ACE Engineering Academy
(Leading institute for ESE/GATE/PSUs)

ESE – 2019 MAINS
OFFLINE TEST SERIES

MECHANICAL ENGINEERING

TEST – 4
SOLUTIONS

All Queries related to **ESE – 2019 MAINS Test Series** Solutions are to be sent to the following email address
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01(a).

Sol: Given that

m = 8 kg ; S = 5.4 /mm ; F = 40 N

Velocity = 1 m/s

Force (F) = $C\dot{x}$ where = $\dot{x}$ velocity

$40 = C \times 1$

$C = 40$ N-s/m

(i) Critical damping coefficient

$$2\xi\omega_n = \frac{C}{m} \qquad [\because \xi = 1 \text{ then } C = C_C]$$

$$2\omega_n = \frac{C_C}{m}$$

$$C_C = 2\sqrt{km} = 2\sqrt{5400 \times 8}$$

$$C_C = 415.692\ \text{N} - \text{S/m}$$

(ii) As we know that

$$2\xi\omega_n = \frac{C}{m} \qquad [\therefore\ \xi\ =\ \text{Damping factor}]$$

ω_n = Natural Frequency

$$2 \times \xi \times \sqrt{\frac{K}{m}} = \frac{C}{m}$$

$$2 \times \xi \times \sqrt{\frac{5400}{8}} = \frac{40}{8}$$

$$\text{Damping factor}(\xi) = 0.0962$$

(iii) Logarithmic decrement

$$(\delta) = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2 \times \pi \times 0.0962}{\sqrt{1-0.0962^2}}$$

$$\delta = 0.6072$$

(iv) Ratio of two consecutive amplitudes

$$\frac{x_n}{x_{n+1}} = e^{\delta} = e^{0.6072}$$

$$\frac{x_n}{x_{n+1}} = 1.83539$$

01(b).

Sol: **Buses:** Digital signals move from one section to another along paths called buses. A bus, in the physical sense, is just a number of parallel conductors along which electrical signals can be carried and are paths which can be shared by all the chips in the system. This is because if separate connections were used between the chips, there would be a very large number of connecting conductors. Using shared connection buses mean that when one chip puts data on the bus, the other chips have to wait their turn until the data transfer is complete before one of them can put its data on the bus.

There are three forms of bus used in a microprocessor system.

1. *Data bus :*

The data associated with the processing function of the CPU is carried by the data bus. Thus, it is used to transport a word to or from the CPU and the memory or the input/output interfaces. Each wire in the bus carries a binary signal, i.e. 0 or 1. Thus with a four-wire bus we might have the word 1010 being carried, each bit being carried by a separate wire in the bus, as:

Word	**Bus wire**
0 (least significant bit)	First data bus wire
1	Second data bus wire
0	Third data bus wire
1 (most significant bit)	Fourth data bus wire

2. *Address bus :*

The address bus carries signals which indicate where data is to be found and so the selection of certain memory locations or input or output ports. Each storage location within a memory device has a unique identification, termed its address, so that the system is able to select a particular instruction or data item in the memory. Each input/output interface also has an address. When a particular address is selected by its address being placed on the address bus, only that location is open to the communications from the CPU. The CPU is thus able to communicate with just one location at a time. A computer with an 8-bit data bus has typically a 16-bit-wide address bus, i.e. 16 wires. This size of address bus enables 216 locations to be addressed; 216 is 65 536 locations and is usually written as 64K, where K is equal to 1024. The more memory that can be addressed, the greater the volume of data that can be stored and the larger and more sophisticated the programs that can be used.

3. *Control bus :*

The signals relating to control actions are carried by the control bus. For example, it is necessary for the microprocessor to inform memory devices whether they are to read data from an input device or write data to an output device. The term READ is used for receiving a signal and WRITE for sending a signal. The control bus is also used to carry the system clock signals; these are to synchronise all the actions of the microprocessor system. The clock is a crystal-controlled oscillator and produces pulses at regular intervals.

01(c).

Sol: Assuming that the controlling force curve F varies linearly with the radius of rotation r. we have

$F = ar + b$

$r = 0.075$ m,

$F = 22.5$ N

$22.5 = 0.075 \times a + b$ ------- (i)

$r = 0.15$ m,

$F = 60$ N,

$60 = 0.15 \times a + b$ -------- (ii)

Solving equation (i) and (ii) , we have

$a = 500$ and $b = -15$

hence, controlling force curve equation is

$$F = 500 \times r - 15$$

Now at $r = 0.1$ m,

Controlling force :

$F = 500 \times 0.1 - 15 = 35$ N

We know that

$F = m\omega^2 r$

$$35 = \frac{7.5}{9.81} \times \omega^2 \times 0.1$$

$\omega = 21.39$ rad/s

$N = 204.26$ rpm

Presently the governor is stable because constant a is positive and constant b is negative.

For isochronous governor,

$$F = a \times r$$

Where $b = 0$

$$\frac{F}{r} = a = \text{constant} = 500$$

Therefore, to make the governor isochronous, the ratio of $\frac{F}{r}$ should be made constant.

For isochronous speed

$F = m\,\omega^2\,r = ar + b$

$$\frac{7.5}{9.81}\times\omega^2\times 0.1 = 500\times 0.1 + 0$$

$\omega = 25.57$ rad/sec

Speed, $N = \dfrac{60\times 25.57}{2\pi}$ = 244.17 rpm

01(d).

Sol: The mass which converts acceleration into the spring displacement is called the seismic mass. The following are some of the types of accelerometer :

1. Displacment seismic accelerometer
2. Strain gauge accelerometer
3. Piezoelectric accelerometer
4. Potentiometric accelerometer
5. LVDT accelerometer

***Displacement Seismic Accelerometer*:**

In this type of accelerometer, the displacement of seismic mass is measured by displacment transducer itself. A spring mass damper is placed inside the casing by placing the damper at the top and spring at the bottom of the casing. Again, the seismic is connected with the displacement transducer by a shaft which is already connected with the casing itself.

When the acceleration or vibration is applied on the casing the mass is displaced which is sensed by the sensor. Therefore, the displacement sensed by the transducer is directly proportional to the acceleration.

The equation of motion can be written as

$$m\ddot{x} = -c(\dot{x}-\dot{y}) - k(x-y) + F_o\sin\omega t$$

Let z = x – y

$$m\ddot{x} + c\dot{z} + kz = F_o\sin\omega t$$

where x is the displacement and $F_o = m\omega^2 y$

Therefore, the relative amplitude z provided by the instrument can be evaluated as

$$z = \frac{m\omega^2 y}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}} = \frac{r^2 y}{\sqrt{(1-r^2)^2+(2\xi r)^2}}$$

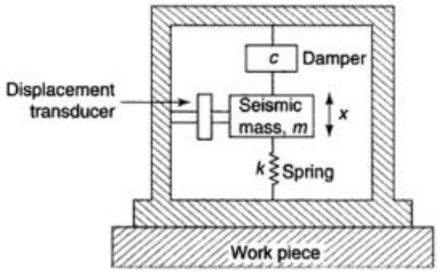


Fig: Seismic accelerometer

and the position of the amplitude can be evaluated from

$$\tan\phi = \frac{2\xi r}{1-r^2}$$

where $r = \dfrac{\omega}{\omega_n}$

and ζ = Damping ratio = $\dfrac{c}{2m\omega_n}$

The steady state amplitude Z of seismic mass with respect to the frame is the measure of acceleration, when the natural freuqnecy of the instrument is high compared to that of the vibration to be measured. The implies

$$\frac{Z}{y} = r^2 \text{if } r << 1$$

$$Z = r^2 y = \frac{\omega^2 y}{\omega_n^2}$$

where $\omega^2 y$ is the acceleration.

The measured value of Z is proportional to the acceleration of the motion to be measured.

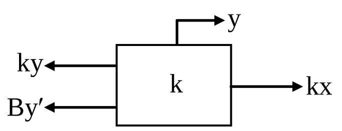
01(e)(i).

Sol:

- Write a set of linearly independent integral differential equations in *(n+1)* variables the input (the independent variable), the output and *(n–1)* other independent variables.
- Convert the equations in frequency domain by taking Laplace transform assuming initial condition as zero.
- Solve the resulting set of *n* linearly independent algebraic equations in the *n* dependent variable for the Laplace transform of the output in terms of the Laplace transform of the input.
- Find the relation between Laplace transform of output and input eliminating intermediate variables. The resultant equation is the transfer function.

01(e)(ii).

Sol: Free body diagram for the given system is,



Algebraic sum of applied and opposing forces is zero.

$$By' + ky - kx = 0 \text{ (or) } By' + ky = kx$$

Laplace transformation

$$\Rightarrow sB\psi(s) + k\psi(s) = kx(s)$$

$$\psi(s)[Bs + k] = k\,x(s)$$

$$TF: \frac{\psi(s)}{x(s)} = \frac{k}{Bs + k}$$

02(a).

Sol: Given: d = 75 mm

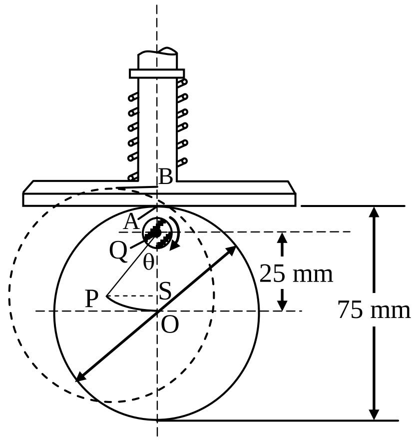
r = QA = 37.5 mm

OQ = 25 mm; m = 2.3 kg;

s = 3.5 N/mm; S = 45 N

1. *Expression for the acceleration of the follower*

The cam in its lowest position is shown by full lines in figure and by dotted lines when it has rotated through an angle θ.



