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ESE – 2019 MAINS
OFFLINE TEST SERIES

ELECTRICAL ENGINEERING

TEST – 2
SOLUTIONS

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1.(a)

Sol: The closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

Given that, $G(s) = \frac{10}{s(s+2)}$

$$\therefore \frac{C(s)}{R(s)} = \frac{\frac{10}{s(s+2)}}{1+\frac{10}{s(s+2)}} = \frac{10}{s(s+2)+10} = \frac{10}{s^2+2s+10} \quad \ldots\ldots\ldots\ldots. (1)$$

The values of damping ratio ξ and natural frequency of oscillation ω_n are obtained by comparing the system transfer with standard form of second order transfer function.

Standard form of second order system $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$ ....(2)

On comparing equation (1) & (2) we get,

$$\omega_n^2 = 10$$

$$\therefore \omega_n = \sqrt{10} = 3.16\,\text{rad/sec}$$

$$2\zeta\omega_n = 2$$

$$\therefore \zeta = \frac{2}{2\omega_n} = \frac{1}{3.162} = 0.316$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = \tan^{-1}\frac{\sqrt{1-0.316^2}}{0.316} = 1.249\,\text{rad}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 3.16\sqrt{1-0.316^2} = 3\,\text{rad/sec}$$

$$\text{Rise Time}, t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-1.249}{3} = 0.63\,\text{sec}$$

Percentage overshoot,

$$\%M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}\times 100 = e^{\frac{-0.316\pi}{\sqrt{1-0.316^2}}}\times 100 = 0.3512\times 100 = 35.12\%$$

$$\text{Peakovershoot} = \frac{35.12}{100} \times 12\,\text{units} = 4.2144\,\text{units}$$

$$\text{Peak time}, t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3} = 1.047\,\text{sec}$$

$$\text{Time constant, } T = \frac{1}{\xi\omega_n} = \frac{1}{0.316 \times 3.162} = 1\text{sec}$$

$\therefore$ For 5% error, settling time, $t_s = 3T = 3\text{sec}$

For 2% error, settling time, $t_s = 4T = 4\text{sec}$

Result

Rise time, $t_r = 0.63\text{sec}$

Percentage overshoot, $\%M_p = 35.12\%$

Peak overshoot $= 4.2144$ units, (for a input of 12 units)

Peak time, $t_p = 1.047$ sec

Settling time, $t_s = 3\text{sec}$ for 5% error

= 4sec for 2% error

1(b).

Sol: (i) $\int_{-\infty}^{\infty} (t^3 - \cos \pi t)\delta(t+1)dt$

From sifting property of impulse signal $\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$

$$\int_{-\infty}^{\infty} (t^3 - \cos \pi t)\delta(t+1)dt = (-1)^3 - \cos\pi(-1)$$

$= -1 + 1$

$= 0$

(ii) $\int_{-\infty}^{\infty}\left(e^{-\pi t}+\sin 10\pi t\right)\delta(2t+1)dt$

From time scaling property $\delta(at+b)=\frac{1}{|a|}\delta\left(t+\frac{b}{a}\right)$

From sifting property of impulse signal $\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$

$$\int_{-\infty}^{\infty}\left(e^{-\pi t}+\sin 10\pi t\right)\delta(2t+1)dt = \frac{1}{2}\int_{-\infty}^{\infty}\left(e^{-\pi t}+\sin 10\pi t\right)\delta\left(t+\frac{1}{2}\right)dt$$

$$=\frac{1}{2}\left[e^{-\pi\left(\frac{-1}{2}\right)}+\sin 10\pi\left(\frac{-1}{2}\right)\right]$$

$$=\frac{1}{2}\left[e^{\frac{\pi}{2}}-\sin 5\pi\right] = \frac{1}{2}\left[e^{\frac{\pi}{2}}\right]$$

$= 2.403$

1(c).

Sol: We will first find out from Routh's array, the range of K for stability. Transfer function of system, i.e.

$$\frac{C(s)}{R(s)}=\frac{K(s+1)}{s^4+2s^3+2s^2+(K+3)s+K}$$

The characteristic equation is given by $[s^4 + 2s^3 + 2s^2 + (K + 3)s + K] = 0$

Routh's array can be written as

s^3	1	2	K
s^4	2	K+3	0
s^2	$\frac{1-K}{2}$	K	
s^1	$\frac{(1-K)(K+3)-4K}{1-K}$	0	
s^0	K		

For stability, from s^2 row, we get $\frac{1-K}{2} > 0$ or $K < 1$

Therefore, the system will be unstable for K = 6.

Also from s^0 row, K > 0 for stability.

From s^1 row,

$$\frac{(1-K)(K+3)-4K}{1-K} > 0 \text{ for stability}$$

Therefore,

$$[(1-K)(K+3)-4K] > 0$$

or, $3 + K - 3K - K^2 - 4K > 0$

or, $3 - 6K - K^2 > 0$

or, $K^2 + 6K - 3 < 0$

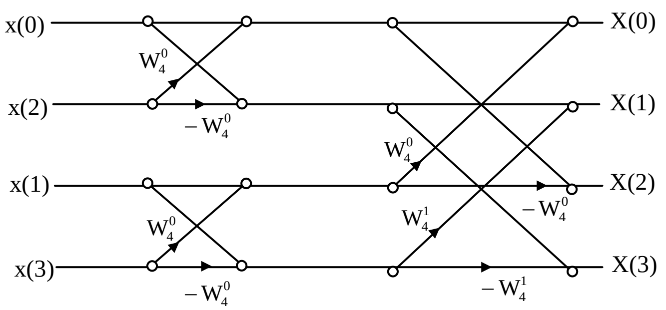
or, $(K - 0.464)(K + 6.464) < 0$

$\therefore$ K < 0.464 or K < – 6.464

K cannot be less than 0. Therefore, second alternative is ruled out. Range of K for stability is 0 < K < 0.464.

1(d).

Sol: (i) **4 - point DIT algorithm:-**

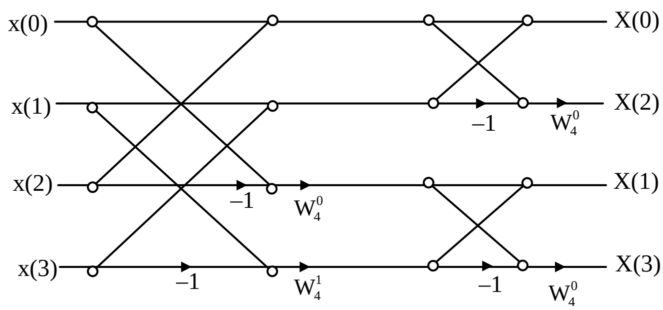


$$W_4^0 = 1, W_4^1 = -j$$

input	Stage 1 outputs	Stage 2 outputs
0	0 + 2 = 2	2 + 4 = 6
2	0 – 2 = –2	–2 + (–j) (–2) =–2 + 2j
1	1 + 3 = 4	2 – 4 = – 2
3	1 – 3 = – 2	– 2 – (–j) (–2) = – 2 – 2j

X(k)={6, –2 + 2j, –2, –2 – 2j}

(ii) **4 point DIF algorithm:**



inputs	Stage 1 outputs	Stage 2 outputs
0	0 + 2 = 2	2 + 4 = 6
1	1 + 3 = 4	2 – 4 = – 2
2	0 – 2 = – 2	– 2 + 2j
3	(1 – 3) (–j) = 2j	– 2 – 2j

X(k)={6, –2 + 2j, –2, –2 – 2j}

1(e).

Sol: $G(s)\,H(s) = \dfrac{1}{s(s+1)}$

Let the $G_c(s) = K_p + sk_d$

Hence, $G(s)\,H(s)\,G_c(s) = \dfrac{K_p + sk_d}{s(s+1)}$

At ω = 2 rad/sec, Phase margin = 40°, hence

$G(j2)\,H(j2)\,G_c(j2) = 1\angle(180° - 40°)$

$G(j2)\,H(j2)\,G_c(j2) = \dfrac{K_p + j2k_d}{j(1+j2)}$

Hence, $-140° = -90° - \tan^{-1}2 + \tan^{-1}\dfrac{2k_d}{k_p}$

or , $-140° = -90° - 63.434° + \tan^{-1}\dfrac{2k_d}{k_p}$

So, $\tan^{-1}\frac{2k_d}{k_p} = 13.434$

or $\frac{2k_d}{k_p} = 0.2388$ --------(1)

$|G(j2)\,H(j2)\,G_C(j2)| = \frac{(k_p + j2k_d)}{j2(1+j2)} = 1$

Therefore, $\frac{\sqrt{k_p^2 + (2k_d)^2}}{2\times 2.236} = 1$

Or, $\sqrt{k_p^2 + (2k_d)^2} = 4.472$

$K_p\sqrt{1+\left(\frac{2k_d}{K_p}\right)^2} = 4.472$ -------- (2)

From (1) and (2), we get

$K_p\sqrt{1+0.2388^2} = 4.472$

$K_p = 4.349$

Putting this value in (1) we get

$K_d = 0.519$

Controller transfer function

$G_c(s) = k_p + k_d s = 4.349 + 0.519s$

2(a)(i)

Sol: $\text{CLTF} = \frac{1}{s^2(s+a)(s+b)+1}$

Open loop transfer function $G(s) = \frac{\text{CLTF}}{1-\text{CLTF}}$

$G(s) = \frac{1}{s^2(s+a)(s+b)+1-1} \Rightarrow \frac{1}{s^2(s+a)(s+b)}$

G(s) has two poles at the origin, therefore type of the system is two.

