



**ACE**  
Engineering Academy  
(Leading institute for ESE/GATE/PSUs)

# **ESE – 2019 MAINS OFFLINE TEST SERIES**



## **ELECTRICAL ENGINEERING**

# **TEST – 5 SOLUTIONS**



01. (a)

**Sol:** Let  $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{bmatrix} = 0$$

$(R_1 + R_3)$

$$\begin{bmatrix} -3-\lambda & 0 & -3-\lambda \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = 0$$

$$-(3+\lambda) \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = 0$$

$$(3+\lambda) [(\lambda^2 - \lambda - 12) + (-4 + 1 - \lambda)] = 0$$

$$(3+\lambda) (\lambda^2 - 2\lambda - 15) = 0$$

$$(3+\lambda) (\lambda - 5) (\lambda + 3) = 0$$

$\therefore \lambda = 5, -3, -3$  are the characteristic roots of 'A'.

At  $\lambda = 5$ :  $(A - \lambda I)X = 0$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \dots\dots\dots (1)$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \sim \begin{bmatrix} -1 & -2 & -5 \\ 2 & -4 & -6 \\ -7 & 2 & -3 \end{bmatrix}$$

$R_3 + (3R_2 - R_1); (R_2 + 2R_1)$



$$\sim \begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & -8 & -16 \end{bmatrix}$$

$$(R_3 - R_2)$$

$$\sim \begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$  (1) reduces to

$$\begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-8y - 16z = 0$$

$$\Rightarrow y = -2z$$

$$\text{Let } z = k_1$$

$$\Rightarrow y = -2k_1$$

$$-x - 2y - 5z = 0$$

$$x = -2y - 5z$$

$$= 4k_1 - 5k_1$$

$$= -k_1$$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k_1 \\ -2k_1 \\ k_1 \end{bmatrix} \text{ is the characteristic vector.}$$

$$\text{At } \lambda = -3: (A - \lambda I)X = O$$

$$\begin{pmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(R_3 + R_1); (R_2 - 2R_1)$$



$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore (x + 2y - 3z) = 0$$

$$\text{Let } y = k_2 \text{ \& } z = k_3$$

$$\Rightarrow x = (3k_3 - 2k_2)$$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3k_3 - 2k_2 \\ k_2 \\ k_3 \end{bmatrix} \text{ is the characteristic vector.}$$

**01. (b)**

**Sol:**

0	16	31
Source Port		Destination Port
UDP Length		UDP Checksum
Data		

**Source port Number (16 bits):**

This field identifies the sending port when meaningful and should be assumed to be the port to reply to if needed. If not used, then it should be zero.

**Destination port Number (16 bits)**

This field identifies the destination port and is required.

**UDP Length (16 bits):**

A 16-bit field that specifies the length in bytes of the entire datagram: header and data. The minimum length is 8 bytes since that's the length of the header. The field size sets a theoretical limit of 65,535 bytes (8 byte header + 65527 bytes of data) for a UDP datagram. The length includes the UDP header, so the minimum size for a UDP datagram is 8 (8 byte header with no data). The practical limit for the data length which is imposed by the underlying IPv4 protocol is 65,507 bytes.



### UDP Checksum

The 16-bit checksum field is used for error-checking of the header and data. The algorithm for computing the checksum is different for transport over IPv4 and IPv6.

01. (c)

**Sol:** AE:  $(m^2 - 1) = 0$

$$m = \pm 1$$

CF:  $y_c = C_1 e^x + C_2 e^{-x}$

PI: Let  $y_p = (y_{p1} + y_{p2})$

$$y_{p1} = \frac{x \sin x}{(D^2 - 1)}$$

$$= x \left( \frac{\sin x}{D^2 - 1} \right) + \left( \frac{2D}{(D^2 - 1)^2} \right) \sin x$$

$$= \frac{x \sin x}{(-1-1)} + \frac{2 \cos x}{(-1-1)^2}$$

$$= -\frac{x}{2} \sin x + \frac{\cos x}{2}$$

$$= \frac{1}{2} (\cos x - x \sin x)$$

$$y_{p2} = \frac{(1 + x^2) e^x}{(D^2 - 1)}$$

$$= e^x \left[ \frac{(1 + x^2)}{(D+1)^2 - 1} \right]$$

$$= e^x \frac{(1 + x^2)}{(D^2 + 2D)}$$

$$= e^x \frac{1}{2D} \left( 1 + \frac{D}{2} \right)$$



$$\begin{aligned}
 &= e^x \frac{1}{2D} \left( 1 + \frac{D}{2} \right)^{-1} (1+x^2) \\
 &= e^x \frac{1}{2D} \left( 1 - \frac{D}{2} + \frac{D^2}{4} - \frac{D^3}{8} \right) (1+x^2) \\
 &= \frac{e^x}{2} \left( \frac{1}{D} - \frac{1}{2} + \frac{D}{4} - \frac{D^2}{8} \right) (1+x^2) \\
 &= \frac{e^x}{2} \left( x + \frac{x^3}{3} - \frac{1}{2} - \frac{x^2}{2} + \frac{1}{4}(2x) - \frac{1}{8}(2) \right) \\
 &= \frac{e^x}{2} \left( \frac{x^3}{3} - \frac{x^2}{2} + \frac{3x}{2} - \frac{3}{4} \right) \\
 &= \frac{e^x}{24} (4x^3 - 6x^2 + 18x - 9)
 \end{aligned}$$

∴ GS/CS:

$$y = (y_c + y_p)$$

$$y = c_1 e^x + c_2 e^{-x} + \frac{1}{2} (\cos x - x \sin x) + \frac{e^x}{24} (4x^3 - 6x^2 + 18x - 9) \text{ is the required solution of the given equation.}$$

**01(d)**

**Sol: Local variable:** A variable which is declared within the function or is an argument passed to a function are known as local variables.

These variables can be used within the functions only, outside the function there is no any existence of such variables.

**Global variables:** A global variable is variable which is accessible in multiple scope or which can be used in entire program.

**Difference between Local & Global variable:**

Local variable	Global variable
1. Declared within functions	1. Declared outside functions
2. Having scope within the function	2. Having scope throughout the program
3. Having lifetime during the function execution	3. Having lifetime till the program execution



**Example:**

```
#include<stdio.h>

int x = 5; //global variable

void fun ( );

void fun ( )
{
    int x = 10; // local variable x of function fun ( )
    printf ("%d \n", x);
}

void main ( )
{
    printf("%d \n", x);    //printing variable x
    fun ( );
    printf("%d \n", x);
}
```

**Output: 5**

10

5

**Explanation:**

Since there is not any local variable x in main ( ) function hence from main ( ) function global variable x (with value 5) is accessed. But in fun ( ) function a local variable x is present hence during the function call x = 10 is printed.