From the geometry of the figure, the displacement of the follower,

$$x = AB = OS = OQ - QS$$

$$= OQ - PQ\cos\theta$$

$$= OQ - OQ\cos\theta \qquad (\because PQ = OQ)$$

$$= OQ(1-\cos\theta) = 25(1-\cos\theta) \text{......(i)}$$

Differentiating equation (i) with respect to t, we get velocity of the follower,

$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt} = \frac{dt}{d\theta} \times \omega$$

(substituting dθ/dt = ω)

........(ii)

Now differentiating equation (ii) with respect to t, we get acceleration of the follower,

$$a = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt} = 25\cos\theta \times \omega$$

$$= 25\omega^2 \cos\theta \, mm/s^2 = 0.025\omega^2 \cos\theta \, m/s^2$$

2. *Cam shaft speed*

Let N = Cam shaft speed in rpm

We know that accelerating force

$$= m.a = 2.3\times 0.025\,\omega^2\cos\theta$$
$$= 0.0575\,\omega^2\cos\theta\,N$$

Now for any value of θ, the algebraic sum of the spring force, weight of the follower and the acceleration force is equal to the vertical reaction between the cam and follower. When this reaction is zero, then the follower will just begin to leave the cam.

∴ S+sx+m.g+m.a = 0

$$45+3.5\times 25(1-\cos\theta)+2.3\times 9.81+0.0575\,\omega^2\cos\theta=0$$
$$45+87.5-87.5\cos\theta+22.56+0.0575\,\omega^2\cos\theta=0$$
$$155.06-87.5\cos\theta+0.0575\,\omega^2\cos\theta=0$$
$$2697-1522\cos\theta+\omega^2\cos\theta=0$$

(Dividing by 0.0575)

$$\omega^2\cos\theta=1522\cos\theta-2697 \text{ or } \omega^2=1522-2697\sec\theta$$

since $\sec\theta\geq +1 \text{ or } \leq -1$, therefore the minimum value of ω^2 occurs when θ = 180°, therefore

$$\omega^2=1522-(-2697)=4219$$

[substituting sec θ = –1]

ω = 65 rad/s

And maximum allowable cam shaft speed,

$$N=\frac{\omega\times 60}{2\pi}=\frac{65\times 60}{2\pi}=621\,\text{r.p.m}$$

Sol: Given: T = (5000+1500 sin 3θ) N-m;

t = 1000 kg-m^2;

N = 300 r.p.m

or $\omega=2\pi\times 300/60=31.42\,\text{rad/s}$

1. Power of the engine:

We know that workdone per revolution

$$=\int_0^{2\pi}(5000+1500\sin 3\theta)d\theta=\left[5000\theta-\frac{1500\cos 3\theta}{3}\right]_0^{2\pi}$$

$$= 10000\pi\,\text{N-m}$$

∴ Mean resisting torque,

$$T_{mean}=\frac{\text{Work done / rev}}{2\pi}=\frac{10000\pi}{2\pi}=5000\,\text{N}-\text{m}$$

We know that power of engine,

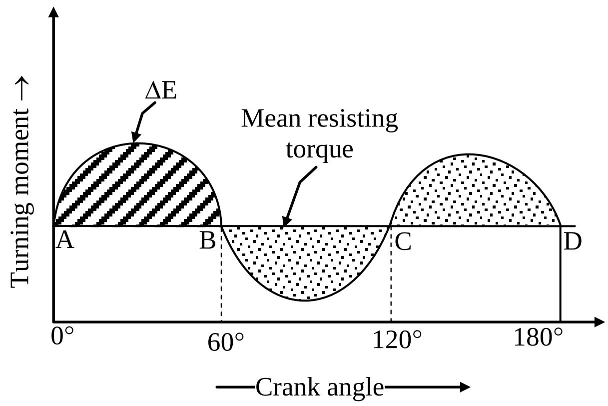
$$P=T_{mean}.\omega=5000\times 31.42=157100\text{ W}=157.1\text{kW}$$

2. Maximum fluctuation of the speed of the flywheel,

Let C_s = Maximum or total fluctuation of speed of the flywheel

(i) When resisting torque is constant

The turning moment diagram is shown in figure. Since the resisting torque is constant, therefore the torque exerted on the shaft is equal the mean resisting torque on the flywheel.



∴ T = T_{mean}

$$5000+1500\sin 3\theta=5000$$
$$1500\sin 3\theta=0 \text{ or } \sin 3\theta=0$$

∴ 3θ = 0° or 180°

θ = 0° or 60°

∴ Maximum fluctuation of energy,

$$\Delta E=\int_0^{60°}(T-T_{mean})d\theta$$

$$= \int_0^{60^\circ} (5000 + 1500\sin 3\theta - 5000)d\theta$$

$$= \int_0^{60^\circ} 1500\sin 3\theta = \left[-\frac{1500\cos 3\theta}{3}\right]_0^{60^\circ} = 1000\ \text{N} - \text{m}$$

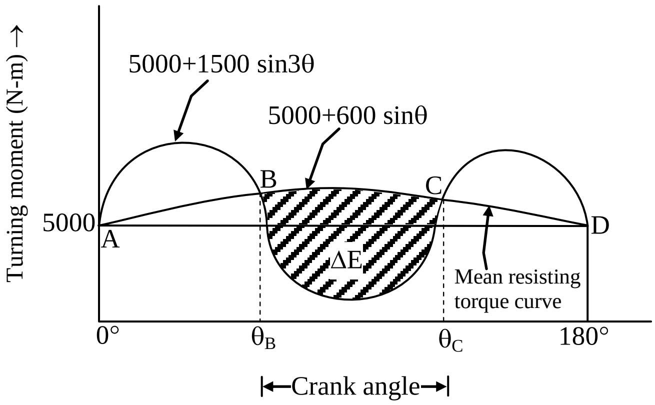
We know that maximum fluctuation of energy (ΔE),

$$1000 = I.\omega^2.C_s = 1000(31.42)^2 C_s = 987216\, C_s$$

$\therefore$ Cs = 1000/987216 = 0.001 or 0.1%

(ii) When resisting torque is (5000+600 sin θ) N-m

The turning moment diagram is shown in figure. Since at points B and C , the torque exerted on the shaft is equal to the mean resisting torque on the flywheel, therefore



5000+1500 sin3θ = 5000+600 sinθ

or 2.5sin 3θ = sin3θ

$$2.5(3\sin\theta - 4\sin^3\theta) = \sin\theta$$

$$(\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta)$$

$$3 - 4\sin^2\theta = 0.4 \quad \text{(Dividing by 2.5 sin}\theta)$$

$$\sin^2\theta = \frac{3-0.4}{4} = 0.65 \text{ or } \sin\theta = 0.8062$$

$\therefore$ θ = 53.7° or 126.3°

i.e θ_B = 53.7°, and θ_C = 126.3°

$\therefore$ Maximum fluctuation of energy,

$$\Delta E = \int_{53.7^\circ}^{126.3^\circ} [(500 + 1500\sin 3\theta) - (5000 + 600\sin\theta)]d\theta$$

$$= \int_{53.7^\circ}^{126.3^\circ} (1500\sin 3\theta - 600\sin 3\theta)d\theta = \left[-\frac{1500\cos 3\theta}{3} + 600\cos\theta\right]_{53.7^\circ}^{126.3^\circ}$$

= –1656N-m

We know that maximum fluctuation of energy (ΔE),

$$1656 = I.\omega^2.C_s = 1000(31.42)^2 C_s = 987216\, C_s$$

$\therefore C_s$ = 1656/987216 = 0.00168 or 0.168%

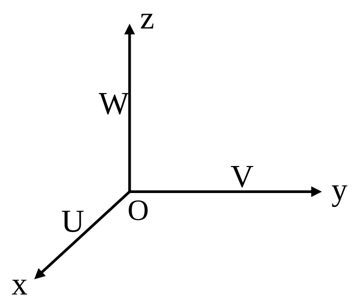
02(c).

Sol: Point (P) undergoes successive rotations as given below.

SET A: (i) R(z, –90°)

(ii) T(5, –3, 4)

(iii) R(y, –90°)



Rotation about fixed axis ⇒ Premultiplication.

⇒ R(y, –90°).T(5, –3, 4). R(z, –90°)

$$R(y, -90^\circ) \Rightarrow \begin{bmatrix} \cos(-90^\circ) & 0 & \sin(-90^\circ) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-90) & 0 & \cos(-90^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T(5, –3, 4) ⇒ $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

⇒ **Resultant:** multiplication of above two matrices.

$$\begin{bmatrix} 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⇒ Multiplied with R(z, –90°)

$$\begin{bmatrix} 0 & 0 & -1 & -4 \\ 0 & 1 & 0 & -3 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Overall transformation matrix.

$$T = \begin{bmatrix} 0 & 0 & -1 & -4 \\ -1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

New coordinate point 'P' after above rotations

$$\begin{bmatrix} 0 & 0 & -1 & -4 \\ -1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -7 \\ 8 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -7 \\ 8 \\ 1 \end{bmatrix}$$

Now, point is $\begin{bmatrix} -6 & -7 & 8 \end{bmatrix}$

SET B:

(i) Rotation about current frame ⇒ Post multiplication R (W, –90°), (ii) T(5, –3, 4), (iii) R(V, –90°)

⇒ R(W, –90°).T(5, –3, 4).R(V, –90°)

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 & -3 \\ -1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & -5 \\ 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3 \\ 8 \\ 1 \end{bmatrix}$$

New point is (0, –3, 8)

SET C:

R(z, –90) → R(U, –90)

Movable axis ⇒ Post multiplication

So, R(z, –90).R(U, –90)

⇒ R(z, –90) × R(U, –90)

Note: Anticlockwise (+ve)

Clockwise (–ve)

Only Rotation, so 3×3 matrix:

R(z, –90)×R(U, –90)

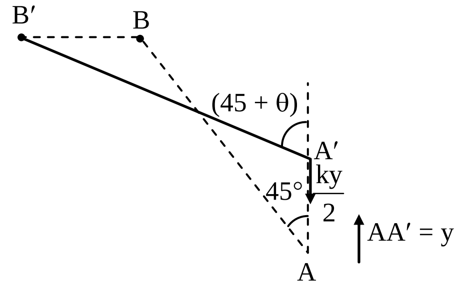
$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

Point (P) with fixed frame

$$\begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 & -4 & 3 \end{bmatrix}$$

03(a).

