\epsilon_0}$$

Since it is a ratio of same quantity,  $\epsilon_r$  has no unit. It is a measure of polarization in the dielectric material.

**(ii) Electric polarization**

Let us consider an atom placed inside an electric field. The centre of positive charge is displaced along the applied field direction while the centre of negative charge is displaced in the opposite direction. Thus a dipole is produced. When a dielectric material is placed inside an electric field such dipoles are created in all the atoms inside. This process of producing electric dipoles which are oriented along the field direction is called polarization in dielectrics.

**(iii) Polarisability ( $\alpha$ )**

When the strength of the electric field  $E$  is increased the strength of the induced dipole  $\mu$  also increases. Thus the induced dipole moment is proportional to the intensity of the electric field.



$$\text{i.e., } \mu = \alpha E$$

Where  $\alpha$ , the constant of proportionality is called polarisability. It can be defined as induced dipole moment per unit electric field.

**(iv) Polarization vector ( $\vec{P}$ )**

The dipole moment per unit volume of the dielectric material is called polarization vector  $\vec{P}$ . If  $\vec{\mu}$  is the average dipole moment per molecule and  $N$  is the number of molecules per unit volume the polarization vector

$$\vec{P} = N\vec{\mu}$$

The dipole moment per unit volume of the solid is the sum of all the individual dipole moments within that volume and is called the polarization  $\vec{P}$  of the solid.

**(v) Electric susceptibility ( $\chi_e$ )**

The polarization vector  $\vec{P}$  is proportional to the total electric flux density  $E$  and is in the same direction of  $E$ . Therefore the polarization vector can be written as

$$P = \epsilon_0 \chi_e E$$

Where the constant  $\chi_e$  is the electric susceptibility

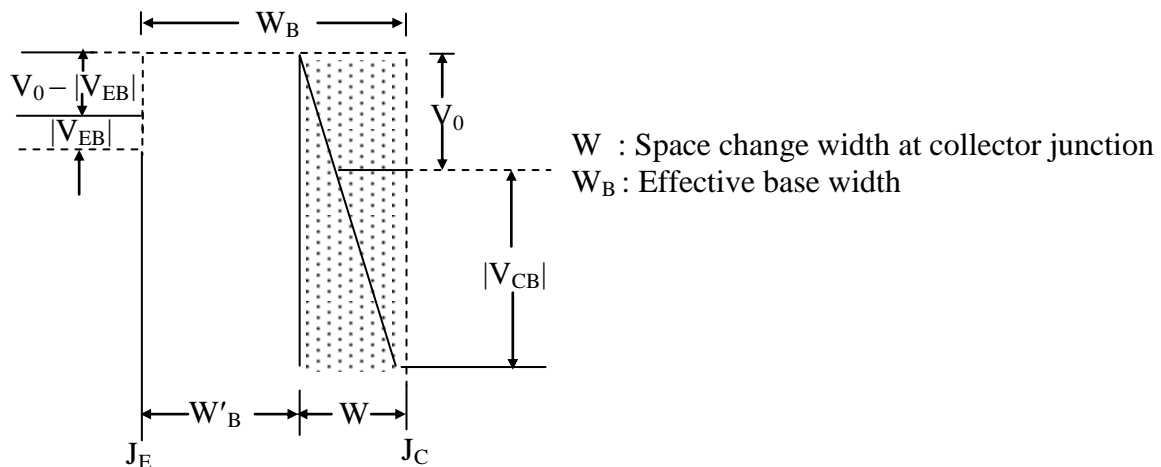
$$\text{Therefore } \chi_e = \frac{P}{\epsilon_0 E} = \frac{\epsilon_0 (\epsilon_r - 1) E}{\epsilon_0 E}$$

$$\chi_e = (\epsilon_r - 1)$$

**01.(d)**

**Sol:** Consider a p-n-p BJT with emitter junction ( $J_E$ ) in forward biased and collector junction ( $J_C$ ) in reverse biased.

The barrier width at  $J_E$  is negligible compared to space charge width at  $J_C$ . As the doping in the base is substantially smaller than that of collector, the penetration of transition region into the base is much larger than into collector. Hence the collector depletion region is neglected and all immobile charge is indicated in the base region.



If the metallurgical base width is  $W_B$ , then the effective electrical base width is  $W'_B = W_B - W$ .

This modulation of effective base width by the collector voltage is known as by the collector voltage is known as base-width modulation or early effect.

The decrease in  $W'_B$  with increasing reverse collector voltage has 3 consequences.

1. There is less chance for recombination within the base region. Hence,  $\alpha$  increases with increasing  $|V_{CB}|$ .
2. The concentration gradient of minority carriers is increased within the base. Since the hole current injected across the emitter is proportional to the gradient of minority carriers at emitter junction, then emitter current  $I_E$  increases with increasing reverse collector voltage.
3. For extremely large voltages, effective electrical base width  $W_B$  may be reduced to zero, causing voltage breakdown in the transistor. This phenomenon is called punch through.

**01(e)**

**Sol:** In a multi-stage amplifier, it is convenient to analyse the last stage (output stage) first. In the present case, the last stage ( $Q_2$  – stage) is emitter follower which has voltage gain of unity. Then the over all gain of two-stage amplifier (product of gains of individual stages) is same as gain of first stage which is common emitter amplifier in fig.

The gain of CE amplifier is

$$A_v = A_{v1} = \frac{r_{cl}}{r_{E1} + r_e'}$$

Since  $R_E = 0$  (for  $Q_1$  transistor),  $r_{E1} = 0$





Then,

$$A_v = \frac{r_{c1}}{r_e}$$

Where  $r_{c1}$  is the effective impedance seen by the collector of transistor  $Q_1$ .

That is,

$$r_{c1} = R_{C1} \parallel Z_{i(\text{base})2}$$

The impedance,  $Z_{i(\text{base})2}$  at the base of transistor  $Q_2$  is,

$$\begin{aligned} Z_{i(\text{base})2} &= \beta \cdot R_{E2} \\ &= 80 \times 3k \end{aligned}$$

$$\text{or } Z_{i(\text{base})2} = 240k\Omega$$

Therefore,

$$\begin{aligned} R_{c1} &= R_{C1} \parallel Z_{i(\text{base})2} \\ &= 10k \parallel 240k \end{aligned}$$

$$(\text{or}) r_{c1} = 9.6 k\Omega$$

Therefore, over all gain of amplifier of fig is

$$A_v = \frac{r_{c1}}{r_e} = \frac{9.6 \times 10^3}{25} = 384$$

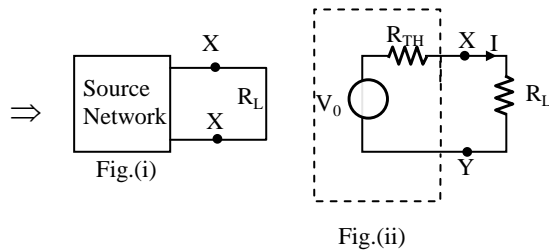
$$\text{Or } A_v = 384$$

## 02(a)

**Sol: (i) Statement:** Compensation theorem states that, “In a linear time invariant network, when the resistance  $R$  of an uncoupled branch, carrying a current ( $I$ ), is changed by ( $\Delta R$ ), the currents in all the branches can be evaluated by assuming an ideal voltage source ( $V_C$ ) such that  $V_C = I(\Delta R)$  in series with ( $R + \Delta R$ ) when all other sources in the network are replaced by their internal resistances.

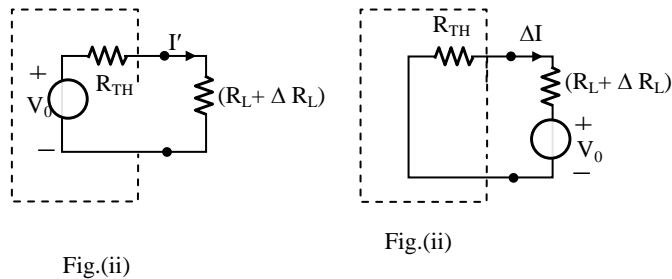
### Proof:

Let us assume a Load  $R_L$  be connected to a DC source network whose Thevenin's equivalent gives ' $V_0$ ' as the Thevenin's voltage and ' $R_{TH}$ ' as Thevenin's resistance as shown in the figure below.



$$\text{Here, } I = \frac{V_0}{R_{TH} + R_L} \dots\dots\dots(1)$$

Let the Load resistance  $R_L$  be changed to  $(R_L + \Delta R_L)$



$$\text{Here, } I' = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} \dots\dots\dots(2)$$

The change of current being termed as  $\Delta I$

$$\text{Therefore, } \Delta I = I' - I \dots\dots\dots(3)$$

Substituting (1) & (2) in equation (3),

$$\Delta I = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} - \frac{V_0}{R_{TH} + R_L}$$

$$\Delta I = - \left[ \frac{V_0}{R_{TH} + R_L} \right] \frac{\Delta R_L}{R_{TH} + R_L + \Delta R_L}$$

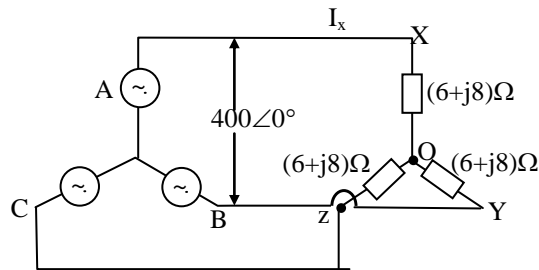
$$\Delta I = - \frac{I \Delta R_L}{R_{TH} + R_L + \Delta R_L}$$

