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ESE – 2019 MAINS
OFFLINE TEST SERIES

MECHANICAL ENGINEERING

TEST – 2
SOLUTIONS

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01(a).

Sol: Given forces are

20 kN, 50 kN, 20 kN, 50 kN

$\therefore$ Resultant force $F_R = 20 + 50 + 20 + 50$

$\vec{F}_R = -140\,\hat{k}\ (\text{kN})$

Moment of the force system about origin,

$$\Sigma M_0 = \vec{r}_1\times\vec{F}_1 + \vec{r}_2\times\vec{F}_2 + \vec{r}_3\times\vec{F}_3 + \vec{r}_4\times\vec{F}_4$$

$$= (4\hat{i}+3\hat{j})\times -50\hat{k} + 10\hat{i}\times -20\hat{k} + 11\hat{j}\times -20\hat{k} + (10\hat{i}+13\hat{j})\times -50\hat{k}$$

$$= 200\,\hat{j} - 150\,\hat{i} + 200\,\hat{j} - 220\,\hat{i} + 500\,\hat{j} - 650\,\hat{i}$$

$$\Sigma M_o = -1020\,\hat{i} + 900\,\hat{j}$$

$$\Sigma M_o = (x\,\hat{i} + y\,\hat{j})\times -140\,\hat{k}$$

$$-1020\,\hat{i} + 900\,\hat{j} = 140x\,\hat{j} - 140\,y\,\hat{i}$$

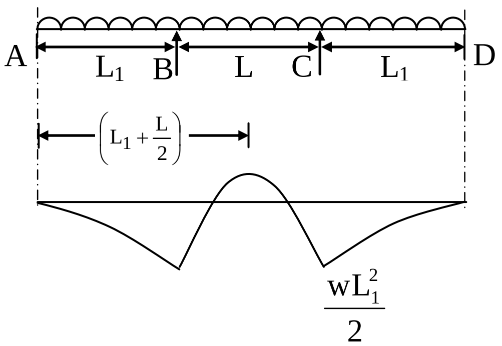
Equating $\hat{i}$ & $\hat{j}$ components,

we get y = 7.29 m & x = 6.43 m

01(b).

Sol: The reaction at the support B and C are given as

$$R_B = R_C = w\left(L_1 + \frac{L}{2}\right)$$



at $L_1 + \frac{L}{2}$ maximum positive bending moment occurs and is equal to

$$M_{max1} = w\left(L_1+\frac{L}{2}\right)\left(\frac{L}{2}\right) - \frac{w}{2}\left(L_1+\frac{L}{2}\right)^2 \quad \text{----(1)}$$

and maximum negative moment occurs at C at distance L_1 from D,

$$M_{max2} = \left(\frac{wL_1^2}{2}\right) \quad \text{----------(2)}$$

$\therefore$ Equating (1) and (2)

$$w\left(L_1+\frac{L}{2}\right)\left(\frac{L}{2}\right) - \frac{w}{2}\left(L_1+\frac{L}{2}\right)^2 = \frac{wL_1^2}{2}$$

$$w\left(L_1+\frac{L}{2}\right)\left[\frac{L}{2} - \frac{L_1}{2} - \frac{L}{4}\right] = \frac{wL_1^2}{2},$$

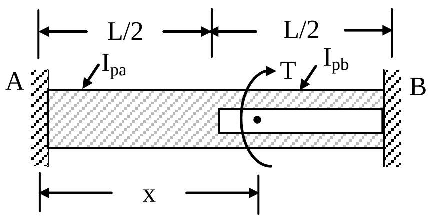
$$\frac{w}{2}\left(L_1+\frac{L}{2}\right)\left(\frac{L}{2} - L_1\right) = \frac{wL_1^2}{2}$$

$$\left(\frac{L}{2}\right)^2 = 2L_1^2$$

$$\therefore\ L_1 = \frac{L}{2\sqrt{2}}$$

01(c).

Sol:



Applying Equilibrium conditions

$T = T_A + T_B \rightarrow (1)$

$\theta_{AB} = 0$

$$\frac{T_A\left(\frac{L}{2}\right)}{G I_{Pa}} + \frac{T_a\left(x-\frac{L}{2}\right)}{G I_{Pb}} - \frac{T_b(L-x)}{G I_{Pb}} = 0$$

If, $T_A = T_B \Rightarrow T_A = T_B = T/2$ $[\because \text{given}]$

$$\frac{\frac{L}{2}}{I_{Pa}}+\frac{x-\left(\frac{L}{2}\right)-L+x}{I_{Pb}}=0$$

$$\frac{\frac{L}{2}}{I_{Pa}}+\frac{2x-\left(\frac{3}{2}\right)L}{I_{Pb}}=0$$

$$2x-\left(\frac{3}{2}\right)L=\frac{-I_{Pb}}{I_{Pa}}\left(\frac{L}{2}\right)$$

$$2x=\frac{3}{2}L-\frac{I_{Pb}}{I_{Pa}}\left(\frac{L}{2}\right)$$

$$\therefore x=\frac{L}{4}\left[3-\frac{I_{Pb}}{I_{Pa}}\right]$$

01(d).

Sol: In such problem, first determine the stresses for each loading condition. Using the principle of superposition, determine the combined stresses and hence state of stress on the most critical section.

Stress due to axial load F = 30 kN

$$\sigma_a=\frac{4F}{\pi d^2}=\frac{4\times 30000}{\pi\times 45^2}=18.863\,N/mm^2$$

Due to bending moment M = 650 Nm

$$\sigma_b=\frac{32M}{\pi d^3}=\frac{32\times 650\times 10^3}{\pi\times(45)^3}=72.657\ N/mm^2$$

Due to torque T = 900 Nm

$$\tau_{xy}=\frac{16T}{\pi d^3}=\frac{16\times 900\times 10^3}{\pi\times(45)^3}=50.30\,N/mm^2$$

Combined stress on an element is

This is a plane stress case,

$\sigma_{xx}=91.52\,MPa$ $\quad\sigma_{yy}=\sigma_{zz}=0$

$\tau_{xy}=50.3\,MPa$ $\quad\tau_{yz}=\tau_{zx}=0$

The principal stresses are

$$\sigma_{1,3}=\frac{\sigma_{xx}+\sigma_{yy}}{2}\pm\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2+\tau^2}$$

$$\sigma_{1,3}=\frac{91.52+0}{2}\pm\sqrt{\left(\frac{91.52-0}{2}\right)^2+(50.3)^2}$$

$$\sigma_{1,3}=45.76\pm 68.00$$

$$\sigma_1=113.76\,MPa$$

$$\sigma_3=-22.24\,MPa$$

$$\sigma_2=0$$

The maximum shear stress theory

$$N=\frac{\sigma_y}{\sigma_1-\sigma_3}=\frac{280}{113.76-(-22.24)}=2.06$$

The distortion energy theory

$$N=\frac{\sigma_y}{\sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}}$$

$$N=\frac{280}{\sqrt{\frac{(113.76-0)^2+[0-(-22.24)]^2+(-22.24-113.76)^2}{2}}}$$

$$N=\frac{280}{126.4}=2.22$$

01(e).

Sol: Given:

F_{max} = 250 kN, $\quad F_{min}=100\,kN$,

$S_e=225\,MPa$, $\quad\sigma_y=280\,MPa$,

factor of safety N = 1.8, w = 120 mm.

Let the plate thickness be t,

Cross sectional area of plate A = w.t

= 120 t mm²

$$\sigma_{max}=\frac{F_{max}}{A}=\frac{250\times 10^3}{120t}=\frac{2.083\times 10^3}{t}$$

$$\sigma_{min}=\frac{F_{min}}{A}=\frac{100\times 10^3}{120t}=\frac{0.833\times 10^3}{t}$$

Mean stress is

$$\sigma_m = \frac{1}{2}\left(\sigma_{max} + \sigma_{min}\right)$$

$$\sigma_m = \frac{1}{2}\left(\frac{2.083\times10^3}{t} + \frac{0.833\times10^3}{t}\right) = \frac{1.458\times10^3}{t}$$

Variable stress is

$$\sigma_a = \frac{1}{2}\left(\sigma_{max} - \sigma_{min}\right)$$

$$\sigma_a = \frac{1}{2}\left(\frac{2.083\times10^3}{t} - \frac{0.833\times10^3}{t}\right) = \frac{0.625\times10^3}{t}$$

Using soderberg design criterion

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{\sigma_y} = \frac{1}{N}$$

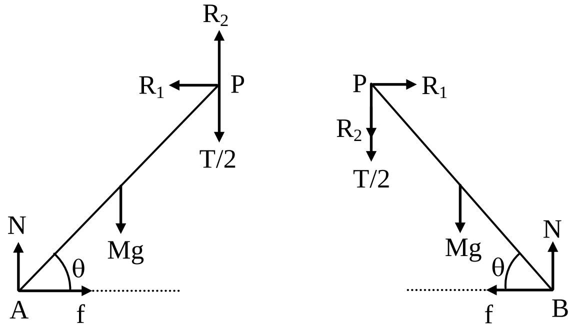
$$\frac{0.625\times10^3}{225t} + \frac{1.458\times10^3}{280t} = \frac{1}{1.8}$$

Solving , we get

t = 14.37 mm = 15 mm

02(a).