**01(e)**

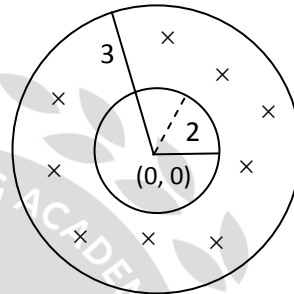
**Sol:** 
$$\frac{z^2 - 1}{(z^2 + 5z + 6)} = 1 + \frac{(-5z - 7)}{(z + 2)(z + 3)}$$
$$= 1 - \frac{(5z + 7)}{(z + 2)(z + 3)}$$



$$\begin{aligned}
 &= 1 - \left[ \frac{A}{(z+2)} + \frac{B}{(z+3)} \right] \\
 &= 1 - \left[ \frac{-3}{(z+2)} + \frac{8}{(z+3)} \right] \\
 &= 1 + \frac{3}{(z+2)} - \frac{8}{(z+3)} \quad \dots (1)
 \end{aligned}$$

$2 < |z| < 3$  represents a ring shaped region as shown below.

$$\begin{aligned}
 \frac{1}{(z+2)} &= \frac{1}{z \left( 1 + \frac{2}{z} \right)} \\
 &= \frac{1}{z} \left( 1 + \frac{2}{z} \right)^{-1} \quad \dots (2) \\
 \frac{1}{(z+3)} &= \frac{1}{3 \left( 1 + \frac{z}{3} \right)} \\
 &= \frac{1}{3} \left( 1 + \frac{z}{3} \right)^{-1} \quad \dots (3)
 \end{aligned}$$



(2) & (3) in (1)

$$\begin{aligned}
 \frac{z^2 - 1}{(z^2 + 5z + 6)} &= 1 + \frac{3}{z} \left( 1 + \frac{2}{z} \right)^{-1} - \frac{8}{3} \left( 1 + \frac{z}{3} \right)^{-1} \\
 &= 1 + \frac{3}{z} \left( 1 - \frac{2}{z} + \frac{4}{z^2} - \dots \right) - \frac{8}{3} \left( 1 - \frac{z}{3} + \frac{z^2}{9} - \dots \right)
 \end{aligned}$$

is the required Laurent's series expansion of the given function.

**02(a)**

**Sol:** Let  $A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$

$$|A - \lambda I| = 0$$





$$\begin{bmatrix} 7-\lambda & 2 & -2 \\ -6 & -1-\lambda & 2 \\ 6 & 2 & -1-\lambda \end{bmatrix} = 0$$

$$(R_3 + R_2)$$

$$\begin{bmatrix} 7-\lambda & 2 & -2 \\ -6 & -1-\lambda & 2 \\ 0 & 1-\lambda & -1-\lambda \end{bmatrix} = 0$$

$$(1-\lambda) \begin{bmatrix} 7-\lambda & 2 & -2 \\ -6 & -1-\lambda & 2 \\ 0 & 1 & 1 \end{bmatrix} = 0$$

$$(C_3 - C_2)$$

$$(1-\lambda) \begin{bmatrix} 7-\lambda & 2 & -4 \\ -6 & -1-\lambda & 3+\lambda \\ 0 & 1 & 0 \end{bmatrix} = 0$$

$$(1-\lambda)(-\lambda^2 + 4\lambda - 3) = 0$$

$$(1-\lambda)(\lambda^2 - 4\lambda + 3) = 0$$

$$(1-\lambda)(\lambda-1)(\lambda-3) = 0$$

$$(1-\lambda)^2(\lambda-3) = 0 \dots \dots \dots (1) \text{ is the characteristic equation}$$

$$(A - I) = \begin{bmatrix} 6 & 2 & -2 \\ -6 & -2 & 2 \\ 6 & 2 & -2 \end{bmatrix}$$

$$(A - I)^2 = \begin{bmatrix} 6 & 2 & -2 \\ -6 & -2 & 2 \\ 6 & 2 & -2 \end{bmatrix} \begin{bmatrix} 6 & 2 & -2 \\ -6 & -2 & 2 \\ 6 & 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 4 & -4 \\ -12 & -4 & 4 \\ 12 & 4 & -4 \end{bmatrix}$$

$$(A-I)^2 (A-3I)$$



$$= \begin{bmatrix} 12 & 4 & -4 \\ -12 & -4 & 4 \\ 12 & 4 & -4 \end{bmatrix} \begin{bmatrix} 4 & 2 & -2 \\ -6 & -4 & 2 \\ 6 & 2 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore (A-I)^2 (A-3I) = 0$$

$\therefore$  Characteristic equation (1) is satisfied by 'A'

Hence Cayley-Hamilton theorem is verified.

$$\text{From (1), } (\lambda - 1)^2 (\lambda - 3) = 0$$

$$(\lambda^2 - 2\lambda + 1)(\lambda - 3) = 0$$

$$(\lambda^3 - 5\lambda^2 + 7\lambda - 3) = 0$$

$$\Rightarrow (A^3 - 5A^2 + 7A - 3I) = 0$$

$$3I = (A^3 - 5A^2 + 7A)$$

$$A^{-1} = \frac{1}{3}(A^2 - 5A + 7I) \quad \dots\dots\dots (2)$$

$$A^2 = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix} \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix}$$

$$\therefore (A^2 - 5A + 7I) = \begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix} - \begin{bmatrix} 35 & 10 & -10 \\ -30 & -5 & 10 \\ 30 & 10 & 5 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -2 & 2 \\ 6 & 5 & -2 \\ -6 & -2 & 5 \end{bmatrix}$$



$$\therefore A^{-1} = \frac{1}{3} \begin{bmatrix} -3 & -2 & 2 \\ 6 & 5 & -2 \\ -6 & -2 & 5 \end{bmatrix}$$

**02(b)**

**Sol:** Let  $f(z) = \frac{e^z}{\cos \pi z}$

$$\cos \pi z = 0$$

$$\Rightarrow z = \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \dots$$

But  $z = \pm \frac{1}{2}$  are the singular points lie inside  $|z| = 1$

$$\begin{aligned} [\text{Residue of } f(z)]_{\text{At } z = \frac{1}{2}} &= \lim_{z \rightarrow \frac{1}{2}} \frac{\left(z - \frac{1}{2}\right) e^z}{\cos \pi z} \left(\frac{0}{0}\right) = \lim_{z \rightarrow \frac{1}{2}} \frac{\left(z - \frac{1}{2}\right) e^z + e^z}{-\pi \sin \pi z} \\ &= \frac{e^{\frac{1}{2}}}{-\pi} = -\frac{1}{\pi} e^{1/2} \end{aligned}$$

$$\begin{aligned} \text{Similarly, } [\text{Residue of } f(z)]_{\text{At } z = -\frac{1}{2}} &= \lim_{z \rightarrow -\frac{1}{2}} \frac{\left(z + \frac{1}{2}\right) e^z}{\cos \pi z} \left(\frac{0}{0}\right) \\ &= \lim_{z \rightarrow -\frac{1}{2}} \frac{\left(z + \frac{1}{2}\right) e^z + e^z}{-\pi \sin \pi z} = \frac{e^{-1/2}}{\pi} \end{aligned}$$

$$\begin{aligned} \therefore \text{ From Cauchy's residue theorem, } \oint_c \frac{e^z}{\cos \pi z} dz &= 2\pi i \left( -\frac{1}{\pi} e^{\frac{1}{2}} + \frac{1}{\pi} e^{-\frac{1}{2}} \right) \\ &= 2i \left( e^{-\frac{1}{2}} - e^{\frac{1}{2}} \right) \\ &= -4i \left( \frac{e^{\frac{1}{2}} - e^{-\frac{1}{2}}}{2} \right) = -4i \sinh \left( \frac{1}{2} \right) \end{aligned}$$



**02(c) (i)**

**Sol:** #include < stdio.h>

```
void main ( )  
{  
    int A [100], i, max;  
    max = A[0];  
    for (i = 1; i < 100; i ++)  
    {  
        if (A[i] > max)  
            max = A[i];  
    }  
    printf("%d", max);  
}
```

**02(c)(ii)**

**Sol:** MM size =  $2^{31}$  Bytes

physical Address bits  $\Rightarrow$  31 bits

cache line size =  $2^{12}$  Bytes

offset bits  $\Rightarrow$  12 bits

cache size =  $2^{20}$  Bytes

No. of cache lines =  $\frac{\text{cache size}}{\text{cache line size}} = 2^8$  lines

Index bits  $\Rightarrow$  8 bits

Tag entry bits = physical Address bits – offset bits – index bits

= (31–12–8)

= 11 bits.