As we know,  $V_C = I \Delta R_L$

$$\Delta I = - \frac{-V_C}{R_{TH} + R_L + \Delta R_L}$$



(ii)



The phase voltage  $V_{XO} = \frac{400}{\sqrt{3}} V$

$$I_X = I_{line} = I_{ph} = \frac{400\sqrt{3}}{6 + j8} = 23.09 \angle -53.13^\circ A$$

In a star connected network,  $I_{line} = I_{ph}$

$$I_x = 23.09 \angle -53.13^\circ A$$

Power factor =  $\cos 53.13^\circ = 0.6 \text{ lag}$

$$\begin{aligned} \text{Real power consumed by the Load} &= \sqrt{3} V_L I_L \cos \phi \\ &= \sqrt{3} (400) (23.09) (0.6) \\ &= 9.598 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Reactive power consumed by the Load} &= \sqrt{3} V_L I_L \sin \phi \\ &= \sqrt{3} (400) (23.09) (0.8) \\ &= 12.797 \text{ Kvar} \end{aligned}$$

**02(b)**

**Sol:** (i) Given that germanium diode is in Reverse biased,

From the circuit,  $I = I_o + I_R$

Differentiate w.r.t to temperature T,

$$\frac{dI}{dT} = \frac{dI_o}{dT} + \frac{dI_R}{dT}$$

As  $I_R$  is Independent of T, i.e.  $\frac{dI_R}{dT} \rightarrow 0$

$$\therefore \frac{dI}{dT} = \frac{dI_o}{dT} \rightarrow (1)$$



Also it is given that, reverse saturation current of diode increases by  $0.11^\circ/\text{c}$ ,

$$\therefore \frac{1}{I_o} \frac{dI_o}{dT} = 0.11 \rightarrow (2)$$

And Temperature dependences of reverse current is  $0.07^\circ/\text{c}$

$$\therefore \frac{1}{I} \frac{dI}{dT} = 0.07 \rightarrow (3)$$

from equation (1), (2) and (3),

$$0.01 I_o = 0.07 I \rightarrow (4)$$

substituting equation (4) in  $I = I_o + I_R$ ,

$$\text{we get, } I_R = \frac{4}{11} I$$

since  $I = 5 \mu\text{A}$

$$I_R = \frac{4}{11} \times 5 \mu\text{A} = \frac{20}{11} \mu\text{A}$$

$$\therefore R = \frac{V}{I_R} = \frac{10 \times 11}{20} \times 10^6 = 5.5 \text{M}\Omega$$

Hence leakage resistance shunting the diode is  $5.5 \text{M}\Omega$

## 02.(b)(ii)

**Sol:** (ii) First we find for  $T = 300 \text{ K}$

$$n_i^2 = N_C N_V \exp\left(\frac{-E_g}{kT}\right)$$

$$\begin{aligned} n_i^2 &= (4.7 \times 10^{17}) (7.0 \times 10^{18}) \exp\left(\frac{-1.42}{0.0259}\right) \\ &= 5.09 \times 10^{12} \end{aligned}$$

So that,  $n_i = 2.26 \times 10^6 \text{ cm}^{-3}$

At  $T = 450 \text{ K}$ :

The value of  $kT$  at  $450 \text{ K}$  is

$$kT = (0.0259) \left(\frac{450}{300}\right) = 0.03885 \text{ eV}$$



$$n_i^2 = (4.7 \times 10^{17}) (7.0 \times 10^{18}) \left( \frac{450}{300} \right)^3 \exp \left( \frac{-1.42}{0.03885} \right) = 1.48 \times 10^{21}$$

So that,  $n_i = 3.85 \times 10^{10} \text{ cm}^{-3}$

Comment:

We may note from this example that the intrinsic carrier concentration increased by over 4 orders of magnitude as the temperature increased by  $150^\circ\text{C}$ .

## 02.(c)

**Sol: (i)** Oscillator: An oscillator is an electronic circuit which produces a continuous, repeated, alternating waveform without any input. Oscillators basically convert unidirectional current flow from a DC source into an alternating waveform which is of the desired frequency, as decided by its circuit components. Oscillators can be called as sine wave generators.

(ii) RC phase-shift Oscillator:-

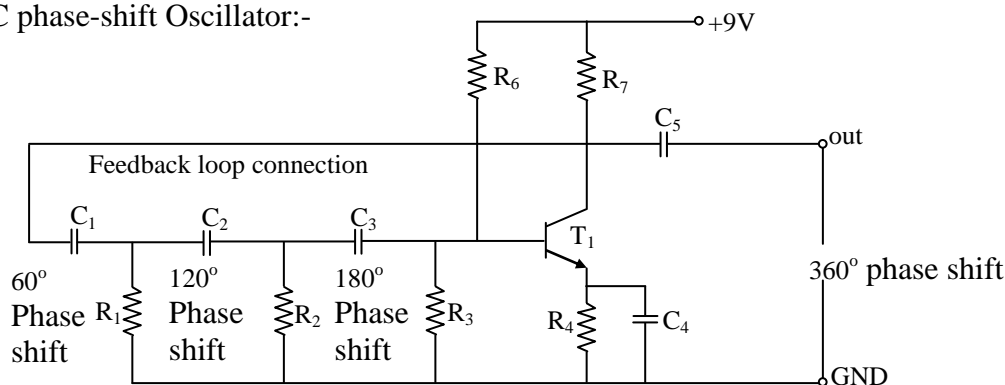


Fig. RC phase shift oscillator using NPN transistor.

It is a linear electronic oscillator circuit that produces a sine wave output. It consists of an inverting amplifier elements, such as Op-amp or transistor with its output fed back to its input through a phase -shift network consisting of capacitors and resistors in a ladder network. The feedback network shifts the phase of the amplifier output by  $180^\circ$  at the oscillation frequency to give positive feedback. The filter produces a phase shift that increase with the frequency. It must have maximum phase-shift of more than  $180^\circ$  at high frequencies so the phase-shift at the desired oscillation frequency can be  $180^\circ$ . The most common phase-shift network cascades three identical resistors, capacitor stages that produce a phase shift is zero at low frequencies and  $270^\circ$  at high frequencies.



Advantages:

- This circuit is very simple and cheap as it comprises resistors and capacitors (not bulky and expensive high-value inductors)
- It provides good frequency stability.
- The output of this circuit is sinusoidal that is quite distortion free.
- The phase-shift oscillator circuit is simple than the wein bridge oscillator circuit because it does not need negative feedback and the stabilization arrangements.
- They have a wide range of frequency (from a few Hz to several hundreds of KHz).
- They are particularly apt for low frequencies say of the order of 1 Hz, so these frequencies can be easily gained by using R and C of large values.

Disadvantages:

- The output is small and It is due to smaller feedback.
- The frequency stability is not as good as that of wein Bridge oscillator.
- It is difficult for the circuit to start oscillations as the feedback is usually small.
- It needs high voltage (12V) battery so as to develop a sufficiently large feedback voltage.

Applications:

- FET phase - shift oscillator is used for generating signals over a wide frequency range. The frequency may be varied from a few Hz to 200 Hz by employing one set of resistor with three capacitors ganged together to vary over a capacitance range in the 1:10 ratio. Similarly the frequency ranges of 200Hz to 2 kHz, 2 kHz to 20 kHz and 20 kHz to 200 kHz can be obtained by using other set of resistors.
- RC phase shift oscillators are used in musical instruments, voice synthesis and in GPS units since they work at all audio frequencies.

**3(a)**

**Sol: Unit cell:** Smallest numbers of atoms are arranged in a repetitive manner.

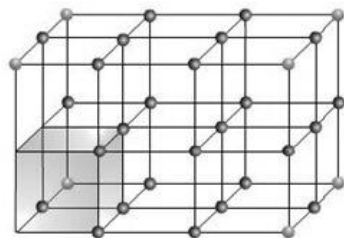


Fig: Space lattice

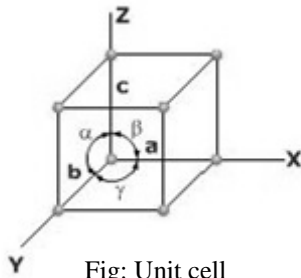


Fig: Unit cell

$a, b, c$  = lattice parameter(s),

$\alpha, \beta, \gamma$  = interfacial angle(s)

**Effective number of atoms / lattice point in cubic unit cell :**

**1. Simple cubic structure:**

$$8 \text{ atoms at the corners} \times \frac{1}{8} = 1 \text{ atom}$$

Therefore, the unit cell of simple cubic structure contains one atom.

**2. Body centered cubic structure (B.C.C):**

$$8 \text{ atoms at the corners} \times \frac{1}{8} = 1 \text{ atom}$$

$$1 \text{ centre atom} = 1 \text{ atom}$$

$$\therefore \text{Total} = 2 \text{ atoms}$$

Therefore, the unit cell of B.C.C. structure contains two atoms.

**3. Face centered cubic structure (F.C.C.) :**

$$8 \text{ atoms at the corners} \times \frac{1}{8} = 1 \text{ atom}$$

$$6 \text{ face centered atoms} \times \frac{1}{2} = 3 \text{ atoms}$$

$$\therefore \text{Total} = 4 \text{ atoms}$$

Therefore, the unit cell of F.C.C. structure contains four atoms.

**Atomic packing factor of Si material:**

Si is a diamond cubic structure material. In diamond cubic structure 8 atoms are arranged at corners and 6 atoms are arranged at face centers and 4 atoms are arranged inside unit cell on 4 body diagonals

with each atom located at  $\frac{a}{4}, \frac{a}{4}, \frac{a}{4}$  from corner.



