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ESE – 2019 MAINS
OFFLINE TEST SERIES

CIVIL ENGINEERING

TEST – 7
SOLUTIONS

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01(a).

Sol: $\sigma_y = \frac{-P}{a^2} \rightarrow$ (i)

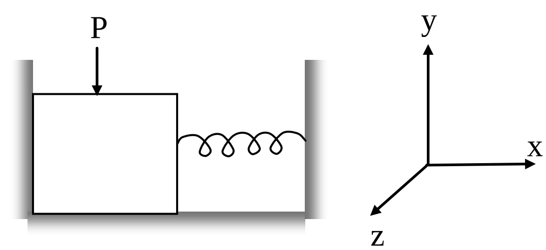
Further, since the cube is completely constrained in Z direction, $\varepsilon_z = 0$

$$-\mu\frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} + \frac{\sigma_z}{E} = 0 \Rightarrow -\mu\sigma_x + \sigma_z = \mu\sigma_y$$

$$-\mu\sigma_x + \sigma_z = \frac{\mu P}{a^2} \qquad \rightarrow \text{(ii)}$$

Now, in x direction, elongation of the cube must be equal to compression of the spring,

$$|\varepsilon_x \times a| = \left|\frac{\sigma_x \times a^2}{k}\right|$$



$$\Rightarrow \varepsilon_x = \frac{\sigma_x \times a}{K}$$

$$\Rightarrow \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} = \frac{\sigma_x a}{K}$$

$$\Rightarrow \sigma_x\left(1 - \frac{Ea}{K}\right) - \mu\sigma_z = -\frac{\mu P}{a^2} \rightarrow \text{(iii)}$$

Solving equation (ii) and (iii) we obtain

$$\left(-\mu\sigma_x + \sigma_z = \frac{-\mu P}{a^2}\right) \times (-\mu)$$

$$\sigma_x\left(1 - \frac{Ea}{K}\right) - \mu\sigma_z = -\mu\frac{P}{a^2}$$

$$\sigma_x\left(\mu^2 - 1 + \frac{Ea}{K}\right) = \frac{\mu(\mu+1)P}{a^2}$$

$$\sigma_x = \left[\frac{\mu(\mu+1)K}{K(\mu^2-1) + Ea}\right]\frac{P}{a^2}$$

From equation (ii), $\sigma_z = \mu\sigma_x - \frac{\mu P}{a^2}$

$$\sigma_z = \frac{\mu P}{a^2}\left\{\left[\frac{\mu(\mu+1)K}{K(\mu^2-1) + Ea}\right] - 1\right\}$$

$$\sigma_z = \left[\frac{(\mu+1)K - Ea}{(\mu^2-1)K + Ea}\right]\frac{\mu P}{a^2}$$

01(b).

Sol: Seismic weight = Seismic weight for floors + for roof

Seismic weight of floor:

Dead load = (12 × 400) = 4800 kN

Live Load:

As per IS 1893; if LL > 3 kN/m^2; 50% has to be considered.

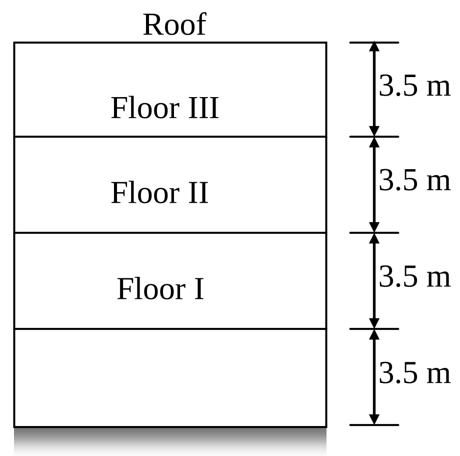
∴ LL = 0.5 × 4 × 400 = 800 Kn

∴ Total load = 4800 + 800 = 5600 Kn

Seismic weight of roof:

As per IS 1893, for roof imposed load need not be considered in the calculation of design seismic force

∴ Total load = 400 × 10 = 4000 kN



∴ Total seismic weight = (3 × 5600) + 4000

= 16800 + 4000

= 20800 kN

(a) Soft storey:

It is the storey in which the lateral stiffness is less than 70% of that in the storey immediately above (or) 80% of the combined stiffness of three stones above.

01(c).

Sol: Free body diagram of the composite bar is shown in figure.

Equilibrium: $\Sigma F_x = 0$

$R_1 + R_2 = P \quad \rightarrow$ (i)

Compatibility:

$\delta L_{overall} = 0 \Rightarrow \delta L_{st} + \delta L_{Al} = 0$

$$\frac{P_{st} \times L_{st}}{E_{st} \times A_{st}} + \frac{P_{A\ell} \times L_{A\ell}}{E_{A\ell} \times A_{A\ell}} = 0$$

$$\Rightarrow \frac{P_{st}}{3 \times A_{st}} + \frac{P_{A\ell}}{A_{A\ell}} = 0$$

$$\Rightarrow \frac{P_{st}}{3 \times 400} + \frac{P_{A\ell}}{600} = 0$$

$$\Rightarrow \frac{P_{st}}{2} + P_{A\ell} = 0 \Rightarrow \frac{R_1}{2} + (-R_2) = 0$$

$\Rightarrow R_1 = 2R_2 \rightarrow$ (ii)

Solving equation (i) and (ii) we obtain

$$R_2 = \frac{P}{3} \text{ and } R_1 = \frac{2P}{3}$$

Now for steel $\sigma_{st} \le \hat{\sigma}_{st}$

$$\Rightarrow \left|\frac{R_1}{400}\right| \le 140 \Rightarrow \left|\frac{2P}{3 \times 400}\right| \le 140$$

$\Rightarrow P \le 84000$ N or $P \le 84$ kN

For aluminium, $\sigma_{Al} \le \hat{\sigma}_{A\ell}$

$$\left|\frac{-R_2}{600}\right| \le 90 \Rightarrow \left|\frac{-P}{3 \times 600}\right| \le 90$$

$\Rightarrow P \le 162000$ N or $P \le 162$ kN

From both conditions, we conclude that maximum value of P that can be used is 84 kN.

01(d).

Sol: In this case the copper is in tension and aluminium is in compression.

Let f_c = Tensile stress in copper and f_a = compressive stress in aluminium

A_c = Area of copper section = 2 (50 × 12.5) = 1250 mm^2

A_a = Area of aluminium section = 50 × 25 = 1250 mm^2

Tension in copper = Compression in aluminium

$f_c A_c = f_a A_a$

$\therefore f_c = f_a$

Since $A_c = A_a$

Actual expansion of copper = Actual expansion of aluminium

$$\alpha_c T\ell + \frac{f_c}{E_c}\ell = \alpha_a T\ell - \frac{f_a}{E_a}\ell$$

$$\alpha_c T + \frac{f_c}{E_c} = \alpha_a T - \frac{f_a}{E_a}$$

$$1.6\times10^{-5}\times40 + \frac{f_c}{1.2\times10^5} = 2.2\times10^{-5}\times40 - \frac{f_a}{0.7\times10^5}$$

$$64 + \frac{f_c}{1.2\times10^5} = 88 - \frac{f_a}{0.7} \qquad \therefore \frac{f_c}{1.2} + \frac{f_a}{0.7} = 24$$

Since $f_c = f_a$

$$f_c\left(\frac{1}{1.2} + \frac{1}{0.7}\right) = 24$$

$$\therefore f_c\left(\frac{1.9}{1.2\times0.7}\right) = 24$$

f_c = 10.61 N/mm^2 and f_a = 10.61 N/mm^2

Force in the aluminium bar = P = $f_a A_a$ = 10.61 × 1250 = 13262.5 N

Let f_s = shear stress in the pins

$$2f_s\frac{\pi d^2}{4} = P$$

$$\therefore f_s = \frac{2P}{\pi d^2} = \frac{2\times13262.5}{\pi\times10^2} = 84.43 \text{ N/mm}^2$$

01(e).

Sol: **Given:** Size of column = 400 × 400

$\therefore$ A = 16×10^4 mm^2

Reinforcement = 8 bars of 16 mm diameter

$A_s = 8 \times \frac{\pi}{4} \times 16^2 = 1608.49$ mm^2

Case: I

Load carrying capacity:

Since minimum eccentricity ≤ 0.05B

$R_i = 0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{se}$

$= 0.4\, f_{ck}\, (A - A_{sc}) + 0.67\, f_y\, A_{sc}$

$= [0.4 \times 20\, (16 \times 10^4 - 1608.49)] + [0.67 \times 415 \times 1608.49]$

$= 1714.37 \times 10^3$ N

= 1714.37 kN

Safe load carrying capacity = $\frac{P_u}{FOS} = \frac{1714.37}{1.5}$

Case II: Load carrying capacity = 1.5 × 1142.9

= 1714.35 kN

$\therefore P_u = 1.5 \times 1714.35 = 2571.52$ kN

$P_u = 0.4\, f_{ck}\, (A - A_{sc}) + 0.67\, f_y\, A_{sc}$

$\Rightarrow 2571.52 \times 10^3 = 0.4 \times 20 \times (A - 1608.49)] + [0.67 \times 415 \times 1608.49]$

$\Rightarrow A = 267144.03$ mm^2

Since it is a square column:

$B^2 = 267144.03$ mm^2

$\therefore$ B = 516.86 mm ~ 517 mm

$\therefore$ Required section is 517 × 517 mm

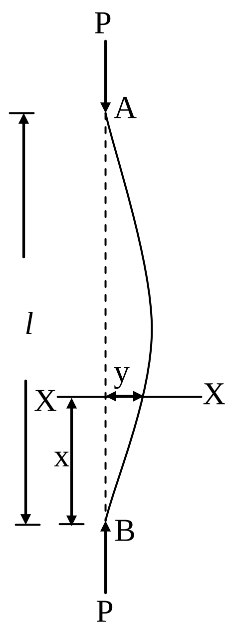
02(a). (i)

Sol: When both ends of the column are pinned or hinged.

Consider a column AB of length *l* and uniform sectional area A, hinged at both the ends A and B. Let P be the crippling load at which the column has just buckled.

Consider a section at a distance x from the end B. Let y be the deflection (lateral displacement) at the section.

The bending moment at the section is given by



$$EI\frac{d^2y}{dx^2} = -Py$$

$$EI\frac{d^2y}{dx^2} + Py = 0$$

$$\therefore \frac{d^2y}{dx^2} + \frac{P}{E\ell}y = 0$$

The solution to the above differential equation is

$$y = C_1\cos\left(x\sqrt{\frac{P}{EI}}\right) + C_2\sin\left(x\sqrt{\frac{P}{EI}}\right)$$

Where C_1 and C_2 are constants of integration

At $x = 0, y = 0 \Rightarrow C_1 = 0$ and at $x = l, y = 0$

$$\therefore \quad 0 = C_2\sin\left(\ell\sqrt{\frac{P}{EI}}\right)$$

Since $C_1 = 0$ we conclude that C_2 cannot be zero

Hence $\sin\left(\ell\sqrt{\frac{P}{EI}}\right) = 0$

$$\therefore \quad \left(\ell\sqrt{\frac{P}{EI}}\right) = 0, \pi, 2\pi, 3\pi, 4\pi$$

Considering the least practical value

$$\ell\sqrt{\frac{P}{EI}} = \pi$$

$$\therefore \quad P = \frac{\pi^2 EI}{\ell^2}$$

02(a). (ii)

Sol: D = 200 mm; d = 150 mm

$$A = \frac{\pi}{4}\left(200^2 - 150^2\right) = 13744.5 \text{ mm}^2$$

$$I = \frac{\pi}{64}\left(200^2 - 150^4\right) = 53689328 \text{ mm}^4$$

$$K^2 = \frac{I}{A} = \frac{53689328}{13744.5} = 3906.24 \text{ mm}^2$$

Since both ends are hinged

Effective length L = *l* = 6000 mm

$$\text{Euler's critical load } P_e = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 8 \times 10^4 \times 53689328}{6000^2} \text{ N}$$

$$= 1177539 \text{ N}$$

$$\text{Rankine's critical load } P_r = \frac{f_c A}{1 + \alpha \frac{\ell^2}{K^2}} = \frac{550 \times 13744.5}{1 + \frac{1}{1600} \times \frac{6000^2}{3906.24}} = 1118263 \text{ N}$$

$$\text{Ratio of Euler's critical load to Rankine's critical load} = \frac{1177539}{1118263} = 1.053$$

For Euler's critical load to be equal to Rankine's critical load

$$\frac{\pi^2 EI}{\ell^2} = \frac{f_c A}{1 + \alpha \frac{\ell^2}{K^2}}$$

$$\frac{\pi^2 \times 8 \times 10^4 \times 53689328}{\ell^2} = \frac{550 \times 13744.5}{1 + \frac{1}{1600} \times \frac{\ell^2}{3906.24}}$$

$$1 + \frac{\ell^2}{6250000} = 1.78325 \times 10^{-7} \ell^2$$

$$l = 7387.17 \text{ mm}$$

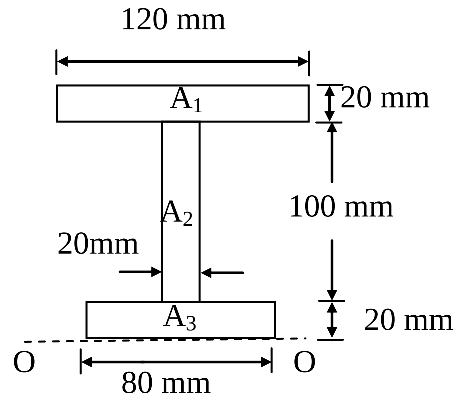
02(b). (i)

Sol:

$A_1 = 2400\ mm^2$

$A_2 = 2000\ mm^2$

$A_3 = 1600\ mm^2$



Moment by Inertia about vertical centroidal axis

$$I_{yy} = \frac{20\times(120)^3}{12} + \frac{100\times 20^3}{12} + \frac{20\times 80^3}{12}$$

$$= 3800000\ mm^4$$

Moment of Inertia about horizontal centroidal axis

Location of horizontal centroidal axis

$$= \frac{A_1\bar{x}_1 + A_2\bar{x}_2 + A_3\bar{x}_3}{A_1 + A_2 + A_3}$$

$$= \frac{(2400\times 130) + (2000\times 70) + (1600\times 10)}{2400 + 2000 + 1600}$$

= 78 mm from datum O – O

$$I_{XX} = I_{xx} \text{ of } A_1 + A_1x_1^2 + I_{xx} \text{ of } A_2 + A_2x_2^2 + I_{xx} \text{ of } A_3 + A_3x_3^2$$

$$= \left(\frac{120\times 20^3}{12}\right) + 2400(130-78)^2 + \left(\frac{20\times 100^3}{12}\right) + 2000(70-78)^2 + \left(\frac{80\times 20^3}{12}\right) + (1600)(78-10)^2$$

$I_{XX} = 15816000\ mm^4$

02(b). (ii)

Sol: Given:

An unknown weight 'W' is falling through a height

L = 3000 mm; Δ = 3.2 mm

h = 20 mm; d = 30 mm, E = 205 × 10^3 MPa

Maximum stress induced = E.ε

$$= E.\frac{\Delta}{L}$$

$$= 205\times 10^3 \times \frac{3.2}{3000}$$

= 218.7 MPa

and we have W.D = strain energy

$$W(h+\Delta)=\frac{\sigma^2}{2E}\times A\times L$$

$$W(20+3.2)=\frac{218.7^2}{2\times 205\times 10^3}\times\frac{\pi}{4}\times 30^2\times 3000$$

$\therefore$ W = 10660 N (or) W = 10.66 kN

02(c).