Sol: Let the block of mass m hangs from the point P by a string attached to the hinges of the two ladders.



Newton's second law gives the tension T in the string as T = mg. By symmetry, string tension pulls down each ladder by a force.

T/2 = mg/2

By Newton's third law, the reaction forces acting on the two ladders at the hinge point P are equal and opposite. These are shown by R_1 and R_2 in the figure. By symmetry, the normal reaction N at A and B are equal, the friction forces f at A and B are equal in magnitude but opposite in direction. Another force acting on both the ladders is their weight Mg which pass through their centre of mass. In equilibrium, the resultant forces on the two ladders are zero i.e.,

$N + R_2 - Mg - mg/2 = 0$ … (1)

$f = R_1$, … (2)

$N - R_2 - Mg - mg/2 = 0$ … (3)

The equation (1) and (3) give $R_2 = 0$ and $N = (M + m/2)g$. In equilibrium, the net torque about any point is zero. Thus, the torque about the point P for the left ladder is

$Mg(L/2)\cos\theta + f L \sin\theta - N L \cos\theta = 0$ … (4)

Substitute $N = (M + m/2)g$ in equation (4) and simplify to get

$$f = \left(\frac{M+m}{2}\right) g \cot\theta$$

02(b).

Sol: To find the reactions R_A and R_B.

To find the reaction R_B, taking moments about A.

$$R_B \times 8 = 4\times3\times\frac{3}{2} + 6 + 4\times6$$

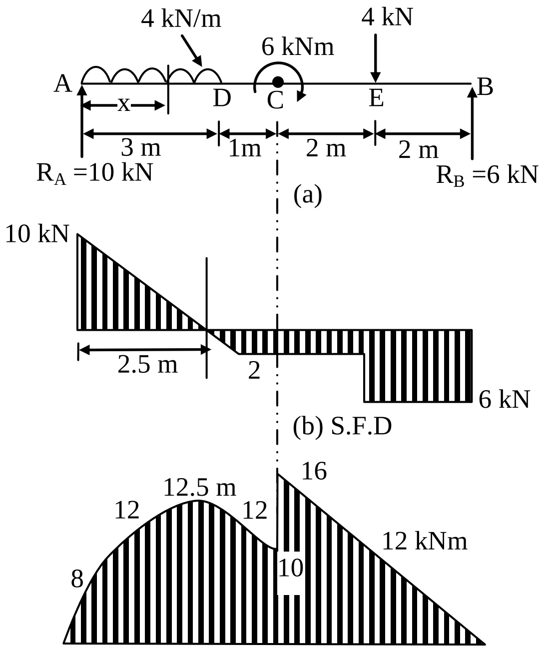
R_B = 6 kN

Similarly, to find the reaction R_A, taking moments about B

$R_A \times 8 + 6 = 4 \times 2 + (4 \times 3)(5 + 1.5)$

$\therefore \quad R_A = 10$ kN

To plot the shear force diagram



(a)
(b) S.F.D
(c) B.M.D

Shear force at any section at a distance x from A is given by

$$S.F|_x = +10 - 4x$$

$$S.F.\text{at } A|_{x=0} = +10\text{kN}$$

$$S.F.\text{at } D|_{x=3m} = +10 - 4 \times 3$$

i.e., = –2 kN

$\therefore$ shear force decreases gradually due to uniformly distributed load, changes its sign from A to D, and becomes zero when (10 –4x) = 0 i.e., at x = 2.5 m.

From D to E, the S.F. remains constant = 2 kN

$$S.F|_E = 10 - 4 \times 3 - 4 = -6\text{kN}$$

$$S.F.\text{at support } B = 6\text{kN} \uparrow$$

The shearing force diagram is as shown in figure.

To plot bending moment diagram

Consider a section xx at a distance x from A.

$$B.M|_x = \left[+10x - 4\frac{x^2}{2}\right] = \left[10x - 2x^2\right] 0 < x < 3$$

$$B.M|_{x=0} = 0$$

$$B.M|_{x=1} = (+10 - 2) = 8\text{kNm}$$

$$B.M|_{x=2} = +20 - 2(2)^2 = 12\,\text{kNm}$$

$$B.M|_{x=3} = +10 \times 3 - 2(3)^2 = 12\,\text{kNm}$$

To find the position of maximum B.M., differentiate the equation and equate to zero (in the portion AD)

i.e., $\dfrac{dM_x}{dx} = 10 - 2(2x) = 0$

$\therefore$ x = 2.5 m form support A.

Hence, B.M is either maximum or minimum when shear force changes its sign, unless and until it is not acted by external B.M.

Magnitude of maximum B.M.

$$= +10 \times 2.5 - 2\,(2.5)^2 = 12.5 \text{ kNm}$$

$$B.M|_{x=4m(\text{left of C})} = +10 \times 4 - 4 \times 3(1.5 + 1)$$
$$= +40 - 30 = 10\,\text{kNm}$$

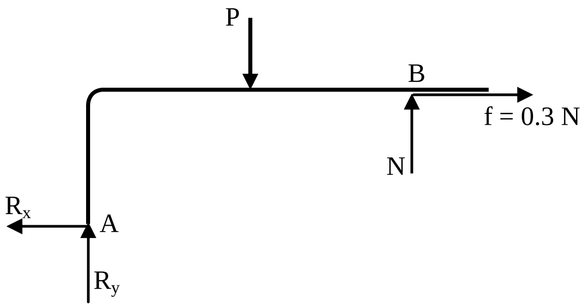
$$B.M|_{x=4m(\text{Right of C})} = +10 \times 4 - 4 \times 3(1.5 + 1) + 6$$
$$= 16\,\text{kNm}$$

$$B.M|_{x=2m\text{ from B}} = 6 \times 2 = 12\,\text{kNm}$$

From the BMD the maximum bending moment occurs at 'C' and is equal to 16 kNm.

02(c)(i).

Sol: F.B.D of brake lever is shown below.



For equilibrium of brake lever,

$\Sigma M_A = 0$

$N\times0.6 - 0.3\,N \times 0.2 - P \times 0.3 = 0$

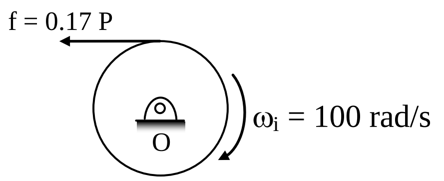
$N = 0.56\,P$

$f = 0.17\,P$

Let t be the time needed for disk to come to rest (let us suppose t > 2 sec)

We know that,

Torque = Rate of change of angular momentum



$$\tau = \frac{d\vec{L}}{dt}$$

$$\tau\,dt = d\vec{L}$$

$$\int 0.17\,P \times 0.15 \times (dt) = I\omega_f - I\omega_i \quad \left[I = \frac{mr^2}{2}\right]$$

$$0.0255\int_0^t P\,dt = -(-0.45\times100) = 45$$

$$\int_0^t P\,dt = 1764.7$$

$$\frac{1}{2}\times500\times2 + 500(t-2) = 1764.7$$

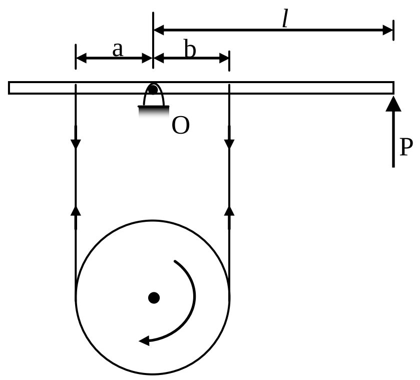
$\Rightarrow$ t = 4.53 sec

Thus, t > 2 sec.

Hence, our assumption is correct.

02(c)(ii).

Sol:



Assuming,

(1) a > b

(2) clockwise rotation

Taking moments about 'O'

$\Sigma M_o = 0$

$P\times l + T_2 a = T_1\times b$ -----(1)

$\frac{T_1}{T_2} = e^{\mu\theta}$ --------(2)

In the above equation the moment due to T_2 is in the same direction as that of moment due to P about O.

Hence, less force is required for braking action. These brakes are known as self-energizing brake. i.e. whenever additional moment is present in the direction of applied moment. The brake is said to be self energizing.

From the above equations (1) and (2) even if operating load P is absent, T_1 and T_2 exists. Hence braking can be obtained. i.e. even without the application of force. Brakes will be automatically applied. Such a situation is known as self locking.

Self-locking is an extreme condition of self energizing brakes.

$\therefore$ The condition for self locking is $P \le 0$

$\therefore$ from equation (1)

$$\frac{T_1 \times b - T_2 a}{\ell} \le 0$$

$$T_1 b - T_2 a \le 0$$

$$\frac{T_1}{T_2} \le \frac{a}{b}$$

when drum rotates in ccw direction

$$\Sigma m_0 = 0$$

$$P \times l + T_1 \times a = T_2 \times b$$

Hence, it leads to self energizing

$$P \le 0$$

$$T_2 \times b - T_1 \times a \le 0$$

$$T_2 b \le T_1 a$$

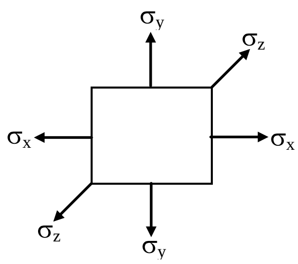
$$\frac{T_1}{T_2} \ge \frac{b}{a}$$

Limiting case $\frac{T_1}{T_2} = \frac{b}{a}$ which is not true.

