ACE
Engineering Academy

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ESE-2019 MAINS TEST SERIES
Question Cum Answer Booklet (QCAB)

Mechanical Engineering **Test-1** **Paper-I**

Time Allowed: 3 Hours **Maximum Marks: 300**

ACE HALL TICKET No. :

HALL TICKET No. : (Issued by UPSC)

NAME OF THE CANDIDATE :

NAME OF THE CENTRE :

BRANCH : BATCH : ROLL No. : MOBILE No. :

TEST CODE : 801

DATE : 30-03-2019

INSTRUCTIONS TO CANDIDATES:

- This Question-cum- Answer (QCA) Booklet contains **80** pages. Immediately on receipt of booklet, please check that this QCA booklet does not have any misprint or torn or missing pages or items, etc. If so, get it replaced by a fresh QCA booklet.
- Candidates must read the instructions on this page and the following pages carefully before attempting the paper.
- Candidates should attempt all questions strictly in accordance with the specified instructions and in the space prescribed under each question in the booklet. Any answer written outside the space allotted may not be given credit.
- Question Paper in detachable form is available at the end of the QCA booklet and can be removed and taken by the candidates after conclusion of the exam

For filling by Examiners only

Question No.	Page No.	Marks
1	03	
2	13	
3	22	
4	31	
5	41	
6	50	
7	59	
8	67	
Grand Total		

Signature of the Invigilator

Signature of the Student

Marks Secured after Scrutiny

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

*There are **EIGHT** questions divided into two sections.*

*Candidate has to attempt **FIVE** questions in all.*

*Question No.**1** and Question No.**5** are compulsory and out of the remaining, any **THREE** are to be attempted choosing at least **ONE** from each section.*

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Diagrams/Figures, wherever required, shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the QCA Booklet must be clearly struck off.

*Answers must be written in **ENGLISH** only.*

DONT'S:

1. Do not write your Name or Roll number or Sr. No. of Question-Cum-Answer-Booklet anywhere inside this Booklet. Do not sign the "Letter Writing" questions, if set in any paper by name, nor append your roll number to it.
2. Do not write anything other than the actual answers to the questions anywhere inside your Question-Cum-Answer-Booklet.
3. Do not tear off any leaves from your Question-Cum-Answer-Booklet. If you find any page missing, do not fail to notify the Supervisor/invigilator.
4. Do not write anything on the Question Paper available in detachable form or admission certificate and write answers at the specified space only.
5. Do not leave behind your Question-Cum-Answer-Booklet on your table unattended, it should be handed over to the Invigilator after conclusion of the exam.

DO'S :

1. Read the instructions on the cover page and the instructions specific to this Question Paper mentioned on the next page of this Booklet carefully and strictly follow them.
2. Write your Roll number and other particulars, in the space provided on the cover page of the Question-Cum-Answer-Booklet.
3. Write legibly and neatly. Do not write in bad/illegible handwriting.
4. For rough notes or calculations the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be valued.
6. Hand over your Question-Cum-Answer-Booklet personally to the invigilator before leaving the examination hall.
7. **Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.**

SECTION - A

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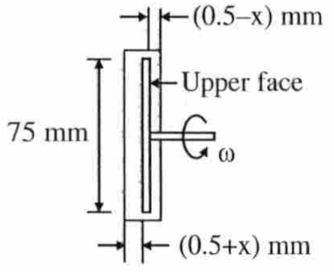
01(a). To damp oscillations, the pointer of a galvanometer is fixed to a circular disk which turns in a container of oil as shown in the figure. For angular velocity, $\omega = 0.3$ rad/s express the damping torque (in N-m) as a function of displacement x (in mm) of the disk from its center position. (The oil has a viscosity of 8×10^{-3} Pa-s? Neglect edge effects).



(12 M)

Assume at any point that the velocity profile is linear ; $\tau = \mu \cdot \frac{dV}{dh}$

For the upper face,

$$\frac{dV}{dh} = \frac{r.\omega}{[(0.5-x)/1000]} = \frac{r\times0.3}{(0.5-x)/1000}$$

$$\tau = \frac{8\times10^{-3} \times r\times 0.3}{(0.5-x)/1000} = \frac{2.4r}{0.5-x}$$

The force dF on dA on the upper face of the disk is then

$$(dF)_{upper} = \tau.dA = \frac{2.4r}{0.5-x}. r.d\theta.dr$$

$$= \frac{2.4r^2}{0.5-x} d\theta.dr$$

The torque dT for dA on the upper face

$$(dT)_{upper} = r \cdot (dF)_{upper} = \frac{2.4\, r^3}{0.5 - x} \, d\theta \cdot dr$$

For lower face,

$$\frac{dv}{dn} = \frac{r \cdot \omega}{(0.5 + x)/1000}$$

Then $$(dT)_{lower} = \frac{2.4\, r^3}{0.5 + x} \, d\theta \cdot dr$$

Total resisting torque on both faces is

$$T = \int_0^{\frac{0.075}{2}} \int_0^{2\pi} \frac{2.4\, r^3}{0.5 - x} \, d\theta \cdot dr + \int_0^{\frac{0.075}{2}} \int_0^{2\pi} \frac{2.4\, r^3}{0.5 + x} \, d\theta \cdot dr$$

$$= \left[\frac{1}{0.5 - x} + \frac{1}{0.5 + x} \right] (2.4)(2\pi) \left[\frac{r^4}{4} \right]_0^{\frac{0.075}{2}}$$

$$= \frac{0.5 + x + 0.5 - x}{0.25 - x^2} \cdot (7.46 \times 10^{-6})$$

$$\boxed{T = \frac{7.46 \times 10^{-6}}{0.25 - x^2}}$$

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01(b). A 10 cm diameter solid cylinder of height 9.5 cm weighing 0.4 kg is immersed in liquid $(\gamma = 8.2\ kN/m^3)$ contained in a tall, upright metal cylinder having a diameter of 12.7 cm. Before immersion the liquid was 7.6 cm deep. What is the rise of original liquid surface? (Assume, g = 10 m/s^2) **(12 M)**

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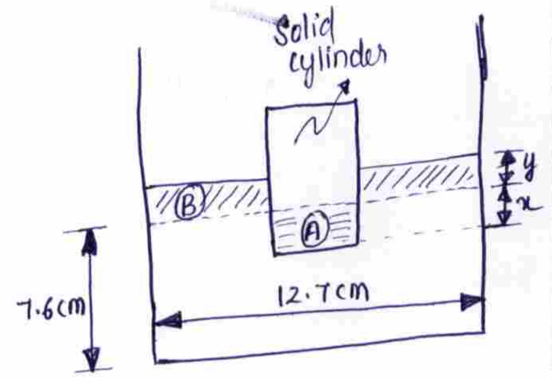

let x = distance Solid cylinder falls below original liquid Surface.

y = distance liquid rises above original liquid surface.

So $x+y = d$, depth of Submergence.

weight of solid cylinder

= weight of volume of liquid displaced.

$$0.4 \times 10 = 8.2 \times 10^3 \times \frac{\pi}{4} \times 10^2 \times 10^{-4} (x+y)$$

$$x+y = 0.0621\ m \quad \text{——— (1)}$$

Candidates must not write on this margin

Volume of liquid in (A) = Volume of liquid in (B)

$$\frac{\pi}{4} \times 10^2 \times 10^{-4} \cdot x = \frac{\pi}{4} \left[12.7^2 - 10^2 \right] \times 10^{-4} \cdot y$$

$$100\, x = 61.29\, y$$

$$x = 0.6129\, y \quad \text{—— (2)}$$

Thus from equation (1)

$$(0.6129 + 1)\, y = 0.0621$$

$$y = \frac{0.0621}{1.6129} = 0.0385 \text{ m}$$

$$\boxed{y = 3.85 \text{ cm}}$$

Thus, the original liquid level rises by 3.85 cm

01(c). Derive an expression for the local velocity u(y) of viscous fluid flow between two stationary flat fixed plates separated by distance 't'. State suitable Assumptions made.

(12 M)

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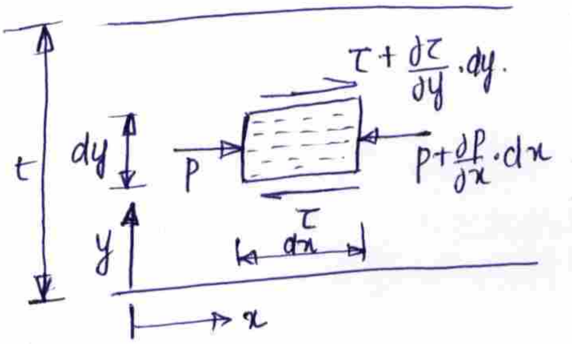

Assumptions

1. Fluid is Newtonian
2. There is no slip or liquid motion at solid Boundaries.
3. Flow is 2-D, steady, laminar flow.
4. Consider unit width of parallel plates shown.

From Newtons Second law of motion: $\Sigma F = m.a$

$$P.dA - \left[P + \frac{\partial P}{\partial x}.dx\right]dA - \tau.dA_{sy} + \left[\tau + \frac{\partial \tau}{\partial y}dy\right]dA_{sy} = 0$$

$$-\frac{\partial P}{\partial x}.dx\,[1 \times dy] + \frac{\partial \tau}{\partial y}.dy\,[1 \times dx] = 0$$

$$\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$$

i.e., pressure gradient in x-direction = Rate of Change of Shear stress in y-direction.

By integration: $\int d\tau = \frac{\partial P}{\partial x}\int dy$

$$\tau = \frac{\partial P}{\partial x}.y + C_1 \quad \text{—— (1)}$$

where C_1 is a constant

As per Newton's law of viscosity

$$\tau = \mu \cdot \frac{\partial u}{\partial y} \quad \text{———} (2)$$

from equations (1) & (2)

$$\mu \cdot \frac{du}{dy} = \frac{dp}{dx} \cdot y + C_1$$

$$du = \frac{1}{\mu} \cdot \frac{dp}{dx} \cdot y \cdot dy + \frac{C_1}{\mu} \cdot dy$$

$$\int du = \frac{1}{\mu} \cdot \frac{dp}{dx} \int y \cdot dy + \frac{C_1}{\mu} \int dy$$

$$u = \frac{1}{\mu} \frac{dp}{dx} \cdot \frac{y^2}{2} + \frac{C_1}{\mu} \cdot y + C_2$$

From no slip condition : $u = 0$ @ $y = 0$

$$C_2 = 0$$

at $y = t$, $u = 0 \Rightarrow C_1 = -\frac{1}{2} \frac{\partial p}{\partial x} \cdot t$

$$u = \frac{1}{2\mu} \left(\frac{\partial p}{\partial x} \right) (y^2 - y \cdot t)$$

Since $t \cdot y > y^2$, it can be written as

$$\boxed{u(y) = \frac{1}{2\mu} \cdot \left(-\frac{\partial p}{\partial x} \right) [t \cdot y - y^2]}$$

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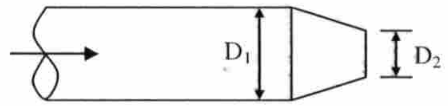
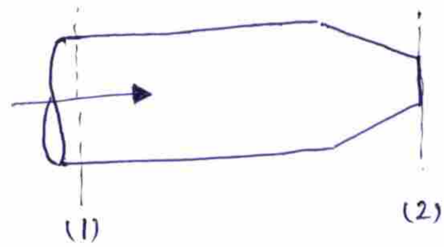
01(d). Water flows steadily through a horizontal nozzle, discharging to atmosphere. At the nozzle inlet, the diameter is D_1 and at the nozzle outlet, the diameter is D_2. Derive an expression for the minimum gauge pressure required at nozzle inlet to produce a given volume flow rate 'Q'. Also find this inlet gauge pressure if $D_1 = 75$ mm, $D_2 = 25$ mm and the desired flow rate is 0.02 m^3/s.



