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ESE – 2019 MAINS
OFFLINE TEST SERIES

CIVIL ENGINEERING

TEST – 2
SOLUTIONS

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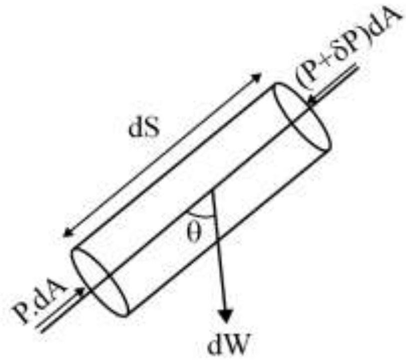
01 (a).

Sol: Assume a differential mass along a streamline



C/s area = dA

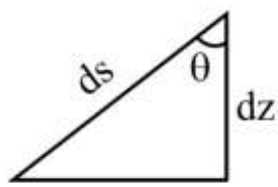
Length = ds

By Eulers equation:

$(F_g + F_p)_s = ma_s$

$\Rightarrow (\overrightarrow{PdA} - (P + \delta p)\ dA) - dW \cos\theta.\ d_m\ a_s$

$\Rightarrow \delta P.\ dA + dW \cos\theta + d_m\ a_s = 0 \rightarrow (1)$



$P = f(s,t) \Rightarrow dp = \frac{\partial p}{\partial s} ds + \frac{\partial p}{\partial t} dt$

If the flow is steady, then $\delta p = \frac{\partial p}{\partial s} ds$

$\cos\theta = \frac{dz}{ds} \Rightarrow dW \cos\theta = \rho g\ ds\ dA\ \frac{dz}{ds}$

$a_s = \frac{\partial V_s}{\partial t} + V_s \frac{\partial V_s}{\partial s}$; ∵ Flow is steady $\Rightarrow a_s = Vs\frac{\partial V_s}{\partial s}$

$\Rightarrow$ From equation (1) $\frac{dp}{ds} dsdA + \rho g\ ds\ dA\ \frac{dz}{ds} + \rho\ ds\ dA\ V_s \frac{dV_s}{ds} = 0$

$$\Rightarrow \frac{dP}{\rho} + vdv + gdz = 0$$

By integration : $\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant}$

If the flow is incompressible, then

$$\frac{P}{\gamma} + \frac{V^2}{2g} + Z = H$$

01 (b).

Sol: Discharge per nozzle = 1 lt/s = 10^{-3} m^3/s

Relative velocity in both nozzle = $\dfrac{Q}{c/s}$

$$= \frac{10^{-3}}{1\times10^{-4}} = 10\,m/s$$

$$\Rightarrow V_{r1} = V_{r2} = 10\text{ m/s}$$

$$V_{r1} = V_1 - U_1 = V_1 - 0.4\omega$$

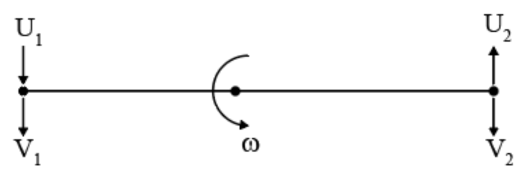
$$\Rightarrow V_1 = V_{r1} + 0.4\,\omega$$

$$= 10 + 0.4\,\omega$$

Similarly $V_{r2} = V_2 + U_2 = V_2 + 0.6\,\omega$

$$\Rightarrow V_2 = V_{r2} - 0.6\,\omega$$

$$= 10 - 0.6\omega$$



Net torque = 0 (∵ Frictionless shaft)

$$\Rightarrow \vec{\dot{m}}_1 v_1 r_1 = \vec{\dot{m}}_2 v_2 r_2 \qquad (\because \dot{m}_1 = \dot{m}_2)$$

(10 + 0.4ω) 0.4 = (10 – 0.6ω) 0.6

$\Rightarrow$ 4 + 0.16 ω = 6 – 0.36 ω

$$\Rightarrow \omega = \frac{2}{0.52} = 3.84 \text{ rad/s}$$

$$N = \frac{60\omega}{2\pi} = 36.728 \text{ rpm}$$

01(c).

Sol:

(i) Total displacement = L

$$U = U_o\left(1-\frac{x}{2L}\right) = \frac{dx}{dt} \Rightarrow dt = \frac{1}{U_o}\frac{dx}{\left(1-\frac{x}{2L}\right)}$$

$$\Rightarrow t = \frac{1}{U_o}\int_O^L \frac{dx}{\left(1-\frac{x}{2L}\right)} \Rightarrow t = \frac{1}{U_o}\times\left.\frac{\ell n\left(1-\frac{x}{2L}\right)}{\left(-\frac{1}{2L}\right)}\right|_o^L$$

$$\Rightarrow t = \frac{-2L}{U_o}\left[\ell n\left(\frac{1}{2}\right) - \ell n 1\right]$$

$$t = \frac{2L\ell n2}{U_o}$$

$$\text{Average velocity} = \frac{L}{t} = \frac{L}{\frac{2L\ell n2}{U_o}} \Rightarrow \text{Average velocity}\} = \frac{U_o}{\ell n4}$$

(ii) $\overline{U} = 0.721\, U_o \rightarrow (1)$

$$a_x = U\frac{\partial u}{\partial x} = U_o\left(1-\frac{x}{2L}\right)U_o\left(\frac{-1}{2L}\right)$$

$$\Rightarrow a_x = \frac{-U_o^2}{2L}\left(1-\frac{x}{2L}\right)$$

@ Initial point (x = 0) $a_x = \frac{-U_o^2}{2L}$

@ Final point (x = L) $a_x = \frac{-U_o^2}{4L} \rightarrow (2)$

01(d).

Sol:

Instrument at	Staff station	Staff readings			H.I (m)	R.L (m)	Remarks
		B.S	I.S	F.S			
O_1	A	0.964			61.614	60.65	B.M
O_2	B	1.632		0.948	62.298	60.666	
O_3	C	1.105		1.153	62.25	61.145	
O_4	D	0.850		1.984	61.116	60.266	

Chainage

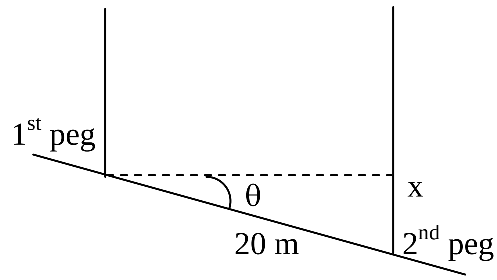
0	1.116	60.000	1^{st} peg
20	1.516	59.600	2^{nd} peg
40	1.916	59.200	3^{rd} peg
60	2.316	58.800	4^{th} peg
80	2.716	58.400	5^{th} peg

$\tan\theta = \frac{x}{20} = \frac{1}{50}$

$x = \frac{20}{50} = 0.4$

R.L of 2nd peg = 60 – 0.4

= 59.6 m



Arithmetic Check:

$\sum$B.S = 4.551 $\sum$B.S – $\sum$F.S = 4.551 – 6.801

$\sum$F.S = 6.801 = – 2.25

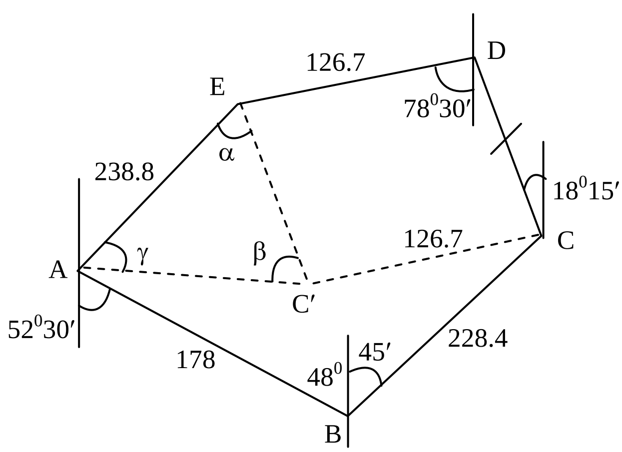
Last RL – 1st R.L = 58.400 – 60.65

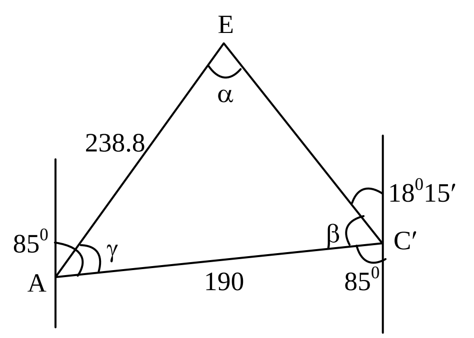
= – 2.25 = $\sum$B.S – $\sum$F.S

Hence checked.