03(a)

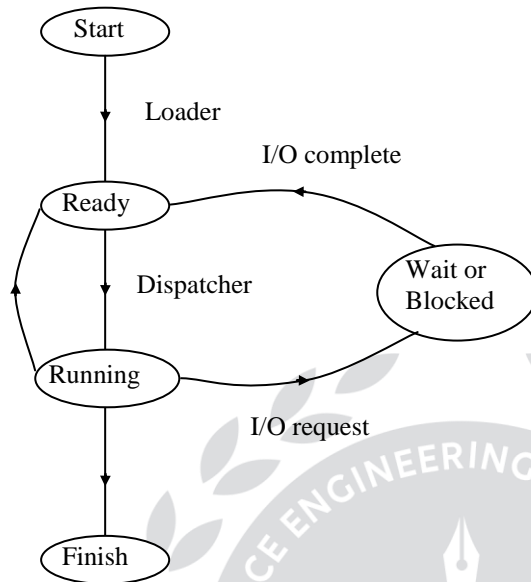
Sol:

Category	Direct mapped cache	Set associative cache	Fully associative cache
1. Associativity	<ul style="list-style-type: none"> <li>A main memory block associated with particular cache line</li> <li>Lowest associativity</li> <li>Associativity is one</li> </ul>	<ul style="list-style-type: none"> <li>A main memory block associated with particular set of K lines</li> <li>K way set associative cache</li> <li>Associativity is K.</li> </ul>	<ul style="list-style-type: none"> <li>A main memory block can go in any of the cache line</li> <li>Associativity is full</li> <li>Associativity is high</li> </ul>
2. Mapping function	Cache line no. $\leftarrow$ (MM block no.) mod (No. of cache lines)	Cache set no. $\leftarrow$ (MM block no.) mod (number of cache sets)	No any mapping function
3. No. of cache miss	More no. of conflict miss due to more no. of collisions	less no. of conflict w.r.t direct mapped cache	less no. of conflict miss w.r.t set associative due to less no. of collisions
4. Cost	<ul style="list-style-type: none"> <li>One comparator</li> <li>less costly due to less hardware requirement</li> </ul>	<ul style="list-style-type: none"> <li>K no. of comparators</li> <li>Relatively more hardware</li> </ul>	<ul style="list-style-type: none"> <li>No. of comparators equal to no. of cache lines</li> <li>More costly due to more hardware requirement</li> </ul>



03.(b)

**Sol:** (i) Process state transition (life cycle) for multitasking operating systems:



**Start state:**

- When a process is requested to execute, then it is in start state
- Process is in secondary memory (like hard disk)

**Ready state:**

- When requested process is loaded by loader into main memory, then it is in ready state
- Ready for execution, waiting for its turn
- If more than one processes are waiting for execution in ready state then process (short term) scheduler does scheduling among them.

**Running state:**

- Selected process by process scheduler initialized by dispatcher on CPU for execution
- Process is executing on CPU it is in running state
- CPU fetches instructions of process from main memory one by one and executes it.

**Wait or Blocked state:**

- When a running process requested for I/O activity (Input/output from secondary devices) then it moves to Blocked state
- Once the process completes its I/O activity it is shifted to ready state



**Finish state:**

- When a running process completed its execution it is in finish state
- Process execute its last exit instruction and request O.S. to remove it from main memory

**03.(b)(ii)**

**Sol:** Special representation for +0 is

$$\begin{array}{ccc} 0 & 00000000 & 00.....00 \\ \text{sign} & \text{Exponent} & \text{Mantissa} \end{array}$$

$$[00000000]_H$$

special representation for -0 is

$$\begin{array}{ccc} 1 & 00000000 & 00...00 \\ \text{sign} & \text{Exponent} & \text{Mantissa} \end{array}$$

$$[80000000]_H$$

**03. (c)**

**Sol:** Let  $\frac{dy}{dx} = xy = f(x, y)$

$$y(0) = 1 \Rightarrow x_0 = 0 \text{ \& } y_0 = 1$$

From modified Euler's method,

We know that,

$$y(0+0.5) = y(x_0+h) = y(x_1) = y_1$$

$$y_1 = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_0 + hf(x_0, y_0))]$$

$$= 1 + \frac{0.5}{2} [f(0,1) + f(0.5, 1 + 0.5f(0,1))]$$

$$= 1 + \frac{0.5}{2} [0 + f(0.5,1)]$$

$$= 1 + \frac{0.5}{2} [0.5]$$

$$= 1 + \frac{0.25}{2} = 1 + 0.125 = 1.125$$



$$y(0 + 2(0.5)) = y(x_0 + 2h) = y(x_2) = y_2$$

$$y_2 = y_1 + \frac{h}{2} [f(x_1, y_1) + f(x_2, y_1 + hf(x_1, y_1))]$$

$$= 1.125 + \frac{0.5}{2} [f(0.5, 1.125) + f(1, 1.125 + 0.5f(0.5, 1.125))]$$

$$= 1.125 + \frac{0.5}{2} [0.5625 + f(1, 1.4063)]$$

$$= 1.125 + \frac{0.5}{2} [0.5625 + 1.4063]$$

$$= 1.7578$$

**04. (a)**

**Sol:**  $f(x) = x \sin x$  is an even function we know that  $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx \dots (1)$  is the Fourier series

of an even function where,  $a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx = \frac{2}{\pi} \int_0^{\pi} x \sin x dx$

$$a_0 = \frac{2}{\pi} [x(-\cos x) - (-\sin x)]_0^{\pi}$$

$$= \frac{2}{\pi} [\pi - 0] = 2 \dots (2)$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x \sin x \cos nx dx$$

$$= \frac{1}{\pi} \int_0^{\pi} x (\sin(n+1)x - \sin(n-1)x) dx$$

$$= \frac{1}{\pi} \left[ \left[ x \left( \frac{-\cos(n+1)x}{(n+1)} \right) - \left( \frac{-\sin(n+1)x}{(n+1)^2} \right) \right] - \right.$$

$$\left. \left[ x \left( \frac{-\cos(n-1)x}{(n-1)} \right) - \left( \frac{-\sin(n-1)x}{(n-1)^2} \right) \right] \right]_0^{\pi}$$





$$= \frac{1}{\pi} \left[ \frac{-\pi \cos(n+1)\pi}{(n+1)} + \frac{\pi \cos(n-1)\pi}{(n-1)} \right]$$

$$= \frac{-(-1)^{n+1}}{(n+1)} + \frac{(-1)^{n-1}}{(n-1)}$$

$$= (-1)^n \left[ \frac{1}{(n+1)} + \frac{1}{(n-1)} \right]$$

$$= \frac{-2(-1)^n}{(n^2-1)} = \frac{2(-1)^{n+1}}{(n^2-1)} \quad \dots (3)$$

(for n=2, 3, 4..)