(12 M)

Given data:
$D_1 = 75mm$, $D_2 = 25mm$
$Q = 0.02\ m^3/sec$

continuity equation between (1) & (2)



$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{Q}{\frac{\pi}{4} D_1^2}$$

$$V_2 = \frac{Q}{A_2} = \frac{Q}{\frac{\pi}{4} D_2^2}$$

Bernoulli's Equation between (1) & (2)

$$\frac{P_{1g}}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_{2g}}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As $Z_1 = Z_2$ and $P_{2g} = 0$

$$\frac{P_{1g}}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

$$P_{1g} = \frac{\rho}{2}\left[\left(\frac{Q}{A_2}\right)^2 - \left(\frac{Q}{A_1}\right)^2\right]$$

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$$P_{1g} = \frac{8\rho Q^2}{\pi^2}\left[\frac{1}{D_2^4} - \frac{1}{D_1^4}\right]$$

$$= \frac{8\rho Q^2}{\pi^2 D_1^4}\left[\left(\frac{D_1}{D_2}\right)^4 - 1\right]$$

For $D_1 = 0.075\,m$; $D_2 = 0.025\,m$

& $Q = 0.02\,m^3/s$.

$$P_{1g} = \frac{8\times10^3\times(0.02)^2}{\pi^2\times0.075^4}\left[\left(\frac{0.075}{0.025}\right)^4 - 1\right]$$

$$\boxed{P_{1g} = 819.776\ kPa}$$

01(e). A Pelton wheel works under following conditions. Nozzle velocity coefficient = 0.97, bucket friction coefficient = 0.9, bucket to jet speed ratio = 0.47, jet deflection angle = 165°. If mechanical losses are 5% runner power, calculate :

(i) Wheel efficiency (ii) Nozzle efficiency

(iii) Mechanical efficiency (iv) Hydraulic efficiency **(3×4 = 12 M)**

Candidates must not write on this margin

(i) wheel efficiency (η_w) :

$$\eta_w = 2\frac{u}{V}\left[1-\frac{u}{V}\right]\left[1+K\cos\beta\right]$$

$$= 2\times 0.47\times[1-0.47]\times[1+0.9\times\cos 15^\circ]$$

$$\boxed{\eta_w = 0.931}$$

(ii) Nozzle efficiency (η_n):

$$\eta_n = \frac{V^2/2g}{H} = \frac{\left[C_v\cdot\sqrt{2gH}\right]^2/2g}{H}$$

$$= C_v^2$$

$$= 0.97^2$$

$$\boxed{\eta_n = 0.941}$$

(iii) Mechanical efficiency (η_m) :

$$\eta_m = \frac{\text{Shaft power (S.P)}}{\text{Runner power (R.P)}} = \frac{\text{R.P} - \text{frictional loss}}{\text{R.P}}$$

$$= 1-0.05$$

$$\boxed{\eta_m = 0.95}$$

(iv) Hydraulic efficiency (η_h):

$$\eta_h = \frac{\text{Runner Power}}{\text{Hydraulic power}}$$

$$= \frac{\text{Kinetic Energy per unit time}}{\text{Hydraulic power}} \times \frac{\text{Runner power}}{\text{Kinetic Energy per unit time}}$$

$$= \eta_n \times \eta_w$$

$$= 0.941 \times 0.931$$

$$\boxed{\eta_h = 0.876}$$

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02(a). Consider a penstock pipe which connects dam reservoir outlet to the hydro power plant turbine. At the end of pipe, flow control valve is provided. Details of pipe are as follows:

L = Length of penstock pipe = 6 km
D = Diameter of pipe = 1 m
f = Darcy's friction factor = 0.025
H = Head above the pipe outlet = 25 m
K_{Valve} = Loss coefficient of valve = 10
K_{Entry} = Loss coefficient at entry of penstock pipe = 0.5

If pipe valve is suddenly opened, find time required to attain 90% steady state discharge. It is given that the expression of time required to establish the steady state in penstock pipe is $t = \frac{L.V_o}{2gH}\log_e\left(\frac{V_o + V}{V_o - V}\right)$

where, V_o = Steady state velocity
V = Attained steady velocity fraction in terms of V_o in time t. **(20 M)**

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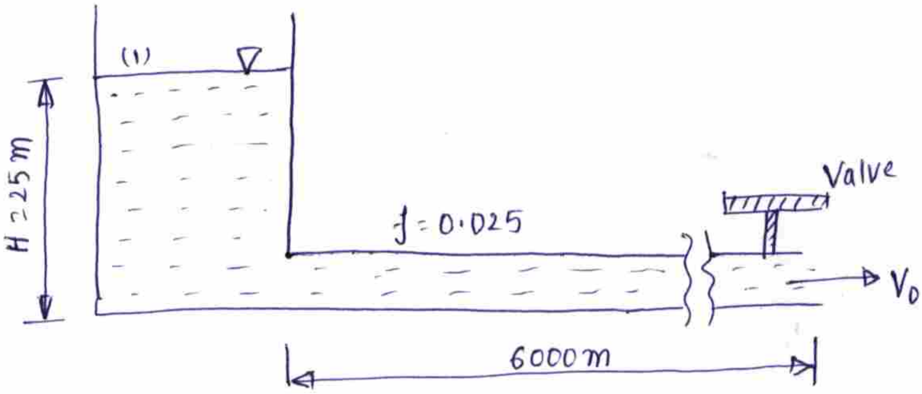

Given

L = 6 km = 6000 m D = 1 m

f = 0.025 H = 25 m

K_{valve} = 10 K_{entry} = 0.5

Using energy equation, at steady state, in between a point on the free surface of water in the dam reservoir and a point on the discharge plane after the control volume :

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 - h_{losses} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

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$$0 + 0 + H - h_{loss} = 0 + \frac{V_0^2}{2g} + 0 \quad [\because V_2 = V_0]$$

$$H - h_{loss} = \frac{V_0^2}{2g}$$

$$H = \frac{V_0^2}{2g} + \underbrace{\frac{f \cdot L \cdot V_0^2}{2gD}}_{h_{friction}} + \underbrace{K_{entry} \cdot \frac{V_0^2}{2g}}_{h_{entry}} + \underbrace{K_{valve} \cdot \frac{V_0^2}{2g}}_{h_{valve}}$$

$$H = \frac{V_0^2}{2g}\left[1 + \frac{f \cdot L}{D} + K_{entry} + K_{valve}\right]$$

$$25 = \frac{V_0^2}{2g}\left[1 + \frac{0.025 \times 6000}{1.0} + 0.5 + 10\right]$$

$$V_0^2 = \frac{25 \times 2 \times 9.81}{161.5}$$

$$V_0 = 1.743 \text{ m/s}$$

Substituting V_0 in the given expression for time required to establish steady state:

$$t = \frac{L \cdot V_0}{2gH} \cdot \log_e\left[\frac{V_0 + V}{V_0 - V}\right]$$

Where, $L = 6000$ m $\qquad H = 25$ m

$V_0 = 1.734$ m/s $\qquad V = 0.9 V_0$

$g = 9.81$ m/s^2

$$t = \frac{6000 \times 1.743}{2 \times 9.81 \times 25} \times \log_e \left[\frac{V_0 + 0.9V_0}{V_0 - 0.9V_0} \right]$$

$$= 21.321 \times \log_e \frac{1.9}{0.1}$$

$$= 21.321 \times 2.9444$$

$$= 62.8 \text{ Sec}$$

$$\boxed{t \simeq 63 \text{ Seconds.}}$$

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02(b). In a 50% reaction turbine the blade inlet angle is 55° and the outlet angle is 20° to the tangent of the blade ring. The steam speed is 260 m/sec. (i) Determine the blade speed for shockless entry and also the stage efficiency. (ii) In case maximum efficiency is to be achieved find the blade inlet angle, blade speed and stage efficiency in that case.

(12 + 8 = 20 M)

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For a 50 % reaction turbine

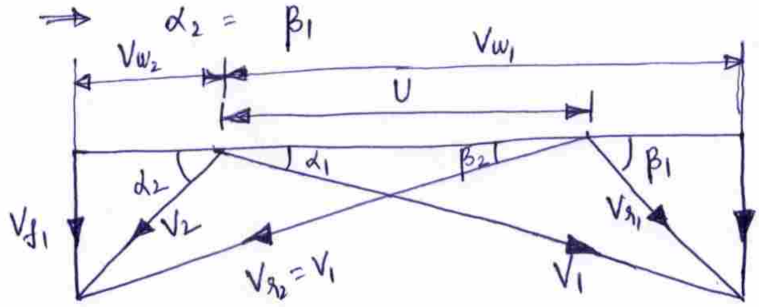
$V_{r_2} = V_1 \quad ; \quad V_{r_1} = V_2$

- Exit angle of fixed blade = Exit angle of moving blade.

$\Rightarrow \alpha_1 = \beta_2$

Inlet angle of fixed blade = Inlet angle of Moving blade

$\Rightarrow \alpha_2 = \beta_1$



Given:

$V_1 = 260 \text{ m/s} \qquad \alpha_1 = 20^\circ \qquad \beta_1 = 55^\circ$

(i) $V_{w_1} = V_1 \cos\alpha_1 = 260 \cos 20^\circ$

$= 244.32 \text{ m/s}$

$\tan\beta_1 = \dfrac{V_1 \sin\alpha_1}{V_{w_1} - U}$

Candidates must not write on this margin

$$\tan 55^\circ = \frac{260 \sin 20^\circ}{244.32 - u}$$

$$244.32 - u = \frac{260 \sin 20^\circ}{\tan 55^\circ}$$

$$u = 244.32 - \frac{260 \sin 20^\circ}{\tan 55^\circ}$$

$$= 182.05 \text{ m/s}$$

$$V_{r_1}^2 = (V_1 \sin \alpha_1)^2 + (V_{w_1} - u)^2$$

$$= (260 \sin 20)^2 + (244.32 - 182.05)^2$$

$$V_{r_1} = 108.56 \text{ m/s} = V_2$$

$$V_{w_2} = V_2 \cos 55^\circ$$

$$= 108.56 \cdot \cos 55^\circ$$

$$= 62.27 \text{ m/s}$$

$$\text{Workdone per kg} = U[V_{w_1} + V_{w_2}]$$

$$= 182.05 [244.32 + 62.27]$$

$$= 55814 \text{ J}$$

$$\text{Input Energy per kg} = \frac{V_1^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2} = \frac{260^2}{2} + \frac{260^2 - 108.56^2}{2}$$

$$= 61707 \text{ J}$$

$$\eta_{stage} = \frac{\text{Work done per kg}}{\text{Input Energy}}$$

$$= \frac{55814}{61707}$$

$$= 0.9045$$

$$\boxed{\eta_{stage} = 90.45\ \%}$$

(iii) Incase of Maximum efficiency

$u = V_1 \cos\alpha_1$ $\quad\quad$ $V_{r_1} = V_1 \sin\alpha_1$

$= 260 \cos 20^\circ$ $\quad\quad$ $= 260 \sin 20^\circ$

$= 244.32$ m/s $\quad\quad$ $= 88.92$ m/s

$\beta_1 = 90^\circ$ $\quad$ $V_{w_1} = 244.32$ m/s

$V_{w_2} = 0$

Work done per kg $= u^2$

$= 244.322^2$

$= 59692$ J

$$\text{Input} = \frac{V_1^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2} = \frac{260^2}{2} + \frac{260^2 - 88.92^2}{2}$$