Hence, the drum when rotated in new direction only behaves as self energizing but not as self-locking.

03(a)(i).

Sol:



$$\frac{\sigma_x - \mu(\sigma_y + \sigma_z)}{E} = \varepsilon_x \text{ ------------- (1)}$$

$$\frac{\sigma_y - \mu(\sigma_x + \sigma_z)}{E} = \varepsilon_y = 0 \text{ --------- (2)}$$

$$\frac{\sigma_z - \mu(\sigma_x + \sigma_y)}{E} = \varepsilon_z = 0 \text{ --------- (3)}$$

Adding eqn. (2) and (3)

$$(\sigma_z + \sigma_y) = 2\mu(\sigma_x) + \mu(\sigma_y + \sigma_z)$$

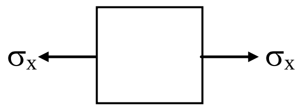
$$(\sigma_z + \sigma_y)[1 - \mu] = 2\mu\sigma_x$$

$$(\sigma_z + \sigma_y) = \frac{2\mu\sigma_x}{(1-\mu)} \text{ --------------(4)}$$

When only σ_x is acting and lateral contraction is completely restricted

$$\varepsilon_y = \varepsilon_z = 0$$

$\therefore$ $\mu_{eff} = 0$ $[\because \varepsilon_y = \varepsilon_z = -\mu_{eff}\frac{\sigma_x}{E_{eff}}]$



Substituting eqn. (4) in eqn. (1)

$$\frac{1}{E}\left[\sigma_x - \frac{2\mu^2}{(1-\mu)}\sigma_x\right] = \varepsilon_x = \frac{\sigma_x}{E_{eff}}$$

$$\frac{\sigma_x}{E}\left[\frac{(1-\mu) - 2\mu^2}{(1-\mu)}\right] = \frac{\sigma_x}{E_{eff}}$$

$$E_{eff} = \frac{(1-\mu)E}{(1+\mu)(1-2\mu)} \quad \text{and} \quad \mu_{eff} = 0$$

03(a)(ii).

Sol: **Poisson's ratio:** It is defined as the negative ratio of lateral strain to longitudinal strain in the member. It is dimensionless quantity.

$$\mu = \frac{-\varepsilon_{lat}}{\varepsilon_{lin}}$$

ε_{lat} = lateral strain

ε_{lin} = linear strain

μ ranges from 0.25 to 0.33 for general engineering materials.

03(b)(i).

Sol: d = 14mm, A = $\frac{\pi}{4}(14)^2 = 154\,\text{mm}^2$

For the maximum load that can be dropped the corresponding extension is ,

$$\Delta\ell = \frac{300\times10^6}{200\times10^9}\times 2 = 3\,\text{mm}$$

Work done by load = P(h + Δℓ)

= P(100+3)×10^{-3} = 103 ×10^{-3} P

Strain energy of the bar

$$= \frac{\sigma^2}{2E}\times \text{volume of bar}$$

$$= \frac{\left(300\times10^6\right)^2}{2\times200\times10^9}\times\left(154\times10^{-6}\right)\times 2$$

$$= \frac{\left(300\times10^6\right)^2}{2\times200\times10^9}\times\left(154\times10^{-6}\right)\times 2$$

= 69.3 Nm

Equating work done to strain energy

103×10^{-3} P = 69.3

$$\Rightarrow \quad P = \frac{69.3}{103\times10^{-3}} = 672.8\text{ N}$$

03(b)(ii).

Sol: Auto – frettage:

It is a process of pre-stressing the cylinder before using it in operation.

When cylinder is subjected to internal pressure, the circumferential stress at the inner surface limits the pressure carrying capacity of the cylinder.

In auto frettage, pre-stressing develops a residual compressive stress at the inner surface. When the cylinder is actually loaded in operation, the residual compressive stress at the inner surface begin to decrease, becomes zero and finally become tensile as the pressure is gradually increased. Thus autofrettage increases the pressure carrying capacity.

03(c).

Sol: T = 250 N-m, μ = 0.35,

$$r_1 = \frac{250}{2} = 125\,\text{mm}, \qquad r_2 = \frac{175}{2} = 87.5\,\text{mm},$$

δ = 5mm,

Load on each spring, $w = 800\text{ N/spring}$

No. of springs = 9

Total load, W = 800×9 = 7200 N

No. of contacting surfaces, n = 2,

(i) When the clutch is new, uniform pressure is applied

$$T = \frac{2}{3}\mu W\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)\times n$$

$$T = \frac{2}{3}\times 0.35\times 7200\times\left(\frac{125^3 - 87.5^3}{125^2 - 87.5^2}\right)\times 2$$

= 541.058 N-m

$$\text{FOS} = \frac{541.058}{250} = 2.164$$

(ii) When initial wear occurred, applying uniform wear theory.

$$T = \mu W\left(\frac{r_1 + r_2}{2}\right)\times n$$

$$= 0.35 \times 7200 \times \left(\frac{125+87.5}{2}\right) \times 2$$

= 535.5 N-m

$$\text{FOS} = \frac{535.5}{250} = 2.142$$

(iii) The spring force required to transmit a torque of

$$250 = \mu W' \left(\frac{r_1 + r_2}{2}\right) \times 2$$

$$= 0.35 \times W' \times \left(\frac{125+87.5}{2}\right) \times 2$$

W′ = 3361.34 N

$$\therefore W'/\text{spring} = \frac{3361.34}{a} = 373.48\,\text{N}$$

If x = wear of the plate

compression of spring, $\delta_{net} = 5 - x$

$$\text{stiffness of spring} = \frac{373.48}{5-x}$$

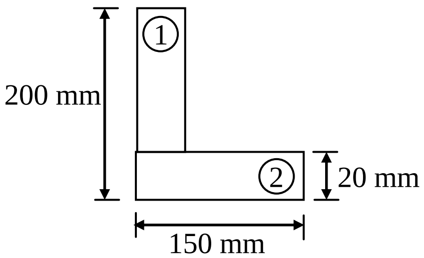
stiffness of spring is also given by $= \frac{800}{5}$

$$\therefore \frac{373.48}{5-x} = \frac{800}{5} \Rightarrow x = 2.67\,\text{mm}$$

When the wear is more than 2.68 the slip will occur.

03(d).

Sol:



As the weld is unsymmetrical, the CG of the weld area is to be determined.

Area - 1: $A_1 = (200-20) \times 20 = 3600\,\text{mm}^2$

Area - 2: $A_2 = 150 \times 20 = 3000\,\text{mm}^2$

Position of the centroidal axis is obtained as

$$b = \frac{A_a \times 100 + A_b \times 10}{A_a + A_b}$$

$$= \frac{3600 \times 110 + 3000 \times 10}{3600 + 3000}$$

b = 64.545 mm

a = 200 – b = 200 – 64.545 = 135.454 mm

The maximum load that can be carried by a single fillet weld is given by

F = 0.707 h l τ_{all}

$$\ell = \frac{F}{0.707\,h\,\tau_{all}} \;......(i)$$

If loads shared by weld A and B are F_a and F_b respectively, then force in the weld A is

$$F_a = 0.707\,h\,\ell_a\,\tau_{all}$$

Force shared by weld B is

$$F_b = 0.707\,h\,\ell_b\,\tau_{all}$$

Taking the moment of the forces about the CG of the weld and equating both (As they are equal but opposite to each other),

We have, $F_a \times a = F_b \times b$

Substituting forces and simplifying

$$0.707\,h\,\ell_a\,\tau_{all} \times a = 0.707\,h\,\ell_b\,\tau_{all} \times b$$

$$\frac{\ell_a}{\ell_b} = \frac{b}{a} = \frac{64.545}{135.454} = 0.476 \quad ....(ii)$$

$$\ell_a = 0.476\,\ell_b$$

From equation (i)

$$\ell = \ell_a + \ell_b = \frac{F}{0.707\,h\,\tau_{all}} \quad .....(iii)$$

The maximum load that can be carried by the steel angle is

$$F = A\sigma_{all} = (A_1 + A_2)\sigma_{all} = 6600 \times 124$$

F = 818400 N.........(iv)

From equation (ii), (iii) and (iv)

$$0.476\,\ell_b + \ell_b = \frac{818400}{0.707 \times 15 \times 94}$$

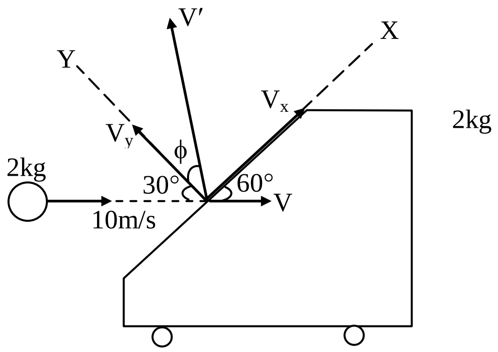
$$\ell_b = 556.212 \text{ mm}$$

$$\ell_a = 0.476 \times 556.212 = 264.757 \text{ mm}$$

12.5 mm additional length should be added for the end correction.

