ACE
Engineering Academy

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ESE-2019 MAINS TEST SERIES
Question Cum Answer Booklet (QCAB)

Civil Engineering **Test-1** – SOLUTIONS **Paper-I**

Time Allowed: 3 Hours **Maximum Marks: 300**

ACE HALL TICKET No. :

HALL TICKET No. : (Issued by UPSC)

NAME OF THE CANDIDATE :

NAME OF THE CENTRE :

BRANCH :

BATCH :

ROLL No. :

MOBILE No. :

TEST CODE : 901

DATE : 30-03-2019

INSTRUCTIONS TO CANDIDATES:

- This Question-cum- Answer (QCA) Booklet contains **80** pages. Immediately on receipt of booklet, please check that this QCA booklet does not have any misprint or torn or missing pages or items, etc. If so, get it replaced by a fresh QCA booklet.
- Candidates must read the instructions on this page and the following pages carefully before attempting the paper.
- Candidates should attempt all questions strictly in accordance with the specified instructions and in the space prescribed under each question in the booklet. Any answer written outside the space allotted may not be given credit.
- Question Paper in detachable form is available at the end of the QCA booklet and can be removed and taken by the candidates after conclusion of the exam

For filling by Examiners only

Question No.	Page No.	Marks
1	03	
2	12	
3	21	
4	32	
5	41	
6	50	
7	60	
8	67	
Grand Total		

Signature of the Invigilator

Signature of the Student

Marks Secured after Scrutiny

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions:

*Answers must be written in **ENGLISH** only.*

*There are **EIGHT** questions divided in **TWO** sections.*

*Candidate has to attempt **FIVE** questions in all.*

*Questions no. **1** and **5** are **compulsory** and out of the remaining **THREE** are to be attempted choosing at least **ONE** question from each section.*

The number of marks carried by a question/part is indicated against it.

Answers must be written in the medium authorized in the Admission Certificate which must be stated clearly on the cover of this Question-Cum-Answer(QCA) Booklet in the space provided. No marks will be given for answer written in a medium other than the authorized one.

Wherever any assumptions are made for answering a question, they must be clearly indicated. Diagrams/figures, wherever required, shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations carry their usual standard meanings. Attempt of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-Cum-Answer Booklet must be clearly struck off.

DONT'S:

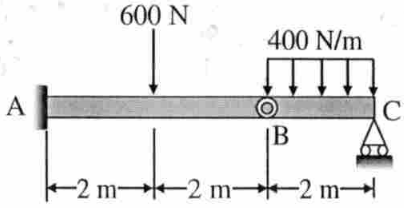
1. Do not write your Name or Roll number or Sr. No. of Question-Cum-Answer-Booklet anywhere inside this Booklet. Do not sign the "Letter Writing" questions, if set in any paper by name, nor append your roll number to it.
2. Do not write anything other than the actual answers to the questions anywhere inside your Question-Cum-Answer-Booklet.
3. Do not tear off any leaves from your Question-Cum-Answer-Booklet. If you find any page missing, do not fail to notify the Supervisor/invigilator.
4. Do not write anything on the Question Paper available in detachable form or admission certificate and write answers at the specified space only.
5. Do not leave behind your Question-Cum-Answer-Booklet on your table unattended, it should be handed over to the Invigilator after conclusion of the exam.

DO'S :

1. Read the instructions on the cover page and the instructions specific to this Question Paper mentioned on the next page of this Booklet carefully and strictly follow them.
2. Write your Roll number and other particulars, in the space provided on the cover page of the Question-Cum-Answer-Booklet.
3. Write legibly and neatly. Do not write in bad/illegible handwriting.
4. For rough notes or calculations the last two blank pages of this booklet should be used. The rough notes should be crossed through afterwards.
5. If you wish to cancel any work, draw your pen through it or write "Cancelled" across it, otherwise it may be valued.
6. Hand over your Question-Cum-Answer-Booklet personally to the invigilator before leaving the examination hall.
7. **Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.**

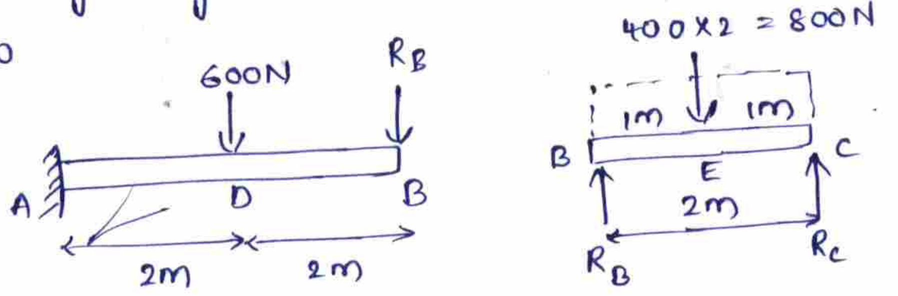
SECTION - A

01(a). The compound beam is fixed at A, pin connected at B, and supported by a roller at C. Draw the shear force diagram for the beam.



(12 M)

Free body diagrams of each beam are shown below



Beam BC

Due to symmetory of the beam reactions can be written as

$$R_B = R_C = \frac{800}{2} = 400N$$

Shear force: $SF_C = -R_C = -400N$

$SF_B = -R_C + 400 \times 2 = -400 + 800$

$= 400N$

Beam AB

Shear Force: $SF_B = 400N$

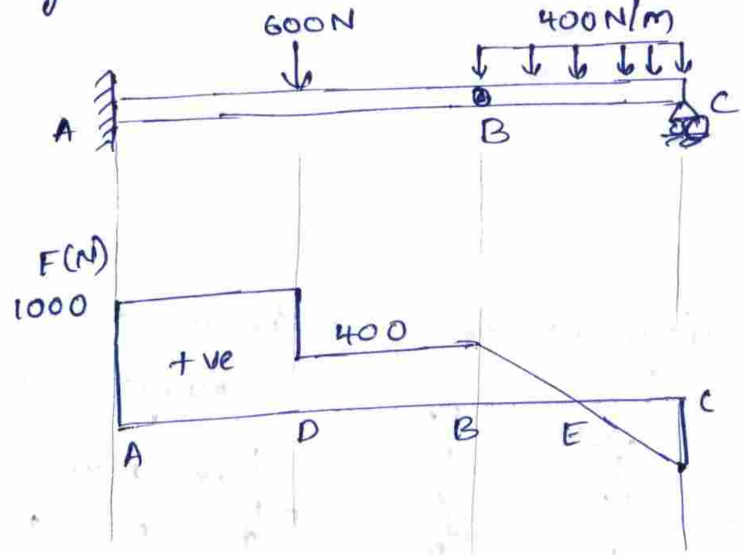
$SF_D^{Right} = SF_B = 400N$

$SF_D^{left} = SF_D^{Right} + 600 = 400 + 600$

$= 1000N$

$SF_A = SF_D^{left} = 1000N$

From the above values calculated as above, shear force diagram can be drawn as shown below

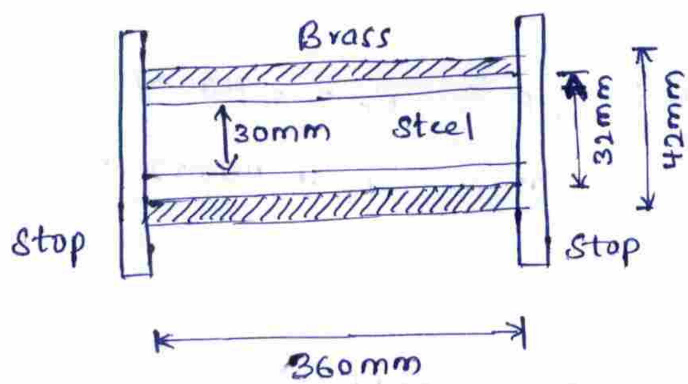


01(b). A steel rod of 30 mm diameter is enclosed in a brass tube of 42 mm external diameter and 32 mm internal diameter. Each is 360 mm long and the assembly is rigidly held between two stops 360 mm apart. The temperature of the assembly is then raised by 50°C. Determine (i) Stresses in the tube and the rod

(ii) Stresses in the tube and the rod if the stops yields by 0.15 mm

E_s = 205 GPa, E_b = 90 GPa,

$\alpha_s = 11 \times 10^{-6}/^\circ C$; $\alpha_b = 19 \times 10^{-6}/^\circ C$ **(5+7 = 12 M)**



$$A_s = \left(\frac{\pi}{4}\right) 30^2 = 225\pi \text{ mm}^2$$

$$A_b = \left(\frac{\pi}{4}\right)\left[42^2 - 32^2\right] = 185\pi \text{ mm}^2$$

Candidates must not write on this margin

(i) when the temperature is raised by 50°c

Stress in the steel rod $= \alpha_s . t . E_s$

$= 11 \times 10^{-6} \times 50 \times 205000$

$\boxed{\sigma_s = 112.75 MPa (compressive)}$

stress in the brass tube $= \alpha_b . t . E_b$

$= 19 \times 10^{-6} \times 50 \times 90000$

$\boxed{\sigma_b = 85.5 MPa (compressive)}$

(ii) If the stops yields by 0.15mm,

$$\Delta_s = (\alpha_s . L . t - 0.15) = \frac{\sigma_s . L}{E_s}$$

(or)

$$\sigma_s = \alpha_s . t . E_s - \frac{0.15 E_s}{L}$$

$$= 112.75 - \frac{0.15 \times 205000}{360}$$

$$= 112.75 - 85.42$$

$\boxed{\sigma_s = 27.33 MPa (compressive)}$

And $\Delta_b = (\alpha_b . L . t - 0.15) = \frac{\sigma_b . L}{E_b}$

(or)

$$\sigma_b = \alpha_b . t . E_b - \frac{0.15 E_b}{L}$$

$$= 85.5 - \frac{0.15 \times 90000}{360}$$

$$= 85.5 - 37.5$$

$\boxed{\sigma_b = 48 MPa (compressive)}$

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01(c). Determine the maximum stress in the steel rod shown in figure caused by a mass of 4 kg falling freely through a distance of 1 m. Consider two cases: One as shown in figure and other when the rubber washer is removed. $E_{steel\ rod}$ = 200 GPa, K_{washer} = 4.5 N/mm

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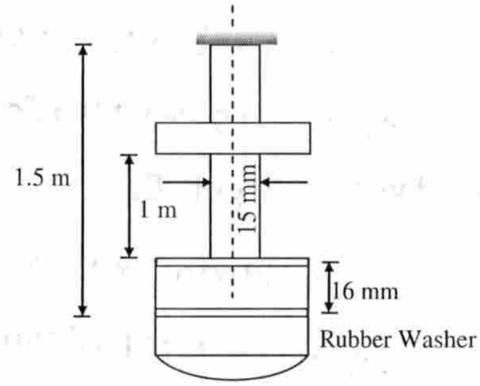

(12 M)

$W = mg = 4 \times 9.81 = 39.24\,N$

i.e $P = 39.24\,N$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(15)^2 = 176.71\ mm^2$

Case 1: Rod with washer:

$(\delta l_{inst}) = (\delta l)_{rod} + (\delta l)_{washer}$

$= \left(\frac{Pl}{AE}\right) + \left(\frac{P}{K}\right)$

$= \left(\frac{39.24 \times 1500}{176.71 \times 200 \times 10^3}\right) + \left(\frac{39.24}{4.5}\right)$

$\boxed{(\delta l_{inst}) = 8.722\ mm}$

$(\sigma_{inst})_{max} = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2h}{\delta l_{inst}}}\right]$

$$= \frac{39.24}{176.71}\left[1+\sqrt{1+\frac{2\times1000}{8.722}}\right]$$

$$\boxed{(\sigma_{inst})_{max} = 3.59\ MPa}$$

Case 2: Rod without Washer:

$$\delta l_{inst} = \frac{Pl}{AE}$$

$$= \frac{39.24\times1500}{176.71\times200\times10^{3}}$$

$$\boxed{\delta l_{inst} = 1.665\times10^{-3}\ mm}$$

$$(\sigma_{inst})_{max} = \frac{W}{A}\left[1+\sqrt{1+\frac{2h}{\delta l_{inst}}}\right]$$

$$= \frac{39.24}{176.71}\times\left[1+\sqrt{1+\frac{2\times1000}{1.665\times10^{-3}}}\right]$$

$$\boxed{(\sigma_{inst})_{max} = 243.6\ MPa}$$

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01(d). **(i)** In a circular shaft of radius 'R' and length L, U is the total strain energy stored. What will be strain energy stored in the central portion of radius R/2? (Consider torsional strain energy) **(4 M)**

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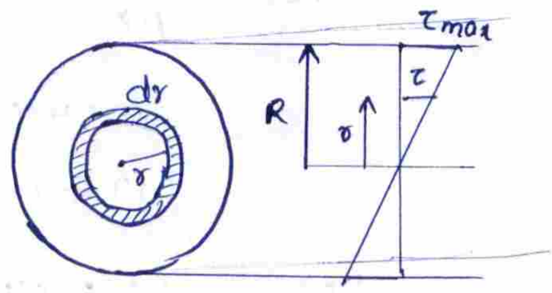
Strain energy stored per unit volume

$$= \frac{\tau^2}{2G}$$

$$dU = \frac{\tau^2}{2G} dV$$

$$U' = \int \frac{\tau^2}{2G}(2\pi r dr).L$$



$$U' = \frac{2\pi L}{2G}\int \tau^2.r.dr$$

$$= \frac{2\pi L}{2G}\int_0^{R/2} \frac{\tau^2 max}{R^2}.r^2.r.dr \qquad \left[\frac{\tau_{max}}{R} = \frac{\tau}{r}\right]$$

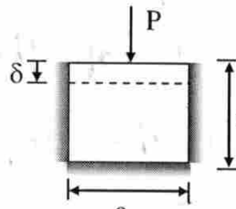
$$= \frac{\pi L}{G}.\frac{\tau^2_{max}}{R^2}\int_0^{R/2} r^3 dr$$

$$= \frac{\pi L}{G}.\frac{\tau^2_{max}}{R^2}\left(\frac{r^4}{4}\right)_0^{R/2}$$

$$= \frac{1}{16}\left[(\pi R^2 L)\frac{\tau^2_{max}}{4G}\right]$$

$$\boxed{U' = \frac{1}{16}U}$$

01(d) (ii) A cube of side "a" is constrained on five faces and the top face is free as shown in figure. Derive an expression for δ in terms of P, a, E and μ.



(8 M)

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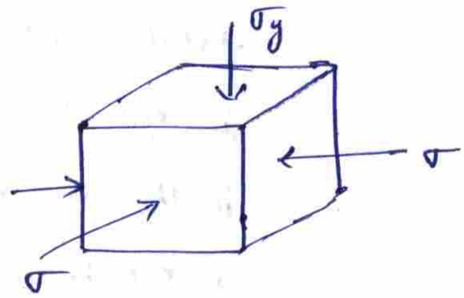
$\sigma_y = -\frac{P}{a^2}$. Let $\sigma_x = \sigma_z = -\sigma$

Since the cube is constrained in x-direction,

$\epsilon_x = 0$



$$\frac{\sigma_x - \mu\cdot\sigma_y - \mu\cdot\sigma_z}{E} = 0$$

$$\sigma_x - \mu\cdot\sigma_y - \mu\cdot\sigma_z = 0 \Rightarrow -\sigma - \mu\left(\frac{-P}{a^2}\right) - \mu(-\sigma) = 0$$

$$-\sigma(1-\mu) + \frac{\mu P}{a^2} = 0$$

$$\sigma = \frac{\mu P}{(1-\mu)a^2} \quad \text{......} \ (i)$$

Now, $\epsilon_y = \frac{-\mu\cdot\sigma_x + \sigma_y - \mu\cdot\sigma_z}{E} = -\frac{\mu}{E}(\sigma_x + \sigma_z) + \frac{\sigma_y}{E}$

$$= -\frac{\mu}{E}(-\sigma - \sigma) + \frac{\sigma_y}{E} = \frac{2\mu\sigma}{E} + \frac{\sigma_y}{E}$$

$$\Rightarrow \epsilon_y = \frac{2\mu^2 P}{(1-\mu)Ea^2} + \frac{-P}{a^2 E} = \frac{P}{Ea^2}\left\{\frac{2\mu^2}{1-\mu} - 1\right\}$$

$$= \frac{P}{Ea^2}\left(\frac{2\mu^2 + \mu - 1}{1-\mu}\right) \Rightarrow \frac{\delta}{a} = \frac{P}{Ea^2}\left(\frac{2\mu^2 + \mu - 1}{1-\mu}\right)$$

$$\boxed{\delta = \frac{P}{Ea}\left(\frac{2\mu^2 + \mu - 1}{1-\mu}\right)}$$

01(e) **(i)** What is critical path? Why the critical path of such importance in project scheduling? **(4 M)**

Critical path is the longest path in a network which covers first and last events of the project network.