Sol:

(i) **Given:** Length of the beam '*l*' = 6 m

Cross section = 300 × 500 mm

$$A = 300\times 500 = 15\times 10^4\ \text{mm}^2$$

$$Z = 300\times\frac{500^2}{6}=12.5\times 10^6\ \text{mm}^2$$

Dead load = $25\times 15\times 10^4\times 10^{-6}$ = 3.75 kN/m

Live load = 4 kN/m

Total load = 7.75 kN/m

Maximum bending moment at midspan = $\frac{w\ell^2}{8}$

$$= 7.75\times\frac{36}{8} = 34.875\ \text{kNm}$$

Case 2: Concrete prestressing force:

Permissible tensile stress in concrete = 0

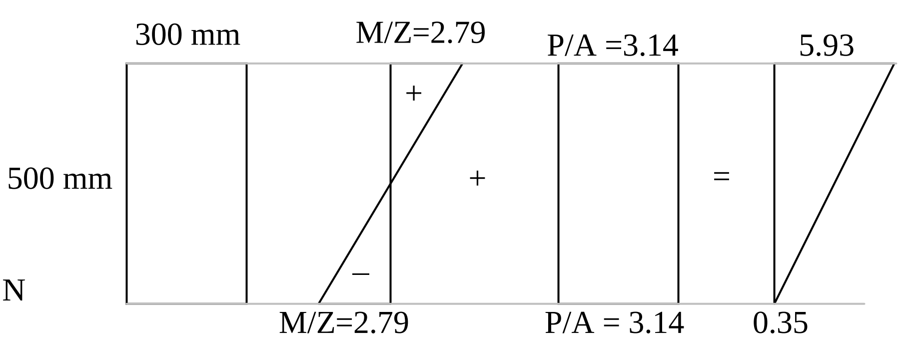
Stress at soffit = 0

$$\frac{-M}{Z}+\frac{P}{A}=0$$

$$\Rightarrow P=\frac{M}{Z}A$$

$$=\frac{34.875\times 10^6\times 15\times 10^4}{12.5\times 10^6}\ \text{N}$$

$= 418.5\times 10^3$ N = 418.5 kN



Area of each wire = $\frac{\pi}{4} \times 10^2 = 78.54\,\text{mm}^2$

$\therefore$ No of wires required = $\frac{418.5 \times 10^3}{1000 \times 78.54} = 5.32 \approx 6$

$\therefore$ 6 wires are required

Prestressing force = 6 × 78.54 × 1000 N = 471.24 kN

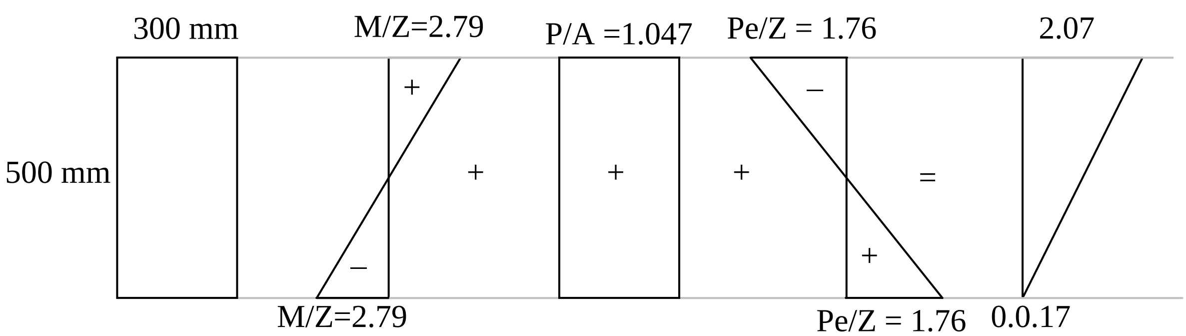
Stress at top = $\frac{P}{A} + \frac{M}{Z}$

$$= \frac{471.24 \times 10^3}{15 \times 10^4} + \frac{34.875 \times 10^6}{12.5 \times 10^6} = 3.14 + 2.79 = 5.93 \text{ MPa}$$

Stress at bottom = $\frac{P}{A} - \frac{M}{Z}$ = 3.14 – 2.79 = 0.35 MPa

Case III:

Eccentric prestressing force:



Stress at soffit = $\frac{M}{Z} + \frac{P}{A} + \frac{Pe}{Z} = 0$

$$\Rightarrow P\left(\frac{1}{A} + \frac{e}{z}\right) = \frac{M}{Z}$$

$$\Rightarrow P\left(\frac{1}{15 \times 10^4} + \frac{140}{12.5 \times 10^6}\right) = \frac{34.875 \times 10^6}{12.5 \times 10^6}$$

$$\Rightarrow \frac{p}{55970.15} = 2.79$$

$\Rightarrow$ P = 156156.71 N

= 156.156 kN

$\therefore$ No, of wires required = $\dfrac{156.156\times10^3}{1000\times78.54}=1.98\approx2$

$\therefore$ 2 Wires are required

$\therefore$ Prestressing force = 2× 78.54 × 1000 N = 157.08 kN

Stress at top = $\dfrac{P}{A}+\dfrac{M}{Z}-\dfrac{Pe}{Z}$

$$=\frac{157.08\times10^3}{15\times10^4}+\frac{34.875\times10^6}{12.5\times10^6}-\frac{157.08\times10^3\times140}{12.5\times10^6}$$

= 1.047 + 2.79 – 1.76

= 2.077 MPa

Stress at bottom = $\dfrac{P}{A}+\dfrac{Pe}{Z}-\dfrac{M}{Z}$

= 1.047 + 1.76 – 2.79

= 0.017 MPa

(ii).

(a) One of the important applications of prestressed concrete is int he construction of bridge of long span. Bridges of long span if constructed using the prestressed concrete girders, depth drastically reduces and construction also required less time.

(b) Prestressed concrete sleepers are widely used in railways as they are economical and are suited for high speeds and heavy traffic density. They provide stable track structure and also required less maintenance.

(c) Prestressed concrete slab system are ideally suited for floor and roof construction of industrial buildings where live loads are of higher order and also uninterrupted floor space is desirable i.e long spans are required.

(d) Liquid retaining structures like circular pipe, tanks and pressure vessel can be constructed using circular prestressing technique. The circumferential hoop compression induced in concrete by prestressing balances the hoop tension developed due to internal fluid pressure. Hence, reinforcement required in circular prestressed elements is less and it also ensures safety against shrinkage cracks in liquid retaining structures.

(e) Other applications are prestressed concrete poles, piles, pavements, prestressed concrete folded plate structures are used for roofs of industrial structures, coal bunkers and cooling towers.

03(a).

Sol:

(i) $Z = 500\ x_1 + 600\ x_2$

subjected to

$3x_1 + 2x_2 \leq 64$ (Line –I)

(Carpentry dept)

$x_1 + 4x_2 \leq 68$ (Line-II)

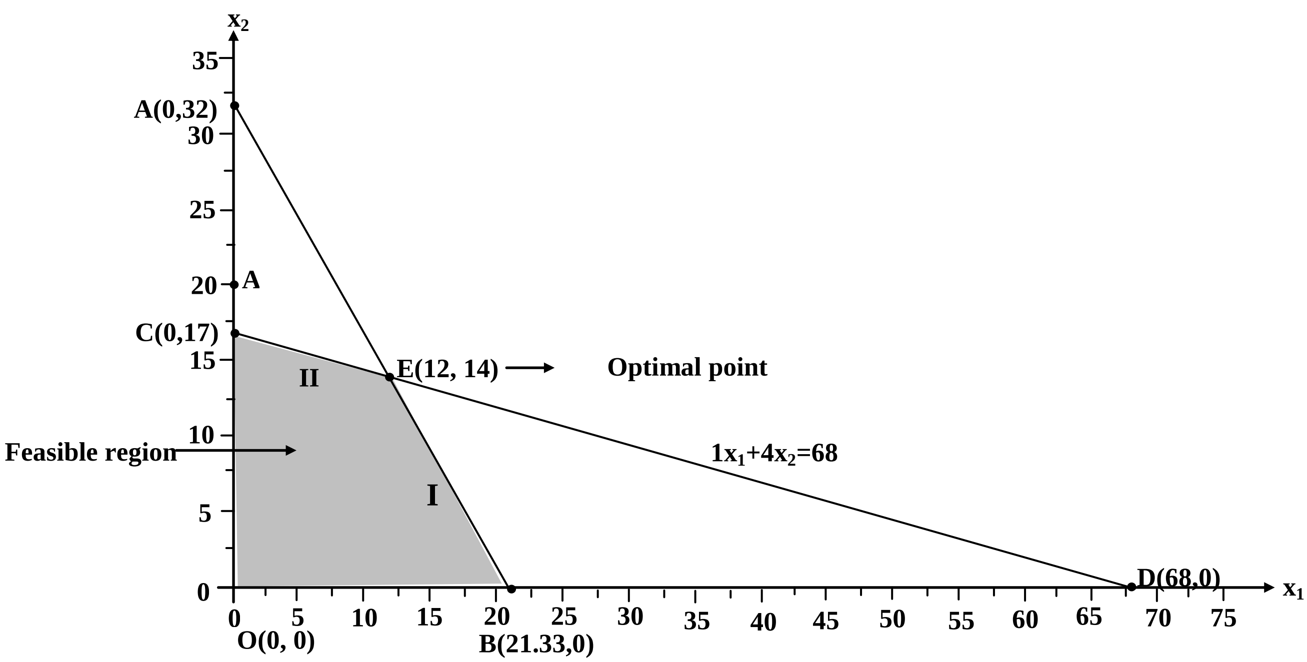
(Painting dept)

$x_1\ ,\ x_2 \geq 0$

Where

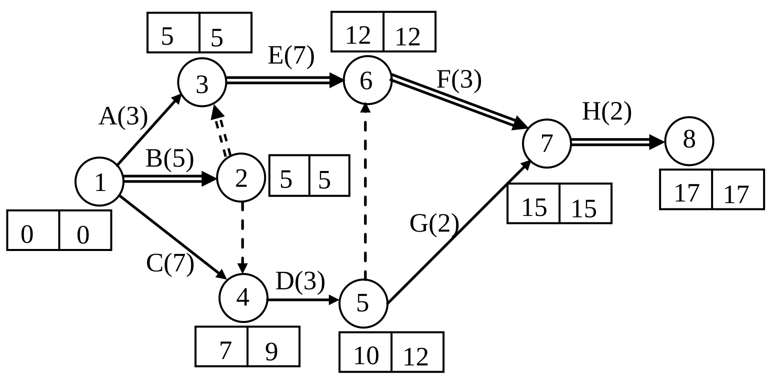
x_1 : No. of windows

x_2 : No. of Doors



(ii)

Activity	$t_e = \frac{t_o + 4t_m + t_p}{6}$
A	3
B	5
C	7
D	3
E	7
F	3
G	2
H	2



Critical path: 1 – 2 – 3 – 6 – 7 – 8

(or)

B – Dummy – E – F – G

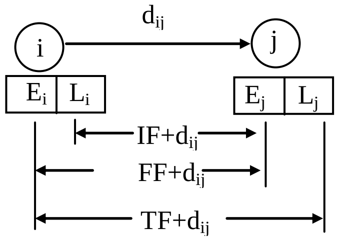
Exp. Completion time of the project = 17 days

Slack @ 1, 2, 3, 6, 7, 8 = 0

Slack @ 4 = 9 – 7 = 2

Slack @ 5 = 12 – 10 = 2

Floats:



All the three floats of B, E, F & H = 0

Activity 'D'

Total float = 12 – 7 – 3 = 2

Free float = 10 – 7 – 3 = 0

Independent float = 0

03(b).