Effective number of atoms

$$n = \frac{1}{8} \times 8 + \frac{1}{2} \times 6 + 4 = 8 \text{ atoms}$$

Relationship between atomic radius (R) and lattice parametrical

$$8R = \sqrt{3}a$$

$$\Rightarrow R = \frac{\sqrt{3}a}{8} = \frac{\sqrt{3} \times 5.43 \times 10^{-10}}{8}$$

$$= 1.1756 \times 10^{-10} \text{ m}$$

$$V_{uc} = a^3 = \left( \frac{8R}{\sqrt{3}} \right)^3$$

$$= (5.431 \times 10^{-10})^3$$

$$\text{Atomic packing factor} = \frac{n \times \frac{4}{3} \pi R^3}{a^3}$$

$$= \frac{8 \times \frac{4}{3} \pi (1.175 \times 10^{-10})^3}{(5.431 \times 10^{-10})^3} = 0.34$$

$$\text{Theoretical density} = \frac{n \times AW}{AN \times V_{uc}}$$

Atomic weight = 28 g/mol

$$= \frac{8 \times 28 \times 10^{-3}}{6.023 \times 10^{23} \times (5.431 \times 10^{-10})^3}$$

$$= 2322 \text{ kg / m}$$

**03.(b)**

**Sol:** (i)  $R = 2\Omega$ ,  $l = 1\text{mH}$ ,  $C = 0.4 \mu\text{F}$

$$V(t) = 20 \sin \omega t$$

For a series R–L–C circuit,

$$(a) \text{ Resonant frequency. } \omega_0 = \frac{1}{\sqrt{LC}}$$





$$= \frac{1}{\sqrt{1 \times 10^{-3} \times 0.4 \times 10^{-6}}}$$

$$= 50,000 \text{ rad/sec}$$

(b) Lower half power frequency,

$$\omega_1 = \frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$= 49009 \text{ rad/sec}$$

Upper half power frequency,

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{\sqrt{LC}}\right)^2}$$

$$= 51009 \text{ rad/sec}$$

(c) Q – factor =  $\frac{1}{R} \sqrt{\frac{L}{C}}$

$$= 25$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L} = 2000 \text{ rad/sec}$$

(d) Amplitude of current at  $\omega_0$  is given by  $I_0 = \frac{V_{\text{rms}}}{R} = \frac{20}{2(\sqrt{2})} = 7.07 \text{ A}$

Amplitude of current at  $\omega_1$  and  $\omega_2$  is given by,  $I = \frac{I_0}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5 \text{ A}$

(ii) The current through the capacitor I given by an expression,

$$I(t) = C \frac{dv}{dt} \text{ where } C = 1 \text{ mF}$$

From  $t = 0$  to  $t = 1 \text{ sec}$

$$V(t) = 10t$$

$$I(t) = 10 \text{ mA}$$

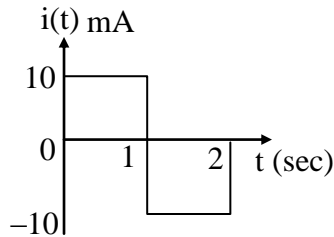
From  $t = 1 \text{ sec}$  to  $t = 2 \text{ sec}$

$$v(t) = -10t + 20$$

$$I(t) = -10 \text{ mA}$$



The current wave form is given as below



**03(c)**

**Sol:** (i) (a)  $f = 4x - 2y + 3z - 4$ , the  $\hat{a}_n = \pm \frac{\nabla f}{|\nabla f|}$  gives the possible unit vectors which are perpendicular to

$f$ . The unit vector with  $-ve$  sign gives the unit vector which is directed from higher value of  $f$  towards the lower value of  $f$ . The unit vector with  $+ve$  sign gives the unit vector which is directed from lower value of  $f$  towards the higher value of  $f$ . We have to determine  $\hat{a}_{21}$ .

In region 1, at point  $P_1 (0,0,100)$ ,  $f_1 = 296$

In region 2, at point  $P_2 (0,0,-100)$ ,  $f_2 = -304$

Hence we have to determine the unit vector from lower value of  $f$  ( $f_2 = -304$ ) towards higher value of  $f$  ( $f_1 = 296$ ). That is with  $+ve$  sign.

$$\text{Hence } \hat{a}_n = \frac{\nabla f}{|\nabla f|} = \frac{4\hat{a}_x - 2\hat{a}_y + 3\hat{a}_z}{\sqrt{(4)^2 + (-2)^2 + (3)^2}} = 0.743\hat{a}_x - 0.371\hat{a}_y + 0.557\hat{a}_z$$

(b) Let point A  $(1,3,z)$  lie on the plane. Then

$$4(1) - 2(3) + 3z = 4, \text{ i.e. } z = 2$$

$$\begin{aligned} \bar{E}_{N1} &= (\bar{E}_1 \cdot \hat{a}_{21}) \hat{a}_{21} \\ &= (7.43 - 7.42 + 22.28)(0.743\hat{a}_x - 0.371\hat{a}_y + 0.557\hat{a}_z) \\ &= 16.56147\hat{a}_x - 8.26959\hat{a}_y + 12.41553\hat{a}_z \text{ V/m.} \end{aligned}$$

$$\begin{aligned} \bar{E}_1 &= \bar{E}_{t1} + \bar{E}_{N1}, \bar{E}_{t1} = \bar{E}_1 - \bar{E}_{N1} \\ \bar{E}_{t1} &= (10\hat{a}_x + 20\hat{a}_y + 40\hat{a}_z) - (16.56147\hat{a}_x - 8.26959\hat{a}_y + 12.41553\hat{a}_z) \\ &= -6.56147\hat{a}_x + 28.26959\hat{a}_y + 27.58447\hat{a}_z \text{ V/m} \end{aligned}$$

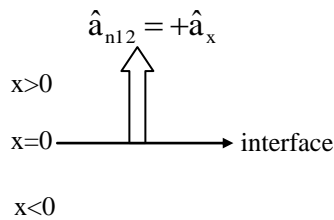
$$\text{Now } \bar{D}_{N1} = \bar{D}_{N2} \text{ or } \epsilon_o \epsilon_{R1} \bar{E}_{N1} = \epsilon_o \epsilon_{R2} \bar{E}_{N2}$$



$$\begin{aligned}\therefore \bar{E}_{N2} &= \frac{\epsilon_{R1}}{\epsilon_{R2}} \bar{E}_{N1} = \frac{3}{8} (16.56147 \hat{a}_x - 8.26959 \hat{a}_y + 12.41553 \hat{a}_z) \\ &= 6.2015 \hat{a}_x - 3.101 \hat{a}_y + 4.656 \hat{a}_z \text{ V/m}\end{aligned}$$

$$\begin{aligned}\therefore \bar{E}_2 &= \bar{E}_{t2} + \bar{E}_{N2} = \bar{E}_{t1} + \bar{E}_{N2} \\ &= -6.56147 \hat{a}_x + 28.26959 \hat{a}_y + 27.58447 \hat{a}_z + 6.2105 \hat{a}_x - 3.101 \hat{a}_y + 4.656 \hat{a}_z \\ &= -0.351 \hat{a}_x + 25.169 \hat{a}_y + 32.240 \hat{a}_z \text{ V/m}\end{aligned}$$

(ii)



Given  $\vec{K} = 10 \hat{a}_z \text{ A/m}$

$\vec{H}_1 = 12 \hat{a}_y \text{ A/m}$

As the interface is  $x = 0$  (or)  $yz$ -plane, hence  $y, z$  are tangential components and  $x$  is normal component

$\vec{H}_{t1} = 12 \hat{a}_y, \vec{H}_{n1} = 0$

$\vec{H}_{t1} - \vec{H}_{t2} = \hat{a}_{n12} \times \vec{K}$

$12 \hat{a}_y - \vec{H}_{t2} = \hat{a}_x \times 10 \hat{a}_z$

$\vec{H}_{t2} = 22 \hat{a}_y$

$\therefore \vec{H}_2 = 22 \hat{a}_y \text{ A/m}$

**04(a)**

**Sol: (i)**

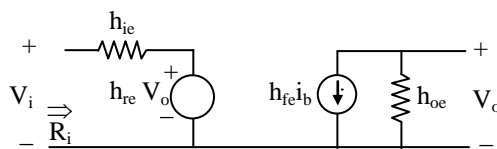


Fig1: CE config equivalent ckt.

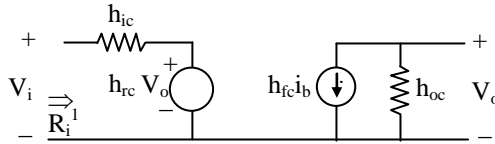


Fig1: CC config equivalent ckt.