$= 63646$ J

$$(\eta_{stage})_{max} = \frac{59692}{63646} = 0.9379 \text{ (or) } 93.79\ \%$$

Candidates must not write on this margin

02(c). (i). An incompressible flow field is given by $\vec{V} = x^2\hat{i} - z^2\hat{j} - 3xz\hat{k}$ with V in m/s and (x, y, z) in meters. If the fluid viscosity is 0.04 Pa.s, evaluate the entire viscous stress tensor at the point (x, y, z) = (3, 2, 1). **(12 M)**

Candidates must not write on this margin

From given velocity field

$u = x^2 \quad ; \quad V = -z^2 \quad ; \quad W = -3xz$

$$\tau_{ij} = \begin{vmatrix} \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{vmatrix}$$

Where

$$\tau_{xx} = 2\mu \cdot \frac{\partial u}{\partial x} = 2\mu \frac{\partial}{\partial x}(x^2)$$

$$= 4\mu x$$

$$\tau_{yy} = 2\mu \cdot \frac{\partial v}{\partial y} = 2\mu \cdot \frac{\partial}{\partial y}(-z^2)$$

$$= 0$$

$$\tau_{zz} = 2\mu \cdot \frac{\partial w}{\partial z} = 2\mu \cdot \frac{\partial}{\partial z}(-3xz)$$

$$= -6\mu x$$

$$\tau_{xy} = \tau_{yz} = \mu\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]$$

$$= \mu\left[\frac{\partial}{\partial y}(x^2) + \frac{\partial}{\partial x}(-z^2)\right]$$

$$= 0$$

$$\tau_{yz} = \tau_{zy} = \mu \cdot \left[\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right]$$

$$= \mu \left[\frac{\partial}{\partial z}(-z^2) + \frac{\partial}{\partial y}(-3xz) \right]$$

$$= -2\mu z$$

$$\tau_{xz} = \tau_{zx} = \mu \left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right]$$

$$= \mu \left[\frac{\partial}{\partial z}(x^2) + \frac{\partial}{\partial x}(-3xz) \right]$$

$$= -3\mu z$$

Now at $(x, y, z) = (3, 2, 1)$ & for $\mu = 0.04$ Pa.s

$$\tau_{ij} = \begin{bmatrix} 0.48 & 0 & -0.12 \\ 0 & 0 & -0.08 \\ -0.12 & -0.08 & -0.72 \end{bmatrix} \text{Pa}$$

Candidates must not write on this margin

Candidates must not write on this margin

02(c). (ii). A velocity field is given by u = Vcosθ and v = Vsinθ and w = 0, where V and θ are constants. Find an expression for the streamlines of this flow. **(8 M)**

The equation of streamlines is given by :

$$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dr}{V}}$$

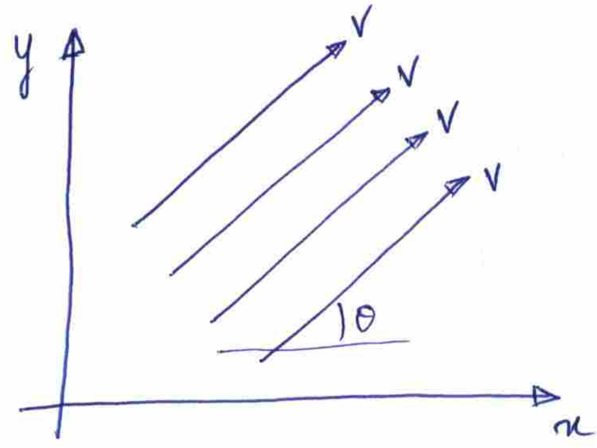
$$\frac{dx}{V\cos\theta} = \frac{dy}{V\sin\theta} = \frac{dz}{0}$$

[Note : $\frac{dz}{0}$ indicates that the streamlines donot vary with z]

$$\frac{dy}{dx} = \frac{V\sin\theta}{V\cos\theta} = \tan\theta$$

$$\boxed{y = x \cdot \tan\theta + c}$$

Hence, the streamlines are straight and inclined at angle θ as shown below.



03(a). A rain drop of diameter 0.3 mm is falling down in air (ρ_{air} = 1.2 kg/m^3 and ν_{air} = 15cs). Determine the velocity of fall. Check the analysis for validity of Stokes law. **(15 M)**

Candidates must not write on this margin

Applying drag concept:

weight of Rain drop = Drag force of spherical Rain drop.

$$W_{droplet} = (F_D)_{sphere}$$

$$\rho_{rain\,water} \cdot g \cdot \forall = C_D \cdot \frac{\rho \cdot A \cdot V^2}{2}$$

where $\forall$ is the volume of rain drop.

$$\rho_{rain\,water} \cdot g \cdot \frac{4}{3}\pi R^3 = \frac{24}{Re} \cdot \frac{\rho \cdot A \cdot V^2}{2}$$

$$\rho_{rain\,water} \cdot g \cdot \frac{\pi}{6} D^3 = 3\pi\mu\, D \cdot V \quad \left[\because Re = \frac{\rho \cdot V \cdot D}{\mu}\right]$$

$$= 3\pi\left[\rho_{air} \cdot \nu_{air}\right] \cdot D \cdot V$$

$$1000 \times 9.81 \times \frac{\pi}{6}\left(0.3 \times 10^{-3}\right)^3 = 3\pi \times \left[1.2 \times 15 \times 10^{-6}\right] \times \left[0.3 \times 10^{-3}\right] \times V$$

$$V = 2.725 \text{ m/s}$$

Reynold's Number (Re) for falling Rain drop in air

$$Re = \frac{\rho \cdot V \cdot D}{\mu}$$

$$= \frac{V \cdot D}{\mu}$$

$$Re_f = \frac{2.725 \times 0.3 \times 10^{-3}}{15 \times 10^{-6}}$$

$$\boxed{Re_f = 54.5 \quad > 0.1}$$

$\Rightarrow$ Stoke's law is not justified.

Candidates must not write on this margin

03(b). An aircraft is flying at an altitude of 10 km (ρ_{air} = 0.17 kg/m^3) at a speed of 900 km/hr and the propulsive efficiency is 60%. Overall efficiency is 20%. Drag on aircraft 6.5 kN. If heating value of fuel is 45 MJ/kg, calculate volume of air handled by compressor. The velocity of gases leaving the nozzle is 575 m/s. **(15 M)**

Candidates must not write on this margin

Given data: $\rho_{air} = 0.17$ kg/m^3, $V_1 = 900$ km/hr

Velocity of air craft

$$V_1 = 900 \times \frac{5}{18}$$

$$= 250 \text{ m/s}$$

Thrust force (T) = 6.5 KN

Thrust power = $T \times V$

$= 6.5 \times 250$

$= 1625$ KW

$$\text{Overall efficiency} = \frac{\text{Thrust Power}}{\text{Energy Supplied}}$$

$$0.2 = \frac{1625}{\dot{m}_f \times 45000}$$

where $\dot{m}_f$ is mass flow rate of fuel.

$$\dot{m}_f = 0.1805 \text{ kg/sec}$$

$$\text{Thrust force} = (\dot{m}_a + \dot{m}_f) V_e - \dot{m}_a \cdot V_1$$

$$= 6500$$

Candidates must not write on this margin

$$(\dot{m}_a + 0.1805)575 - \dot{m}_a \times 250 = 6500$$

where $\dot{m}_a$ is mass flow rate of air.

$$\dot{m}_a = 19.68 \text{ kg/sec.}$$

$$\rho = \frac{P}{RT} = 0.17 \text{ kg/m}^3$$

Volume handled by compressor $= \dfrac{19.68}{0.17}$

$$\boxed{\dot{V} = 115.77 \text{ m}^3/\text{sec.}}$$

03(c). A centrifugal pump has an efficiency of 80 percent with specific speed of 2323 (units of rpm, m^3/hr, and meters). The impeller diameter is 200 mm and the volume flow rate 68 m^3/hr of water at 1170 rpm. To obtain a higher flow rate, the pump is to be fitted with a 1750 rpm motor.

(i) Use pump laws to find performance characteristics of pump at the higher speed.

(ii) Also, show that the specific speed remains constant for the higher operating speed.

(10 + 5 = 15 M)

Candidates must not write on this margin

Given data:

$N_s = 2323,$ $D = 200 mm,$ $Q_1 = 68 m^3/hr$

$N_1 = 1170 rpm,$ $N_2 = 1750 rpm,$ $Q_2 = ?$

$H_2 = ?$ $P_2 = ?$

$$N_s = \frac{N \cdot \sqrt{Q}}{H^{3/4}} = \frac{N_1 \cdot \sqrt{Q_1}}{H_1^{3/4}}$$

$$2323 = \frac{1170 \cdot \sqrt{68}}{H_1^{3/4}}$$

$$H_1 = 6.68 m$$

$$Q \propto N \cdot D^3 \Rightarrow \frac{Q_2}{Q_1} = \frac{N_2}{N_1} \cdot \left(\frac{D_2}{D_1}\right)^3$$

$$Q_2 = Q_1 \cdot \frac{N_2}{N_1} \quad [\because D_1 = D_2]$$

$$Q_2 = 68 \times \frac{1750}{1170}$$

$$Q_2 = 101.7 m^3/hr$$

Candidates must not write on this margin

$H \propto N^2 \cdot D^2$

$\Rightarrow \quad H \propto N^2 \quad [\because D = \text{constant}]$

$$\frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2$$

$$H_2 = H_1 \cdot \left(\frac{N_2}{N_1}\right)^2$$

$$= 6.68 \times \left(\frac{1750}{1170}\right)^2$$

$$H_2 = 14.94 \text{ m}$$

$$P_1 = \rho Q_1 g \cdot H_1$$

$$= 10^3 \times \frac{68}{3600} \times 9.81 \times 6.68$$

$$= 1.24 \text{ kW}$$

Also $\quad P \propto N^3 \cdot D^5$

$\Rightarrow \quad P \propto N^3 \quad [\because D = \text{constant}]$

$$\frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3$$

$$P_2 = P_1 \cdot \left(\frac{N_2}{N_1}\right)^3$$

Candidates must not write on this margin

$$P_2 = 1.24 \times \left(\frac{1750}{1170}\right)^3$$

$$= 4.15 \text{ kW}$$

$$\boxed{N_s = \frac{N_2 \sqrt{Q_2}}{H_2^{3/4}}}$$

$$= \frac{1750 \cdot \sqrt{101.7}}{14.94^{3/4}}$$

$$\boxed{N_s \simeq 2323}$$

Thus, the specific speed remains constant for the higher operating speed.

03(d). An air vessel is fitted on delivery side of a reciprocating pump. (i) What is the ratio of velocity of fluid in delivery pipe to the maximum velocity in delivery pipe without air vessel ? (ii) Show that pumping power saved by the use of air vessel is 84.8%.