Features are: (1) It decides the project completion time.

(2) It contains zero float activities and totat float along the critical path is zero,

(3) It contains zero slack events and total slack along the critical path is zero.

(4) It is useful for the project crashing in the case of CPM network.

(5) It is used to decide the project duration distribution.

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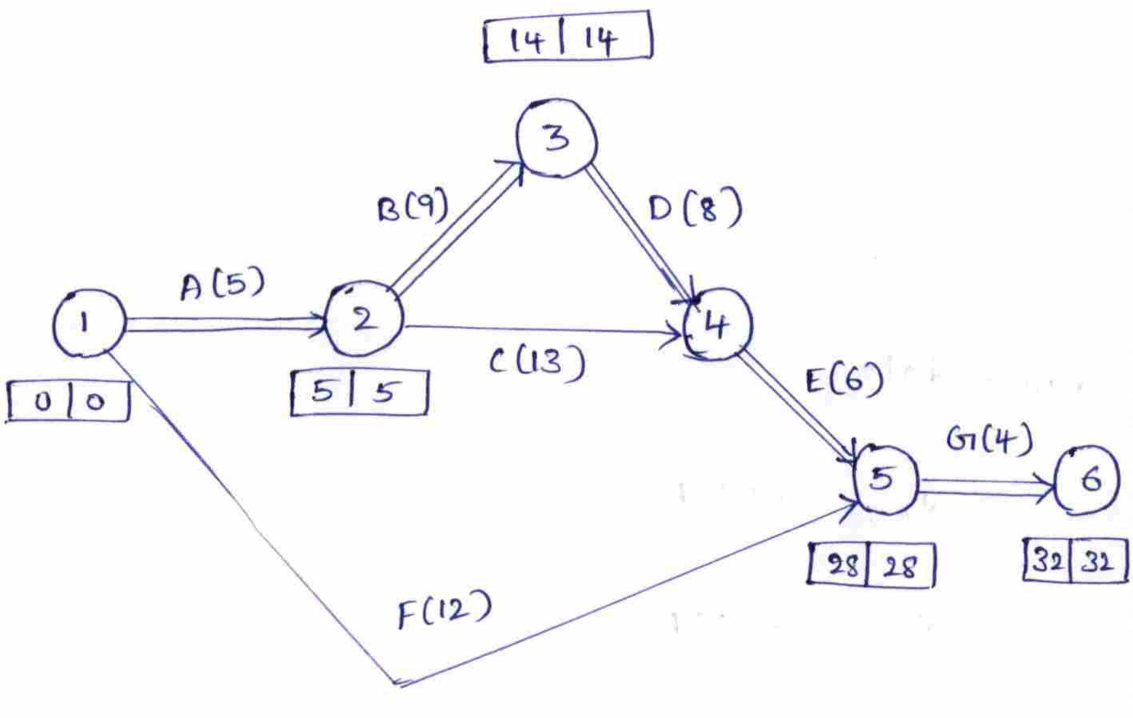
01(e). **(ii)** A project consists of 7 jobs with the following details.

Job	Optimistic time (days)	Pessimistic time (days)	Most likely time (days)	Predecessor
A	3	7	5	-
B	7	11	9	A
C	4	18	14	A
D	4	12	8	B
E	4	8	6	C, D
F	5	19	12	-
G	2	6	4	E, F

Draw the network and determine the critical path, and its expected duration (T_e). What is the probability of completing the project in 'T_e' days? **(8 M)**

Job	$t_E = \frac{t_o + 4t_m + t_p}{6}$
A	5
B	9
C	13
D	8
E	6
F	12
G	4



Critical path : 1 – 2 – 3 – 4 – 5 – 6

(A – B – D – E – G)

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Candidates must not write on this margin

Expected completion time of the project

$(T_E) = 32$ days

Probability to complete the project in due date

$(x) = T_E$

$$Z = \frac{x - \mu}{\sigma}$$

$$= \frac{32 - 32}{\sigma} = 0$$

$P(Z) = 0.5$ or 50%

02(a). For the given state of stress, determine :

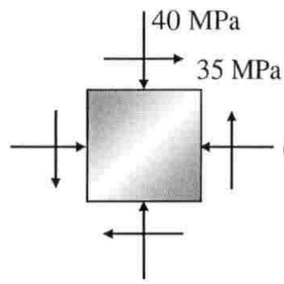
(i) the principal planes,

(ii) the principal stresses,

(iii) the orientation of the planes of maximum in-plane shearing stress,

(iv) the maximum in-plane shearing stress,

(v) the normal stress corresponding to maximum in-plane shear stress plane.



(5×4 = 20 M)

Given data

$\sigma_x = -60 MPa$

$\sigma_y = -40 MPa$

$\tau_{xy} = 35 MPa$

(i) Principal planes orientation are given by,

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= \frac{2(35)}{-60+40}$$

$$= -3.50$$

$$2\theta_p = -74.05^\circ$$

$$\boxed{\theta_p = -37.03^\circ,\ 52.97^\circ}$$

(ii) Principal ~~planes~~ stresses are given by,

$$\sigma_{max,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-60-40}{2} \pm \sqrt{\left(\frac{-60+40}{2}\right)^2 + 35^2}$$

$$= -50 \pm 36.4 \text{ MPa}$$

Candidates must not write on this margin

$$\boxed{\begin{aligned} \sigma_{max} &= -13.60 MPa \\ \sigma_{min} &= -86.4 MPa \end{aligned}}$$

(iii) Maximum shear plane orientation is given by,

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$= -\frac{-60+40}{2(35)}$$

$$= 0.2857$$

$$2\theta_s = 15.95^\circ$$

$$\boxed{\theta_s = 7.97^\circ, 97.97^\circ}$$

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(iv) Maximum in-plane shearing stress is given by,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-60+40}{2}\right)^2 + 35^2}$$

$$\boxed{\tau_{max} = 36.4 \text{ MPa}}$$

(v) Normal stress corresponding to maximum in-plane shear stress plane is given by,

$$\sigma' = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$= \frac{-60-40}{2}$$

$$\boxed{\sigma' = -50 \text{ MPa}}$$

02(b). The non-prismatic cantilever circular bar shown has an internal cylindrical hole from 0 to x, so that the net polar moment of inertia of the cross section for segment 1 is $\left(\frac{7}{8}\right)I_p$. Torque T is applied at x and torque T/2 is applied at x = L. Assume that G is constant.

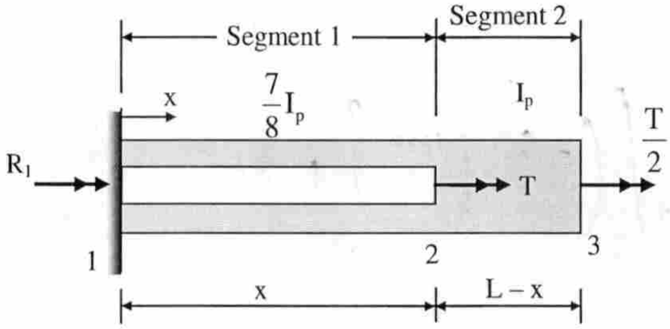
(i) Find reaction moment R_1.

(ii) Find internal torsional moments T_i in segments 1 & 2.

(iii) Find x required to obtain twist at joint 3 of $\theta_3 = \frac{TL}{GI_p}$.

(iv) What is the rotation at joint 2, θ_2?

(v) Draw the torsional moment (TMD: T(x), $0 \le x \le L$) diagram.



(5×4 = 20 M)

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(i) Reaction torque R_1:

$$\Sigma M_x = 0$$

$$R_1 = -\left(T + \frac{T}{2}\right)$$

$$\boxed{R_1 = -\frac{3}{2}T}$$

(ii) Internal moments in segments 1 and 2:

$$\boxed{T_1 = -R_1}$$

$$\boxed{\begin{aligned} T_1 &= 1.5\,T \\ T_2 &= \frac{T}{2} \end{aligned}}$$

(iii) 'x' required to obtain twist at joint 3:

$$\theta_3 = \Sigma \frac{T_i L_i}{G I_{p_i}}$$

$$\frac{TL}{GI_p} = \frac{T_1 x}{G\left(\frac{7}{8} I_p\right)} + \frac{T_2 (L-x)}{G I_p}$$

$$\frac{TL}{GI_p} = \frac{\left(\frac{3}{2}\right) T x}{G\left(\frac{7 I_p}{8}\right)} + \frac{\left(\frac{T}{2}\right)(L-x)}{G I_p}$$

$$L = \frac{\left(\frac{3}{2}\right) x}{\left(\frac{7}{8}\right)} + \frac{1}{2}(L-x)$$

$$L = \frac{17}{14} x + \frac{1}{2} L$$

$$x = \frac{14}{17}\left(\frac{L}{2}\right)$$

$$\boxed{x = \frac{7}{17} L}$$

Candidates must not write on this margin

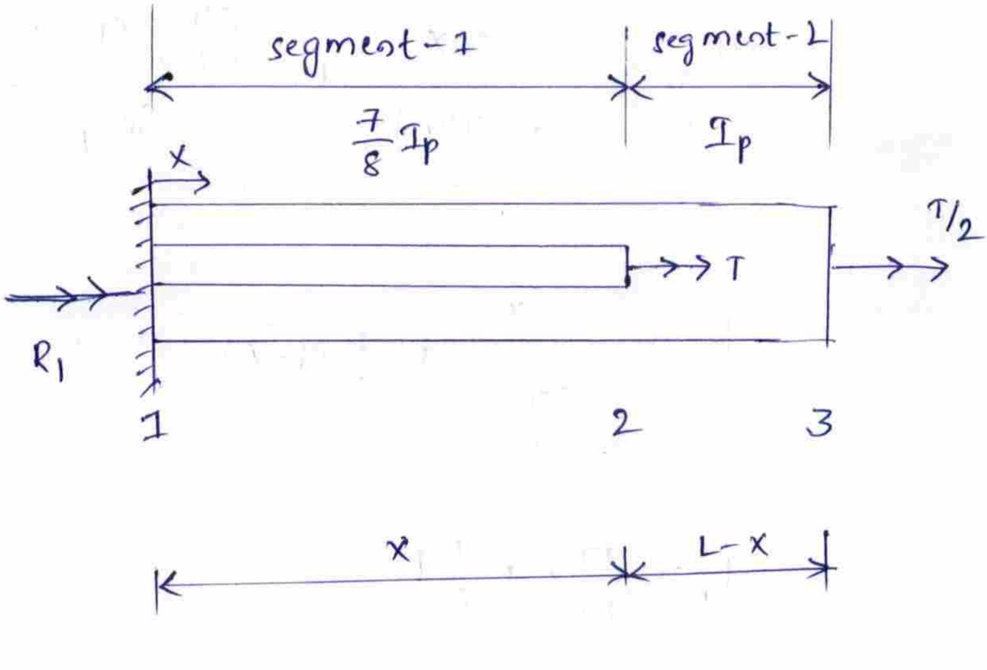
(iv) Rotation at joint 2 for x value in (c) :

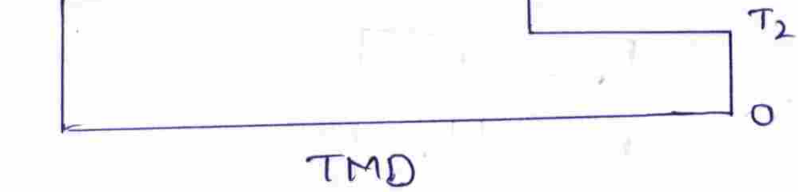
$$\theta_2 = \frac{T_1 x}{G\left(\frac{7}{8} I_p\right)}$$

$$= \frac{\left(\frac{3}{2}T\right)\left(\frac{7}{17}L\right)}{G\left(\frac{7}{8} I_p\right)}$$

$$\boxed{\theta_2 = \frac{12TL}{17GI_p}}$$

(v) Torsional moment diagram





TMD

02(c). **(i)** A hydraulic excavator back hoe is excavating a basement for a building. Heaped capacity of bucket is 1.5 cu yd (0.8 cum near higher side). The material is common earth. The job efficiency is estimated to be 50 min/hr. The machines maximum depth of cut is 7.6 m and the average digging depth is 4 m. Average swing angle is 90°. Estimate the hourly production in bank measure.

Consider,

Bucket fill factor (for common earth, loam) = 0.9

Swing – depth factor for Back hoe at 90° angle of swing = 1.1

Standard cycle per hour = 135 cycles / hour . **(6 M)**

Candidates must not write on this margin

Given data

Bucket capacity $(V) = 0.8 m^3$

Job efficiency factor $(K_1) = \frac{50}{60} = 0.833$

Bucket fill factor $(K_2) = 0.9$

Swing - depth factor $(K_3) = 1.1$

Cycle Time $(C.T) = 135$ cycles/hr

$$= \frac{3600}{135} = 27 \text{ seconds}$$

Production rate in Bank Cubic Meters per hour (BCM per hour)

$$= \left(\frac{\text{Bucket capacity } (m^3)}{\text{cycle time (sec)}} \times 3600\right) \times K_1 K_2 K_3$$

$$= \left(\frac{0.8 (m^3) \times 3600}{27 \text{ (sec)}}\right) \times 0.833 \times 0.9 \times 1.1$$

$$= \boxed{88 \text{ BCM/hour}}$$

02(c). **(ii)** Write the difference between a Dozer and a loader? **(10 M)**

Candidates must not write on this margin

(1) DOZER :- A dozer is a heavy construction equipment with large metal bucket on its front, used in construction sites. It is basically a crawler equipment associated with push a large quantity of soil, sand, loose soils with small boulders and any earth materials.

(2) LOADER :- It is also called front end bucket loader, works like a dozer to push and ~~sho~~ shove. The main function is to load materials.

(3) A Dozer has two primary tools, the front heavy metal blade and back end ripper, designed to push and shove materials on the ground such as dirt, sand, rubble, and etc. Its back end ripper breaks down hard earth materials.

(4) A loader is equipped with square bucket on the front. Its main function is to scoop or load materials from the ground such as debris, soil, sand, rocks and gravels etc.

(5) Cost of Dozer is higher when compared to the cost of Loader.

(6) More power rating engine necessary for Dozer when compared to front end Loader.

02(c). **(iii)** Describe the Dozer production estimate ? **(4 M)**

Candidates must not write on this margin

For Dozer earth work equipment

Let H = Blade Height of the pile measured

W = Blade width of the pile measured

L = Blade length of the pile measured

∴ (1) Blade Load $(LM^3) = H(m) \times W(m) \times L(m)$

(2) Dozer cycle time = Fixed cycle time + variable cycle time.

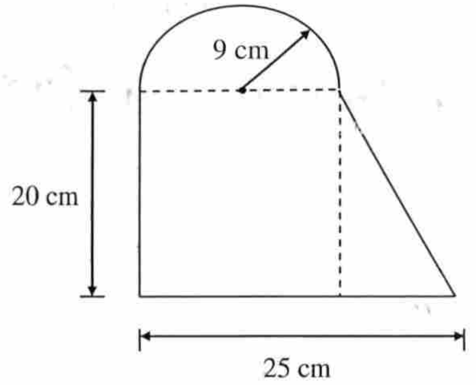
→ Fixed cycle time for manoeuvering, changing gears, start loading and dump.

→ Variable cycle time is the time required to doze and return.

→ output of dozer production (BCM)

$$= \frac{\text{Blade capacity (LCM)} (m^3)}{\text{cycle time (sec)}} \times 3600 \times (\text{Factors multiplied})$$

03(a). **(i)** Compute 2nd moment of area of a plane lamina shown in figure about an axis parallel to its base and passing through the centroid in cm^4.