For n = 1  $\rightarrow a_1 = \frac{2}{\pi} \int_0^{\pi} x \sin x \cos x \, dx$

$$= \frac{1}{\pi} \int_0^{\pi} x \sin 2x \, dx$$

$$a_1 = \frac{1}{\pi} \left[ x \left( \frac{-\cos 2x}{2} \right) - \left( \frac{-\sin 2x}{4} \right) \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left[ \frac{-\pi}{2} - 0 \right] = \frac{-1}{2} \quad \dots (4)$$

$\therefore$  From (1)

$$f(x) = \frac{1}{2} a_0 + a_1 \cos x + \sum_{n=2}^{\infty} a_n \cos nx$$

$$x \sin x =$$

$$1 - \frac{1}{2} \cos x + (a_2 \cos 2x + a_3 \cos 3x + a_4 \cos 4x + \dots)$$

$$x \sin x =$$

$$1 - \frac{1}{2} \cos x + \left[ \frac{-2 \cos 2x}{1.3} + \frac{2 \cos 3x}{2.4} - \frac{2 \cos 4x}{3.5} + \dots \right]$$

Put  $x = \frac{\pi}{2}$  in the above



$$\frac{\pi}{2} = 1 + \left[ \frac{2}{1.3} - \frac{2}{3.5} - \dots \right]$$

$$\left[ \frac{\pi}{2} - 1 \right] = 2 \left[ \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots \right]$$

$$\therefore \left( \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots \right) = \frac{(\pi - 2)}{4}$$

**04(b)**

**Sol:** (i) The given equations can be represented in the form as  $AX = B$  as follows

$$\begin{bmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$$

The Augmented matrix

$$(AB) = \begin{bmatrix} 3 & -1 & 4 & 3 \\ 1 & 2 & -3 & -2 \\ 6 & 5 & \lambda & -3 \end{bmatrix}$$

$(R_3 - 2R_1); (3R_2 - R_1)$

$$\sim \begin{bmatrix} 3 & -1 & 4 & 3 \\ 0 & 7 & -13 & -9 \\ 0 & 7 & \lambda - 8 & -9 \end{bmatrix}$$

$(R_3 - R_2)$

$$\sim \begin{bmatrix} 3 & -1 & 4 & 3 \\ 0 & 7 & -13 & -9 \\ 0 & 7 & \lambda + 5 & 0 \end{bmatrix}$$

If  $\lambda \neq 5$ , then  $\rho(A) = 3$ ,  $\rho(AB) = 3$

and number of variables = 3

Hence, the system has unique solution as shown below

$$z = 0$$



$$7y - 13z = -9 \Rightarrow y = \frac{-9}{7}$$

$$3x - y + 4z = 3 \Rightarrow 3x = 3 + y$$

$$= 3 - \frac{9}{7} = \frac{12}{7}$$

$$x = \frac{4}{7}$$

$$\therefore x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4/7 \\ -9/7 \\ 0 \end{bmatrix}$$

If  $\lambda = -5$ , then  $\rho(A) = 2$ ;  $\rho(AB) = 2$

The system has infinite number of solutions and given by

$$7y - 13z = -9$$

$$\text{Let } z = k \Rightarrow 7y = 13k - 9$$

$$y = \left( \frac{13k - 9}{7} \right)$$

$$3x - y + 4z = 3 \Rightarrow 3x = 3 + y - 4z$$

$$= 3 + \left( \frac{13k - 9}{7} \right) - 4k$$

$$= \left( \frac{12 - 15k}{7} \right)$$

$$\Rightarrow x = \left( \frac{4 - 5k}{7} \right)$$

$$\therefore x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{4 - 5k}{7} \\ \frac{13k - 9}{7} \\ k \end{bmatrix}$$

By giving various values of 'k', the system has infinite number of solutions.



**04(b)(ii)**

**Sol:** AE:  $(m^2 - 4m + 4) = 0$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

CF:  $y_c = (c_1 + c_2x)e^{2x}$

PI:  $y_p = \frac{8x^2e^{2x} \sin 2x}{(D+2-2)^2}$

$$= 8e^{2x} \left( \frac{x^2 \sin 2x}{D^2} \right)$$

$$= 8e^{2x} \frac{1}{D} \left[ \int x^2 \sin 2x \, dx \right]$$

$$= 8e^{2x} \frac{1}{D} \left[ x^2 \left( \frac{-\cos 2x}{2} \right) - 2x \left( \frac{-\sin 2x}{4} \right) + 2 \left( \frac{\cos 2x}{8} \right) \right]$$

$$= 2e^{2x} \frac{1}{D} (-2x^2 \cos 2x + 2x \sin 2x + \cos 2x)$$

$$= 2e^{2x} \int (\cos 2x + 2x \sin 2x - 2x^2 \cos 2x) \, dx$$

$$= 2e^{2x} \left[ \left( \frac{\sin 2x}{2} \right) + 2 \left( x \left( \frac{-\cos 2x}{2} \right) - \left( \frac{-\sin 2x}{4} \right) \right) - 2 \left( x^2 \left( \frac{\sin 2x}{2} \right) - 2x \left( \frac{-\cos 2x}{4} \right) + 2 \left( \frac{-\sin 2x}{8} \right) \right) \right]$$

$$y_p = 2e^{2x} \left( \frac{\sin 2x}{2} - x \cos 2x + \frac{1}{2} \sin 2x - x^2 \sin 2x \right.$$

$$\left. - x \cos 2x + \frac{\sin 2x}{2} \right)$$

$$= 2e^{2x} \left( \frac{3}{2} \sin 2x - 2x \cos 2x - x^2 \sin 2x \right)$$

$$= e^{2x} [(3 - 2x^2) \sin 2x - 4x \cos 2x]$$

**GS/CS:**

$$y = (y_c + y_p)$$

$$y = (c_1 + c_2x)e^{2x} + e^{2x}[(3 - 2x^2) \sin 2x - 4x \cos 2x] \text{ is the required solution.}$$



04(c)

**Sol:** (i)  $f(x) = (x + \sqrt{x} - 3) = 0$

$$\Rightarrow f'(x) = 1 + \frac{1}{2\sqrt{x}}$$

Given  $x_0 = 2$

We know that  $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$= x_0 - \frac{(x_0 + \sqrt{x_0} - 3)}{\left(1 + \frac{1}{2\sqrt{x_0}}\right)}$$

$$= x_0 - \frac{2\sqrt{x_0}(x_0 + \sqrt{x_0} - 3)}{(2\sqrt{x_0} + 1)}$$

$$x_1 = \frac{6\sqrt{x_0} - x_0}{2\sqrt{x_0} + 1}$$

$$= \frac{6\sqrt{2} - 2}{2\sqrt{2} + 1} = \frac{6.484}{3.828} = 1.6938$$

Similarly,  $x_2 = \frac{6\sqrt{x_1} - x_1}{2\sqrt{x_1} + 1}$

$$= \frac{6\sqrt{1.694} - 1.694}{2\sqrt{1.694} + 1}$$

$$= \frac{6.1152}{3.6031} = 1.6972$$

$$x_3 = \frac{6\sqrt{x_2} - x_2}{2\sqrt{x_2} + 1}$$

$$= \frac{6\sqrt{1.697} - 1.697}{2\sqrt{1.697} + 1}$$

$$= \frac{6.1191}{3.6054} = 1.6972$$

$\therefore$  The required root of the given equation is 1.6972.