(7 + 8 = 15 M)

Candidates must not write on this margin

After air vessel, Velocity remains constant and is given by

$$V_{da} = \frac{Q}{A_d} = \frac{A_p . L N/60}{A_d}$$

$$= \frac{A_p}{A_d} . 2r . \frac{\omega}{2\pi}$$

$$\left[\because L = 2r \ \& \ \omega = \frac{2\pi N}{60} \right]$$

$$V_{da} = \frac{A_p}{A_d} . \frac{r\omega}{\pi} \text{ ——— } (1)$$

Velocity in delivery pipe before installing air vessel is given by :

$$V_d = \frac{A_p}{A_d} . r\omega . \sin\theta$$

$$(V_d)_{max} = \frac{A_p}{A_d} . r\omega \text{ ——— } (2)$$

$$\frac{①}{②} \Rightarrow \frac{V_{da}}{(V_d)_{max}} = \frac{1}{\pi}$$

Candidates must not write on this margin

Head loss in delivery pipe before installing air vessel is given by

$$h_{f1} = \frac{2}{3} h_{fdmax} = \frac{2}{3} \frac{f \cdot L_d \cdot V_{dmax}^2}{2g D_d}$$

Head loss in delivery pipe after installing air vessel is given by

$$h_{f2} = \frac{f \cdot L_d \cdot V_{da}^2}{2g D_d}$$

$$\frac{h_{f2}}{h_{f1}} = \frac{3}{2} \times \left[\frac{V_{da}}{V_{dmax}}\right]^2$$

$$= \frac{3}{2} \times \frac{1}{\pi^2}$$

$$= 0.152$$

$$\% \text{ Saving in power} = \frac{h_{f1} - h_{f2}}{h_{f1}} \times 100$$

$$= \frac{1 - 0.152}{1} \times 100$$

$$\boxed{\% \text{ Saving in power} = 84.8\,\%}$$

04(a). A hydraulic ram pump delivers 5 lit/s to a destination which is 18 m above the ram. Source of the water is 4 m above the ram pump and discharge passing through the waste valve is 75 lit/s. If head loss in the supply pipe and delivery pipe are 0.5 m and 1.5 m respectively then find D'Aubuisson efficiency
(i) by ignoring the frictional losses,
(ii) by considering the frictional losses. **(6 + 6 = 12 M)**

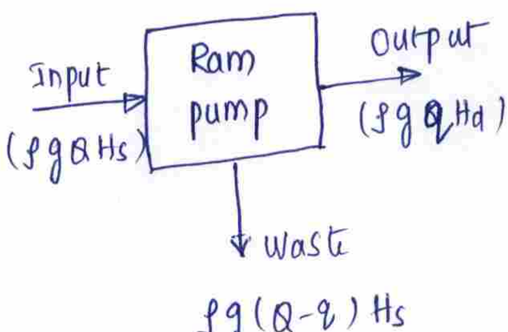
Candidates must not write on this margin

D' Aubuisson efficiency is given by

$$\eta_d = \frac{\text{Hydraulic power supplied to destination}}{\text{Hydraulic power taken from Source.}}$$

(i) By ignoring frictional losses

$$\eta_d = \frac{q \cdot H_d}{Q \cdot H_s}$$

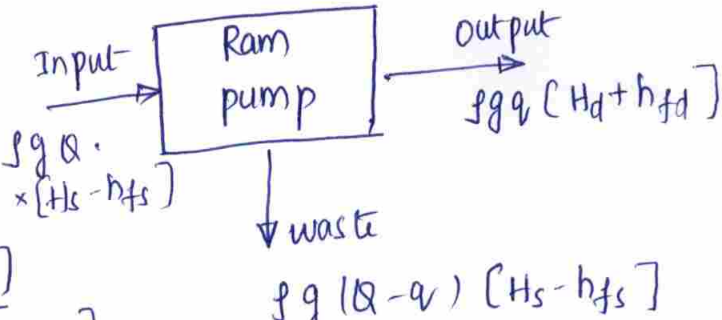


$$= \frac{5 \times 18}{(5+75) \times 4}$$

$$= 28.13\ \%$$

(ii) By considering frictional losses.

$$\eta_d = \frac{q\,[H_d + h_{fd}]}{Q\,[H_s - h_{fs}]}$$



$$= \frac{5\,[18 + 1.5]}{(5+75)\,[4 - 0.5]}$$

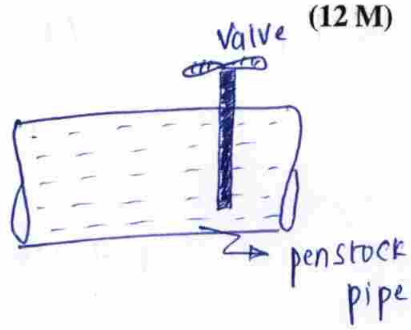
$$= 34.8\ \%$$

Candidates must not write on this margin

04(b). In order to analyse water hammer in a penstock pipe of hydel plant, velocity of sound or pressure wave velocity (C) is expressed in terms of bulk modulus of elasticity (K) and mass density (ρ) of water. Prove that $C = \sqrt{K/\rho}$. *(Don't use dimensional analysis)*

(12 M)

Candidates must not write on this margin



Consider a penstock pipe carrying water with velocity V with a control valve which suddenly starts closing. Under such conditions, momentum of water starts destroying.

let C = Velocity of sound

V = Velocity of water

ρ = mass density of water

K = Bulk Modulus of elasticity

After time interval Δt, every particle of water moves with V in right direction and C is velocity of sound (Accoustic velocity of water).

In this water medium

Impulse = Momentum Change.

$$I = m \cdot \Delta V = \Sigma F \cdot \Delta t = (\Delta p \cdot A) \cdot \Delta t \quad \text{—— (1)}$$

We know that

$$\boxed{K = \frac{\Delta P}{-\frac{d\forall}{\forall}} = \frac{\text{Change in pressure}}{\text{Volumetric Strain}}}$$

$$\Delta p = -K \cdot \frac{\Delta \forall}{\forall} = -K \cdot \left[\frac{A \cdot (-V \cdot \Delta t)}{A \cdot C \cdot \Delta t} \right]$$

Candidates must not write on this margin

$$\Delta p = K \cdot \frac{V}{C}$$

Replacing Δp in equation (i) : it reduces to

$$I = K \cdot \frac{V}{C} \cdot A \cdot \Delta t$$

$$\rho \cdot \forall \cdot [V - 0] = K \cdot \frac{V}{C} \cdot A \cdot \Delta t$$

$$\rho \cdot C \cdot \cancel{V} \cdot \cancel{A} \cdot \cancel{\Delta t} = K \cdot \frac{\cancel{V}}{C} \cdot \cancel{A} \cdot \cancel{\Delta t}$$

$$\rho \cdot C = \frac{K}{C}$$

$$C^2 = \frac{K}{\rho}$$

Velocity of sound, in penstoke pipe $C = \sqrt{\frac{K}{\rho}}$

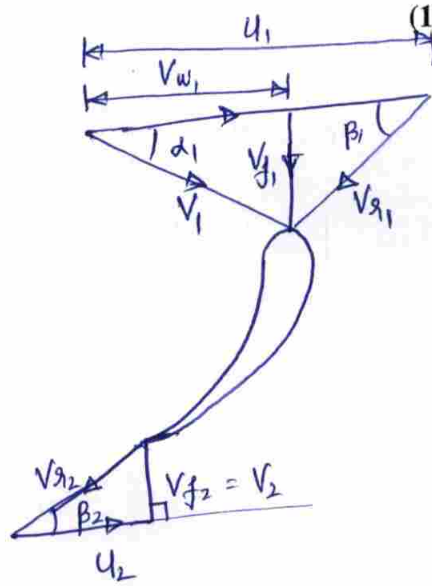
$$C = \sqrt{\frac{\text{Bulk modulus of elasticity of water}}{\text{mass density of water}}} \ \text{m/s}$$

where K is in N/m^2

ρ is in kg/m^3

04(c). At a particular cross section of a Kaplan turbine inlet guide vane angle and exit blade angle are equal and their value is equal to 25°. What is the value of blade angle at inlet and hydraulic efficiency ? Assume the turbine is discharging water into atmosphere and ignore the frictional losses in turbine. **(12 M)**

Candidates must not write on this margin



$\alpha_1 = \beta_2$

$\therefore \tan \alpha_1 = \tan \beta_2$

$$\frac{V_{f_2}}{V_{w_1}} = \frac{V_{f_2}}{u_2} \quad \text{——— (1)}$$

for kaplan turbine

$$V_{f_1} = V_{f_2} = \frac{4Q}{\pi\left[D_t^2 - D_h^2\right]}$$

$V_{w_1} = u_2$

and $u_1 = u_2 = r\omega$

$\therefore V_{w_1} = u_1 = u_2$ ——— (2)

$$\tan \beta_1 = \frac{V_{f_1}}{u_1 - V_{w_1}} = \frac{V_{f_2}}{u_1 - u_1}$$

$= \infty$

$\beta_1 = 90^\circ$

Hydraulic efficiency is given by

$$\eta_h = \frac{V_{w_1} \cdot u_1}{g \cdot H} \quad \text{——— (3)}$$

Candidates must not write on this margin

By Energy Balance,

Input head = Output head + head lost

As internal frictional losses are ignored only loss ~~percent~~ present is kinetic energy loss at exit.

$$H = He + \frac{V_2^2}{2g}$$

$$= \frac{u_1 \cdot V_{w_1}}{g} + \frac{(u_2 \tan\beta_2)^2}{2g}$$

$$= \frac{u_1^2}{g}\left[1 + \frac{\tan^2\beta_2}{2}\right]$$

$$\therefore \eta_h = \frac{u_1 \cdot V_{w_1}}{g \cdot H}$$

$$= \frac{u_1^2}{\cancel{g} \cdot \frac{u_1^2}{\cancel{g}}\left[1 + \frac{\tan^2\beta_2}{2}\right]}$$

$$= \frac{1}{1 + \frac{\tan^2 25^\circ}{2}}$$

$$\boxed{\eta_h = 0.902 = 90.2\ \%}$$

04(d). A centrifugal compressor handles 200 kg/min of air whose inlet pressure and temperature are 1 bar and 290 K with inlet velocity of 100 m/s. The conditions after the compression in impeller are 1.7 bar and 350K at 260 m/s. calculate

(i) Isentropic efficiency of compressor.

(ii) Power required to run compressor (kW) **(8 + 4 = 12 M)**

Candidates must not write on this margin

From steady flow energy equation

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} - W_{act}$$

$$W_{act} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2000} \quad kJ/kg$$

$$= C_p [T_2 - T_1] + \frac{V_2^2 - V_1^2}{2000}$$

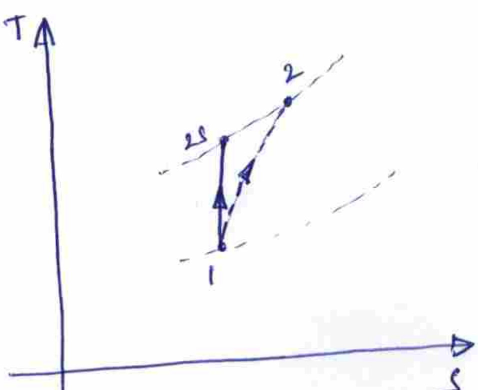
$$= 1.005 [350 - 290] + \frac{260^2 - 100^2}{2000}$$

$$W_{act} = 89.1 \ kJ/kg$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{2s} = 290 \cdot (1.7)^{\frac{0.4}{1.4}}$$

$$= 337.474 \ K$$



$$W_{isentropic} = C_p [T_{2s} - T_1] + \frac{V_2^2 - V_1^2}{2000}$$

$$= 1.005 [337.474 - 290] + \frac{260^2 - 100^2}{2000}$$

$$= 76.511 \ kJ/kg$$

Candidates must not write on this margin

(i) Isentropic efficiency

$$\eta_{isen} = \frac{W_{isentropic}}{W_{actual}}$$

$$= \frac{76.511}{89.1}$$

$$\boxed{\eta_{isen} = 85.87\ \%}$$

(iii) Power , $P = \dot{m} \times W_{act}$

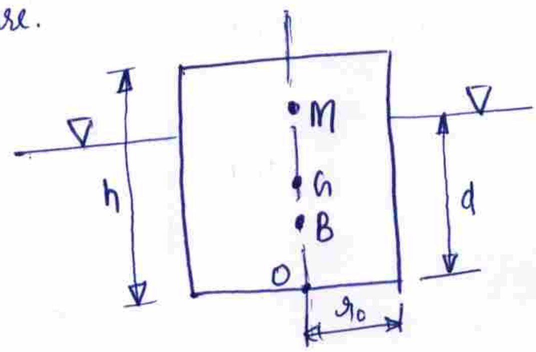
$$= \frac{200}{60} \times 89.1$$

$$\boxed{P = 297\ kW.}$$

04(e). What are the proportions of radius to height (r_o/h) of a right circular cylinder of specific gravity, S so that it will float in water with end faces horizontal in stable equilibrium ? **(12 M)**

Candidates must not write on this margin

with reference to the figure.