Sol:



If system is displaced from equilibrium position, writing equation for one of the bar.

$\Rightarrow\ T + I\ddot{\theta} = 0$

$I = \dfrac{m\ell^2}{3}$ (I about hinge point A)

If y is the small vertical displacement of spring then

Torque, $T = \dfrac{ky\ell}{2}\sin\left(\dfrac{\pi}{4}+\theta\right) + I\ddot{\theta} = 0$

$\Rightarrow\ y = \ell\cos\dfrac{\pi}{4} - \ell\cos\left(\dfrac{\pi}{4}+\theta\right)$

$\Rightarrow y \approx \left(\ell\sin\dfrac{\pi}{4}\right)\theta$

$$\frac{k\ell^2\sin\frac{\pi}{4}\left(\sin\frac{\pi}{4}+\theta\right)\theta}{2} + I\ddot{\theta} = 0$$

$\sin\left(\dfrac{\pi}{4}+\theta\right)\theta \approx \left(\sin\dfrac{\pi}{4}\right)\theta$ (for very small θ)

$$\frac{k\ell^2\left(\sin^2\frac{\pi}{4}\right)\theta}{2} + \frac{m\ell^2}{3}\ddot{\theta} = 0$$

$$\omega = \sqrt{\frac{3}{2}\frac{k\sin^2\frac{\pi}{4}}{m}}$$

$$= \sqrt{\frac{3\times1050\times\left(\frac{1}{\sqrt{2}}\right)^2}{2\times1.5}} = 22.91 \text{ rad/s}$$

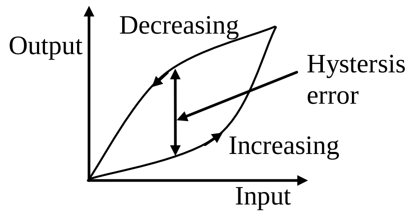
03(b).

Sol: ***Sensor:*** Sensor is a device used to measure different parameters in any mechatronic system, which is to control. By physical contact it converts non electrical parameters to electrical signal.

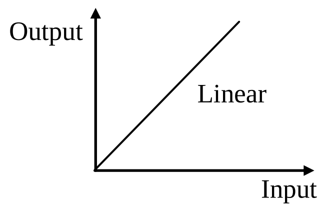
(a) *Accuracy:* Closeness of measured value to the true value of system.

(b) *Sensitivity:* Ratio of output to input of any system (or) sensor.

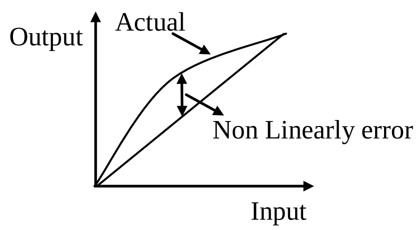
(c) *Hysteresis error:* It is the difference between sensor outputs, when sensor is subjected to loading (increasing input) and unloading (decreasing input).



(d) *Non-Linearity error:* If the sensor has linear relationship between input and output it can be shown as straight line as below.



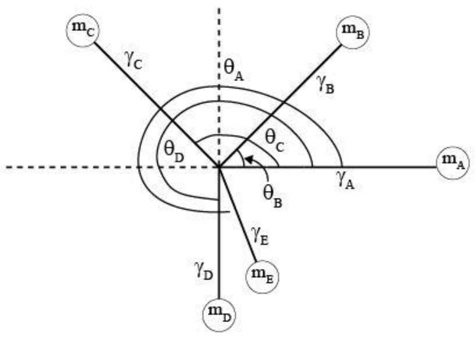
But in reality the maximum difference from straight line to actual values is known as Non-linearity error.



(e) *Resolution:* The smallest input difference, any sensor can measure is known as resolution.

03(c).

Sol:



Given that ($\gamma = r$)

$m_A = 12$ kg $\quad r_A = 40$ mm $\quad \theta_A = 0°$ (reference plane)

$m_B = 10$ kg $\quad r_B = 50$ mm $\quad \theta_B = 60°$

$m_C = 18$ kg $\quad r_C = 60$ mm $\quad \theta_C = 135°$

$m_D = 15$ kg $\quad r_D = 30$ mm $\quad \theta_D = 270°$

$m_e = ?$ $\quad r_e = 100$ mm $\quad \theta_e = ?$

(Balancing mass)

For static balancing of the shaft

$\Sigma mr\omega^2 = 0 \quad [\omega_A = \omega_B = \omega_C = \omega_D = \omega_e]$

$\Sigma mr = 0$

$\Sigma F_X = 0$

$m_A r_A \cos\theta_A + m_B r_B \cos\theta_B + m_C r_C \cos\theta_C + m_D r_D \cos\theta_D + m_e r_e \cos\theta_e = 0$

$12\times 40 \cos 0° + 10\times 50 \cos 60° + 18\times 60 \cos 135° + 15\times 30 \cos 270° + m_e 100 \cos\theta_e = 0$

$100\, m_e \cos\theta_e = 33.67$ ……. (i)

$\Sigma F_y = 0$

$m_A r_A \sin\theta_A + m_B r_B \sin\theta_B + m_C r_C \sin\theta_C + m_D r_D \sin\theta_D + m_e r_e \sin\theta_e = 0$

$12\times 40 \sin 0° + 10\times 50 \sin 60° + 18\times 60 \sin 135° + 15\times 30 \sin 270° + m_e 100 \sin\theta_e = 0$

$100\, m_e \sin\theta_e = -746.68$ …..... (ii)

From equation (i) and (ii)

$$\frac{100 m_e \sin\theta_e}{100 m_e \cos\theta_e} = \frac{-746.68}{33.67}$$

$$\tan\theta_e = -22.17$$

$$\theta_e = -87.42 \text{ or } 272.58°$$

Put the value of θ_e in equation (i)

$100\, m_e \cos(272.58) = 33.67$

$\Rightarrow \quad m_e = 7.48$ kg

03(d).

Sol:

(i) Position

$$P_x = 1 + \frac{\sqrt{3}}{2}, \quad P_y = \frac{1}{2}, \quad P_3 = 0$$

(ii) Joint angles θ_1, θ_2 from inverse kinematics:

$$P_x = \ell_1 c_1 + \ell_2 c_{12} = 1 + \frac{\sqrt{3}}{2}$$

$$P_y = \ell_1 s_1 + \ell_2 s_{12} = \frac{1}{2}$$

$$\text{So, } \theta_2 = \cos^{-1}\left[\frac{P_x^2 + P_y^2 - \ell_1^2 - \ell_2^2}{2\ell_1\ell_2}\right]$$

values of P_x, P_y and $\ell_1 = \ell_2 = 1$ are substituted.

Then,

$$\theta_2 = \cos^{-1}\left[\frac{\left(1+\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2 - (1)^2}{2\times 1\times 1}\right]$$

$$= \cos^{-1}[0.866],$$

So, $\theta_2 = 30°$

Its elbow up, $\theta_2 = +30°$,

Its elbow down, $\theta_2 = -30°$

$\Rightarrow$ Next θ, value from orientation (3×3) matrix.

$$\begin{bmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\cos(\theta_1 + \theta_2) = \frac{\sqrt{3}}{2}$, So $\theta_1 + \theta_2 = 30°$

$\sin(\theta_1 + \theta_2) = \frac{1}{2}$, So, $\theta_1 + \theta_2 = 30°$

So, $\theta_1 + \theta_2 = 30°$

$\rightarrow$ elbow up, $\theta_2 = +30°$, then $\theta_1 = 0°$

$\rightarrow$ elbow down, $\theta_2 = -30°$, then

$\theta_1 + \theta_2 = 30°$, (or) $\theta_1 - 30° = 30°$

$\Rightarrow \theta_1 = 60°$

So elbow up : $\theta_1 = 0$, $\theta_2 = +30°$

elbow down : $\theta_1 = 60°$, $\theta_2 = -30°$

(iii) Home position and orientation

Home position at $\theta_1 = \theta_2 = 0°$

$P_x = \ell_1 c_1 + \ell_2 \frac{c}{2}$ = 1.cos0° + 1.cos0° = 2

$P_y = \ell_1 s_1 + \ell_2 \frac{s}{2}$ = 1.sin0° + 1.sin0° = 0

$P_z = 0$; $\Rightarrow$ (2, 0, 0)

Home orientation

$$\begin{bmatrix} \cos 0° & -\sin 0° & 0 \\ \sin 0° & \cos 0° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

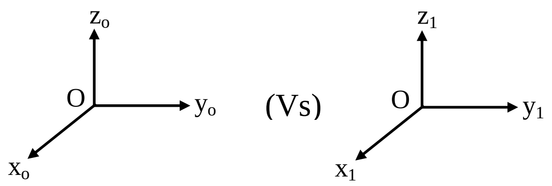
04(a).

Sol: (i).