04(a).

Sol:



Let us take X – Y axes as shown above.

Let the velocity of sphere after collision is v_y along y-axis and v_x along x-axis.

Let the velocity of carriage be v (m/s)

Now, along x-direction there is no force on the sphere, so its velocity remains constant along x-direction.

$V_x = 10 \cos 60 = 5$ ....(i)

Also coefficient of restitution,

$$e = \frac{v\cos 30 + v_y}{10\cos 30}$$

$v_y + v\cos 30 = 0.6 \times 10 \cos 30 = 5.2$ ....(ii)

Also by using linear momentum conservation along the direction of motion of carriage.

$2 \times 10 = 2 \times v_x \times \cos 60 - 2 \times v_y \cos 30 + 10v$

$20 = 5 - 1.732 v_y + 10v$

$1.732 v_y - 10v = -15$ ....(iii)

From (i), (ii) and (iii), we get

$v_y = 3.4$ m/s and $v = 2.08$ m/s

$$\text{Thus, } v' = \sqrt{(v_x)^2 + (v_y)^2}$$

$$= \sqrt{5^2 + 3.4^2} = 6.04 \text{ m/s}$$

$$\phi = \tan^{-1}\left(\frac{v_x}{v_y}\right) = 55.78$$

$\Rightarrow \theta = \phi + 55.78 = 30 + 55.78 = 85.78°$

Again by using conservation of mechanical energy of ‘carriage + spring’ system

$$\frac{1}{2} \times 10 \times v^2 = \frac{1}{2} \times k \times \delta^2$$

$$\frac{1}{2} \times 10 \times 2.08^2 = \frac{1}{2} \times 1600 \times \delta^2$$

$\Rightarrow \delta = 164$ mm

04(b).

Sol: P = 7.5 kW, Speed = 300 rpm

Pressure angle $\phi = 20°$,

Pitch circle diameter of the gear

$$D_p = 150 \text{ mm}$$

Torque is obtained as follows

$$T = \frac{P \times 9550}{\text{speed}} = \frac{7.5 \times 9550}{300} = 238.75 \text{ N-m}$$

From the gear dimension, torque that gear can transmit is given by,

$$T = F_t \times \frac{D_p}{2}$$

Hence, the tangential force acting on the gear is

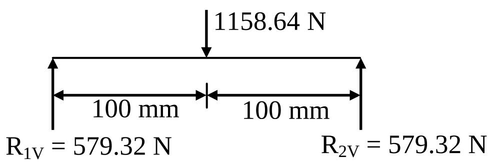
$$F_t = \frac{2T}{D_p} = \frac{2 \times 238.75 \times 1000}{150} = 3183.33\,N$$

Radial load on the shaft is given as

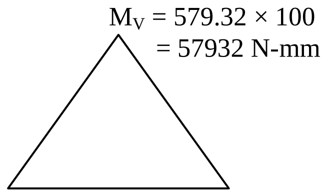
$$F_r = F_t \tan\phi \;= 3183.33 \times \tan 20 = 1158.64\,N$$

The line diagram of the shaft and loading is shown.

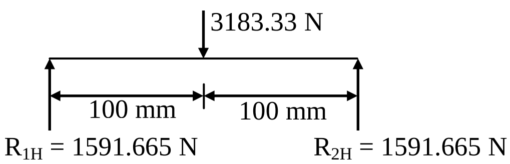
Separately this forces along with reactions at the bearing is shown in figure below. Reactions are obtained from force and moment equilibrium conditions.



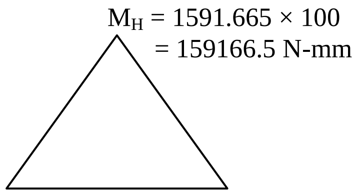
(Vertical Force diagram)



(Vertical Bending moment diagram)



(Horizontal Force diagram)



(Horizontal Bending moment diagram)

The resultant bending moment is

$$M = \sqrt{(M_H)^2 + (M_V)^2}$$

$$= \sqrt{(159166.5)^2 + (57932)^2}$$

= 169381.965 N-mm

The equivalent torque,

$$T_e = \sqrt{M^2 + T^2}$$

$$= \sqrt{(169381.965)^2 + (238750)^2}$$

$$T_e = 292731.639\,N - mm$$

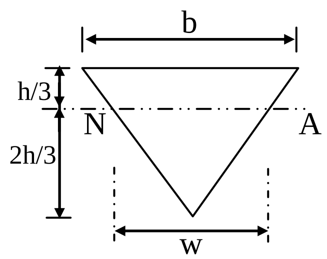
From torsion relation,

$$\tau_{all} = \frac{16 T_e}{\pi d^3} \Rightarrow d = \sqrt[3]{\frac{16 \times 292731.639}{\pi \times 45}}$$

$\Rightarrow$ d = 32.116 mm

04(c)(i).

Sol:



$$\tau = \frac{FA\bar{y}}{I_{NA} w}$$

Where, w = width of the section

$$A\bar{y} = \frac{1}{2} \times \left(\frac{2}{3}b\right)\left(\frac{2}{3}h\right) \times \left[\frac{1}{3} \times \left(\frac{2}{3}h\right)\right]$$

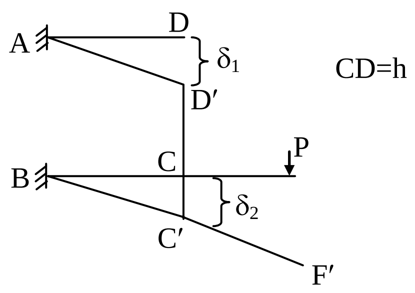
$$A\bar{y} = \frac{4}{81} bh^2$$

$$I_{NA} = \frac{bh^3}{36}, \quad w = \frac{2}{3}b$$

$$\tau_{NA} = \frac{FA\bar{y}}{I_{NA} \times w} = \frac{F \times \frac{4}{81}bh^2}{\frac{bh^3}{36} \times \frac{2}{3}b} = \frac{8}{3}\frac{F}{bh}$$

04(c)(ii).

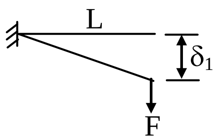
Sol:



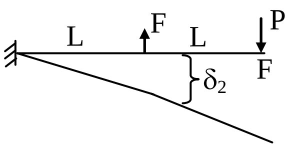
δ = elongation of CD

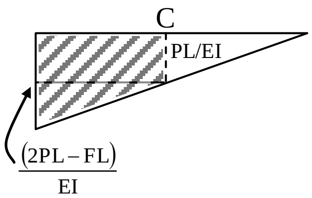
Deflection/Displacement compatibility conditions is $\delta_2 - \delta_1 = \delta$

Difference in deflection of beams at C and D is equal to the elongation of steel rod CD.



$$\delta_1 = \frac{FL^3}{3EI}$$





Using moment area method

δ_2 = moment of shaded area about C

$$\delta_2 = \frac{PL^3}{2EI} + \frac{1}{2}\frac{(PL - FL)}{EI}L.\frac{2L}{3}$$

$$= \frac{5PL^3}{6EI} - \frac{FL^3}{3EI}$$

Elongation of steel rod due to tensile force F is $\delta = \dfrac{Fh}{EA}$

$$\delta_2 - \delta_1 = \delta \Rightarrow \frac{5PL^3}{6EI} - \frac{FL^3}{3EI} - \frac{FL^3}{3EI} = \frac{Fh}{EA}$$

$$F = \frac{\frac{5}{6}\frac{PL^3}{EI}}{\frac{2L^3}{3EI} + \frac{h}{EA}}$$

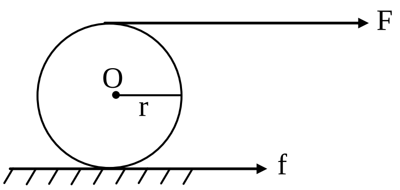
$$= \frac{\left(\frac{5 \times 50 \times 10^3 \times 2000^3}{6 \times 24 \times 10^{12}}\right)}{\left(\frac{2 \times 2000^3}{3 \times 24 \times 10^{12}} + \frac{5000}{200 \times 10^3 \times 300}\right)}$$

= 45454.5 N

$$\delta_1 = \frac{FL^3}{3EI} = \frac{45454.5 \times 2000^3}{3 \times 24 \times 10^{12}} = 5.051 \text{ mm}$$

05(a).

Sol:



The situation is shown in figure. As the force F rotates the sphere, the point of contact has a tendency to slip towards left so that the static friction on the sphere will act towards right. Let r bet the radius of the sphere and a be the linear acceleration of centre of sphere. The angular acceleration about the centre of the sphere is α = a/r, as there is not slipping.