$\bar{OG} = \frac{h}{2}$

weight of the cylinder = Buoyant force.

$\Rightarrow S \times \gamma_w \times A_{cyl} \times h = \gamma_w \times A_{cyl} \times d$

where d is depth of submergence.

so $d = S.h$

Thus $\bar{OB} = \frac{d}{2} = \frac{S.h}{2}$

$\bar{BM} = \frac{I}{\forall} = \frac{\pi}{4} r_0^4 \times \frac{1}{\pi r_0^2 . d}$

$= \frac{r_0^2}{4S.h}$

Then $\bar{GM} = \bar{BM} - \bar{BG}$

$= \bar{BM} - [\bar{OG} - \bar{OB}]$

$= \frac{r_0^2}{4S.h} - \left[\frac{h}{2} - \frac{S.h}{2}\right]$

$$\overline{GM} = \frac{r_0^2}{4 \cdot S \cdot h} - \frac{h}{2}[1-S]$$

for stable equilibrium, $\overline{GM} \geqslant 0$

Thus $\frac{r_0^2}{4 \cdot S \cdot h} \geqslant \frac{h}{2}[1-S]$

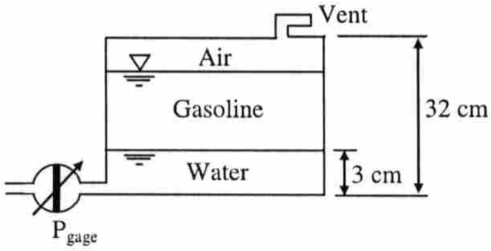
or $\boxed{\frac{r_0}{h} \geqslant \sqrt{2S[1-S]}}$

Candidates must not write on this margin

SECTION - B

Candidates must not write on this margin

05(a). The reading of an automobile fuel gage is proportional to the gage pressure at the bottom of the tank as shown in the figure. If the tank is 32 cm deep and is contaminated with 3 cm of water, how many centimetres of air remains at the top when the gage indicates "full" ? Use $\gamma_{gasoline} = 6670 \text{ N/m}^3$ and $\gamma_{air} = 11.8 \text{ N/m}^3$. (Assume, $g = 10 \text{ m/s}^2$)



(12 M)

When full of gasoline

$$P_{gauge} = 6670 \times 0.32$$
$$= 2134 \text{ Pa}$$

with water added,

$$2134 = 10^4 \times 0.03 + 6670\,[(0.32 - 0.03) - h] + 11.8h$$
$$= 300 + 1934.3 - 6670h + 11.8h$$

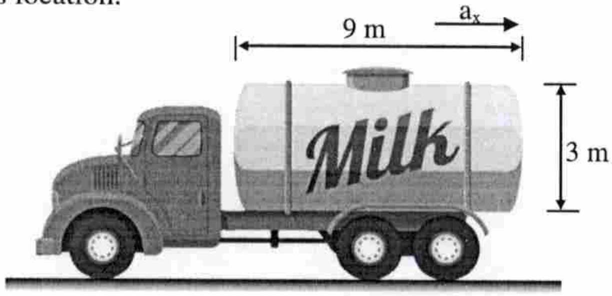
$$6658.2h = 100.3$$

$$h = 0.015 \text{ m}$$

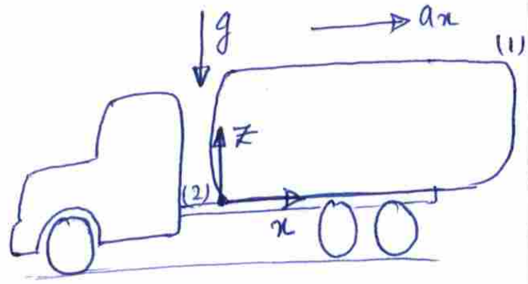
$$\boxed{h = 1.5 \text{ cm}}$$

05(b). Milk with a density of 1020 kg/m^3 is transported on a level road in a 9 m long, 3 m diameter cylindrical tanker. The tanker is completely filled with milk (no air space), and it decelerates at 2.5 m/s^2. If the minimum pressure in the tanker is 100 kPa, determine the maximum pressure and its location.

Candidates must not write on this margin



(12 M)



Assumption

* The deceleration remains constant
* Milk is an incompressible substance.

We take x and z axis as shown.

The horizontal deceleration is in the x-direction and thus a_x is positive.

Also, There is no acceleration in the z direction thus $a_z = 0$

The pressure difference between two point ① & ② in an incompressible fluid in linear rigid body motion is given by

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho (g + a_z) \times (z_2 - z_1)$$

(or) $$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho g [z_2 - z_1]$$

The first term is due to deceleration in the horizontal direction and the resulting compression effect towards the front of the tanker, while the second term is simply the hydrostatic pressure that increases with depth.

Therefore, we reason that the lowest pressure in the tank will occur at point ① [upper front corner] and the higher pressure at point ② [lower rear corner]

Thus

$$\Delta P_{max} = P_2 - P_1$$

$$= -\rho a_x [x_2 - x_1] - \rho g [z_2 - z_1]$$

$$= -\rho [a_x [x_2 - x_1] + g [z_2 - z_1]]$$

Since

$x_1 = 9\,m \qquad z_1 = 3m$

$x_2 = 0\,m \qquad z_2 = 0m$

$$\Delta P_{max} = -1020 [2.5 \times -9 + 9.81 \times -3]$$

$$\boxed{\Delta P_{max} = 53.0 \text{ kPa}}$$

05(c). A pipe of 800 mm in diameter carries water with turbulent flow velocity profile as:

$$u(y) = 4 + 0.25 \log_e(y)$$

The shear stress at a point 100 mm from the wall is measured as 1.2 Pa.

Determine (i) Turbulence dynamic viscosity (η)

(ii) Prandtl mixing length

(iii) Turbulence constant. **(5 + 5 + 2 = 12 M)**

Candidates must not write on this margin

Given

$$u(y) = 4 + 0.25 \log_e y$$

$$\frac{du}{dy} = 0 + 0.25 \times \frac{1}{y}$$

$$= \frac{0.25}{y}$$

(i) $$\tau_{y=0.1m} = \eta \cdot \left.\frac{du}{dy}\right|_{y=0.1m}$$

$$1.2 = \eta \times \frac{0.25}{0.1}$$

$$\boxed{\eta = 0.48 \ N.s/m^2}$$

(ii) Using Prandtl mixing length

$$\tau = \rho \cdot l^2 \cdot \left(\frac{du}{dy}\right)^2$$

$$1.2 = 1000 \times l^2 \times \left(\frac{0.25}{0.1}\right)^2$$

$$l = 0.01386 \ m$$

$$\boxed{l = 13.86 \ mm}$$

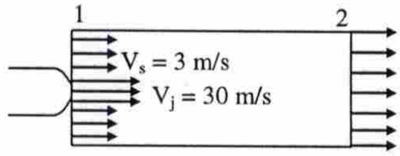
(iii) Turbulence constant $= \frac{l}{y}$

$= \frac{13.86}{100}$

$= 0.1386$

Candidates must not write on this margin

05(d). A water jet pump has jet area $A_j = 0.01\ m^2$ and jet speed $V_j = 30$ m/s. The jet is within a secondary stream of water having speed $V_s = 3$ m/s. The total area of duct (sum of jet and stream area) is $0.075\ m^2$. The water is thoroughly mixed and leaves the pump exit with the pressure rise $P_2 - P_1$.



(i) What is the speed (V_2) at the pump exit ?

(ii) What is the pressure rise ($P_2 - P_1$) ?

(6 + 6 = 12 M)

Candidates must not write on this margin

(i) Continuity Equation

$$A \cdot V = \text{constant}$$

$$A_s \cdot V_s + A_j \cdot V_j = A_2 \cdot V_2 \quad \text{—— (1)}$$

But $A_1 = A_2 = 0.075\ m^2$

$$A_s = A_2 - A_j$$

$$= 0.075 - 0.01$$

$$= 0.065\ m^2$$

Substituting A_s, A_j & A_2 in equation (1)

$$0.065 \times 3 + 0.01 \times 30 = 0.075 \times V_2$$

$$V_2 = 6.6\ m/s$$

(ii) Momentum equation (x-direction)

$$\Sigma F = \sum_{out} \dot{m} V - \sum_{in} \dot{m} V$$

[Bernoulli equation is not valid because mixing is a viscous process]

Candidates must not write on this margin

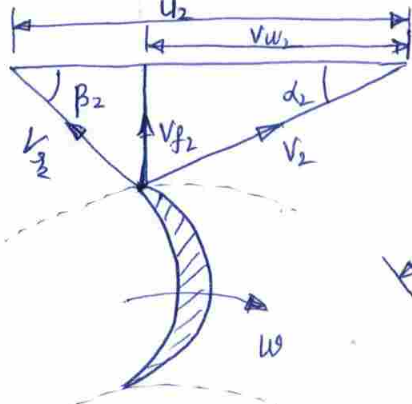
$$P_2 A_2 - P_1 A_1 = \rho \left[A_s V_s^2 + A_j V_j^2\right] - \rho A_2 V_2^2$$

$$P_2 - P_1 = \frac{\rho}{A_2} \left[A_s V_s^2 + A_j V_j^2 - A_2 V_2^2\right]$$

$$= \frac{10^3}{0.075} \left[0.065 \times 3^2 + 0.01 \times 30^2 - 0.075 \times 6.6^2\right]$$

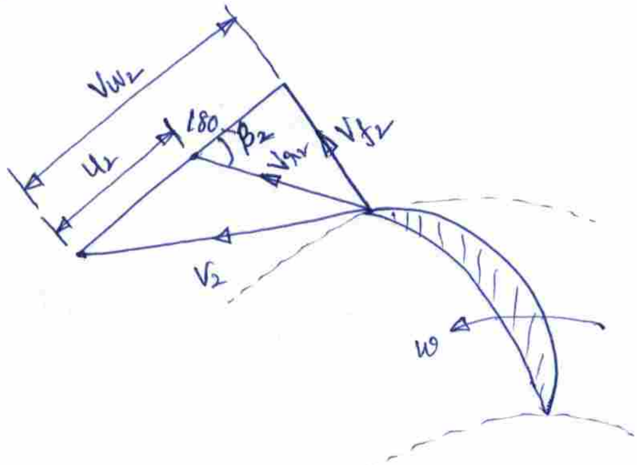
$$\boxed{P_2 - P_1 = 84.2 \text{ kPa}}$$

05(e). A backward vane centrifugal pump has exit blade angle of 45°. Discharge through the pump is such that radial component of absolute velocity is 25% peripheral velocity. The pump is made to run in opposite direction with same speed while maintaining same discharge. What is the ratio of theoretical head developed by the pump when running as per designed direction to that in reverse direction ? **(12 M)**