Position error coefficient: $k_p = \lim_{s\to 0} G(s) = \lim_{s\to 0}\frac{1}{s^2(s+a)(s+b))} = \infty$

Velocity error coefficient: $k_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{1}{s(s+a)(s+b)} = \infty$

Acceleration error coefficient: $K_a = \lim_{s\to 0} s^2G(s) = \lim_{s\to 0} \frac{1}{(s+a)(s+b)} = \frac{1}{ab}$

For input 2u(t): $e_{ss} = \frac{A}{1+k_p} = \frac{2}{1+\infty} = 0$

For input $4t^2$ u(t): $e_{ss} = \frac{A}{k_a} = \frac{8}{1/ab} = 8ab$

For input tu(t): $e_{ss} = \frac{A}{K_V} = \frac{1}{\infty} = 0$

(ii) The overall Transfer function is given by $\frac{C(s)}{R(s)} = \frac{\frac{K}{s(sT+1)}}{\left[1+\frac{K}{s(sT+1)}.1\right]} = \frac{K/T}{\left(s^2+\frac{1}{T}s+\frac{K}{T}\right)}$

The characteristic equation is $s^2 + \frac{1}{T}s + \frac{K}{T} = 0$

$\therefore \quad \omega_n = \sqrt{K/T}$ and $2\zeta\omega_n = 1/T$

$\therefore \quad \zeta = \frac{1}{T}.\frac{1}{2\omega_n} = \frac{1}{T}.\frac{1}{2\sqrt{K/T}} = \frac{1}{2\sqrt{KT}}$

Let K_1 be the forward path gain when $M_{p1} = 60\%$ and the corresponding damping ratio be ζ_1.

Since, $M_{p1} = e^{-\frac{\zeta_1\pi}{\sqrt{1-\zeta_1^2}}} \times 100\%$

$\therefore\ 60 = e^{-\frac{\zeta_1\pi}{\sqrt{1-\zeta_1^2}}} \times 100$

or $\log_e(0.6) = -\frac{\zeta_1\pi}{\sqrt{1-\zeta_1^2}}\log_e(e)$

$\therefore \quad \zeta_1 = 0.158$

Let K_2 be the forward path gain when $M_{p2} = 20\%$ and the corresponding damping ratio be ζ_2.

Since $M_{p2} = e^{-\frac{\zeta_2\pi}{\sqrt{1-\zeta_2^2}}} \times 100\%$

$\therefore 20 = e^{-\frac{\zeta_2\pi}{\sqrt{1-\zeta_2^2}}} \times 100$

From the above relation the value of ζ_2 can be calculated as $\zeta_2 = 0.447$

Assuming time constant T to be constant

$$\zeta_1 = \frac{1}{2}.\frac{1}{\sqrt{K_1 T}} \quad \text{and} \quad \zeta_2 = \frac{1}{2}.\frac{1}{\sqrt{K_2 T}}$$

$$\frac{\zeta_1}{\zeta_2} = \frac{1}{2}.\frac{1}{\sqrt{K_1 T}} \times \frac{2\sqrt{K_2 T}}{1}$$

Hence, $\dfrac{K_2}{K_1} = \left(\dfrac{\zeta_1}{\zeta_2}\right)^2 = \left(\dfrac{0.158}{0.447}\right)^2 = \dfrac{1}{8}.$

2(b)(i).

Sol: The given differential equation is:

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

Taking Laplace transform on both sides, we get

$$[s^2Y(s) - sy(0) - y^1(0)] + 4\,[sY(s) - y(0)] + 3Y(s) = [sX(s) - x(0)] + 2X(s)$$

Neglecting the initial conditions, we have

$$s^2Y(s) + 4sY(s) + 3Y(s) = sX(s) + 2X(s)$$

i.e. $(s^2 + 4s + 3)\,Y(s) = (s + 2)\,X(s)$

$$\therefore\ Y(s) = \frac{s+2}{s^2+4s+3}X(s) = \frac{s+2}{(s+1)(s+3)}X(s)$$

Given, $x(t) = e^{-t}\,u(t)$

$$\therefore\ X(s) = \frac{1}{s+1}$$

$$\therefore\ Y(s) = \frac{s+2}{(s+1)(s+3)}\left(\frac{1}{s+1}\right) = \frac{s+2}{(s+1)^2(s+3)}$$

Taking partial fractions, we get

$$Y(s) = \frac{s+2}{(s+1)^2(s+3)} = \frac{A}{(s+1)^2} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$Y(s) = \frac{1/2}{(s+1)^2} + \frac{1/4}{s+1} - \frac{1/4}{s+3}$$

Taking inverse Laplace transform on both sides, we get the response

$$y(t) = \left(\frac{1}{2}te^{-t} + \frac{1}{4}e^{-t} - \frac{1}{4}e^{-3t}\right)u(t)$$

2(b)(ii)

Sol: Energy of the signal, $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= \left[\int_{-2}^{0} (t-2)^2 dt + \int_{0}^{2} (2-t)^2 dt\right]$$

$$= \int_{-2}^{0} (t^2 - 4t + 4)dt + \int_{0}^{2} (4 + t^2 - 4t)dt$$

$$= \left[\frac{t^3}{3} - \frac{4t^2}{2} + 4t\right]_{-2}^{0} + \left[4t + \frac{t^3}{3} - \frac{4t^2}{2}\right]_{0}^{2}$$

$$= \frac{64}{3} \text{ joules}$$

Power of the signal, $P = \underset{T\to\infty}{\text{Lt}} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T\to\infty}{\text{Lt}} \frac{1}{2T}\left[\int_{-2}^{0} (t-2)^2 dt + \int_{0}^{2} (2-t)^2 dt\right]$$

$$= \underset{T\to\infty}{\text{Lt}} \frac{1}{2T}\left[\frac{64}{3}\right] = 0$$

Since energy is finite and power is zero, it is an energy signal.

The given signal is a non-periodic finite duration signal. So it has finite energy and zero average power. So it is an energy signal.

2(c)(i)

Sol: Given that,

$M_p = 30\% = 0.3$ and $t_s = 0.4\,\text{sec}$

We know that, $\%M_P = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$

$$0.3 = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$-1.2 = -\frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\therefore \zeta = 0.35$$

$$\phi = \cos^{-1}(\zeta) = \cos^{-1}(0.35) = 69^0$$

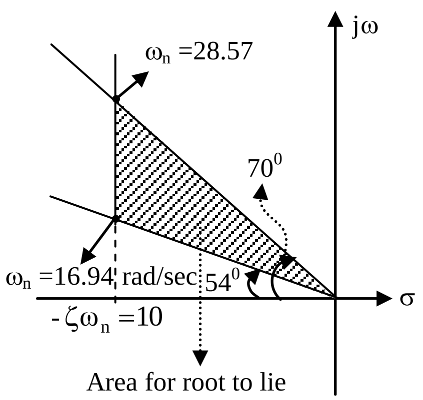
If $M_p = 10\% = 0.1,\ \zeta = 0.59$

$$\phi = \cos^{-1}(\zeta) = \cos^{-1}(0.59) = 53.8^0$$

Settling time t_s (For 2% tolerance band) $= \frac{4}{\zeta\omega_n}$

$\zeta = 0.35$: $\omega_n = \frac{4}{0.35\times0.4} = 28.57$ rad/sec

$\zeta = 0.59$: $\omega_n = \frac{4}{0.59\times0.4} = 16.94$ rad/sec



2(c)(ii).

Sol: CE = $1 + \frac{K(s+1)}{s^3 + as^2 + 2s + b} = 0$

$s^3 + as^2 + (K+2)s + (K+b) = 0$

s^3	1	K + 2
s^2	a	K+ b
s^1	$\frac{a(K+2)-(K+b)}{a}$	0
s^0	K + b	

Given,

$\omega_n = 3$

$\Rightarrow s^1$ row = 0

s^2 row is A.E

$a(K+2) - (K+b) = 0$

$a = \dfrac{K+b}{K+2}$

A.E = $as^2 + K + b = 0$

$= \left(\dfrac{K+b}{K+2}\right)s^2 + K + b = 0$

$(K+b)\left(\dfrac{s^2}{K+2}+1\right) = 0$

$s^2 + K + 2 = 0$

$s = \pm j\sqrt{(K+2)}$

$\omega_n = \sqrt{K+2} = 3$

K = 7

$a = \dfrac{K+b}{K+2} = \dfrac{7+b}{9}$

$\therefore 9a - b = 7$

3(a)(i).

Sol: $X(z) = \dfrac{z^3}{(z-2)(z-1)^2}$

$$\frac{X(z)}{z} = \frac{z^2}{(z-2)(z-1)^2} = \frac{A}{z-2} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

$$\frac{X(z)}{z} = \frac{4}{z-2} - \frac{3}{z-1} - \frac{1}{(z-1)^2}$$

$$X(z) = 4\frac{z}{z-2} - 3\frac{z}{z-1} - \frac{z}{(z-1)^2}$$

Apply IZT.

Then, $x(n) = 4(2)^n u(n) - 3u(n) - n\,u(n)$

3(a)(ii)

Sol: Statement of duality property is

If $x(t) \xleftrightarrow{FT} X(\omega)$

then $X(t) \xleftrightarrow{FT} 2\pi x(-\omega)$

Proof: By definition,

Inverse Fourier transform is given by $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$

$\therefore 2\pi x(t) = \int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$

Replace t with '-t'

or $2\pi x(-t) = \int_{-\infty}^{\infty} X(\omega)e^{-j\omega t}d\omega$

Interchanging t and ω, we have

$$2\pi x(-\omega) = \int_{-\infty}^{\infty} X(t)e^{-j\omega t}dt = FT[X(t)]$$

$\therefore$ F[X(t)] =2π x(–ω)

i.e, $X(t) \xleftrightarrow{FT} 2\pi x(-\omega)$

For even functions, x(–ω)=x(ω)

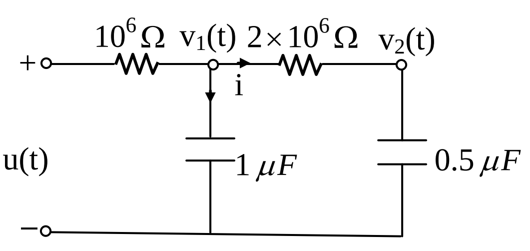
3(b)(i).