(a) ${}_oT^1 \Rightarrow$ Frame 1 with ref to F_o

$\Rightarrow F_1(x_1\ y_1\ z_1)$ w.r. to $F_o(x_o\ y_o\ z_o)$ position is $(5, 4, 0)^T$,

Orientation angles is



	x_1	y_1	z_1
x_o	0	90	90
y_o	90	0	90
z_o	90	90	0

Cosine values has taken

Same orientation with only position change

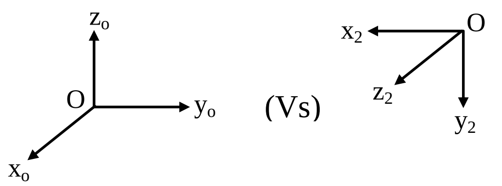
So,

$${}_oT^1 = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) ${}_oT^2 \Rightarrow$ Frame 2 w.r.t F_o

Positioning $(0, 5, 6)^T$

Orientation angles is



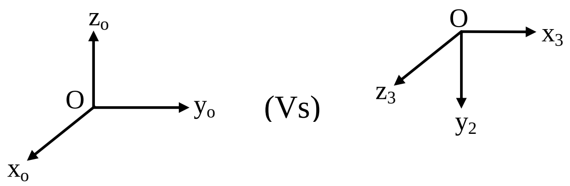
	x_2	y_2	z_2
x_o	90°	90°	0°
y_o	180°	90°	90°
z_o	90°	180°	0°

$${}_oT^2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 5 \\ 0 & -1 & 0 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) $_oT^3 \Rightarrow$ Frame 3 w.r.t F_o

position is $(5, 7, 8)^T$

Orientation angles is



	x_3	y_3	z_3
x_o	90°	0°	90°
y_o	0°	90°	90°
z_o	90°	90°	180°

Cosine angles taken

$$_oT^3 = \begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 7 \\ 0 & 0 & -1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note: above 15 Marks (3×5)

(ii). Frame 3 with reference to frame 1 $\Rightarrow {}_1T^3$

$$\left[{}_1T^3 = {}_1T^o \times {}_oT^3\right]$$

So $_1T^3 = {}_1T^0 \times {}_oT^3$

$$\left[{}_1T^0\right] = \left[{}_0T^1\right]^{-1}$$

$$_0T^1 = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & {}_0R_1 & \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} {}_0D_1 \\ 1 \end{bmatrix}$$

If

$$_1T_2 = \begin{bmatrix} & {}_1R_2 & & {}_1D_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ then its inversing}$$

$$\left[{}_1T^2\right]^{-1} = \begin{bmatrix} {}_1R_2^T & (-)({}_1R_2^T)({}_1D_2) \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

So, $({}_0T_1)^{-1}$ is calculated with above relationship as

$$\left[{}_1T_1\right]^{-1} = \begin{bmatrix} {}_1R_1^T & (-)({}_0R_1^T)({}_0D_1) \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = {}_0R_1^T \quad (\therefore \text{ Unity matrix})$$

So, final 4×4 matrix is

$$({}_0T_1)^{-1} = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = {}_1T_0$$

Now, finally

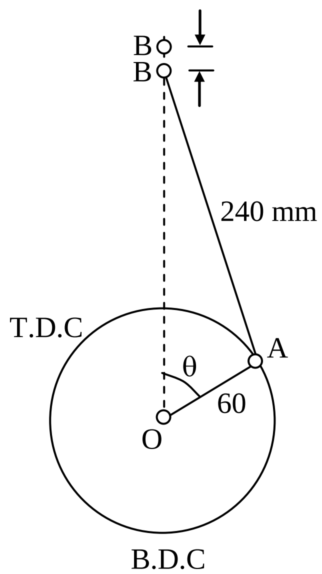
$$_1T_3 = {}_1T_0 \times {}_0T_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 & 5 \\ 1 & 0 & 0 & 7 \\ 0 & 0 & -1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & -1 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

04(b).

Sol:



r = 0.06m, l = 0.24 m,
N = 1800 rpm, m = 1.2 kg,
n = 0.24/0.06 = 4, d = 0.08 m

$$\omega = \frac{2\pi\times 1800}{60} = 188.5\,\text{rad/s}$$

Draw the configuration for the given position to some scale and obtain angle θ which is found to be 43.5°

$$\sin\beta = \frac{\sin\theta}{n} = \frac{\sin 43.5^o}{4} = 0.1721$$

or β = 9.91°

Force due to gas pressure,

$$F_p = \text{Area}\times\text{Pressure} = \frac{\pi}{4}(d)^2\times p$$

$$= \frac{\pi}{4}(0.08)^2\times 800\times 10^3 = 4021\text{ N}$$

Accelerating force,

$$F_b = mr\omega^2\left(\cos\theta + \frac{\cos\theta}{n}\right)$$

$$= 1.2\times 0.06\times(188.5)^2\left[\cos 43.5^o + \frac{(\cos 87^o)}{4}\right]$$

$$= 1889\text{ N}$$

(i) Force on the piston,

$$F = F_p + mg - F_b$$

$$= 4021 + 1.2\times 9.81 - 1889 = 2144\text{ N}$$

(ii) Thrust in the connecting rod,

$$F_c = \frac{F}{\cos\beta} = \frac{2144}{\cos 9.91^o} = 2176\text{ N}$$

(iii) Thrust on the sides of cylinder walls,

$$F_n = F\tan\beta = 2176\tan 9.91^o = 380\text{ N}$$

(iv) The above values are zero at the speed when the force on the piston F is zero.

$$F = F_p - mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right) + mg$$

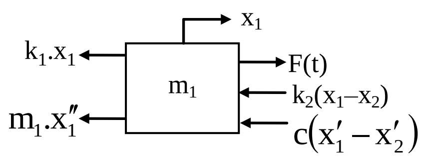
$$0 = 4021 - 1.2\times 0.06\,\omega^2\left(\cos 43.5^o + \frac{\cos 87^o}{4}\right) + 1.2\times 9.81$$

$$0.05317\,\omega^2 = 4032.8, \omega = 75849$$

$$\frac{2\pi N}{60} = 275.4, N = 2630\text{ rpm}$$

04(c).

Sol: As per the D Alembert's principle, algebraic sum of applied forces and opposing forces isequal to zero. Free body diagram on mass m_1 is drawn as :



So, the algebraic sum of forces is equal to zero.

$$F(t) = m_1x_1'' + c(x_1' - x_2') + k_1x_1 + k_2(x_1 - x_2)$$

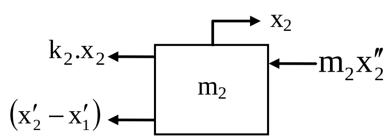
$$F(t) = m_1x_1'' + cx_1' + (k_1 + k_2)x_1 - cx_2' - k_2x_2$$

Laplace transformation is taken, then

$$F(t) = x_1(s)[m_1s^2 + cs + (k_1 + k_2)] - x_2(s)[cs + k_2]$$

..........(1)

Next free body diagram for mass (m_2) is drawn as :



$$m_2x_2'' + c(x_2' - x_1') + k_2x_2 = 0$$

L.T is taken, then

$$m_2s^2\times x_2(s) + csx_2(s) - csx_1(s) + k_2x_2(s) = 0$$

$$x_2(s)[m_2s^2 + cs + k_2] = csx_1(s)........(2)$$

To get $F(s)[VS]x_1(s)$ relation, equation 2 is substituted in equation 1.

$$x_2(s)=\frac{[cs]x_1(s)}{m_2s^2+cs+k_2}$$

$$F(s)=x_1(s)\left[m_1s^2+cs+(k_1+k_2)-x_1(s)\frac{(cs)}{m_2s^2+cs+k_2}\right]$$

$$F(s)=x_1(s)\left[\frac{(m_1s^2+cs+k_1+k_2)(m_2s^2+cs+k_2)-cs}{m_2s^2+cs+k_2}\right]$$

(i)

$$\frac{x_1(s)}{F(s)}=\frac{m_2s^2+cs+k_2}{(m_1s^2+cs+k_1+k_2)(m_2s^2+cs+k_2)-cs}$$

(ii) Next from equation (2)

$$x_1(s)=x_2(s)\left(\frac{m_2s^2+cs+k_2}{cs}\right)\text{..........(3)}$$

Substituted in equation (1)

$$F(s)=x_2(s)\left(\frac{m_2s^2+cs+k_2}{cs}\right)(m_1s^2+cs+k_1k_2)-x_2(s)(cs+k_2)$$

$$F(s)=\frac{x_2(s)}{cs}[(m_2s^2+cs+k_2)(m_1s^2+cs+k_1+k_2)-(cs)(cs+k_2)]$$

$$\frac{x_2(s)}{F(s)}=\frac{cs}{[(m_2s^2+cs+k_2)(m_1s^2+cs+k_1+k_2)-cs(cs+k_2)]}$$

05(a).

Sol: The four basic configurations are:

(i) Cartesian (rectangular) configuration – all three P joints.

(ii) Cylindrical configuration – one R and two P joints.

(iii) Polar (spherical) configuration – two R and one P joint.

(iv) Articulated (Revolute or Jointed-arm) Configuration – all three R joints.

(i) **Cartesian (Rectangular) Configuration:** This is the simplest configuration with all three prismatic joints. It is constructed by three perpendicular slides, giving only linear motions along the three principal axes. There is an upper and lower limit for movement of each link. Consequently, the endpoint of the arm is capable of operating in a cuboidal space, called workspace.

(ii) **Cylindrical Configuration:** The cylindrical configuration uses two perpendicular prismatic joints and a revolute joint. The difference from the Cartesian one is that one of the prismatic joint is replaced with a revolute joint. One typical construction is with the first joint as revolute. The rotary joint may either have the column rotating or a block revolving around a stationary vertical cylindrical column. The vertical column carries a slide that can be moved up or down along the column. The horizontal link is attached to the slide such that it can move linearly, in or out, with respect to the column. This results in a RPP configuration. The arm endpoint is, thus, capable of sweeping a cylindrical space. To be precise, the workspace is a hollow cylinder. Usually a full 360° rotation of the vertical column is not permitted due to mechanical restriction imposed by actuators and transmission elements.

The cylindrical configuration offers good mechanical stiffness and the wrist positioning accuracy decreases as the horizontal stroke increase. It is suitable to access narrow horizontal cavities and, hence, is useful for machine loading operation.