For the linear motion of centre,

$$F + f = ma \qquad \ldots \text{(i)}$$

and for the rotational motion about the centre,

$$Fr - fr = I\alpha = \left(\frac{2}{5}mr^2\right)\left(\frac{a}{r}\right)$$

or, $$F - f = \frac{2}{5}ma \qquad \ldots \text{(ii)}$$

from (i) and (ii),

$$2F = \frac{7}{5} ma \text{ (or) } a = \frac{10F}{7m}$$

05(b).

Sol: Given data:

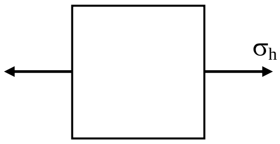
P = 300 kPa, d = 200 mm, t = 7 mm

Case (I): Hoop stress,

$$\sigma_h = \frac{Pd}{2t} = \frac{300 \times 10^3 \times 200}{2 \times 7} = 4.29 \times 10^6 \text{ Pa}$$

Longitudinal stress, $\sigma_L = 0$

As top end of the cylinder is not constrained, there will not be any longitudinal stress in the wall.

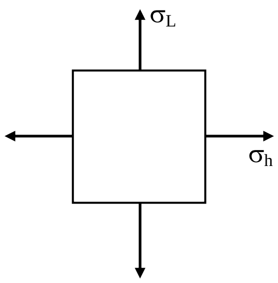


Case (II): Hoop stress,

$$\sigma_h = \frac{Pd}{2t} = 4.29 \times 10^6 \text{ Pa}$$

Longitudinal stress,

$$\sigma_L = \frac{Pd}{4t} = \frac{\sigma_h}{2} = 2.14 \times 10^6 \text{ Pa}$$



05(c).

Sol: The weld is subjected to direct shear and bending stresses.

Throat Area,

$$A = 0.707 h\ell = 0.707 \times h \times 2\pi \times r$$

$$A = 0.707 \times h \times 2\pi \times 40$$

$$A = 177.69 \text{ h mm}^2$$

Direct shear is

$$\tau = \frac{F}{A} = \frac{15000}{177.69h} = \frac{84.42}{h}$$

Bending moment is

$$M = Fx = 15000 \times 200 = 3 \times 10^6 \text{ N.mm}$$

For circular section, the section modulus is

$$Z = \frac{\pi t D^2}{4}$$

$$= \frac{\pi \times 0.707h \times (80)^2}{4} = 3553.769 \text{ h mm}^3$$

Bending stress is,

$$\sigma = \frac{M}{Z} = \frac{3 \times 10^6}{3553.769h} = \frac{844.174}{h}$$

According to maximum shear stress theory

$$\frac{\sigma_y}{2N} = \frac{\sigma_{all}}{2} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

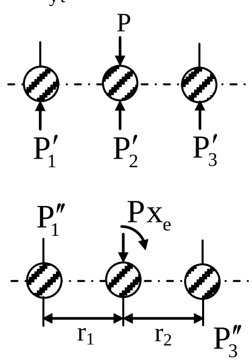
$$\frac{143}{2} = \sqrt{\left(\frac{844.174}{2h}\right)^2 + \left(\frac{84.42}{h}\right)^2}$$

Solving, we get

$\Rightarrow$ h = 6.02 mm

05(d)(i)

Sol: P = 5kN, $s_{yt} = 380 \text{ N/mm}^2$, $(f_s) = 3$



Step 1: Permissible shear stress,

$$\tau = \frac{s_{sy}}{(fs)} = \frac{0.5 s_{yt}}{(fs)} = \frac{0.5 \times 380}{3} = 63.33\,N/mm^2$$

Step 2: primary and secondary\ shear forces.

The centre of gravity of three bolts will be at the centre of bolt - 2

The primary and secondary shear forces are shown in figure.

$$P_1' = P_2' = P_3' = \frac{P}{3} = \frac{5000}{3} = 1666.67\,N$$

$$P_1'' = P_3'' = \frac{(P_e)(r_1)}{(r_1^2 + r_3^2)} = \frac{(5000 \times 305) \times 75}{(75^2 + 75^2)}$$

$$= 10166.67\text{ N}$$

Step 3:- Resultant shear force

The resultant shear force on the bolt 3 is maximum

$$P_3 = P_3' + P_3''$$

$$= 1666.67 + 10166.67 = 11833.34\,N$$

Step 4:- Size of bolts

$$\tau = \frac{P_3}{A} \Rightarrow 63.33 = \frac{11833.34}{\frac{\pi}{4} d_c^2}$$

Solving, we get $d_c = 15.42$

As we know that,

$$d = \frac{d_c}{0.8} = \frac{15.42}{0.8} = 19.26\,\text{or}\,20\,mm$$

The standard size of the bolts is M20.

05(d)(ii).

Sol: The critical section is the section at support where bending stress is maximum. The maximum bending moment is given by

$$M_{max} = 30\times10^3\times600 - 40\times10^3\times200$$

$$= 10\times10^6\text{ N.mm } (\circlearrowright)$$

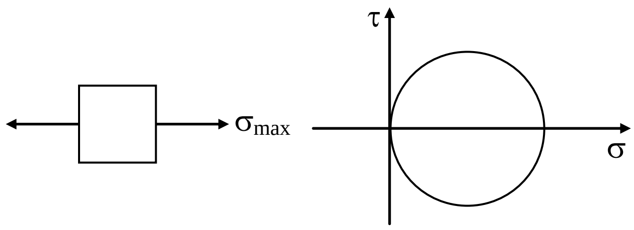
At upper most layer from neutral layer the bending stress as well as direct stress are tensile in nature hence they will get added.

$$\sigma_{max} = \frac{M_{max} y}{I} + \frac{P}{A}$$

$$= \frac{10\times10^6 \times 75}{\left(\frac{50\times150^3}{12}\right)} + \frac{40\times10^3}{(150\times50)}$$

$$= 53.33\text{ MPa} + 5.33\text{ MPa} = 58.66\text{ MPa}$$

The state of stress at the critical element and corresponding Mohr's circle is shown below.



$$\therefore\ \tau_{max} = \frac{\sigma_{max}}{2} = 29.33\,MPa$$

05(e).

Sol: $\sigma_x = 110$ MPa ,

At θ = 32°, $\sigma_{x1} = 37$ MPa (tension)

At θ = 48°, $\sigma_{x1} = -12$ MPa (compression)

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

For θ = 32°, $\sigma_{x1} = 37$ MPa

$$37\,MPa = \frac{110 + \sigma_y}{2} + \frac{110 - \sigma_y}{2} \times \cos(64^o) + \tau_{xy}\sin(64^o)$$

$0.28081\,\sigma_y + 0.89879\,\tau_{xy} = -42.11041$ MPa ----- (1)

For θ = 48°, $\sigma_{x1} = -12$ MPa

$$-12=\frac{110+\sigma_y}{2}+\frac{110-\sigma_y}{2}\times\cos(96^o)+\tau_{xy}\sin(96^o)$$

$0.55226\ \sigma_y + 0.99452\ \tau_{xy} = -61.25093$ MPa ----- (2)

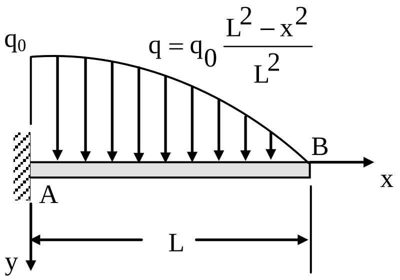
Solve eq. (1) and (2), we get

$\sigma_y = -60.7$ MPa,

$\tau_{xy} = -27.9$ MPa .

06(a).

Sol:



We know that

$$EI\frac{d^2y}{dx^2}=-M$$

Fourth – order differential equation of the deflection curve (load equation)

$$EI\frac{d^4y}{dx^4}=q$$

$$\therefore EI\frac{d^4y}{dx^4}=\frac{q_0}{L^2}\left[L^2-x^2\right]$$

Integrating w.r.to x on both sides we get

$$EI\frac{d^3y}{dx^3}=\frac{q_0}{L^2}\left[L^2x-\frac{x^3}{3}\right]+c_1\to(1)$$

Again Integrating w.r.to x on both sides we get

$$EI\frac{d^2y}{dx^2}=\frac{q_0}{L^2}\left[\frac{L^2x^2}{2}-\frac{x^4}{12}\right]+c_1x+c_2\to(2)$$

From (1) $\Rightarrow \frac{d^3y}{dx^2}=\frac{dM}{dx}=$ shear force

At x = L $\Rightarrow$ Load = 0 $\Rightarrow$ shear force = 0

$$\therefore 0=\frac{q_0}{L^2}\left[L^3-\frac{L^3}{3}\right]+c_1$$

$$\therefore c_1=\frac{-2}{3}q_0L$$

From (2) $\Rightarrow \frac{d^2y}{dx^2}=\frac{M}{EI}$

At x = L $\Rightarrow$ bending moment is zero

$$\therefore 0=\frac{q_0}{L^2}\left[\frac{L^4}{2}-\frac{L^4}{12}\right]-\frac{2}{3}q_0L^2+c_2$$

$$\therefore c_2=\frac{2}{3}q_0L^2-\frac{5}{12}q_0L^2$$

$$\therefore c_2=\frac{q_0L^2}{4}$$

$$\therefore EI\frac{d^2y}{dx^2}=\frac{q_0}{L^2}\left[\frac{L^2x^2}{2}-\frac{x^4}{12}\right]-\frac{2}{3}q_0Lx+\frac{q_0}{4}L^2$$