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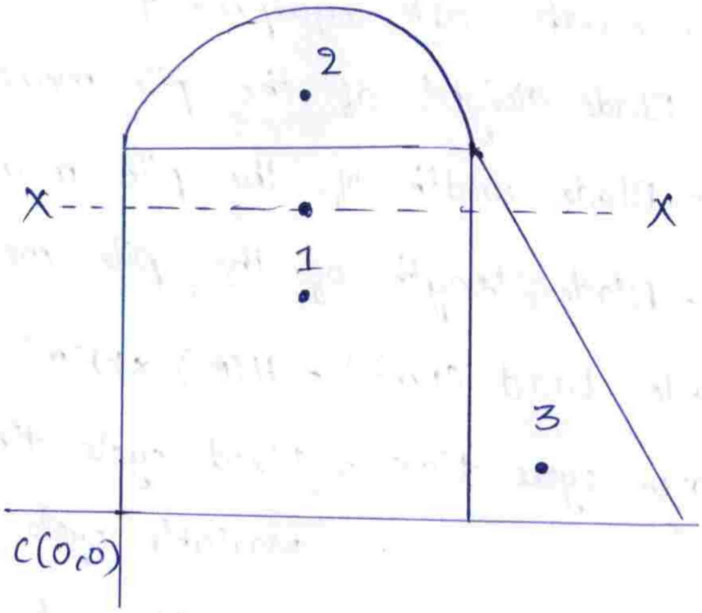

→ Consider point 'C' as origin

Let $\bar{y}_c$ be the distance of centroid above the base of plane lamina

$$\bar{y}_c = \frac{A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3}{A_1 + A_2 + A_3}$$

$$\bar{y}_c = \frac{20\times18\times10 + \frac{\pi 9^2}{2}\times\left(20 + \frac{4\times9}{3\pi}\right) + \frac{7\times20}{2}\times\frac{20}{3}}{(20\times18) + (7\times20/2) + (\pi\times9^2/2)}$$

$\bar{y}_c = 12.73\,cm$

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We have to find 2nd moment of area about x-x axis

$$I_1 = \frac{bd^3}{12} + A_1 y^2$$

$$= \frac{18 \times 20^3}{12} + 20 \times 18 \times 2.73^2$$

$$= 14683.04 \text{ cm}^4$$

$$I_2 = \frac{\pi r^4}{4} + A_2 y^2$$

$$= 0.1098 \times 9^4 + \frac{\pi \times 9^2}{2} \times \left\{ \frac{4 \times 9}{3\pi} + (20 - 12.73) \right\}^2$$

$$= 16367.93 \text{ cm}^4$$

$$I_3 = \frac{bh^3}{36} + A_3 y^2$$

$$= \frac{7 \times 20^3}{36} + \frac{1}{2} \times 20 \times 7 \times \left(12.73 - \frac{20}{3}\right)^2$$

$$= 4129.04 \text{ cm}^4$$

So, $I = I_1 + I_2 + I_3$

$$= 14683.04 + 16367.93 + 4129.04$$

$$\boxed{I = 35180 \text{ cm}^4}$$

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03(a). **(ii)** Derive the expression for distortion energy per unit volume for a body subjected to a uniform stress state, given by the principal stresses σ_1 and σ_2 with third principal stress σ_3 being zero. **(8 M)**

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According to Von-Misses criterion,

For triaxial stress at a point, total strain energy

$$U = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\mu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right] \quad ---(1)$$

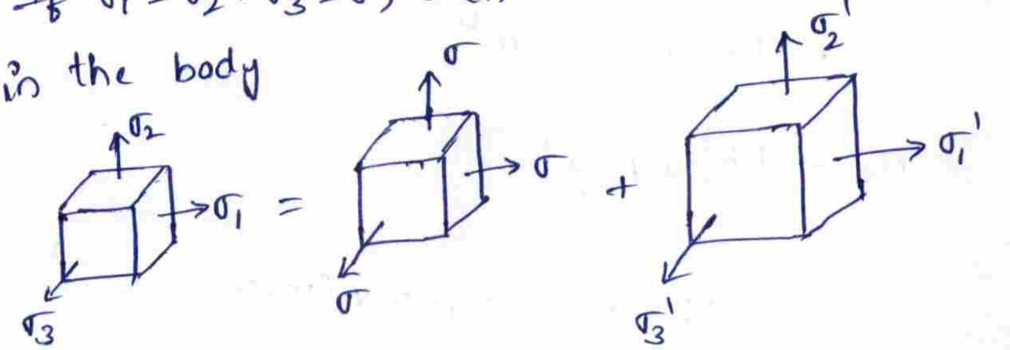
volumetric strain, $\epsilon_v = \epsilon_1 + \epsilon_2 + \epsilon_3$

$$= \frac{1-2\mu}{E}(\sigma_1 + \sigma_2 + \sigma_3)$$

Condition for zero volumetric change is,

$$\sigma_1 + \sigma_2 + \sigma_3 = 0$$

If $\sigma_1 = \sigma_2 = \sigma_3 = \sigma$, then there will be no distortion in the body



we, have $\sigma = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$; from fig: $\sigma_1 = \sigma + \sigma_1'$

$\sigma_2 = \sigma + \sigma_2'$

Adding all three principal stresses

$\sigma_3 = \sigma + \sigma_3'$

$$\sigma_1 + \sigma_2 + \sigma_3 = 3\sigma + \sigma_1' + \sigma_2' + \sigma_3'$$

$\Rightarrow \sigma_1' + \sigma_2' + \sigma_3' = 0$ i.e stress causing only distortion and no volume changes.

$\Rightarrow$ substitute $\sigma_1 + \sigma_2 + \sigma_3 = \sigma$ in (1)

$$u_1 = \frac{3(1-2\mu)}{E}\sigma^2$$

$$U_v = \frac{1-2\mu}{6E}(\sigma_1 + \sigma_2 + \sigma_3)^2$$

$\therefore$ $U_d = U - U_v$

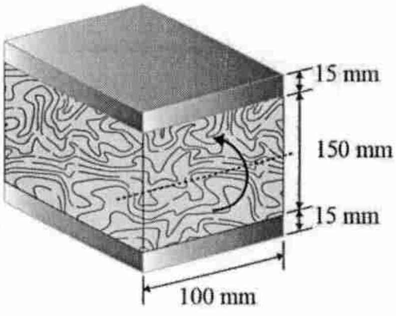
$$\boxed{U_d = \frac{1+\mu}{6E}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]}$$

03(b). The wooden section of the beam is reinforced with two steel plates as shown in the figure below. The beam is subjected to an internal moment of M = 30 kN-m.

(i) Determine the maximum bending stresses developed in the steel plates and wood beams.

(ii) Sketch the stress distribution over the combined cross section.

(Take E_w = 10 GPa and E_{st} = 200 GPa)



(14 + 6 = 20 M)

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Given data

$$M = 30 \text{ kN-m}$$

$$E_\omega = 10 \text{ GPa}$$

$$E_{st} = 200 \text{ GPa}$$

(i) To analyse the composite section, let us first transform it to fully steel section

Modular ratio,

$$m = \frac{E_\omega}{E_{st}} = \frac{10}{200} = 0.05$$

Thus

$$b_{st} = m \times b_\omega = 0.05 \times 100$$

$$= 5 \text{ mm}$$

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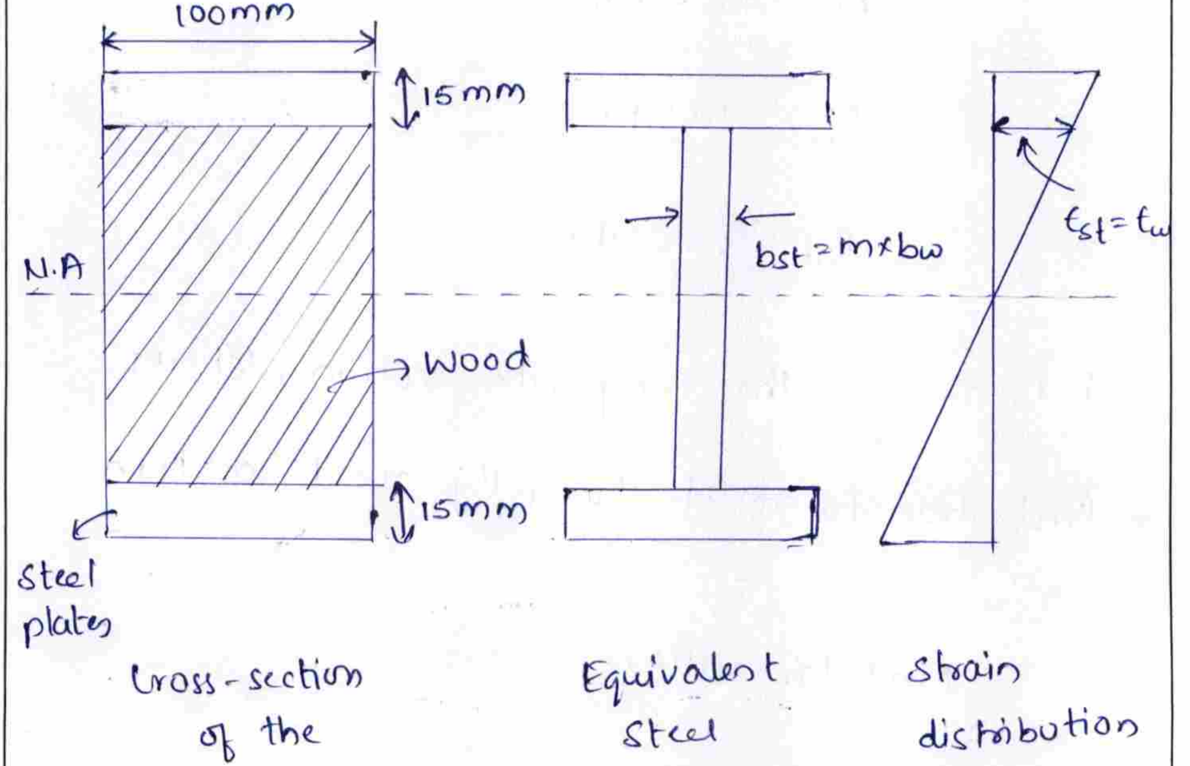
Moment of Inertia of I-section is given by

$$I = \frac{100 \times (180)^3}{12} - \frac{(100-5) \times 150^3}{12}$$

$$= 2.19 \times 10^7 \ mm^4$$



Cross-section of the beam

Equivalent steel section

strain distribution

→ By using flexural formula,

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

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$$\therefore (\sigma_b)_{max} = \frac{M}{I} \times y_{max}$$

⇒ For the steel plate:

$$(\sigma_b)_{y=90mm} = \frac{30\times10^6}{2.19\times10^7} \times 90 = 123.29\ MPa$$

$$(\sigma_{st})_{max} = 123.29\ MPa$$

$$(\sigma_b)_{y=75mm} = \frac{30\times10^6}{2.19\times10^7} \times 75 = 102.74\ MPa$$

⇒ for the wooden section:

WKT strain distribution is linear for the given composite section.

$$(\epsilon_{st})_{y=75mm} = (\epsilon_\omega)_{y=75mm}$$

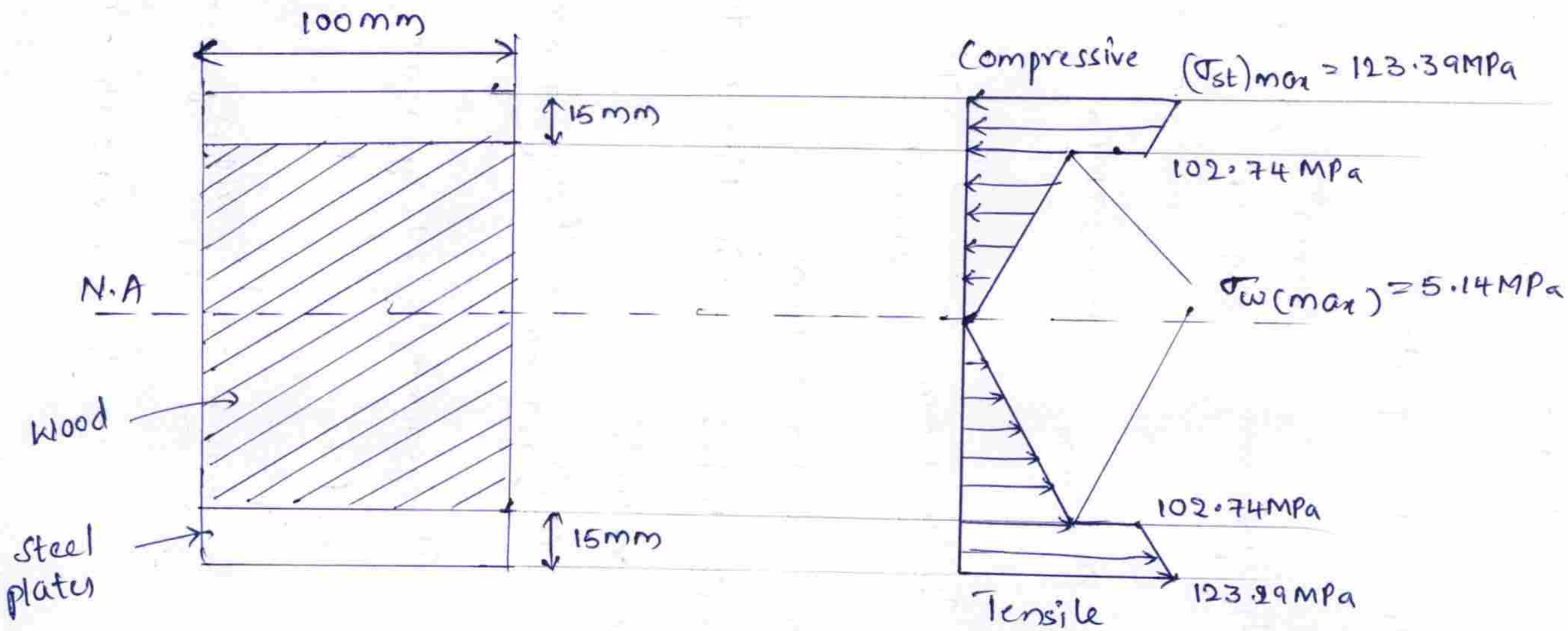
$$\therefore \frac{(\sigma_{st})_{y=75mm}}{E_{st}} = \frac{(\sigma_\omega)_{y=75mm}}{E_\omega}$$

$$\therefore (\sigma_\omega)_{y=75mm} = \frac{E_\omega}{E_{st}} \times (\sigma_{st})_{y=75mm}$$

$$= 0.05 \times 102.74 = 5.14\ MPa$$

$$(\sigma_\omega)_{max} = 5.14\ MPa$$

(ii) The stress distribution will be symmetric about neutral axis (i.e compression above the N.A and tensile below the N.A) as shown below:



Cross-section of the beam

stress distribution

03(c). Consider the following project data :

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Predecessor(s)	-	A	A	A	B	B	C	E	C	D	I,J	F,G,H	K	L,M
Duration (days)	7	5	6	3	8	4	2	5	6	3	5	7	3	4