(iii) **Polar (Spherical) Configuration:** The polar configuration consists of a telescope link (prismatic joint) that can be raised or lowered about a horizontal revolute joint. These two links are mounted on a rotating base. This arrangement of joints, known as RRP configuration, gives the capability of moving the arm end-point within a partial spherical shell space as work volume.

This configuration allows manipulation of objects on the floor because its shoulder joint allows its end-effector to go below the base. Its mechanical stiffness is lower than Cartesian and cylindrical configurations and the wrist positioning accuracy decreases with the increasing radial stroke. The construction is more complex. Polar arms are mainly employed for industrial applications such as machining, spray painting and so on. Alternate polar configuration can be obtained with other joint arrangements such as RPR, but PRR will not give a spherical work volume.

(iv) **Articulated (Revolute or Jointed-arm) Configuration:** The articulated arm is the type that best simulates a human arm and a manipulator with this type of an arm is often referred as an anthropomorphic manipulator. It consists of two straight links, corresponding to the human "forearm" and "upper arm" with two rotary joints corresponding to the "elbow" and "shoulder" joints. These two links are mounted on a vertical rotary table corresponding to the human waist joint.

This configuration (RRR) is also called revolute because three revolute joints are employed. The work volume of this configuration is spherical shaped, and with proper sizing of links and design of joints, the arm endpoint can sweep a full spherical space. The arm endpoint can reach the base point and below the base. This anthropomorphic structure is the most dexterous one, because all the joints are revolute, and the positioning accuracy varies with arm endpoint location in the workspace. The range of industrial applications of this arm is wide.

05(b).

Sol: Tangent to cam profile at θ = 30°

$$= \left(\frac{dy}{dx}\right)_{\theta=30^\circ} = \tan\beta$$

$$= \left(\frac{dy/d\theta}{dx/d\theta}\right)_{\theta=30^\circ} = \left(\frac{5\cos\theta}{-15\sin\theta}\right)_{\theta=30^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}}{-3\times\frac{1}{2}} = \frac{-\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$$

$$\beta = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = 150^\circ$$

= angle of tangent to cam profile wrt x-axis

Angle of normal to cam profile wrt x-axis = $\beta - 90^\circ = 60^\circ$

Angle of line of follower wrt x-axis at

$$(\theta = 30^\circ) = \tan^{-1}\left(\frac{y}{x}\right)_{\theta=30^\circ} = \gamma$$

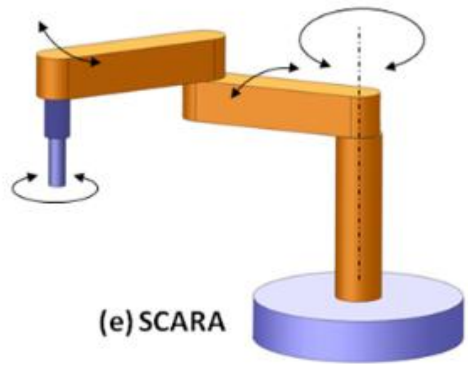
$$\gamma = \tan^{-1}\left(\frac{12.5}{12.99}\right) = 43.9^\circ$$

Pressure angle α = angle between normal and line of follower = 60° – 43.9° = 16.1°

05(c).

Sol: SCARA

Its full form is 'Selective Compliance Assembly Robot Arm'. It is similar in construction to the jointer-arm robot, except the shoulder and elbow rotational axes are vertical. It means that the arm is very rigid in the vertical direction, but compliant in the horizontal direction.



Robot wrist assemblies consist of either two or three degrees-of-freedom. A typical three-degree-of-freedom wrist joint is depicted in Figure. The roll joint is accomplished by use of a T-joint. The pitch joint is achieved by recourse to an R-joint. And the yaw joint, a right-and-left motion, is gained by deploying a second R-joint.

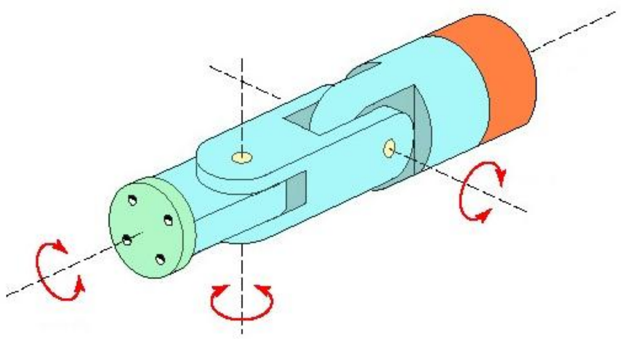
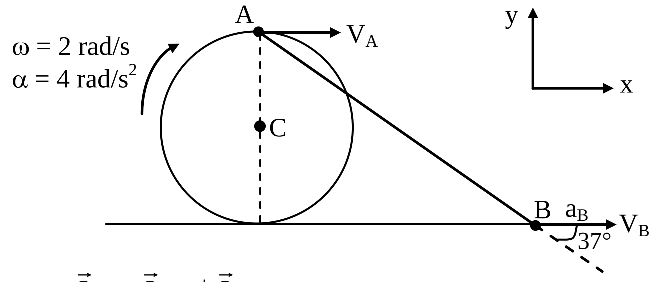


Fig: Robotic wrist joint

The SCARA body-and-arm configuration typically does not use a separate wrist assembly. Its usual operative environment is for insertion-type assembly operations where wrist joints are unnecessary. The other four body-and-arm configurations more-or-less follow the wrist-joint configuration by deploying various combinations of rotary joints viz. type R and T.

05(d).

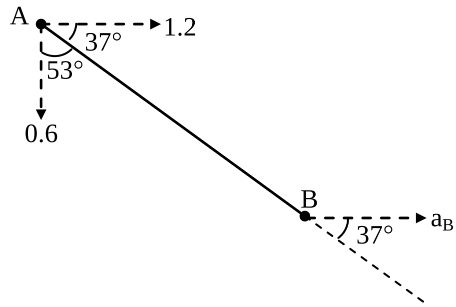
Sol:



$$\vec{a}_A = \vec{a}_{AC} + \vec{a}_C$$
$$= \left(r\alpha\hat{i} - \omega^2 r\hat{j}\right) + \left(r\alpha\hat{i}\right)$$
$$= \left(0.15\times 4\hat{i} - 2^2\times 0.15\hat{j}\right) + \left(0.15\times 4\hat{i}\right)$$
$$= 1.2\ \hat{i} - 0.6\ \hat{j}$$

Also, $V_A = V_B$

$\therefore \quad \omega_{AB} = 0$



Now, since rod AB has zero angular velocity,

$$\therefore\ \left(\vec{a}_{BA}\right)_{\text{along link AB}} = \omega^2\left(r_{BA}\right) = 0$$

$\therefore$ 1.2 cos37 + 0.6 cos53 = a_B cos37

($\because$ Component of acceleration of point A along AB is equal to component of acceleration of point B along AB).

$\therefore$ a_B = 1.65 m/s^2

05(e).

Sol:

(i) **Work volume:** The workspace is defined as the space enclosing the entire set of points representing the maximum extent or reach of the robot end-effector in all three spatial directions. The workspace of a manipulator is also defined as the total volume swept out by the end-effector as the manipulator executes all possible motions. The workspace describes the working volume of the manipulator. It defines what positions the manipulator can and cannot reach in space, with the former being included within the workspace boundary.

In Cartesian arm configuration the workspace represents the portion of space around the base of the manipulator that can be accessed by the arm endpoint.

The work space of Cartesian configuration is Cuboidal as shown in below figure.

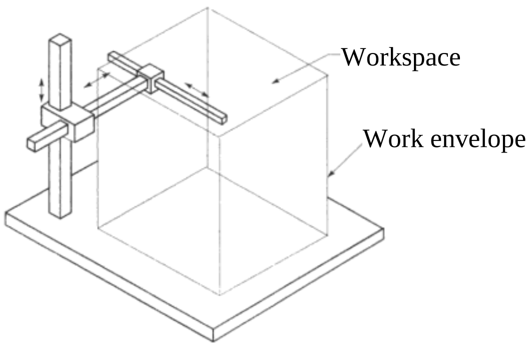


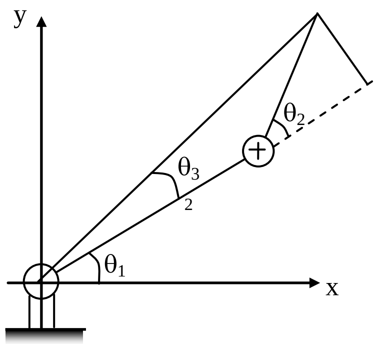
Fig: A 3 DOF Cartesian arm configuration and its workspace

(ii) & (iii) Reachable work space (RWS) and Dexterous work space (DWS)

The workspace is often broken down into a reachable workspace and a dexterous workspace. The reachable workspace is the entire set of points reachable by the manipulator, where as the dexterous workspace consists of those points that the manipulator can reach with an arbitrary orientation of the end-effector. The reachable workspace describes the volume in space within which the manipulator end-effector's tool center point (TCP) can reach. The dexterous workspace is a subset of the reachable workspace based on both the position and orientation reachability of the end-effector. That is, at each point in the dexterous workspace, the end-effector can be arbitrarily oriented. Therefore the dexterous workspace is a subset of the reachable workspace.

06(a).