CE config connection;

similarly for CC

$$R_i = \left. \frac{V_{be}}{i_b} \right|_{V_{ce}=0} \quad R_i^1 = \left. \frac{V_{bc}}{i_b} \right|_{V_{cc}=0} = h_{ic} \quad (1)$$

But from definition of h-parameters,

$$h_{ie} = \left. \frac{V_{be}}{i_b} \right|_{V_{ce}=0} \rightarrow (2)$$

And in small signal hybrid model of transistors,

$$h_{ie} = h_{ic} \rightarrow (3)$$

from (1), (2) & (3)

$$\therefore R_i = R_i^1$$

i.e input impedances of two circuits are identical

(ii)

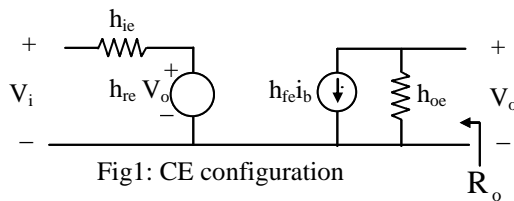


Fig1: CE configuration

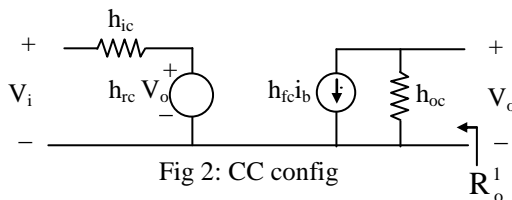
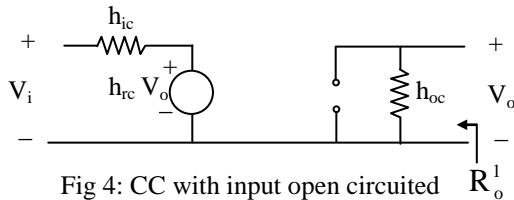
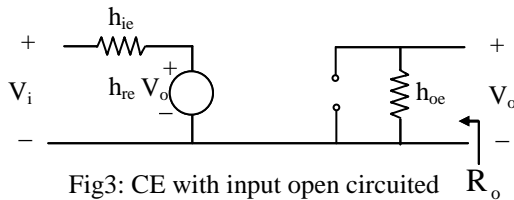


Fig 2: CC config

When input is open circuited  $\Rightarrow i_b = 0$

$\therefore$  configurations become,



Thus the output impedance of both circuits with input open circuited is given by,

$$R_o = h_{oe} \text{ \& } R_o^1 = h_{oc}$$

But according to small signal hybrid model.

$$h_{oe} = h_{oc}$$

$$\therefore R_o = R_o^1$$

The output impedances of both the circuits are identical.

#### 04.(b)

**Sol: (i)** The electrical properties of ceramic products vary from the low loss, high frequency dielectric to semiconductors. Electrical insulator fall into two general classical electrical porcelain for both high and low tension service and the special bodies such as steatite, rutile, cordierite, high alumina, and clinoestatite for high frequency insulation.

#### Dielectric constant:

Dielectric constant is the ratio of the capacitance of a dielectric compared to the capacitance of air under the same conditions.

A low dielectric constant contributes to low power loss and low loss factor; a high dielectric constant permits small physical size.

The dielectric constant for electrical porcelain varies between 4.1 and 11.0. Some special bodies have reported values of several thousands.

Porcelain has large positive temperature coefficient.

Rutile bodies have large negative coefficient.



By combining capacitor dielectrics having different temperature coefficients it is possible to reduce effect of the temperature change.

**Volume and surface resistivity:**

A volume resistivity of  $10^6$  ohms/cm<sup>3</sup> is considered the lower limit for an insulating material. At room temperature practically all ceramic materials exceed this lower limit. As the temperature of ceramic materials is raised, the volume resistivity decreases; the volume resistivity of soda-lime glasses decreases rapidly with temperature, whereas some special bodies are good insulators (above  $10^6$  ohm/cm<sup>3</sup>) at 700°C. Crystallized alumina has a volume resistivity of 500 ohms/cm<sup>3</sup> at 1600°C.

Surface resistivity for dry, clean surface is  $10^{12}$  ohms/cm<sup>2</sup>, At 98% humidity, the surface resistivity may be  $10^{11}$  ohms/cm<sup>2</sup> for a glazed or  $10^9$  ohms/cm<sup>2</sup> for an unglazed piece. The presence of dissolved gases and other deposits also tends to decrease the surface resistivity of ceramic materials.

**High temperature super conductors:**

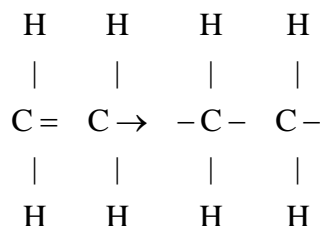
Mixed oxide of Ba, Ca, Cu exhibit super conduction at about 100°K.

**(ii) Polymers are giant molecules made by linking**

Together a large number of small molecules called monomers. The reaction by which monomer units combine to form polymers is termed polymerization these are mainly three kinds of polymerization processes

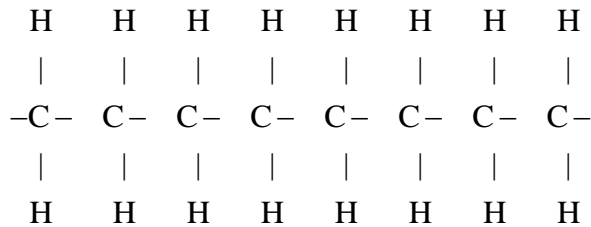
- (i) Addition polymerization
- (ii) condensation polymerization and
- (iii) copolymerization.

At high temperature and in the presence of catalyst like sulphuric acid or zinc chloride ethylene molecule polymerize to form polyethylene or polythene during polymerization, the double bond is opened up into two single bond





The monomers are bonded together end –to-end in a polymerization reaction



The degree of polymerization defines the number of repeating monomers in the chain

Polymers have a wide range of user electrical insulation ropes and filaments carriages, sound proofing materials, vacuum seats, chemical wave, human body implants coating for flying panes polymeric clothing etc.

**04.(c)**

**Sol: (i)** First general solution

$$i(t) = I_{ss}(t) + I_{tr}(t)$$

$$i(t) = \frac{100}{\sqrt{5^2 + 5^2}} \sin(500t + \phi - \theta) + Ae^{-t/\tau}$$

$$|z| = \sqrt{5^2 + 5^2} = \sqrt{50}$$

$$\theta = \tan^{-1} \left[ \frac{\omega L}{R} \right] = \tan^{-1}[1] = 45^\circ$$

$$T = \frac{L}{R} = \frac{0.01}{5} = \frac{1}{500}$$

$$i(t) = 2\sqrt{50} \sin(500t + \phi - 45^\circ) + Ae^{-500t}$$

$$\text{At } t = 0 \rightarrow i = 0$$

$$0 = 2\sqrt{50} \sin(\phi - 45^\circ) + A$$

$$\text{So, } A = -2\sqrt{50} \sin(\phi - 45^\circ)$$

$$i(t) = 2\sqrt{50} \sin(500t + \phi - 45^\circ) - 2\sqrt{50} \sin(\phi - 45^\circ) e^{-500t}$$

$$\text{Now put } \phi = 0^\circ$$

$$i(t) = 2\sqrt{50} \sin(500t - 45^\circ) + 2\sqrt{50} e^{-500t}$$

$$i(t) = 14.14 \sin(500t - 45^\circ) + 10e^{-500t}$$

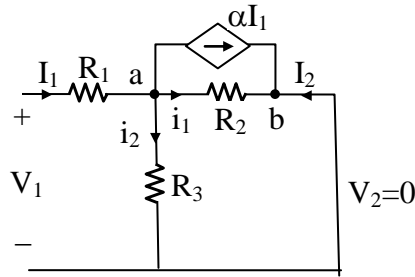


$$(ii) V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

**When  $V_2 = 0$**

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} \quad h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0}$$



At node a,

$$I_1 = i_2 + i_1 + \alpha I_1$$

$$i_2 + i_1 = (1 - \alpha) I_1$$

$$i_1 = (1 - \alpha) I_1 \frac{R_3}{R_2 + R_3} \dots\dots\dots(1)$$

$$i_2 = (1 - \alpha) I_1 \frac{R_2}{R_2 + R_3} \dots\dots\dots(2)$$

Applying KVL

$$V_1 = I_1 R_1 + i_2 R_3$$

$$V_1 = I_1 R_1 + (1 - \alpha) I_1 \cdot \frac{R_2 R_3}{R_2 + R_3}$$

$$h_{11} = \frac{V_1}{I_1} = R_1 + (1 - \alpha) \frac{R_2 R_3}{R_2 + R_3}$$

At node -b

$$\alpha I_1 + i_1 + I_2 = 0$$

$$I_2 = -\alpha I_1 - (1 - \alpha) I_1 \frac{R_3}{R_2 + R_3}$$

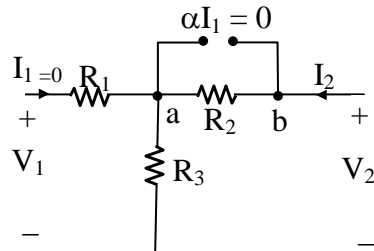
$$h_{21} = \frac{I_2}{I_1} = -\alpha - (1 - \alpha) \frac{R_3}{R_2 + R_3} = \frac{-(\alpha R_2 + R_3)}{R_2 + R_3}$$





When  $I_1 = 0$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \left. \frac{I_2}{V_2} \right|_{I_1=0}$$



$$V_1 = I_2 R_3$$

$$V_2 = I_2 (R_2 + R_3)$$

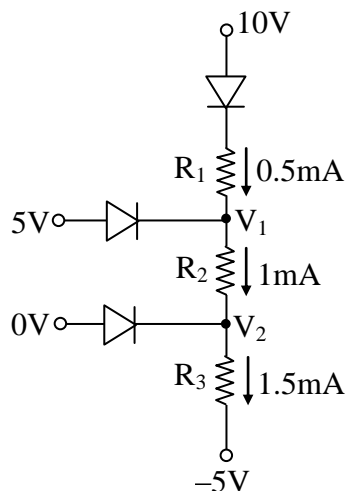
$$h_{12} = \frac{V_1}{V_2} = \frac{R_3}{R_2 + R_3}$$

$$h_{22} = \frac{I_2}{V_2} = \frac{1}{R_2 + R_3}$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} R_1 + (1-\alpha) \frac{R_2 R_3}{R_2 + R_3} & \frac{R_3}{R_2 + R_3} \\ -\frac{(\alpha R_2 + R_3)}{R_2 + R_3} & \frac{1}{R_2 + R_3} \end{bmatrix}$$