04(c)

**Sol:** (ii) The given equation is in Euler-Cauchy's homogeneous equation form

$$\text{Let } x = e^z \Rightarrow z = \log x \text{ \& } D = \frac{d}{dz}$$

Then, the given equation reduces to

$$D(D-1)(D-2)y + 3D(D-1)y + 8y = 65 \cos z$$

$$(D^3 - 3D^2 + 2D + 3D^2 - 3D + D + 8)y = 65 \cos z$$

$$(D^3 + 8)y = 65 \cos z$$

$$\text{AE: } (m^3 + 8) = 0$$

$$(m + 2)(m^2 - 2m + 4) = 0$$

$$\therefore m = -2, m = \frac{2 \pm \sqrt{-12}}{2}$$

$$= (1 \pm \sqrt{3}i)$$

$$\text{CE: } y_c = c_1 e^{-2z} + e^z (c_2 \cos \sqrt{3}z + c_3 \sin \sqrt{3}z)$$

$$= \frac{c_1}{x^2} + x(c_2 \cos \sqrt{3}(\log x) + c_3 \sin \sqrt{3}(\log x)) \text{ PI: } y_p = \frac{65 \cos z}{(D^3 + 8)}$$

$$= 65 \left[ \frac{\cos z}{(D^3 + 8)} \right]$$

$$= 65 \frac{\cos z}{(-1)D + 8}$$

$$= -65 \frac{\cos z}{(D - 8)}$$

$$= -65 \frac{(D + 8)}{(D^2 - 64)} \cos z$$

$$= -65 \frac{(D + 8) \cos z}{(-1 - 64)}$$

$$= -\sin z + 8 \cos z$$

$$= (8 \cos z - \sin z)$$

$$= [8 \cos (\log x) - \sin (\log x)]$$



**CS/GS:**

$$y = (y_c + y_p)$$

$= \frac{c_1}{x^2} + x(c_2 \cos \sqrt{3}(\log x) + c_3 \sin \sqrt{3}(\log x)) + [8 \cos(\log x) - \sin(\log x)]$  is the required solution of the given equation.

**05(a)**

**Sol:** Disk scheduling algorithms:

1. FCFS
2. SSTF
3. Elevator

**1. FCFS disk scheduling algorithm:**

- First come first served, scheduled the request as they arrived
- Simple, easy to implement
- No chance of starvation
- May cause more seek time and more no. of times head changing its direction

**2. SSTF disk scheduling algorithm:**

- Shortest seek time first (Nearest cylinder Next)
- Schedule the request next which is nearest from current head position (shortest seek time)
- Produce minimum average seek time
- Chance of starvation
- May cause more no. of times head change its direction

**3. Elevator:**

- Head moving from one end to other and service request in that order
- No chance of starvation
- Minimum no. of times head change its direction (only at ends)
- Types of elevator disk scheduling algorithm are
  - a) SCAN
  - (b) C-SCAN
  - c) Look
  - (d) C-Look



05(b)

**Sol:** Let the side of each of the squares cut off be 'x' cm. So that the height of the box is 'x' cm as shown below.

∴ The volume of the box

$$V = (18 - 2x)(12 - 2x)x$$

$$= (216x - 60x^2 + 4x^3)$$

$$V = 4(54x - 15x^2 + x^3) \dots (1)$$

$$\frac{dv}{dx} = 0 \Rightarrow 4(54 - 30x + 3x^2) = 0$$

$$12(18 - 10x + x^2) = 0$$

$$x = \frac{10 \pm \sqrt{100 - 72}}{2}$$

$$= 5 \pm \sqrt{7}$$

$$= 5 \pm 2.65$$

∴ x = 7.65 and 2.35 are the stationary points of (1)

$$\frac{d^2v}{dx^2} = 4(6x - 30)$$

At x = 7.65,  $\frac{d^2v}{dx^2} > 0$  and x = 7.65 is an inadmissible value as no box is possible for this value

At x = 2.35,  $\frac{d^2v}{dx^2} < 0$

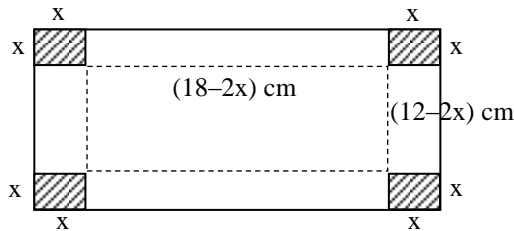
At x = 2.35, we get maximum volume and is given by V = 2.35(13.3) (7.3)

$$= 228.16 \text{ cm}^3$$

And the dimension are length = 13.3 cm

breath = 7.3 cm

And height = 2.35 cm

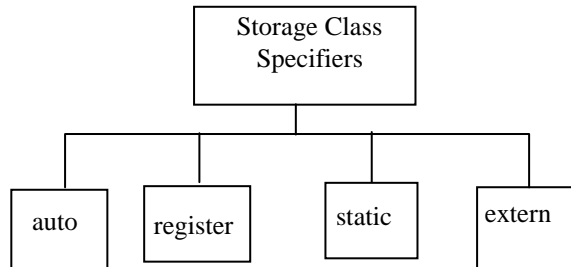






05(c)

**Sol:** We defined the storage class of an object using one of four specifiers: auto, register, static, and extern.



**Storage Class Specifiers**

### **Auto variables:**

A variable with an auto specification has the following storage characteristic:

- The memory is allocated automatically at the declaration and automatically vanished, once it goes out of the block.
- Life time of the variables is upto that block only, where it is declared.
- When the values are not initialized to variables then that variable contain JUNK values.

### **Register Variable:**

A register storage class is the same as the auto class with only one difference. The declaration includes a recommendation to the compiler to use a central processing unit (CPU) register for the variable instead of a memory location.

### **Static Variables:**

A static variable in this context can be referred to only in the block it is defined. The extent, however, is static; the computer allocates storage for this variable only once.

A static variable can be initialized where it is defined, or it can be left uninitialized. If initialized, it is initialized only once. If it is not initialized, its value will be initialized to zero. Note however, that it is initialized only once in the execution of the program.

### **External Variables:**

A variable declared with a storage class of *external* has a file scope: the extent is static, but the linkage is external.



An external variable must be declared in all source files that reference it, but it can be defined only in one of them.

**05(d)**

**Sol:** Let  $f(xyz) = (x^2 + y^2 + z^2) + \lambda(xyz - a^3)$

$$\frac{\partial f}{\partial x} = 0 \Rightarrow (2x + \lambda yz) = 0 \quad \dots (1)$$

$$\frac{\partial f}{\partial y} = 0 \Rightarrow (2y + \lambda xz) = 0 \quad \dots (2)$$

$$\frac{\partial f}{\partial z} = 0 \Rightarrow (2z + \lambda xy) = 0 \quad \dots (3)$$

From (1) & (2)  $(2x^2 + \lambda xyz) = (2y^2 + \lambda xyz)$

$$\therefore x^2 = y^2$$

$$\Rightarrow x = y \quad \dots (4)$$

Similarly from (2) & (3)

$$(2y^2 + \lambda xyz) = (2z^2 + \lambda xyz)$$

$$\therefore y^2 = z^2$$

$$\Rightarrow y = z \quad \dots (5)$$

Given that  $xyz = a^3$

$$\Rightarrow x^3 = a^3 \text{ (from (4) \& (5)),}$$

$$x = a, y = a, z = a$$

$$\therefore \text{The minimum value of } (x^2 + y^2 + z^2) = 3a^2$$

**05(e)**

**Sol:** Program size = 256 MB =  $2^{26}$  Bytes

$$\text{Page size} = 1\text{KB} = 2^{10} \text{ Bytes}$$

$$\text{No. of pages in program} = \left\lceil \frac{\text{Prog.size}}{\text{page size}} \right\rceil = 2^{16} \text{ pages}$$

$$\text{Page Table}_A \text{ size} = \text{No. of entries} * \text{PTE size}$$



$$= 2^{16} \times 4 \text{ Bytes} = 2^{18} \text{ Bytes}$$

$$\text{No. of pages in } PT_A = \left\lceil \frac{PT_A \text{ size}}{\text{pagesize}} \right\rceil = 2^8 \text{ pages}$$

Page Table<sub>B</sub> size = No. of entries \* PTE size

$$= 2^8 \times 4 \text{ Bytes} = 2^{10} \text{ Bytes}$$

Page Table<sub>B</sub> size = page frame size

So minimum two level paging is required

**06(a)(i)**

**Sol:**  $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2) \dots (1)$  is a linear first order partial differential equation.