Again Integrating w.r.to x on both sides, we get

$$\therefore EI\frac{dy}{dx}=\frac{q_0}{L^2}\left[\frac{L^2x^3}{6}-\frac{x^5}{60}\right]-\frac{2}{3}q_0\frac{Lx^2}{2}+\frac{q_0L^2x}{4}+c_3\to(3)$$

$$\therefore EI(y)=\frac{q_0}{L^2}\left[\frac{L^2x^4}{24}-\frac{x^6}{360}\right]-\frac{q_0Lx^3}{9}+\frac{q_0L^2x^2}{8}+c_3x+c_4\to(4)$$

From (3)

At x = 0 $\Rightarrow$ dy/dx = 0

$0 = c_3 \Rightarrow c_3 = 0$

From (4)

At x = 0 $\Rightarrow$ y = 0

$\therefore\ c_4 = 0$

$$\therefore EI(y)=\frac{q_0}{L^2}\left[\frac{L^2x^4}{24}-\frac{x^6}{360}\right]-\frac{q_0Lx^3}{9}+\frac{q_0L^2x^2}{8}$$

At free end, x = L, y = y_b

$$EI(y_b) = \frac{q_0}{L^2}\left[\frac{L^6}{24} - \frac{L^6}{360}\right] - \frac{q_0L^4}{9} + \frac{q_0L^4}{8}$$

$$\therefore EI(y_b) = q_0L^4\left[\frac{1}{24} - \frac{1}{360} - \frac{1}{9} + \frac{1}{8}\right]$$

$$\therefore y_b = \frac{19q_0L^4}{360EI}$$

From (3)

$$EI\left(\frac{dy}{dx}\right) = \frac{q_0}{L^2}\left[\frac{L^2x^3}{6} - \frac{x^5}{60}\right] - \frac{q_0Lx^2}{3} + \frac{q_0L^2x}{4}$$

At free end x = L, dy/dx = θ_b

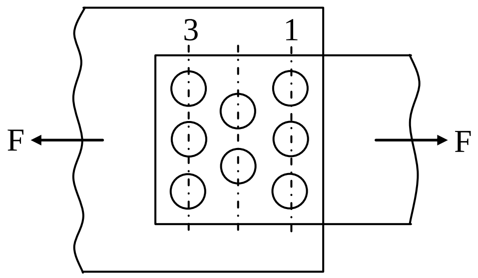
$$EI(\theta_b) = \frac{q_0}{L^2}\left[\frac{L^5}{6} - \frac{L^5}{60}\right] - \frac{q_0L^3}{3} + \frac{q_0L^3}{4}$$

$$EI(\theta_b) = q_0L^3\left[\frac{1}{6} - \frac{1}{60} - \frac{1}{3} + \frac{1}{4}\right]$$

$$\therefore \theta_b = \frac{q_0L^3}{15EI}$$

06(b).

Sol: The joint is shown in figure.



As the plate thickness is less than 8mm, the diameter of rivet can be obtained by equating shearing strength to crushing strength.

No. of rivets for pitch, n = 3.

Shearing resistance: As it a lap joint, the rivet will be in single shear,

Shear resistance is

$$F_s = \frac{\pi}{4}d^2\sigma_s n = \frac{\pi}{4}\times d^2\times 90\times 3.............(i)$$

Crushing resistance is

$$F_c = dt\sigma_c n = d\times 7\times 150\times 3..........(ii)$$

equating (i) and (ii)

$$\frac{\pi}{4}\times d^2\times 90\times 3 = d\times 7\times 150\times 3$$

Solving , we get d = 14.86 ≈15 mm

Taking hole size as 15 mm, diameter of the rivet will be 14.0 mm.

Pitch of the point: Pitch of the lap joint is obtained by equating tearing to shearing or crushing resistance.

Equating tearing to shearing resistance , we get

$$(p-d)t\sigma_t = \frac{\pi}{4}d^2\sigma_s n$$

$$(p-15)\times 7\times 10 = \frac{\pi}{4}\times 15^2\times 90\times 3$$

Solving, we get

p = 76.96 mm ≈77mm

- Tearing efficiency of the joint:-

$$\eta_t = \frac{p-d}{p} = \frac{77-15}{77} = 80.5\%$$

Which is acceptable as per design code

- Back pitch is given as

$$p_b = 0.33p + 0.67d = 0.33\times 77 + 0.67\times 15$$

$$p_b = 35.46 \approx 36mm$$

- Marginal distance,

$$m = 1.5d = 1.5\times 15 = 22.5mm$$

Failure Analysis,

To perform the failure analysis, we should calculate the strength against each failure separately. The combined failure mode is then analyzed considering the possible moles of failures.

Tearing strength,

$$f_t = (p-d)t\sigma_t = (77-15)\times 7\times 110 = 47740\text{ N}$$

shearing strength,

$$F_s = \frac{\pi}{4}d^2\sigma_s n = \frac{\pi}{4}\times 15^2\times 90\times 3 = 47713\text{ N}$$

Crushing strength,

$$F_c = dt\sigma_c n = 15\times 7\times 150\times 3 = 47250\text{ N}$$

Strength of the unpunched plate,

$$F = pt\sigma_t = 77\times 7\times 110 = 59290\text{ N}$$

As the tearing strength is highest among all the three, the possible mode of failure may be due to the shearing and crushing. Hence, all such possible modes are analyzed.

Shearing along 1-1 and crushing along 2–2 and 3–3 or reverse

$$F_{s/c23} = \frac{\pi}{4}d^2\sigma_s k n_s + dt\sigma_c n_c$$

$$= \frac{\pi}{4}\times 15^2\times 90\times 1\times 1 + 15\times 7\times 150\times 2$$

= 47404 N

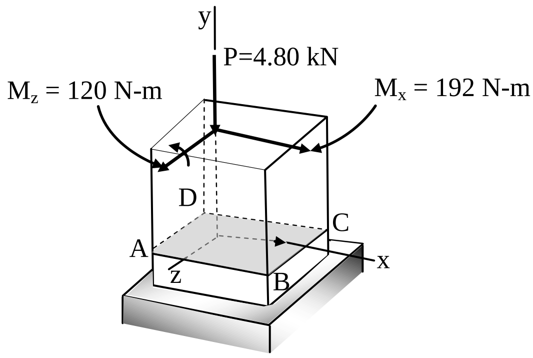
If any combination, the strength will be more than the visual failure i.e tearing, shearing or crushing. Hence, efficiency of the joint is

$$\eta = \frac{47250}{59290} = 79.69\%$$

06(c).

Sol:

(a) Stresses :



The given eccentric load is replaced by an equivalent system consisting of a centric load P and two couples M_x and M_z represented by vectors directed along the principal centroidal axes of the section. Thus

$$M_x = (4.80\text{ kN})(40\text{ mm}) = 192\text{ N}-\text{m}$$

$$M_z = (4.80\text{ kN})(60\text{ mm}-35\text{ mm}) = 120\text{ N}-\text{m}$$

Compute the area and the centroidal moments of inertia of the cross section:

$$A = (0.080\text{ m})(0.120\text{ m}) = 9.60\times 10^{-3}\text{m}^2$$

$$I_x = \frac{1}{12}(0.120\text{ m})(0.080\text{ m})^3 = 5.12\times 10^{-6}\text{m}^4$$

$$I_z = \frac{1}{12}(0.080\text{ m})(0.120\text{ m})^3 = 11.52\times 10^{-6}\text{m}^4$$

The stress σ0 due to the centric load P is negative and uniform across the section:

$$\sigma_0 = \frac{P}{A} = \frac{-4.80\text{ kN}}{9.60\times 10^{-3}\text{m}^2} = -0.5\text{MPa}$$

The stresses due to the bending couples M_x and M_z are linearly distributed across the section with maximum values equal to

$$\sigma_1 = \frac{M_x z_{max}}{I_x}$$
$$= \frac{(192\ \text{N}-\text{m})(40\ \text{mm})}{5.12\times10^{-6}\ \text{mg}4} = 1.5\ \text{MPa}$$

$$\sigma_2 = \frac{M_2 x_{max}}{I_z}$$
$$= \frac{(120\ \text{N}-\text{m})(60\ \text{mm})}{11.52\times10^{-6}\ \text{m}^4} = 0.625\ \text{MPa}$$

The stresses at the corners of the section are

$$\sigma_y = \sigma_0 \pm \sigma_1 \pm \sigma_2$$

Where the signs must be determined from Fig 4.57 b. Noting that the stresses due to M_x are positive at C and D and negative at A and B, and the stresses due to M_z are positive at B and C and negative at A and D, we obtain

$$\sigma_A = -0.5 - 1.5 - 0.625 = -2.625\ \text{MPa}$$
$$\sigma_B = -0.5 - 1.5 + 0.625 = -1.375\ \text{MPa}$$
$$\sigma_C = -0.5 + 1.5 + 0.625 = +1.625\ \text{MPa}$$
$$\sigma_D = -0.5 + 1.5 - 0.625 = +0.375\ \text{MPa}$$

(b) **Neutral Axis:** The stress will be zero at a point G between B and C, and at a point H between D and A. Since the stress distribution is linear,

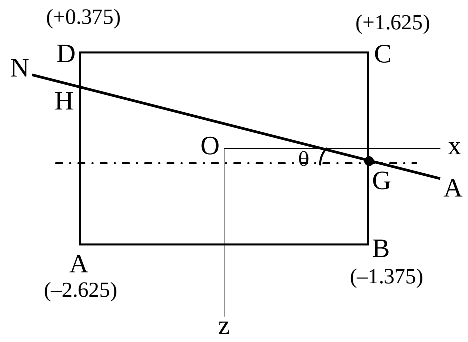
$$\frac{BG}{80\ \text{mm}} = \frac{1.375}{1.625 + 1.375}$$

BG = 36.7 mm

$$\frac{HA}{80\ \text{mm}} = \frac{2.625}{2.625 + 0.375}$$

HA = 70 mm

The neutral axis can be drawn through points G and H. The distribution of the stresses across the section is shown in figure.



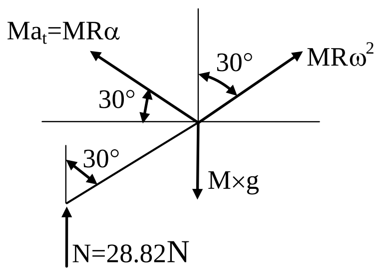
$$\tan\theta = \frac{HA - BG}{AB}$$

$$\tan\theta = \frac{70 - 36.7}{120}$$

$\Rightarrow$ θ = 15.5 w.r.t x-axis

07(a).

Sol:



When θ = 30°

$$\sum T = I\alpha$$

$$\therefore M\times g\times\frac{L}{2}\times\sin 30 = \frac{ML^2}{3}\times\alpha$$

$$\alpha = \frac{3}{2}\times9.81\times\frac{\sin 30}{2} = 3.68\frac{\text{rad}}{\text{s}^2}$$

Using ΣFy = 0

$$\therefore\ M\times g = N + MR\omega^2\times\cos 30 + MR\alpha\times\sin 30$$

$$4\times9.81 = 28.82 + 4\times1\times\omega^2\times\cos30 + 4\times1\times3.68\times\sin30$$

$\therefore$ ω = 0.94 rad/s

using conservation of energy between θ = θ_o and θ = 30°

$$\Delta KE = \Delta PE$$

$$\therefore \frac{1}{2}\times I\times[\omega^2-0] = M\times g\times\frac{L}{2}\times\{\cos\theta_o-\cos 30\}$$

$$\therefore \frac{1}{2}\times\frac{ML^2}{3}\times\omega^2 = M\times g\times\frac{L}{2}\times\{\cos\theta_o-\cos 30\}$$

$$\therefore \frac{1}{2}\times\frac{4\times 2^2}{3}\times 0.94^2 = 4\times 9.81\times\frac{2}{2}\times\{\cos\theta_o-\cos 30\}$$

$$\therefore \theta_o = 22.17^o$$

07(b).

Sol: N = 110 rpm, P = 30 kW

$$P = \frac{2\pi NT}{60} \Rightarrow T = \frac{60\times 30\times 10^3}{2\pi\times 110}$$

T = 2605.67 N-m

$$\tau = \frac{16T}{\pi d^3} = 80$$

$$d^3 = \frac{16\times 2605.67\times 10^3}{\pi\times 80}$$

d = 54.95 mm ≈ 55 mm

If this shaft replaced by hollow shaft

$$\frac{D_1}{D_2} = 2 \Rightarrow k = \frac{D_2}{D_1} = 0.5$$

$$\tau = \frac{16T}{\pi D_1^3(1-k^4)}$$

$$D_1^3 = \frac{16\times 2605.67\times 10^3}{\pi(1-0.5^4)\times 80}$$

D_1 = 56.14 ≈ 57 mm

D_2 = 28.5 mm

Weight of solid shaft (W_1) = $\rho\frac{\pi}{4}D^2L\times g$

Weight of hollow shaft

(W_2) = $\rho\frac{\pi}{4}(D_1^2-D_2^2)L\times g$

$$\frac{W_1-W_2}{W_1}\times 100 = \%\,saving$$

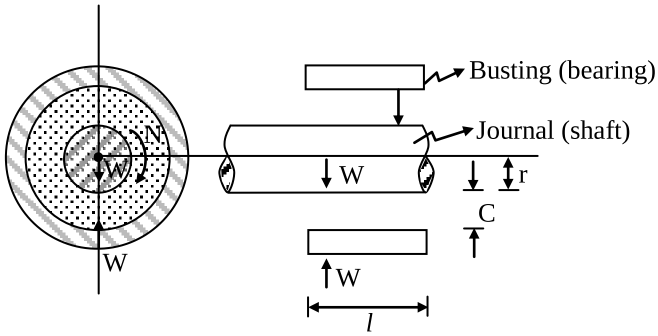
$$\%\ saving = \frac{D^2-(D_1^2-D_2^2)}{D^2}\times 100$$

$$\%\ saving = \left[1-\frac{57^2}{55^2}(1-0.5^2)\right]\times 100 = 19.44\%$$

07(c).

Sol: Assumptions:

- The Shaft is concentric
- Bearing carrier very small load, that the clearance space is completely filled with oil.
- No leakage
- Velocity profile is linear.



Let r = radius of shaft

c = radial clearance

l = length of bearing

n = speed of shaft in rev/s

μ = dynamic viscosity

shearing stress in the lubricant, $\tau = \mu.\frac{u}{y}$

But surface velocity $u = 2\pi r.n\ m/s$

$$\therefore \tau = \mu.\frac{2\pi r.n}{c}......(1)$$

∴ Shear force acting over the surface

$$F_s = \tau.A_s$$

$$= \mu.\frac{2\pi rn}{c}.2\pi r\ell = \frac{4\pi^2 r^2\mu n\ell}{c}$$

Frictional torque due to lubricant

$$T = F_s.r = \frac{4\pi^2 r^3 \mu n\ell}{c} ....(2)$$

Let W be the load acting on the bearing

$\therefore$ Bearing pressure $P = \frac{W}{2r\ell}$

$$W = P.2r\ell$$

But frictional force due to loading against the resistance of lubrication.

$$F_f = f\,.\,W = f\,2r\,\ell\,.p$$

where f is coefficient of friction

$$\text{frictional torque } T_f = F_f \times r$$

$$T = f\ 2r^2\ell.P ....(3)$$

Equation (2) and (3) we get

$$\frac{4\pi^2.r^3.\mu.n.\ell}{c} = f\ 2r^2.\ell.P$$

After rearranging the terms, $f = 2\pi^2.\frac{\mu n}{P}.\frac{r}{c}$

This equation is called Petroff equation.

If we observe carefully $\frac{\mu.n}{P}$ and $\frac{r}{c}$ are have no units.

$\therefore$ f depends on 2 dimensional numbers,

$$\frac{\mu.n}{P} \text{ and } \frac{r}{c}$$

Generally for the surfaces in contact, coefficient of friction remains constant. Hence the combination $\frac{\mu.n}{P}$ and $\frac{r}{c}$ should be constant for a given bearing. Hence lead to the discovery of sommerfeld number.

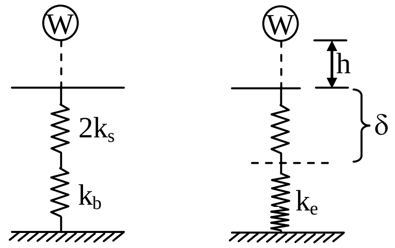
Sommerfeld number (S) = $\frac{\mu.n}{P}.\left(\frac{r}{c}\right)^2$

07(d)(i).

Sol: Using conservation of energy,

Change in potential energy of ball = Change in potential energy of springs + Change in potential energy of beam

Simply supported beam can be considered as a spring of stiffness k_b which is in series connection with two supports springs of stiffness k



$$K_b = \frac{48EI}{L^3}$$

$$K_b = \frac{48 \times 200 \times 10^9 \times 0.05^4}{1^3 \times 12}$$

$$= 5\times10^6 \text{ N/m}$$

$$\frac{1}{K_e} = \frac{1}{K_b} + \frac{1}{2K_s}$$

$$K_e = \frac{2K_sK_b}{2K_s + K_b} = \frac{2\times300000\times5\times10^6}{2\times300000+5\times10^6}$$

$$K_e = 535714.3 \text{ N/m}$$

$$W(h+\delta) = \frac{1}{2}K_e\delta^2$$

$$K_e\delta^2 - 2W\delta - 2Wh = 0$$

$$\delta = \frac{2W \pm \sqrt{4W^2 + 8WhK_e}}{2K_e}$$

$$\delta = \frac{W}{K_e}\left(1+\sqrt{1+\frac{2hK_e}{W}}\right)$$

$\Rightarrow \delta = 6.78$ mm

07(d)(ii).