Sol:



(i) $x = l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2)$,

$y = l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2)$ is Position $P_w(x, y)$

$x^2+y^2 = \ell_1^2 + \ell_2^2 + 2\ell_1\ell_2\cos\theta_2$

(ii) $\cos\theta_2 = \left[\dfrac{x^2+y^2-\ell_1^2-\ell_2^2}{2\ell_1\ell_2}\right]$

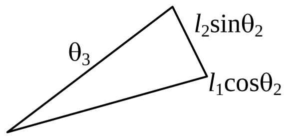
$$\theta_2 = \cos^{-1}\left[\frac{x^2+y^2-\ell_1^2-\ell_2^2}{2\ell_1\ell_2}\right] \text{----------(1)}$$

$$\cos\theta_2 = \left[\frac{x^2+y^2-\ell_1^2-\ell_2^2}{2\ell_1\ell_2}\right]$$

$$\sin\theta_2 = \pm\sqrt{1-\cos^2\theta_2}$$

$$\tan\theta_2 = \frac{\sin\theta_2}{\cos\theta_2};$$

So, $\theta_2 = \tan^{-}\dfrac{\sin\theta_2}{\cos\theta_2}$; ----------(1)



$$\tan(\theta_1+\theta_3) = \frac{y}{x}$$

and $\quad \tan\theta_3 = \dfrac{\ell_2\sin\theta_2}{\ell_1+\ell_2\cos\theta_2}$

We know that,

$$\tan(A-B) = \frac{\tan A-\tan B}{1+\tan A\tan B}$$

So, $\quad \theta_1+\theta_3 = \tan^{-1}\left(\dfrac{y}{x}\right)$

$$\theta_3 = \tan^{-1}\left(\frac{\ell_2 s_2}{\ell_1+\ell_2 c_2}\right)$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right)-\tan^{-1}\left(\frac{\ell_2 s_2}{\ell_1+\ell_2 c_2}\right)$$

$$\tan\theta_1 = \frac{\dfrac{y}{x}-\dfrac{\ell_2\sin\theta_2}{\ell_1+\ell_2\cos\theta_2}}{1+\dfrac{y}{x}\left[\dfrac{\ell_2\sin\theta_2}{\ell_1+\ell_2\cos\theta_2}\right]}$$

$$= \frac{y[\ell_1+\ell_2\cos\theta_2]-x.\ell_2\sin\theta_2}{x(\ell_1+\ell_2\cos\theta_2)+y(\ell_2\sin\theta_2)}$$

$$\theta_1 = \tan^{-1}\left[\frac{y(\ell_1+\ell_2\cos\theta_2)-x\ell_2\sin\theta_2}{x(\ell_1+\ell_2\cos\theta_2)+y(\ell_2\sin\theta_2)}\right]$$

-----------(2)

(iii) $\begin{array}{l|l|l} \ell_1=12 & x=15.7 & \theta_1=? \\ \ell_2=10 & y=12.6 & \theta_2=? \end{array}$

$$\theta_2 = \cos^{-1}\left[\frac{x^2+y^2-\ell_1^2-\ell_2^2}{2\ell_1\ell_2}\right]$$

$$\cos\theta_2 = \frac{43}{64}$$

$\theta_2 = 47.78°$

$$\theta_1 = \tan^{-1}\left[\frac{y(\ell_1+\ell_2\theta_2)+x.\ell_2\sin\theta_2}{x(\ell_1+\ell_2\cos\theta_2)-y(\ell_2\sin\theta_2)}\right]$$

$$= \left[\frac{12.6(12+10\cos 47.78)+15.7(10)\sin(47.78)}{15.7(12+10\cos 47.78)-12.6(10)\sin(47.78)}\right]$$

$$= \tan^{-1}\left[\frac{352.1388}{200.588}\right]$$

$$= \tan^{-1}(+1.755)$$

$\theta_1 = 60.325°$

(d) Forward Kinematics:

$x = \ell_1\cos\theta_1+\ell_2\cos(\theta_1-\theta_2)$

$= 12\cos 60.325° + 10\cos(60.325° - 47.78°)$

$= 15.7$

$y = \ell_1\sin\theta_1+\ell_2\sin(\theta_1-\theta_2)$

$= 12560.325 + 10\sin(60.325° - 47.78°)$

$= 12.6$

06(b).

Sol: Given : $\phi = 20°$; $\quad t = 30$;

$T = 50$; $\quad m = 4$; $\quad N_1 = 1000$ r.p.m,

or $\quad \omega_1 = 2\pi\times 1000/60 = 104.7\,\text{rad/s}$

1. Sliding velocities at engagement and at disengagement of pair of a teeth,

First of all, let us find the radius of addendum circles of the smaller gear and the larger gear,

We know that,

Addendum of the smaller gear,

$$= \frac{m.t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}-1\right]$$

$$= \frac{4\times 30}{2}\left[\sqrt{1+\frac{50}{30}\left(\frac{50}{30}+2\right)\sin^2 20^\circ}-1\right]$$

$$= 60(1.31-1) = 18.6\,\text{mm}$$

and addendum of the larger gear,

$$= \frac{m.T}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi}-1\right]$$

$$= \frac{4\times 50}{2}\left[\sqrt{1+\frac{30}{50}\left(\frac{30}{50}+2\right)\sin^2 20^\circ}-1\right]$$

$$= 100(1.09-1) = 9\,\text{mm}$$

Pitch circle radius of the smaller gear,

$$r = m.t/2 = 4\times 30/2 = 60\,\text{mm}$$

∴ Radius of addendum circle of the smaller gear,

$$r_A = r + \text{Addendum of the smaller gear}$$

$$= 60 + 18.6 = 78.6\,\text{mm}$$

Pitch circle radius of the larger gear,

$R = m.T/2 = 4\times50/2 = 100$ mm

∴ Radius of addendum circle of the larger gear,

$R_A = R$ + Addendum of the larger gear

$= 100+9 = 109$ mm

We know that the path of approach (i.e, path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(109)^2-(100)^2\cos^2 20^\circ} - 100\sin 20^\circ$$

$$= 55.2 - 34.2 = 21\,\text{mm}$$

And the path of recess (i.e path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(78.6)^2-(60)^2\cos^2 20^\circ} - 60\sin 20^\circ$$

$$= 54.76 - 20.52 = 34.24\,\text{mm}$$

Let

$$\omega_2 = \text{Angular speed of the larger gear in rad/s}$$

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t}$ or

$$\omega_2 = \frac{\omega_1\times t}{T} = \frac{104.7\times 30}{50} = 62.82\,\text{rad/s}$$

∴ Sliding velocity at engagement of a pair of teeth

$= (\omega_1 + \omega_2)KP = 104.7+62.82)21$

$= 3518$ mm/s

And sliding velocity at disengagement of a pair of teeth,

$$= (\omega_1+\omega_2)PL = (104.7+62.82)34.24$$

$$= 5736\,\text{mm/s}$$

2. Contact ratio

We know that the length of the arc of contact,

$$= \frac{\text{Length of the contact}}{\cos\phi} = \frac{KP+PL}{\cos\phi}$$

$$= \frac{21+34.24}{\cos 20^\circ} = 58.78\,\text{mm}$$

And Circular pitch $= \pi\times m$

$= 3.142\times4 = 12.568$ mm

$$\text{Contact ratio} = \frac{\text{Length of arc of contact}}{\text{Circular pitch}}$$

$$= \frac{58.78}{12.568} = 4.67 \approx 5$$

06(c).

Sol: The various components of a robot are enumerated and discussed below:

1. Base

2. Manipulator arm

3. End – effectors

4. Actuators and transmissions

5. Controller

6. Sensors

1. Base:

- The base may be fixed or mobile.

2. Manipulator arm:

- The most obvious mechanical configuration of the robot is the manipulator arm.

- There are several designs of the arm to facilitate movement within the work envelope with maximum possible load and speed with high precision and repeatability.
- The simplest robot may be a two or three axes arm. The axis is meant to understand independent movement or degree of freedom (DOF).
- A robotic manipulator arm consists of several separate links making a chain. The arm is located relative to the ground on either a fixed base or a movable base. It has a free-end where an end-deflector or gripper or sometimes a specialized tool holder (for holding, say, a welding gun) or any powered device (say, a drill) is attached.
- In a fixed base, six degrees of freedom robot, the first three links of the manipulator constitute the body and they help to place the end-effector at a desired location inside its work environment or working volume. The remaining three links make up the wrist of the manipulator and are used to define the orientation of the manipulator end points.

3. End – effector:

- Robot end – effector is the gripper or end of arm tooling mounted on the wirst of the robot manipulator arm.
- A robot performs a variety of tasks for which various tooling and special grippers are required to be designed.
- A robot manipulator is flexible and adaptable, but its end – effector is task – specific.
- A gripper designed for picking up a tool to be fitted to a CNC machine tool is not suitable for welding a railway vagon.

The wide range of gripping methods include:

(i) Mechanical clamping;
(ii) Magnetic gripping;
(iii) Vaccum (suction) gripping.

4. Actuators and Transmissions:

ACTUATORS:

The robot arm can be put to a desired motion with its payload if actuator modules are fitted in to provide power drives to the systems.

There are three different types of power drives in common use. They are:

(i) Pneumatic drives:

- These systems use compressed air to move the robot arm.
- The pneumatic systems may employ a linear actuator, i.e, double acting cushioned cylinders or it may employ rotary actuators like vane motors However, linear actuators are more popular.
- The advantages of pneumatic actuators are: Simple construction, relatively smaller payloads, the mass inertia and delayed response of the robot arm due to the sponginess and reduced repeatability.
- Non-servo robots can be built up with pneumatically powered actuators.

(ii) Hydraulic drives:

- In a hydraulic system, the electric motor pumps fluid (oil) from a reserve tank to the hydraulic actuators which are, in general, double acting piston-cylinder assemblies. Fluid at a higher pressure passes through control valves before its entry into the linear actuators. On the other hand, rotary actuator comprising some motors or hydraulic motors which rotate continuously may also be employed.

- The hydraulic drives have high payload capacities and are relatively easy to maintain. They are, however, rather expensive and not as accurate as electric drives.

(iii) Electrical drives:

- These drives are clean and quiet with a high degree of accuracy and reliability. They also offer a wide range of payload capacity, accompanied by an equally wide range of costs.
- D.C servo motors, Brushless D.C motors, Reversible A.C servo motors and Stepper motors are important electrical drives.