AE's or SE's:

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$

↓

(2)

↓

(3)

↓

(4)

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$

$$= \frac{(x dx + y dy + z dz)}{(x^2 y^2 - x^2 z^2 + y^2 z^2 - y^2 x^2 + z^2 x^2 - z^2 y^2)}$$

$$\therefore (x dx + y dy + z dz) = 0$$

$$\Rightarrow \left( \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} \right) = \frac{a}{2}$$

$$\therefore (x^2 + y^2 + z^2) = a \dots \dots \dots (5)$$

Similarly, from (2), (3) & (4)

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$$



$$= \frac{\left( \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} \right)}{\left( y^2 - z^2 + z^2 - x^2 + x^2 - y^2 \right)}$$

$$\therefore \left( \frac{1}{x} dx + \frac{1}{y} dy + \frac{1}{z} dz \right) = 0$$

$$\Rightarrow (\log x + \log y + \log z) = \log b$$

$$\therefore xyz = b \quad \dots\dots\dots (6)$$

Hence, from (5) & (6)

$f(x^2 + y^2 + z^2, xyz) = 0$  is the required solution of the given equation.

**06(a)(ii)**

**Sol:** Let  $\phi(x, y, z) = (x^2 + y^2 - 16)$

$$\Rightarrow \bar{n} = \frac{\nabla \phi}{|\nabla \phi|} = \frac{2x\bar{i} + 2y\bar{j}}{\sqrt{4x^2 + 4y^2}}$$

$$= \frac{(x\bar{i} + y\bar{j})}{\sqrt{x^2 + y^2}} = \frac{(x\bar{i} + y\bar{j})}{4}$$

$$\iint_S (\bar{A} \cdot \bar{n}) ds = \iint_R (\bar{A} \cdot \bar{n}) \frac{dydz}{|\bar{n} \cdot \bar{i}|}$$

(Where 'R' is the projection of 'S' in yz - plane)

$$= \iint_R \frac{1}{4} (xz + xy) \left( \frac{x}{4} \right) dydz$$

$$= \iint_R (y + z) dydz$$

$$= \int_0^5 \left[ \int_0^4 (y + z) dy \right] dz$$

$$= \int_0^5 \left( \frac{y^2}{2} + zy \right)_0^4 dz$$

Rough:

$$z: 0 \rightarrow 5$$

In yz - plane

$$x = 0,$$

$$\Rightarrow x^2 + y^2 = 16$$



$$\begin{aligned}
 &= \int_0^5 (8 + 4z) dz \\
 &= \left[ 8z + 4 \left( \frac{z^2}{2} \right) \right]_0^5 \\
 &= 40 + 2(25) = 90 \text{ cu-uts}
 \end{aligned}$$

**06(a)(iii)**

**Sol:**  $f(z) = (u + iv) \quad \dots (1)$

$$2 \times (1) \rightarrow 2f(z) = (2u + 2iv) \quad \dots (2)$$

$$-i \times (1) \rightarrow -if(z) = (-iu + v) \quad \dots (3)$$

$$(2) + (3) \rightarrow (2-i)f(z) = (2u + v) + i(2v - u)$$

$$\text{i.e. } F(z) = (U + iV) \quad \dots (4)$$

We know that  $F'(z) = \left( \frac{\partial U}{\partial x} + i \frac{\partial V}{\partial x} \right)$

$$= \left( \frac{\partial U}{\partial x} - i \frac{\partial U}{\partial y} \right) \quad \dots (5)$$

Given that  $(2u + v) = U = e^x (\cos y - \sin y)$

$$\begin{aligned}
 \frac{\partial U}{\partial x} &= e^x (\cos y - \sin y) \\
 &= e^z \quad \dots (6)
 \end{aligned}$$

$$\begin{aligned}
 \frac{\partial U}{\partial y} &= e^x (-\sin y - \cos y) \\
 &= -e^z \quad \dots (7)
 \end{aligned}$$

(6) & (7) in (5)

$$F'(z) = e^z - i(-e^z)$$

$$= (1 + i)e^z$$

$$\Rightarrow F(z) = (1 + i)e^z + ic$$

$$\text{i.e. } (2-i)f(z) = (1 + i)e^z + ic$$



$$\begin{aligned}\therefore f(z) &= \frac{(1+i)}{(2-i)} e^z + \frac{ic}{(2-i)} \\ &= \frac{(1+i)(2+i)}{5} e^z + \frac{i}{5} (2+i)c \\ &= \frac{(1+3i)}{5} e^z + \frac{(2i-1)}{5} c\end{aligned}$$

is the required analytic function.

**06(b)**

**Sol:** [3F000000]<sub>H</sub>

0   01111110   00.....00  
sign Exponent Mantissa

$$+ (1.0) \times 2^{(126-127)}$$

$$+ (0.1)_2$$

$$+ (0.5)_{10}$$

[BF800000]<sub>H</sub>

1   01111111   00....00  
S Exponent Mantissa

$$- (1.0) \times 2^{(127-127)}$$

$$- (1.0)_2$$

$$- (1.0)_{10}$$

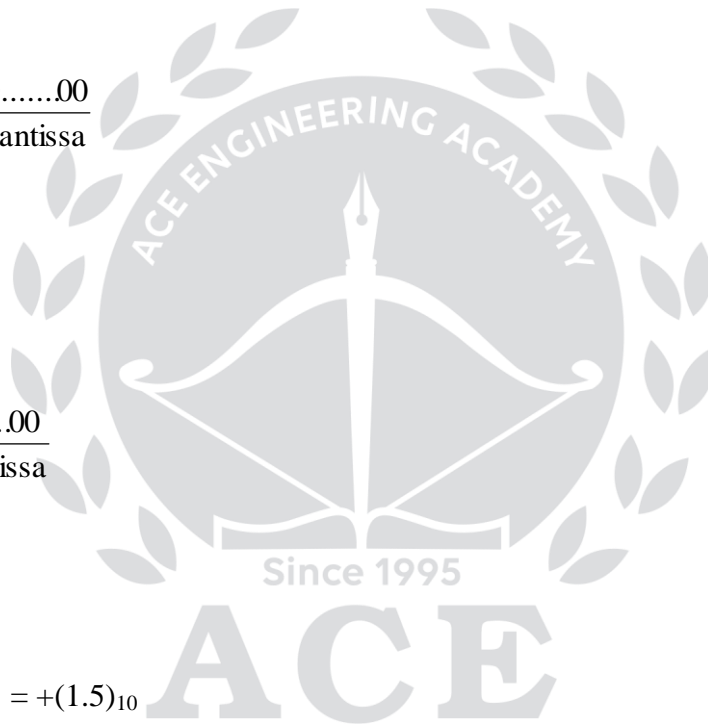
$$+ (0.5)_{10} - [-(1.0)_{10}] = + (1.5)_{10}$$

$$+ (0.1)_2 - [-(1.0)_2] = + (1.1)_2$$

$$+ (1.1) \times 2^0$$

0   01111111   100.....00  
sign Exponent Mantissa

[3FC00000]<sub>H</sub>

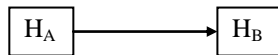




06(c)(i)

**Sol: Simplex:**

- Communication channel only send information in single direction
- One way communication
- Radio station is simplex channel



**Duplex:**

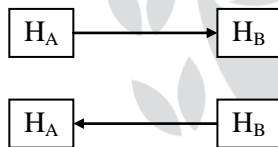
Duplex communications are of two types:

i) Half Duplex

ii) Full Duplex

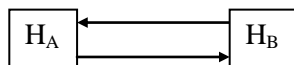
**i) Half Duplex:**

- Either side communication at one time
- Information can be send in any direction, but at one time only in one direction
- Walkie-talkie is half duplex channel