(i) Draw a project network and find out critical path

(ii) Determine all the three floats of all activities **(2×10 = 20 M)**

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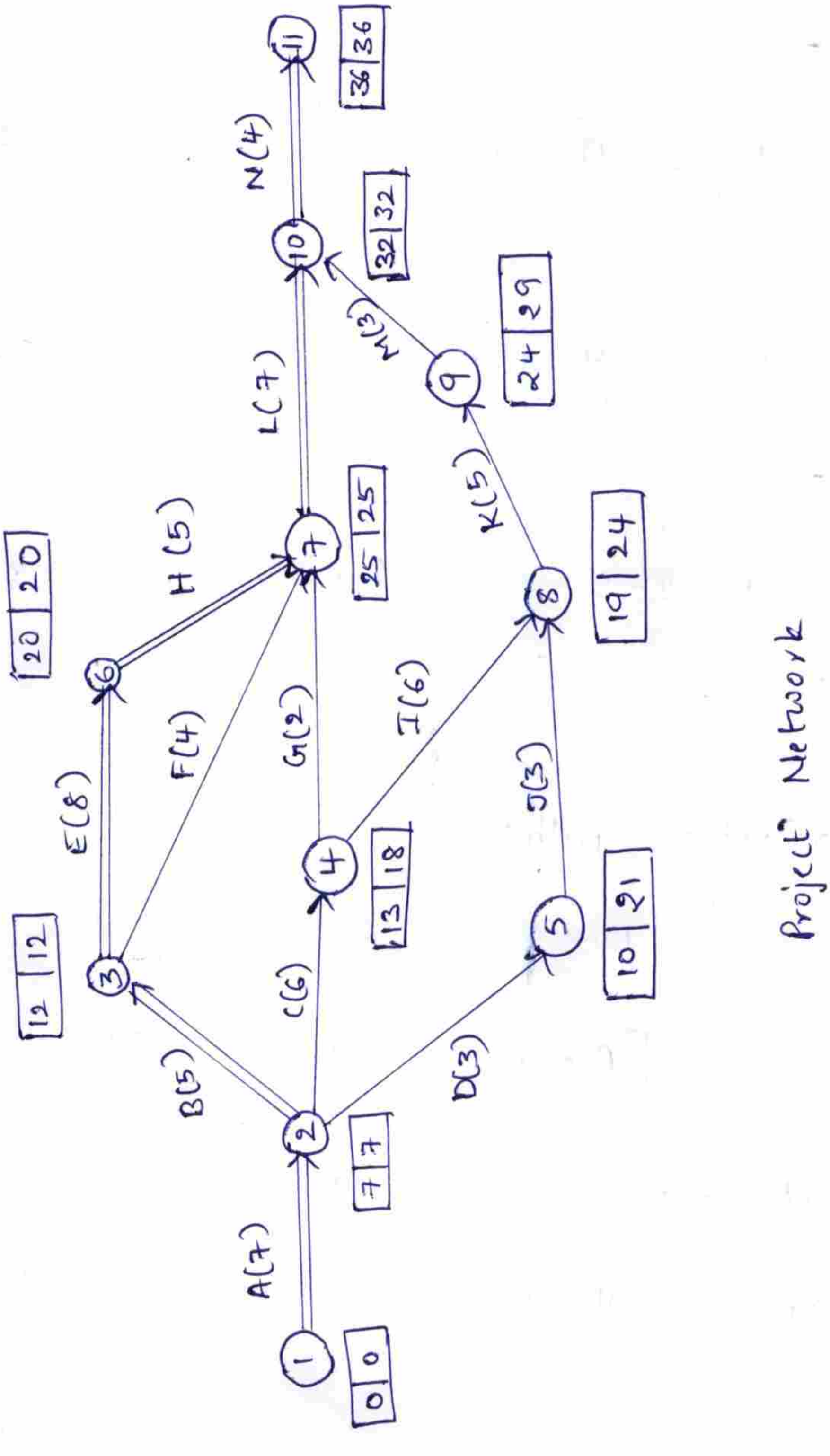

Path	duration(days)
1 – 2 – 3 – 6 – 7 – 10 – 11	36 d
1 – 2 – 4 – 7 – 10 – 11	26 d
1 – 2 – 4 – 8 – 9 – 10 – 11	32 d
1 – 2 – 5 – 8 – 9 – 10 – 11	25 d

⇒ Critical path :-

1 – 2 – 3 – 6 – 7 – 10 – 11

[A – B – E – H – L – N]

⇒ Project completion time = 36 days

⇒ Floats of A, B, E, H, L, N = 0 [critical events]

Activity	Total Float $= L_j - E_i - d_{ij}$	Free Float $= E_j - E_i - d_{ij}$	Independent Float $= E_j - L_i - d_{ij}$
C	$18 - 7 - 6 = 5$	$13 - 7 - 6 = 0$	0
D	$21 - 7 - 3 = 11$	$10 - 7 - 3 = 0$	0
F	$25 - 12 - 4 = 9$	9	9
G	$25 - 13 - 2 = 10$	$25 - 13 - 2 = 10$	$25 - 18 - 2 = 5$
I	$24 - 13 - 6 = 5$	$19 - 13 - 6 = 0$	0
J	$24 - 10 - 3 = 11$	$19 - 10 - 3 = 6$	$19 - 21 - 3 = 0$ (−ve indicates zero)
K	$29 - 19 - 5 = 5$	$24 - 19 - 5 = 0$	0
M	$32 - 24 - 3 = 5$	5	$32 - 29 - 3 = 0$

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04(a). Three pinned-end columns of the same material but different shapes have the same length and the same cross-sectional area as shown in the figure below. The columns are free to buckle in any direction. The columns have cross section as follows: (1) a circle, (2) a square, and (3) an equilateral triangle. Determine the ratios $P_1 : P_2 : P_3$ of the critical loads for these columns.

(1) (2) (3)

(20 M)

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Given data

- Material of each section is same.
- Length of each section is same.
- Cross-sectional area of each section is same.

Critical buckling load is given by,

$$P = \frac{\pi^2 E I}{L_e^2}$$

Here, E and Le are constant.

$$\therefore \quad P \propto I$$

(1) circle:

$$I_1 = \frac{\pi d^4}{64} \quad \text{and} \quad A = \frac{\pi d^2}{4}$$

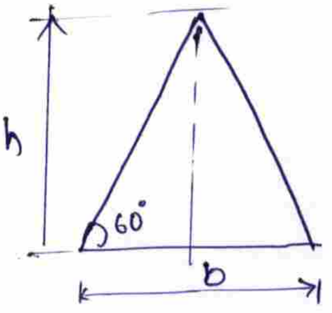
$$\therefore \quad I_1 = \frac{A^2}{4\pi}$$

(2) Square (Let side = a)

$$I_2 = \frac{a^4}{12} \quad \text{and} \quad A = a^2$$

$\therefore\ I_2 = \dfrac{A^2}{12}$

(3) Equilateral triangle:



$I_3 = \dfrac{bh^3}{36}$ and $A = \frac{1}{2}bh$

from geometry,

$\sin 60° = \dfrac{h}{b}$

$\therefore\ h = b\sin 60° = \dfrac{\sqrt{3}\,b}{2}$

$\therefore\ A = \frac{1}{2} \times b \times \frac{\sqrt{3}}{2} b$

$\therefore\ A = \dfrac{\sqrt{3}b^2}{4}$

$\therefore\ b^2 = \dfrac{4A}{\sqrt{3}}$

Now, $I_3 = \dfrac{b\left(\frac{\sqrt{3}b}{2}\right)^3}{36} = \dfrac{3\sqrt{3}}{8\times 36} \times b^4 = \dfrac{3\sqrt{3}}{8\times 36} \times \left(\dfrac{4A}{\sqrt{3}}\right)^2$

$= \dfrac{\sqrt{3}}{18} A^2$

Thus, $P_1 : P_2 : P_3 = I_1 : I_2 : I_3$

$= \dfrac{A^2}{4\pi} : \dfrac{A^2}{12} : \dfrac{\sqrt{3}}{18} A^2$

$\boxed{P_1 : P_2 : P_3 = 0.08 : 0.083 : 0.096}$

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04(b). **(i)** A project is composed of eleven activities. The time estimates are given below.

Activity	t_o	t_p	t_m
1 – 2	7	17	9
1 – 3	10	60	20
1 – 4	5	15	10
2 – 5	50	110	65
2 – 6	30	50	40
3 – 6	50	90	55
3 – 7	1	9	5
4 – 7	40	68	48
5 – 8	5	15	10
6 – 8	20	52	27
7 – 8	30	50	40

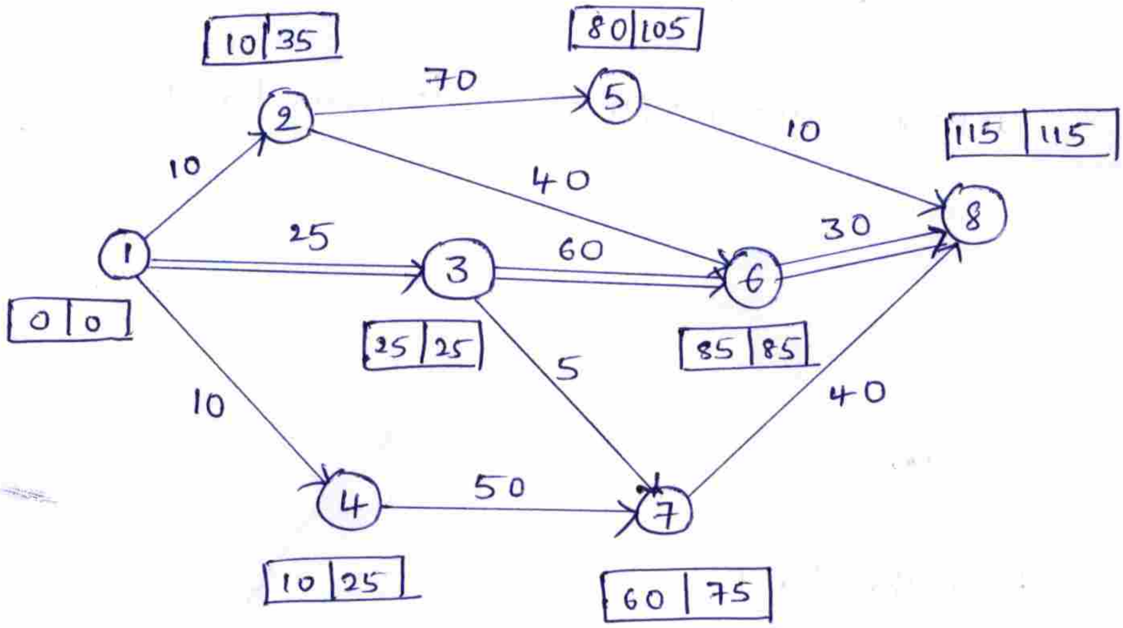
Find (i) Critical path and Expected duration of the project

(ii) Slacks of events and Project duration distribution **(2×5 = 10 M)**

Candidates must not write on this margin

Activity	$t_E = \frac{t_o + 4t_m + t_p}{6}$
1 – 2	$\frac{7+4(9)+17}{6} = 10$
1 – 3	$\frac{10+4(20)+60}{6} = 25$
1 – 4	10
2 – 5	$\frac{50+4(65)+110}{6} = 70$
2 – 6	40
3 – 6	$\frac{50+4(55)+90}{6} = 60$
3 – 7	5
4 – 7	$\frac{40+4(48)+68}{6} = 50$

5 — 8	10
6 — 8	$\frac{20+4(27)+52}{6} = 30$
7 — 8	40



*Critical path : 1 – 3 – 6 – 8

*Expected completion time of the project = 115 days

*Slacks of events :

1, 3, 6, 8 = 0 (critical events)

2 = 35 – 10 = 25

4 = 25 – 10 = 15

5 = 105 – 80 = 25

7 = 75 – 60 = 15

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* Project duration distribution ($\mu \pm 3\sigma$)

$$\sigma = \sigma_{cp} = \sqrt{\left(\frac{60-10}{6}\right)^2 + \left(\frac{90-50}{6}\right)^2 + \left(\frac{52-20}{6}\right)^2}$$

$$= \sqrt{69.44 + 44.44 + 28.44}$$

$$= 11.92$$

$$\mu \pm 3\sigma = 115 \pm 3(11.92) = \boxed{79.24 \text{ to } 150.76}$$

04(b). **(ii)** A bolt is under an axial thrust of 9 kN and a transverse shear force of 5 kN. Calculate the diameter of bolt according to the theory of maximum strain energy. Take S_{yt} = 250 N/mm^2, μ = 0.3, and factor of safety = 2.8. Assume uniform distribution of normal and shear stress on cross section. **(10 M)**

Axial thrust $= P = 9 \times 10^3$ N

Shear Force $= S = 5 \times 10^3$ N

$$\therefore \text{Axial stress} = \sigma_x = -\frac{9\times10^3}{\frac{\pi d^2}{4}} = \frac{-36\times10^3}{\pi d^2} \text{ N/mm}^2$$

$$\text{Shear stress} = \tau = \frac{5\times10^3}{\frac{\pi d^2}{4}} = \frac{20\times10^3}{\pi d^2} \text{ N/mm}^2$$

$\therefore$ Principal stresses are,

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

Candidates must not write on this margin

$$= \left(\frac{\frac{-36\times10^3}{\pi d^2}}{2}\right) \pm \sqrt{\left(\frac{\frac{-36\times10^3}{\pi d^2}}{2}\right)^2 + \left(\frac{20\times10^3}{\pi d^2}\right)^2}$$

$$= \left(-\frac{18}{\pi d^2} \pm \frac{26.90}{\pi d^2}\right)\times10^3$$

$$\therefore \sigma_1 = \frac{8.9\times10^3}{\pi d^2} \text{ and } \sigma_2 = -\frac{44.9\times10^3}{\pi d^3}$$

According to maximum strain energy theory

$$\sigma_1^2 + \sigma_2^2 - 2\mu\sigma_1\sigma_2 = \left(\frac{S_{yt}}{FOS}\right)^2$$

$$\left(\frac{8.9\times10^3}{\pi d^2}\right)^2 + \left(\frac{-44.9\times10^3}{\pi d^2}\right)^2 - 2(0.3)\left(\frac{8.9\times10^3}{\pi d^2}\right)\left(\frac{-44.9\times10^3}{\pi d^2}\right)$$

$$= \left(\frac{250}{2.8}\right)^2$$

$$\Rightarrow \left(\frac{79.21\times10^6}{\pi^2 d^4}\right) + \left(\frac{2016.01\times10^6}{\pi^2 d^4}\right) + \left(\frac{239.76\times10^6}{\pi^2 d^4}\right)$$

$$= 7971.93$$

$$\therefore \pi^2 d^4 = 292900.21$$

$$d^4 = 29676.99$$

$$\boxed{d = 13.12\ mm}$$

04(c). (i) Explain different types of depreciation? **(8 M)**

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Depreciation :- Depreciation expense is used in accounting to allocate the cost of a tangible asset over its useful life. It is the reduction of value in an asset over time due to usage, wear, tear or obsolescence.

There are several types of depreciation expense and different formulas for determining the book value of an asset.

(i) Straight-Line Depreciation : It is common and simple method of the calculating the expense.

$$\boxed{\text{Depreciation Expense} = \frac{\text{cost} - \text{salvage value}}{\text{useful life}}}$$

(ii) Double declining - balance depreciation:

It results in larger expense in the earlier years as opposed to the later years of an asset's useful life.

$$\boxed{\text{Depreciation Expense} = \text{Beginning book value} \times \text{Rate of depreciation}}$$

(iii) Units of production - depreciation:

$$\boxed{\text{Depreciation Expense} = \frac{(\text{Number of units produced})}{(\text{Life in number of units})} \times (\text{cost} - \text{salvage value})}$$

04(c) **(ii)** The initial cost of a piece of construction equipment is Rs.35,00,000. It has useful life of 10 years. The estimated salvage value of the equipment at the end of useful life is Rs.5,00,000. Calculate the annual depreciation and book value of the construction equipment using straight-line method and double-declining balance method. **(12 M)**

Candidates must not write on this margin

Initial cost of the construction equipment

$= P = Rs: 35,00,000$

Estimated salvage value $= SV = Rs. 5,00,000$

Useful life $= n = 10$ years

⇒ For straight line method:

$$D_m = D_1 = D_2 \ldots = D_9 = D_{10} = \frac{3,50,000 - 5,00,000}{10}$$

$$= Rs. 3,00,000$$

Book value at the end of 1st year

$= BV_1 = Rs. \cancel{350}\ 35,00,000 - Rs. 3\,00\,000$

$= Rs: 32\,00\,000$

Book value at the end of 2nd year

$= BV_2 = Rs. 32\,00\,000 - Rs. 3\,00\,000$

$= Rs. 29,00,000$

⇒ For Double Declining Balancing method:

$$d_m = \frac{2}{n} = \frac{2}{10} = 0.2$$

Depreciation for 1st year $= D_1$

$= 35\,00\,000 \times (1-0.2)^{1-1} \times 0.2$

$= Rs. 7\,00\,000$

Depreciation for 2nd year $= D_2$

$= 3{\cdot}50\,000 \times (1-0.2)^{2-1} \times 0.2$

= Rs. 5 60 000.

Book value at the end of 1^{st} year

BV_1 = Rs. 35 00 000 − Rs. 7 00 000

= Rs. 28 00 000

Book value at the end of 2^{nd} year

BV_2 = Rs. 28 00 000 − Rs. 5 60 000

= Rs. 22 40 000

⇒ Tabulation of Depreciation and book value of the construction equipment.