Sol: Given:

Clear dimensions = 3m × 4m

Live load = 2 kN/m^2

Floor finish = 1 kN/m^2

M-20 grade concrete, Fe 415 steel

Aspect ratio $= \frac{\ell_y}{\ell_x} = \frac{4}{3} = 1.33 < 2$

$\therefore$ Two – way slab

Depth required from deflection criteria for continuous slabs : $\frac{\ell_x}{40}$

$= \frac{\ell_x}{40 \times 0.8}$ if Fe415 is used

$D = \frac{3000}{40 \times 0.8} = 93.75$ mm

$\therefore$ Provide D = 125 mm; effective cover = 25 mm

$\therefore$ d = 100 mm

Step 1: Load calculation:

Assuming 1 m width:

Dead load = 25 $\times$ 0.125 = 3.125 kN/m

Live load = 2 kN/m

Floor finish = 1 kN/m

Total load = 6.125 kN/m

Factored load = 1.5 $\times$ 6.125 = 9.18 kN/m

Step 2:

Effective length:

$$\left.\begin{aligned} \ell_x = \ell_{x_o} + d \\ \ell_{x_o} + w \end{aligned}\right\} \text{minimum}$$

$$\left.\begin{aligned} = 3 + 0.1 \\ = 3 + 0.3 \end{aligned}\right\} = 3.1 \text{ m}$$

Similarly l_y = 4 + 0.1 = 4.1 m

$$\frac{\ell_y}{\ell_x} = \frac{4.1}{3.1} = 1.322$$

Moment coefficient calculation:

For short span at midspan:

$$\alpha_x^{(+)} = 0.043 + \left(\frac{1.322 - 1.3}{1.4 - 1.3}\right) \times (0.044 - 0.043)$$

= 0.04322

For short span at continuous edge

$$\alpha_x^{(-)} = 0.056 + \left(\frac{1.322 - 1.3}{1.4 - 1.3}\right)(0.059 - 0.056)$$

= 0.0566

For long span: $\alpha_y = 0.035$

$$\therefore\ M_x^{(+)} = \alpha_x^{(+)} w \ell_x^2 = 0.04322 \times 9.18 \times 3.1^2 = 4.07\,\text{kNm}$$

$$M_x^{(-)} = \alpha_x^{(-)} w \ell_x^2 = 0.0566 \times 9.18 \times 3.1^2 = 4.99\,\text{kNm}$$

$$M_y = \alpha_y w \ell_x^2 = 0.035 \times 9.18 \times 3.1^2 = 3.087\,\text{kNm}$$

$\therefore$ Maximum BM = 4.99 kNm

Step-3: Check for depth

Effective depth required = $\sqrt{\dfrac{M_u}{0.138 f_{ck} b}}$

$$= \sqrt{\frac{4.99\times10^6}{0.138\times20\times1000}}$$

= 42.52 mm

Provided depth = 100 mm

Hence safe

Step 4: Area of steel required

(a) At continuous support of short span:

$$A_x^{(-)} = \frac{0.5 f_{ck} bd}{f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}bd^2}}\right]$$

$$= \frac{0.5\times20}{415}\times1000\times100\left[1-\sqrt{1-\frac{4.6\times4.99\times10^6}{20\times1000\times100^2}}\right]$$

$= 142.49\ \text{mm}^2$

Spacing of 10 mm bars = $\dfrac{1000}{142.49}\times\dfrac{\pi}{4}\times10^2 = 551.195$ mm

Maximum permissible space minimum = {3d or 300 mm}

= min { 300, 300}

= 300 mm

∴ Provide 10 mm bars @ 300 mm c/c

(b) At centre of short span:

$$A_{st_x}^{(+)} = \frac{0.5\times20}{415}\times1000\times100\left[1-\sqrt{1-\frac{4.6\times4.07\times10^6}{20\times1000\times100^2}}\right]$$

$= 115.55\ \text{mm}^2$

Spacing of 10 mm bars = $\dfrac{1000}{115.55}\times\dfrac{\pi}{4}\times10^2$ = 679.68 > 300 mm

∴ Provide 10 mm bars @ 300 mm c/c

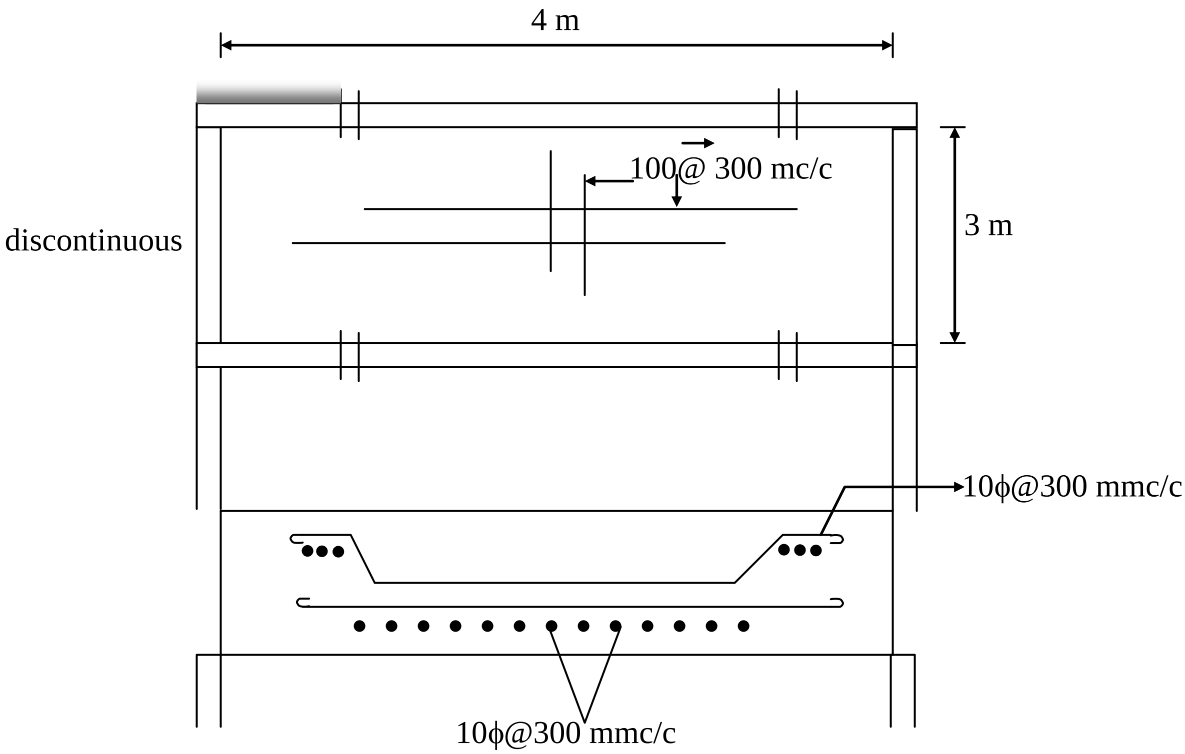
(c) At centre of long span:

$$A_{sty}^{(+)} = \frac{0.5\times 20}{415}\times 1000\times 100\left[1-\sqrt{1-\frac{4.6\times 3.087\times 10^6}{20\times 1000\times 100^2}}\right]$$

$= 87.11 \text{ mm}^2$

Spacing of 10 mm bars = $\frac{1000}{87.11}\times\frac{\pi}{4}\times 10^2 = 901.53 > 300\,\text{mm}$

$\therefore$ Provide 10 mm bars @ 300 mm c/c



03(c).

Sol: **Principal Stresses:**

$$\sigma_{1,2} = \sigma_{avg} \pm R = \frac{\sigma_x+\sigma_y}{2}\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\Rightarrow \sigma_{1,2} = \frac{40+(-20)}{2}\pm\sqrt{\left(\frac{40-(-20)}{2}\right)^2+40^2}$$

$$\Rightarrow \sigma_{1,2} = 10\pm\sqrt{30^2+40^2} = 10\pm 50$$

$\Rightarrow \sigma_1 = 60$ MPa, $\sigma_2 = -40$ MPa

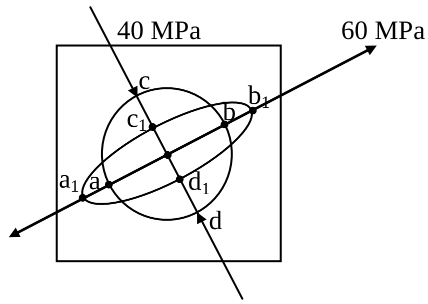
Using generalized Hooke's Law, we get principal strain as follows

$$\varepsilon_1 = \frac{\sigma_1 - \mu\sigma_2}{E} = \frac{60 - (0.3 \times -40)}{200000} = \frac{72}{200000}$$

$\Rightarrow \varepsilon_1 = 36 \times 10^{-5}$

$$\varepsilon_2 = \frac{-\mu\sigma_1 + \sigma_2}{E} = \frac{-0.3 \times 60 - 40}{200000} = \frac{-58}{200000}$$

$\Rightarrow \varepsilon_2 = -29 \times 10^{-5}$



Major Axis = $a_1b_1 = ab + \delta(ab) = ab\left[1 + \frac{\delta(ab)}{ab}\right]$

Major = $200(1 + \varepsilon_1) = 200[1 + 36 \times 10^{-5}]$

$\Rightarrow$ Major = 200.072 mm

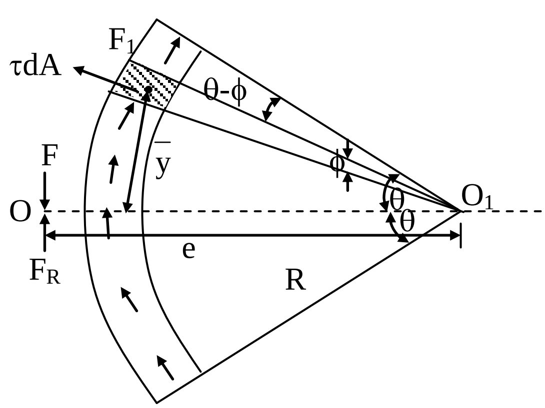
Minor Axis = $c_1d_1 = cd + \delta(cd) = cd\left[1 + \frac{\delta(cd)}{cd}\right]$

Minor axis = $200[1 + (-29) \times 10^{-5}]$

$\Rightarrow$ Minor axis = 199.942 mm

04(a). (i)

Sol:



From geometry, $F_1 = \int \tau \, dA$

$$= \int \frac{FA\bar{y}}{I.t}. \, dA$$

$$I = 2R\theta t.\frac{R^2}{2}\left(\frac{\theta - \sin\theta\cos\theta}{\theta}\right)$$

$$= R^3 \, t \, (\theta - \sin\theta \cos\theta)$$

$$A = R\,(\theta - \phi)\,t$$

$$dA = R\, d\phi \,.\, t$$

$$\bar{y} = R\sin\phi$$

Moment of F_1 about 'O_1' is given by

$$= \int_{-\theta}^{\theta} \frac{F}{It}.R(\theta-\phi)t.\,R\sin\phi.R\; d\phi\; t.R$$

$$= \frac{FR^4t}{I}\left[\int_{-\theta}^{+\theta}(\theta-\phi)\sin\phi \, d\phi\right]$$

$$= \frac{FR^4t}{I}\left[\left(-(\theta-\phi)\cos\phi\right)_{-\theta}^{+\theta} - \int_{-\theta}^{+\theta}\cos\phi \;\, d\phi\right]$$

$$= \frac{FR^4t}{I}\left[2\theta\cos\theta - 2\sin\theta\right]$$

$$= \frac{2FR^4t}{I}(\theta\cos\theta - \sin\theta)$$

$$= \frac{2FR^4t(\theta\cos\theta - \sin\theta)}{R^3t(\theta - \sin\theta\cos\theta)}$$

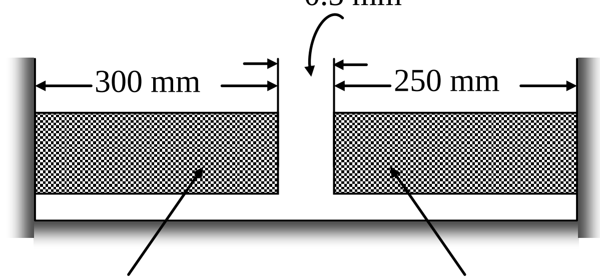
$$= \frac{2FR(\theta\cos\theta - \sin\theta)}{\theta - \sin\theta\cos\theta} = -F_R.e = -F.e$$

$$\therefore e = \frac{-2R(\theta\cos\theta - \sin\theta)}{(\theta - \sin\theta\cos\theta)}$$

or $$e = \frac{2R(\sin\theta - \theta\cos\theta)}{(\theta - \sin\theta\cos\theta)}$$

04(a) (ii)

Sol: The ends of the rods will thrust against each other at the higher temperature. Both the rods will be subjected to compressive stresses. Let f_a and f_b be the compressive stresses in the aluminium and steel rods.



Compressive force in aluminium rod = Compressive force in steel rod

$$f_a A_a = f_s A_s$$

$$\therefore f_s = \frac{A_a}{A_s}.f_a = \frac{2000}{800} f_a$$

$$\therefore f_s = 2.5\ f_a$$

Actual expansion of aluminium rod + Actual expansion of steel rod = 0.5 mm

$$\alpha_a T\ell_a - \frac{f_a}{E_a}\ell_a + \alpha_s T\ell_s - \frac{f_s}{E_s}\ell_s = 0.5$$

$$23\times10^{-6}(140-20)300 - \frac{f_a}{0.75\times10^{-5}}\times300 + 17.3\times10^{-6}(140-20)250 - \frac{f_s}{1.9\times10^{5}}\times250 = 0.5$$

$$0.828 - 0.004\ f_a + 0.519 - 0.001316\ f_s = 0.5$$

$$1.348 - 0.004\ f_a - 0.001316\ f_s = 0.5$$

$$0.004\ f_a + 0.001316\ f_s = 0.847$$

$$0.00729\ f_a = 0.847$$

$\therefore$ f_a = 116.187 N/mm^2 (compressive) and f_s = 2.5 ×116.187 = 290.466 N/mm^2 (Compressive)

Actual change in length (increase in length) of aluminium bar

$$= \alpha_a T\partial\ell_a - \frac{f_a}{E_a}\ell_a = 23\times10^{-6}(140-20)300 - \frac{116.187}{0.75\times10^{5}}\times300$$

= 0.828 – 0.465 = 0.363 mm

Actual change in length (increase in length) of steel bar

= 0.5 – 0.363 = 0.137 mm.