**05.(a)**

Sol: (i) The circuit is as follows





$$V_1 = 5 - 0.6 = 4.4\text{V}$$

$$R_1 = \frac{10 - 0.6 - 4.4}{0.5\text{m}} = 10\text{K}\Omega$$

$$R_2 = \frac{V_1 - V_2}{1\text{m}} = \frac{4.4 - (-0.6)}{1\text{m}} = 5\text{K}\Omega$$

$$R_3 = \frac{V_2 - (-5)}{1.5\text{m}} = \frac{-0.6 + 5}{1.5\text{m}} = 2.93\text{K}\Omega$$

$$(ii) V_{TH} = \frac{3.6(20)}{11 + 3.6} = 4.93\text{V} > V_z$$

$$R_{TH} = 11 \parallel 3.6 = 2.71\text{K}\Omega$$

$$i_z = \frac{4.93 - 4}{2.71\text{K}} = 0.34\text{mA}$$

$$\therefore Q\text{-point} = (i_z, V_{TH}) = (0.34 \text{ mA}, 4.93\text{V})$$

**05.(b)**

**Sol:** Nanomaterials having wide range of applications in the field of electronics, fuel cells, batteries, agriculture, food industry, and medicines, etc... It is evident that nanomaterials split their conventional counterparts because of their superior chemical, physical, and mechanical properties and of their exceptional formability.

**(i) Fuel cells:**

A fuel cell is an electrochemical energy conversion device that converts the chemical energy from fuel (on the anode side) and oxidant (on the cathode side) directly into electricity. The heart of fuel cell is the electrodes. Microbial fuel cell is a device in which bacteria consume water-soluble waste such as sugar, starch and alcohols and produces electricity plus clean water. Carbon nanotubes (CNTs) have chemical stability, good mechanical properties and high surface area, making them ideal for the design of sensors and provide very high surface area due to its structural network. Since carbon nanotubes are also suitable supports for cell growth, electrodes of microbial fuel cells can be built using of CNT. Due to three-dimensional architectures and enlarged electrode surface area for the entry of growth medium, bacteria can grow and proliferate and get immobilized. Multi walled CNT scaffolds could offer self-supported structure with large surface area through which hydrogen producing bacteria (e.g., E. coli) can eventually grow and proliferate. Also CNTs and MWCNTs have been reported to be biocompatible for different eukaryotic cells.



**(ii) Catalysis:**

Higher surface area available with the nanomaterial counterparts, nano-catalysts tend to have exceptional surface activity. For example, reaction rate at nano-aluminum can go so high, that it is utilized as a solid-fuel in rocket propulsion. Nano-aluminum becomes highly reactive and supplies the required thrust to send off payloads in space.

**(iii) Phosphors for High-Definition TV:**

The resolution of a television, or a monitor, depends greatly on the size of the pixel. These pixels are essentially made of materials called “phosphors,” which glow when struck by a stream of electrons inside the cathode ray tube (CRT). The resolution improves with a reduction in the size of the pixel, or the phosphors. Nano-crystalline zinc selenide, zinc sulfide, cadmium sulfide, and lead telluride synthesized by the sol-gel techniques are candidates for improving the resolution of monitors.

**(iv) Next-Generation Computer Chips:**

The microelectronics industry has been emphasizing miniaturization, whereby the circuits, such as transistors, resistors, and capacitors, are reduced in size. By achieving a significant reduction in their size, the microprocessors, which contain these components, can run much faster, thereby enabling computations at far greater speeds.

**(v) Elimination of Pollutants:**

Nanomaterials possess extremely large grain boundaries relative to their grain size. Hence, they are very active in terms of their chemical, physical, and mechanical properties. Due to their enhanced chemical activity, nanomaterials can be used as catalysts to react with such noxious and toxic gases as carbon monoxide and nitrogen oxide in automobile catalytic converters and power generation equipment to prevent environmental pollution arising from burning gasoline and coal.

**(vi) Sun-screen lotion:**

Prolonged UV exposure causes skin-burns and cancer. Sun-screen lotions containing nano-TiO<sub>2</sub> provide enhanced sun protection factor (SPF) while eliminating stickiness. The added advantage of nano skin blocks (ZnO and TiO<sub>2</sub>) arises as they protect the skin by sitting onto it rather than penetrating into the skin. Thus they block UV radiation effectively for prolonged duration. Additionally, they are transparent, thus retain natural skin color while working better than conventional skin-lotions.



05.(c)

Sol:

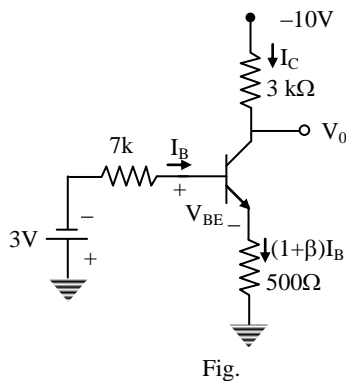


Fig.

(i) KVL for base-emitter loop of BJT:

$$-3V - (I_B \times 7k\Omega) - V_{BE} - ((1+\beta)I_B \times 500\Omega) = 0$$

$$I_B (7k + 101 \times 0.5k) = -3.7V$$

$$\therefore I_B = \frac{-3.7V}{57.5k\Omega} = -0.0643478mA$$

$$= -64.3478\mu A$$

**Note:** The transistor used in the circuit is an n-p-n transistor and the base current,  $I_B$  is found to be negative, hence the transistor is in cut-off region.

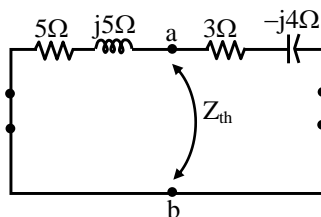
(ii) Since the transistor in the given circuit is in cutoff region,  $I_C = 0$

$$\therefore V_0 = -10V$$

05.(d)

Sol: In an AC network, maximum power transfer takes place when,  $Z_L = \text{Conjugate of } (Z_{Th})$

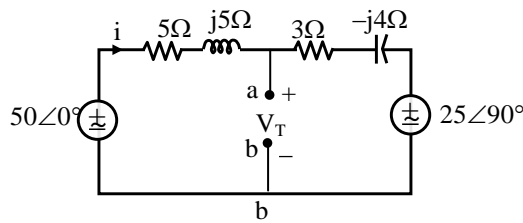
In the given network,  $Z_{Th}$  can be calculated by



$$Z_{TH} = Z_{ab} = (5 + j5) \parallel (3 - j4)$$

$$= (4.23 - j1.15)\Omega$$

$$Z_L = \text{Conjugate } (Z_{TH}) = 4.23 + j1.15\Omega$$



$$i = \frac{50\angle 0^\circ - 25\angle 90^\circ}{(5 + j5) + (3 - j4)}$$

$$= (5.77 - j3.846) \text{ A}$$

$$V_{TH} = 50\angle 0^\circ - (5 + j5) \times (5.77 - j3.846)$$

$$= 9.805 \angle -78.69^\circ$$

$$\begin{aligned} \text{Maximum power transferred to Load} &= \frac{V_{TH}^2}{4R_{TH}} \\ &= \frac{(9.805)^2}{4 \times 4.23} = 5.68 \text{ W} \end{aligned}$$

**05.(e)**

**Sol:** (a) we have

$$\vec{E} = - \left[ \frac{\partial}{\partial x} (50xyz) \hat{a}_x + \frac{\partial}{\partial y} (50xyz) \hat{a}_y + \frac{\partial}{\partial z} (50xyz) \hat{a}_z \right]$$

$$= -50(yz \hat{a}_x + xz \hat{a}_y + xy \hat{a}_z)$$

$$\therefore |\vec{E}| = 50 \sqrt{y^2 z^2 + x^2 z^2 + x^2 y^2}$$

$$W_E = \frac{\epsilon_o}{2} \int |\vec{E}|^2 dV$$

$$= \frac{\epsilon_o}{2} \int_0^2 \int_0^2 \int_0^2 2500(y^2 z^2 + x^2 z^2 + x^2 y^2) dx dy dz$$

$$= 1250 \epsilon_o \left\{ \left[ x \right]_0^2 \left[ \frac{y^3}{3} \right]_0^2 \left[ \frac{z^3}{3} \right]_0^2 + \left[ \frac{x^3}{3} \right]_0^2 \left[ y \right]_0^2 \left[ \frac{z^3}{3} \right]_0^2 + \left[ \frac{x^3}{3} \right]_0^2 \left[ \frac{y^3}{3} \right]_0^2 \left[ z \right]_0^2 \right\}$$

$$= 1250 \epsilon_o (14.222 + 14.222 + 14.222) = 0.472 \mu\text{J}$$

$$(b) \text{ Energy density} = \frac{\epsilon_o}{2} |\vec{E}|^2 = \frac{\epsilon_o}{2} \times 2500(y^2 z^2 + x^2 z^2 + x^2 y^2)$$



The center of the cube is  $\left(\frac{0+2}{2}, \frac{0+2}{2}, \frac{0+2}{2}\right) = (1,1,1)$

Hence energy density at the center of the cube is equal to

$$1250 \epsilon_0 (1+1+1) = 3750\epsilon_0 \text{ J/m}^3$$

$$\begin{aligned} W_E &= \int \text{Energy density} \times dV = \int_0^2 \int_0^2 \int_0^2 3750\epsilon_0 dx dy dz \\ &= 3750\epsilon_0 [x]_0^2 [y]_0^2 [z]_0^2 = 0.2656 \mu\text{J} \end{aligned}$$

**06.(a)**

**Sol: (i)**  $B = \mu H$

$$= \mu_0 \mu_r H$$

$$\text{i. e. } B = \mu_0 \mu_r H + \mu_0 H - \mu_0 H$$

$$= \mu_0 H + \mu_0 H (\mu_r - 1)$$

$$= \mu_0 H + \mu_0 M$$

Where the magnetisation M is equal to H

$$(\mu_r - 1)$$

$$\text{i.e. } B = \mu_0(H + M) \text{ -----(1)}$$

The first term on the right side of Eq. (1) is due to external field. The second term is due to the magnetization.