Designed direction

Case - A



Reverse direction

Case- B

$$(\tan \beta_2) = \frac{V_{f_2}}{u_2 - V_{w_2}}$$

$$\tan 45^\circ = \frac{0.25\, u_2}{u_2 - V_{w_2}}$$

$$(V_{w_2})_A = (u_2)_A - 0.25\, u_{2A}.$$

$$V_{w_{2A}} = 0.75\, u_{2A} \quad \text{—}(1)$$

$$\tan (180-\beta_2) = \frac{V_{f_2}}{V_{w_2} - u_2}$$

$$\tan 45^\circ = \frac{0.25\, u_2}{V_{w_2} - u_2}$$

$$V_{w_2} = u_2 + 0.25\, u_2.$$

$$V_{w_2 B} = 1.25\, u_{2B} \quad \text{—}(2)$$

Candidates must not write on this margin

$$\frac{He_A}{He_B} = \frac{\left[U_2 V_{w_2}/g\right]_A}{\left[U_2 V_{w_2}/g\right]_B}$$

$$= \frac{0.75\, U_{2A}^2}{1.25\, U_{2B}^2}$$

$$= \frac{0.75}{1.25}$$

$$\boxed{\frac{He_A}{He_B} = 0.6}$$

06(a). The following data refers to a Francis Turbine:

Working head = 25 m

Power developed= 2555 kW

The overall efficiency = 90 %

The diameter and width at inlet are 1310 mm and 380 mm

The diameter and width at outlet are 1100 mm and 730 mm

Angle made by relative velocity at inlet with positive direction of blade velocity is 135°

The whirl is zero at exit. [Assume $\eta_v = 0.98$, $\eta_m = 0.97$]

Determine:

(i) Head extracted by the runner , (ii) Guide blade angle,

(iii) Runner speed, (iv) Runner blade angle at outlet

(v) Specific speed, **(4 × 5 = 20 M)**

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The flow rate $Q = \dfrac{P}{\eta_o H \cdot \gamma}$

$$= \frac{2555 \times 10^3}{0.9 \times 9.81 \times 25 \times 1000}$$

$$= 11.58 \ m^3/s$$

(i) Hydraulic efficiency $= \dfrac{\text{Overall efficiency}}{\text{Mechanical efficiency} \times \text{Volumetric efficiency}}$

$$\eta_H = \frac{0.9}{0.98 \times 0.97}$$

$$\frac{u_1 V_{w_1}}{g H} = 0.9468$$

Head extracted by runner $= \dfrac{u_1 V_{w_1}}{g} = \eta_H \times H$

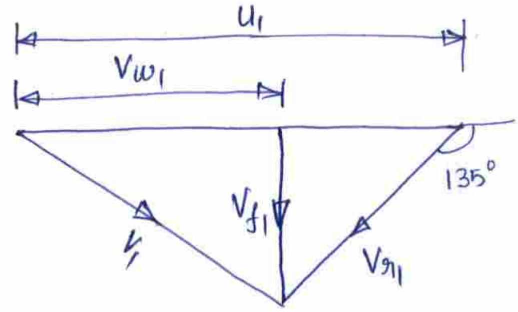
$$= 0.9468 \times 25$$

$$= \underline{23.67 \ m}$$

(ii)

$u_1 \cdot V_{w_1} = 0.9468 \times 9.81 \times 25$

$= 232.2$ — ①



The flow velocity at inlet

$$V_{f_1} = \frac{Q}{A_i} = \frac{11.58}{\pi \times 1.31 \times 0.38} = 7.4 \text{ m/s}$$

$$\tan(180 - 135) = \frac{V_{f1}}{u_1 - V_{w_1}}$$

$u_1 - V_{w_1} = 7.4$ m/s $\Rightarrow V_{w_1} = u_1 - 7.4$ — ②

from ① & ② $\quad u_1 [u_1 - 7.4] = 232.2$

solving for u_1 $\Rightarrow$ $u_1 = \dfrac{7.4 \pm \sqrt{7.4^2 + 4 \times 232.2}}{2}$

$u_1 = 19.38$ m/s

$V_{w_1} = u_1 - 7.4 = 19.38 - 7.4$

$= 11.98$ m/s

Guide Blade angle, $\tan \alpha_1 = \dfrac{7.4}{11.98}$

$$\boxed{\alpha_1 = 31.7^0}$$

Candidates must not write on this margin

Candidates must not write on this margin

(iii) $u_1 = \frac{\pi D N}{60}$

The runner speed, $N = \frac{19.38 \times 60}{\pi \times 1.31}$

$\boxed{N = 282.5 \text{ rpm}}$

(iv) $V_{f_2} = \frac{Q}{A_0} = \frac{11.58}{\pi \times 1.1 \times 0.73}$

$= 4.59 \text{ m/s}$

Blade velocity at outlet $u_2 = \frac{\pi D_2 N}{60}$

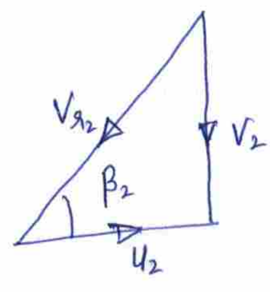
$= \frac{\pi \times 1.1 \times 282.5}{60}$

$= 16.27 \text{ m/s}$

The ~~ex~~ exit triangle is right angled

$\tan \beta_2 = \frac{4.59}{16.27}$

$\boxed{\beta_2 = 15.75^\circ}$



(v) Specific speed $= \frac{N\sqrt{P}}{H^{5/4}}$

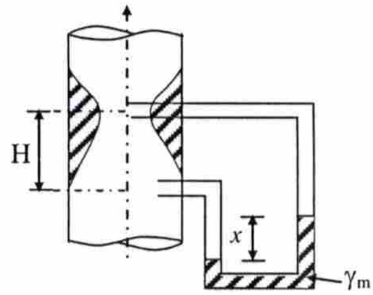
$N_s = \frac{282.5 \cdot \sqrt{2555}}{25^{5/4}}$

$\boxed{N_s = 255}$

06(b). A venturimeter is installed in a pipeline of 400 mm diameter. The throat diameter is 1/3rd of pipe diameter. The pressure in the water pipeline is 1.405 kgf/cm^2 gauge and vacuum in throat is 37.5 cm of mercury. If 4% of differential head is lost between gauges then find

(i) coefficient of the discharge of the venturimeter

(ii) the discharge in pipeline.



(10 + 10 = 20 M)

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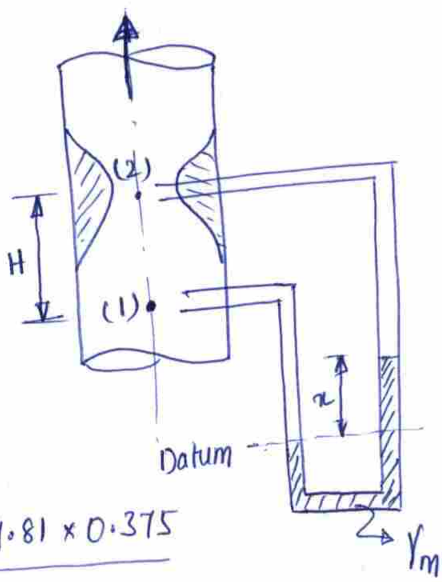
$$d_1 = 400\ mm = 0.4\ m$$

$$d_2 = \frac{400}{3}\ mm = 0.133\ m$$

(i) $P_1 = 1.405\ \frac{kgf}{cm^2}$

$$= \frac{1.405 \times 9.81}{10^{-4}}\ Pa$$

$$P_2 = -13600 \times 9.81 \times 0.375\ Pa$$



$$h = \frac{P_1 - P_2}{\rho g}$$

$$= \frac{1.405 \times 9.81 \times 10^{+4} + 13600 \times 9.81 \times 0.375}{9810}$$

$$= 19.15\ m \text{ of water}$$

$$h_L = 4\% \text{ of } h$$

$$= 0.04\ h$$

Candidates must not write on this margin

$$C_d = \sqrt{\frac{h - h_L}{h}}$$

$$= \sqrt{1 - \frac{h_L}{h}}$$

$$= \sqrt{1 - 0.04}$$

$$\boxed{C_d = 0.979}$$

(ii) $$Q = \frac{C_d \cdot A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \cdot \sqrt{2gh}$$

$$= \frac{0.979 \times \frac{\pi}{4} \times 0.4^2 \times \frac{\pi}{4}\ 0.133^2}{\sqrt{\left[\frac{\pi}{4} \cdot 0.4^2\right]^2 - \left[\frac{\pi}{4} \cdot 0.133^2\right]^2}}$$

$$\boxed{Q = 0.266\ m^3/s}$$

Candidates must not write on this margin

06(c). Velocity profile in a laminar boundary layer is given by :

$$\frac{u}{U_\infty} = \sin\left(\frac{\pi}{2}.\frac{y}{\delta}\right).$$

Prove that the coefficient of drag is equal to $\frac{1.31}{\sqrt{Re_L}}$ **(20 M)**

Candidates must not write on this margin

$$\text{Drag force} = \int_0^L \tau_0 \cdot B \cdot dx$$

where $\tau_0 = \rho \cdot U_\infty^2 \cdot \frac{d\theta}{dx}$ [von-Karman Momentum Integral equation]

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left[1 - \frac{u}{U_\infty}\right] dy$$

$$= \int_0^\delta \sin \frac{\pi}{2}.\frac{y}{\delta} \left[1 - \sin \frac{\pi}{2}.\frac{y}{\delta}\right] dy$$

$$= \left[- \frac{\cos \frac{\pi}{2}.\frac{y}{\delta}}{\pi/2\delta} - \frac{y}{2} + \frac{\sin \pi \cdot \frac{y}{\delta}}{\pi/\delta} \right]_0^\delta$$

$$= 0 + \frac{2\delta}{\pi} - \frac{\delta}{2} + 0 - 0$$

$$= \left[\frac{2}{\pi} - \frac{1}{2}\right] \delta$$

$$\theta = \frac{4-\pi}{2\pi} \cdot \delta$$

$$\theta \approx \tau_0 = \rho \cdot U_\infty^2 \cdot \frac{d}{dx}\left[\frac{4-\pi}{2\pi}\right]\delta$$

Candidates must not write on this margin

$$\tau_0 = \frac{4-\pi}{2\pi} \cdot \rho \cdot U_\infty^2 \cdot \frac{d\delta}{dx} \quad \text{———} \quad (1)$$

But $\tau_0 = \mu \cdot \frac{du}{dy}$ [Newton's law of Viscosity]

$$= \mu \cdot \frac{d}{dy}_{y=0} \left[U_\infty \cdot \sin \frac{\pi}{2} \cdot \frac{y}{\delta} \right]$$

$$\tau_0 = \frac{\mu \cdot U_\infty \cdot \pi}{2\delta} \quad \text{———} \quad (2)$$