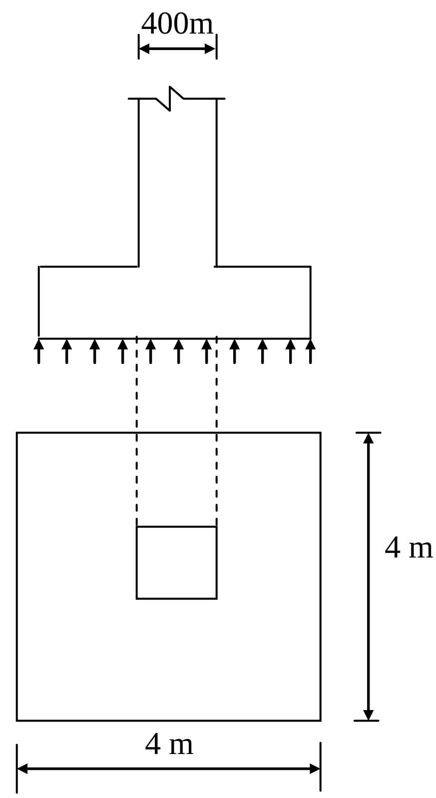
04(b).

Sol: **Given:** load from column = 1200 kN

Column size = 300 × 300 mm

SBC of soil = 110 kN/m^2

Self weight of footing = 100 kN



Step 1 : Size of footing

Total load = 1200 + 100 = 1300 kN

Area of footing required $= \frac{1300}{110} = 11.818\ m^2$

Since it is a square footing side = $\sqrt{11.818} = 3.43$ m

∴ Provide footing of size 4.0 m × 4.0 m

Net upward soil pressure at ultimate loads

$$= \frac{1200 \times 1.5}{4 \times 4} = 112.5\ kN/m^2$$

$$= 0.112\ N/mm^2$$

Step 2 : Effective depth of footing:

(a) From bending moment inertia

Critical section for bending moment is at the face of the column.

$$BM = \frac{P_o B}{8}(B-b)^2$$

$$\Rightarrow M_u = 112.5 \times 4 \frac{(4-0.4)^2}{8} = 729\ kNm$$

Effective depth required = $\sqrt{\frac{M_u}{0.138 f_{uk} b}} = \sqrt{\frac{729 \times 10^6}{0.138 \times 2.5 \times 4000}}$

= 229.839 mm ----------(1)

(b) From one-way shear

Critical section is at a distance 'd' is from the face of the column

$$V_u = P_o B\left(\frac{B-b}{2} - d\right)$$

$$= 112.5 \times 4\left(\frac{4-0.4}{2} - d\right)$$

= 450 (1.8 – d)

Shear stress (τ_v) = $\dfrac{V_u}{bd} = \dfrac{450(1.8-d)}{4d}$

Shear strength of concrete = 0.35 MPa = 350 N/m^2

To be safe in shear:

$\tau_v \le 0.35$ MPa

$\Rightarrow \dfrac{450}{4d}(1.8-d) \le 350$

$\Rightarrow 810 - 450\,d \le 1400\,d$

$\Rightarrow d \ge 0.437$ m

$\therefore \ge 437$ mm ----------(2)

(c) From two way shear

Critical section is at a distance $\dfrac{d}{2}$ from face of column.

Punching shear force $V = P_o\left(B^2 - (b+d)^2\right)$

$= 112.5\,(4^2 - (0.4 + d)^2)$

Punching shear stress = $\tau_v = \dfrac{112.5\left(4^2-(0.4+d)^2\right)}{4(0.4+d)d}$

Shear resistance of concrete = $k_s\,\tau_c$

$\tau_c = 0.25\sqrt{f_{ck}} = 0.25\sqrt{25} = 1.25\ \text{MPa} = 1250\ \text{kN/m}^2$

To be safe in two-way shear:

$\tau_v \le \tau_c$

$\Rightarrow \dfrac{112.5\left(4^2-(0.4+d)^2\right)}{4(0.4+d)d} \le 1250$

$\Rightarrow 4^2 - (0.4 + d)^2 \le 44.45\,(0.4 + d)d$

$\Rightarrow 15.84 - d^2 - 0.8\,d^2 \le 17.78\,d + 44.44\,d^2$

$\Rightarrow 45.44\,d^2 + 18.58\,d - 15.84 \ge 0$

d ≥ 0.420 m

≥ 420 mm --------- (3)

From (1), (2) and (3) effective depth required ≥ 437 mm

$\therefore$ Provide d = 450 mm, effective cover = 50 mm

$\therefore$ D = 500 mm

Step 3: Area of steel required

$$A_{SA} = \frac{0.5f_{ck}bd}{f_y}\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}bd^2}}\right]$$

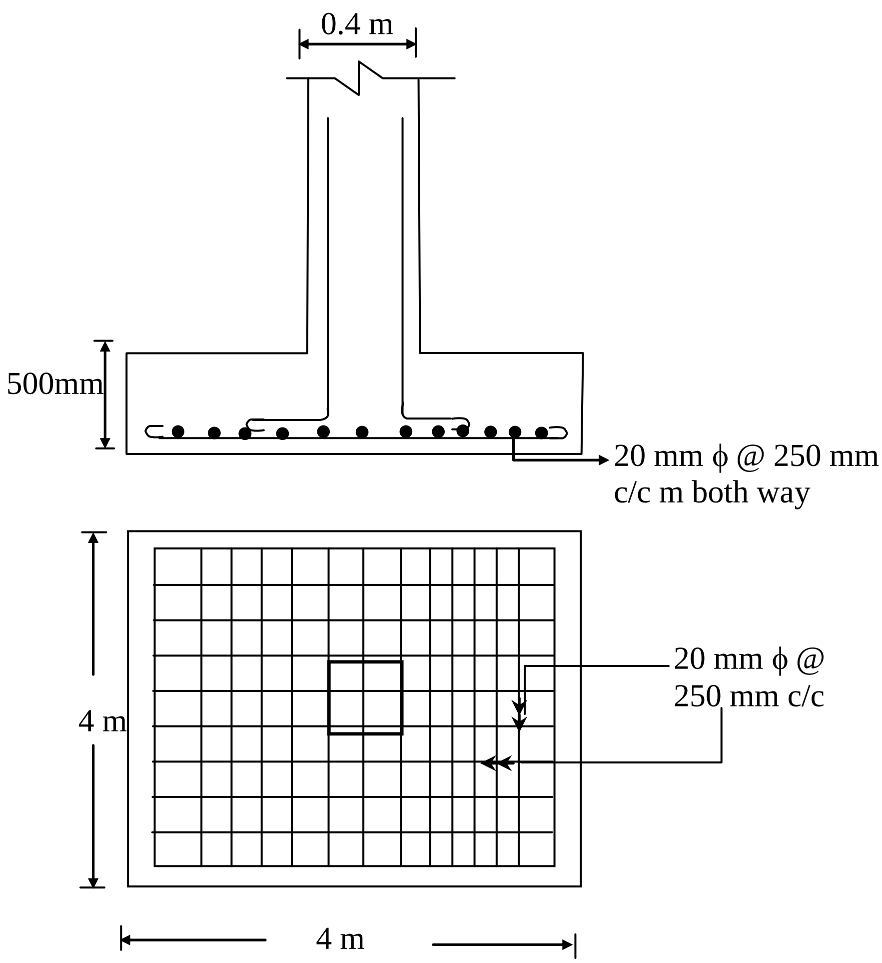
$$= \frac{0.5\times 25\times 4000\times 450}{415}\left[1-\sqrt{1-\frac{4.6\times 729\times 10^6}{25\times 4000\times 450^2}}\right]$$

$= 4692.2\ mm^2$

Using 20 mm bars; spacing required = $\frac{4000}{4692.2}\times\frac{\pi}{4}\times 20^2$

= 267.85 mm

∴ Provide 20 mm bars @ 250 mm c/c



04(c).

Sol: **Given:**

Cantilever tube length L = 120 mm

Axial tension P = 9 kN

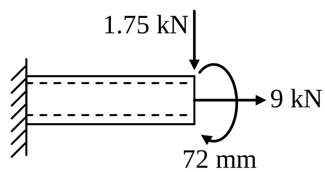
Torsional moment T = 72 N.m

Bending load F = 1.75 kN at free end

Yield strength of material = 267 MPa

Outside diameter = 50 mm

Factor of safety N = 4



Analysis:

Thickness of tube :

Axial stress, $\sigma_a = \frac{P}{A} = \frac{9\times10^3}{\pi\times50\times t}$ [Area of tube = πDt]

$$\sigma_a = \frac{57.29}{t}\,\text{N/mm}^2$$

Bending stress, $\sigma_b = \frac{M\times y}{I}$

Polar moment of Area, $J = \frac{\pi}{4}D^3\times t = 98174.77\ t\ \text{mm}^4$

$$J = 2I$$

$$\therefore\ I = 49087.38\ t\ \text{mm}^4$$

$$\sigma_b = \frac{P\times L\times y}{I} = \frac{1.75\times10^3\times120\times25}{49087.38t} = \frac{106.95}{t}\,\text{N/mm}^2$$

Tensile stress, $\sigma = \sigma_a + \sigma_b$

$$= \frac{57.29}{t} + \frac{106.95}{t}$$

$$= \frac{164.24}{t}\,\text{N/mm}^2$$

Torsional shear stress, $\frac{T}{J} = \frac{\tau}{R}$

$$\tau = \frac{T \times R}{J} = \frac{72 \times 10^3 \times 25}{98174.8t}$$

$$\tau = \frac{18.335}{t} \text{ N/mm}^2$$

Maximum stress $\sigma_1 = \frac{\sigma_t}{2} + \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$

$$= \frac{164.24}{2 \times t} + \sqrt{\left(\frac{164.24}{2t}\right)^2 + \left(\frac{18.335}{t}\right)^2}$$

$$= \frac{82.12}{t} + \frac{84.14}{t} = \frac{166.26}{t}$$

Yield strength of material = 267 MPa

N = 4

Allowable stress = $\frac{267}{4}$ = 66.75 N/mm^2

$$\sigma_1 = \frac{166.26}{t} = 66.75$$

t = 2.49 mm

05(a).

Sol: Given:

p = 8 MPa

Rod diameter = d = 25 mm

Plunger diameter = D = 80 mm

Length = L = 2500 mm

E = 210 × 10^3 MPa

$$A_{plunger} = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times 80^2 = 5026.55 \text{ mm}^2$$

$$A_{rod} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 25^2 = 490.87 \text{ mm}^2$$

Sudden force on plunger = water pressure × Area of plunger

P = 8 × 5026.55 = 40.21 kN

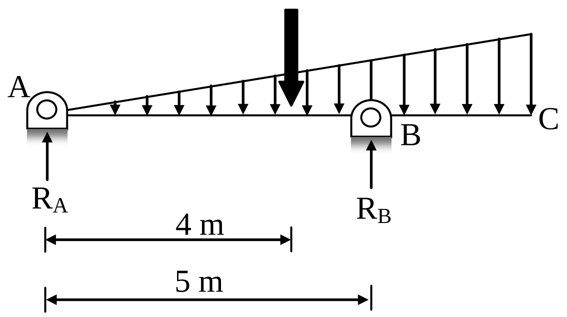
∴ Force on rod = Force on plunger = 40.21 kN

$$\therefore (\sigma_{inst})_{max} = \frac{2P}{A_{rod}} = \frac{2\times 40.21\times 10^3}{490.87} = 163.84\ \text{N/mm}^2$$

$$(\delta\ell_{inst}) = \frac{\sigma L}{E} = \frac{163.84\times 2500}{210\times 10^3} = 1.95\ \text{mm}$$

05(b).

Sol: Reactions



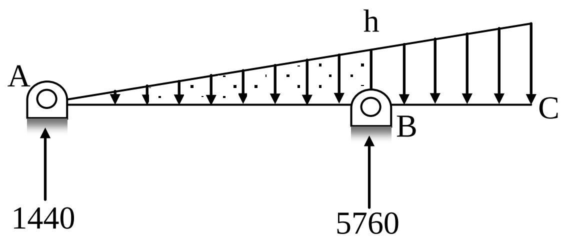
$\Sigma M_A = 0$

$$R_B \times 5 = \left(\frac{1}{2}\times 6\times 2400\right)\times 4$$

$R_B = 5760$ N

$\Sigma F_y = 0 \Rightarrow R_A + R_B$

$$= \frac{1}{2}\times 6\times 2400 \ \Rightarrow R_A = 1440\ \text{N}$$



Shear force calculation:

$F_A = 0,\ \ F_{A^+} = 1440$ N

$$F_{B^-} = 1440 - \left(\frac{1}{2}\times 5\times h\right)$$

$$\Rightarrow F_{B^-} = 1440 - \left(\frac{1}{2}\times 5\times \frac{2400}{6}\times 5\right) = -3560$$

$$\Rightarrow F_{B^+} = -3560 + 5760 = 2200$$

$F_C = 0$

Location of Point D where SF = 0

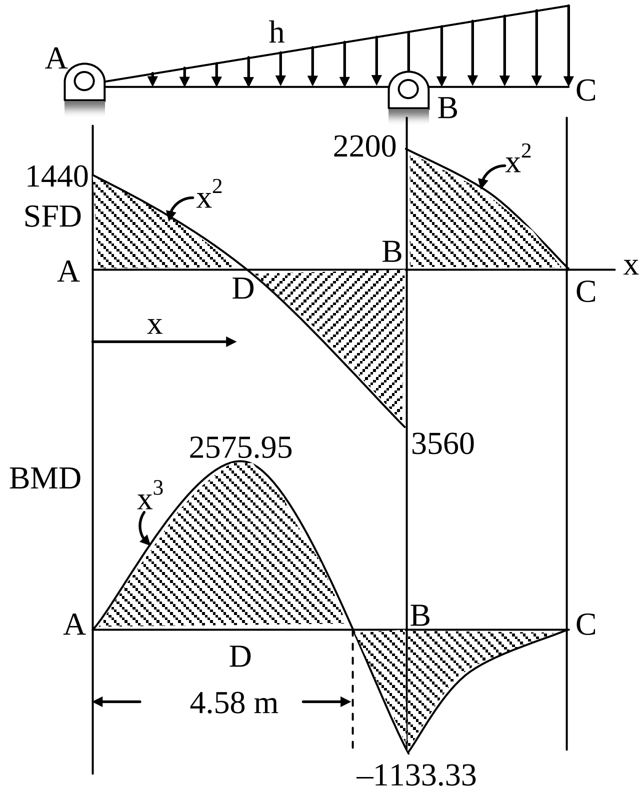
$$F_D = 1440 - \left(\frac{1}{2}\times x\times h\right)$$

$$\Rightarrow F_D = 1440 - \left(\frac{1}{2}\times x\times \frac{2400}{6}x\right)$$

$\Rightarrow 0 = 1440 - 200\,x^2$

$\Rightarrow x = 2.6833$ m from A

Bending moment calculation:



$M_A = 0$

$\Rightarrow M_D = 1440 \times x - \left(\frac{1}{2} \times x \times h\right) \times \frac{x}{3}$

$\Rightarrow M_D = 1440x - \frac{1}{2} \times x \times \frac{2400}{6} x \times \frac{x}{3}$

$\Rightarrow M_D = 1440x - 66.67\, x^3$

$\Rightarrow M_D = 1440 \times 2.6833 - 66.67 \times (2.6833)^3$

$\Rightarrow M_D = 2575.95$ Nm

$M_B = 1440 \times 5 - \left(\frac{1}{2} \times 5 \times \frac{2400}{6} \times 5\right) \times \frac{5}{3}$

$= -1133.33$

$M_C = 0$

Point of concraflexure:

$1400x - 66.67x^3 = 0$

$x = 4.58$ m

05(c).