01 (e).

Sol:





A rough sketch of the traverse is drawn as shown in Fig. above Lines CD and EA are to be brought adjacent. For this, draw a line parallel to CD through E and a line parallel to DE through C. EC' = CD and the two lines with missing measurements are brought adjacent. CC' = DE. The traverse ABCC'A is a closed traverse and the length and bearing of C'A can be found. The calculations are shown in Table below

Table Calculation of length and bearing of C'A

Line	Length	Bearing	Latitude	Departure
AB	178.6	$S52^0 30'E$	-108.73	141.69
BC	228.4	$N48^0 45'E$	150.60	171.72
CC"	126.7	$S78^0 30'W$	-25.26	-124.15
C'A	-	-	-	-

Latitude of C'A = -(- 108.75 + 150.60 - 25.26) =-16.61

Departure of C'A = - (141.69 + 171.72 - 124.15) = - 189.26

Length of C'A = $\sqrt{(-16.61)^2 + (-189.26)^2} = 190\text{m}$

Bearing of C'A: tan θ = (-189.26)/(-16.61), θ = $S48^0$ 00'00"W

We can now solve the triangle C'AE to obtain the length of CD and the bearing of EA. In triangle C'AE, shown enlarged in Fig below,

$\beta = 180^0 - 18^0\ 15' - 85^0\ 00' = 76^0 45'$

Applying the sine rule,

$190/\sin\alpha = 238.8/\sin\beta$, $\alpha = 50^0 45'$, $\gamma = 180^0 - \alpha - \beta = 52^0 30'$

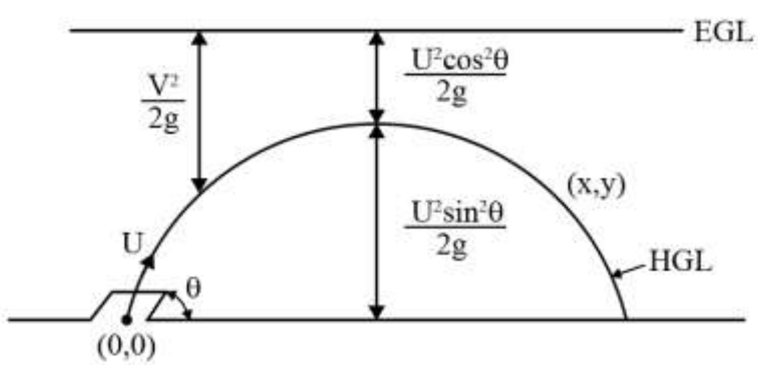
$EC'/\sin 52^0 30' = 238.8/\sin 76^0 45'$

$EC' = CD = 238.8 \times 52^0 30' / \sin 76^0 45' = 194.6$ m

Length of CD = 194.6 m; bearing of AE = $85^0 - \gamma = 85^0 - 52^0 30' = 32^0 30'$; bearing of EA = $S30^0 30'W$.

02 (a).

Sol:



(i) $a_x = 0$ $U_x = U\cos\theta$

$a_y = -g$ $U_y = U\sin\theta$

$S_x = U_x t + \frac{1}{2}a_x t^2$

$\Rightarrow x = U\cos\theta . t$

$\Rightarrow t = \frac{x}{U\cos\theta} \rightarrow (1)$

$S_y = U_y t + \frac{1}{2}a_y t^2$

$\Rightarrow y = U\sin\theta\frac{x}{U\cos\theta} - \frac{1}{2}g\left(\frac{x}{U\cos\theta}\right)^2$

$\Rightarrow y = x\tan\theta - \frac{1}{2}g\frac{\sec^2\theta}{U^2}.x^2$

$\tan\theta = \tan 30 = \frac{1}{\sqrt{3}} = 0.577$

$\frac{1}{2}g\frac{\sec^2\theta}{U^2} = \frac{1}{2}\times 9.81\times\frac{\sec^2 30}{8^2} = 0.102$

$\Rightarrow$ equation of trajectory is

$y = 0.577x - 0.102x^2 \rightarrow (1)$

(ii) **$\Rightarrow$ Maximum height**

$y_{max} = \frac{U^2\sin^2\theta}{2g}$

$= \frac{8^2\times\frac{1}{4}}{2\times 9.81}$

$\Rightarrow y_{max} = 0.815\text{ m} \rightarrow (2)$

(iii) **At the top of the trajectory**

$V = U\cos\theta$

$\Rightarrow D_1^2 V_1 = D_2^2 V$

$\Rightarrow 10^2\times 8 = D_2^2\left(8\times\frac{3}{4}\right)$

$\Rightarrow D_2 = 11.54\text{ cm}$

(iv) Horizontal range:

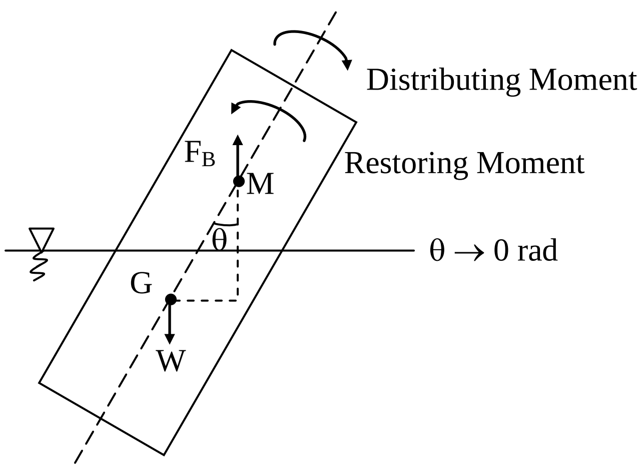
$$x = \frac{U^2 \sin 2\theta}{g}$$

$$= \frac{8^2 \times \sin 60}{9.81}$$

x = 5.65 m

02 (b). (i)

Sol:



Disturbing moment = I α = $mk^2\alpha$

Restoring moment = –W.GM sin θ

$\because \theta \to 0$ rad

$\Rightarrow \sin\theta \to \theta$

$\Rightarrow$ Restoring moment = – W.GM. θ = – mg. GM. θ

For Equilibrium – mg GM θ = $mk^2\ \alpha$

$$\Rightarrow \theta = -\left(\frac{k^2}{gGM}\right)\alpha$$

The above equation represents simple harmonic motion where $\frac{k^2}{g.GM} = \omega^2$ & $T = \frac{2\pi}{\omega}$

$$\therefore T = 2\pi\sqrt{\frac{k^2}{gGM}}$$

(ii) .

Sol:

(a) $u = 4xt + yz;\ \dfrac{\partial u}{\partial x} = 4t$

$v = 4t - 2xyz; \dfrac{\partial v}{\partial y} = -2xz$

$w = xz^2 - 4zt; \dfrac{\partial w}{\partial z} = 2xz - 4t$

$\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 4t - 2xz + 2xz - 4t = 0$

$\therefore$ Continuity equation is satisfied

(b) $\dfrac{Du}{Dt} = \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} + w.\dfrac{\partial u}{\partial z}$

$= 4x + (4xt + yz)4t + (4t - 2xyz)z + (xz^2 - 4zt)y$

$= 4 + 64 + 8$

$\dfrac{Du}{Dt} = 76$ units

$\dfrac{Dv}{Dt} = \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} + w\dfrac{\partial v}{\partial z}$

$= 4 + (4xt + yz)(-2yz) + (4t - 2xyz)(-2xz) + (xz^2 - 4zt)(-2xy)$

$= 4 - 16$

$= -12$ units

$\dfrac{Dw}{Dt} = \dfrac{\partial w}{\partial t} + u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} + w\dfrac{\partial w}{\partial z}$

$= -4z + (4xt+yz)(z^2) + (4t - 2xyz)\,0 + (xz^2 - 4zt)(2xz - 4t)$

$= -4 + 8 + (-7)(-6)$

$= 46$ units

$\vec{a} = \left(76\hat{i} - 12\hat{j} + 46\hat{k}\right)$ units

02 (c).

Sol:

Line	**Length (m)**	**W.C.B.**	**R.B.**	**Quadrant**
AB	89.31	$45^0\ 10'$	$45^0\ 10'$	NE
BC	219.76	$72^0\ 05'$	$72^0\ 05'$	NE
CD	151.18	$161^0\ 52'$	$18^0\ 08'$	SE
DE	159.10	$228^0\ 43'$	$48^0\ 43'$	SW
EA	232.26	$300^0\ 42'$	$59^0\ 18'$	NW

Line AB: Latitude = $89.31 \cos 45^0\ 10' = 62.967$

Departure = $89.31 \sin 45^0\ 10' = 63.335$

Line BC: Latitude = $219.76 \cos 72^0\ 05' = 67.605$

Departure = $219.76 \sin 72^0\ 05' = 209.102$

Line CD: Latitude = $-151.18 \cos 18^0\ 08' = -143.671$

Departure = $151.18 \sin 18^0\ 08' = 47.051$

Line DE: Latitude = $-159.10 \cos 48^0\ 43' = -104.971$

Departure = $159.10 \sin 48^0\ 43' = 119.556$

Line EA: Latitude = $232.26 \cos 59^0\ 18' = 118.578$

Departure = $-232.26 \sin 59^0\ 18' = -199.709$

Summation of latitudes and departures should be equal to zero.