Year	Straight-line method		Double-declining balancing method	
	Depreciation D_m (Rs)	Book value BV_m (Rs)	Depreciation D_m (Rs)	Book value BV_m (Rs)
0	—	35 00 000	—	35 00 000
1	3 00 000	32 00 000	7 00 000	28 00 000
2	3 00 000	29 00 000	5 60 000	22 40 000
3	3 00 000	26 00 000	4 48 000	17 92 000
4	3 00 000	23 00 000	3 58 400	14 33 600
5	3 00 000	20 00 000	2 86 720	11 46 880
6	3 00 000	17 00 000	2 29 376	9 17 504
7	3 00 000	14 00 000	18 3500.80	734003.20
8	3 00 000	11 00 000	146800.64	587202.56
9	3 00 000	8 00 000	117440.51	469762.05
10	3 00 000	5 00 000	93952.41	375809.64

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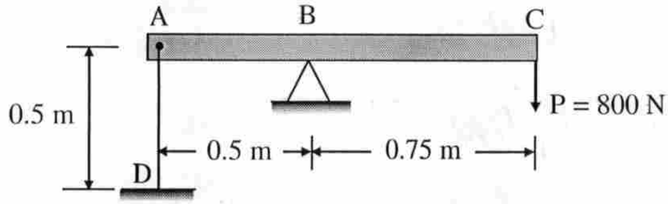
SECTION - B

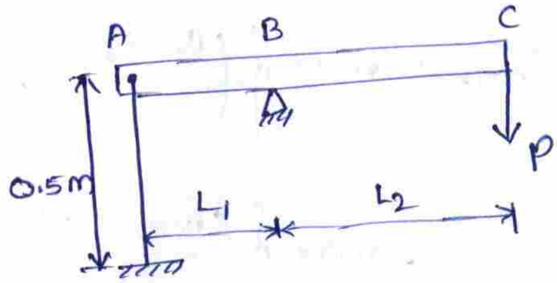
05(a). A beam ABC having flexural rigidity EI = 75 kN-m^2 is loaded by a force P = 800 N at end C and tied down at end A by a wire having axial rigidity EA = 900 kN. What is the deflection at point C when the load P is applied?



(12 M)

Beam tied down by a wire.

Given data:



$EI = 75\ kN\text{-}m^2$

$EA = 900\ kN$

$P = 800\ N$

$H = 0.5m,\ L_1 = 0.5m,\ L_2 = 0.75m$

Consider BC as a cantilever beam $\Rightarrow \delta_C' = \dfrac{PL_2^3}{3EI}$

consider AB as a simple beam

$M_0 = P.L_2 \qquad \theta_B' = \dfrac{M_0 L_1}{3EI} = \dfrac{P.L_1L_2}{3EI}$

Consider the stretching of the wire AD

$\delta_A' = [\text{force in } AD]\left[\dfrac{H}{EA}\right] = \dfrac{P.L_2}{L_1}\cdot\dfrac{H}{EA}$

Deflection 'δ_C' of point $C = \delta_C' + \delta_B'\left(\dfrac{L_2}{L_1}\right) + \theta_B'(L_2)$

$$= \frac{P.L_2^3}{3EI} + \frac{P.L_2^2 H}{EA L_1^2} + \frac{P.L_1L_2^2}{3EI}$$

$$= \frac{800\times(0.75)^3}{3\times75\times10^3} + \frac{800\times0.75^2\times0.5}{900\times10^3\times0.5^2} + \frac{800\times0.5\times0.75^2}{3\times75\times10^3}$$

$$\boxed{\delta_C = 3.50mm}$$

05(b). **(i)** A certain steel has proportionality limit of 3000 kg/cm^2 in simple tension. It is subjected to principal stresses of 1200 kg/cm^2 (tensile), 600 kg/cm^2 (tensile) and 300 kg/cm^2 (compression). Find the factor of safety according to maximum shear stress theory ? **(6 M)**

Candidates must not write on this margin

Given

$S_{yt} = 3000\ kg/cm^2$

$\sigma_1 = 1200\ kg/cm^2$ (tensile)

$\sigma_2 = 600\ kg/cm^2$ (tensile)

$\sigma_3 = -300\ kg/cm^2$

$$\tau_{max} = max\left\{\left(\frac{\sigma_1 - \sigma_2}{2}\right), \left(\frac{\sigma_2 - \sigma_3}{2}\right), \left(\frac{\sigma_3 - \sigma_1}{2}\right)\right\}$$

$$= max\left\{\left(\frac{1200 - 600}{2}\right), \left(\frac{600 + 300}{2}\right), \left(\frac{-300 - 1200}{2}\right)\right\}$$

$$= max\{300, 450, 750\}$$

$\therefore \tau_{max} = 750\ kg/cm^2$

According to maximum shear stress theory

$$\tau_{max} \leq \frac{S_{yt}}{2\,FOS}$$

$$750 \leq \frac{3000}{2\,FOS}$$

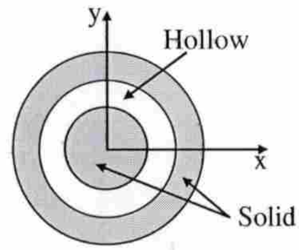
$$FOS \leq \frac{3000}{2 \times 750}$$

$$FOS \leq 2$$

$$\therefore \boxed{FOS = 2}$$

Candidates must not write on this margin

05(b). **(ii)** Calculate the moment of inertia I_x for the hollow composite circular area shown in the figure. The origin of the axes is at the center of the concentric circles, and the three diameters are 20 mm, 40 mm, and 60 mm.



(6 M)

Given data:

$d_1 = 20mm$, $d_2 = 40mm$

$d_3 = 60mm$

Moment of inertia of a given composite section is calculated by

$$I_x = \frac{\pi}{64}\left[d_1^4 + (d_3^4 - d_2^4)\right]$$

$$= \frac{\pi}{64}\left[20^4 + (60^4 - 40^4)\right]$$

$$\boxed{I_x = 5.18 \times 10^5 \ mm^4}$$

05(c). A hollow column, 400 mm external diameter and 300 mm internal diameter, is hinged at both ends. If the length of column is 5 m, E = 0.75 × 10^5 N/mm^2, factor of safety 5, Rankine's constant 1/1600 and crushing stress 587 N/mm^2, find the safe load the column can carry without buckling. Use Euler's and Rankine's formulae. **(12 M)**

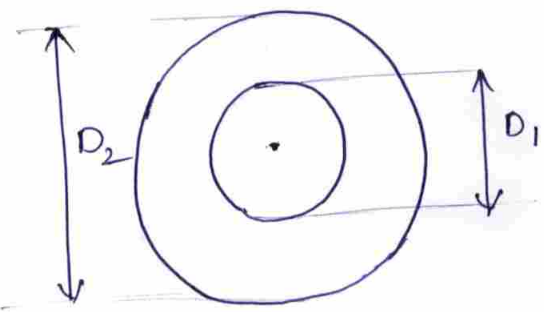
Given data

$D_1 = 300mm$ $D_2 = 400mm$

$L_e = L = 5m$

$E = 0.75 \times 10^5$ MPa

FOS = 5



Rankine's constant $(\alpha) = \frac{1}{1600}$

Candidates must not write on this margin

Crushing stress $(\sigma_c) = 587\ N/mm^2$

* **According to Euler's formula**

$$P_{cr} = \frac{\pi^2 E I}{L^2}$$

$$P_{safe} = \frac{\pi^2 E I}{L^2 (FOS)}$$

$$= \frac{\pi^2 \times 75 \times 10^3}{(5000)^2 \times 5} \times \left[\frac{\pi}{64} (400^4 - 300^4) \right]$$

$$= \frac{\pi^2 \times 75 \times 10^3}{5 \times (5000)^2} \times (858.6 \times 10^6)$$

$$P_{safe} = 5084425.4\ N$$

$$\boxed{P_{safe} = 5084\ KN}$$

* **According to Rankine's formula**

$$P = \frac{\sigma_c . A}{1 + c \left(\frac{L}{k}\right)^2}$$

where k = Radius of gyration

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We know that,

$$I = AK^2 = \frac{\pi}{4}(D_1^2 - D_2^2)K^2$$

$$\because K = 125 mm$$

Now,
$$P_{safe} = \frac{\frac{\sigma_c}{FOS} \cdot A}{1 + c\left(\frac{L}{K}\right)^2}$$

$$= \frac{\frac{587}{5} \times \frac{\pi}{4}(D_1^2 - D_2^2)}{1 + \frac{1}{1600}\left(\frac{5000}{125}\right)^2}$$

$$= \frac{6451130}{2}$$

$$= 3225565 N$$

$$\boxed{P_{safe} = 3225.5 kN}$$

05(d). Define line of balance and mention its application. Cash flows for a company are given in the table.

End of year	0	1	2	3	4
Project x	–50,000	5,000	17,500	30,000	42,500
Project y	–50,000	40,000	15,000	15,000	15,000

Using Net Present Value method, find which project is favourable? Consider 10% discounted rate per annum. **(12 M)**

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⇒ Line of Balance :- (LOB) is a management control process used in construction where the project contains blocks of repetitive work activities, such as roads, pipelines, tunnels, railways and highrise buildings.

* LOB collects, measures and present information relating to time, cost and completion, and presents it against a specific plan.

* LOB itself is a graphic device that enables a manager to see at a single glance which activities of an operation are "in balance".

⇒ Net present value method:-

* The net present worth for project X =

$-50,000 + 5,000 \times \left(\frac{P}{F}, 10\%, 1\right) + 17,500 \times \left(\frac{P}{F}, 10\%, 2\right) + 30000 \times (P/F, 10\%, 3) + 42,500 \times \left(\frac{P}{F}, 10\%, 4\right)$

$NPW = -50\,000 + 5\,000 \times 0.9091 + 17\,500 \times 0.82645 + 30,000 \times 0.75131 + 42500 \times 0.6830$

$$\boxed{NPW = Rs.\ 20{,}575}$$

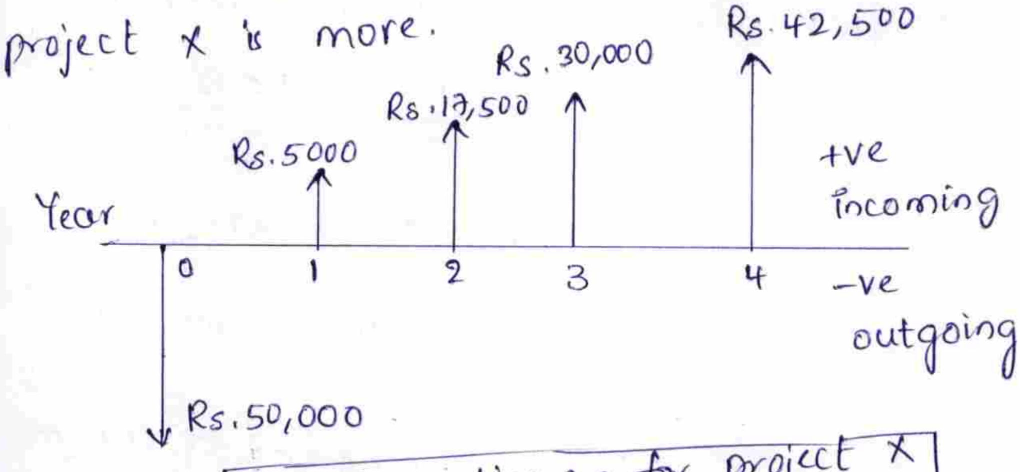
* The net present worth for project Y

$$= -50\,000 + 40\,000 \times \left(\frac{P}{F}, 10\%, 1\right) + 15\,000 \times \left(\frac{P}{F}, 10\%, 2\right) + 15{,}000 \times \left(\frac{P}{F}, 10\%, 3\right) + 15{,}000 \times \left(\frac{P}{F}, 10\%, 4\right)$$

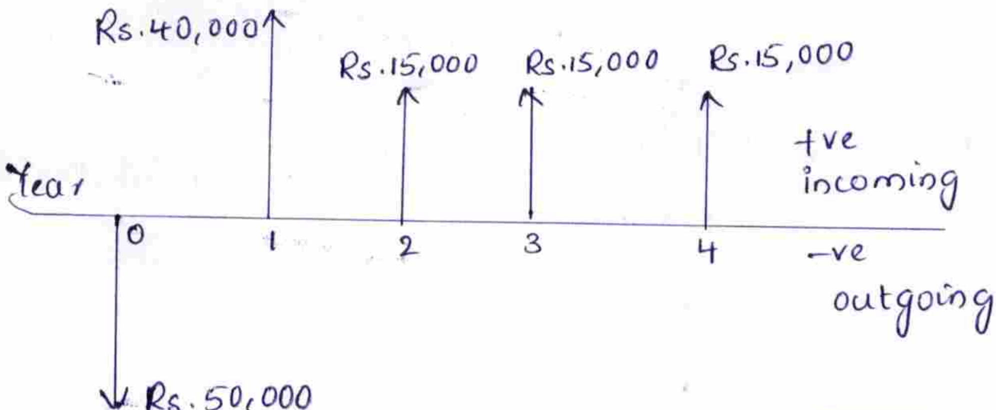
$$NPW = -50{,}000 + 40{,}000 \times 0.9091 + 15\,000 \times 0.8264 + 15{,}000 \times 0.7513 + 15{,}000 \times 0.6830$$

$$\boxed{NPW = Rs.\ 20{,}274}$$

Hence, choose $\boxed{X}$ since the net present worth for project X is more.



cash flow diagram for project X



Cash flow diagram for project Y

05(e). **(i)** A Dozer is consider for estimation of loose cubic meter dozing the soil excavation work. The measurement from the Blade Load test

Blade height, $H_1 = 1.5$ m & $H_2 = 1.58$ m

Blade width, $B_1 = 2.1$ m & $B_2 = 2.13$ m

Blade Length, L = 3.84 m

Manufacturer's catalogue blade load (where H, B & L are in feet)

Blade load = 0.0139 HBL (LCY)

(a) What is the blade capacity in loose cubic meter for the tested material.

(b) What is blade capacity in Bank cubic meter for the test material, if soil load factor is 0.8 (25% swell). **(7 M)**

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$$\text{Mean Height } (H) = \frac{H_1 + H_2}{2} = \frac{1.5 + 1.58}{2}$$

$$= 1.54 m$$

$$\text{Mean width } (B) = \frac{B_1 + B_2}{2} = \frac{2.1 + 2.13}{2} = 2.12 m$$

(a) Blade Load (LCY) = 0.0139 HBL (where H, B, L and are in 'feet', as per manufacturer catalogue)

$$= (0.0139)\left(\frac{1.54}{0.3048}\right)\left(\frac{2.12}{0.3048}\right)\left(\frac{3.84}{0.3048}\right)$$

$= 6.15$ LCY (Loose cubic yard)

$= 6.15 \times (3 \text{ feet})^3$ (Loose cubic feet)

$= 6.15 \times (3 \times 0.3048)^3$ (LCM)

$= \boxed{4.7 \text{ LCM}}$ (Loose cubic meter)

(b) Blade Load (BCM)

= Blade load (LCM) × soil load factor

$= 4.7 \times 0.8$

$= \boxed{3.76 \text{ (BCM)}}$

05(e). **(ii)** Write a note on loaders construction equipment with line diagrams. **(5 M)**

Candidates must not write on this margin

→ Loader is a tractor equipment with a front end bucket called as loader, front end loader (or) bucket loader. Both wheel loaders and track mounted loaders are available.

→ Loaders are used:

(1) To excavate soft to medium, hard material.

(2) Loading haul units and hopers.

(3) Stock piling materials.