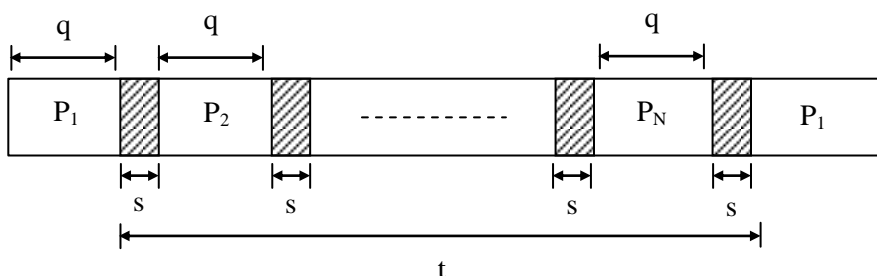
**ii) Full duplex:**

- Data can be transmitted in both directions simultaneously
- Telephone is full duplex channel



06(c)(ii)

**Sol:**





$$t = (N-1) \times q + N \times S$$

$$q = \frac{(t - N \times S)}{(N - 1)}$$

**07(a)(i)**

**Sol: Page Replacement Policy:**

- It occurs when there is no free frame in main memory to bring the requested page.
- Page replacement is to find some page in memory, which is not really in use and swap it out to bring requested page.
- Need a page replacement algorithm which will result in minimum number of page faults.
- Same page may be brought into memory several times.
- Page replacement completes separation between logical memory and physical memory i.e. large virtual memory can be provided on a smaller physical memory.

**Steps in Page Replacement**

- Finding the location of the desired page on disk.
- Finding a Free Frame if there is a free frame, then use it.
- If there is no free frame, then use a page replacement algorithm to select a victim frame (the one to be replaced).
- Bringing the desired page into the (newly) free frame.
- Updating the page and frame tables.
- Restarting the process

Page Replacement policies are of 3 types:

1. FIFO
2. LRU
3. Optimal

**1. FIFO page replacement policy:**

- Replace the page which enters first
- First in first out
- Simple, easy to implement
- May suffer with belady anomalies





## 2. LRU page replacement policy:

- Replace the page which is least recently used
- Gives less number of page fault w.r.t. FIFO, user programs follows principle of 'Locality of Reference'

## 3. Optimal page replacement policy:

- Replace the page which has very less reference in the future
- It gives best result (less page fault) among all page replacement policies

### 07(a)(ii)

**Sol:** #include < stdio. h>

void main ( )

{

int a = 10, b = 20;

printf ("%d % d", a, b);

a = a + b;

b = a – b;

a = a – b;

printf ("%d % d", a, b);

}

### 07(b)(i)

**Sol:** Let  $f(z) = u(x, y) + iv(x, y)$  be the required analytic function.

Given  $u(x, y)$

$$= (x \sin x \cos hy - y \cos x \sin hy)$$

$$\text{We know that } f'(z) = \left( \frac{\partial u}{\partial x} + \frac{i \partial v}{\partial x} \right)$$

$$= \left( \frac{\partial u}{\partial x} - \frac{i \partial u}{\partial y} \right) \dots (1)$$

$$\frac{\partial u}{\partial x} = \cos hy (x \cos x + \sin x) - y \sinh y (-\sin x) = (z \cos z + \sin z) \dots (2)$$



$$\frac{\partial u}{\partial y} = x \sin x \sin hy - \cos x (y \cosh y + \sin hy) = 0 \quad \dots (3)$$

(2) & (3) in (1)

$$f'(z) = z \cos z + \sin z$$

$$\begin{aligned} \Rightarrow f(z) &= \int (z \cos z + \sin z) dz \\ &= [z(\sin z) - (-\cos z)] - \cos z + ic \\ &= z \sin z + ic \text{ is the required analytic function.} \end{aligned}$$

**07(b)(ii)**

**Sol:** Let  $f(z) = \phi(x,y) + i\psi(x,y) \quad \dots (1)$

be the required analytic function.

Let  $\phi(x,y) = c$

$$\begin{aligned} \Rightarrow d\phi &= \left( \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy \right) = 0 \\ &= \left( \frac{\partial \psi}{\partial y} dx - \frac{\partial \psi}{\partial x} dy \right) = 0 \quad \dots (2) \end{aligned}$$

$$\frac{\partial \psi}{\partial y} = \frac{(x^2 + y^2)(-1) - (-y)(2y)}{(x^2 + y^2)^2} = \frac{y^2 - x^2}{(x^2 + y^2)^2} \quad \dots (3)$$

$$\frac{\partial \psi}{\partial x} = \frac{y}{(x^2 + y^2)^2} (2x) \quad \dots (4)$$

(3) & (4) in (2)

$$d\phi = \left[ \frac{y^2 - x^2}{(x^2 + y^2)^2} dx - \frac{2xy}{(x^2 + y^2)^2} dy \right] = 0$$

It is always an exact differential equation.

$\therefore \phi = \frac{x}{(x^2 + y^2)} + c$  is the required velocity potential function.



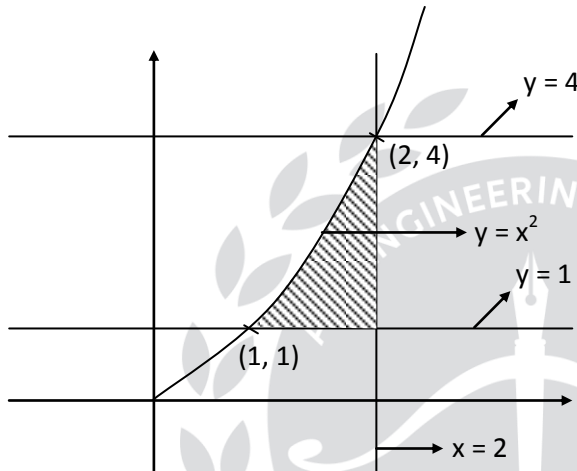
07(c)

**Sol:** Let  $I = \int_1^4 \int_{\sqrt{y}}^2 (x^2 + y^2) dx dy \quad \dots (1)$

$$x : \sqrt{y} \rightarrow 2 \text{ \& } y = 1 \rightarrow 4$$

$$\Rightarrow x = \sqrt{y} \text{ \& } x = 2 \quad y = 1 \text{ \& } y = 4$$

The limits of integration can be changed as follows



$x = \sqrt{y}$  or  $y^2 = x^2$  intersecting the lines  $y = 1$  at  $(1, 1)$  and  $y = 4$  at  $(2, 4)$  and area bounded is shaded above.

Then  $y : 1 \rightarrow x^2$  and  $x : 1 \rightarrow 2$

$\therefore$  (1) becomes

$$\begin{aligned} I &= \int_1^2 \left[ \int_1^{x^2} (x^2 + y^2) dy \right] dx \\ &= \int_1^2 \left( x^2(y) + \frac{y^3}{3} \right)_{y=1}^{y=x^2} dx \\ &= \int_1^2 \left[ \left( x^4 + \frac{x^6}{3} \right) - \left( x^2 + \frac{1}{3} \right) \right] dx \end{aligned}$$



$$\begin{aligned} &= \left[ \frac{x^5}{5} + \frac{x^7}{21} - \frac{x^3}{3} - \frac{x}{3} \right]^2 \\ &= \frac{1}{105} [21x^5 + 5x^7 - 35x^3 - 35x]^2 \\ &= \frac{1}{105} [(672 + 640 - 280 - 70) - (21 + 5 - 35 - 35)] \\ &= \frac{1006}{105} \text{ cu.uts} \end{aligned}$$

**08(a)(i)**

**Sol: Single tasking O.S:**

- CPU execute one process at one time
- Next process starts its execution only after completion of current process.
- Serial execution
- When running process perform its I/O, CPU remains idle
- Poor CPU utilization

**Multiprogramming O.S:**

- More than one user program reside in ready state (ready to execute)
- When a running process goes for I/O activity, CPU start executing next process
- Better CPU utilization
- It is non preemptive (by default)

**Multitasking O.S**

- Preemptive multiprogramming O.S.
- More than one process is executed by CPU
- Concurrent execution
- Next process start its execution before completion of current process



**08(a)(ii)**

**Sol: Deadlock:**

- Indefinite wait for a resource which is acquired by some other process
- 4 necessary and sufficient conditions for deadlock to be occurred.