$\Sigma L = 62.967 + 67.605 - 143.671 - 104.971 + 118.578$

or $\Sigma L = +0.508$

$\Sigma D = 63.335 + 209.102 + 47.051 - 119.556 - 199.709$

or $\Sigma D = +0.223$

Closing error. $e = \sqrt{(\Sigma L)^2 + (\Sigma D)^2}$

$= \sqrt{(0.508)^2 + (0.223)^2}$

$= 0.5547$

$\tan\theta = \Sigma D / \Sigma L$

or $\tan\theta = \dfrac{0.223}{0.508}$

or $\theta = 23^0 42'$

The direction (bearing) of the closing error is N$23^0 42'$ E. Since, the error in latitude as well as in departure are both positive the corrections will be negative.

Corrections:

Perimeter of the traverse = 89.31 + 219.76 + 151.18 + 159.10 + 232.26 = 851.61 m

Line AB:

Correction in latitude $= -0.508 \times \dfrac{89.31}{851.61} = -0.0532$

Correction in departure $= -0.223 \times \dfrac{89.31}{851.61} = -0.0233$

Line BC:

Correction in latitude $= -0.508 \times \dfrac{219.76}{851.61} = -0.131$

Correction in departure $= -0.223 \times \dfrac{219.76}{851.61} = -0.0575$

Line CD:

Correction in latitude $= -0.508 \times \dfrac{151.18}{851.61} = -0.090$

Correction in departure $= -0.223 \times \dfrac{151.18}{851.61} = -0.0395$

Line DE:

Correction is latitude $= -0.508 \times \dfrac{159.10}{851.61} = -0.0949$

Correction in departure $= -0.223 \times \dfrac{159.10}{851.61} = -0.0416$

Line EA:

Correction in latitude $= -0.508 \times \dfrac{232.26}{851.61} = -0.1385$

Correction in departure $= -0.223 \times \dfrac{232.26}{851.61} = -0.0608$

Correct Latitudes and Departures:

Line AB:

Latitude = 62.967 - 0.0532 = 62.9138

Departure = 63.335 -0.0233 = 63.3117

Line BC:

Latitude = 67.605 - 0.131 = 67.474

Departure = 209.102 -0.0575 = 209.0445

Line CD:

Latitude = -143.671 - 0.090 = -143.761

Departure = 47.051 - 0.0395 = 47.0115

Line DE:

Latitude = -104.971 - 0.0949 = - 105.0659

Departure = -119.556 - 0.0416 = - 119.5976

Line EA:

Latitude = 118.578 - 0.1385 = 118.4395

Departure = -199.709 - 0.0608 = -199.7698

$\Sigma L = 62.9138 + 67.474 - 143.761 - 105.0659 + 118.4395$

$= 0.0004 \simeq 0$

$\Sigma D = 63.3117 + 209.0445 + 47.0115 - 119.5976 - 199.7698$

$= 0.0003 \simeq 0$

03(a).

Sol:

(i) Given,

Velocity profile : $\frac{u}{U_\infty} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$

We calculate θ (momentum thickness) in terms of δ :

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy = \int_0^\delta \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\left[1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right]dy$$

$$= \int_0^\delta \left[2\left(\frac{y}{\delta}\right) - 5\left(\frac{y}{\delta}\right)^2 + 4\left(\frac{y}{\delta}\right)^3 - \left(\frac{y}{\delta}\right)^4\right]dy$$

$$= \left[\frac{y^2}{\delta} - \frac{5}{3}\frac{y^3}{\delta^2} + \frac{y^4}{\delta^3} - \frac{1}{5}\frac{y^5}{\delta^4}\right]_0^\delta$$

$$\theta = \frac{2\delta}{15} \text{ ------- (1)}$$

From von-Karman momentum integral equation for flat plate,

$$\frac{d\theta}{dx} = \frac{\tau_o}{\rho U_\infty^2} \text{ ----- (2)}$$

where, $\tau_o = \mu\left(\frac{du}{dy}\right)_{y=0} = \frac{2\mu U_\infty}{\delta}$ ------- (3)

Substituting the expressions for τ_o and θ from equations (3) and (1) into equation (2), we have

$$\frac{d}{dx}\left(\frac{2\delta}{15}\right) = \frac{2\mu U_\infty}{\delta \times \rho U_\infty^2}$$

$$\frac{2}{15}\frac{d\delta}{dx} = \frac{2\mu}{\rho U_\infty} \times \frac{1}{\delta}$$

$$\delta\, d\delta = \frac{15\mu}{\rho U_\infty} dx$$

Integrating,

$$\frac{\delta^2}{2} = \frac{15\mu}{\rho U_\infty} \times x$$

$$\delta^2 = 30\frac{x^2}{\frac{\rho U_\infty x}{\mu}}$$

or $$\delta = \frac{\sqrt{30}\,x}{\sqrt{Re_x}}$$

$$\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}}$$ ---- Proved.

(ii) Taking an elementary area, dA on the flat plate, we can write for drag force on that element as :

$dF = \tau_o\, dA = \tau_o\,(B\times dx)$

$$= \frac{2\mu U_\infty}{\delta} B\times dx$$

$$= 2\mu U_\infty B \times \frac{\sqrt{\frac{\rho U_\infty x}{\mu}}}{5.48\times x} dx$$

$$dF = \frac{2\mu U_\infty B}{5.48}\times\sqrt{\frac{\rho U_\infty}{\mu}}\,x^{-\frac{1}{2}}\,dx$$

Integrating, the drag force on one side of the flat plate is,

$$F = \frac{2\mu U_\infty B}{5.48}\sqrt{\frac{\rho U_\infty}{\mu}}\int_0^L x^{-1/2}\,dx$$

$$F = \frac{4\mu U_\infty B}{5.48}\sqrt{\frac{\rho U_\infty L}{\mu}}$$

$$F = 0.73\mu U_\infty B\sqrt{Re_L}$$

03(b).

Sol: Given data:

H_g = 500 m, D_P = 0.8 m,

d = 0.1 m, D = 1.8 m,

f = 0.03, θ = 180°–165° = 15°,

C_v = 0.95, L = 1500 m

(i) $h_f = \frac{fLQ^2}{12.1D_P^5}$, $V = C_V\sqrt{2gH}$

$$\therefore\ H = \frac{V^2}{2g\times C_V^2} = \frac{Q^2}{2g\times\left(\frac{\pi}{4}\right)\times d^4\times C_V^2}$$

Now, $H_g = H + h_f = \frac{Q^2}{12.1d^4C_V^2} + \frac{fLQ^2}{12.1D_P^5}$

$$\therefore\ 500 = \frac{Q^2}{12.1}\left[\frac{1}{0.1^4\times0.95^2} + \frac{0.03\times1500}{0.8^5}\right]$$

⇒ Discharge through a nozzles,

$Q = 0.7344\ m^3/s$

(ii) $H = H_g - h_f = 500 - \frac{0.03\times1500\times0.7344^2}{12.1\times0.8^5}$

⇒ Net head on the turbines, H = 493.9 m

(iii) $V_1 = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.7344}{\frac{\pi}{4}\times0.1^2} = 93.5$ m/s

For maximum efficiency

$$u = \frac{V_1}{2} = \frac{93.5}{2} = 46.75\ \text{m/s}$$

But, $u = \frac{\pi DN}{60}$ i.e. $46.75 = \frac{\pi\times1.8\times N}{60}$

⇒ Wheel speed, N = 496 rpm

(iv) Theoretical maximum wheel efficiency is given by

$$\eta_{w,\max} = \frac{1+K\cos\theta}{2}$$

$$= \frac{1+1\times\cos 15}{2} = 0.983$$

(The blade friction coefficient (K) is assumed be '1')

The overall discharge (Q_o) is given by

$Q_o = 4Q = 4\times0.7344$

$= 2.9376\ m^3/s$

The maximum power developed is given by:

$$\text{Power developed} = \eta_{w,\max}\rho Q\frac{V_1^2}{2}$$

$$= 0.983\times1000\times2.9376\times\frac{93.5^2}{2}$$

$\Rightarrow$ Power developed = 12.622 MW

03(c).