Sol:

(i) **ABC Analysis**: This is based on Pareto's Law, which says that in any large group there are 'significant few' and 'insignificant many'. For example, only 20 percent of the items may be accounting for 80 percent of the total material cost procured by a construction organization. Here, the 20 percent constitute the 'significant few' that require utmost attention.

To prepare on ABC – type curve, we may follow a simple procedure:

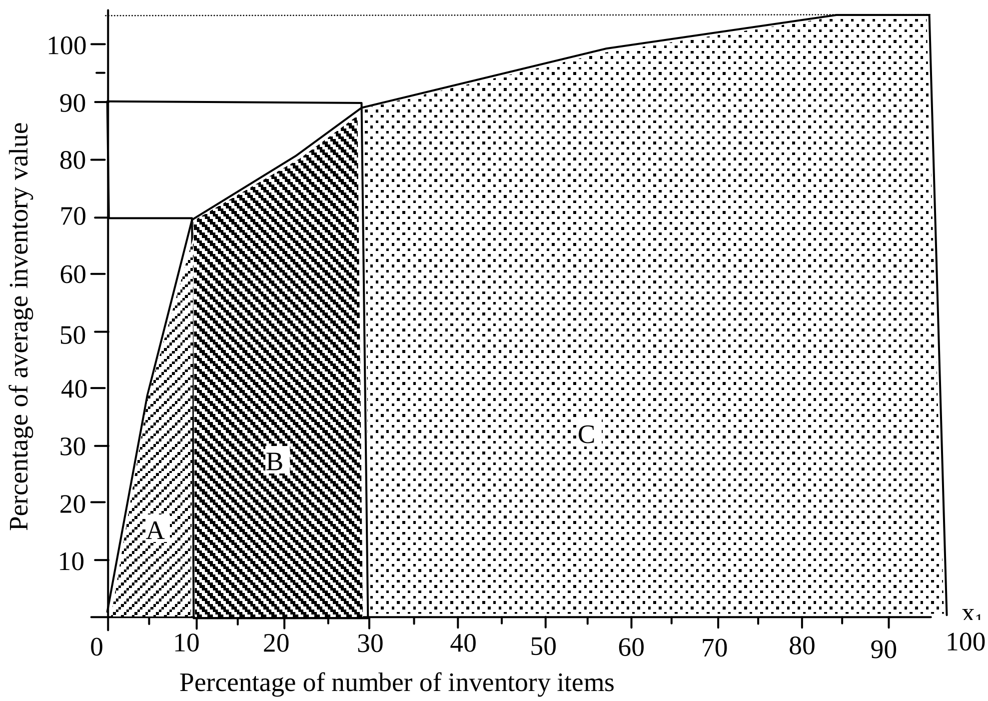
1. Different materials required for the project are identified and their estimated quantities worked out. The quantity estimate could be on the basis of either annual consumption or the project's total requirement.

2. The unit rates of materials are estimated

3. The usage values for each of the materials are obtained by multiplying the estimated quantities and their unit rates. These values are converted into percentage of total annual usages cost or total project cost, as the case may be.

4. The percentage usage cost for each of the materials is arranged in the descending order of their ranking, starting with the first rank, i.e., highest to lowest usage value. The cumulative percentage usage value is also calculated.

5. A curve as shown in Figure is plotted, and points on the curve at which there are perceptible sudden changes of slopes are identified. In the absence of such sharp points, cut-off points corresponding to the top 10 percent and the next 20 percent or so are marked as a general indicator of A, B and C type of materials.



6. According to an empirical approach, ‘A’ class items account for about 70 percent of the usage value, ‘B’ class items for about 20 percent of the usage value, and ‘C’ class items for about 10 percent of the usage value. In terms of numbers, ‘A’ class items constitute about 10 percent of total items, ‘B’ class items about 20 percent of total items, and ‘C’ class items about 70 percent of total items. These percentages are indicative only and can vary depending on a number of factors.

Upon classification of material into A, B and C types, suitable inventory policies can be decided. Corresponding to each type of materials, the implications on inventory policy are mentioned below:

Item type 'A'. The salient features are:

- Accurate forecast of quantities needed.
- Involvement of senior level for purchasing
- Ordering is on requirement basis.
- Enquiries for procurement need to be sent to a large number of suppliers
- Strict degree of control is required, preferably monitoring on a weekly basis
- Low safety stock is needed.

Item type 'B'. The salient features are:

- Approximate forecast of quantities needed.
- Requires involvement of middle level for purchasing
- Ordering is on EOQ basis
- Enquiries for procurement need to be sent to three to five reliable suppliers
- Moderate degree of control required, preferable monitoring on a monthly basis
- Moderate safety stock needed.

Item type 'C'. The salient features are:

- No need of forecasting; even rough quantity estimate is sufficient
- Junior-level staff is authorized to order purchase
- Bulk ordering is preferred
- Quotations from even two to three reliable suppliers are sufficient.
- A relatively relaxed degree of control is sufficient, and monitoring can be done on a quarterly basis.
- Adequate safety stock can be maintained.

(ii) VED Analysis:

This analysis attempts to classify items into three categories depending on the consequences of material stock-out when demanded. As stated earlier, the cost of shortage may vary according to the seriousness of such a situation.

Thus, the items are classified into V(vital), E (essential) and D (desirable) categories. Vital items are the most critical having extremely high opportunity cost of shortage and must be available in stock when demanded. Essential items are quite critical with substantial cost associated with shortage and should be available in stock by and large. Desirable group of items do not have serious consequences if not available when demanded, but these can be stocked items.

Obviously, the percentage risk of shortage with the vital group of items has to be kept quite small, thus calling for a high level of service. With 'essential' category we can take a relatively higher risk of shortage, and for 'desirable' category, even higher. Since even a C – class item may be vital or an A – class item may be desirable, we should carry out a two – way classification of items grouping them in nine distinct group as A-V, A-E, A-D, B-V, B-E, B-D, C-V, C-E and C-D. We can then determine the aimed service level for each of these nine categories and plan for inventories accordingly.

Vital group comprises those items for the want of which the production will come to a stop - for example, power in the factory. Essential group features those items for whose non-availability the stock-out cost is very high. Desirable group contains items whose non – availability causes no immediate loss of production; the stock cost involved is very less and their absence may only cause minor disruption in the production for a short time.

The steps used for classifying materials as vital, essential and desirable are given below:

Step 1: Factors such as stock-out case, lead time, nature of items, and sources of supply are identified and considered for VED analysis.

Step 2: Assign points or weightages to the factors according to the importance they have to the company, as shown above.

Step 3: Divide each factor into three degrees and allocate points to each degree.

Step 4: Prepare categorization plan to provide basis for classification of items – for example, items scoring between 100 and 160 can be classified under desirable items; items between 161 and 230 can be classified under essential items; and items between 231 and 300 can be classified under vital items.

Step 5: Specify the degree and allocate weightages to all the factors.

Step 6: Evaluate and find the final score for every item, and specify the type of item.

(iii) FSN Analysis:

Not all items are required with the same frequency. Some materials are required quite regularly some are required very occasionally, and yet some others may have become obsolete.

And might not have been demanded for years together, FSN analysis groups them as fast-moving, slow- moving and non-moving (dead stock), respectively. Inventory policies and models for the three categories have to be different. Most inventory models in literature are valid for the fast – moving items exhibiting a regular movement (consumption) pattern. Many spare parts come under the slow – moving category, which have to be managed on a different basis. For non-moving dead stock, we have to determine optimal stock disposal rules rather than inventory provisioning rules. Categorization of materials into these three types on value and critical usage enables us to adopt the right type of inventory policy to suit a particular situation.

'F' items are those items that are fast-moving – i.e., in a given period of time, say a month or a year, they have been issued a number of times. However, 'fast-moving' does not necessarily means that these items are consumed in large quantities.

'S' items are those items that are slow-moving – in the sense that in the given of time they have been issued in a very limited number.

'N' or non-moving items are those that are not at all issued for a considerable period of time.

Thus, the stores department, which is concerned with the moving of items, would prefer to classify items in the categories F-S-N, so that they can manage, operate and plan stores activities accordingly. For example, for efficient operations it would be necessary that fast-moving items are stored as near as possible to the point of issue, for these to be issued with minimum of handling. Also, such items must be stored at the floor level, avoiding higher heights. Thus, if the items are slow-moving or issued once in while in a given period of time, they can be stored in the interior of the stores and even at greater heights because handling of these items becomes rare. Further, it is necessary for the stores in charge to know about non-moving items for various reasons meant mentioned below:

1. Non-moving items mean unnecessary blockage of money which affect the rate of returns of the company.
2. Non-moving items also occupy valuable space in the stores without any usefulness and therefore, it becomes necessary to identify these items and find reasons for their non-moving status. If justified, recommendation may be made to top management for the speedy disposal so that company operations are performed efficiently.

To some extent, inventory control can be exercised on the basis of FSN analysis. For example, fast-moving items can be controlled more severely, particularly when their value is also high. Similarly, slow – moving items may not be controlled and reviewed very frequently since their consumption may not be frequent and their value may not be high.

05(d).

Sol: **Assumptions in the Limit State method of design in flexure :**

i) Plane sections normal to the axis of the member remain plane after bending

ii) The tensile strength of concrete is ignored

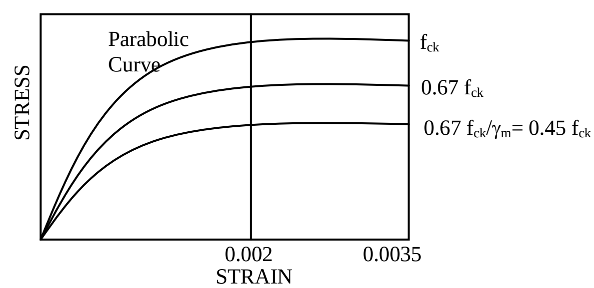
iii) The maximum strain in concrete at the outermost compression fiber is 0.0035

iv) The compressive stress strain curve may be assumed to be rectangular, trapezoidal, parabola or any other shape which results in the prediction of strength in substantial agreements with the results of tests.

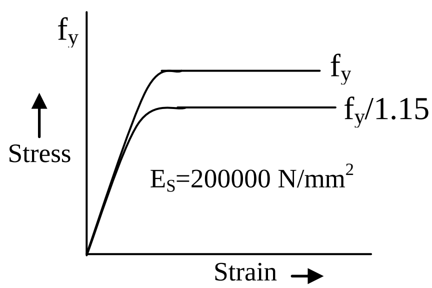
- An acceptable stress strain curve (rectangular- parabolic) is shown aside.
- Compressive strength of concrete in the structure is assumed to be 0.67 times the characteristic strength of concrete.

- The partial strength of concrete in addition to it.
- Therefore, the design strength of concrete is $0.67\ f_{ck} / 1.5$ i.e. $0.446\ f_{ck} \approx 0.45\ f_{ck}$

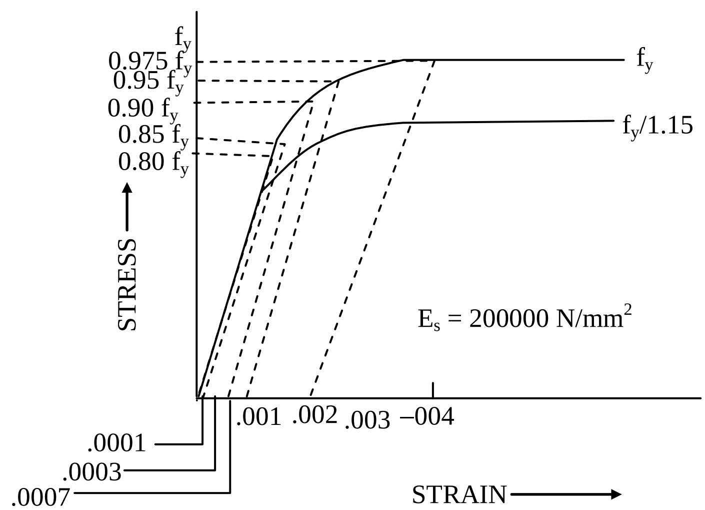
v) The design stress in reinforcement is derived from the stress strain curves given below for mild steel and cold worked deformed bars respectively. The partial factors of safety 'γ_m' equal to 1.15 is applied to the strength of reinforcement. Therefore the design strength of reinforcement is $f_y / 1.5$. i.e. $0.87\ f_y$



σ – ε curve for concrete



Steel Bar with Definite Yield Point



Cold Worked Deformed Bar

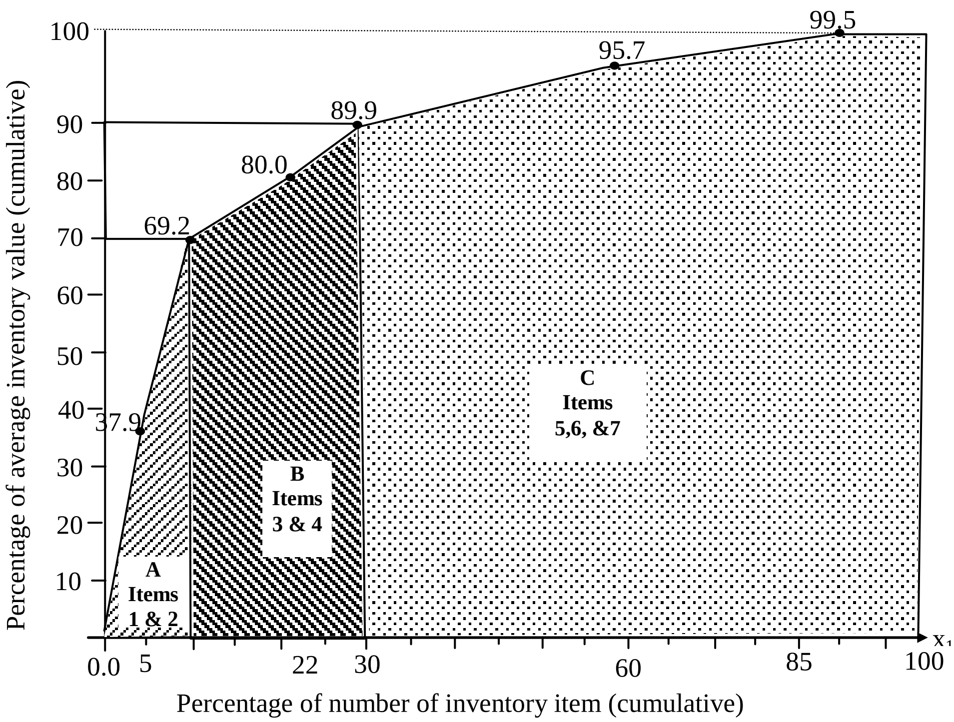
(vi) The maximum strain in the tension reinforcement in the section at failure should not be less than $0.002 + \dfrac{0.87 f_y}{E_S}$

05(e).