- Mutual exclusion
- Hold and wait
- Circular wait
- No preemption

**i) Mutual Exclusion:**

Processes should be mutually exclusive for a non sharable resources

**ii) Hold and wait:**

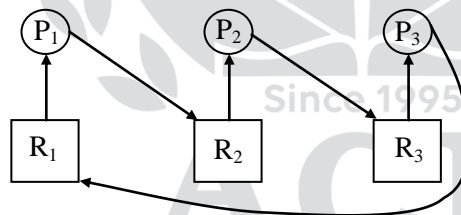
Processes must hold some resource (s) and waiting for some other resource(s)

**iii) Circular wait:**

Hold and wait should be in cyclic fashion

**iv) No preemption:**

Processes releases it's acquired resources by own.



To overcome from deadlock:

- Deadlock Prevention
- Deadlock Avoidance
- Detection and Recovery



**08(a)(iii)**

**Sol: Recursion:**

- Process of repeating items in a self similar way
- When a function call itself
- Main problem is divided into smaller subproblems of same type

**Types of recursion:**

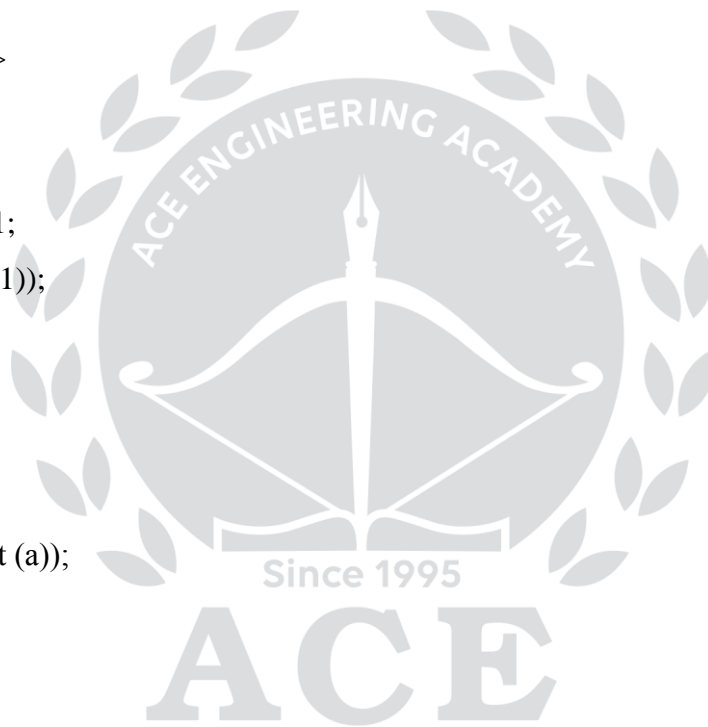
1. Single vs multiple recursion
2. Direct vs Indirect recursion

**Example:**

```
#include <stdio.h>

int fact (int n)
{
    if (n == 1) return 1;
    return (n * fact (n-1));
}

void main ()
{
    int a = 5;
    printf("%d", fact (a));
}
```



**08(b)(i)**

**Sol:** Avg. seek time = 100ms

3600 Revolutions → 1 minute

$$\begin{aligned} 1 \text{ Revolution time} &\rightarrow \frac{60 \times 10^3 \text{ ms}}{3600} \\ &\rightarrow 16.66 \text{ ms} \end{aligned}$$

$$\text{Avg. Rotational latency} = \frac{\text{one revolution time}}{2} = 8.33 \text{ ms}$$



$$\text{Data transfer rate} = \frac{\text{One track size}}{\text{One revolution time}} = \frac{32\text{KB}}{16.66\text{ms}}$$

$$\text{File transfer time} = \frac{\text{File size}}{\text{Data Transfer Rate}} = \frac{16.66\text{ms}}{4} = 4.165\text{ms}$$

$$\begin{aligned}\text{Disk access time} &= \text{Avg. seek time} + \text{Avg. rotational latency} + \text{File transfer time} \\ &= 100\text{ms} + 8.33\text{ms} + 4.165\text{ms} \\ &= 112.495\text{ms.}\end{aligned}$$

**08(b)(ii)**

**Sol:**  $T_P = \text{Distance} \times \text{signal speed} = 2\text{ km} \times 5\text{ms/km}$   
 $= 10\text{ ms}$

$$T_t \geq 2T_P$$

$$\begin{aligned}\text{Minimum Frame size} &= 2T_P \times \text{DTR} \\ &= 10\text{ms} \times 1\text{ mbps} \\ &= 10^4\text{ bits} \\ &= 1250\text{ Bytes}\end{aligned}$$

**08(c)**

**Sol:**  $G(x) = x^3 + 1 = 1 \cdot x^3 + 0 \cdot x^2 + 0 \cdot x + 1 \cdot x^0$

Divisor = 1001, Frame = 110110011011

$$\begin{array}{r} 1001 \overline{) 110110011011} \\ \text{Bitwise} \quad 1001 \quad : \quad : \\ \text{X-OR} \quad \quad 1001 \quad : \quad : \\ \quad \quad \quad 1001 \quad : \quad : \\ \hline \quad \quad \quad 1101 \quad : \\ \quad \quad \quad 1001 \quad : \\ \hline \quad \quad \quad 1001 \\ \quad \quad \quad 1001 \\ \hline \quad \quad \quad 0 \end{array}$$

If receiver find remainder is zero after modulo 2 division it means no any error detected in the frame, receiver accept the received frame.



**08(c)(ii)**

**Sol: CPU special registers:**

1. PC                      2. IR                      3. MAR                      4. MDR                      5. AC

**1. Program counter (PC):**

It is program counter that holds the address of the next instruction to be fetched; It's size is equal to the address bus size of the processor. After fetching an instruction, PC content is automatically incremented to point the address of the next instruction to be fetched

**2. Instruction Register (IR):**

- It is an instruction Register that holds the opcode of the instruction after it's fetching
- After fetching an instruction, opcode will be placed in IR (from MDR) later it sends to control Register for completing it's Decode and execution.
- Size of the IR equal to the MDR size

**3. Memory Address Register (MAR):**

- It is a memory address register that holds the Address of the memory Register
- In Basic processor, it is connected to the address Bus and it's size is equal to the address Bus size
- $\mu$ p places the memory address in this register while performing memory read and memory write operation.
- During instruction fetch; MAR is used to hold program address and during data read or Data write operations MAR holds the Data memory Register Address.

**4. Memory Data Register (MDR):**

- It is used to hold the content of the memory Register while performing Read/write operation from memory
- It is connected to Data Bus.
- It's size is equal to the Data bus size while fetching the instruction, initially opcode of the instruction is placed in MDR

**5. Accumulator (AC):** The accumulator is an internal CPU register used as the default location to store any calculations performed by the **arithmetic and logic unit**.