Sol: Number of state variables=Order of the system

Order of the system=Number of energy storage elements=2

$V_1(t), V_2(t)$ are the state variables

By using Nodal Analysis



$$10^{-6}\frac{dv_1(t)}{dt} = -i + \frac{u(t)-v_1(t)}{10^6}$$

$$\frac{dv_1(t)}{dt} = -i\times10^6 + u(t) - v_1(t)$$

$$= -\left(\frac{v_1(t)-v_2(t)}{2\times10^6}\right)\times10^6 + u(t) - v_1(t)$$

$$= -0.5v_1(t) + 0.5v_2(t) + u(t) - v_1(t) \text{-----}(1)$$

$$\frac{V_2(t) - V_1(t)}{2\times10^6} + 0.5\times10^{-6}\frac{dV_2(t)}{dt} = 0$$

$$\frac{10^{-6}}{2}\frac{dV_2(t)}{dt} = \frac{v_1(t) - v_2(t)}{2\times10^6}$$

$$\dot{V}_2 = \frac{dv_2(t)}{dt} = v_1(t) - v_2(t)$$

$$\therefore \begin{bmatrix}\dot{V}_1\\ \dot{V}_2\end{bmatrix} = \begin{bmatrix}-1.5 & 0.5\\ 1 & -1\end{bmatrix}\begin{bmatrix}V_1(t)\\ V_2(t)\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}u(t)$$

3(b)(ii).

Sol: Open loop transfer function

$$G(s)\,H(s) = \frac{K(s-3)(s-5)}{(s+1)(s+2)}$$

To find break in (or) break away points

1 + K G(s) H(s) = 0

(s+1)(s+2) + K (s–3)(s–5) = 0

$$K = \frac{-(s+1)(s+2)}{(s-3)(s-5)}$$

$$\frac{dK}{ds} = 0$$

$$= \frac{(s^2-8s+15)(2s+3)-(s^2+3s+2)(2s-8)}{(s^2-8s+15)^2}$$

$\Rightarrow (s^2-8s+15)(2s+3)-(s^2+3s+12)(2s-8) = 0$

$\Rightarrow (2s^3 - 16s^2 + 30s + 3s^2 - 24s + 45) -$

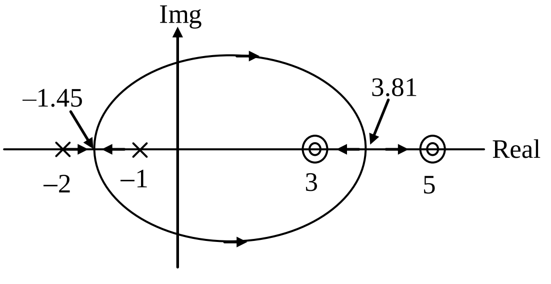
$(2s^3 + 6s^2 + 4s) - 8s^2 - 24s - 16$

$\Rightarrow 11s^2 - 26s - 61 = 0$

By solving,

s = 3.81 and –1.45

break points are 3.81 and –1.45



Root locus

s = –1.45 lies on root locus and it is break away point.
s = 3.81 lies on root locus and as it is between two zeros and it is break in point.

As root locus start from open loop pole

(k = 0) and it will end at it's open loop zero (k = ∞).

As on the root locus between two open loop poles, the roots moves towards each other as the gain factor K is increased till they are coincident. At the coincident point the value of K is maximum as far as the portion of the root locus between two open loop poles is concerned. At the break away point the value of gain K is maximum and at break-in point the value gain K is minimum.

3(c)(i).

Sol:

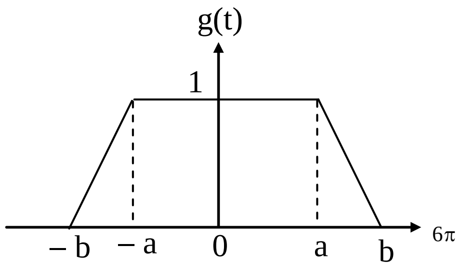
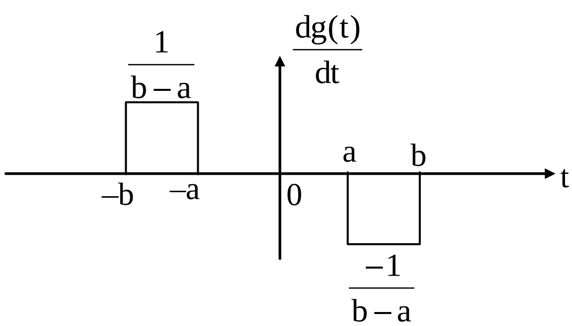
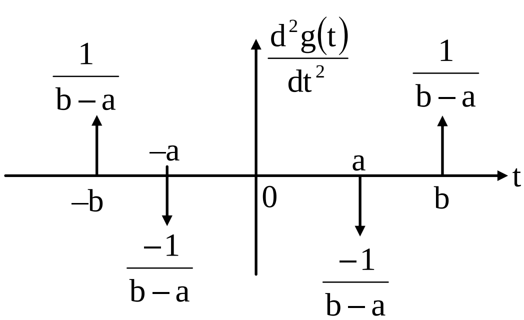


Fig.





$$g(t) = \frac{1}{b-a}[r(t+b) - r(t+a) - r(t-a) + r(t-b)]$$

$$\frac{d}{dt}g(t) = \frac{1}{b-a}[u(t+b) - u(t+a) - u(t-a) + u(t-b)]$$

$$\frac{d^2}{dt^2}g(t) = \frac{1}{b-a}[\delta(t+b) - \delta(t+a) - \delta(t-a) + \delta(t-b)]$$

Using time shifting and time differentiation properties

Time shifting property of Fourier transform is $x(t-t_0) \leftrightarrow e^{-j\omega t_0}X(\omega)$

Time differentiation property of Fourier transform is $\frac{dx(t)}{dt} \leftrightarrow j\omega X(\omega)$

$\delta(t-t_0) \leftrightarrow e^{-j\omega t_0}$

$$(j\omega)^2 G(\omega) = \frac{1}{b-a}\left[e^{jb\omega} - e^{j\omega a} - e^{-j\omega a} + e^{-jb\omega}\right] \Rightarrow G(\omega) = \frac{-2}{(b-a)\omega^2}[\cos(b\omega) - \cos(a\omega)]$$

3(c)(ii).

Sol: Given y(n) = x(n) + 2x(n – 1) –4x(n – 2) + x(n–3)

Apply z-transform on both sides.

$Y(z) = X(z) + 2z^{-1}X(z) - 4z^{-2}X(z) + z^{-3}X(z)$

$H(z) = \frac{Y(z)}{X(z)} = 1 + 2z^{-1} - 4z^{-2} + z^{-3}$ ---------(1)

We know that, $H(z) = \sum_{n=-\infty}^{\infty} h(n) z^{-n}$ ---------(2)

Compare (1) and (2) h(0) = 1, h(1) = 2, h(2) = –4, h(3) = 1

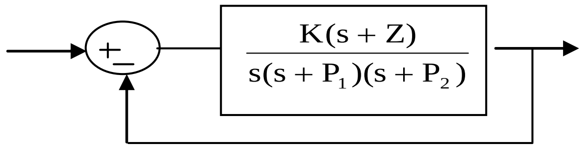
h(n) = {1, 2, –4, 1}.

4(a).

Sol: Root locus diagram:

Root loci diagram (RLD) is the graphical method of analyzing and designing a system

Root Locus Diagram (RLD) is a plot of loci of roots of the characteristic equation while gain 'K' is varied from 0 to ∞.



(ii) No. of root locus branches=2(P>Z) P = 2, Z = 1

No. of Asymptotes N = P – Z= 1

$$\text{Angle of Asymptotes}= \frac{(2l+1)180^0}{P-Z} \quad l= 0$$

$$= \frac{(2(0)+1)180^0}{1} = 180^0$$

Here, only one asymptote is present, therefore centroid is not required.

Break Point:

CE is $1+KG_1(s)H_1(s)=0$

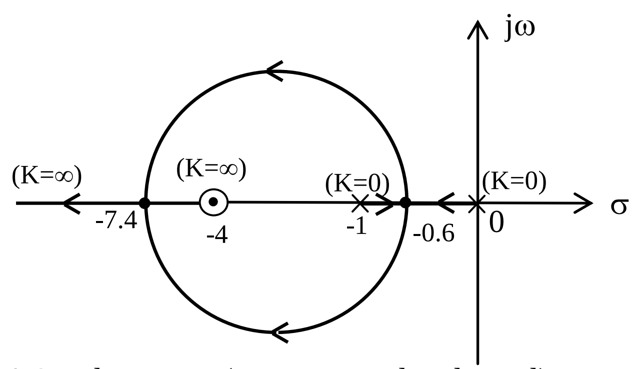
$$K = \frac{-1}{G_1(s)H_1(s)}$$

$$G_1(s)H_1(s) = \frac{s+4}{s(s+1)}$$

$$\frac{dK}{ds} = \frac{d}{ds}\left(\frac{-1}{G_1(s)H_1(s)}\right) = 0$$

$$\frac{d}{ds}\left(-\frac{s(s+1)}{s+4}\right) = 0$$

$$\frac{(s+4)(2s+1)-(s^2+s)(1)}{(s+4)^2} = 0$$



$s^2 + 8s + 4 = 0$ s = –0.6 and s = –7.4

The system is critically damped when s= –0.6 and s = –7.4 (roots are real and equal)

$$K = \frac{(0.6)(0.4)}{3.4} = 0.71 \text{ (at } s = -0.6)$$

$$K = \frac{(7.4)(6.4)}{3.4} = 14 \,(\text{at } s = -7.4)$$

For 0 < K< 0.71 , 14< K <∞ roots are real and distinct . Therefore the system is over damped

For K= 0.71 , K=14 , roots are real and equal .Therefore the system is critically damped.