Sol: Wahl factor :

$$k_w = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

where,

c = spring index = $\frac{D}{d}$

D = mean coil diameter,

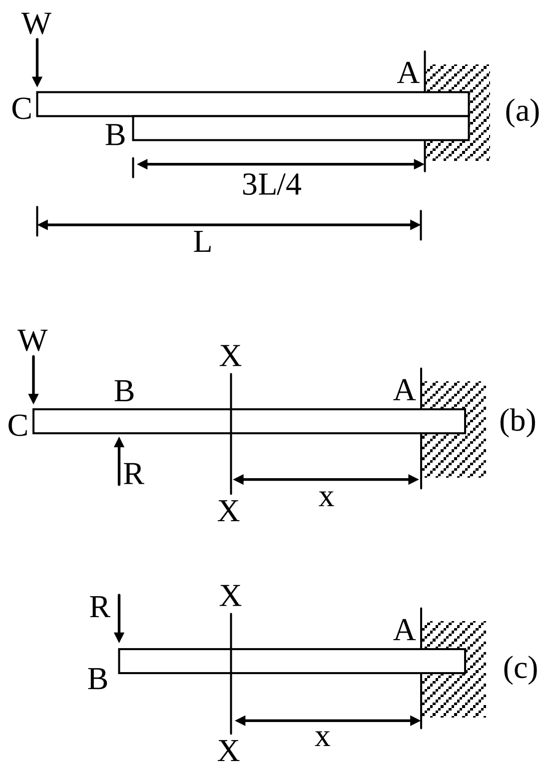
d = wire diameter

The objectives of series and parallel combinations are as follows :

- To save the space
- To change the rate of spring at a certain deflection
- To provide a fail- safe system

08(a).

Sol:



The deflection at B of the top bar can be found using Maxwell's reciprocal theorem. The downward deflection at B due to W at C is the same as the deflection at C due to a load W at B.

$$\therefore \quad y_c = \frac{W\left(\frac{3}{4}L\right)^3}{3EI} + \frac{W\left(\frac{3}{4}L\right)^2}{2EI}\frac{L}{4}$$

$$= \frac{27WL^3}{128EI} \text{due to W at B}$$

The upward deflection at B due to R is

$$\frac{W\left(\frac{3}{4}L\right)^3}{3EI} = \frac{9RL^3}{64EI}$$

Hence the net deflection of the upper bar at B is

$$\frac{27WL^3}{128EI} - \frac{9RL^3}{64EI} \quad \text{i.e.,} = \frac{9L^3}{128EI}[3W-2R]$$

Also downward deflection of the lower bar due to R at B = $\frac{9RL^3}{64EI}$

If the beams are in contact at B

$$\frac{9RL^3}{64EI} = \frac{9L^3}{128EI}[3W-2R]$$

$$\therefore \quad R = \frac{3}{4}W$$

Hence deflection at C

$$= \frac{WL^3}{3EI} - \frac{R\left(\frac{3}{4}L\right)^3}{3EI} - \frac{R\left(\frac{3}{4}L\right)^2}{2EI}\frac{L}{4}$$

$$= \frac{WL^3}{EI}\left[\frac{1}{3} - \frac{27}{256} - \frac{27}{512}\right] = \frac{269}{1536}.\frac{WL^3}{EI}$$

To find the maximum gap between the bars

It is first necessary to derive the deflection equation of each beam for the range AB. A section xx at a distance x from A will be considered even though moments due to forces to the left of x are caused. By this procedure, the constants of integration will all be zero. For the upper beam

$$EI\frac{d^2y}{dx^2} = -W(L-x)+\left(\frac{3}{4}W\right)\left(\frac{3}{4}L-x\right)$$

$$= -\frac{7}{16}WL+\frac{1}{4}Wx$$

$$EI\frac{dy}{dx} = -\frac{7}{16}WLx+\frac{1}{4}W\frac{x^2}{2}+C_1$$

At x = 0, $\frac{dy}{dx}=0$

$\therefore$ $C_1 = 0$

$$EI\frac{dy}{dx} = -\frac{7}{16}WLx+\frac{W}{8}x^2$$

$$EIy = -\frac{7WLx^2}{32}+\frac{Wx^3}{24}+C_2$$

At x = 0, y = 0

$\therefore$ $C_2 = 0$

$$\therefore EIy_{upper} = -\frac{7}{32}WLx^2+\frac{Wx^3}{24}$$

For the lower bar

$$EI\frac{d^2y}{dx^2} = -\frac{3}{4}W\left(\frac{3}{4}L-x\right) = -\frac{9}{16}WL+\frac{3}{4}Wx$$

$$EI\frac{dy}{dx} = -\frac{9}{16}WLx+\frac{3}{4}W\frac{x^2}{2}+C_1$$

At x = 0, $\frac{dy}{dx}=0$

$\therefore$ $C_1 = 0$

$$EI\frac{dy}{dx} = -\frac{9}{16}WLx+\frac{3}{8}Wx^2$$

$$EIy = -\frac{9}{16}W\frac{x^2}{2}+\frac{3}{8}W\frac{x^3}{3}+C_2$$

At x = 0, y = 0

$C_2 = 0$

$$\therefore EIy_{lower} = -\frac{9WLx^2}{32}+\frac{3Wx^3}{24}$$

$\therefore$ The gap Δ between the bars is given by

$EI\Delta = EI\,(y_{lower} - y_{upper})$

$$= -\frac{1}{16}WLx^2+\frac{1}{12}Wx^3$$

For maximum gap $\frac{d\Delta}{dx}=0$

i.e., $0 = -\frac{WLx}{8}+\frac{Wx^2}{4}$

$\therefore$ $x = \frac{L}{2}$

$$\therefore EI(\Delta_{max}) = -\frac{1}{16}WL\left(\frac{L^2}{4}\right)+\frac{W}{12}\left(\frac{L^3}{8}\right)$$

$$= WL^3\left[-\frac{1}{64}+\frac{1}{96}\right] = -\frac{WL^3}{192EI}$$

$\therefore$ Maximum gap = $\frac{WL^3}{192EI}$

08(b).

Sol: Power P = T.ω

Torque $T = \frac{P}{\omega}$

But torque (T) = Tangential force × radius

$T = F_t \times r \times t$

$F_t = \frac{T}{r} = \frac{P}{rw}$

But, module, $m = \frac{d}{z} = \frac{2r}{z}$

where z = no. of teeth.

$$r = \frac{mz}{2}$$

$$\Rightarrow F_t = \frac{2P}{mz\omega}$$

$$\text{Tangential force}(F_t) = \frac{2\times 15\times 10^3}{1.25\times 10^{-3}\times 54\times 132}$$

$$= 3367 \text{ N}$$

Lewis equation for the design of gear is used to find safety factor.

$$F_t = \sigma_b.m.b.y$$

When design correction factors are considered the equation becomes.

$$\sigma_b = \frac{k_m.k_v.F_t}{m.b.y}$$

Where,

σ_b = maximum bending stress in gear teeth

k_m = load distribution factor

k_v = dynamic factor

F_t = tangential force

m = module

b = face width of teeth

y = Lewis form factor

$$\sigma_b = \frac{1.2\times 1.32\times 3367}{1.25\times 60\times 0.475} = 149.71\text{MPa}$$

$$\text{Factor of safety } (f_s) = \frac{\text{failure stress}}{\text{operating stress}}$$

$$= \frac{380}{149.71} \approx 2.54$$

08(c).

Sol: $s_{ut} = 660 \text{ N/mm}^2$,

$s_e = 280 \text{ N/mm}^2$

Step 1: Construction of S-N diagram

$$0.9s_{ut} = 0.9\times 660 = 594 \text{ N/mm}^2$$

$$\log_{10}(0.9s_{ut}) = \log_{10}(594) = 2.7738$$

$$\log_{10} S_e = \log_{10}(280) = 2.4472$$

$$\log_{10}\sigma_1 = \log_{10}(350) = 2.5441$$

$$\log_{10}\sigma_2 = \log_{10}(400) = 2.6021$$

$$\log_{10}\sigma_3 = \log_{10}(500) = 2.6990$$

The S-N curve is shown in figure

Step 2: Calculation of N_1, N_2 and N_3

$$\overline{EF} = \frac{\overline{DB}\times\overline{AE}}{\overline{AD}}$$

$$= \frac{(6-3)(2.7738-\log_{10}\sigma)}{(2.7738-2.4472)}\text{.............(i)}$$

And, $\log_{10} N = 3 + \overline{EF}$..............(ii)

From (i) and (ii)

$$\log_{10} N = 3 + 9.1855(2.7738 - \log_{10}\sigma)$$

$$\log_{10}(N_1) = 39.1855(2.7738 - 2.5441)$$

$$N_1 = 128798$$

$$\log_{10}(N_2) = 3 + 9.1855(2.7738 - 2.6021)$$

$$N_2 = 37770$$

$$\log_{10}(N_3) = 3 + 9.1855(2.7738 - 2.6990)$$

$$N_3 = 4865$$

Step 3: Fatigue life of component

$$\frac{\alpha_1}{N_1} + \frac{\alpha_2}{N_2} + \frac{\alpha_3}{N_3} = \frac{1}{N}$$

$$\frac{0.85}{128798} + \frac{0.12}{37770} + \frac{0.03}{4865} = \frac{1}{N}$$

N = 62723 cycles