5. **Controller:** The "controller" provides the intelligence that is necessary to control the manipulator system.

   It looks at the sensory information and computes the control commands that must be sent to the actuators to carry out the specified tasks. It generally includes:

   (i) Memory to store the control program and the state of the robot system obtained from the sensors.

   (ii) A computational unit that computes the control commands.

   (iii) The appropriate hardware to interface with the external world (sensors and actuators)

   (iv) The hardware for a user interface.

- The "user interface" allows the use of a human operator to monitor or control the operation of the robot.
- It must have a display that shows the status of the system.
- It must also have an input device that allows the human to enter commands to the robot.
- The user interface may be a 'personal computer' with the 'appropriate software' or a 'tech pendant'.

6. **Sensors :** The sensors perform the following functions:

(i) internal to act as feedback devices to direct further actions of the manipulator arm and the end effector (gripper),

(ii) external to interact with robot working environment

07(a).

Sol: A traditional approach is sequential design of electrical/electronic components and system and mechanical system separately and interconnect these systems together .

It has many problems/drawbacks as :

- Original characteristics and operating conditions of independently designed systems will change due to actual loading and interaction.
- Perfect impedance matching of two independently designed systems are almost impossible.
- Sequential design takes more time.
- It has low flexibility to change.
- So, mechatronics is an integrated or simultaneous approach to engineering

design. It supports concurrent or integration of engineering across traditional boundaries. Its design process consists of three aspects.

(a) modelling and simulation

(b) prototyping

(c) deployment

(a) Modelling and simulation: Here physical systems are represented by suitable models and computer simulator methods are used. Modelling and design can be done by understanding the system and its function of objectives.

(b) Prototyping: It is the process of replacing with actual hardware in above modelling and simulation.

Ex: Sensors, actuators, controllers are interfaced with input and output signals and connected with models.

(c) Deployment: It is associated with final product design with development of software and testing life cycle.

Modeling and simulation
Recognition of need
Conceptual design
Mathematical modeling
Sensors and actuators
Development of detailed models
Control system design
Optimization

→

Prototyping
Hardware design and
Simulation
Design optimization

→

Deployment
Deployment of software
Manufacturers and testing
Lifecycle

07(b).

Sol: m = 2200 kg, r = 0.033 m,

R = 80 m, h = 0.55m, w = 1.5 m

$I_w = 2.4\,kg\,m^2, I_e = 1.2\,kg\,m^2$

$G = \frac{\omega_e}{\omega_w} = 3$

(i) Reaction due to weight

$$R_w = \frac{mg}{4} = \frac{2200 \times 9.81}{4} = 5395.5\,N(upwards)$$

(ii) Reaction due to gyroscopic couple

$$C_w = 4I_w \frac{v^2}{rR} = 4 \times 2.4 \times \frac{v^2}{0.33 \times 80} = 0.364v^2$$

$$C_e = I_e G \omega_w \omega_p$$

$$= 1.2 \times 3 \times \frac{v^2}{0.33 \times 80} = 0.136v^2$$

$$\therefore C_G = C_w + C_e = 0.364v^2 + 0.136v^2 = 0.5v^2$$

Reaction on each outer wheel,

$$R_{G0} = \frac{C_G}{2w} = \frac{0.5v^2}{2 \times 1.5} = 0.167v^2(up\,wards)$$

Reaction on each inner wheel,

$$R_{Gi} = 0.167v^2(downwards)$$

(iii) reaction due to centrifugal couple,

$$C_c = \frac{mV^2}{r}h = \frac{2200 \times v^2}{80} \times 0.55 = 15.125v^2$$

Reaction on each outer wheel,

$$R_{co} = \frac{C_e}{2w} = \frac{15.125v^2}{2\times1.5} = 5.042v^2(\text{upwards})$$

Reaction on each inner wheel,

$$R_{ci} = 5.042v^2(\text{downwards})$$

For maximum safe speed, the condition is

$$R_w = R_{Gi} + R_{ci}$$

$$5395.5 = (0.167 + 5.042)v^2$$

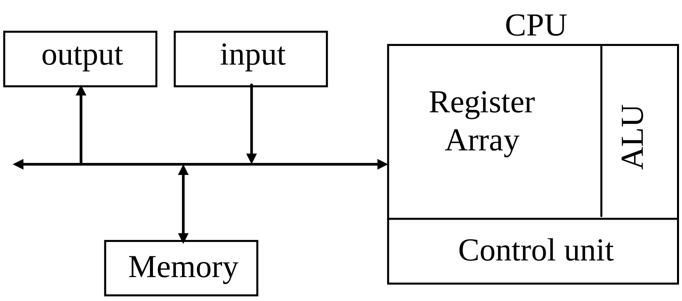
$$v^2 = 135.8 \Rightarrow v = 32.18\,\text{ms}$$

$$\text{or } v = \frac{32.18\times3600}{1000} = 115.9\,\text{km/h}$$

07(c).

Sol: The micro processor based system consists of the following segments/parts :

(i) Central Processing Unit (CPU)
(ii) Input and Output
(iii) Memory
(iv) Buses



Block diagram

(i) Central Processing Unit (CPU): It processes the Input data based on program and generates result in output.

CPU consists of three parts as

(a) Arithmetic and Logic Unit (ALU) : ALU performs computing functions such as arithmetic [Addition, Subtraction and logic operations (AND, OR….)].

(b) *Register unit :* In this a group of registers are used to store data temporarily during the execution of program.

(c) *Control unit :* This provides necessary timing and control signals for all operations during data processing.

(ii) I/P and O/P: These interfaces provide communication between processor with peripherals in outside world. The selection of I/P and O/P depends on the application and usage of mechatronic system.

(iii) Memory: It stores program and data in binary form and appears as one or more integrated circuits (IC). The size of memory depends on application of mechatronic system.

Memory is divided into

(a) ROM (Read only memory): the program/data permanently and available in ROM / PROM / EPROM / EEPROM.

(b) RAM(Random Access Memory): It is Read/Write Memory, used to store the currently operated data temporarily. It is also available in SRAM / DRAM.

(iv) Buses are numbers of conductor or tracks on printed circuit boards (PCB) to carry digital signals.

There are three types of buses :

(a) Address bus is used to identity the peripheral with unique address for each either for input (or) output (or) memory registers.

(b) Data bus carries the data between processor and peripherals during data processing. Based on length of data bus

(4 bit/8/16/32….etc), speed of processor depends.

(c) **Control bus** carries control signals relating to timing [clock signals] and controls actions to coordinate various activities.

In brief, micro processor communicates with all peripherals (memory and input and output) with buses based on timing and control signals and performs all computing tasks specified in a programme.

07(d).

Sol: With the origin at O, the motion of the wheel in the vertical direction can be written as y = Ycos(2πvt/L), where v is the velocity of the vehicle and, at t = 0, it was at A. The equation of motion of the vehicle in terms of x and y explained in figure.

$$m\ddot{x} + k(x - y) = 0$$

Or, $m\ddot{x} + kx = kY\cos\omega t$

with $\omega = 2\pi V/L$

(i) For resonance, $\omega = \omega_n$ Or

$$2\pi V/L = (k/m)^{1/2}$$

Or, $k = 4\pi^2 mV^2/L^2 = 34\times10^4$ N/m.

(ii) As $\zeta = 0$.

Thus, $X = \pm Y.(1/(1-r^2))$

Y and X are given to be 0.025 m and 0.006 m, respectively. Since X is less than Y, only the negative sign will yield a sensible result.

So, $r^2 = 1 + Y/X = 5.17$

Or, $r = 2.275$

Hence, $\dfrac{\omega}{\omega_n} = 2.275$

Or, $\omega_n = \left(\dfrac{k}{m}\right)^{1/2} = \dfrac{2\pi V}{L\times 2.275}$

Or, $k = 6.57 \times 10^4$ N/m.

The rider will leave his seat if at any stage the downward acceleration of the vehicle is more than 9. In the steady state, the maximum acceleration (both in the downward and upward directions) is given by

$\omega^2 X = \left(4\pi^2 V^2 / L^2\right).X = 4.6$ m/s^2 < g.

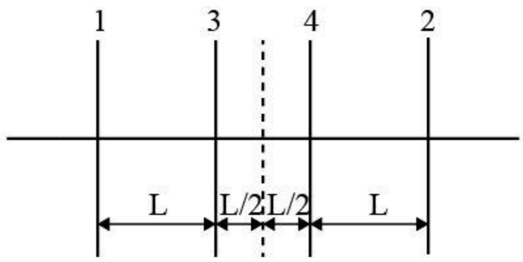
Hence, the rider will not leave his seat.

08(a)(i).

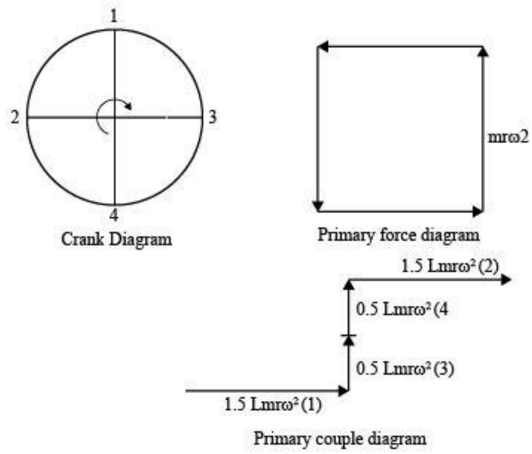
Sol: Given that:

For a two stroke four cylinder in line engine the firing order is $1 \rightarrow 2 \rightarrow 4 \rightarrow 3$

Cylinder Dimensions:

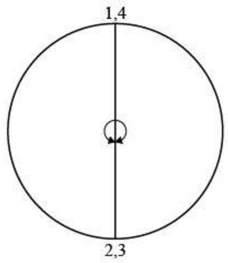


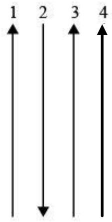
Primary Crack:



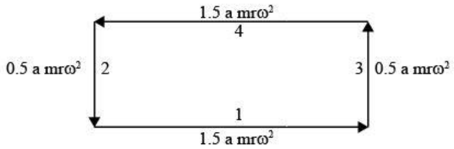
In this two stroke four stroke in line engine the primary force diagram is closed polygon so that engine is balanced in primary force but the primary coupled is not balanced because there is no closed polygon.