Sol: Gradient = 1 in 250

Distance between pegs = 25 m

$\therefore$ Fall between the RLs of two consecutive pegs = $\frac{1}{250}\times 25 = 0.10\text{m}$

Height of instrument method is used and calculations are carried out as shown in Table 8.5.

Table 8.5

Station	Distance	BI	IS	FS	HI	RL	Remarks
BM						100.00	BM
				1.225		101.900	CP_1
		3.125			103.125		
				3.290		99.640	CP_2
		1.030			102.930		
				2.085		98.850	CP_3
		1.295			100.935		
			1.500			99.205	Peg.1
		1.855			100.705		
			1.600			99.105	Peg.2
			1.700			99.005	Peg.3

Last Peg			1.800 1.900	2.000		98.905 98.805 98.705	Peg.4 Peg.5 Peg.6
	Σ	7.305		8.600			
ΣBS - ΣFS = 7.305 - 8.600=-1.295 Last RL - First RL =-1.295 Checked							

Note: As usual to RL of CP_3 back sight is added to get height of instrument. It is 100.705.

Now we know, HI - IS = RL of peg 1

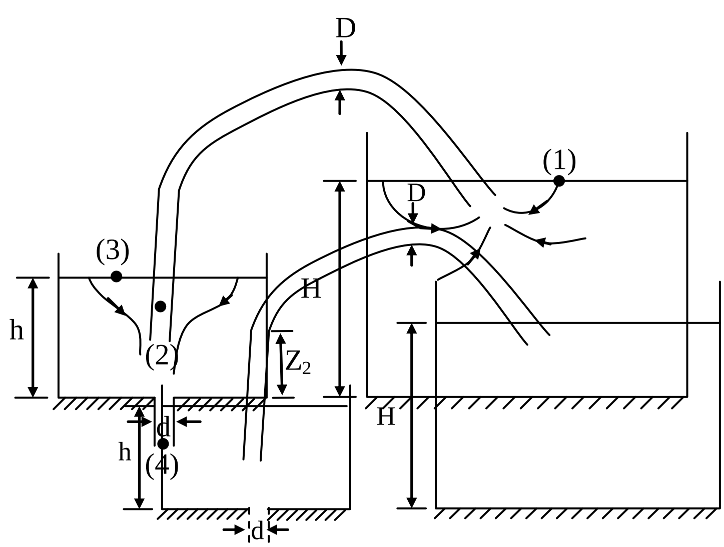
∴ IS for peg 1 = HI - RL of peg 1

= 107.705 - 99.205 = 1.5

Then onwards RL of consecutive pegs is 0.1 m less than previous and hence intermediate sights are 0.1 m more than previous, table is completed and arithmetic check applied.

04. (a)

Sol:



Applying Bernoulli's equation between (1) and (2) we get

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

i.e. $0 + 0 + H = \dfrac{P_2}{\rho g} + \dfrac{V_2^2}{2g} + Z_2$

i.e. $H = \frac{P_3 + (h - Z_2)\rho g}{\rho g} + \frac{V_2^2}{2g} + Z_2$

[$\because$ Hydrostatic law is valid between (2) and (3) as streamlines are parallel]

$\therefore \quad H = \frac{0 + (h - Z_2)\rho g}{\rho g} + \frac{V_2^2}{2g} + Z_2$

$H = h - Z_2 + \frac{V_2^2}{2g} + Z_2$

$V_2 = \sqrt{2g(H - h)}$ ----------(1)

Similarly applying Bernoulli's equation between (3) and (4) we get

$\frac{P_3}{\rho g} + \frac{V_3^2}{2g} + Z_3 = \frac{P_4}{\rho g} + \frac{V_4^2}{2g} + Z_4$

i.e. $0+0+h = 0+\frac{V_4^2}{2g} + 0$

$\therefore \quad V_4 = \sqrt{2gh}$ ---------(2)

As water level in smaller tank remains same

$Q_{in} = Q_{out}$

$\frac{\pi}{4} \times D^2 \times V_2 = \frac{\pi}{4} \times d^2 \times V_4$

$\frac{V_4}{V_2} = \left(\frac{D}{d}\right)^2$

From (1) and (2)

$\frac{\sqrt{2gh}}{\sqrt{2g(H - h)}} = \left(\frac{D}{d}\right)^2$

$\frac{H - h}{h} = \left(\frac{d}{D}\right)^4$

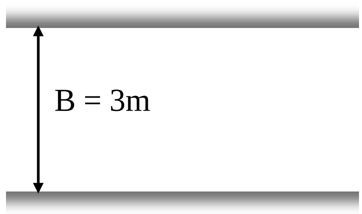
$\frac{H}{h} = 1 + \left(\frac{d}{D}\right)^4$

$\frac{h}{H} = \frac{1}{1 + \left(\frac{d}{D}\right)^4}$

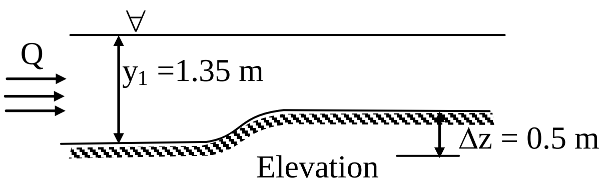
04.(b)

Sol:

(i) Plan



L-section ---------------------------TEL



$Q = 10\ m^3/s$

Upstream condition:

$$V_1 = \frac{Q}{By_1} = \frac{10}{3\times1.35}$$

$V_1 = 2.47$ m/s

$$F_{r_1} = \frac{V_1}{\sqrt{gy_1}}$$

$$= \frac{2.47}{\sqrt{9.81\times1.35}}$$

$$F_{r_1} = 0.67 < 1$$

Flow is subcritical

As the flow is subcritical, when channel gets rises by Δz

(1) $\Delta Z < \Delta Z_{critical} \Rightarrow$ No change in U/S water level & D/S water level drops

(2) $\Delta Z = \Delta Z_c \Rightarrow$ Critical condition attains on D/S

(3) $\Delta Z > \Delta Z_c \Rightarrow$ U/S water level rises & D/S condition remains at critical state

At critical conditions:

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = \frac{Q}{B} = \frac{10}{3} = 3.33 \text{ m}^2\text{/sec}$$

$$y_c = \left(\frac{3.33^2}{9.81}\right)^{1/3}$$

y_c = 1.042 m

$E_1 = E_c + \Delta Z_c$

$$y_1 + \frac{V_1^2}{2g} = \frac{3}{2}y_c + \Delta z_c$$

$$1.35 + \frac{2.47^2}{2\times 9.81} = \frac{3}{2}\times 1.042 + \Delta z_c$$

$\therefore\ \Delta z_c = 0.098$ m

Given hump $\Delta z = 0.25\text{ m} > \Delta z_c$

As the provided hump is more than critical height, u/s water level gets disturbed & d/s condition remains at critical state.

Hence $E_1' = E_c + \Delta z$

$$y_1' + \frac{V_1'^2}{2g} = E_c + \Delta z$$

$$y_1' + \frac{Q^2}{2gB^2y_1'^2} = E_c + \Delta z$$

$$y_1' + \frac{10^2}{2\times 9.81\times 3^2\times y_1'^2} = \frac{3}{2}\times 1.042 + 0.25$$

$$y_1' + \frac{0.567}{y_1'^2} = 1.813$$

By trial & Error

$y_1' = 1.59\text{m}$ and $y_1' = 0.72$ m

As U/S water level should rise

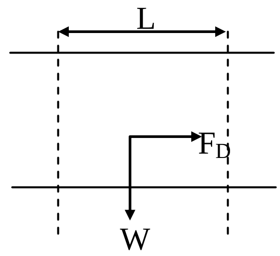
$\therefore\ y'_1 = 1.59$ m

By solving $y'_1 = 1.59$ m

$$y_2 = y_c = 1.042 \text{ m}$$

(ii) Condition for uniform flow:

As channel bed slope is small



$\sin\theta = \theta = S_o$

$\tau_o \rightarrow$ Boundary shear stress

Driving force, $F_D = F_R$, Resisting force

$W \sin\theta = \tau_o \times$ surface area

$\gamma \times A \times L \times S_o = \tau_o \times P \times L$

$\therefore \tau_o = \gamma R S_o$

From Manning's $V = \frac{1}{n} R^{2/3} S_o^{1/2}$

$$S_o = \frac{V^2 n^2}{R^{4/3}}$$

$$\tau_o = \gamma R S_o$$

$$= \gamma R \frac{V^2 n^2}{R^{4/3}}$$

$$\tau_o = \frac{\gamma V^2 n^2}{R^{1/3}}$$

04 (c).

Sol: $L = 60$ m $R = 300$ m $\Delta = 60^0$

Chainage of V = 1750.50 m

Peg interval 15 m for transition curve

30 m for circular curve

Shift, $s = \frac{L^2}{24R} = \frac{60^2}{24 \times 300} = 0.5\text{m}$

Tangent length $TV = (R + s)\tan\frac{\Delta}{2} + \frac{L}{2}$

$$= (300 + 0.5)\tan\frac{60}{2} + \frac{60}{2} = 203.5 \text{ m}$$

Chainage of T = 1750.5 - 203.5

= 1547.0 m

Spiral angle $\phi_1 = \frac{L}{2R} \times \frac{180}{\pi} = \frac{60}{2 \times 300} \times \frac{180}{\pi} = 5.73^0$

Length of circular curve $= R(\Delta - 2\varphi_1) \times \frac{\pi}{180} = 300(60 - 2 \times 5.73) \times \frac{\pi}{180} = 254.16$ m

Chainage of common point E

=1547 + 60 = 1607 m

Chainage of end of circular curve

=1607+254.16

=1861.16m

Chainage of end of combined curve

= 1861.16+60=1921.16 m

Deflection angles:

(a) For first cubic parabola deflection angle = $= 573 \frac{\ell^2}{RL}$ min utes

Point	Chainage	I	Deflection angle
T	1547.0	0	0
1	1560.0	13.0	5'23"
2	1575	28.0	25'0"
3	1590	43.0	58'52"
4	1605	58.0	$1^0$47'5"
E	1607	60.0	$1^0$54'24"

(b) For circular curve, deflection angle 1718.87 $\frac{C}{R}$ minutes.

Point	Chainage	Chord length	δ	Deflection angle
E	1607	0	0	0
1	1620	17	97'24"	$1^0$37'24"
2	1650	30	$2^0$51'53"	$4^0$29'17"
3	1680	30	$2^0$51'53"	$7^0$21'10"
4	1710	30	$2^0$51'53"	$10^0$13'03"
5	1740	30	$2^0$51'53"	$13^0$04'56"
6	1770	30	$2^0$51'53"	$15^0$56'49"

7	1800	30	$2^0 51'53''$	$18^0 48'42''$
8	1830	30	$2^0 51'53''$	$21^0 40'35''$
9	1860	30	$2^0 51'53''$	$24^0 32'28''$
End	1861.16	1.16	$0^0 6'39''$	$24^0 39'07''$

05. (a)

Sol: Given:

$$\frac{u_{max} - V}{V^*} = 3.75$$

We know

$\tau_w = \rho V^{*2}$ -----------(1) (Definition of V^*)

The head loss by Darcy-Weisbach equation is given by

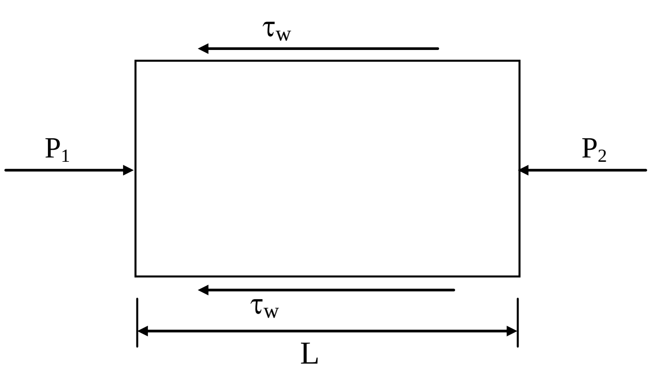
$h_f = \frac{fLV^2}{2gD}$ ------------(2) (Definition of f)

Consider the equilibrium of fluid mass flowing through the pipe

$$(P_1 - P_2) \times \frac{\pi}{4} D^2 = \tau_w \times \pi DL$$

$$P_1 - P_2 = \frac{4\tau_w L}{D}$$

$\therefore \quad h_f = \frac{P_1 - P_2}{\rho g} = \frac{4\tau_w L}{\rho g D}$ ---------- (3)



equating equation (2) and (3)

$$\frac{4\tau_w L}{\rho g D} = \frac{fLV^2}{2gD}$$

$\tau_w = \frac{f}{8}\rho V^2$ --------- (4)

Now, from equation (1) and (4) we get

$$\rho V^{*2} = \frac{f}{8}\rho V^2$$

$$V^* = \sqrt{\frac{f}{8}} \times V$$

$$\frac{u_{max} - V}{V^*} = \frac{u_{max} - V}{\sqrt{\frac{f}{8}}V} = 3.75$$

$$\frac{u_{max}}{V} - 1 = 3.75 \times \sqrt{\frac{f}{8}}$$

$$\frac{u_{max}}{V} = 1 + 3.75 + \frac{1}{\sqrt{8}}\sqrt{f}$$

$$= 1 + 1.33\sqrt{f}$$

05(b).

Sol: For free vortex motion:

$$Vr = C \Rightarrow V = C/r \Rightarrow V_1 = \frac{C}{r_1} \;\&\; V_2 = \frac{C}{r_2}$$

By Bernoulli's equation

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

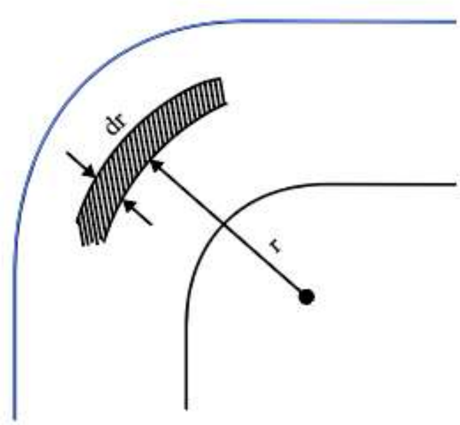
$$\Rightarrow \frac{P_2 - P_1}{\gamma} = \frac{V_1^2 - V_2^2}{2g} \Rightarrow \frac{10}{9.81} = \frac{V_1^2 - V_2^2}{2 \times 9.81}$$

$$\Rightarrow 20 = \left(\frac{C}{0.4}\right)^2 - \left(\frac{C}{0.7}\right)^2 \Rightarrow C^2\left(\frac{1}{0.4^2} - \frac{1}{0.7^2}\right) = 20$$

$$\Rightarrow C = \sqrt{\frac{20}{\left(\frac{1}{0.4^2} - \frac{1}{0.7^2}\right)}}$$

$$\Rightarrow C = 2.18 \text{ m}^2/\text{s}$$

$$dQ = VdA = \frac{C}{r}.dr \times 1$$



$$\Rightarrow Q = C\int_{0.4}^{0.7} \frac{dr}{r}$$

$$\Rightarrow Q = 2.18 \; \ell nr\Big|_{0.4}^{0.7}$$

$$\Rightarrow Q = 1.2198 \text{ m}^3/\text{s}$$

$\therefore$ Discharge is 1219.8 *lt*/sec

05(c).

Sol: Given data:

$\frac{D_m}{D_p} = \frac{1}{5}$, $H_m = 15$ m,

$H_P = 35$ m, $N_p = 420$ rpm,

$P_m = 260$ kW, $\eta_p = 1.03\ \eta_m$

We know that :

$$u \propto V \propto \sqrt{H}$$

i.e., $ND \propto \sqrt{H}$

or, $\frac{N_m}{N_p} \times \frac{D_m}{D_p} = \sqrt{\frac{H_m}{H_p}}$

$$\frac{N_m}{420} \times \frac{1}{5} = \sqrt{\frac{15}{35}}$$

$\Rightarrow N_m = 1375$ rpm

Under ideal condition, the model and prototype have same efficiency. Assuming model and prototype work under homologous conditions.

$$P \propto QH \propto \left(D^2\sqrt{H}\right) \times H$$

$$\propto D^2 H^{3/2}$$

$$\therefore \frac{P_m}{P_p} = \left(\frac{D_m}{D_p}\right)^2 \times \left(\frac{H_m}{H_p}\right)^{\frac{3}{2}}$$

$$\frac{260}{P_p} = \left(\frac{1}{5}\right)^2 \times \left(\frac{15}{35}\right)^{\frac{3}{2}}$$

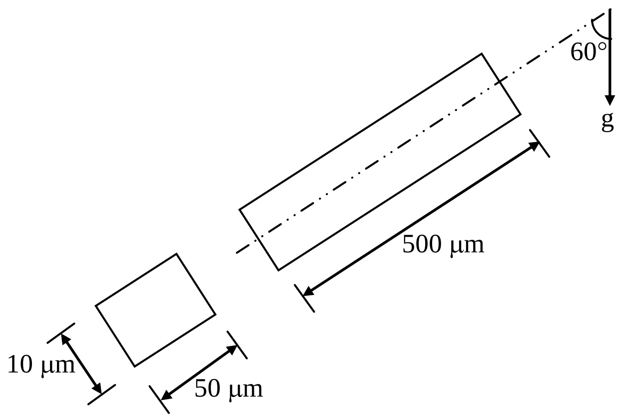
$P_p = 23.18$ MW

The actual protope power will be 3% more than the power predicted by similarity low (given) :

Then, $(P_p)_{actual} = 1.03 \times 23.18 = 23.86$ MW

05 (d).

Sol:



For rectangular channel the diameter is taken as hydraulic diameter (D_h) as

$$D_h = \frac{4A}{P} = \frac{4\times 50\times 30}{2\times(50+30)} = 37.5\,\mu m$$

The velocity of the flow is,

$$V = \frac{Q}{A} = \frac{2.6\times 10^6}{50\times 30} = 1733\,\mu m/s$$

$$= 1.733 \times 10^{-3}\ m/s$$

The Reynolds number of the flow is,

$$Re = \frac{\rho V D_h}{\mu} = \frac{1000\times 1.733\times 10^{-3}\times 37.5\times 10^{-6}}{0.8\times 10^{-3}}$$

$$= 0.081$$

$\Rightarrow$ Flow is laminar

For laminar flow through inclined pipe pressure drop is given by

$$\Delta P^* = \frac{32\mu VL}{D^2} \text{ where } P^* = p + \rho g z$$

$$\therefore \quad (P_1 - P_2) + \rho g (Z_2 - Z_1) = \frac{32\mu VL}{D^2}$$

i.e. $P_1 - P_2 = \frac{32\mu VL}{D^2} + \rho g (Z_2 - Z_1)$

$$= \frac{32\times 0.8\times 10^{-3}\times 1.733\times 10^{-3}\times 500\times 10^{-6}}{(37.5\times 10^{-6})^2} + 9810\times 500\times 10^{-6}\cos 60$$

$$= 15.77 + 2.45 = 18.22\ Pa$$

(% of pressure drop due to gravity)

$$= \frac{2.45}{18.22} \times 100 = 13.44\%$$

for laminar flow friction factor is function of Reynolds number only and is given by

$$f = \frac{64}{Re} = \frac{64}{0.081} = 790.1$$

05(e).

Sol: For Construction of Buildings:

- Buildings should be founded on hard bedrock and never on loose soils or fractured rocks. This is so because loose ground can easily expose to earthquake vibrations.
- Foundation should be of same depth throughout for continuity.
- Buildings situated near hill sides, near steep slopes, on undulating ground or on marshy ground always suffer more when earthquake occurs. Therefore these situations may be avoided.
- Buildings should have light walls.
- Different parts of a building should be well tied together so that the whole structure behaves like a single unit to the vibrations.
- Proper proportionate of cement and mortar should be used.
- Doors and windows should be kept to a minimum and they should not be in vertical rows but preferably along the diagonals.
- The building should have uniform height and additional features such as parapets, cantilevers, domes and arches are undesirable.
- Buildings should have flat RCC roofs and they should be designed not to yield to lateral stress.
- Projections above the roofs are undesirable.

For Construction of Dams:

- Dams being very costly projects their consideration in seismic areas need careful study to ensure their safety precautionary measures which are as follows:
- Forces in the dam due to reservoir water and due to the dam's weight are to counter balanced by introducing additional stress in the design of the dam.

- Design of the dam is to be made such that during an earthquake they move along with the foundations below.
- Dams should not ordinarily be built along or across the faults because possible slipping along these planes during earthquakes will introduce additional complications.
- The resonance factor value (vibrations due to sound) should be given due consideration because a coincidence in the period of vibration of the dam and the earthquake vibrations can produce cumulative effects.

06(a).

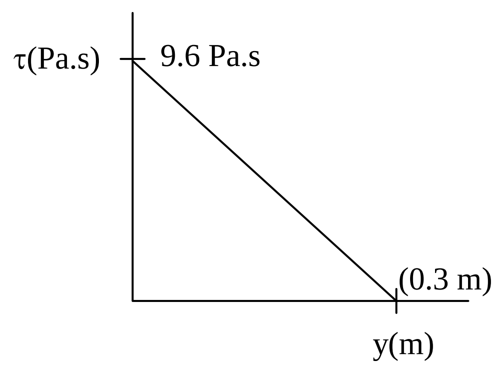
Sol: Given:

Parabolic velocity profile and U /y= 30 cm, = 1.8 m/s

$\mu = 0.8$ Pa.s

To find :

τ @ y = 0 cm, 15 cm & 25 cm



Since the velocity profile is parabolic

$U = Ay^2 + By + C$

@ y = 0; u = 0 (No slip condition)

$\therefore$ C = 0

$\Rightarrow u = Ay^2 + By$

@ y = 0.3 m; U = 1.8 m/s

$\Rightarrow 1.8 = 0.09\,A + 0.3\,B$

$\Rightarrow 3A + 10\,B = 60$

@ y = 0.3 m ; $\frac{du}{dy} = 0$

$\Rightarrow 2Ay + B\big|_{0.3m} = 0$

$\Rightarrow 0.6A + B = 0$

$\Rightarrow B = -0.6\,A$

$\therefore A = -20\ \&\ B = 12$

$\therefore u = -20y^2 + 12y$

$$\frac{\partial u}{\partial y} = -40y + 12$$

@ y = 0; $\frac{du}{dy} = 12s^{-1} \Rightarrow \tau = 0.8 \times 12$

$= 9.6$ Pa.s $\rightarrow$ (1)

@ y = 0.15 m ; $\frac{du}{dy} = 6s^{-1} \Rightarrow \tau = 0.8 \times 6 = 4.8$ Pa.s $\rightarrow$ (2)

@ y = 0.25 m ; $\frac{du}{dy} = 2s^{-1} \Rightarrow \tau = 0.8 \times 2 = 1.6$ Pa.s $\rightarrow$ (3)

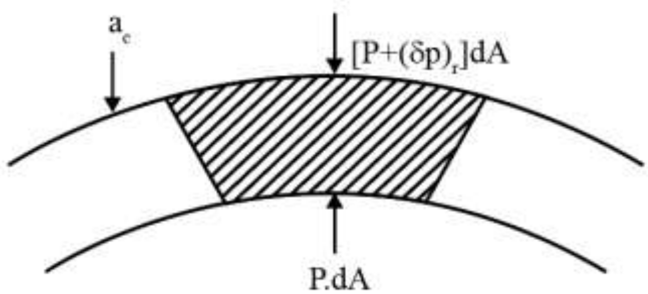
06(b).

Sol: Let us assume a differential fluid element at a distance "r" from the centre, having a thickness of dr

$\Rightarrow (P + (\delta P)_r)\, dA - P\, dA\, .\, dm\, a_c$

$\frac{\partial P}{\partial r} dr.dA = \rho dA\, dr\, \omega^2 r$

$\Rightarrow \frac{\partial P}{\partial r} = \rho \omega^2 r$



P = f(r, z)

$\Rightarrow dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz$

$\frac{\partial P}{\partial r} = \rho \omega^2 r ; \frac{\partial P}{\partial z} = -\rho g$

$dP = \rho \omega^2 r.\, dr - \rho g dz$

Equation of isobars (Lines of constant pressure) :

dP = 0

$\Rightarrow \omega^2\, r\, dr = g dz$

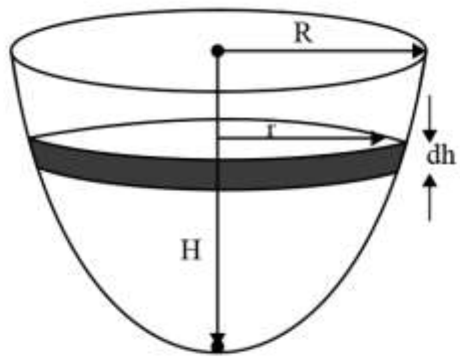
$\because \omega$ = constant in forced vortex motion

$\Rightarrow \omega^2 = \int_{r=o}^{r=r} r.dr = g \int_{z=0}^{z=h} dz$

$$\Rightarrow \frac{\omega^2 r^2}{2} = gh \Rightarrow h = \frac{\omega^2 r^2}{2g}$$

$\because h \propto r^2 \Rightarrow$ The lines of constant pressure (isobars) are parabolic

Volume of parabolaid:



$$d\forall = \pi r^2\, dh$$

$$r^2 = \frac{2gh}{\omega^2}$$

$$\Rightarrow \int d\forall = \frac{\pi 2g}{\omega^2}\int_6^H h\,dh$$

$$\Rightarrow \forall = \frac{2\pi g}{\omega^2}\frac{h^2}{2}\Bigg|_0^H$$

$$= \pi H^2 = \frac{g}{\omega^2}$$

$$= \pi \frac{\omega^2 R^2}{2g}.H.\frac{g}{\omega^2}$$

$$= \frac{\pi R^2 H}{2}$$

$\forall$ Parabolaid = $\frac{1}{2}$ $\forall$ Circumscribing cylinder

Hence Proved

06(c).

Sol: Determine the mean μ of the measurements, residuals υ, and υ^2 as shown in Table below

Number of observation n = 10

(i) Standard deviation

$$\sigma = \pm\sqrt{\frac{\Sigma \upsilon^2}{n-1}}$$

$$= \pm\sqrt{\frac{4.800\times 10^{-3}}{10-1}} = \pm 0.023\text{m.}$$

Table

Measurements (x) (m)	**Residual (υ=μ-x)**	υ^2
53.56	-0.004	1.600×10^{-5}
53.52	0.036	1.296×10^{-3}
53.54	0.016	2.560×10^{-4}
53.58	-0.024	5.760×10^{-4}
53.55	0.006	3.600×10^{-5}
53.60	-0.044	1.936×10^{-3}
53.54	0.016	2.560×10^{-4}
53.57	-0.014	1.960×10^{-4}
53.54	0.016	3.600×10^{-5}
53.56	-0.004	1.960×10^{-4}
$\mu = \frac{535.56}{10} = 53.556$	$\sum\upsilon = -0.000$	$\sum\upsilon^2 = 4.800\times 10^{-3}$

(ii) standard error of the mean

$$\sigma_m = \pm\sqrt{\frac{\Sigma\upsilon^2}{n(n-1)}}$$

$$= \pm\sqrt{\frac{4.800\times 10^{-3}}{10\times(10-1)}} = \pm\mathbf{0.007m.}$$

(iii) Most probable error

$e = \pm\, 0.6745\sigma$

$= \pm\, 0.6745 \times 0.023 =$ **± 0.016 m**

(iv) Most Probable error of the mean

$e_m = \pm\, 0.6745\, \sigma_m$

$= \pm\, 0.6745 \times 0.007 =$ **± 0.005 m**

(v) Variance

$V = \sigma^2$

$= (\pm\, 0.023)^2 =$ **0.005 m²**

(vi) Maximum error

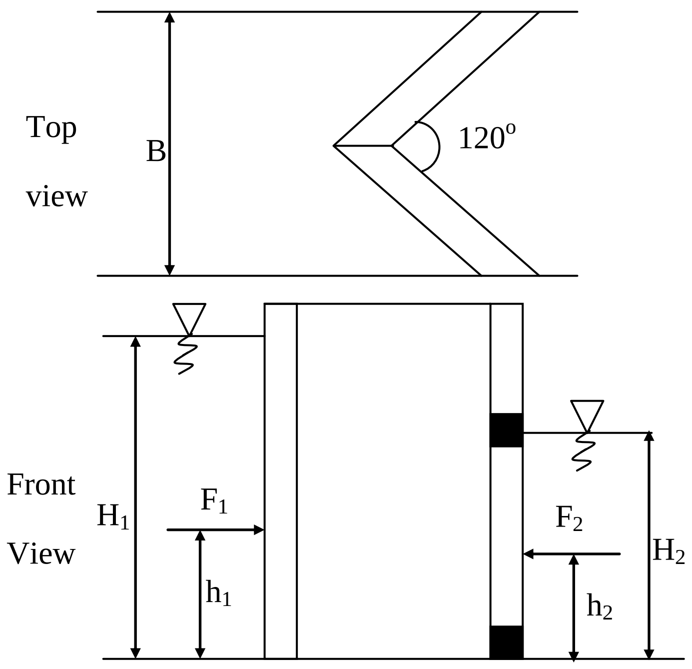
$e_{max} = \pm 3.29\ \sigma$

$e_{max} = \pm\ 3.29 \times 0.023$

$= \pm$ **0.07567 m**

07(a).

Sol:



Given:

B = 8 m

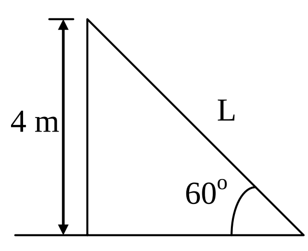
$H_1 = 9$m

$H_2 = 6$ m

$\gamma = 9.81$ kN/m^3

For each gate :

$L = 4/\sin 60 = \frac{8}{\sqrt{3}}$



$F_1 = \gamma\, A_1\, \bar{h}_1 = 9.81 \times \left(\frac{8}{\sqrt{3}} \times 9\right) \times \frac{9}{2}$

$\Rightarrow F_1 = 1835$ kN

$F_2 = \gamma A_2 \bar{h}_2 = 9.81 \times \left(\frac{8}{\sqrt{3}} \times 6\right) \times \frac{6}{2}$

$\Rightarrow F_2 = 815.58$ kN

$F = F_1 - F_2$

$\Rightarrow F = 1019.412$ Kn

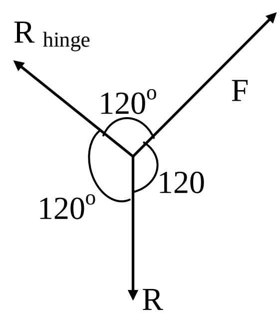
By Varignons Thorem:

$F_1h_1 - F_2h_2 = F.h$

$$\Rightarrow h = \left(\frac{(1835 \times 3) - (815.58 \times 2)}{1019.412}\right)$$

$\Rightarrow h = 3.8$ m

By Lamis Theorem for each gate:



$$\frac{F}{\sin 120} = \frac{R_{hinge}}{\sin 120}$$

$\Rightarrow R_{hinge} = 1019.412$ kN

By Varignons theorm:

$R_1 + R_2 = R_{hinge}$

$R_1 (0) + R_2 (6) = R_{hinge}(3.8)$

$R_1 = 373.7844$ kN

$R_2 = 645.6276$ kN

07(b).

Sol: Given data:

$H_s = 2$ m, $H_d = 12$ m,

$Q = 0.3\ m^3/s$, $h_{fs} = 0.5$ m,

$h_{fd} = 3$ m, $P_{atm} = 98$ kPa,

$P_v = 2.3$ kPa, NPSHR = 6.3 m,

Margin = 0.5m, d = 15 cm.

$$H_a = \frac{P_{atm}}{\rho g} = \frac{98 \times 10^3}{1000 \times 9.81} = 10 \text{ m}$$

$$H_v = \frac{P_v}{\rho g} = \frac{2.3 \times 10^3}{1000 \times 9.81} = 0.23 \text{ m}$$

NPSHA = $H_a - H_s - H_v - h_{fs}$

$= 10 - 2 - 0.23 - 0.5 = 7.27$ m

NPSHR + Margin = 6.3 + 0.5 = 6.8 m

NPSHA> NPSHR + Margin

⇒ The installation is safe with respect to cavitation.

The manometric head is given by :

$$H_m = H_s + H_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g} = 2+12+0.5+3+\frac{\left(\frac{0.03}{(\pi/4)\times 0.15^2}\right)^2}{2\times 9.81} = 17.65 \text{ m}$$

The pumping power is then,

$$P = \frac{\rho g Q H_m}{\eta_o} = \frac{9810\times 0.03\times 17.65}{0.75} = 6.925 \text{ kW}$$

07(c).

Sol:

(i) The discharge per unit width over spillway 'q' is

$$q = \frac{2}{3}C_d\sqrt{2g}H^{3/2}$$

H = 143 – 140 = 3 m

$$q = \frac{2}{3}\times 0.735\times\sqrt{2\times 9.81}\times 3^{3/2}$$

$\therefore$ q = 11.28 m^2/s

E_1 = 143 – 106 = 37 m

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{q^2}{2gy_1^2} = 37$$

$$y_1 + \frac{11.28^2}{2\times 9.81\times y_1^2} = 37$$

$$y_1 + \frac{6.485}{y_1^2} = 37$$

$$y_1^3 - 37y_1^2 + 6.485 = 0$$

By trial & error method, y_1 = 0.421 m

$$V_1 = \frac{q}{y_1} = \frac{11.28}{0.421} = 26.79 \text{ m/s}$$

$$F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{26.79}{\sqrt{9.81 \times 0.421}} = 13.18$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8F_1^2}\right) = \frac{1}{2}\left(-1 + \sqrt{1 + 8 \times (13.18)^2}\right)$$

$$\frac{y_2}{y_1} = 18.15$$

$\therefore$ y_2 = 7.64 m

Tail water level elevation = 106 + 7.64 = 113.64 m

(ii) Energy loss, $E_L = \frac{(y_2 - y_1)^3}{4y_1y_2}$

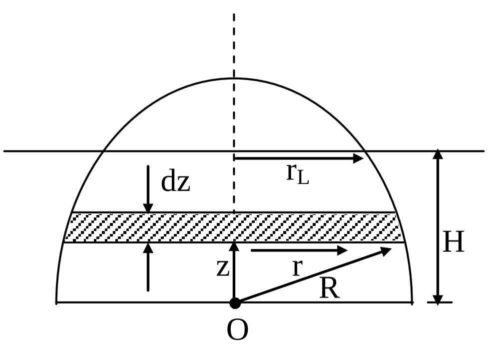
$$E_L = \frac{(7.64 - 0.421)^3}{4 \times 7.64 \times 0.421}$$

E_L = 29.24 m

% Initial energy lost $\Rightarrow \frac{E_L}{E_1} = \frac{29.24}{37} \times 100$ = 79%

08(a).