Sol:

Item	Units	% of total	Cumulative % of total item	Unit cost (Rs)	Total cost (Rs. lakh)	% of total cost	Cum % of total cost
1	10000	5.00	5	121.50	12.15	37.9	37.9
2	10000	5.00	10	100.00	10.00	31.2	69.2
3	24000	12.00	22	14.50	3.48	10.9	80.0
4	16000	8.00	30	19.75	3.16	9.9	89.9
5	60000	30.00	60	3.10	1.86	5.8	95.7
6	50000	25.00	85	2.45	1.225	3.8	99.5
7	30000	15.00	100	0.50	0.15	0.5	100.5
Total	200000	100.00			32.025	100.00	

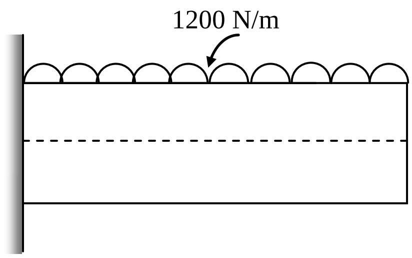
A graph as in figure is drawn between cumulative percentage of cost and cumulative percentage of numbers. From the graph, it is clear that about 70 percent cost. It is consume in 10 percent of the inventory item. These are for items 1 and 2. Thus, items 1 and 2 are class.



'A' items. Similarly, items 3 and 4 have a cost share of about 20 percent (89.9 percent – 69.2 percent) and an inventory share of 20 percent. Thus, items 3 and 4 are Class 'B' items. Finally, we can see that items 5, 6 and 7 belongs to Class 'C' as their cost share is about 10 percent (100 percent – 89.9 percent) and inventory share is 70 percent.

06(a).

Sol:



SF at fixed support = 1200 × 3

F = 3600 N

Bending moment at fixed support, $M = (1200 \times 3) \times \frac{3}{2} = 5400\ \text{Nm}$

Effect of SF:

Now, shear stress at point 15 mm above neutral axis due to shear force of 3600 N is given as

$$\tau = \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right) = \frac{6 \times 3600}{40 \times 60^3}\left(\frac{60^2}{4} - 15^2\right)$$

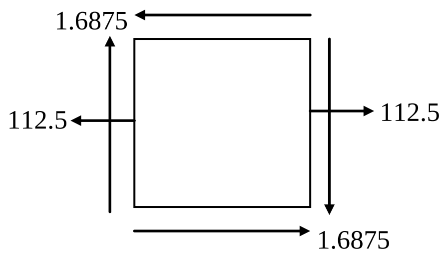
τ = 1.6875 MPa

Effect of BM:

Normal stress at point 15 mm above neutral axis due to bending moment of 5400 Nm is calculated as follows

$$\sigma = \frac{M}{I}y = \frac{5400 \times 10^3}{40 \times \frac{60^3}{12}} \times 15 = 112.5\ \text{MPa}$$

Now, the state of stress at given point is known and is represented in figure



Principal Stresses Calculation:

$\sigma_{1,2} = \sigma_{avg} \pm R$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{112.5 + 0}{2} \pm \sqrt{\left(\frac{112.5 - 0}{2}\right)^2 + (1.6875)^2}$$

$\sigma_{1,2} = 56.25 \pm 56.2753$

$\sigma_1 = 112.5253$ MPa and $\sigma_2 = -0.0253$ MPa

06(b). (i)

Sol: $E = 2\,G\,(1 + \mu)$

$105 = 2G\,(1 + 0.3)$

$G = 40.38$ GPa

$$G = \frac{\text{shear stress}}{\text{shear strain}}$$

$$0.404 \times 10^5\ \text{N/mm}^2 = \frac{\tau}{3.98 \times 10^{-4}}$$

$$\tau = 16.08\ \text{N/mm}^2$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$T = \frac{\tau J}{R}$$

$$= \frac{16.08 \times \frac{\pi}{32} \times 60^4}{30}$$

$T = 681.63$ Nm

$T = 0.681$ kNm

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 800 \times 0.681}{60}$$

$P = 57.02$ kW

06(b). (ii)

Sol: Given: S_{yt} = 240 MPa; μ = 0.3

$\sigma_1 = \sigma$; $\sigma_2 = 0.5\sigma$; $\sigma_3 = 0$

1. Maximum principal strain theory:

$$\varepsilon_1 \le \frac{S_{yt}}{E}$$

$$\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} \le \frac{S_{yt}}{E}$$

$$\Rightarrow \quad \sigma_1 - \mu\sigma_2 - \mu\sigma_3 \le S_{yt}$$

$$\sigma - (0.3)\,(0.5\sigma) - (0.3)\,(0) \le 240$$

$$0.85\sigma \le 240$$

$$\therefore\ \sigma = 282.35\ \text{N/mm}^2$$

2. Maximum shear stress theory:

$$\tau_{max} \le \frac{S_{yt}}{2}$$

$$\frac{\sigma_1 - \sigma_3}{2} \le \frac{S_{yt}}{2}$$

$$\frac{\sigma - 0}{2} \le \frac{240}{2}$$

$$\sigma = 240\ \text{N/mm}^2$$

3. Maximum strain energy theory:

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \le \frac{S_{yt}^2}{2E}$$

$$\Rightarrow \quad \left[\sigma^2 + (0.5\sigma)^2 + 0^2 - 2(0.3)(\sigma(0.50) + 0 + 0)\right] \le 240^2$$

$$\therefore \quad 0.95\sigma^2 \le 240^2$$

$$\sigma = 246.23\ \text{N/mm}^2$$

06(c).

Sol: Given: $\phi = 30^o$, $\gamma_{soil} = 18$ kN/m^3, $\gamma_{cone} = 25$ kN/m^3

SBC of soil = 150 kN/m^2

$\mu = 0.5$

Active earth pressure coefficient $K_a = \dfrac{1-\sin\phi}{1+\sin\phi}$

$$= \frac{1-\sin 30}{1+\sin 30} = \frac{1}{3}$$

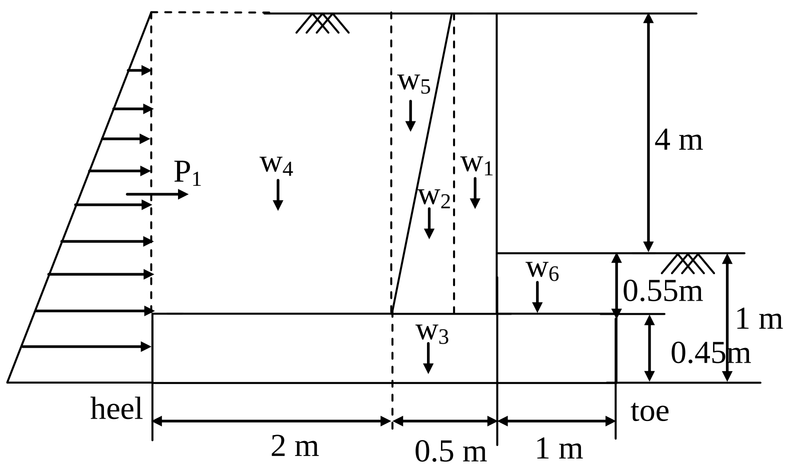
Total height of retaining wall H = 4 + 1 = 5 m

Considering unit width:

Active earth pressure on the wall $= \dfrac{1}{2}K_a\gamma H^2$

$$= \frac{1}{2}\times\frac{1}{3}\times 18\times 5^2$$

$$= 75 \text{ kN at } \frac{H}{3} = \frac{5}{8} \text{ from toe}$$



S.No	Designation	Force (kN)	Leveraram (m)	Moment about toe (kNm)
1.	W_1	25 × 0.25 × 4.55 = 28.43 kN	1+0.25 = 1.125 m	31.98
2.	W_2	$\frac{1}{2}\times 0.25\times 4.55\times 25 = 14.22$ kN	$1+0.25+\frac{0.25}{3}=1.33$	18.96
3.	W_3	25 × 3.5 × 0.45 = 39.375 kN	$\frac{3.5}{2}=1.75$	68.91
4.	W_4	18 × 2 × 4.55 = 163.8	$1.5+\frac{2}{2}=2.5$	409.5
5	W_5	$18\times\frac{1}{2}\times 0.25\times 4.55 = 10.24$	$1.25+\frac{2}{3}(0.25) = 1.417$	14.51
6.	W_6	18× 0.55 × 1 = 9.9	$\frac{1}{2}=0.5$m	4.95
		ΣW = 265.96 kN		$\Sigma M_R = 548.81$

(i) Check for Overturning:

Overturning moment = $P.\frac{H}{3}$

$$M_o = 75\times\frac{5}{3} = 125 \text{ kN}$$

Resisting moment $\Sigma M_R = 548.8$ kNm

FOS against overturning $= \frac{548.81}{125} = 4.39 > 2$

Hence safe against overturning

(ii) Check For Sliding:

ΣH = P = 75 kN

ΣW = 265.96 kN

FOS against sliding $= \frac{\mu\Sigma W}{\Sigma H} = \frac{0.5\times 265.96}{75} = 1.773 > 1.5$

Hence safe against sliding

(iii) Check for tension Failure:

Net moment M = $M_R - M_o$ = 548.81 – 125 = 423.81 kNm

$$\bar{x} = \frac{M}{\Sigma V} = \frac{423.81}{265.96} = 1.593 \text{ m from toe}$$

$$e = \frac{b}{2} - \bar{x} = \frac{3.5}{2} - 1.593 = 0.156 \text{ m}$$

$$\frac{b}{6} = \frac{3.5}{6} = 0.583 \text{ m}$$

$$e < \frac{b}{6}$$

∴ no tension failure

(iv) Check for Maximum Pressure:

$$P_{max} = \frac{\Sigma W}{b}\left(1 + \frac{6e}{b}\right)$$

$$= \frac{265.96}{3.5}\left(1 + \frac{6 \times 0.156}{3.5}\right)$$

= 96.31 kN/m^2 < SBC of soil = 150 kN/m^2

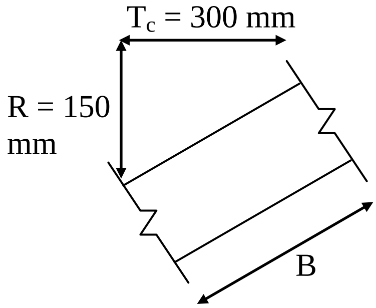
$$P_{min} = \frac{\Sigma W}{b}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{265.96}{3.5}\left(1 - \frac{6 \times 0.156}{3.5}\right) = 55.66 \text{ kN/m}^2$$

Since P_{max} < SBC of soil, safe against crushing failure of foundation soil

07(a).

Sol: Effective span(c/c between supports) = $3+1.5+\frac{0.3}{2}-\frac{0.3}{2}$ = 4.5 m



Thickness of waist slab and landing = 230 mm

Effective cover = 30 mm

Effective depth = 230 – 30 = 200 mm

Step 1: Load Calculation:

a) On going

(i) Self weight of waist slab = on plan area

$$=\frac{WB}{T}\times\gamma_{cone}$$

$$\left[B=\sqrt{R^2+T^2}=\sqrt{150^2+300^2}=335.4\text{ m}\right]$$

$$=\frac{0.23\times0.335}{0.3}\times25$$

= 6.42 kN/m^2

(ii) Self weight of single step = $\left(\frac{1}{2}RT\gamma_{cone}\right)\times\frac{1}{\sigma}$

$$=\frac{R}{2}\gamma_{cone}$$

$$=0.15\times\frac{2.5}{2}$$

= 1.875 kN/m^2

(iii) Floor finish = 1 kN/m^2

(iv) Live load = 3 kN/m^2

Total load = 12.295 kN/m^2

Factored load = 1.5 × 12.298 = 18.44 kN/m^2

b) Loads on Landing:

(i) Self weight of slab = 25 × 0.23 = 5.75 kN/m^2

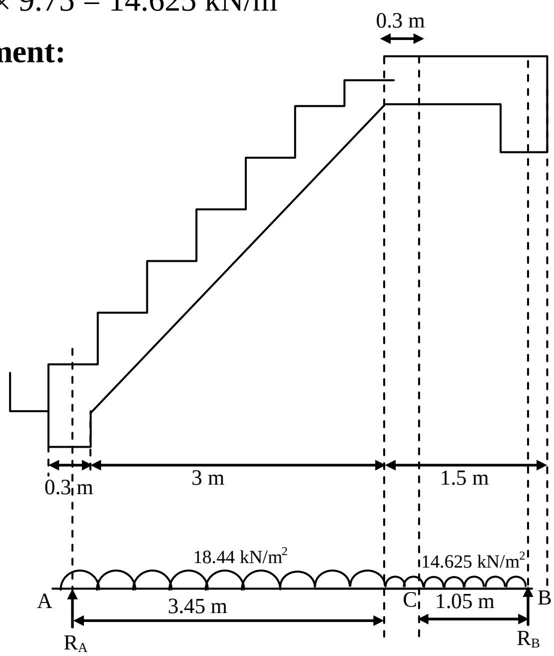
(ii) Finishes = 1 kN/m^2

(iii) Live load = 3 kN/m^2

Total load = 9.75 kN/m^2

Factored load = 1.5 × 9.75 = 14.625 kN/m^2

Step 2: Design Moment:



For equilibrium: $R_A + R_B = (18.44 \times 3.45) + (14.625 \times 1.05)$

$= 78.97$ kN/m

$$\Sigma M_B = 0 \Rightarrow R_A (3.45 + 1.05) - 18.44 \times 3.45 \times \left(1.05 + \frac{3.45}{2}\right) - 14.625 \times 1.05 \times \frac{1.05}{2} = 0$$

$\Rightarrow R_A = 41.02$ kN/m

$R_B = 37.94$ kN/m

At section of maximum BM, SF = 0

Assuming section is at in 'AC' at a distance 'x' from A

$\Rightarrow 41.02 - 18.44x = 0$

$\Rightarrow x = 2.224 \text{ m} < 3.45 \text{ m}$

Hence assumption is correction

Maximum BM = $41.02 \times 2.224 - 18.44 \times \frac{2.2242}{2}$

= 45.645 kNm/m

Step 3: Reinforcement Calculation:

$$A_{st} = 0.5\frac{f_{ck}}{f_y}bd\left[1-\sqrt{1-\frac{4.6M_u}{f_{ck}bd^2}}\right]$$

Considering unit width

$$= 0.5 \times \frac{20}{415} \times 1000 \times 200\left[1-\sqrt{1-\frac{4.6 \times 45.645 \times 10^6}{20 \times 1000 \times 200^2}}\right]$$

= 680.47 mm^2

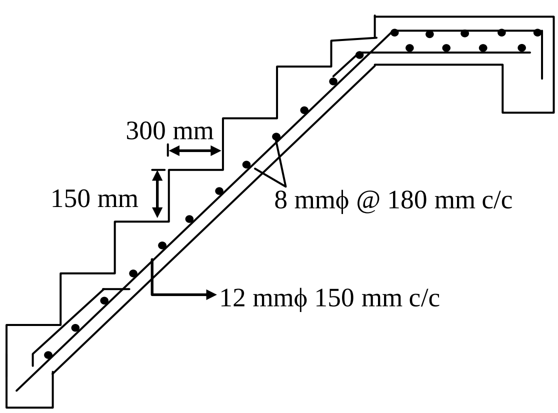
Using 12 mmϕ bars - spacing required $= \frac{1000}{680.47} \times \frac{\pi}{4} \times 12^2$ = 166.20 mm

∴ Provide 12 mmϕ bars @ 150 mm c/c

Distribution reinforcement required = $\frac{0.12}{100} \times 1000 \times 230$ = 276 mm^2

Using 8 mm ϕ bars; Spacing required $\frac{1000}{276} \times \frac{\pi}{4} \times 8^2$ = 182.12 mm

∴ Provide 8 mm ϕ bars @ 180 mm c/c



07(b). (i)

Sol: Production of the bull dozer = Maximum production × correction factors

$$= 430\left(\frac{m^3}{hr}\right)\text{loose}\times(0.84)(0.6)(0.8)(1.2)(0.87)$$

= 181 m^3 loose / hour

$\therefore$ Production of the bull dozer (Bank cumic meter) = 181 m^3 $\frac{\text{loose}}{\text{time}}$ × Swell factor = (0.769)

= 139.19 Bank cubic meter / hour

07(b). (ii)

Sol: Clearing vegetation and over burden & trees/plants is necessary before moving and shaping the site ground.

Step 1: Determine the size of (A) area to clear (in sq. meters)

Step 2: Determine the size and number of dozers available (n)

Step 3: Determine the time required (D) (hours/sq. kms) for clearing based on dozer size

Step 4: Determine efficiency factor $E = \frac{\text{Actual Working minutes per hour}}{\text{60 minutes working hour}}$

Step 5: Determine the operator factor (O) from tables like day light and night hours worked. It varies from Day time (0.6 to 1.0) and Night time (0.45 to 0.75)

$\therefore$ Total time (hours) = $\frac{D\times A}{E\times O\times n}$

07(b). (iii)

Sol: Major components and controls on excavator vary among makers and models. The following are as follows

1. Under carriage
2. Track chain
3. Engine/prime move compartment
4. Operator's cab
5. Boom
6. Dipper stick

7. Dipper (or) Bucket
8. Hoist unit
9. Mast
10. Track rollers
11. Track frame
12. Support rollers

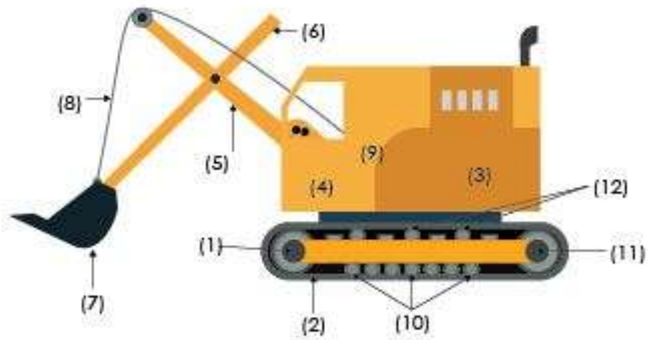


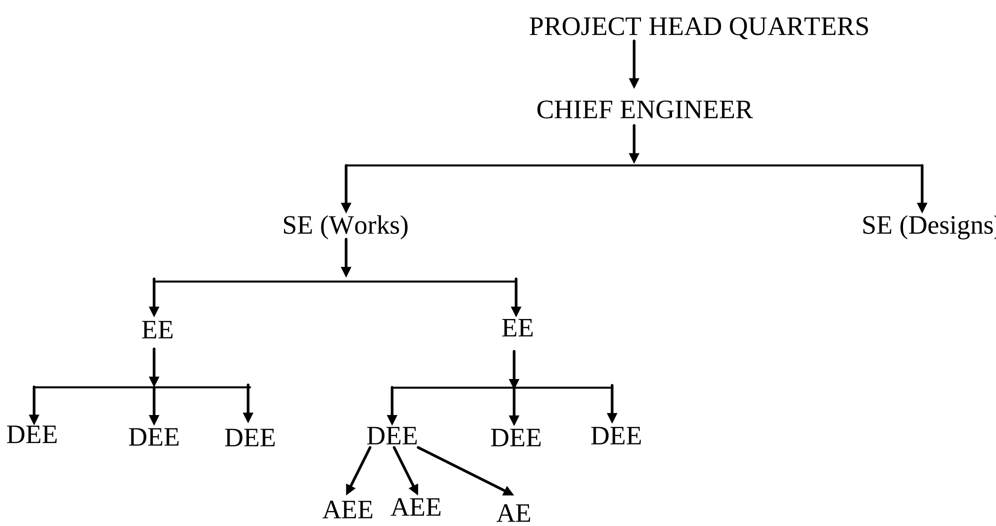
Figure shows the excavator with different parts.

07(b). (iv)

Sol: Line organisation :

It is an organisation where in the responsibility is distributed from top to bottom in a triangular pattern. It is also known as Military organisation.

It is generally used for Civil Engineering constructions which are not of very large magnitude.



Merits:

1. The command and control is very effective.
2. Responsibilities at all levels are defined and fixed
3. The system enables quick decision to be taken at all levels.
4. Each individual carries out order of his next superior and thus helps in maintaining rigid discipline.

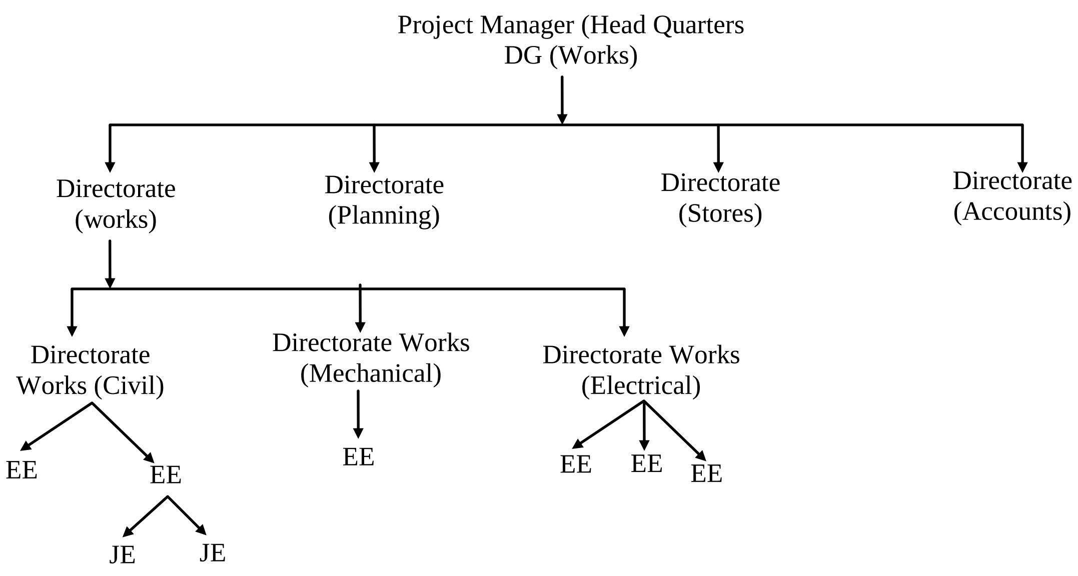
Demerits:

1. Difficult to interchange of work due to rigidity of the structure.
2. Communications are routed through a number of intermediate channels.
3. In line organisation setup, the top executive has to take decision in respect of all matters related to the project. The system is therefore not suitable for large projects requiring specialist services, though for routine type of projects, it may be quite useful.
4. The persons doing important jobs are liable to get over worked.

02. Line and staff organisation:

Line and staff organisation is the one which has the benefits of line organisation such as rigidity in discipline and stability of the structure are retained and the expert knowledge of the specialists is added to it.

This system is generally used for very large Civil Engineering construction works.



Merits:

1. Expert advice is available at all levels and maximum use of specialists is made.
2. The operations are efficient and hence method economical inspite of additional expenditure on experts.
3. The staff at site is competent to take decisions and hence work is not delayed.

Demerits:

1. If the duties of experts are not properly defined, there may be misunderstanding between the staff and line workers.
2. The expert advice given by specialists may get distorted by the time it is passed on to field worker.

07(c).

Sol: Size of the footing

Area of the footing

W : Column load

W′ : Self weight of the footing (10% of column load)

$$A = \frac{W + W'}{q_o}$$

$$= \frac{1000 + 100}{200} = 5.5\ m^2$$

Side of square footing $= \sqrt{5.5} = 2.34$ m

Provided size of the footing = 2.5 m × 2.5 m

Net upward soil pressure

$$p_o = \frac{\text{Column load}}{\text{Provided area of footing}}$$

$$= \frac{1000}{2.5 \times 2.5} = 160\ kN/m^2 < 200\ kN/m^2$$

Effective depth of the footing

The following are the governing factors to decide the depth of the footing

- Maximum bending moment
- One way shear force

- Punching shear force

(i) From Maximum bending moment:

The critical section for bending moment lies at the face of the column

Bending moment

$$M = \frac{P_o B}{8}[B - b]^2$$

$$= \frac{160 \times 2.5}{8}[2.5 - 0.3]^2 = 242 \text{ kNm}$$

Factored bending moment $M_u = 1.5$ m

$$= 1.5 \times 242$$

$$= 363 \text{ kN-m}$$

Effective depth required $d = \sqrt{\frac{M_u}{R_u B}}$

$$= \sqrt{\frac{363 \times 10^6}{0.138 \times 20 \times 2500}}$$

$$= 229.36 \text{ mm}$$

(ii) From one way shear

Critical section for one way shear lies at a distance d from the face of the column.

Where d: Effective depth of the footing

Shear force

$$V = P_o B\left[\frac{B}{2} - \frac{b}{2} - d\right]$$

$$= 160 \times 2.5\left[\frac{2.5}{2} - \frac{0.3}{2} - d\right]$$

Factored shear force

$$V_u = 1.5\ V$$

Nominal shear stress

$$\tau_v = \frac{V_u}{Bd}$$

From safety consideration

$$\tau_v \le k\tau_c$$

Where k is constant it depends on thickness of the footing

In general thickness of the footing more than 300 mm hence k = 1

$\tau_v = \tau_c$

$$\frac{1.5 \times 160 \times 2.5\left[\frac{2.5}{2} - \frac{0.3}{2} - d\right]}{2.5d} = 350$$

$$d = 0.447 \text{ m}$$

$$= 447 \text{ mm}$$

(iii) Check for two way shear

For checking the maximum of the above two effective depths is taken

The critical section for two way shear lies at $\frac{d}{2}$ from the peripheral of column face

Punching shear force

$V = P_o \times [B^2 - (b + d)^2]$

$= 160\ [2.5^2 - (0.3 + 0.447)^2]$

$= 910.7$ kN

$V_u = 1.5$ V

$= 1.5 \times 910.7$

$= 1366.05$ kN

Punching shear stress

$$\tau_v = \frac{V_u}{b_o \times d}$$

Where b_o perimeter = 4 (b + d)

$$\tau_v = \frac{1366.05 \times 10^3}{4(300 + 447)447}$$

$= 1.02 \text{N/mm}^2$

shear strength of concrete

$$\tau_c = 0.25\sqrt{f_{ck}}$$

$= 0.25\sqrt{20}$

$= 1.118 \text{ N/mm}^2$

$K_s = 0.5 + \beta_c \not> 1$

$\beta_c = \dfrac{300}{300} = 1$

$\therefore K_s = 1$

$\tau_v < K_s\,\tau_c$ $\therefore$ safe

Provided D = 560 mm

d = 500 mm > 477 mm

$\therefore$ It is an under reinforced section

Area of tension steel required

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6 M_{UD}}{f_{ck} B d^2}}\right] Bd$$

$$= \frac{0.5\times 20}{415}\left[1-\sqrt{1-\frac{4.6\times 363\times 10^6}{20\times 2500\times 500^2}}\right] 2500\times 500$$

$= 2083.89\ mm^2$

Spacing

Use 12 mm diameter bars

$S = B \times \dfrac{a_{st}}{A_{st}}$

$$= 2500\times\frac{\frac{\pi}{4}\times 12^2}{2083.89}$$

$= 135.23$

Provided 12 mm diameter @ 130 mm c/c.

08(a).

Sol: Parameters:

σ_{cbc} = 8.5 MPa

$$m = \frac{280}{3\sigma_{cbc}} = \frac{180}{3\times 8.5} = 10.98 \cong 11$$

$$k = \frac{m\sigma_{cbc}}{m\sigma_{cbc} + \sigma_{st}} = \frac{11\times 8.5}{11\times 8.5 + 150} = 0.384$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.314}{3} = 0.872$$

$$Q = \frac{1}{2}\sigma_{cbc} jk = \frac{1}{2}\times 8.5\times 0.384\times 0.872$$

= 1.423

Design of long wall:

$$\frac{L}{B} = \frac{8}{3} = 2.667 > 2$$

$\therefore$ $\frac{L}{B} > 2$; long walls are designed as cantilevers fixed at base and subjected to triangularly distributed load.

- Maximum bending moment at the base of these wall = $\frac{\omega H^3}{6} = \frac{9.81\times 3.53}{6} = 70.1\text{kNm}$
- Depth of the section required

M = Qbd^2

$$\Rightarrow d = \sqrt{\frac{M}{Qb}}$$

$$\Rightarrow d = \sqrt{\frac{70.1\times 10^6}{1.423\times 1000}} = 221.95\text{mm}$$

$\therefore$ Provide d = 225 mm; off. Cover = 35 mm

$\therefore$ D = 226 + 35 = 260 mm

Area of steel required:

$$A_{st} = \frac{M}{\sigma_{st} jd} = \frac{70.1\times10^6}{150\times0.372\times225} = 2381.92\text{mm}^2$$

Using 20mm ϕ bars, spacing required = $\frac{1000}{2381.92}\times\frac{\pi}{4}\times20^2 = 131.89\text{mm}$

$\therefore$ Provide 20 mmϕ bars @ 130 mm C/C is vertical direction.

Curtailment of bars:

BM α h^3 ['h' is distance from top)

Distance where alternate bars can be curtailed.

$$\frac{h^3}{H^3} = \frac{1}{2} \Rightarrow R^3 = \frac{(3.5)^3}{2}$$

$\Rightarrow$ h = 2.77 m from top

= 0.72 m from bottom

As per code: - actual curtailment is

= 0.72 + 12ϕ

$= 0.72 + \frac{12\times20}{1000}$

= 0.96 m

$\therefore$ Curtail alternate bars at 0.96 m from base.

Horizontal reinforcement:

Intensity of pressure at 1m above base = w(H – h);

$\left\{h = \max\left(\frac{H}{4},1\right)\right\}$ = 9.81(3.5 – 1) = 24.525 kN/m^2

Direct tensile force due to shortwall on long walk

$$T = \omega(H-h)\frac{1}{2}$$

$$T = 9.81(3.5-1)\times\frac{3}{2} = 36.78\text{kN}$$

Area of steel required = $\dfrac{T}{\sigma_{st}}$

$= \dfrac{36.78 \times 1000}{150}$

$= 245.25\ \text{mm}^2$

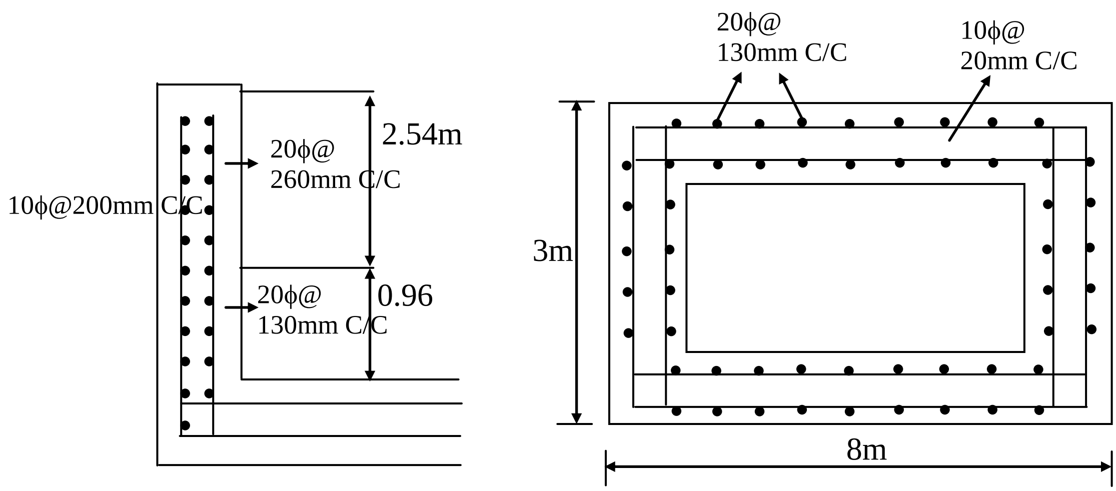
Minimum reinforcement = $\dfrac{0.3}{100} \times 1000 \times 260 = 780\,\text{mm}^2$

$\therefore$ 390 mm^2 steel may be provided on each face.

Using 10 mmϕ bars; spacing required = $\dfrac{1000}{390} \times \dfrac{\pi}{4} \times 10^2$

$= 201.38$ mm

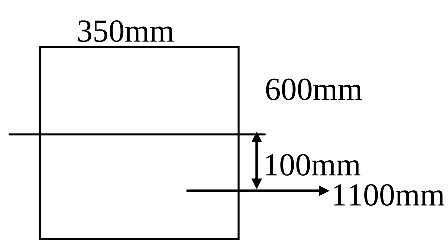
$\therefore$ Provide 10 mmϕ bars @ 200 mm C/C on each face in horizontal direction.



Section through long wall

08(b).

Sol: Prestressing force P_0= 1100 kN



Modular ratio = m = $\dfrac{E_S}{E_C}$

$E_C = 5000\sqrt{F_C} = 5000\sqrt{50}$

$= 35365.34$ MPa

$$\therefore \quad M = \frac{2\times10^5}{36355.34} = 5.65$$

(i) Loss due to elastic shortening = m f_C

f_c = stress in concrete at level of steel

$$= \frac{P}{A} + \frac{Pe^2}{I}$$

$$= \frac{1100\times10^3}{350\times600} + \frac{1100\times10^3\times100^2\times12}{350\times600^3}$$

= 5.238 + 1.746

= 6.984 MPa

$\therefore$ Loss due to elastic shortening = 5.65 × 6.984

= 39.46 MPa

Loss in prestressing force due to elastic shortening = 39.46 × 1100 N

= 43.406 kN

(b) loss due to shrinkage = E_x E_S

$E_{sc = 3 \times 10}{}^{-4}$

$\therefore$ = 3 × 10^{-4} × 2 × 10^5k

= 60 MPa

Loss in prestress = 60 × 1100 N = 66 kN

(c) Loss due to creep = ϕ m f_C

ϕ = 1.6, m = 5.65

f_2 = stress in concrete at level of steel

Prestressing force after elastic deformation = 1100 – 4.3406

= 1056.6 kN

$$f_C = \frac{P}{A} + \frac{Pe^2}{I}$$

$$= \frac{1056.6\times10^3}{350\times600} + \frac{1056.\times100^2\times12\times10^3}{350\times600^3} \quad = 5.03 + 1.677 = 6.708 \text{ MPa}$$

$\therefore$ Loss due to creep = 1.6 × 5.65 × 6.708 = 60.64 MPa

Loss in prestress = 60.64 × 1100 N = 66.704 kN

(d) Loss due to relaxation = 3% = 0.03 × 1100

= 33 kN

$\therefore$ Total loss = 43.406 + 66 + 66.704 + 33

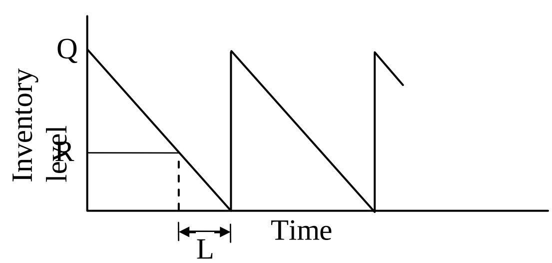
= 209.11 kN

Percentage loss = $\frac{209.11}{1100} \times 100 = 19.01\%$

08(c).

Sol: Economic Order Quantity (EOQ) Model: (Basic Purchase Model)

The EOQ model provides answers on how much to order. Figure shows the behaviour of EOQ model. The reorder point R and the quantity to be ordered, Q, are shown in the figure, as is the lead time L. The ordered quantity derived from this model is known as economic order quantity, EOQ.



Inventory behaviour under EOQ model

It is usually less expensive to purchase (and transport) or produce a bunch of material at once than to order it in small quantities. If orders for large quantities are specified, there will be fewer orders placed. For purchasing, this means that quantity discounts and transportation efficiencies may be realized. The other side of the coin, however, is that larger lot sizes result in more inventory, and inventory is expensive to hold. EOQ model attempts to specify a balance between these opposing costs. This aspect is shown graphically in figure where it is clear that there is a decrease in cost associated with increase in order quantity, while there is increase in cost with increase of inventory.

The total cost is given by the sum of inventory carrying cost and ordering cost.

Total cost TC = Ordering cost + Carrying cost

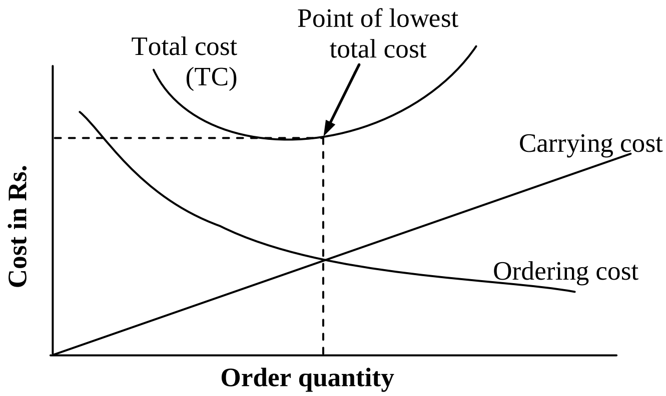


Fig: Total cost curve - EOQ model

- The derivation of EOQ is based on a number of assumptions such as:

- Demand is deterministic and continuous at a constant rate.
- The process continues infinitely.
- No constraints are imposed on quantities ordered, storage capacity, budget, etc.
- Replenishment is instantaneous (the entire order quantity is received all at one time as soon as the order is released).
- All costs are time-invariant.
- There are no shortages of items.
- The quantity discounts are not available.
- There is negligible or deterministic lead time.

- The following notations are used to develop the EOQ model:

  D = Demand rate; unit/year

  A = Ordering cost; ₹ /order

  C = Unit cost; ₹/unit of item

  I = Inventory carrying charges per year

  H = Annual cost of carrying inventory/unit item

  Q = Order quantity; number of units per lot

It is assumed that demand is at a uniform rate. Thus, the average inventory required would be $\frac{(0+Q)}{2}=\frac{Q}{2}$ throughout the year.

The total number of orders placed would be $\frac{D}{Q}$ per year

Order cost per year = Number of orders placed per year × Cost per order

Ordering cost per year $\frac{A\times D}{Q}$

Carrying cost per year = $\frac{\text{Order quantity}\times\text{Unit cost of item}\times\text{Annual cost to carry}}{2}$

Carrying cost per year = $\frac{C\times I\times Q}{2}=\frac{H\times Q}{2}$

where

H = C × I

Using the notations mentioned above, we can write the expression of TC as

$$TC=\frac{A\times D}{Q}+\frac{H\times Q}{2}$$

For optimum Q, one needs to find the particular value of Q which will minimize total cost

This can be done by differentiation, and one gets

$$EOQ=\sqrt{\frac{2\times\text{Order cost}\times\text{Demand}}{\text{Inventory carrying cost}}}$$

$$EOQ=\sqrt{\frac{2\times A\times D}{I\times C}}$$

- **Minimum inventory cost @ EOQ**, Total ordering cost = Total carrying cost

$\Rightarrow TIC_{min}$ = TIC @ EOQ = 2 × Total Carrying cost @ EOQ

$$=\sqrt{2DC_oC_c}$$

08(c). (ii)

Sol: Given data:

Discharge capacity of the pump $= \dfrac{3000\times10^{-3}}{2\times60}\ m^3/sec$

$= 0.025\ m^3/sec$

Head on the pump = pipe length × gradient

$H = 1000\ (m)\times\dfrac{1}{10}$

H = 100 m

h_L = Head losses = 10% of head pump

$h_L = 0.1\times100 = 10$ m

Efficiency of pump system (η_P) = 0.7

Efficiency of a pump $= \dfrac{O/P}{I/P} = \dfrac{\rho gQ(H+h_L)}{\text{Input power}}$

$$0.7 = \frac{1060\times9.81\times0.025\times(100+10)}{P_{input}}$$

$$\therefore\ P_{input} = \frac{1060x9.81x0.025x110}{0.7} = 40851KW = 55.5HP$$