For 0.71 <K< 14 , roots are complex and distinct . Therefore the system is under damped.

4(b)

Sol : Advantages of state space model

1. It can be applicable for multi input and multi output systems.
2. It can be applicable to linear and non linear, time variant and time-in variant systems.
3. By using state space model, we have to predict the internal states of the system
4. It can be applicable to both the stable and unstable systems.

State variable equations:

$$\dot{x}_1(t) = -x_1(t) + x_2(t) + U(t)$$

$$\dot{x}_2(t) = -2x_2(t) - U(t)$$

$$y(t) = x_1(t) + x_2(t)$$

These are in matrix form

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} U(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\dot{X}(t) = AX(t) + B.U(t)$$

Solution

$$X(t) = e^{At}.X(0) + \int_0^t e^{A(t-\tau)}.BU(\tau)d\tau$$

$e^{At} = \phi(t)$ is called as state transition matrix which is calculated in form

$$e^{At} = L^{-1}[SI - A]^{-1}$$

Where $A = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$

$$SI - A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} s+1 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{|sI - A|}\begin{bmatrix} s+2 & 1 \\ 0 & s+1 \end{bmatrix}$$

$$|sI - A| = (s+1)(s+2)$$

$$\therefore \varphi(s) = [sI - A]^{-1} = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{(s+1)(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+1} - \frac{1}{s+2} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$L^{-1}[\varphi(s)] = \varphi(t) = e^{At}$$

$$e^{At} = L^{-1}[sI - A]^{-1} = \begin{bmatrix} e^{-t} & e^{-t} - e^{-2t} \\ 0 & e^{-2t} \end{bmatrix}$$

This is state transition matrix

4(c)

Sol: (i) Nyquist stability criteria(NSC) :

It states that (–1, j0) critical point should be encircled (in ACW) as many number of times as the number of poles of G(s) H(s) in the right half of s – plane by the Nyquist plot, if the Nyquiest contour is defined in the clock wise direction

N = P –Z

Here,

N = Number of encirclements of (–1, j0)

critical point by the Nyquist plot

P = Number of right hand open loop poles

(or) No.of right half of s – plane poles of

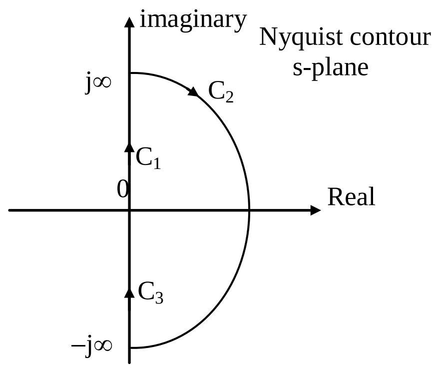
F(s) and F(s) = 1+G(s) H(s)

Z = Number of right half of s – plane poles of CLTF (or) number of right half of

s – plane zeros of F(s)

Hence, z = 0 ⇒ stable system. i.e. N = P is called the Nyquist stability criteria

(ii) Mapping of a Nyquist contour into G(s) H(s) plane is a Nyquist plot



→ Mapping of section C_1, i.e positive imaginary axis substitute s = jω, $0 \le \omega < \infty$

$$G(j\omega)\,H((j\omega) = \frac{K(j\omega+1)}{(j\omega+0.5)(j\omega-2)}$$

$$= K\sqrt{\frac{\omega^2+1}{(\omega^2+0.25)(\omega^2+4)}}$$

$$\angle \tan^{-1}\omega - \left[\tan^{-1}\frac{\omega}{0.5} + 180 - \tan^{-1}\frac{\omega}{2}\right]$$

$\omega = 0 \rightarrow K\angle{-180°}$

$\omega = \omega_{pc} = 0.707 \rightarrow 0.67\,K\,\angle{-180°}$

$\omega = \infty \rightarrow 0\angle{-90°}$

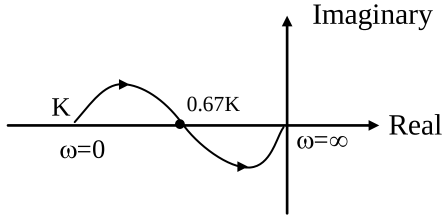
Point of intersection of the plot with respect to negative real axis

$\angle\, G(j\omega)\, H((j\omega) = -180°$ at $\omega = \omega_{pc}$

$$-180° + \tan^{-1}\omega_{pc} + \tan^{-1}\frac{\omega_{pc}}{2} - \tan^{-1}\frac{\omega_{pc}}{0.5} = -180°$$

$$\tan^{-1}\omega_{pc} + \tan^{-1}\frac{\omega_{pc}}{2} = \tan^{-1}\frac{\omega_{pc}}{0.5}$$

$$\Rightarrow \frac{\omega_{pc} + \frac{\omega_{pc}}{2}}{1 - \frac{\omega^2_{pc}}{2}} = \frac{\omega_{pc}}{0.5}$$



$$\Rightarrow 1.5 = 2 - \omega_{pc}^2$$

$$\Rightarrow \omega_{pc} = \sqrt{0.5} \Rightarrow \omega_{pc} = 0.707$$

$$|G(j\omega_{pc})\,H(j\omega_{pc})| = POI = \frac{K\sqrt{\omega_{pc}^2 + 1}}{(\omega_{pc}^2 + 0.25)(\omega_{pc}^2 + 4)}$$

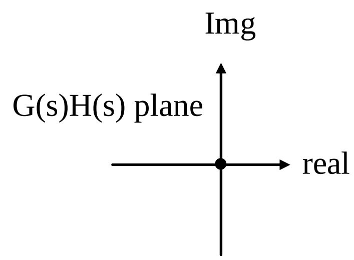
$\Rightarrow$ POI = 0.67 K

$\rightarrow$ Mapping of section C_2 of Nyquist contour i.e. radius 'R' semi circle.

Substitute $S = \underset{R\to\infty}{Lt}\ R.e^{j\theta}$ $90 \geq \theta \geq -90°$

$$G(R.e^{j\theta})\,H(R.e^{j\theta}) = \frac{K(\text{Re}^{j\theta}+1)}{(\text{Re}^{j\theta}+0.5)(\text{Re}^{j\theta}-2)} \cong 0$$

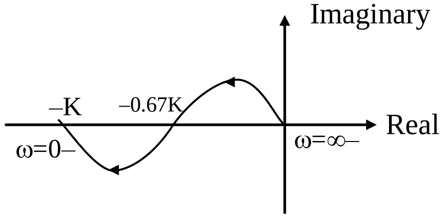
It maps to the origin



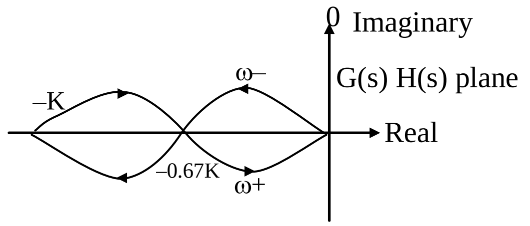
$\rightarrow$ Mapping of section C_3 of Nyquist contour i.e. negative real axis

Substitute $s = j\omega$, $-\infty \leq \omega \leq 0$

It is the image of the section C_1 and it is drawn such that the Nquist plot symmetrical w.r.t. real axis.



Combining all the above three sections the Nyquist plot is drawn below.



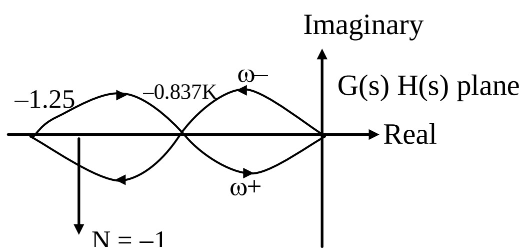
POI = a = 0.67K = 0.67 × 1.25 = 0.8375

N = P – Z, a = 0.8375

Z = P – N;   N = – 1, P =1

Z = 1– (–1) = 2

Number of RHP = 2 ∴ unstable



∴ For stability 0.67 K > 1

$$K > \frac{1}{0.67}$$

K >1.5

∴ Stable for K >1.5

5(a).

Sol: Given X(z) = log $(1 - 0.5z^{-1})$

$$\frac{dX(z)}{dz} = \frac{0.5z^{-2}}{1-0.5z^{-1}}$$

multiply both sides with –'z'

$$-z\frac{dX(z)}{dz} = \frac{-0.5z^{-1}}{1-0.5z^{-1}}$$

Apply IZT to above equation.

$$nx(n) \leftrightarrow -z\frac{dX(z)}{dz}$$

$$(0.5)^n u(n) \leftrightarrow \frac{1}{1-0.5z^{-1}}$$

Then, $nx(n) = -0.5(0.5)^n u(n)\big|_{n=n-1}$

$$x(n) = \frac{-(0.5)^n u(n-1)}{n}$$

5(b)

Sol: **(i) Gain Margin (GM):** It is the gain which can be varied before the system becomes unstable. If the gain increases GM decreases and if the gain is doubled GM becomes half.

$$GM = \frac{1}{|G(j\omega)H(j\omega)|}\bigg|_{\omega=\omega_{pc}}$$

$$GM(dB) = 20\ \log\frac{1}{|G(j\omega_{pc})H(j\omega_{pc})|}$$

Where ω_{pc} = phase cross over frequency, it is the frequency at which phase angle of G(s) H(s) is $-180°$

i.e Arg G(s) H(s) = $-180|_{\omega = \omega pc}$

Phase Margin (PM): It is the phase that can be varied before the system becomes unstable.