Equating equations (1) & (2)

$$\frac{4-\pi}{\cancel{2}\pi} \cdot \rho \cdot U_\infty^2 \cdot \frac{d\delta}{dx} = \frac{\mu \cdot U_\infty \cdot \pi}{\cancel{2}\delta}$$

$$\delta \cdot d\delta = \frac{\pi^2}{4-\pi} \cdot \frac{\mu}{\rho \cdot U_\infty^{\cancel{2}}} \cdot \cancel{U_\infty} \cdot dx$$

$$\delta \cdot d\delta = 11.5 \cdot \frac{\mu}{\rho \cdot U_\infty} \, dx.$$

$$\frac{\delta^2}{2} = 11.5 \frac{\mu}{\rho U_\infty} \cdot x + C$$

Boundary condition

At $x = 0$: $\delta = 0$ $\Rightarrow$ $C = 0$

$$\delta = \frac{4.796\, x}{\sqrt{Re_x}}$$

Candidates must not write on this margin

$\therefore$ Drag force $F_D = \int_0^L \tau_0 \cdot B \cdot dx$

$$= \int_0^L \frac{\mu \cdot U_\infty \cdot \pi}{2\delta} \cdot B \cdot dx$$

$$= \int_0^L \frac{\mu \cdot U_\infty \cdot \pi}{2 \cdot \frac{4.796x}{\sqrt{Re_x}}} \cdot B \cdot dx$$

$$= \int_0^L 0.327 \cdot \mu \cdot U_\infty \cdot B \cdot \sqrt{\frac{\rho \cdot U_\infty}{\mu}} \cdot \frac{1}{\sqrt{x}}\, dx$$

$$= 0.655 \cdot \mu \cdot U_\infty \cdot B \cdot \sqrt{\frac{\rho \cdot U_\infty}{\mu}} \cdot \sqrt{L}$$

$$= 0.655\, \mu \cdot U_\infty \cdot B \cdot \sqrt{Re_L}$$

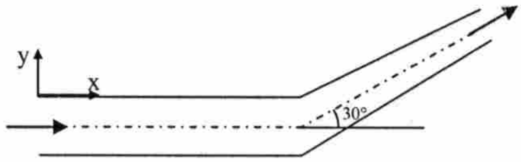
$$\therefore C_D = \frac{F_D}{\frac{1}{2} \cdot \rho \cdot A\, U_\infty^2} = \frac{F_D}{\frac{1}{2} \cdot \rho \cdot B \cdot L \cdot U_\infty^2}$$

$$= \frac{0.655 \times \mu \cdot \cancel{U_\infty} \cdot \cancel{B} \cdot \sqrt{Re_L}}{\frac{1}{2} \times \rho \cdot \cancel{B} \cdot L \cdot U_\infty^{\cancel{2}}}$$

$$= \frac{2 \times 0.655 \cdot \sqrt{Re_L}}{Re_L} \quad \left(\because Re_L = \frac{\rho \cdot U_\infty \cdot L}{\mu}\right)$$

$$\boxed{C_D = \frac{1.31}{\sqrt{Re_L}}}$$ Hence proved.

07(a). A reducing elbow is used to deflect water flow at a rate of 14 kg/s in a horizontal pipe upward 30° while accelerating it as shown in the figure. The elbow discharges water into the atmosphere. The cross sectional area of the elbow is 113 cm^2 at the inlet and 7 cm^2 at the outlet. The elevation difference between the centres of the outlet and inlet is 30 cm. The weight of the elbow and the water in it is considered to be negligible.

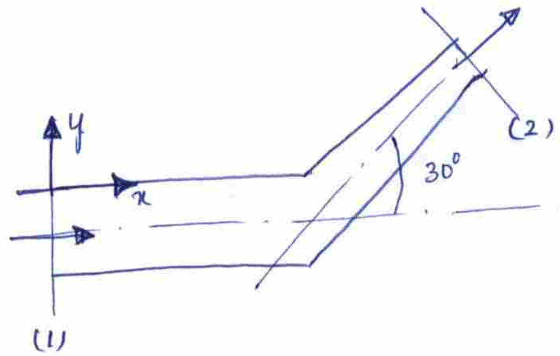


Determine:

(i) The gauge pressure at the centre of the inlet of the elbow.

(ii) The anchoring force needed to hold the elbow in place. **(7 + 8 = 15 M)**

$$\dot{m} = \rho \cdot A_1 V_1$$



$$\Rightarrow V_1 = \frac{\dot{m}}{\rho \cdot A_1}$$

$$= \frac{14}{10^3 \times (0.0113 \, m^2)}$$

$$= 1.24 \, m/s$$

Continuity Equation : $A_1 V_1 = A_2 V_2$

$$V_2 = \frac{113}{7} \times 1.24 = 20 \, m/s$$

(i) Bernoulli Equation :

$$\frac{P_{1g}}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_{2g}}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$P_{1g} = \rho g [z_2 - z_1] + \frac{\rho}{2} [V_2^2 - V_1^2]$$

$$= 10^3 \times 9.81 \times 0.3 + \frac{10^3}{2} [20^2 - 12.4^2]$$

$$= 202.2 \, kPa$$

(ii) Momentum equation :

x - direction :

$$P_{1g} \cdot A_1 + F_x - P_{2g} A_2 \cos\theta = \dot{m}[V_2 \cos\theta - V_1]$$

$$F_x = \dot{m}[V_2 \cos\theta - V_1] - P_{1g} \cdot A_1$$

$$= 14[20 \cos 30 - 1.24] - [202.2 \times 10^3 \times 0.0113]$$

$$= 225.13 - 2284.86$$

$$F_x = -2059.73 \text{ N.}$$

y - direction :

$$F_y - P_{2g} \cdot A_2 \sin\theta = \dot{m}[V_2 \sin\theta - 0]$$

$$F_y = \dot{m} \; V_2 \sin\theta \qquad [\because P_{2g} = 0]$$

$$= 14[20 \times \sin 30^\circ]$$

$$F_y = 140 \text{ N}$$

Resultant Force

$$F_R = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(-2059.73)^2 + 140^2}$$

$$\boxed{F_R = 2064.5 \text{ N}}$$

Candidates must not write on this margin

07(b). The velocity distribution in laminar pipe flow is given by

$$V(r) = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

where, V_{max} = Centre line velocity,

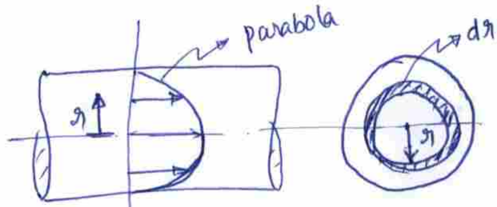
V = Velocity at radius 'r' from central axis

R = Radius of pipe

Prove that the kinetic energy factor is equal to 2 while the momentum correction factor is equal to $\frac{4}{3}$. **(15 M)**

Candidates must not write on this margin

Consider an elementry ring at radius r and thickness dr



$$Q = A \cdot \bar{V} = \int V(r) \cdot dA = \int_0^R V(r) \cdot 2\pi r \cdot dr$$

$$\pi R^2 \cdot \bar{V} = \int_0^R V_{max}\left(1 - \frac{r^2}{R^2}\right) 2\pi r\, dr = V_{max} \cdot 2\pi \cdot \frac{R^2}{4}$$

$$\bar{V} = \frac{V_{max}}{2} \qquad \text{where } \bar{V} = \text{Average velocity.}$$

(i) Kinetic energy factor (α) = $\frac{\text{Actual K.E}}{\text{K.E based on Avg. velocity.}}$

$$\alpha = \frac{1}{A}\int_0^R \left[\frac{V_{actual}}{V_{avg}}\right]^3 \cdot dA$$

$$= \frac{1}{A}\int_0^R \left[V_{max}\left(1 - \frac{r^2}{R^2}\right) \Big/ \frac{V_{max}}{2}\right]^3 \cdot 2\pi r \cdot dr$$

$$= \frac{1}{\pi \cdot R^2} \cdot 16 \int_0^R \left[1 - \frac{r^2}{R^2}\right]^3 \cdot r \cdot dr \cdot \pi$$

$$= \frac{16\pi}{\pi R^2} \cdot \int_0^R \left[1 - \frac{3r^2}{R^2} + 3\cdot\frac{r^4}{R^4} - \frac{r^6}{R^6}\right] r \cdot dr$$

$$\alpha = \frac{16}{R^2}\left[\frac{r^2}{2} - \frac{3r^4}{4R^2} + \frac{3r^6}{6R^4} - \frac{r^8}{8R^6}\right]_0^R$$

$$= \frac{16}{R^2} \times 0.125 \cdot R^2$$

$\boxed{\alpha = 2}$ Hence proved

(ii) Momentum Correction factor (β) = $\dfrac{\text{Actual Momentum}}{\text{Momentum Calculated by Avg. Velocity.}}$

$$\beta = \frac{1}{A}\int_0^R \left[\frac{V_{actual}}{\bar{V}}\right]^2 \cdot dA$$

$$= \frac{1}{\pi R^2}\int_0^R \left[V_{max}\left[1 - \frac{r^2}{R^2}\right] \Big/ \frac{V_{max}}{2}\right]^2 \cdot 2\pi r \cdot dr$$

$$= \frac{8}{R^2}\int_0^R \left[1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4}\right] \cdot r \cdot dr$$

$$= \frac{8}{R^2}\left[\frac{r^2}{2} - \frac{r^4}{2R^2} + \frac{r^6}{6R^4}\right]_0^R$$

$$= \frac{8}{R^2} \cdot \frac{1}{6} R^2$$

$\boxed{\beta = \frac{4}{3}}$ Hence proved.

Candidates must not write on this margin

07(c). Derive an expression for the relation between larger diameter (D_2) and smaller diameter (D_1) of a pipe line at sudden expansion section for maximum pressure rise. Ignore major loss.

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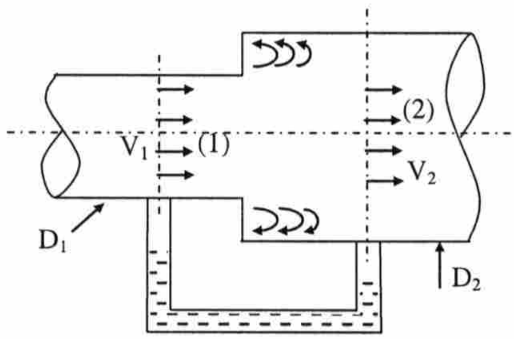


(15 M)

Applying Energy equation between (1) & (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{L\,expansion}$$

But $Z_1 = Z_2$

$$h_{L\,expansion} = \frac{(V_1 - V_2)^2}{2g}$$

$$\therefore \quad P_2 - P_1 = \Delta P = \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{(V_1 - V_2)^2}{2g}\right] \cdot \rho \cdot g$$

$$= \rho \cdot g \cdot \frac{V_1^2}{2g}\left[1 - \left(\frac{V_2}{V_1}\right)^2 - \left[1 - \frac{V_2}{V_1}\right]^2\right]$$

Continuity equation between (1) & (2)

$$Q = A_1 V_1 = A_2 V_2$$

$$\frac{V_2}{V_1} = \left(\frac{D_1}{D_2}\right)^2 = k^2 \quad \left[\text{let } \frac{D_1}{D_2} = k\right]$$

$$\Delta P = \frac{\rho . V_1^2}{2}\left[1 - K^4 - \left[1-K^2\right]^2\right]$$

$$= \frac{\rho . V_1^2}{2}\left[2K^2 - 2K^4\right]$$

$$= \rho . V_1^2 \left[K^2 - K^4\right]$$

For Maximum pressure rise across Suddin expansion

$$\frac{d}{dK}(\Delta P) = 0$$

$$2K - 4K^3 = 0$$

$$K\left[1 - 2K^2\right] = 0$$

$$1 - 2K^2 = 0 \qquad \left[\because K \neq 0\right]$$

$$K^2 = \frac{1}{2}$$

$$K = \frac{1}{\sqrt{2}}$$

$$\frac{D_1}{D_2} = \frac{1}{\sqrt{2}}$$

$$\boxed{D_2 = \sqrt{2}\, D_1}$$

Candidates must not write on this margin

07(d). A two dimensional flow is described in the Lagrangian system as:

$$x = x_o e^{-kt} + y_o\left(1 - e^{-2kt}\right)$$

$$y = y_o e^{kt}$$

Find (i) the equation of pathline of the particle and

(ii) the velocity components in Eulerian system. **(7 + 8 = 15 M)**

Candidates must not write on this margin

(i) pathline of the particle is found by eliminating "t" from equations describing motion, as follows.

$$e^{kt} = \frac{y}{y_0}$$

Hence, $x = x_0 \frac{y_0}{y} + y_0\left[1 - \left(\frac{y_0}{y}\right)^2\right]$

$$x = \frac{x_0 \cdot y_0}{y} + y_0 - \frac{y_0^3}{y^2}$$

$$xy^2 = x_0 \cdot y_0 \cdot y + y_0 y^2 - y_0^3$$

$$(x - y_0)y^2 - x_0 \cdot y_0 \cdot y + y_0^3 = 0$$

This is the required equation of pathline.