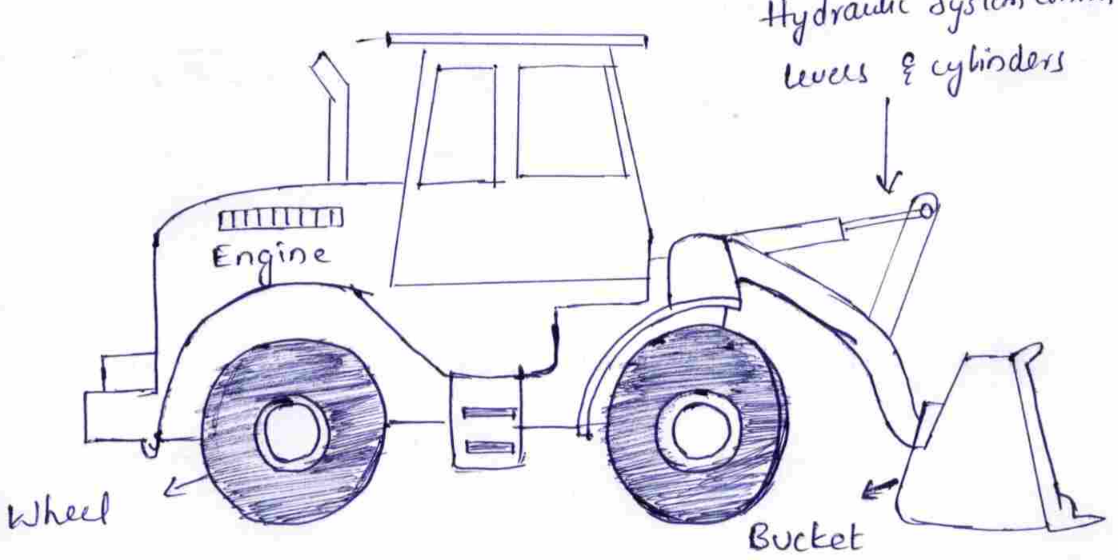
(4) Back filling ditches

(5) Moving concrete and other construction materials

→ Production of loader (m^3/hr) = Bucket load (in m^3) × cycles per hour

→ Cycle time = Basic cycle time + Travel time + Fixed time for gear shift etc.

→ Line Diagram of Loader.



06(a). At a point on the surface of a machine the material is in biaxial stress with $\sigma_x = 32$ MPa, and $\sigma_y = -50$ MPa as shown in figure (a). Figure (b) shows an inclined plane *aa* cut through the same point in the material but oriented at an angle θ.

(i) Determine the value of the angle θ between zero and 90° such that no normal stress acts on plane *aa*.

(ii) Sketch a stress element having plane *aa* as one of its sides and show all stresses acting on the element.

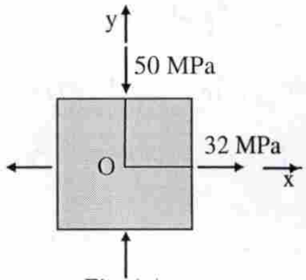


Fig. (a)

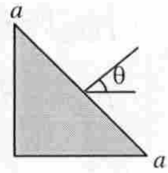


Fig. (b)

(2×10 = 20 M)

Candidates must not write on this margin

Given data

$\sigma_x = 32\,MPa$, $\sigma_y = -50\,MPa$, $\tau_{xy} = 0$

(i) Normal stress at any angle 'θ' is given by

$$\sigma_\theta = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

Given that $\sigma_\theta = 0$

$$0 = \frac{32-50}{2} + \frac{32+50}{2}\cos 2\theta + 0$$

$$41\cos 2\theta = 9$$

$$2\theta = \cos^{-1}(9/41)$$

$$2\theta = 77.32^\circ \quad (\because 0^\circ < \theta < 90^\circ)$$

$$\boxed{\theta = 38.66^\circ}$$

(ii) For the stress element which is at angle $\theta = 38.66^\circ$, shear stress is given by

$$\tau_\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$= \frac{32+50}{2} \sin 2\theta - 0$$

$$= \frac{32+50}{2} \sin (2 \times 38.66)^\circ = 40 \text{ MPa}$$

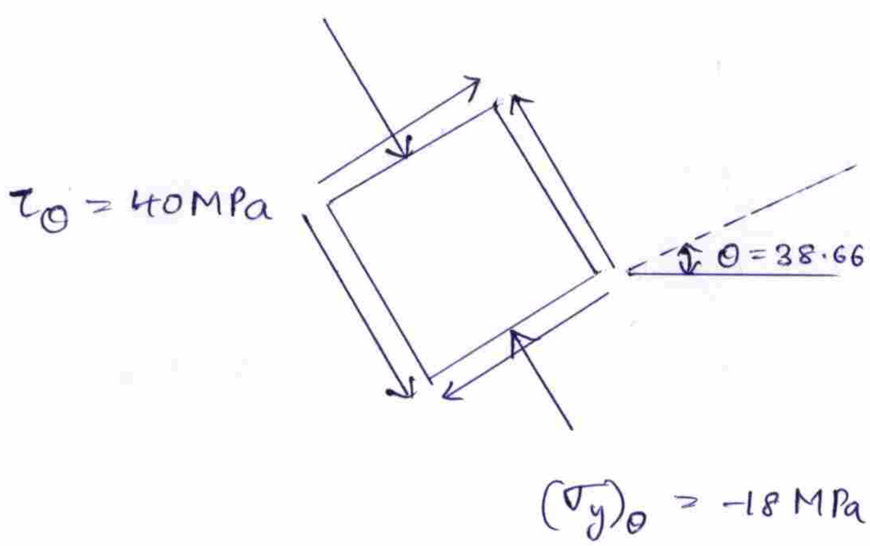
Also, stress invariant

$$I_1 = \sigma_x + \sigma_y = (\sigma_x)_\theta + (\sigma_y)_\theta$$

$$32 - 50 = 0 + (\sigma_y)_\theta$$

$$(\sigma_y)_\theta = -18 \text{ MPa}$$

The stress element having plane 'aa' as one of its side is shown below:



06(b) **(i)** Consider a project with the following data.

Activity	Predecessor	t_o	t_m	t_p
A	-	6	8	10
B	-	3	5	7
C	-	3	4	5
D	A	8	11	14
E	B	2	9	10
F	B, C	2	6	10
G	D, E	2	3	10
H	F, G	4	8	12

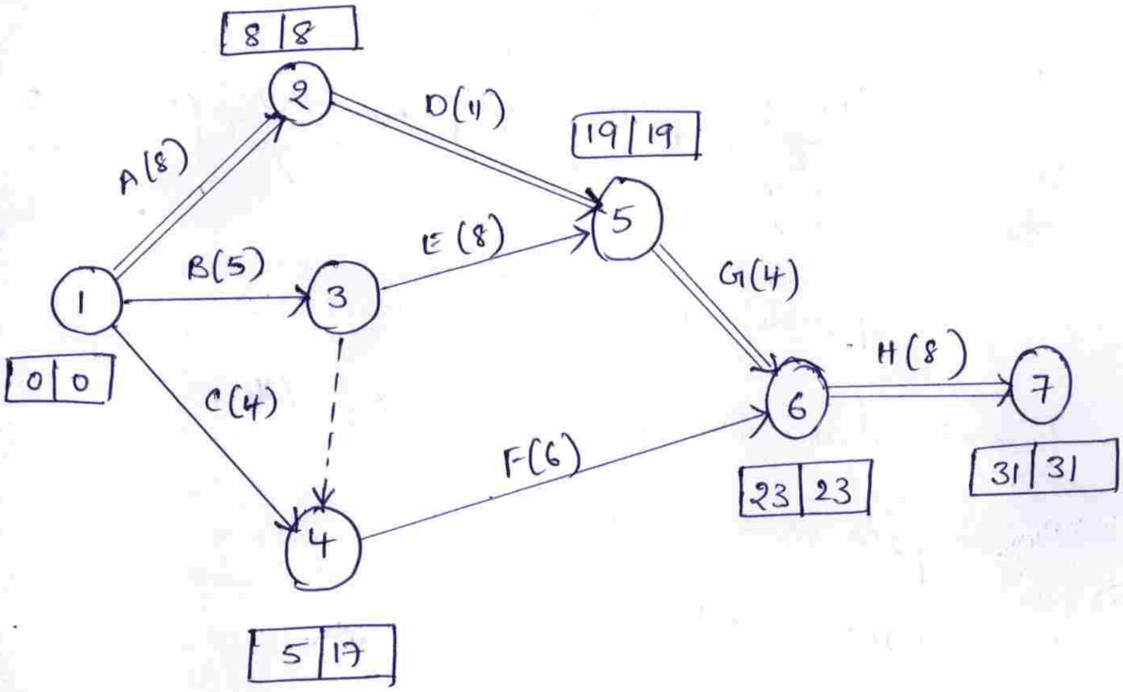
Candidates must not write on this margin

Determine,

(i) Critical path and Expected completion time of the project

(ii) Standard deviation of critical path and Project duration distribution. **(2 × 5 = 10 M)**

Activity	$t_E = \frac{t_o + 4t_m + t_p}{6}$
A	8
B	5
C	4
D	11
E	$\frac{2+4(9)+10}{6} = 8$
F	6
G	$\frac{2+4(3)+10}{6} = 4$
H	8



Path	Duration (days)
A - D - G - H	31
B - E - G - H	25
C - F - H	18
B-dummy - F - H	19

Critical path : $\boxed{A - D - G - H}$

Expected completion time of the project

$$= \boxed{31 \text{ days}}$$

Standard deviation of critical path (σ_{cp})

$$\sigma_{cp} = \sqrt{V_A + V_D + V_G + V_H}$$

Candidates must not write on this margin

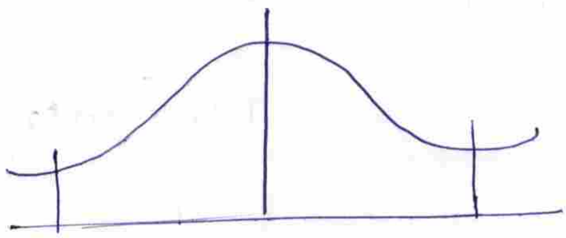
$$\sigma_{cp} = \sqrt{\left(\frac{10-6}{6}\right)^2 + \left(\frac{14-8}{6}\right)^2 + \left(\frac{10-2}{6}\right)^2 + \left(\frac{12-4}{6}\right)^2}$$

$$= \sqrt{0.44 + 1 + 1.77 + 1.77} = \boxed{2.23}$$

Project duration distribution $(\mu \pm 3\sigma)$

$$\mu \pm 3\sigma = 31 \pm 3(2.23)$$

$$\mu \pm 3\sigma = \boxed{24.31 \text{ to } 37.69}$$



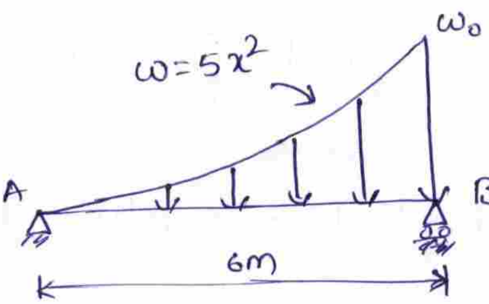
06(b) **(ii)** A simply supported beam of span 6 m is carrying a distributed load w = 5 x^2 N/m, where x is distance in 'm' measured from left support. Develop SF and BM equations. **(10 M)**

Equivalent Load

W = Area $= \frac{1}{3} \times 6 \times \omega_0$

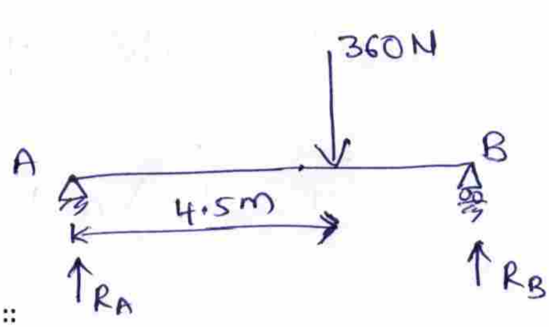
$= \frac{1}{3} \times 6 \times (5 \times 6^2)$

$W = 360\,N$



'W' acting at $\frac{3}{4} \times 6 = 4.5\,m$ from 'A'.

$\Sigma M_A = 0$



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$$6R_B = 360 \times 4.5$$

$$R_B = 270\,N$$

$$\Sigma f_y = 0$$

$$R_A + R_B = 360\,N$$

$$R_A = 90\,N$$

$\Rightarrow$ Shear force equation

$$\frac{dF}{dx} = -\omega \Rightarrow \frac{dF}{dx} = -5x^2$$

$$\int dF = -\int 5x^2 . dx$$

$$\Rightarrow F = -\frac{5x^3}{3} + C$$

At $x=0$, $F = 90N \Rightarrow 90 = -0 + C \Rightarrow C = 90N$

$$\boxed{F = -\frac{5x^3}{3} + 90}$$

$\Rightarrow$ Bending Moment Equation

$$\frac{dM}{dx} = F \Rightarrow \int dm = \int F.dx$$

$$M = \int -\frac{5x^3}{3} + 90 . dx$$

$$M = \frac{-5x^4}{12} + 90x + C$$

At $x=0$, $M=0 \Rightarrow 0 = 0 + 0 + C \Rightarrow C = 0$

$$\boxed{M = -\frac{5x^4}{12} + 90x}$$

06(c). A project consists of 8 activities with the following activity durations and costs.

Activity	Normal		Crash	
	Duration	Cost	Duration	Cost
1 – 2	5	2000	2	2600
1 – 3	6	2200	3	3100
2 – 4	4	3100	2	3900
2 – 6	7	2500	4	4000
3 – 5	5	3500	3	3900
4 – 5	4	1500	2	2300
4 – 6	6	3000	3	4200
5 – 6	7	2000	4	2900

$\Sigma N_c = 19,800$

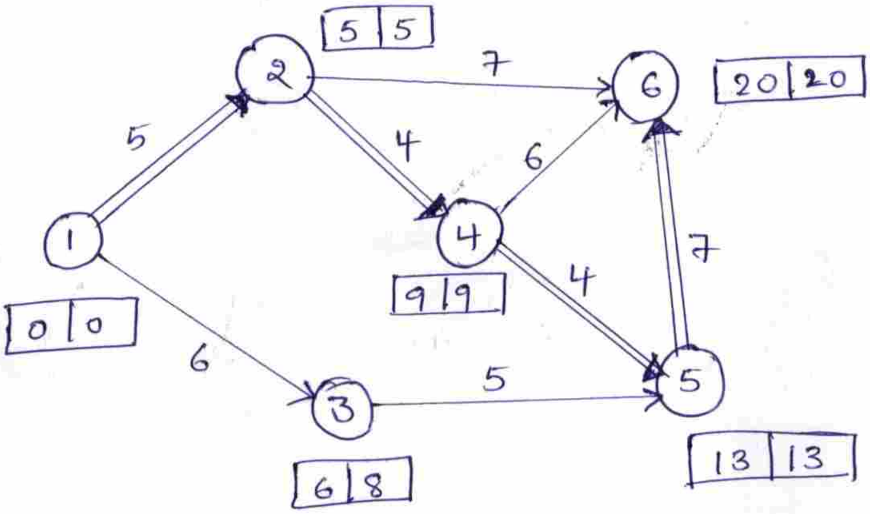
Indirect cost = Rs. 400 /day

Determine Optimum project duration and Minimum project cost. **(20 M)**

Candidates must not write on this margin

Total Normal cost = 19800

Activity	cost slope $= \frac{\Delta C}{\Delta T}$
1 – 2	$\frac{2600-2000}{5-2} = 200$
1 – 3	$\frac{3100-2200}{6-3} = 300$
2 – 4	$\frac{3900-3100}{4-2} = 400$
2 – 6	$\frac{4000-2500}{7-4} = 500$
3 – 5	$\frac{3900-3500}{5-3} = 200$
4 – 5	$\frac{2300-1500}{4-2} = 400$
4 – 6	$\frac{4200-3000}{6-3} = 400$
5 – 6	$\frac{2900-2000}{7-4} = 300$



Path	duration (days)
1-2-6	12
1-2-4-6	15
1-2-4-5-6	20
1-3-5-6	18

Crashing requirement
$= 20 - 18 = 2$ days

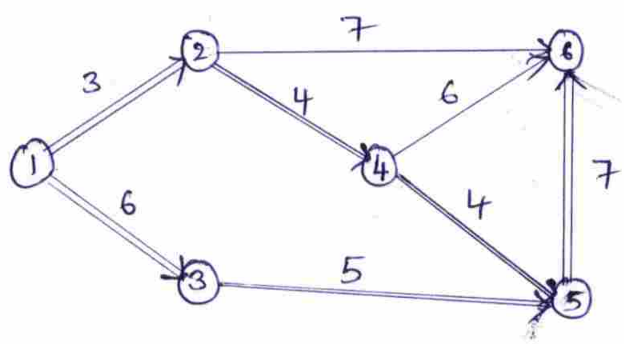
Options for crashing purpose

option	cost slope (Rs)
1-2	200
2-4	400
4-5	400
5-6	300

Here crashing is economical for activity 1-2
(As crashing cost/day = 200 and indirect cost/day = 400)

Crashing possibility of activity 1-2
$= N_T - C_T = 5 - 2 = 3$ days

Hence we need to crash 1-2 by 2-days only



Path	duration (days)
1 – 2 – 6	10
1 – 2 – 4 – 6	13
1 – 2 – 4 – 5 – 6	18
1 – 3 – 5 – 6	18

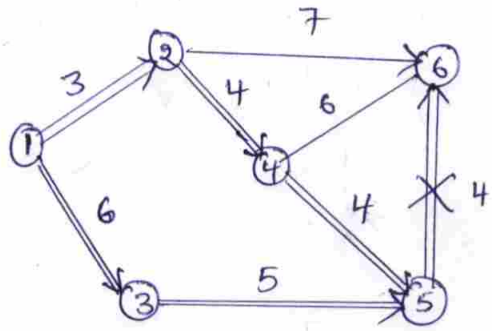
Crashing requirement
= 18 − 13 = 5 days

For crashing purpose →

option	cost slope
5 – 6	300
1–2 & 1–3	200+300 = 500
1–2 & 3–5	200 + 200 = 400
2–4 & 1–3	400 + 300 = 700
2–4 & 3–5	400 + 200 = 600
4–5 & 1–3	400 + 300 = 700
4–5 & 3–5	400 + 200 = 600

So, we crash 5 – 6 by 3days.