PM = 180 + $\angle G(j\omega) H(j\omega)|_{\omega = \omega gc}$

The phase angle is measured in clock wise direction where ω_{gc} = Gain cross over frequency, it is the frequency at which magnitude of G(s) H(s) is unity (or) 0dB

$|G(j\omega) H(j\omega)|_{\omega = \omega gc} = 1$

(ii) Given that G(s) = $\frac{K}{s(1+0.1s)(1+s)}, (K>1)$

(i) Given that resonance peak $M_r = 1.4$

$$M_r = \frac{1}{2\varepsilon\sqrt{1-\varepsilon^2}}$$

$$1.4 = \frac{1}{2\varepsilon\sqrt{1-\varepsilon^2}}$$

$\varepsilon \sqrt{1-\varepsilon^2} = 0.357$

$\Rightarrow \varepsilon^2 (1 - \varepsilon^2) = 0.127$

$\varepsilon^2 - \varepsilon^4 = 0.127$

$\varepsilon^4 - \varepsilon^2 + 0.127 = 0$

Let X = ε^2, then $x^2 - x + 0.127 = 0$

$x_1 = 0.85 \Rightarrow \varepsilon = 0.921$

$x_2 = 0.149 \Rightarrow \varepsilon = 0.386$

Neglecting the insignificant pole because time constant is very small. The equivalent transfer function is G(s) = $\frac{K}{s(1+s)}$. Hence characteristic equation is

$s(1+s) + k = 0 \Rightarrow s^2 + s + k = 0$

$\omega_n = \sqrt{K}$ and $2\varepsilon\omega_n = 1$

For $\varepsilon = 0.386$, $2\times 0.386 \times \sqrt{K} = 1$

$\Rightarrow K = 1.67$

For $\varepsilon = 0.921$, $2\times 0.921 \times \sqrt{K} = 1$

K = 0.294

As, 'K' should be greater than 1, K = 1.67

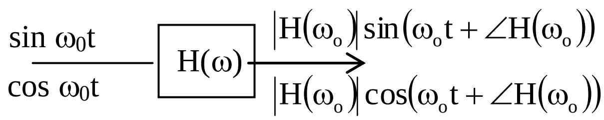
5(c)

Sol: $\frac{dy(t)}{dt} + 4y(t) = x(t)$

Apply Fourier transform to the above differential equation

$$j\omega Y(\omega) + 4Y(\omega) = X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{4 + j\omega}$$



Given x(t) = sin(4π t) + cos (6π t+ π /4)

$$|H(\omega)| = \frac{1}{\sqrt{16+\omega^2}}, \angle H(\omega) = -\tan^{-1}(\omega/4)$$

$x_1(t) = \sin(4\pi t)$

At $\omega_0 = 4\pi$;

$$|H(\omega_0)| = \frac{1}{\sqrt{16+(4\pi)^2}}$$

$$= \frac{1}{\sqrt{173.7536}} = \frac{1}{13.18} = 0.075$$

$$\angle H(\omega_o) = -\tan^{-1}\left(\frac{4\pi}{4}\right)$$

$$= -\tan^{-1}(\pi) = -72.346^0 = -0.4\pi$$

$y_1(t) = 0.075 \sin(4\pi t - 72.34^0)$

$x_2(t) = \cos(6\pi t+\pi/4)$

At $\omega_0 = 6\pi$;

$$|H(\omega_0)| = \frac{1}{\sqrt{16+(6\pi)^2}}$$

$$= \frac{1}{\sqrt{371.3}} = \frac{1}{19.25} = 0.051$$

$$\angle H(\omega_0) = -\tan^{-1}\left(\frac{6\pi}{4}\right) = -\tan^{-1}(1.5\pi) = -78.01^0 = -0.43\pi$$

$y_2(t) = 0.05\cos(6\pi t + \pi/4 - 78.01^0)$

$y(t) = 0.075\sin(4\pi t - 72.34^0) + 0.05\cos(6\pi t + \pi/4 - 78.01^0)$

$$y(t) = 0.075\sin(4\pi t - 0.4\pi) + 0.05\cos\left(6\pi t + \frac{\pi}{4} - 0.43\pi\right)$$

5(d)

Sol: (i) Open Loop T.F

$$G(s)H(s) = \frac{K}{s(s+4)(s+5)}$$

Point of intersection of asymptotes $= \dfrac{\sum \text{poles} - \sum \text{zeros}}{P - Z} = \dfrac{-9}{3} = -3$

(ii) O.L.T.F $G(s)H(s) = \dfrac{K}{s(s+4)(s+5)}$

C.E = $1 + G(s)H(s) = 0$

$s(s+4)(s+5) + K = 0$

$(s^2 + 4s)(s+5) + K = 0$

$s^3 + 5s^2 + 4s^2 + 20s + K = 0$

$s^3 + 9s^2 + 20s + K = 0$

s^3	1	20
s^2	9	K
s^1	$\dfrac{9\times 20 - K}{9}$	0
s^0	K	

For marginally stable system

$9 \times 20 = K$

$K = 180$

$9s^2 + 180 = 0$

$\therefore$ Point of intersection $= \pm j\sqrt{20}$

5(e)

Sol: The procedure used in impulse invariant method is H(s) → h(t) → h(nT) → H(z)

$$H(s) = \sqrt{2}\left[\frac{1/\sqrt{2}}{\left(s+\frac{1}{\sqrt{2}}\right)^2 + \left(1/\sqrt{2}\right)^2}\right], h(t) = \sqrt{2}.e^{-t/\sqrt{2}}.\sin\left(t/\sqrt{2}\right)u(t)$$

$$h(nT) = \sqrt{2}.e^{\frac{-nT}{\sqrt{2}}}.\sin\left(\frac{nT}{\sqrt{2}}\right)u(nT)$$

$$T = 1\text{sec},\ h(n) = \sqrt{2}.e^{-n/\sqrt{2}}.\sin\left(\frac{n}{\sqrt{2}}\right)u(n)$$

$$a^n \sin(\omega_0 n)u(n) \leftrightarrow \frac{az^{-1}\sin(\omega_0)}{1-2az^{-1}\cos(\omega_0)+a^2z^{-2}}$$

$$a = e^{-\frac{1}{\sqrt{2}}}\ ,\ \omega_0 = \frac{1}{\sqrt{2}}$$

$$H(z) = \sqrt{2}\left[\frac{e^{-1/\sqrt{2}}.z^{-1}.\sin\left(1/\sqrt{2}\right)}{1-2e^{-1/\sqrt{2}}.z^{-1}\cos\left(1/\sqrt{2}\right)+e^{-\sqrt{2}}.z^{-2}}\right]$$

$$H(z) = \frac{0.453z^{-1}}{1-0.7497z^{-1}+0.2432z^{-2}}$$

6(a)(i)

Sol: **(a)** Given T.F = $\frac{2}{s(3s+2)}$

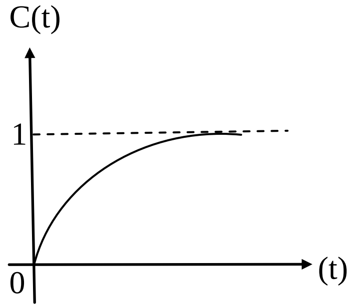
$$= \frac{1}{s} - \frac{1}{\left(s+\frac{2}{3}\right)}$$

By applying Inverse Laplace Transform on both sides, we get

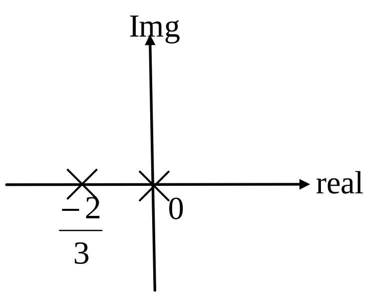
Impulse response, c(t) = L^{-1}[TF] = $L^{-1}\left[\frac{1}{S} - \frac{1}{(S+2/3)}\right]$

$$c(t) = \left(1 - e^{-\frac{2}{3}t}\right)u(t)$$

Impulse response:



Pole – Zero plot:



The given system is bounded for any bounded input. Therefore it is stable.

(b) Given T.F. = $\frac{6s}{(s^2+9)^2}$

The given transfer function can be written as

$$T.F. = \frac{-d}{ds}\left(\frac{3}{s^2+9}\right) \text{-------(1)}$$

If x (t) $\xleftrightarrow{L.T} X(s)$ then

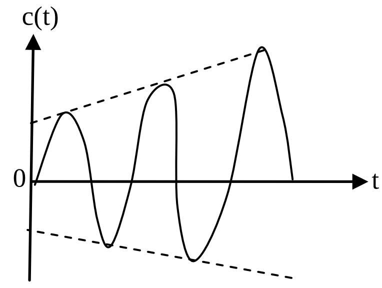
tx(t) $\xleftrightarrow{L.T} \frac{-d}{ds}X(s)$

By applying Inverse Laplace Transform on both sides, we get Impulse Response,

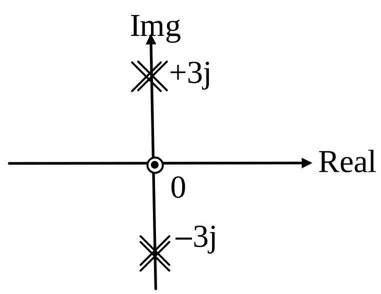
$$c(t) = L^{-1}\,[T.F] = L^{-1}\left[\frac{d}{ds}\left(\frac{3}{s^2+9}\right)\right]$$

$$c(t) = t\sin 3t\; u(t) \qquad \left[\because L^{-1}\left[\frac{3}{s^2+9}\right] = \sin 3t\; u(t)\right]$$

Impulse Response:



Pole – zero plot:



The given system is unbounded as $t \to \infty$. Therefore, it is unstable.

6(a)(ii)

Sol: PID controller will have transfer function

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s$$

The combination of proportional control action, integral action and derivative control action is called PID control action

Proportional controller stabilizes the gain but produces a steady state error. The integral controller reduces (or) eliminates the steady state error. The derivative controller reduces the rate of change of error.

The derivative control acts on rate of change of error and not on the actual error signal. The derivative control action is effective only during transient periods, so it doesn't produce corrective measures for only constant error. Hence derivative controller is never used alone.

6(a)(iii)

Sol: (a) Effect of lead network: (Any 3 points)

- BW increases, rise time and setting time decreases, transient response is improved (faster)
- Steady state response is not affected.
- Improves the stability i.e gain and phase margins are improved
- M_r decreases
- Noise is introduced

Effect of lag network: (Any 3 points)

- Steady state response is improved
- ω_{ge} is reduced, BW decreases
- Rise time increases, transient response becomes sluggish /slower
- Signal to noise ratio is improved
- Improves the value of zeta, phase and gain margins are improved
- M_r decreases

(b) Phase lead compensation increases bandwidth as rise time is decreased. In phase lead controller, a zero is at right side of pole, to the forward path transfer function. The general effect is to add more damping to the closed loop system. The rise time, setting times are reduced. So, given statement is wrong

6(b) (i)

Sol: Initial value theorem:

The initial value theorem enables us to calculate the initial value of a function x(t), i.e. x(0) directly from its transform X(s) without the need for finding the inverse transform of X(s). It states that

If $x(t) \xleftrightarrow{LT} X(s)$

then, $\underset{t\to 0}{Lt}\, x(t) = x(0) = \underset{s\to\infty}{Lt}\, sX(s)$

Proof: Given,L[x(t)] = X(s)

we have, $L\left[\frac{dx(t)}{dt}\right] = \int_0^{\infty} \frac{dx(t)}{dt} e^{-st} dt = sX(s) - x(0^-)$

Taking limit $s \to \infty$ on both sides, we get

$$\underset{s\to\infty}{Lt}\, L\left[\frac{dx(t)}{dt}\right] = \underset{s\to\infty}{Lt}\left[\int_0^\infty \frac{dx(t)}{dt} e^{-st} dt\right] = \underset{s\to\infty}{Lt}\left[sX(s) - x(0)\right]$$

$$\text{i.e. } 0 = \underset{s\to\infty}{Lt}\, sX(s) - x(0)$$

$$\therefore\ x(0) = \underset{s\to\infty}{Lt}\, sX(s)$$

$$x(0) = \underset{t\to 0}{Lt}\, x(t) = \underset{s\to\infty}{Lt}\, sX(s)$$

Final value Theorem

The final value theorem enables us to determine the final value of a function x(t), i.e. $x(\infty)$ directly from its Laplace transform X(s) without the need for finding the inverse transform of X(s). It states that

$$\text{If } x(t) \xleftrightarrow{LT} X(s)$$

$$\text{then } \underset{t\to\infty}{Lt}\, x(t) = x(\infty) = \underset{s\to 0}{Lt}\, sX(s)$$

Proof: Given, L[x(t)] = X(s)

$$\text{We have, } L\left[\frac{dx(t)}{dt}\right] = \int_0^\infty \frac{dx(t)}{dt} e^{-st} dt = sX(s) - x(0^-)$$

Taking the limit $s \to 0$ on both sides, we obtain

$$\underset{s\to 0}{Lt}\left[\int_0^\infty \frac{dx(t)}{dt} e^{-st} dt\right] = \underset{s\to 0}{Lt}\left[sX(s) - x(0^-)\right]$$

$$\therefore \int_0^\infty \frac{dx(t)}{dt} dt = \underset{s\to 0}{Lt}\left[sX(s) - x(0^-)\right]$$

$$\left[x(t)\right]_0^\infty = \underset{s\to 0}{Lt}\left[sX(s) - x(0^-)\right]$$

$$\text{i.e. } x(\infty) - x(0^-) = \underset{s\to 0}{Lt}\left[sX(s) - x(0^-)\right]$$

$$x(\infty) = \underset{s\to 0}{Lt}\, sX(s)$$

(ii) v(t) = – (t – 2) [u(t) – u(t – 2)]

= – t u(t) + 2 u(t) + (t – 2) u(t – 2) = 2 u(t) – r(t) + r(t – 2)

where r(t) = t u(t)

6(c).

Sol: (i) The periodic waveform shown in Figure with period 2π is half of a sine wave with period 2π.

$$x(t)=\begin{cases} A\sin\omega t = A\sin\frac{2\pi}{2\pi}t = A\sin t & ; \quad 0\le t\le \pi \\ 0 & ; \quad \pi\le t\le 2\pi \end{cases}$$

Now the fundamental period T = 2π

Fundamental frequency $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

Let, $t_0 = 0$, $t_0 + T = T = 2\pi$

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{2\pi}\int_0^{2\pi} x(t)dt = \frac{1}{2\pi}\int_0^{\pi} A\sin t dt$$

$$= \frac{A}{2\pi}[-\cos t]_0^{\pi} = \frac{A}{2\pi}[-(\cos\pi - \cos 0)] = \frac{2A}{2\pi} = \frac{A}{\pi}$$

$$\therefore \ a_0 = \frac{A}{\pi}$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\, dt = \frac{2}{2\pi}\int_0^{2\pi} x(t)\cos nt\, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} A\sin t\cos nt\, dt = \frac{A}{\pi}\int_0^{\pi}\sin t\cos nt\, dt$$

$$= \frac{A}{2\pi}\int_0^{\pi}[\sin(1+n)t + \sin(1-n)t]dt = \frac{A}{2\pi}\left[-\frac{\cos(1+n)t}{1+n} - \frac{\cos(1-n)t}{1-n}\right]_0^{\pi}$$

$$= -\frac{A}{2\pi}\left[\frac{\cos(1+n)\pi - \cos 0}{1+n} + \frac{\cos(1-n)\pi - \cos 0}{1-n}\right]$$

$$= -\frac{A}{2\pi}\left\{\left[\frac{(-1)^{n+1}-1}{1+n}\right] + \frac{(-1)^{1-n}-1}{1-n}\right\}$$

For odd n, $a_n = -\frac{A}{2\pi}\left[\frac{1-1}{1+n} + \frac{1-1}{1-n}\right] = 0$

For even n, $a_n = -\frac{A}{2\pi}\left[\frac{-1-1}{1+n} + \frac{-1-1}{1-n}\right] = -\frac{A}{2\pi}\left[\frac{-2}{n+1} + \frac{2}{n-1}\right] = -\frac{2A}{\pi(n^2-1)}$

$$\therefore \ a_n = -\frac{2A}{\pi(n^2-1)} \text{ (for even n)}$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t dt = \frac{2}{2\pi}\int_0^{2\pi} x(t)\sin nt\, dt$$

$$= \frac{1}{\pi}\int_0^{\pi} A\sin t \sin nt\, dt = \frac{A}{\pi}\int_0^{\pi} \sin t \sin nt\, dt$$

$$= \frac{A}{2\pi}\int_0^{\pi}\left[\cos(n-1)t - \cos(n+1)t\right]dt$$

$$= \frac{A}{2\pi}\left[\frac{\sin(n-1)t}{n-1} - \frac{\sin(n+1)t}{n+1}\right]_0^{\pi}$$

This is zero for all values of n except for n = 1

$$\text{For } n = 1, \quad b_1 = \frac{A}{2\pi}$$

Therefore, the Trigonometric Fourier series is:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$= a_0 + b_1 \sin t + \sum_{n=1}^{\infty} a_n \cos nt \;= \frac{A}{\pi} + \frac{A}{2\pi}\sin t - \sum_{n=\text{even}}^{\infty} \frac{2A}{\pi(n^2-1)}\cos nt$$

(ii)

Butterworth filter	Chebyshev filter
(1) The magnitude response of this filter decreases monotonically as the frequency Ω increases form 0 to ∞	(1) The magnitude response of this filter exhibits ripples in PB (or) SB according to the type
(2) The transition band is more in butterworth filter when compare to chebyshev filter.	(2)The transition band is less in chebyshev filter when compare to butterworth filter.
(3) Poles of this filter mapped onto circle in s-plane.	(3) Poles of this filter maps on to ellipse in s-plane
(4) For the same specifications the number of poles in butterworth filter are more compared to chebyshev filter i.e order of chebyshev filter is less compared to butterworth. It is the greatest advantage of chebyshev filter because less number of components will be required to construct the filter.	

7(a).

Sol: x(n) = {1,1,1,1,0,0,0,0}

$$X(k)=\sum_{n=0}^{N-1}x(n)e^{-j\frac{2\pi}{N}nk} \quad k = 0 \text{ to } N-1$$

$$X(k)=\sum_{n=0}^{7}x(n)e^{-j\frac{2\pi}{8}nk} \quad k = 0 \text{ to } 7$$

X(0) = 4

X(1) = 1 – j2.414

X(2) = 0

X(3) = 1 – j0.414

X(4) = 0

X(5) = 1 + j0.414

X(6) = 0

X(7) = 1 + j2.414

X(k) = {4, 1 – j2.414, 0, 1 – j0.414, 0, 1 + j0.414, 0 , 1 + j2.414}

Given, $x_1(n)$ = {1,0,0,0,0,1,1,1}

$x_1(n) = x((n + 3))_8$

From circular shift in time domain property $DFT[x((n-m))_N] = e^{-j\frac{2\pi}{N}mk}X(k)$

$$DFT[x((n+3))_8] = e^{j3\frac{\pi k}{4}}X(k) = X_1(k)$$

$X_1(0) = X(0) = 4$

$$X_1(1) = X(1).e^{j\frac{3\pi}{4}} = 1 + j2.413$$

$$X_1(2) = X(2).e^{j\frac{3\pi}{2}} = 0$$

$$X_1(3) = X(3).e^{j\frac{9\pi}{4}} = 1 + j0.414$$

$$X_1(4) = X(4).e^{j3\pi} = 0$$

$$X_1(5) = X(5).e^{j\frac{15\pi}{4}} = 1 - j0.414$$

$$X_1(6) = X(6).e^{j\frac{9\pi}{2}} = 0$$

$$X_1(7) = X(7).e^{j\frac{21\pi}{4}} = 1 - j2.413$$

$X_1(k)$ = {4, 1 + j2.413, 0, 1 + j0.414, 0, 1 – j0.414, 0, 1–j2.413}

Given $x_2(n)$ = {0, 0, 1, 1, 1, 1, 0, 0}

$x_2(n) = x((n-2))_8$

From circular shift in time domain property $\quad DFT[x((n-m))_N] = e^{-j\frac{2\pi}{N}mk} X(k)$

$$X_2(k) = e^{-j\frac{\pi k}{2}} X(k)$$

$X_2(0) = X(0) = 4$

$$X_2(1) = X(1).e^{-j\frac{\pi}{2}} = -2.414 - j$$

$X_2(2) = 0$

$$X_2(3) = X(3)\,.\,e^{-j\frac{3\pi}{2}} = 0.4141 + j$$

$X_2(4) = 0$

$$X_2(5) = X(5).e^{-j\frac{5\pi}{2}} = 0.414 - j$$

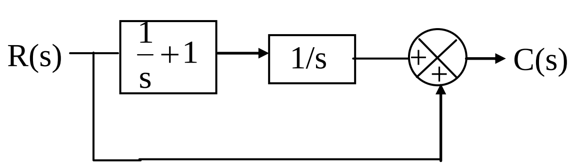
$X_2(6) = 0$

$$X_2(7) = X(7).e^{-j\frac{7\pi}{2}} = -2.414 + j$$

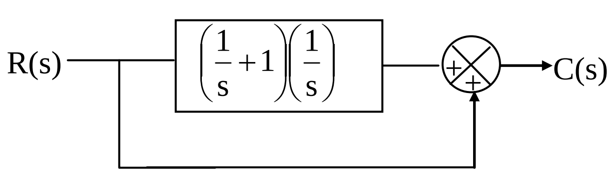
$X_2(k)$ = {4, –2.414 –j, 0, 0.414 + j, 0, 0.414 –j, 0, –2.414 + j}

7(b).

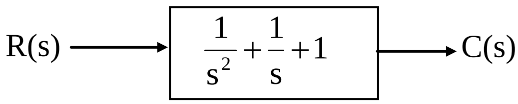
Sol: (i) The given block diagram is



Step I: Cascade the two series blocks

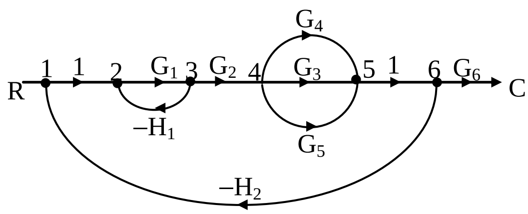


Step II: Summing the overall parallel paths



$\therefore$ Transfer function $\frac{C(s)}{R(s)}=\frac{1}{s^2}+\frac{1}{s}+1$

(ii)



The No. of forward paths = 3

$$\therefore \frac{C}{R}=\frac{\sum_{K=1}^{\Delta} M_K \Delta_K}{\Delta}=\frac{M_1\Delta_1+M_2\Delta_2+M_3\Delta_3}{\Delta}$$

Forward path gains,

$M_1 = G_1\, G_2\, G_3\, G_6 \quad \Delta_1 = 1–0 = 1$

$M_2 = G_1\, G_2\, G_4\, G_6 \quad \Delta_2 = 1–0 = 1$

$M_3 = G_1\, G_2\, G_5\, G_6 \quad \Delta_3 = 1–0 = 1$

Individual loops,

$L_1 = – G_1\, H_1$

$L_2 = – G_1\, G_2\, G_3\, H_2$

$L_3 = – G_1\, G_2\, G_4\, H_2$

$L_4 = – G_1\, G_2\, G_5\, H_2$

Non – touching loops = None

$\Delta = 1– (L_1 + L_2 + L_3 + L_4)$

$$\therefore \frac{C}{R}=\frac{G_1G_2G_3G_6+G_1G_2G_4G_6+G_1G_2G_5G_6}{1+G_1H_1+G_1G_2G_3H_2+G_1G_2G_4H_2+G_1G_2G_5H_2}$$

7(c).

Sol: Initial slope = – 6 dB/octave

i.e there is one pole at origin (or) one integral term portion of transfer function

$$G(s) = \frac{K}{S}$$

At ω = 2 rad/sec, slope is changed to 0 dB/oct

$\therefore$ change is slope = present slope – previous slope

= 0 – (– 6) = 6dB/octave

$\therefore$ There is a real zero at corner frequency ω_1 = 2 rad/sec

$$1 + sT_1 = \left(1+\frac{s}{\omega_1}\right) = 1+\frac{s}{2}$$

At ω = 10 rad/sec, slope is changed to – 6dB/octave

$\therefore$ Change in slope = – 6 – 0

= – 6dB/octave

$\therefore$ There is a real pole at corner frequency, ω_2 = 10 rad/s

$$\frac{1}{1+sT_2} = \frac{1}{\left(1+\frac{s}{\omega_2}\right)} = \frac{1}{\left(1+\frac{s}{10}\right)}$$

At ω = 50 rad/sec, slope is changed to – 12 dB/octave

$\therefore$ Change in slope = – 12– (–6)

= – 6 dB/octave

$\therefore$ There is a real pole at corner frequency ω_3 = 50 rad/sec

$$\frac{1}{1+sT_3} = \frac{1}{1+\frac{s}{\omega_3}} = \frac{1}{1+\frac{s}{50}}$$

At ω = 100 rad/sec, the slope changed to – 6dB /octave.

$\therefore$ Change in slope = – 6 – (–12) = 6dB/octave

$\therefore$ There is a real zero at corner frequency ω_4 = 100 rad/sec

$$\therefore (1 + sT_4) = 1 + \frac{s}{\omega_4} = 1+\frac{s}{100}$$

Hence, transfer function = $\dfrac{K\left(1+\frac{s}{2}\right)\left(1+\frac{s}{100}\right)}{S\left(1+\frac{s}{50}\right)\left(1+\frac{s}{10}\right)}$

$$= \frac{K(s+2)(s+100)}{s(s+50)(s+10)}\frac{\frac{1}{2}.\frac{1}{100}}{\frac{1}{50}.\frac{1}{10}}$$

$$= \frac{2.5K(s+2)(s+100)}{s(s+10)(s+50)}$$

In the given bode plot, at ω = 1 rad/sec

Magnitude = 20 dB

– 20 = 20 logK – 20logω

– 20 = 20log K

⇒ K = 0.1

$\therefore$ Transfer function $\dfrac{C(s)}{R(s)} = \dfrac{0.25(s+2)(s+100)}{s(s+10)(s+50)}$

8(a)

Sol: **(i)** Given , $x(t) = e^{-at^2}$

The Fourier transform of the given signal is:

$$X(\omega) = F\left[e^{-at^2}\right] = \int_{-\infty}^{\infty} e^{-at^2} e^{-j\omega t} dt = \int_{-\infty}^{\infty} e^{-\left(at^2 + j\omega t\right)} dt$$

$$= e^{\frac{-\omega^2}{4a}} \int_{-\infty}^{\infty} e^{-\left(t\sqrt{a} + \left(j\omega/2\sqrt{a}\right)\right)^2} dt$$

Let, $p = t\sqrt{a} + \dfrac{j\omega}{2\sqrt{a}}$

$\therefore \quad dp = \sqrt{a}dt$

$$\therefore \quad X(\omega) = \frac{e^{-\omega^2/4a}}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-p^2} dp = \frac{e^{-\omega^2/4a}}{\sqrt{a}}\sqrt{\pi} \qquad \left[\because \int_{-\infty}^{\infty} e^{-p^2} dp = \sqrt{\pi}\right]$$

$$= \sqrt{\frac{\pi}{a}} e^{-\omega^2/4a}$$

$$\therefore F\left[e^{-at^2}\right] = \sqrt{\frac{\pi}{a}}e^{-\omega^2/4a} \text{ or } e^{-at^2} \xleftrightarrow{FT} \sqrt{\frac{\pi}{a}}e^{-\omega^2/4a}$$

(ii) Given, Gaussian modulated signal $x(t) = e^{-at^2}\cos\omega_c t = e^{-at^2}\left(\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right)$

$$\therefore X(\omega) = \frac{1}{2}\left\{\left[F\left(e^{-at^2}e^{j\omega_c t}\right)\right] + \left[F\left(e^{-at^2}e^{-j\omega_c t}\right)\right]\right\}$$

By using frequency shifting property $\left[\text{i.e. } e^{-j\omega_0 t}x(t) \xleftrightarrow{FT} X(\omega+\omega_0)\right]$.

We have,

$$F\left(e^{-at^2}e^{j\omega_c t}\right) = F\left(e^{-at^2}\right)\Big|_{\omega=\omega-\omega_c}$$

and $F\left(e^{-at^2}e^{-j\omega_c t}\right) = F\left(e^{-at^2}\right)\Big|_{\omega=\omega+\omega_c}$

$$\therefore X(\omega) = \frac{1}{2}\left[\sqrt{\frac{\pi}{a}}e^{-(\omega-\omega_c)^2/4a} + \sqrt{\frac{\pi}{a}}e^{-(\omega+\omega_c)^2/4a}\right]$$

8(b)

Sol: Here the impulse response and input are:

$h(t) = e^{-2t}u(t) = e^{-2t} \quad t > 0$

and $x(t) = e^{-4t}[u(t)-u(t-2)] = e^{-4t} \quad 0 < t < 2$

The output of the circuit y(t) can be obtained by convolution of x(t) and h(t).

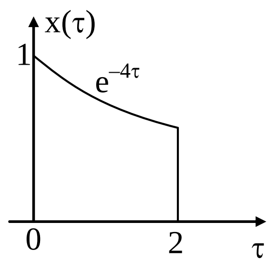
$$y(t) = x(t)*h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

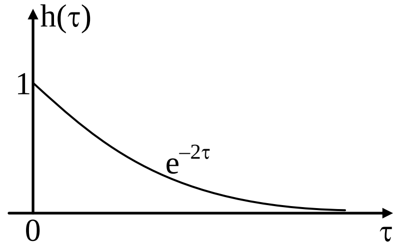
Writing x(t) and h(t) in terms of τ, we have

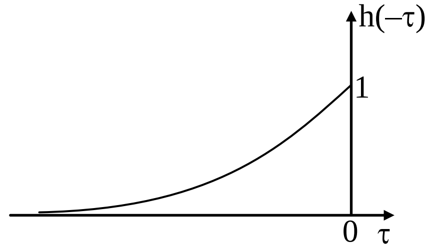
$x(\tau) = e^{-4\tau} \quad 0 < \tau < 2$

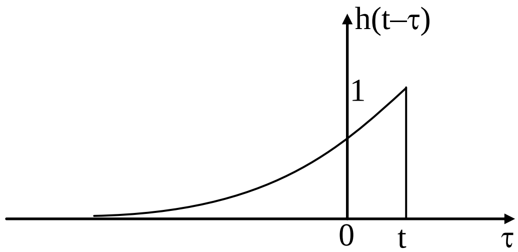
and $h(\tau) = e^{-2\tau} \quad \tau > 0$

Figure shows the plots of $x(\tau)$, $h(\tau)$ and $h(-\tau)$ w.r.t. 'τ'.









The plots of x(τ) and h(t – τ) drawn on the same time axis are shown in Figure for t < 0. The plots do not overlap.

y(t) = 0 (for t < 0)

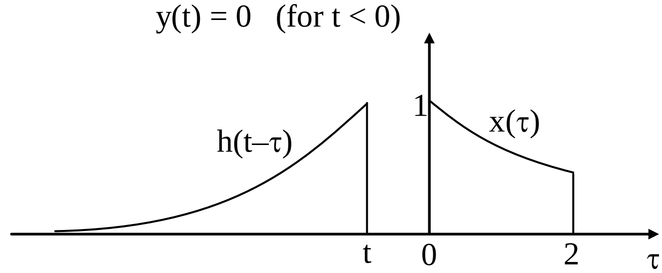


Figure. Plots of x(τ) and h(t–τ) for t < 0

For 0 < t < 2

Figure shows the plots of x(τ) and h(t – τ) for 0 < t < 2 drawn on the same time axis.

Observe that there is an overlap between x(τ) and h(t–τ) as shown by the shaded area only for 0 to t.

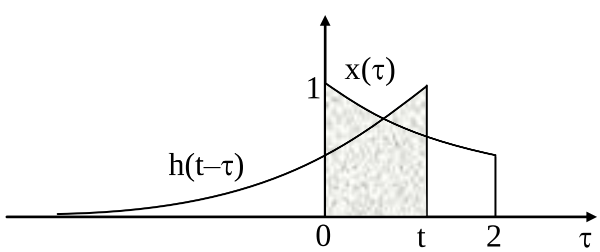


Figure. Plots of x(τ) and h(t–τ) when there is an overlap For 0 < t < 2

We can write the convolution as:

$$y(t) = \int_{-\infty}^{0} (0) h(t-\tau) d\tau + \int_{0}^{t} x(\tau) h(t-\tau) d\tau + \int_{t}^{2} x(\tau)(0) d\tau \; = \int_{0}^{t} x(\tau) h(t-\tau) d\tau$$

$$= \int_{0}^{t} \left(e^{-4\tau}\right)\left(e^{-2(t-\tau)}\right) d\tau = e^{-2t}\int_{0}^{t} e^{-2\tau} d\tau \; = e^{-2t}\left[\frac{e^{-2\tau}}{-2}\right]_0^t = e^{-2t}\left(\frac{e^{-2t}-1}{-2}\right) = \frac{1}{2}e^{-2t}\left(1-e^{-2t}\right)$$

$$\therefore y(t) = \frac{1}{2}e^{-2t}\left(1-e^{-2t}\right) \qquad \text{(for } 0 < t < 2\text{)}$$

For t > 2

Now, consider the case t > 2. For t > 2, the plots of x(τ) and h(t – τ) drawn on the same time axis are shown in Figure. In this figure, observe that x(τ) and h(t – τ) overlap only for 0 < t < 2 as shown by the shaded area.

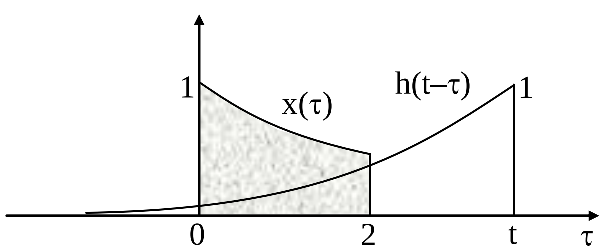


Figure. Plots of x(τ), and h(t–τ) for t > 2

Hence, we can write the convolution equation as:

$$y(t) = \int_{-\infty}^{0} (0)\times h(t-\tau) d\tau + \int_{0}^{2} x(\tau) h(t-\tau) d\tau + \int_{2}^{t} (0)\times h(t-\tau) d\tau$$

$$= \int_{0}^{2} x(\tau) h(t-\tau) d\tau = \int_{0}^{2} e^{-4\tau} e^{-2(t-\tau)} d\tau = e^{-2t}\int_{0}^{2} e^{-2\tau} d\tau \; = e^{-2t}\left[\frac{e^{-2\tau}}{-2}\right]_0^2 = e^{-2t}\left(\frac{e^{-4}-1}{-2}\right)$$

$$= \frac{1}{2}e^{-2t}\left(1-e^{-4}\right) \qquad \text{for } t > 2$$

Thus, we obtained the convolution as follows:

$$y(t) = \begin{cases} 0 & ; \quad \text{for } t < 0 \\ \frac{1}{2}\left(1-e^{-2t}\right)e^{-2t} & ; \quad \text{for } 0 < t < 2 \\ \frac{1}{2}\left(1-e^{-4}\right)e^{-2t} & ; \quad \text{for } t > 2 \end{cases}$$

This function is plotted in Figure.

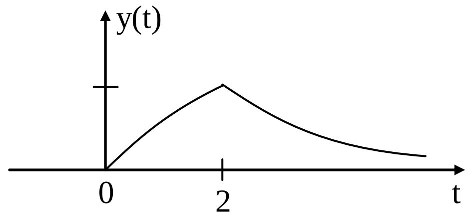


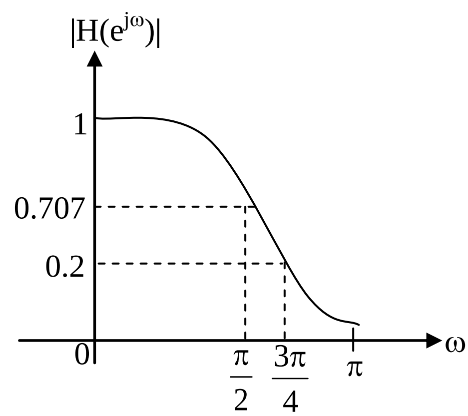
Figure. Plot of y(t)

At t = 2, the value of $y(t)=\frac{1}{2}\left(1-e^{-4}\right)e^{-4}=0.009$

In above figure we observe that y(t) increases from t = 0 to t = 2. It has the maximum value at t = 2. The y(t) decays exponentially.

8(c)

Sol: Given specifications of digital filter are



$$\frac{1}{\sqrt{1+\xi^2}}=0.707$$

$$\frac{1}{\sqrt{1+\lambda^2}}=0.2$$

$$\omega_p=\frac{\pi}{2},\quad \omega_s=\frac{3\pi}{4}$$

Step 1: Convert digital filter specifications to analog filter specifications

$$\frac{1}{\sqrt{1+\xi^2}}=0.707$$

$$\frac{1}{\sqrt{1+\lambda^2}}=0.2$$

$$\Omega_p=\frac{2}{T}\tan\left(\frac{\omega_p}{2}\right)$$

$$\Omega_s=\frac{2}{T}\tan\left(\frac{\omega_s}{2}\right)$$

Step 2: Design normalized analog low pass filter

$$\frac{\Omega_s}{\Omega_p} = \frac{\tan(\omega_s/2)}{\tan\left(\frac{\omega_p}{2}\right)} = 2.414$$

$$N \geq \frac{\log\left[\frac{\lambda}{\xi}\right]}{\log\left[\frac{\Omega_s}{\Omega_p}\right]}$$

$\lambda = 4.898,\ \xi = 1$

$N \geq 1.803$

Round off to next higher integer. So, N = 2

$$\Omega_c = \frac{\Omega_p}{(\xi)^{1/N}} = \Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = 2\text{rad/s}$$

The transfer function of N = 2 order with Ω_c = 1 rad/sec

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

Step 3: Convert normalized analog low pass filter to un-normalized analog low pass filter using analog to analog frequency transformation.

$$H_a(s) = H(s)|_{s \to s/\Omega_c}$$

$$H_a(s) = \frac{4}{s^2 + 2.828s + 4}$$

Step 4: Convert above analog filter to digital filter using analog to digital frequency transformation i.e., bilinear transformation

Using bilinear Transformation $H(z) = H_a(s)\Big|_{S \to \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$

$$H(z) = \frac{0.2929\left[1 + z^{-1}\right]^2}{1 + 0.1716z^{-1}}$$