Sol:



Let R be the radius of semicircle

H be the depth of submergence

r_L be the radius at the water line

$\Sigma Fy = 0$

$\Rightarrow W = F_B \Rightarrow \gamma_b \frac{2\pi}{3} R^3 = \gamma_f \forall \Rightarrow \forall = \frac{S_b}{S_f} \frac{2\pi}{3} R^3$

$$OG = \frac{\int \pi rzdz}{\frac{2\pi}{3}R^3} \Rightarrow OG = \frac{\int_o^R \pi\left(R^2 - z^2\right)zdz}{\frac{2\pi}{3}R^3} = \frac{3}{2R^3}\int_o^R \left(R^2 \times z - z^3\right)dz$$

$$\Rightarrow OG = \frac{3}{2R^3}\left[\frac{R^2z^2}{2} - \frac{z^4}{4}\right]_O^R$$

$$\Rightarrow OG = \frac{3}{2R^3}\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$\Rightarrow OG = \frac{3}{8}R$$

$$OB = \frac{\int_o^H \pi r^2 zdz}{\frac{S_b}{S_f}.\frac{2\pi}{3}R^3} = \frac{S_f}{S_b}\frac{3}{2R^3}\int_0^H \left(R^2 - z^2\right)zdz$$

$$\Rightarrow OB = \frac{S_f}{S_b}\frac{3}{2R^3}\left[\frac{R^2z^2}{2} - \frac{z^4}{4}\right]_0^H = \frac{S_f}{S_b}\frac{3}{2R^3}\left[\frac{R^2H^2}{2} - \frac{H^4}{4}\right]$$

$$\Rightarrow OB = \frac{S_f}{S_b}.\frac{3R}{8}\frac{H^2}{R^2}\left(2 - \frac{H^2}{R^2}\right)$$

$$I = \frac{\pi}{4}r_i^4$$

$$GM = \frac{I}{\forall} - BG = \frac{I}{\forall} - \left(OG - OB\right) = \frac{I}{\forall} - OG + OB$$

$$\Rightarrow GM = \frac{\frac{\pi}{4}r_i^4}{\frac{S_b}{S_f}\frac{2\pi}{3}R^3} - \frac{3}{8}R + \frac{S_f}{S_b}\frac{3R}{8}\frac{H^2}{R^2}\left(2 - \frac{H^2}{R^2}\right)$$

$$= \frac{3R}{8}\left[\frac{S_f}{S_b}\frac{r_i^4}{R^4} - 1 + \frac{S_f}{S_b}\left(\frac{R^2 - r_i^2}{R^2}\right)\left(2 - \left(\frac{R^2 - r_i^2}{R^2}\right)\right)\right] = \frac{3R}{8}\left[\frac{S_f}{S_b}\frac{r_i^4}{R^4} - 1 + \frac{S_f}{S_b}\left(1 - \frac{r_i^4}{R^4}\right)\right]$$

$$GM = \frac{3R}{8}\left(\frac{S_f}{S_b} - 1\right)$$

08(b).

Sol: **<u>SENSORS:</u>** Sensors are the devices used for making observations. They consist of sophisticated lenses with filter coating. They are designed to operate on different spectoral bands of visible and infrared. each sensor has its own characteristics for detecting reflected energy. The following are the four main characteristics of sensors:

1. Special Resolution
2. Spectral Resolution
3. Temporal Resolution, and
4. Radiometric Resolution.

Special Resolution: It may be defined as the smallest object that can be detected and distinguished. Its measure is the instantaneous field of view (IFOV) which is the area of the surface which is viewed by a sensor at a given time, Spatial resolution 72.5 m × 72.5 m indicates that an area of 72.5 m × 72.5 m is represented by a pixel of the image. IRS 1C and 1D can provide much finer details since they have station resolution of 5.8 m × 5.8 m. Spatial resolution plays an important role in identifying various features on the earth's surface.

Spectral Resolution: It is the width of the spectral band in which image is taken. There can be more number of bands (multiband) to cover wider area. Narrower the spectral resolution more is the required number of bands to cover the required area. The use of narrower band widths allow better identification and classification of the objects. It is important to select the correct spectral resolution for the type of information to be obtained from the image.

Temporal Resolution: Temporal resolution refers to observing the same object on different dates. This is also called multistage observation. Obviously, more frequent remote sensing captures changes in environmental phenomena. High temporal resolution helps in studying conditions of crops, deforestation, etc. They are useful to identify fires and volcanoes also.

Radiometric Resolution: Radiometric resolution is the smallest difference in radiant energy that can be detected by a sensor. It is applicable to both photographs and digital images. Higher contrast films can resolve smaller differences for digital images, radiometric resolution refers to the number of discrete levels into which the digital signal may be divided during the analog to digital conversion.

TYPES OF SENSORS:

The sensor systems can be classified as

1. Multispectral Imaging Sensor Systems
2. Thermal Remote Sensing Systems, and
3. Microwave Radar Sensing Systems.

Multispectral Imaging Sensor System: They use cameras along with filters to capture photos in the visible band. This way electromagnetic energy can be recorded by scanning ground. They need sunlight to operate and hence may be grouped under passive system. Television cameras are also passive system. Images of TV cameras are formed as pattern of electrical charges in an image plate which is scanned by an electron beam and converted to electrical signal.

Thermal Remote Sensing System: These detectors also use passive energy, i.e. energy of sunlight. The temperature of air reduces as we go away from the earth's surface. Hence, by using thermal scanners it is possible to identify at what distance from the earth surface electromagnetic energy is reflected. It is very useful to find the clouds and trace cloud movements. These systems operate in either 3 to 5 μm or 8 to 14 μm range of wavelength.

Microwave Remote Sensing System: These are the remote sensing system which rely on the microwaves generated, beamed to earth and recorded on their return. Hence, they may be called as active systems. They work on microwave length of electromagnetic energy. Radar and antenna are used in these systems. Micro-waves beamed from satellites penetrate atmosphere without getting distorted but return with depleted energy due to absorption and transmittance. The reflected energy is measured by these sensors and used to identify the object that reflect the energy.

08(c).

Sol:

(i) For a closed traverse $\sum L = 0$, $\sum D = 0$

Total error in latitude $e_L = 240 + 160 - 239 - 160$

$= 1$

Total error in departure $e_D = 239 + 160 - 160 - 240$

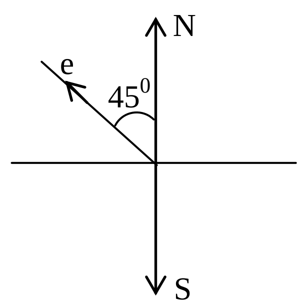
$= -1$

Magnitude of closing error = $\sqrt{(e_D)^2 + (e_L)^2} = \sqrt{(1)^2 + (-1)^2} = \sqrt{2} = 1.414\,\text{m}$

Direction of closing error $\delta = \tan^{-}\left(\frac{e_D}{e_L}\right) = \tan^{-1}\left(\frac{-1}{1}\right) = 45^0$

$\therefore$ Departure is negative, it is lying on the 4th quadrant W.C.B of closing error = $360^0 - 45^0$

$= 315^0$



(ii) Arithmetic sum of latitude = 240 + 160 + 239 + 160 = 799

Total error in latitude = 1

Correction to latitude of AB $= \frac{-1 \times \text{Latitude of AB}}{\text{Arithmetic sum of latitude}}$

$= \frac{-1 \times 160}{799} = -0.2002\,\text{m}$

Arithmetic sum of departure = 239 + 160 + 160 + 240 = 799

Total error in departure = – 1

Correction to departure of AB = $\frac{+1 \times \text{Departure of AB}}{\text{Arithmetic sum of departure}}$

$= \frac{1 \times 239}{799} = 0.2991$ m

Hence corrected consecutive coordinates of station B are

Northing = 160 – 0.2002 = 159.7998 m

Easting = 239 + 0.2991 = 239.2991 m

(iii) Independent coordinates of station B; if those of A are (80, 80)

Northing = 80 + 159.7998 = 239.9998 m ≈ 239.80 m

Easting = 80 + 239.2991 = 319.2991 m ≈ 319.30 m