(ii) The x- component of velocity,

$$u = \frac{dx}{dt} = \frac{d}{dt}\left[x_0 e^{-kt} + y_0\left[1 - e^{-2kt}\right]\right]$$

$$= -k \cdot x_0 \cdot e^{-kt} + 2ky_0 \cdot e^{-2kt}$$

$$= -k\left[x - y_0\left[1 - e^{-2kt}\right]\right] + 2ky_0 e^{-2kt}$$

$$u = -kx + ky_0 \left[1 + e^{-2kt}\right]$$

$$\boxed{u = -kx + ky \left[e^{-kt} + e^{-3kt}\right]}$$

The y-component of velocity

$$V = \frac{dy}{dt} = \frac{d}{dt}\left[y_0 e^{kt}\right]$$

$$= y_0 \cdot k \cdot e^{kt}$$

$$\boxed{V = k \cdot y}$$

Candidates must not write on this margin

08(a). A horizontal pipe of 100 mm diameter carries water at the rate of 3000 *lpm*. The relation between average height of the protrusions on the pipe surface, k, friction factor (f) and radius of pipe (R) is given as : $\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{R}{k}\right) + 1.74$

Candidates must not write on this margin

Find the following for k = 0.15 mm :

(i). Friction factor

(ii). Wall shear stress of pipe

(iii). Centreline velocity

(iv). State type of pipe surface i.e. smooth or rough.

(v). State type of hydro dynamically surface. **(5 + 10 + 3 + 4 + 3 = 25 M)**

$d = 100mm = 0.1\ m$

$Q = \frac{3000}{1000 \times 60} = 0.05\ m^3/s$

$k = 0.15\ mm$

(i) friction factor(f):

$$\frac{1}{\sqrt{f}} = 2\log_{10}\frac{R}{k} + 1.74$$

$$= 2\log_{10}\frac{0.1/2}{0.15\times10^{-3}} + 1.74$$

$$= 2\times 2.523 + 1.74$$

$$\therefore \boxed{f = 0.0217}$$

(ii) Wall Shear Stress (τ_{wall}):

Mean or Average velocity $V = \frac{Q}{A}$

$$= \frac{0.05}{\frac{\pi}{4}\times 0.1^2} = 6.37\ m/s$$

Shear velocity $V^* = V.\sqrt{\frac{f}{8}}$

$= 6.37 \times \sqrt{\frac{0.0217}{8}}$

$= 0.332 \ m/s$

$V^* = \sqrt{\frac{\tau_{wall}}{\rho}}$

$\tau_{wall} = \rho . V^{*2}$

$= 1000 \times 0.332^2$

$\boxed{\tau_{wall} = 110 \ N/m^2}$

(iii) Centre line Velocity $[V_{CL} = V_{max}]$

$V_{max} = V . [1 + 1.43\sqrt{f}]$

$= 6.37 \ [1 + 1.43 . \sqrt{0.0217}]$

$\boxed{V_{max} = 7.7 \ m/s}$

(iv) Laminar Sub Layer Thickness

$\delta' = \frac{11.6 \ \nu}{V^*} = \frac{11.6 \times 1 \times 10^{-6}}{0.332}$

$= 3.5 \times 10^{-5} \ m$

$= 0.035 \ mm$

Candidates must not write on this margin

Candidates must not write on this margin

$k = 0.15m$ & $\delta' = 0.035$ mm

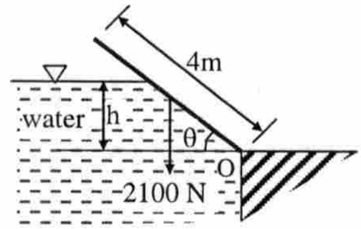
$k > \delta'$. Hence pipe Surface is rough.

(V) $\frac{k}{\delta'} = \frac{0.15}{0.035} = 4.286$

$0.25 < \frac{k}{\delta'} < 6.0$

pipe boundary Surface is hydro-dynamically transition.

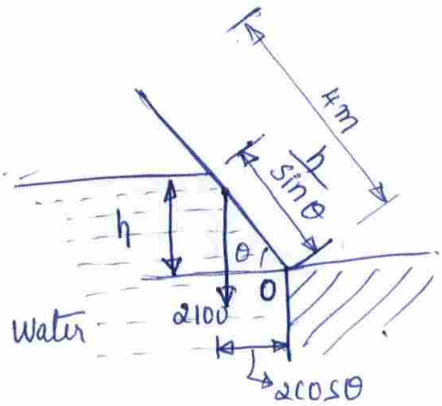
08(b). The plane gate in figure, weighs 2.1 kN/m normal to the paper, and its centre of gravity is 2 m from the hinge at O. Consider a unit width of gate, as shown in figure. (Take, $g = 10 \text{ m/s}^2$)



(i) Find h as a function of θ for equilibrium of the gate.

(ii) Is the gate in stable equilibrium for any values of θ ? **(7 + 8 = 15 M)**

Consider a unit width of gate, as shown in figure below.



(i) $F = \gamma \cdot \bar{h} \cdot A$

$$F_x = 10 \times 1000 \times \frac{h}{2} \times \frac{h}{\sin\theta}$$

$$= 5000h^2 / \sin\theta$$

$$\Sigma M_0 = 0$$

$$\frac{5000\,h^2}{\sin\theta} \cdot \frac{h/\sin\theta}{3} - 2100 \cdot \frac{4}{2} \cos\theta = 0$$

$$h^3 = 2.52 \sin^2\theta \cdot \cos\theta$$

$$h = 1.361 \left[\sin^2\theta \cdot \cos\theta\right]^{1/3}$$

(ii) From part (i) $\Sigma M_0 = \dfrac{1667 h^3}{\sin^2\theta} - 4200 \cos\theta$

$$\frac{dM}{d\theta} = -3334\, h^3 \cdot \sin^{-3}\theta \cdot \cos\theta + 4200 \sin\theta$$

Substituting $h = 1.361 \left[\sin^2\theta \cdot \cos\theta\right]^{1/3}$

$$\frac{dm}{d\theta} = -3334 \times 1.361^3 \cdot \frac{\cos^2\theta}{\sin\theta} + 4200 \sin\theta$$

$$= -8405 \frac{\cos^2\theta}{\sin\theta} + 4200 \sin\theta$$

for stability $\frac{dm}{d\theta} < 0$

$$-8405 \times \frac{\cos^2\theta}{\sin\theta} + 4200 \sin\theta < 0$$

This occurs for $\theta \leq 54.74°$ [upper limit]

for the lower limit [when water spills over the top of the gate], $h = 4\sin\theta$

$$\Sigma M_0 = \frac{1667 h^3}{\sin^2\theta} - 4200 \cos\theta$$

Substituting, $h = 4\sin\theta$,

$$\Sigma M_0 = \frac{1667 \cdot (4\sin\theta)^3}{\sin^2\theta} - 4200 \cos\theta$$

$$= 106688 \sin\theta - 4200 \cos\theta$$

In this case $\Sigma M_0 = 0$, $\tan\theta = \frac{4200}{106688}$

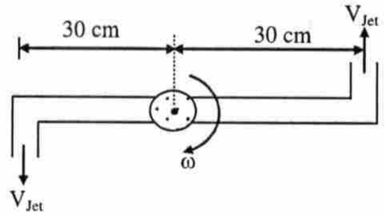
$$\theta = 2.25^0$$

For stable equilibrium,

θ must be between $2.25°$ and $54.74°$

Candidates must not write on this margin

08(c). Figure shows a lawn sprinkler with two jets, each located at 30 cm from centre. (diameter of each jet is 1 cm)



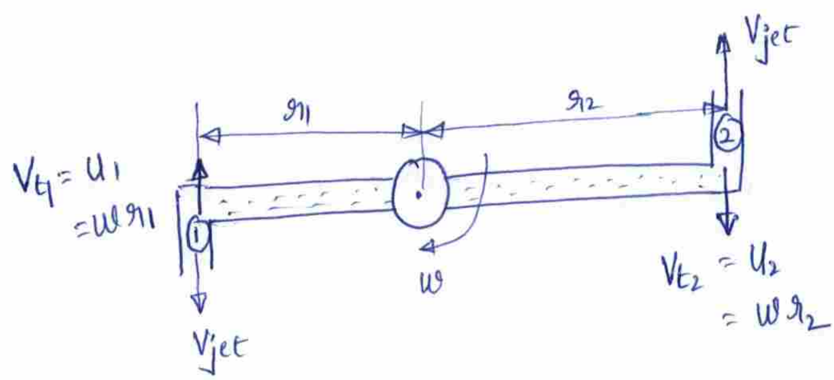
(i) Assuming no friction, find the speed of rotation for a discharge of 2.5 lit/s.

(ii) Find torque required to hold the sprinkler stationary.

(iii) What will be its steady rotation rate if it has retarding friction torque of 1.5 N-m?

(10 + 5 + 5 = 20 M)

Candidates must not write on this margin



Moment of Momentum Equation:

Retarding torque on Rotating System by fluid = Rate of change of Angular momentum of fluid.

$$T = \Sigma(\rho Q_{in} V_{in} r_{in}) - \Sigma(\rho Q_{out} \cdot V_{out} \cdot r_{out})$$

But $u_1 = u_2 = u = r.\omega \quad [\because r_1 = r_2 = r]$

$$Q_1 = Q_2 = \frac{Q}{2}$$

$$V_{r_1} = V_{r_2} = V_r = \frac{Q}{2A}$$

where A is Jet area $= \frac{\pi}{4} d^2$

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Substituting in equation (1)

$$T = 0 - \left[\rho \frac{Q}{2} \cdot (U - V_r) \cdot r + \rho \cdot \frac{Q}{2} \cdot (U - V_r) \cdot r \right]$$

$$= - \left[\rho \cdot Q \cdot r \, (U - V_r) \right] \quad \text{——— (2)}$$

(i) No friction $\Rightarrow T = 0$; $\omega = ?$

$$0 = - \left[\rho \cdot Q \cdot r \cdot \left(r\omega - \frac{Q}{2A} \right) \right]$$

$$\omega = \frac{Q}{2 \times \frac{\pi}{4} d^2 \cdot r} = \frac{2.5 \times 10^{-3}}{2 \times \frac{\pi}{4} \times 0.01^2 \times 0.3}$$

$$\boxed{\omega = 53.05 \text{ rad/s}}$$

(ii) For $\omega = 0$; $U = 0$ $T = ?$

From equation (2)

$$T = - \rho \cdot Q \cdot r \left(\frac{-Q}{2A} \right)$$

$$= \frac{\rho \cdot Q^2 \cdot r}{2A}$$

$$= \frac{10^3 \times \left[2.5 \times 10^{-3} \right]^2 \times 0.3}{2 \times \frac{\pi}{4} \times 0.01^2}$$

$$\boxed{T = 11.937 \text{ N} \cdot \text{m}}$$

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(iii) For $T = 1.5\ N\cdot m$; $\omega = ?$

from equation (2)

$$1.5 = -\rho \cdot Q \cdot r \left[r\omega - \frac{Q}{2A} \right]$$

$$= -\left[10^3 \times 2.5 \times 10^{-3} \times 0.3 \left[0.3\,\omega - \frac{2.5 \times 10^{-3}}{2 \times \frac{\pi}{4} \times 0.01^2} \right] \right]$$

$$\boxed{\omega = 46.39\ rad/s}$$