Thus the magnetic induction (B) in a solid is

$$B = \mu_0(H + M)$$

$$\text{Hence } \mu_0 = \frac{B}{H + M}$$

The relative permeability

$$\begin{aligned} \mu_r &= \frac{\mu}{\mu_0} = \frac{B/H}{B/H + M} \\ &= \frac{H + M}{H} \\ &= 1 + \frac{M}{H} \end{aligned}$$

$$\mu_r = 1 + \chi$$



(ii) Given data:

Magnetization  $M = 2300 \text{ Am}^{-1}$

Flux density  $B = 0.00314 \text{ Wb m}^{-2}$

The magnetic flux density  $B = \mu_0 (M+H)$

The magnetizing force  $H = \frac{B}{\mu_0} - M = \frac{0.00314}{4\pi \times 10^{-7}} - 2300 = 198.7326 \text{ Am}^{-1}$

The susceptibility  $\chi = \frac{M}{H} = \mu_r - 1$

Where  $\mu_r$  is the relative permeability, i.e.,  $\mu_r = \frac{M}{H} + 1 = \frac{2300}{198.723} + 1 = 12.57334$

The magnetizing force  $H = 198.7326 \text{ Am}^{-1}$

The relative permeability  $\mu_r = 12.57334$

(iii)

**Types of imperfections or defects:**

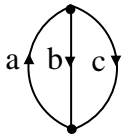
The various types of crystal imperfections are:

1. Point defects
  - a) Vacancies
  - b) Displacement of atoms
  - c) Impurities/ Inclusions
  - d) Frankel defect
  - e) Schottky defect
2. Line defects:
  - a) Edge Dislocation
  - b) Screw Dislocation
3. Surface or Grain boundaries defects
  - a) Grain boundaries
  - b) Tilt boundaries
  - c) Twin boundaries
  - d) Stacking faults
4. Volume defects:

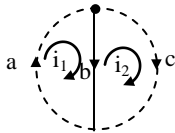


06.(b)

**Sol:** Graph of the given network is



Tree



Tie – set matrix

$$[B] = \begin{matrix} i_1 \\ i_2 \end{matrix} \begin{bmatrix} a & b & c \\ 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[Z_b] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3 \times 3} \quad [I_{\text{links}}] = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}_{2 \times 1}$$

$$[V_s] = \begin{bmatrix} +4 \\ -3i_x \\ -3 \end{bmatrix} \quad [I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3 \times 1}$$

**LHS:**

$$[B][Z_b][B]^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}_{2 \times 3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}_{3 \times 3} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}_{3 \times 2}$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}_{2 \times 2}$$

$$[B][Z_b][B]^T [I_{\text{line}}] = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 2i_1 - i_2 \\ -i_1 + 3i_2 \end{bmatrix}_{2 \times 1} \text{ ----- (1)}$$

**RHS:**

$$[B][V_s] - [B][Z_b][I_s] = \begin{bmatrix} 4 & -3i_x \\ 3i_x & -3 \end{bmatrix}_{2 \times 1} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$





$$= \begin{bmatrix} 4 & -3i_x \\ 3i_x & -3 \end{bmatrix}_{2 \times 1} \text{-----} (2)$$

LHS = RHS

$$\begin{bmatrix} 2i_1 - i_2 \\ -i_1 + 3i_2 \end{bmatrix} = \begin{bmatrix} 4 - 3i_x \\ 3i_x - 3 \end{bmatrix} \text{--- (3)}$$

Link equation:  $i_2 = i_x$  -----(5)

Solving (3) (4) and (5)

$$i_1 = 3A, i_2 = -1A, i_x = -1A$$

Now branch currents are given by

$$[B]^T [I_{link}] = [I_b]$$

$$\begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} j_a \\ j_b \\ j_c \end{bmatrix}$$

$$j_a = 3A, j_b = 4A, j_c = -1A$$

**06.(c)**

**Sol: (i)** As  $V_{AB} = V_{BC} = V_{CA} = 100$  V and KVL,

$V_{AB} + V_{BC} + V_{CA} = 0$ , they must be phase displaced by  $120^\circ$  with each other

$$\text{Let } V_{AB} = 100 \angle 0^\circ$$

$$V_{BC} = 100 \angle 120^\circ$$

$$V_{CA} = 100 \angle -120^\circ$$

$$\text{Line current, } I_L = 5 \angle \theta \text{ A}$$

$$|V_{BC}| = |I_L| \times |j\omega L|$$

$$100 = 5 \times (2\pi \times 50 \times L)$$

$$L = \frac{1}{5\pi} \text{ H}$$

$$L = 63.66 \text{ mH}$$

$$V_{BC} = I_L \times (j\omega L)$$

$$100 \angle 120^\circ = 5 \angle \theta \times 20 \angle 90^\circ$$

$$\theta = 30^\circ$$



$$I_L = 4.33 + j2.5$$

$$\text{Since, } V_{AB} = 100 \angle 0^\circ$$

$$R = \frac{100}{4.33} = 23.09 \Omega$$

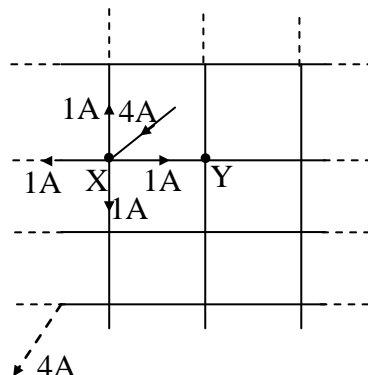
$$X_C = \frac{100}{2.5} = 40 \Omega$$

$$C = \frac{1}{2\pi \times 50 \times 40} = 79.57 \mu F$$

$$\text{Power consumed} = \frac{V_{AB}^2}{R} = \frac{100^2}{23.09} = 433.08 W$$

### (ii) Step –1

Inject a current of +4 at 'X' and collect at " $\infty$ "

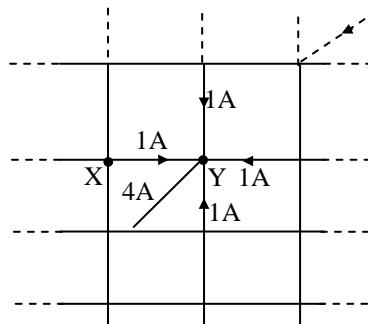


$$V'_{XY} = 1(1\Omega) = 1V,$$

$$I'_{XY} = 1A$$

### Step2:

Inject a current of 4A at ' $\infty$ ' and collect at 'Y'





$$V_{XY}'' = 1(1\Omega) = 1V,$$

$$I_{XY}'' = 1A$$

By superposition principle,

$$I_{XY} = I_{XY}' + I_{XY}'' = 1 + 1 = 2A$$

**07.(a)**

**Sol: (i)** Piezo means pressure. Piezoelectricity implies the production of electric field in a crystal by applying pressure.

- When certain crystals are subjected to stress (or) tension in a specific direction, they produce opposite kinds of charges at the opposite faces in a perpendicular direction.
- The charges produced on each face are always equal in magnitude and are proportional to the total force applied.
- Materials which gets polarized when subjected to mechanical deformation are called piezoelectric materials.
- Only those crystal classes which don't have center of inversion (that is non-Centro-symmetrical crystals) show this piezoelectric property. There are 20 point groups in nature which exhibit this phenomenon.
- In inverse piezoelectric effect, when an electric field applied to the crystal in a specific direction, the crystal gets strained in a perpendicular direction. This is called electrostriction.
- Piezoelectricity is related to symmetry of crystal and it is excluded from all structures having a centre of symmetry because in a Centro symmetric system, the opposite ends in every direction are identical and hence the application of tension or compression produces no polarization at all.

**Application of piezoelectric materials:**

- We can produce high frequency oscillations in quartz or barium titanate. An alternating potential difference of 50,000 volt at a frequency of 3 MHz is imposed between the foils at the two ends of a quartz crystal which begins to vibrate rapidly. If the natural frequency of the mechanical vibrations of them, on is equal to applied ac, then on account of resonance, the vibrations are well maintained, such an arrangement is used for stabilizing frequencies in transmitting radio stations and also as standards of frequency.



- Ultrasonic are used in Non-Destructive Testing (NDT) of materials in the industry and as a diagnostic aid in medicine,
- In the construction of SONAR (Sound Navigation and Ranging), in under water communication, in the construction of crystal microphones, phonograph reproducers and sound pickups, the piezoelectric materials are widely used.

The SAW devices are performing similar functions as bulk acoustic wave devices like delay lines, dispersive delay lines, frequency filters, oscillator in the fields like communications and radar, since both depend on the ability to store energy, information on the slowly propagating acoustic wave.