After crashing 5 – 6 by 3days.

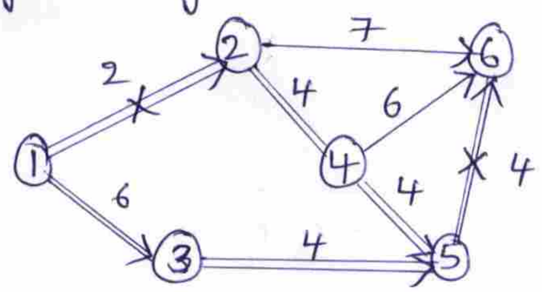


Path	duration
1 – 2 – 6	10
1 – 2 – 4 – 6	13
1 – 2 – 4 – 5 – 6	15
1 – 3 – 5 – 6	15

Crashing requirement
= 15 − 13 = 2 days.

Option	Cost Slope
1-2 & 1-3	200 + 300 = 500
1-2 & 3-5	200 + 200 = 400
2-4 & 1-3	400 + 300 = 700
2-4 & 3-5	400 + 200 = 600
4-5 & 1-3	400 + 300 = 700
4-5 & 3-5	400 + 200 = 600

By crashing 1-2 and 3-5 by 1 day



Path	duration
1-2-6	9
1-2-4-6	12
1-2-4-5-6	14
1-3-5-6	14

Crashing requirement
= 14 - 12 = 2 days

for crashing purpose

option	cost-slope
2-4 & 1-3	400+300 = 700
2-4 & 3-5	400 + 200 = 600
4-5 & 1-3	400 + 300 = 700
4-5 & 3-5	400 + 200 = 600

Here crashing cost/day is more than indirect cost/day

∴ stop crashing

∴ optimum project duration = $\boxed{14 \text{ days}}$

Minimum project cost = Normal-cost + crashing-cost + $\frac{\text{indirect-cost}}{\text{day}}$ × optimum project duration

= 19,800 + [200×2 + 300×3 + 400×1] + 400×14 = $\boxed{\text{Rs. } 27,100/-}$

07(a). A project has 11 activities with the following 3 time estimates.

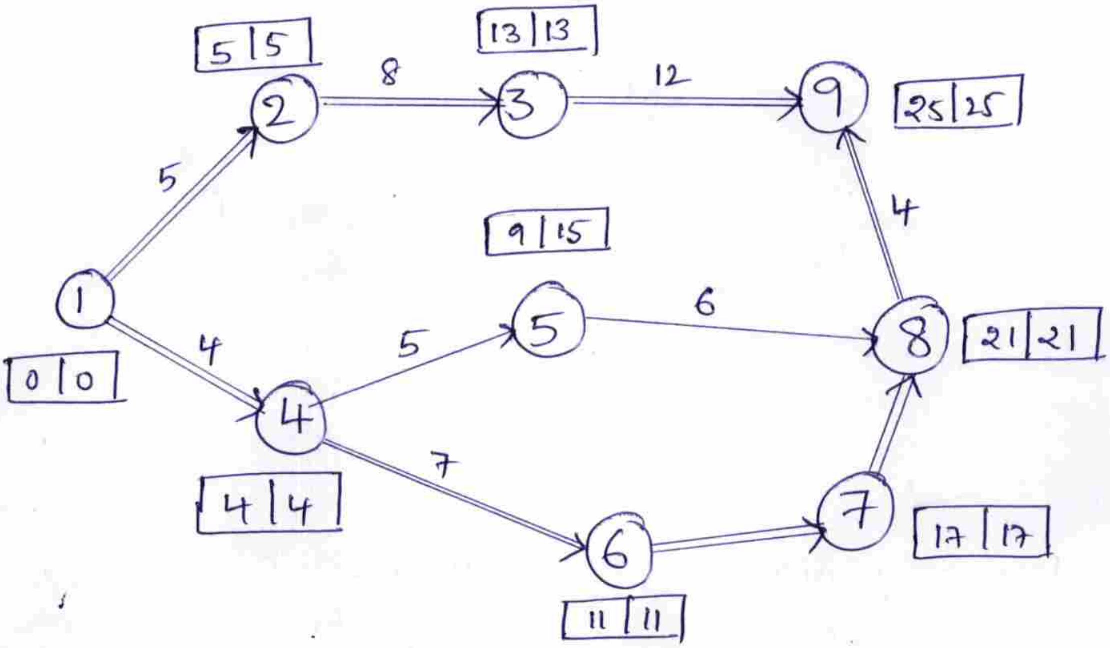
Activity	Optimistic time	Most likely time	Pessimistic time
1 - 2	3	5	7
1 - 4	2	3	10
2 - 3	2	9	10
3 - 4	4	8	12
3 - 9	8	12	16
4 - 5	2	5	8
4 - 6	4	7	10
5 - 8	3	6	9
6 - 7	4	6	8
7 - 8	2	4	6
8 - 9	1	4	7

Find (i) Critical path duration

(ii) Slacks of all events

(iii) Probability to complete the project in time **(6 + 6 + 8 = 20 M)**

Activity	$t_e = \frac{t_o + 4t_m + t_p}{6}$
1 – 2	5
1 – 4	$\frac{2+4(3)+10}{6} = 4$
2 – 3	$\frac{2+4(9)+10}{6} = 8$
3 – 4	8
3 – 9	12
4 – 5	5
4 – 6	7
5 – 8	6
6 – 7	6
7 – 8	4
8 – 9	$\frac{1+4(4)+7}{6} = 4$



Path	duration
1 – 2 – 3 – 9	25
1 – 4 – 5 – 6 – 8 – 9	19
1 – 4 – 6 – 7 – 8 – 9	25

Slacks of events:

@ 1, 2, 3, 9 = 0 (critical events)

@ 1, 4, 6, 7, 8, 9 = 0 (critical events)

@ 5 = 15 – 9 = 6

⇒ Critical paths duration:

1 – 2 – 3 – 9 = 25 days

1 – 4 – 6 – 7 – 8 – 9 = 25 days

⇒ Standard deviation of critical path (σ_{cp})

$$\sigma_{cp} = \max \{ \sigma_{1-2-3-9}, \sigma_{1-4-6-7-8-9} \}$$

$$\sigma_{1-2-3-9} = \sqrt{\left(\frac{7-3}{6}\right)^2 + \left(\frac{10-2}{6}\right)^2 + \left(\frac{16-8}{6}\right)^2} = 1.256$$

$$\sigma_{1-4-6-7-8-9} = \sqrt{\left(\frac{10-2}{6}\right)^2 + \left(\frac{10-4}{6}\right)^2 + \left(\frac{8-4}{6}\right)^2 + \left(\frac{6-2}{6}\right)^2 + \left(\frac{7-1}{6}\right)^2}$$

$$= 2.156$$

⇒ Probability to complete the project in time (i.e. in critical path duration) =

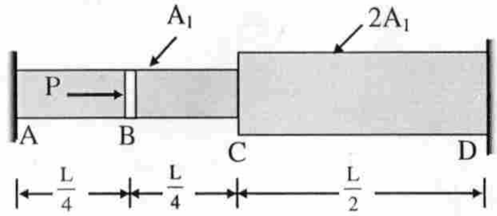
$$Z = \frac{X-\mu}{\sigma} = \frac{25-25}{\sigma} = 0 \Rightarrow P(Z) = \boxed{50\%}$$

07(b). The axially loaded bar ABCD shown in the figure is held between rigid supports. The bar has cross-sectional area A_1 from A to C and $2A_1$ from C to D.

(i) Derive formulas for the reactions R_A and R_D at the ends of the bar.

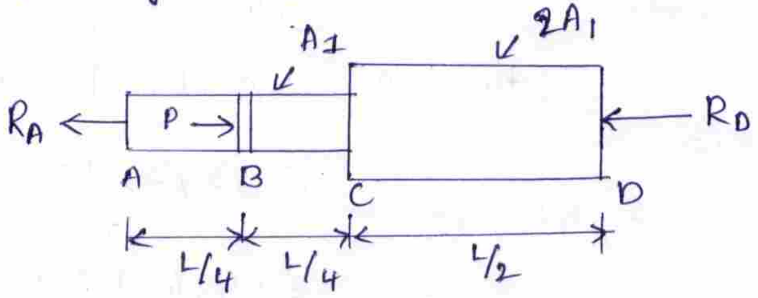
(ii) Determine the displacements δ_B and δ_C at points B and C, respectively.

(iii) Draw a diagram in which the abscissa is the distance from the left-hand support to any point in the bar and the ordinate is the horizontal displacement δ at that point.



(20 M)

Free body diagram of bar is shown below:

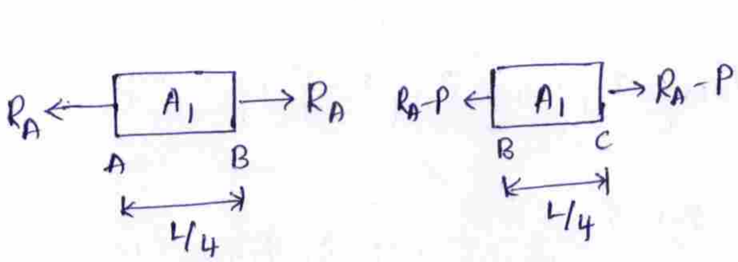


Equation of equilibrium

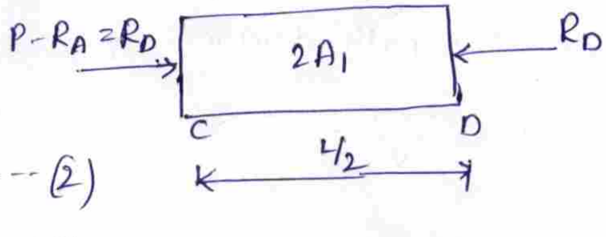
$$\Sigma F_{Hor} = 0$$

$$R_A + R_D = P \text{ ------ } (1)$$

The free body diagram of each section is shown below:



Equation of compatability



$$\delta_{AB} + \delta_{BC} + \delta_{CD} = 0 \text{ --- } (2)$$

Positive means elongation

Force displacement equations,

$$\delta_{AB} = \frac{R_A(L/4)}{EA_1} \text{ ------ } (3)$$

$$\delta_{BC} = \frac{(R_A - P)(L/4)}{EA_1} \text{ ------ } (4)$$

$$\delta_{CD} = -\frac{R_D(L/2)}{E(2A_1)} \text{ ------ } (5)$$

By substituting equations (3), (4), and (5) into equation (2):

$$\frac{R_A L}{4EA_1} + \frac{(R_A - P)(L)}{4EA_1} - \frac{R_D L}{4EA_1} = 0 \text{ ---- } (6)$$

Solve simultaneously equations (1) and (6)

$$\therefore (P - R_D) + (P - R_D - P) - R_D = 0$$

(i) Reactions:

$$\boxed{R_A = \frac{2P}{3}} \text{ and } \boxed{R_D = \frac{P}{3}}$$

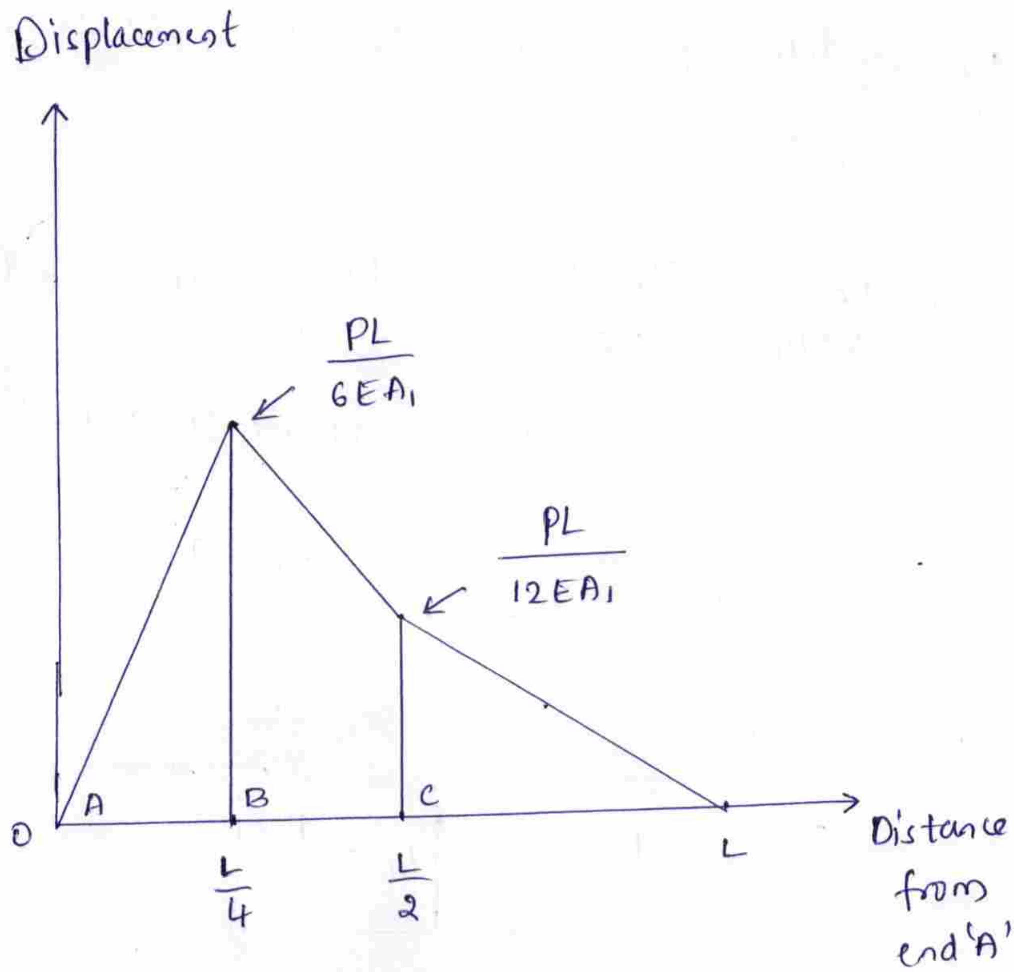
Candidates must not write on this margin

(ii) Displacements at points B and C:

$$\delta_B = \delta_{AB} = \frac{R_A L}{4EA_1} = \boxed{\frac{PL}{6EA_1}} \text{ (To the right)}$$

$$\delta_C = |\delta_{CD}| = \frac{R_D L}{4EA_1} = \boxed{\frac{PL}{12EA_1}} \text{ (To the right)}$$

(iii) Axial displacements diagram (ADD):