Secondary Crack:





Secondary force diagram



In this secondary crack diagram the secondary force is unbalanced but secondary couple is balanced because for secondary force the force are parallel to each other and secondary coupled is closed polygon.

08(a)(ii).

Sol: Given that

Arm (a) = 240 mm

Mass of ball (m) = 5 kg

Mass of sleeve (M) = 18 kg

Ball radius at minimum speed (r_1) = 150 mm

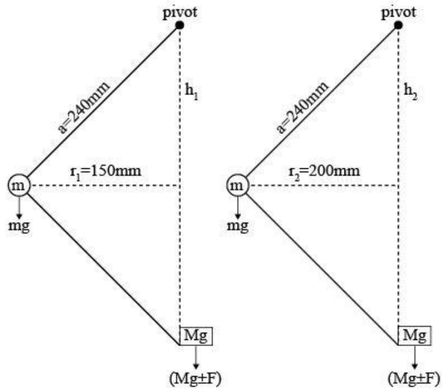
Ball radius / path at maximum speed (r_2) = 200 mm

For porter governor

$$N^2 = \frac{895}{4}\left(\frac{2mg + (mg+F)(1+k)}{2mg}\right)$$

If 1st case we assume K = 1 and F = 0

$$N^2 = \frac{895}{h}\left(\frac{m+M}{m}\right)$$



Case I:

Then for min speed

$$h_1 = \sqrt{a^2 - r_1^2} = \sqrt{240^2 - 150^2}$$

= 187.35 mm = 0.187 m

$$\text{Then } N_{min}^2 = \frac{895}{h_1}\left(\frac{m+M}{m}\right) = \frac{895}{0.187}\left(\frac{5+18}{5}\right)$$

N_{min} = 148.24 rpm

For max. speed

$$h_2 = \sqrt{a^2 - r_2^2} = \sqrt{240^2 - 200^2} = 132.66 \text{ mm}$$

$$\text{Then } N_{max}^2 = \frac{895}{0.133}\left(\frac{5+18}{5}\right)$$

$$N_{max} = 176.16 \text{ rpm}$$

Range of speed = $N_{max} - N_{min}$

= 176.16 − 148.24

= 27.925 rpm

Case II:

F = 10N K = 1

(i) For minimum speed r = 150 min as the radii decrease, the F is negative and h_1 = 187.25 mm.

$$N_{min}^2 = \frac{895}{h_1}\left[\frac{mg + \left(\frac{Mg - f}{2}\right)(1+k)}{mg}\right]$$

$$= \frac{895}{0.187}\left(\frac{5\times 9.8 + 18\times 9.8 - 10}{5\times 9.8}\right)$$

$$N_{min} = 145.05\,\text{rpm}$$

(ii) For r = 200 mm and h2 = 132.66 mm as the friction act positive

$$N_{max}^2 \frac{895}{0.133}\left(\frac{5\times 9.8 + 18\times 9.810}{5\times 9.8}\right)$$

$$N_{max} = 180\,\text{rpm}$$

Then,
$$N_{mean} = \frac{N_{max}+N_{min}}{2}$$
$$= \frac{180+145.05}{2} = 162.52\,\text{rpm}$$

Coefficient of sensitiveness

$$= \frac{N_{max}-N_{min}}{N_{mean}} = \frac{180-145.05}{162.52} = 0.215$$

08(b).

Sol: Euler angle representation means rotation with reference to current ovable frame ⇒ Post multiplicaiton.

⇒ Roll → Pitch → Roll

⇒ R(z, δ) → R(ψ, λ) → R(z, α)

⇒ R(z, δ).R(ψ, λ).R(z, α)

$$\Rightarrow \begin{bmatrix} C\delta & -S\delta & 0 \\ S\delta & C\delta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} C\lambda & 0 & S\lambda \\ 0 & 1 & 0 \\ -S\lambda & 0 & C\lambda \end{bmatrix}\begin{bmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Post multiplication is considered.

$$= \begin{bmatrix} C\delta C\lambda & -S\delta & C\delta S\lambda \\ S\delta C\lambda & C\delta & S\delta S\lambda \\ -S\lambda & 0 & C\lambda \end{bmatrix}\begin{bmatrix} C\alpha & -S\alpha & 0 \\ S\alpha & C\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\delta C\lambda C\alpha - S\delta S\alpha & -C\delta C\lambda S\alpha - S\delta S\alpha & C\delta S\lambda \\ S\delta C\lambda C\alpha + C\delta S\alpha & -S\delta C\lambda S\alpha + C\delta C\alpha & S\delta S\lambda \\ -S\lambda C\alpha & S\lambda S\alpha & C\lambda \end{bmatrix}$$

In above matrix

Cδ = cosδ, Cλ = cosλ, Cα = cosα

Sδ = sinδ, Sα = sinα, Sλ = sinλ

In brief, robot tool moves time to time with reference to base, by using different actuators at all joints in Robot anatomy. Different actuators get signals from controller based on sensors feed back signals. Robot tool position and orientation is maintained time to time as per application of Robot.

08(c).

Sol:

(i) When a pair of gear transmits power, the normal force is passed through common normal to the two involutes at the point of contact, which is also a tangent to the base circles of mating gear pair. If by any reason, any of the two surfaces is not involute, the two surfaces could not touch each other tangentially and transmission of power would not be proper. The mating of non−conjugate gear teeth will violate the fundamental law of gearing and this non−conjugate action is called interference.

There are following methods to prevent interference:

1. **Under cutting:** When gear teeth are manufactured by a generating process, a portion of tooth flank which causes interference is cut away by the cutting tools.
2. **Stubbed tooth:** When a portion of a tooth near the top is cut away such a tooth is called stubbed tooth. Such a measure prevents interference but it reduces the contact ratio.

3. **Number of teeth:** Interference in a gear pair can be avoided by increasing number of teeth on the gears.

(ii) Minimum number of teeth on pinion:

r = Radius of pinion

R = Radius of gear

r_A = radius of addendum circle on pinion

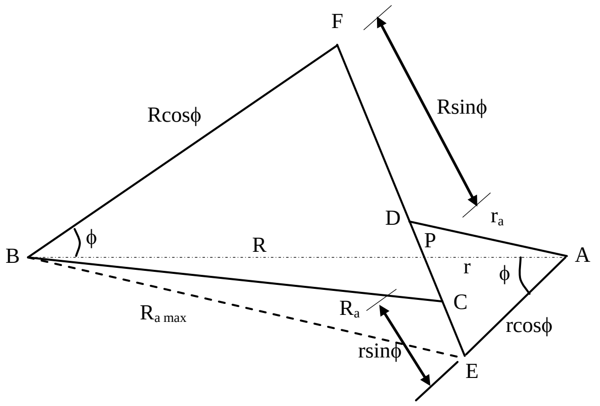
R_A = radius of addendum circle on gear

G = gear ratio = $\frac{r}{R} = 1$

A_P = standard addendum = 1

φ = pressure angle = 20°

The maximum value of the addendum radius of the wheel to avoid interference can be up to BE.



Dimensions for involute gears in engage

we$(BE)^2 = (BF)^2 + (FE)^2$

$= (BF)^2 + (FP + PE)^2$

$= (R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2$

$= R^2\cos^2\varphi + R^2\sin^2\varphi + r^2\sin^2\varphi + 2rR\sin^2\varphi$

$= R^2(\cos^2\varphi + \sin^2\varphi) + \sin^2\varphi\,(r^2 + 2rR)$

$= R^2 + (r^2 + 2rR)\sin^2\varphi$

$$= R^2\left[1 + \frac{1}{R^2}\left(r^2 + 2rR\right)\sin^2\varphi\right]$$

$$= R^2\left[1 + \left(\frac{r^2}{R^2} + \frac{2r}{R}\right)\sin^2\varphi\right]$$

$$BE = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi}$$

Therefore, the maximum value of the addendum of the wheel can be equal to (BE – pitch circle radius) or

$$a_{w\max} = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi} - R$$

$$= R\left[\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi} - 1\right]$$

Let, t = number of teeth on the pinion

T = number of teeth on the wheel

Now, $R = \frac{mT}{2}, r = \frac{mt}{2}$

Hence, $$a_{w\max} = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\varphi} - 1\right]$$

$$= \frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right]$$

Let the adopted value of the addendum in some case be a_w times the module of teeth. Then this adopted value of the addendum must be less than the maximum value of the addendum to avoid interference.

i.e

$$\frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right] \ge a_w m$$

(or) $$T \ge \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1}$$

In the limit,

$$T = \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1}$$

This gives the minimum number of teeth on the wheel for the given values of the gear ratio, the pressure angle and the addendum coefficient a_w.

The minimum number of teeth on the pinion is given by, $t = \frac{T}{G}$

For $a_w = 1$, i.e., when the addendum is equal to one module,

$$T \geq \frac{2}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varphi} - 1}$$

For equal number of teeth on the pinion and the wheel, G = 1 and

$$T_{min} = \frac{2}{\sqrt{1+3\sin^2\varphi} - 1}$$

Put the value

$$T_{min} = \frac{2\times 1}{\sqrt{1+1(1+2)\sin^2\varphi - 1}}$$

= 12.32 teeth = 13 teeth