Given data is:

Dimensions = 5 mm × 5 mm × 1.25 mm,

Force  $F = 5\text{N}$ ,

Charge sensitivity = 150 pC/N,

Voltage sensitivity,  $g = 12 \times 10^{-3} \text{ Vm/N}$ ,

Permittivity,  $\epsilon = 12.5 \times 10^{-9} \text{ F/m}$

Charge,  $Q = ?$

Capacitance,  $C = ?$

$$\begin{aligned}\text{Pressure } P &= \frac{F}{\text{area}} \\ &= \frac{5}{25 \times 10^{-6}} \\ &= 0.2 \text{ MN / m}^2\end{aligned}$$

$$\begin{aligned}\text{Voltage, } V &= g \cdot t \cdot P \\ &= 12 \times 10^{-3} \times 1.25 \times 10^{-3} \times 0.2 \times 10^6 \\ &= 3 \text{ Volts}\end{aligned}$$

$$\begin{aligned}\text{Charge, } Q &= \text{Charge sensitivity} \times \text{Force} \\ &= 150 \times 10^{-12} \times 5 = 750 \text{ pC}\end{aligned}$$

$$\begin{aligned}\text{Capacitance } C &= Q / V \\ &= \frac{750 \text{ pC}}{3} \\ &= 250 \text{ pF}\end{aligned}$$



**07.(a)**

**Sol: (ii)** Given data:

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ Fm}^{-1}$$

$$\epsilon_r = 1.0000684$$

$$N = 2.7 \times 10^{25} \text{ atoms per m}^3$$

$$\alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

$$\alpha_e = \frac{8.854 \times 10^{-12} (1.0000684 - 1)}{2.7 \times 10^{25}}$$

$$\alpha_e = \frac{8.854 \times 10^{-12} \times 0.684 \times 10^{-4}}{2.7 \times 10^{25}}$$

$$= 2.243 \times 10^{-41} \text{ Fm}^2$$

**07.(b)**

**Sol: (i)** Given  $V_{BE(sat)} = 0.8 \text{ V}$

$$\beta_{(sat)} = 100$$

$$V_{CE(sat)} = 0.2 \text{ V}$$

for transistor remains in saturation

$$I_{Bmin} = \frac{5 - 0.8}{200 \text{ k}\Omega} = 21 \mu\text{A}$$

$$\text{and } I_{Cmin} = \mu I_B = 2.1 \text{ mA}$$

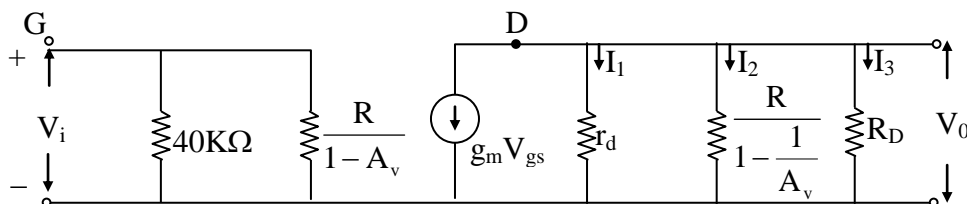
$$\text{Hence } I_{Csat} \leq 2.1 \text{ mA}$$

$$I_{Csat} = \frac{10 - 0.2}{R_C} \leq 2.1 \text{ mA}$$

$$R_C \geq 4.66 \text{ k}\Omega$$

$\therefore$  Minimum value of  $R_C = 4.66 \text{ k}\Omega$

(ii) Drawing the equivalent circuit of given FET,





$$(R_D = 10K\Omega, R = 50K\Omega)$$

From the circuit, using KCL at drain terminal, we have

$$\begin{aligned} -g_m V_{gs} &= I_1 + I_2 + I_3 \\ &= \frac{V_0}{r_d} + \frac{V_0}{\frac{R A_v}{A_v - 1}} + \frac{V_0}{R_0} \\ -g_m V_{gs} &= V_0 \left[ \frac{1}{r_d} + \frac{1}{R_D} + \frac{A_v - 1}{R A_v} \right] \\ -g_m &= \frac{V_0}{V_{gs}} \left[ \frac{1}{r_d} + \frac{1}{R_D} + \frac{A_v - 1}{R A_v} \right] \\ &= A_v \left[ \frac{1}{r_d} + \frac{1}{R_D} \right] + \frac{A_v - 1}{R} \quad \left( \because \frac{V_0}{V_{gs}} = A_v \right) \\ -g_m + \frac{1}{R} &= A_v \left[ \frac{1}{R} + \frac{1}{r_d} + \frac{1}{R_D} \right] \\ A_v \left[ \frac{1}{50K\Omega} + \frac{1}{5K\Omega} + \frac{1}{10K\Omega} \right] &= \frac{-30}{5K\Omega} + \frac{1}{50K\Omega} \quad (\because \mu = g_m r_d) \\ \therefore A_v &= -18.7 \end{aligned}$$

**07.(c)**

**Sol:** Garnet is a group of minerals that have been used since the Bronze Age as gemstones and abrasives.

Garnets species are found in many colors including red, orange, yellow, green, blue, purple, brown, black, pink and colorless.

Garnets are nano silicates having the general formula  $X_3Y_2(SiO_4)_3$ . The X site is usually occupied by divalent cations ( $Ca^{2+}, Mg^{2+}, Fe^{2+}, Mn^{2+}$ ) and the Y site by trivalent cations ( $Al^{3+}, Fe^{3+}, Cr^{3+}, Mn^{3+}, V^{3+}$ ) in an octahedral/tetrahedral framework with  $[SiO_4]^{4-}$  providing the tetrahedral.

They crystallize in the isometric system, having three axes that are all of equal length and perpendicular to each other.

Garnets do not show cleavage, so when they fracture under stress, sharp irregular pieces are formed.

Because the chemical composition of garnet varies, the atomic bonds in some species are stronger than in others. As a result, this mineral group shows a range of hardness on the Mohs Scale of about 6.5 to 7.5.



Garnets can be made in the lab by powdering  $\text{MgO}$ ,  $\text{Al}_2\text{O}_3$ , and  $\text{SiO}_2$ . The powder is then placed inside gold or platinum tubes which are welded shut. (Gold and platinum are used since they do not melt and corrode at high temperatures and do not suffer oxidation). The capsule is then placed in a hydraulic press and brought to a pressure of 80-50,000 atm. An electrical current is run through the sample to attain temperatures of 1200 to 1400°C.

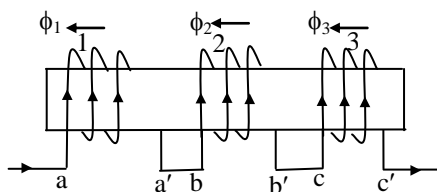
Yttrium iron garnet (YIG),  $\text{Y}_3\text{Fe}_2(\text{FeO}_4)_3$ , another important garnet, the five iron(III) ions occupy two octahedral and three tetrahedral sites, with the yttrium (III) ions coordinated by eight oxygen ions in an irregular cube. The iron ions in the two coordination sites exhibit different spins, resulting in magnetic behaviour. YIG is a ferrimagnetic material having a Curie temperature of 550 K. By substituting specific sites with rare earth elements, for example, Gadolinium, interesting magnetic properties can be obtained.

### Applications of Garnets:

- Gadolinium gallium garnet,  $\text{Gd}_3\text{Ga}_2(\text{GaO}_4)_3$ , which is synthesized for use in magnetic bubble memory.
- Yttrium aluminium garnet (YAG),  $\text{Y}_3\text{Al}_2(\text{AlO}_4)_3$ , is used for synthetic gemstone. When doped with neodymium ( $\text{Nd}^{3+}$ ), these Y Al-garnets are useful as the lasing medium in lasers.
- The Garnet group is a key mineral in interpreting the genesis of many igneous and metamorphic rocks.
- Garnets are also useful in defining metamorphic facies of rocks.
- Pure crystals of garnet are used as gemstones.
- Garnet sand is a good abrasive, and a common replacement for silica sand in sand blasting
- Mixed with very high pressure water, garnet is used to cut steel and other materials in water jets.
- Garnet sand is also used for water filtration media.

08.(a)

Sol: (i)



Fig(1)



Let the self inductance of each coil be  $L$  and  $L = 0.2H$  (given)

Mutual inductance between, Coil – 1 and Coil– 2 =  $M_{12}$

Coil – 2 and Coil– 3 =  $M_{23}$

Coil – 3 and Coil– 1 =  $M_{31}$

In fig(i)

Let the effective inductances be  $L_{\text{eff1}}$ ,  $L_{\text{eff2}}$  and  $L_{\text{eff3}}$  and their expressions are given as

$$L_{\text{eff1}} = L + M_{12} + M_{31}$$

$$L_{\text{eff2}} = L + M_{12} + M_{23}$$

$$L_{\text{eff3}} = L + M_{31} + M_{23}$$

First two in series

$$L_{\text{eff1}} + L_{\text{eff2}} = 2L + 2M_{12} + M_{31} + M_{23} = 0.6$$

$$2M_{12} + M_{31} + M_{23} = 0.2 \dots\dots(1)$$

When all coils are in series

$$\begin{aligned} L_{\text{eff1}} + L_{\text{eff2}} + L_{\text{eff3}} &= 3L + 2M_{12} + 2M_{23} + 2M_{31} = 1 \\ &= M_{12} + M_{23} + M_{31} = 0.2 \text{ -----}(2) \end{aligned}$$

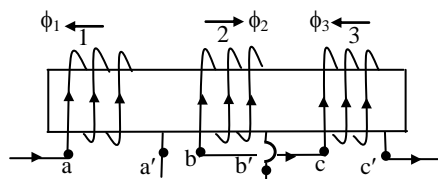
**When terminals of first two coils are interchanged**

**Initial connections**

$a' - b$  and  $b' - c$

**New connections**

$a' - b'$  and  $b - c$



Fig(1)

$$M_{12} \rightarrow -Ve, \quad M_{23} \rightarrow -Ve, \quad M_{31} \rightarrow +ve$$

So,

$$L_{\text{eff1}} = L - M_{12} + M_{31}$$

$$L_{\text{eff2}} = L - M_{12} - M_{23}$$

$$L_{\text{eff3}} = L - M_{23} + M_{31}$$

$$L_{\text{eff1}} + L_{\text{eff2}} + L_{\text{eff3}} = 3L - 2M_{12} - 2M_{23} + 2M_{31} = 0.5$$





$$2M_{12} + 2M_{23} - 2M_{31} = 0.1$$

$$M_{12} + M_{23} - M_{31} = 0.05 \text{ -----(3)}$$

Solving equations (1), (2) and (3)

$$(1) \& (2) \rightarrow M_{12} = 0$$

$$\text{then } M_{23} + M_{31} = 0.2$$

$$M_{23} - M_{31} = 0.05$$

$$M_{23} = 0.125,$$

$$M_{31} = 0.075$$

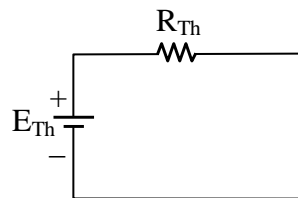
$$K_1 = \frac{M_{12}}{\sqrt{LL}} = 0,$$

$$K_2 = \frac{M_{23}}{\sqrt{LL}} = \frac{0.125}{0.2} = 0.625$$

$$K_3 = \frac{M_{31}}{\sqrt{LL}} = \frac{0.075}{0.2} = 0.375$$

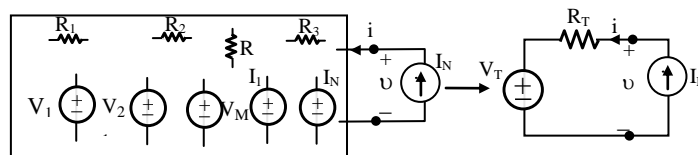
**(ii) Statement:**

Any two-terminal, linear bilateral dc networks can be replaced by an equivalent circuit consisting of a voltage source and a series resistor as shown in the fig (a)



**Proof:**

Let us consider a linear bilateral dc network



The above linear network is composed of

- Any number of resistors



- M' voltage sources :  $V_1$ -----  $V_M$
- 'N' current sources:  $I_1$  -----  $I_{N-1}$  internal current sources and one external current source  $I_N$  connected to the port. By applying superposition theorem,

$$v_m = A_m V_m \text{----- (1) where}$$

$A_m$  is a scale factor which is a dimensionless ratio of resistances and  $M = 1, 2, \dots, M$

$$v_n = I_n R_n \text{----- (2) where}$$

$R_n$  is some resistance expression and

$$n = 1, 2, \dots, N-1$$

$v_N = I_N R_N \dots\dots (3)$   $R_N$  is the equivalent resistance looking into the port when all internal sources are suppressed.

$$v = \sum_{m=1}^M v_m + \sum_{n=1}^{N-1} v_n + v_N$$

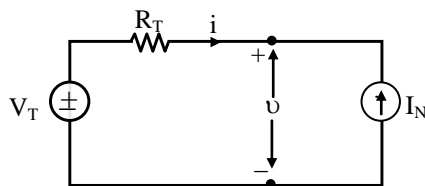
$$v = \sum_{m=1}^M A_m V_m + \sum_{n=1}^{N-1} I_n R_n + I_N R_N \text{----- (4)}$$

In the equation (4)

$$\text{Open circuit voltage, } v_{oc} = \sum_{m=1}^M A_m V_m + \sum_{n=1}^{N-1} I_n R_n$$

$$v = v_{oc} + I_N R_N \text{----- (5)}$$

The equation (5) can be represented as



Where  $V_T = v_{oc}$  ----- Thevenin's voltage

$R_T = R_N$ ----- Thevin's resistance

$I = I_N$  ----- current into or out of the port

$$v = V_T + i R_T$$



08.(b)

**Sol: (i)** For multi-stage amplifier consisting of  $n$  identical amplifying stages, the lower cut off  $f_{1(\text{multi})}$  is expressed as,

$$f_{1(\text{multi})} = \frac{f_1}{\sqrt{2^{1/n} - 1}} \text{ -----(1)}$$

Where  $f_1$  is lower cut-off of individual stages.

And the upper cutoff  $f_{2(\text{multi})}$  is expressed as,

$$f_{2(\text{multi})} = f_2 \sqrt{2^{1/n} - 1} \text{ -----(2)}$$

Where  $f_2$  is upper cut-off of individual stages.

Therefore, the lower cut-off of 2-stages amplifier (Eq(1)) is

$$f_{1(2\text{-stage})} = \frac{100}{\sqrt{2^{1/2} - 1}} = \frac{100}{0.643}$$

$$\text{Or } f_{1(2\text{-stage})} = 155.5 \text{ Hz}$$

Similarly, the upper cutoff of the 2-stage amplifier (Eq 2), is

$$f_{2(2\text{-stage})} = f_2 \sqrt{2^{1/2} - 1} = 20 \times 0.643 \text{ kHz}$$

$$\text{Or } f_{2(2\text{-stage})} = 12.8 \text{ kHz}$$

Thus the band width gets reduced in a multi-stage amplifier.

(ii) The value of gain of feedback network,  $B$  can be obtained using the basic feedback relation,

$$A_{\text{FB}} = \frac{A}{1 + A\beta}$$

Since  $A_{\text{FB}} = 100$  and  $A = 200$ , we have

$$100 = \frac{200}{1 + 200 \times \beta} \approx \frac{1}{\beta}$$

$$\text{Or } \beta = \frac{1}{100}$$

Further, the value of  $\beta$  is given by the ratio,

$$\beta = \frac{R_E}{R_C}$$

$$\text{or, } R_E = \beta R_C = \frac{6 \times 10^3}{100} = 60 \Omega$$

$$R_E = 60 \Omega$$



(iii)

BJT	MOSFET
1. Low input resistance	1. High input resistance
2. High Transconductance	2. Low Transconductance
3. Bipolar device	3. Unipolar
4. Used in amplifiers & current mirrors	4. Used as a Switch
5. Takes large area on a chip	5. Relatively less area

**08.(c)**

**Sol: (i)** Given:  $R_H = 3.66 \times 10^{-4} \text{ m}^3/\text{coulomb}$ ;  
 $\rho = 8.93 \times 10^{-3} \Omega\text{-m}$ ;  
 $B = 0.8 \text{ T}$ ;

Find Hall angle?

$$\text{Hall angle } \tan \theta_H = \frac{E_H}{E_x} = \mu B$$

$$\tan \theta_H = \mu B = \frac{R_H}{\rho} \times B \quad \left( \because \mu = \frac{R_H}{\rho} \right)$$

$$\tan \theta_H = \frac{3.66 \times 10^{-4}}{8.93 \times 10^{-3}} \times 0.8$$

$$= 0.0327$$

$$\theta_H = \tan^{-1}(0.0327)$$

$$\theta_H = 1.8729^\circ$$

**(ii) Paramagnetism:**

- Materials, whose atoms possess permanent dipole moments, are paramagnetic in nature. In the absence of external field, these atomic dipoles are randomly oriented such that net magnetization (M) is zero.
- When an external magnetic field is applied, these atomic dipoles tend to align themselves in the direction of the field, producing net magnetization 'M' in the direction of H, and positive susceptibility.



- Paramagnetic atoms and ions include particles having one electron over and above a completed shell (alkali group), atoms of transition elements, ions of the rare earth elements with incomplete shell etc.
- The magnitude of alignment of atomic dipoles with external field direction is limited by temperature.
- As the temperature increases, thermal agitation of atoms increases and magnetization decreases, there by decreasing susceptibility.
- Thus paramagnetic susceptibility is inversely proportional to temperature

$$\chi \propto \frac{1}{T} \text{ (Curie's law)}$$

$$\text{(or)} \quad \chi = \frac{C}{T} \text{ (or)} \quad \chi = \frac{\mu^2}{3kT}$$

C → Curie's constant;

$\mu$  → Atomic dipole moment,

T → Absolute temperature

k → Boltzman's constant

Thus paramagnetic susceptibility  $\chi$  is +ve and small and depends on temperature.

**Examples:** Liquid Oxygen, Oxygen, Air, Aluminum, Ebonite, Platinum, Tungsten etc.

### **Ferromagnetism & Ferromagnetic materials:**

- These are generally crystalline solids, which are magnetized independent of any external field. i.e., M has finite value even when the external H is zero. (Spontaneous magnetization)
- When an external H is applied, large M is produced in the direction of H. Thus ferromagnetic susceptibility is positive and large. This can be understood in terms of uncompensated electron spins.
- According to quantum theory, strong exchange forces between neighboring spinning electrons tend to make spins parallel (ferromagnetic) or anti-parallel (anti-ferromagnetic) depending upon the number of electrons and the distance between individual electrons.
- Weiss domain theory is most successful in explaining ferromagnetism.

**Examples:** Iron, Cobalt, Nickel, Gaddlinium.