07(c). A vertical composite bar rigidly fixed at the upper end consists of a steel rod of 16 mm diameter enclosed in a brass tube of 16 mm internal diameter and 24 mm external diameter each being 2 m long. Both are fixed together at the ends. The tie bar is suddenly loaded by a weight of 8 kN falling through a distance of 4 mm. Determine the maximum stresses in steel rod and brass tube. [Take, E_s = 205 GPa, E_b = 100 GPa]

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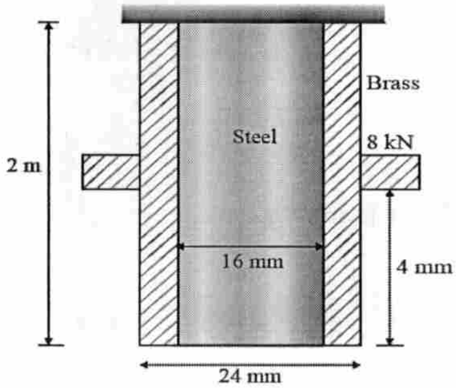

(Diagram not to scale)

(20 M)

Steel

$d_s = 16\,mm$

$W = 8000\,N$

$E_s = 205\,GPa$

Brass

$d_{bi} = 16\,mm$

$d_{bo} = 24\,mm$

$L = 2000\,mm$

$h = 4\,mm$

$E_b = 100\,GPa$

$$A_s = \frac{\pi d_s^2}{4} = \frac{\pi}{4} 16^2 = 64\pi\ mm^2$$

$$A_b = \frac{\pi(24^2 - 16^2)}{4} = 80\pi\ mm^2$$

Let the extension of bar = x mm

$$\therefore\ \sigma_s = \frac{E_s \cdot x}{L} \quad \text{and} \quad \sigma_b = \frac{E_b \cdot x}{L}$$

Strain energy of the bar

$$= \frac{\sigma_s^2}{2E_s} \cdot A_s \cdot L + \frac{\sigma_b^2}{2E_b} \cdot A_b \cdot L$$

$$= \frac{E_s^2 x^2}{L^2 \cdot 2E_s} \times A_s \cdot L + \frac{E_b^2 x^2}{L^2 2E_b} \cdot A_b \cdot L$$

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$$= \frac{x^2}{2L}\left(E_s A_s + E_b A_b\right)$$

$$= \frac{x^2}{2\times 2000}\left(205\times10^3\times 64\pi + 100\times10^3\times 80\pi\right)$$

$$= 16588x^2 \text{ N-mm}$$

P.E lost by the weight $= W(h+x)$

$$= 8000(4+x) \text{ N-mm}$$

$$16588x^2 = 8000(4+x)$$

$$x^2 - 0.482x - 1.929 = 0$$

$$\therefore x = 1.651\text{mm}$$

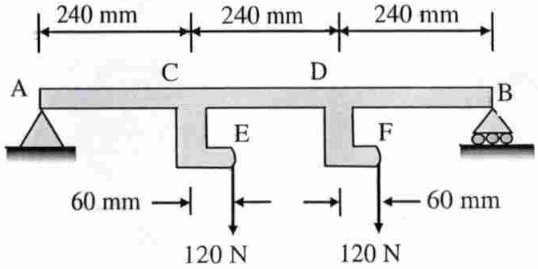
$$\therefore \sigma_s = \frac{E_s x}{L} = \frac{205\times10^3\times1.651}{2000} = \boxed{169.2\text{MPa}}$$

$$\sigma_b = \frac{E_b x}{L}$$

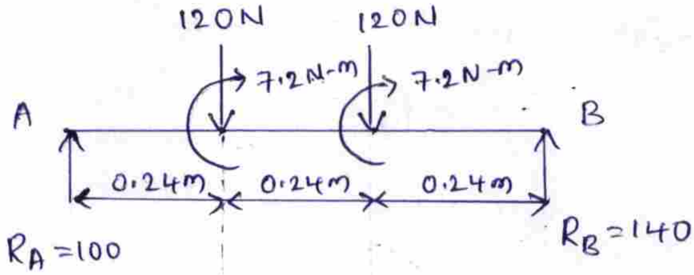
$$= \frac{100\times10^3\times1.651}{2000}$$

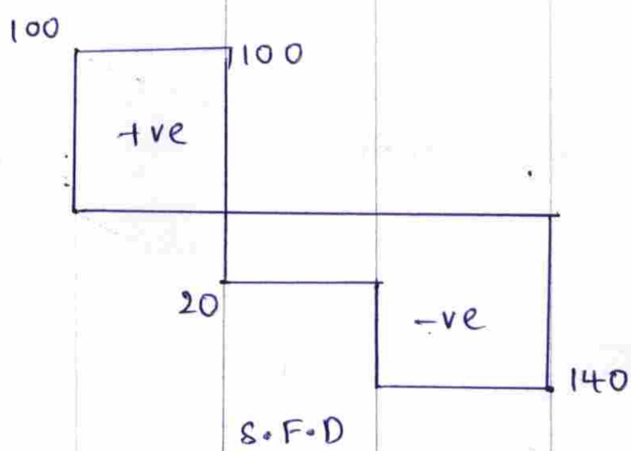
$$= \boxed{82.54\text{MPa}}$$

08(a). Draw the shear and bending moment diagrams for the beam and loading shown, and determine the maximum absolute value (i) of the shear force, (ii) of the bending moment.

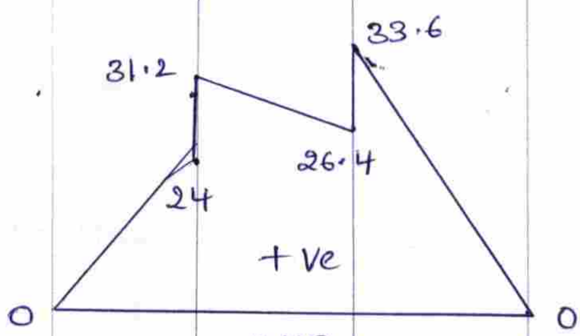


(2×10 = 20 M)





S.F.D



BMD

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Taking moment about point B,

$$+\circlearrowleft \Sigma M_B = 0$$

$$-0.72A + (0.42)(120) + (0.24)(120) - 7.2 - 7.2 = 0$$

$$A = 100N$$

Under equilibrium

$$\Sigma F_x = 0$$

$$\therefore A + B = 120 + 120$$

$$100 + B = 240$$

$$B = 140 \text{ N}$$

<u>Shear Force:</u>

$$V_A = 100N$$

$$(V_C)_{Left} = V_A = 100 \text{ N}$$

$$(V_C)_{Right} = 100 - 120 = -20 \text{ N}$$

$$(V_D)_{left} = (V_C)_{right} = -20N$$

$$(V_D)_{right} = -20 - 120 = -140N$$

$$(V_B) = (V_D)_{right} = -140N$$

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Bending Moment:

$$M_A = 0$$

$$(M_C)_{left} = 100 \times 0.24 = 24 \text{ N.m}$$

$$(M_C)_{right} = 24 + 7.2 = 31.2 \text{ N.m}$$

$$(M_D)_{left} = 100 \times 0.48 + 7.2 - 120 \times 0.24$$

$$= 26.4 \text{ N.m}$$

$$(M_D)_{right} = 26.4 + 7.2 = 33.6 \text{ N.m}$$

$$M_B = 0$$

Maximum absolute value of shear force $\boxed{|V| = 140 \text{ N}}$

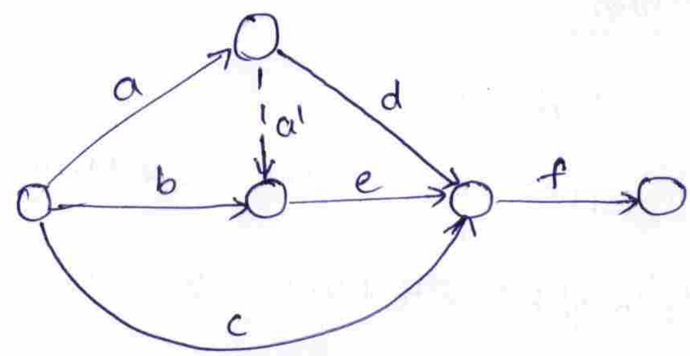
Maximum absolute value of bending moment $\boxed{|M| = 33.6 \text{ N.m}}$,

08(b). Explain bidding process for the project, find minimum duration and optimum duration.

Activity	Immediate predecessor(s)	Days		Rupees	
		NT	CT	NC	CC
A	-	8	7	4000	5000
B	-	6	5	3000	3600
C	-	14	12	7000	9000
D	A	7	5	3500	4500
E	A, B	7	6	4200	4800
F	C, D, E	2	1	1000	1500

Overhead cost is 600 Rs /day. **(20 M)**

Activity	NT	CT	NC	CC	Slope
a	8	7	4000	5000	1000
b	6	5	3000	3600	600
c	14	12	7000	9000	1000
d	7	5	3500	4500	500
e	7	6	4200	4800	600
f	2	1	1000	1500	500



a' : dummy activity

Path	Sum	Stages		
		I	II	III
a – d – f	17	16	15	14
a – a' – e – f	17	16	15	14
b – e – f	15	14	14	13
c – f	16	15	15	14

Normal project duration = 17 days

Total direct cost = ΣNC = Rs. 28,700

Total indirect cost = 600 × 17 = Rs. 10,200

Total project cost = Rs. 38,900

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Stage I :

Target = 16 days

Options	Slope
a	1000
f	500
d & e	500 + 600 = 1100

→ most economical

Crashing 'f' by 1 day is best option.

Total project cost

= 38,900 + 500 − 600

= Rs. 38,800

Stage – II

Target = 15 days

Options	slope
a	1000
d & e	1100

→ most economical

Crashing 'a' by 1 day is the best option.

Total project cost = 38,800 + 1000 − 600
= Rs. 39,200

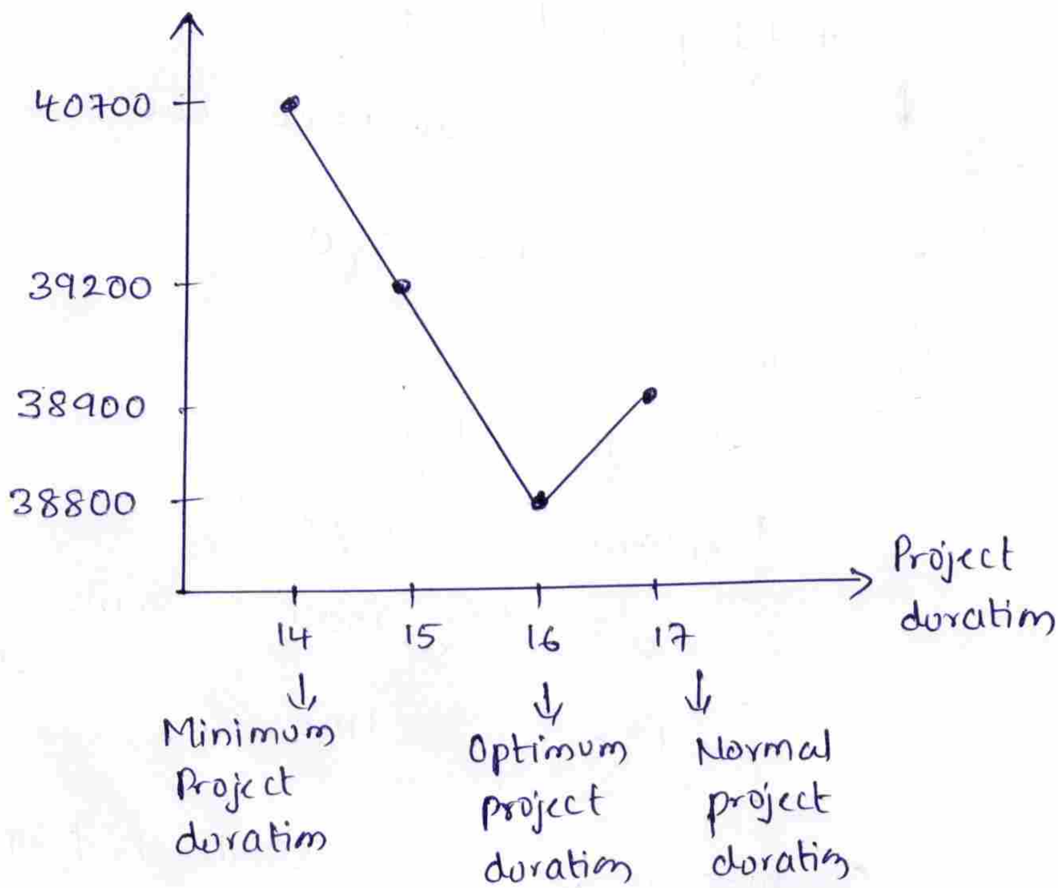
Minimum project cost is Rs. 38,800.

Stage – III :- Target 14 days

options	slope
d & e & c	1000 + 500 + 600 = 2100

Total project cost = 39,200 + 2100 − 600
= Rs. 40,700

since a–a'–e–f
cannot be further crashed, 14 days is the minimum project duration.



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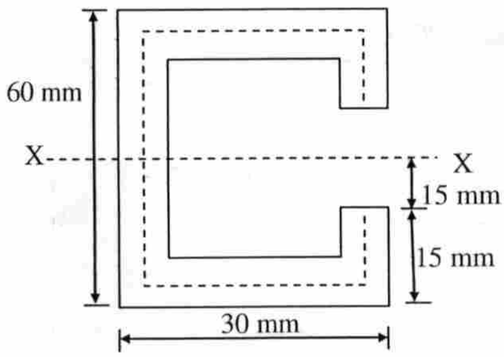
08(c). A beam section as shown in figure has a constant thickness of 2 mm. The shear force at the section is 8 kN. Sketch the shear stress distribution in the section of beam.



(Diagram not to scale) **(20 M)**

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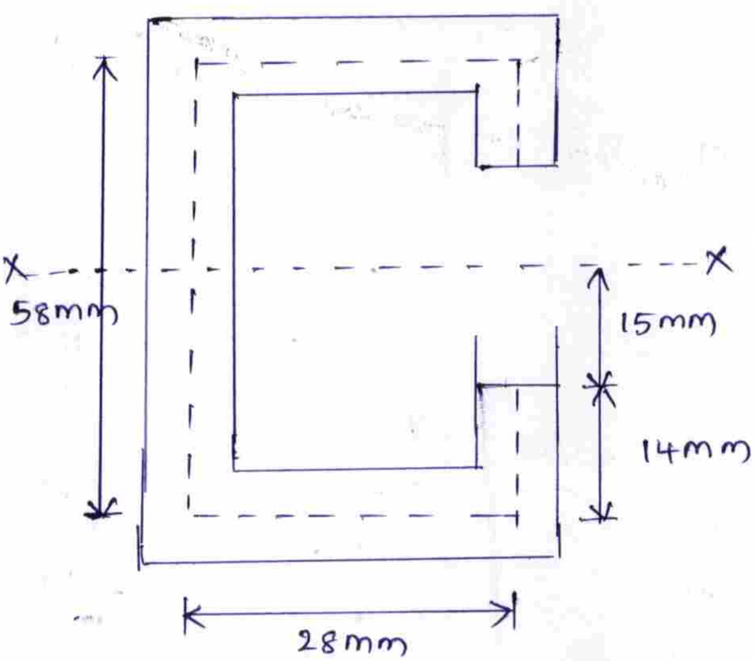
Given

$F = 8 kN$

$t = 2 mm$



$$I_{xx} = \frac{2 \times 58^3}{12} + 2\left(\frac{28 \times 2^3}{12} + 28 \times 2 \times 29^2\right)$$

$$+ 2\left(\frac{2 \times 14^3}{12} + 2 \times 14 \times 22^2\right)$$

$$I_{xx} = 154766 mm^4$$

Assuming there is no twisting in the beam;

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Shear stresses:

$$\tau_1 = F \cdot \frac{A\bar{y}}{Ib} = 0$$

$$\tau_2 = \frac{8\times10^3 \times (2\times14\times22)}{154766\times2} = \boxed{15.92\ MPa}$$

$$\tau_3 = 15.92 + \frac{8\times10^3\times(28\times2\times29)}{154766\times2} = \boxed{57.89 MPa}$$

$$\tau_4 = 57.89 + \frac{8\times10^3\times(29\times2\times14.5)}{154766\times2} = \boxed{79.62 MPa}$$

Sketch: