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GATE – 2019

Questions with Detailed Solutions

ELECTRONICS & COMMUNICATION ENGINEERING

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Subject wise weightage

S. No.	Name of the Subject	1 Mark (Q)	2 Marks (Q)	Total Marks
1	Verbal Ability	3	1	5
2	Numerical Ability	2	4	10
3	Engineering Mathematics	5	3	11
4	Network Theory	1	2	5
5	Control Systems	2	4	10
6	Digital Circuits	3	1	5
7	Signals & System	2	3	8
8	EDC & VLSI	4	8	20
9	Analog Circuits	1	2	5
10	EMT	3	3	9
11	Communication Systems	4	4	12
Total No. of. Marks		**30**	**70**	**100**

Subject Experts,
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Section : General Aptitude

01. The boat arrived ______ dawn.
(a) Under (b) at (c) on (d) in

01. Ans: (b)

Sol: Use preposition 'at' dawn

End of Solution

02. When he did come home, she ______ him lying dead on the roadside somewhere.
(a) concluded (b) notice (c) looked (d) pictured

02. Ans: (d)

Sol: Pictured means to have a thought, understanding or idea about something or someone.

End of Solution

03. Five different books (P, Q, R, S, T) are to be arranged on a shelf. The books R and S are to be arranged first and second, respectively from the right side of the shelf. The number of different orders in which P, Q and T may be arranged is ______.
(a) 6 (b) 2 (c) 12 (d) 120

03. Ans: (a)

Sol: Five different books = P, Q, R, S and T from the given data, the above books are arranged as follows

__ __ __ S R

P Q T
P T Q
Q P T
Q T P
T P Q
T Q P

} 6 ways

Another Method:

From the given data R and S places are fixed and remaining three books can be arranged in 3! ways (i.e.) $3 \times 2 \times 1 = 6$ ways.

Hence Option (1) is correct.

End of Solution

04. The strategies that the company _______ to sell its products ______ house-to-house marketing.
(a) use, includes (b) uses, include (c) uses, including (d) used, includes

04. Ans: (b)

Sol: Sub + Verb agreement

'The Company' is singular, so 'uses' and the 'strategies' is plural, so 'include'(The company uses, the strategies include).

05. It would take one machine 4 hours to complete a production order and another machine 2 hours to complete the same order. If both machines work simultaneously at their respective constant rates, the time taken to complete the same order is _____ hours.

(a) 3/4 (b) 2/3 (c) 7/3 (d) 4/3

05. Ans: (d)

Sol: Machine one (M_1) can take to complete production = 4 hours

Second Machine (M_2) can take to complete production = 2 hours

M_1 = 4 hours, 1 hours = $\frac{1}{4}^{th}$ production

M_2 = 2 hours, 1 hours = $\frac{1}{2}^{th}$ production

In one hour $(M_1 + M_2) = \frac{1}{4} + \frac{1}{2} = \frac{1+2}{4} = \frac{3}{4}^{th}$

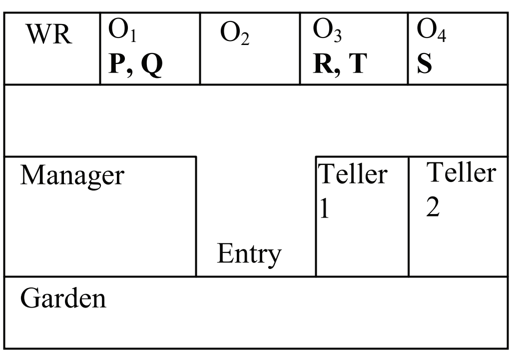
$\therefore$ M_1 and M_2 can take to complete production = $\frac{4}{3}$ hours

End of Solution

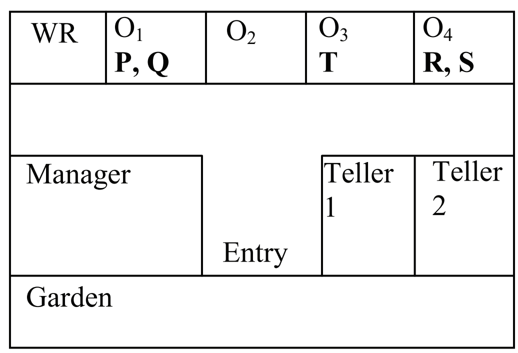
06. Five people P, Q, R, S and T work in a bank. P and Q don't like each other but have to share an office till T gets a promotion and moves to the big office next to the garden. R, who is currently sharing an office with T wants to move to the adjacent office with S, the handsome new intern. Given the floor plan, what is the current location of Q, R and T?

(O = Office, WR = Washroom)

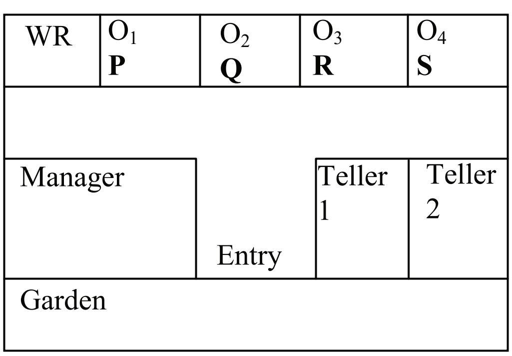
(a)



(b)



(c)



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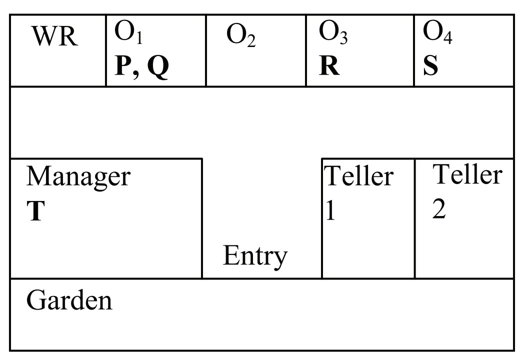


06. Ans: (a)

Sol: Before getting promotion 'T' sharing with R and P and Q's are working together means they are in same office.

Option '2' is not correct due to T is sharing with R (i.e.) before getting promotion T is not worked alone.

Option '3' is not correct due to 'T' place of work is not defined.

Option '4' is also not correct due to after 'T' getting promotion P and Q is are not working together.

End of Solution

07. Four people are standing in a line facing you. They are Rahul, Mathew, Seema and Lohit. One is an engineer, one is a doctor, one a teacher and another a dancer. You are told that:

1. Mathew is not standing next to Seema
2. There are two people standing between Lohit and the engineer
3. Rahul is not a doctor
4. The teacher and the dancer are standing next to each other
5. Seema is turning to her right to speak to the doctor standing next to her

Who among them is an engineer?

(a) Mathew (b) Rahul
(c) Seema (d) Lohit

07. Ans: (a)

Sol: Four peoples are Rahu, Mathew, Seema and Lohit and in the group one engineer, one is a doctor, one a teacher and another a dancer.

Statement 1:

Seema Mathew

Statement 2:

Lohit ___ ___ ___ ___

Engineer

Statement 3:

Rahul $\neq$ doctor

Statement 4:

Teacher (or) Dancer Dancer (or) Teacher

Statement 5:

____ Seema

Doctor

From above conditions, the following line can be formed

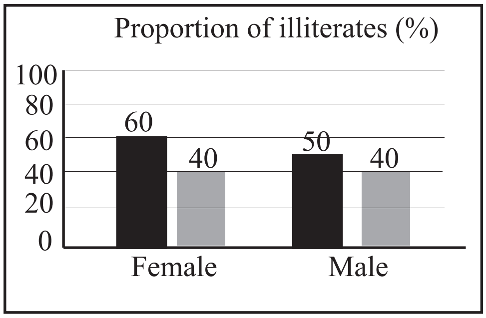
Lohit	Seema	Rahul	Mathew
↓	↓	↓	↓
Doctor	Teacher/Dancer	Teacher/Dancer	Engineer

From above, an engineer in the group is Mathew.

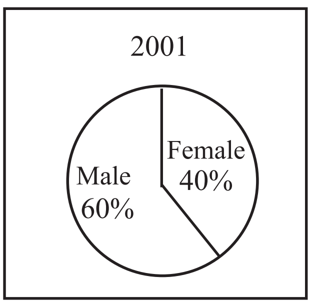
Hence option '1' is correct.

08. The bar graph in Panel(a) shows the propagation of male and female illiterates in 2001 and 2011. The proportions of males and females in 2001 and 2011 are given in Panel(b) and (c), respectively. The total population did not change during this period.

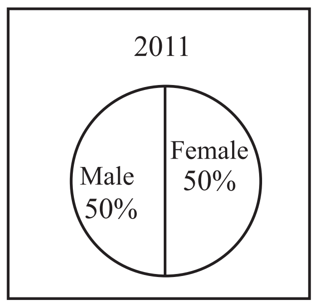
The percentage increase in the total number of literates from 2001 to 2011 is _____.



Panel (a)



Panel (b)



Panel (c)

(a) 35.43 (b) 30.43 (c) 34.43 (d) 33.43

08. Ans: (b)

Sol: Panel (a), Bar diagram represents, proportion of illiterates (%) dark shaded represents female and male illiterates in 2001 and light shaded represents female and male illiterates in 2011. Panel (b) and panel (c) male and females in 2001 and 2011 respectively.

Assume population in 2001 = 100 nos

from given data, population in 2011 also 100.

From the given Bar charts and pie charts, the following table can be possible.

	2001		2011	
	Males	Females	Males	Females
Total	60	40	50	50
Illiterates	50% of 60 = 30	60% of 40 = 24	40% of 50 = 20	40% of 50 = 20
Literates	60 - 30 = 30	40 - 24 = 16	50 - 20 = 30	50 - 20 = 30

Total literates in 2001 = 30 + 16 = 46

Total literates in 2011 = 30 + 30 = 60

$\therefore$ The percentage increase in the total number of literates from 2001 to 2011 $= \frac{60-46}{46} \times 100$

$= \frac{14}{60} \times 100 = 30.43\%$

09. "Indian history was written by British historians – extremely well documented and researched, but not always impartial. History had to serve its purpose: Everything was made subservient to the glory of the Union Jack. Latter-day Indian scholars presented a contrary picture."

From the text above, we can infer that:

Indian history written by British historians _____

(a) was not well documented and researched and was always biased

(b) was well documented and not researched but was always biased

(c) was well documented and researched but was sometimes biased

(d) was not well documented and researched and was sometimes biased

09. Ans: (c)

Sol: Other choices are irrelevant

End of Solution

10. Two design consultants, P and Q, started working from 8 Am for a client. The client budgeted a total of USD 3000 for the consultants. P stopped working when the hour hand moved by 210 degrees on the clock. Q stopped working when the hour hand moved by 240 degrees. P took two tea breaks of 15 minutes each during her shift, but took no lunch break. Q took only one lunch break for 20 minutes, but no tea breaks. The market rate for consultants is USD 200 per hour and breaks are not paid. After paying the consultants, the client shall have USD ____ remaining in the budget.

(a) 000.00

(b) 166.67

(c) 300.00

(d) 433.33

10. Ans: (b)

Sol: P and Q started work at 8 am

P makes an angle of 210°

Q makes an angle of 240°

hours hand makes an angle of 30° for each hours (i.e.) $\frac{360^\circ}{12} = 30^\circ$

P →210° and Q →240°

Tea break time of P = 15 min × 2 = 30 min

Lunch break time of Q = 20 min

P's working hours including breaks = $\frac{210}{30} = 7$ hours

Q's working hours including breaks = $\frac{240}{30} = 8$ hours

P's net working hours (excluding breaks) = 7 hrs - 30 min = 6 hrs 30 min

Q's net working hours (excluding breaks) = 8 hrs - 20 min = 7 hrs 40 min

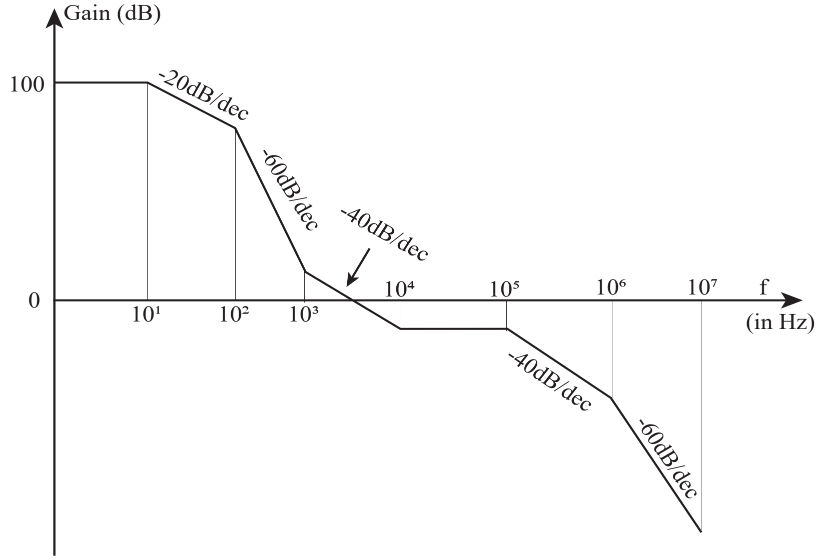
P's and Q's total working hours = 6 hrs 30 min + 7 hrs 40 min = 14 hrs 10 min

Total amount payed to consultants = $14 \text{ hrs} \times 200 + \frac{10}{60} \times 200 = 2800 + 33.34 = 2833.33$

The remaining amount with client from the budget = 3000 - 2833.33 = 166.67 USD

Section : Electronics & Communication Engineering

01. For an LTI system, the Bode plot for its gain is as illustrated in the figure shown. The number of system poles N_p and the number of system zeros N_z in the frequency range 1 Hz $\le f \le 10^7$ Hz is

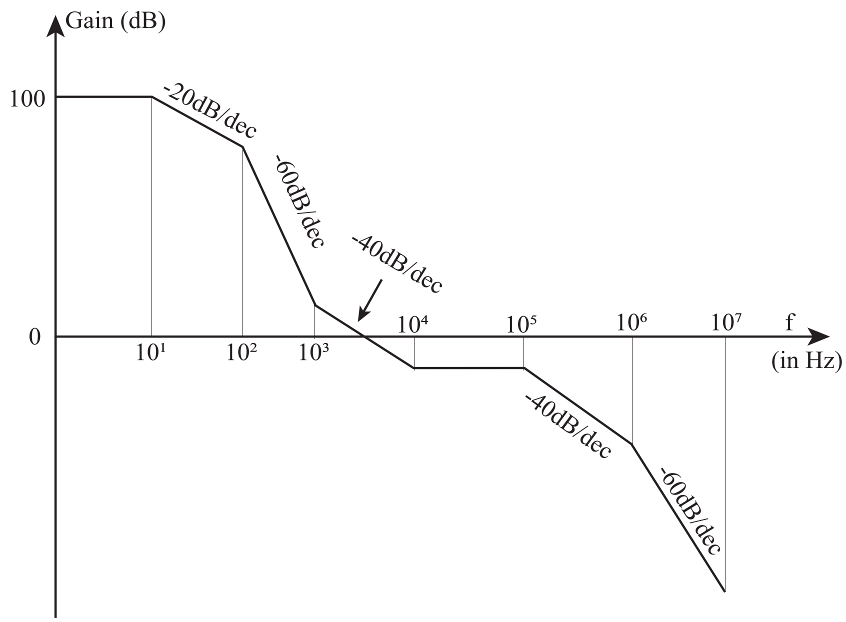


(a) $N_p = 4, N_z = 2$ (b) $N_p = 6, N_z = 3$

(c) $N_p = 7, N_z = 4$ (d) $N_p = 5, N_z = 2$

01. Ans: (b)

Sol:



At f = 10Hz → 1pole (– 20 dB/dec)

$f = 10^2$ Hz → (–40 dB/dec) → 2 poles

$f = 10^3$ Hz → (+20 dB/dec) → 1 zero

$f = 10^4$ Hz → (+40 dB/dec) → 2 zeros

$f = 10^5$ Hz → (–40 dB/dec) → 2 poles

$f = 10^6$ Hz → (–20 dB/dec) → 1 pole

$N_p = 6$, $N_Z = 3$

End of Solution

02. Let Y(s) be the unit-step response of a causal system having a transfer function $G(s) = \frac{3-s}{(s+1)(s+3)}$

that is, $Y(s) = \frac{G(s)}{s}$. The forced response of the system is

(a) u(t)

(b) 2u(t)

(c) $2u(t) - 2e^{-t}\,u(t) + e^{-3t}\,u(t)$

(d) $u(t) - 2e^{-t}\,u(t) + e^{-3t}\,u(t)$

02. Ans: (a)

Sol: Y(s) → Unit step response of a causal system

$$Y(s) = \frac{G(s)}{s} = \frac{3-s}{(s+1)(s+3)s}$$

$$= \frac{-2}{s+1} + \frac{1}{s+3} + \frac{1}{s}$$

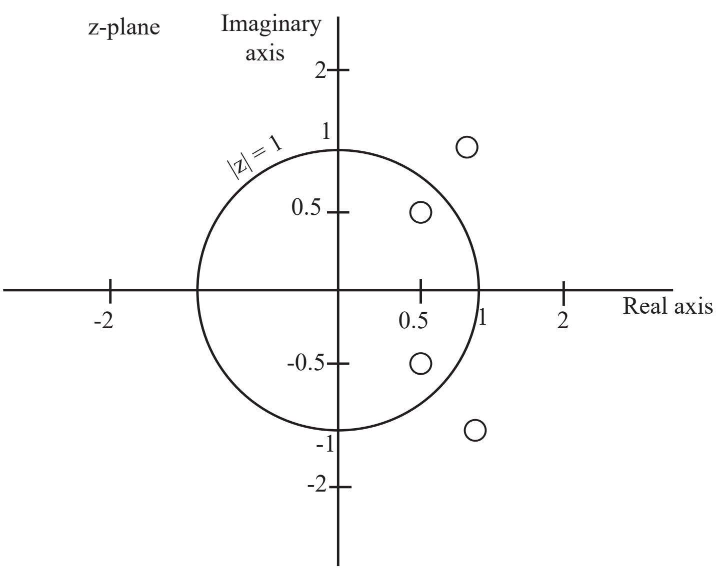
$\downarrow_{ILT}$

$y(t) = -2\,e^{-t}\,u(t) + e^{-3t}\,u(t) + u(t)$. So, the forced response of the system is u(t)

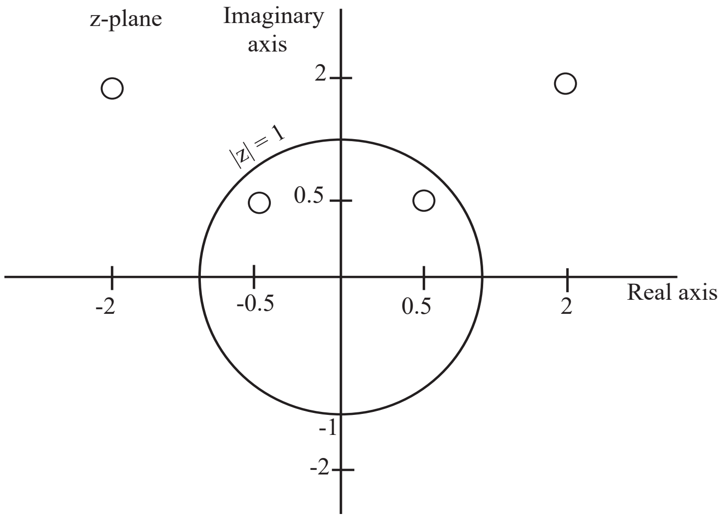
End of Solution

03. Let H(z) be the z-transform of a real-valued discrete-time signal h[n]. If P(z) = H(z) $H\left(\frac{1}{z}\right)$ has a zero at $z = \frac{1}{2} + \frac{1}{2}j$, and P(z) has a total of four zeros, which one of the following plots represents all the zeros correctly?

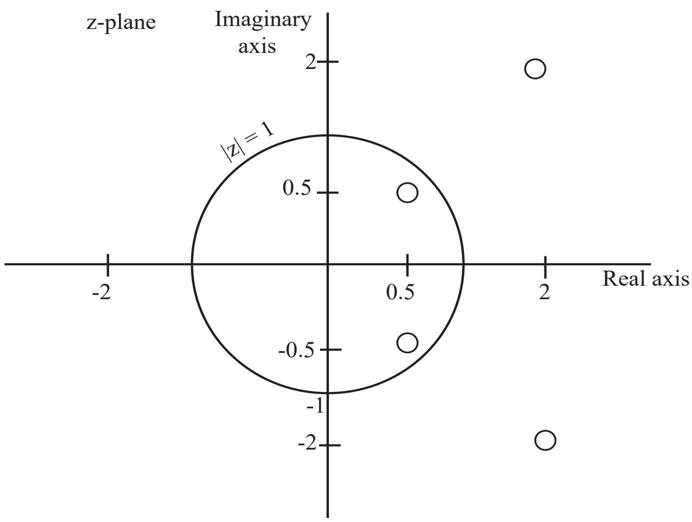
(a)



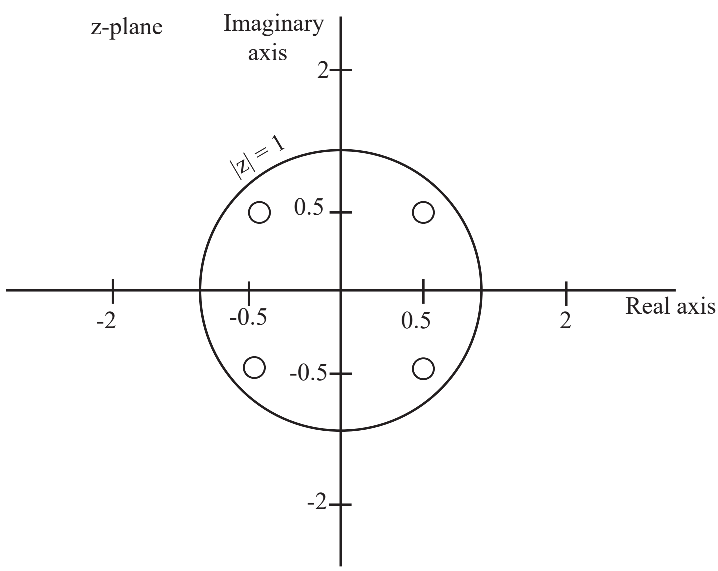
(b)


z-plane
Imaginary axis
2
|z| = 1
0.5
-2
-0.5
0.5
2
Real axis
-1
-2

(c)


z-plane
Imaginary axis
2
|z| = 1
0.5
-2
0.5
2
Real axis
-0.5
-1
-2

(d)


z-plane
Imaginary axis
2
|z| = 1
0.5
-2
-0.5
0.5
2
Real axis
-0.5
-2

03. Ans: (a)

Sol: H(z) is Z-transform of real valued discrete time signal h(n)

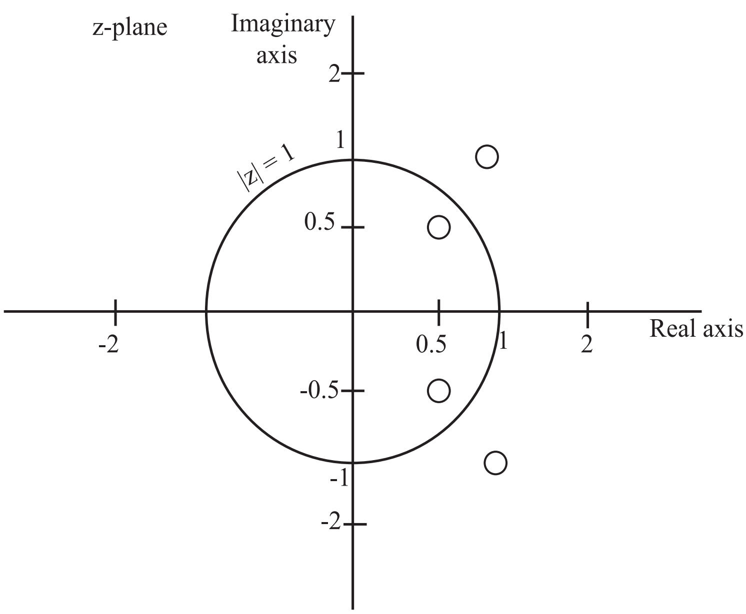
If H(z) has a zero at $z_1 = \frac{1}{2} + \frac{j}{2}$

The remaining zeros are $z_2 = z_1^* = \frac{1}{2} - \frac{j}{2}$

$$z_3 = \frac{1}{z_1} = 1 - j$$

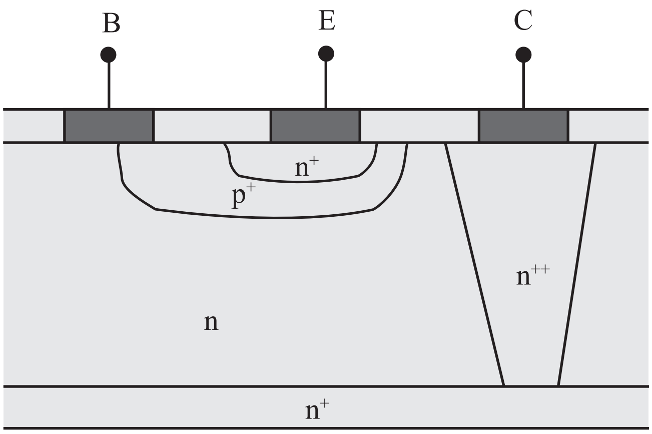
$$z_4 = \left(\frac{1}{z_1}\right)^* = 1 + j$$

The pole zero plot is

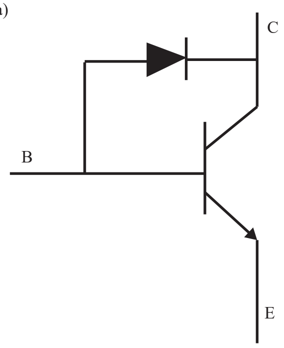


End of Solution

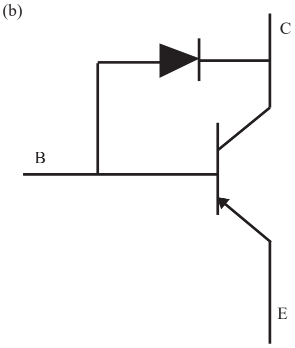
04. The correct circuit representation of the structure shown in the figure is



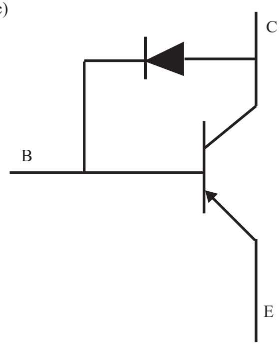
(a)



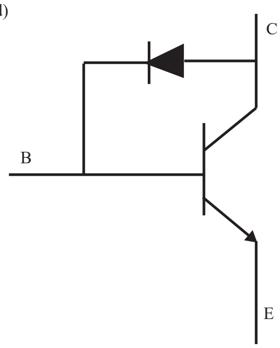
(b)



(c)



(d)



04. Ans: (a)

Sol: As per the fabrication structure given BJT is npn transistor with base to collector of nn^+ material means act as P-N diode from base to collector i.e at base usually 'n' material acts as p-type with respect to n^{++} and n^{++} act as n-type material with respect to n-material

End of Solution

05. The families of curves represented by the solution of the equation $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$

For n = –1 and n = +1, respectively, are

(a) Circles and Hyperbolas

(b) Parabolas and Circles

(c) Hyperbolas and Circles

(d) Hyperbolas and Parabolas

05. Ans: (c)

Sol: Given $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^n$

$\Rightarrow y^n\, dy + x^n\, dx = 0$ (use variable separable method)

By Integrating $\frac{y^{n+1}}{n+1} + \frac{x^{n+1}}{n+1} = \frac{c^{n+1}}{n+1}$ For $(n \neq -1)$

$\Rightarrow y^{n+1} + x^{n+1} = c^{n+1}$

Clearly for n = 1 $x^2 + y^2 = c^2 \Rightarrow$ circle

For n = –1 $\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{-1} = -\frac{y}{x}$

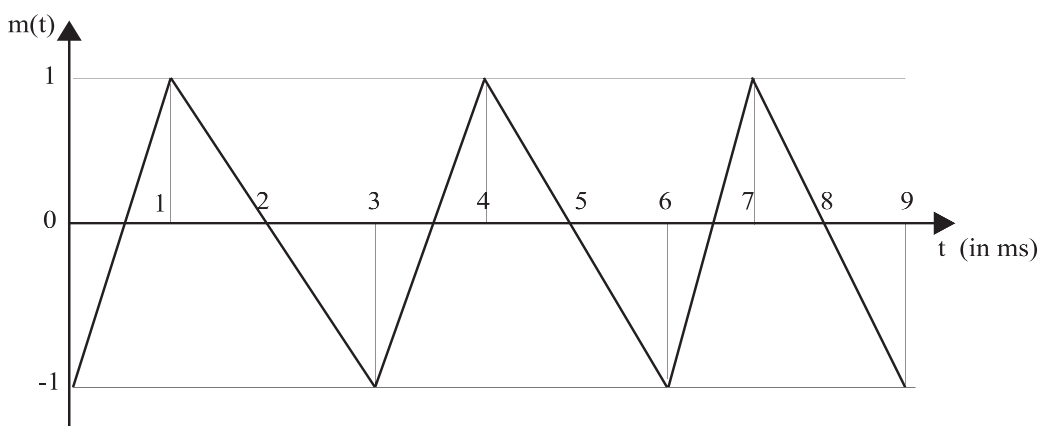
$\Rightarrow x\, dy + y\, dx = 0$

$\Rightarrow d(xy) = 0$

By Integrating $\rightarrow xy = c \rightarrow$ Hyperbola

End of Solution

06. The baseband signal m(t) shown in the figure is phase-modulated to generate the PM signal $\phi(t) = \cos(2\pi f_c t + k\, m(t))$. The time t on the x-axis in the figure is in milliseconds. If the carrier frequency is $f_c = 50$ kHz and $k = 10\pi$, then the ratio of the minimum instantaneous frequency (in kHz) to the maximum instantaneous frequency (in kHz) is ______ (rounded off to 2 decimal places).



06. Ans: 0.75

Sol: $f_i = f_c + \frac{k}{2\pi}\frac{d}{dt}m(t)$

$f_{max} = f_c + \frac{k}{2\pi}\frac{d}{dt}m(t)_{max}$

$f_{min} = f_c + \frac{k}{2\pi}\frac{d}{dt}m(t)_{min}$

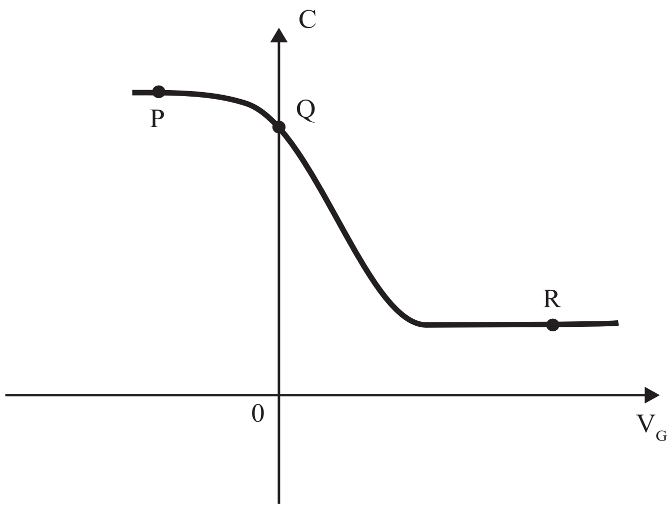
given $f_c = 50kHz, k = 10\pi \Rightarrow \frac{k}{2\pi} = 5$

$\left.\frac{dm(t)}{dt}\right|_{min} = -1k, \left.\frac{dm(t)}{dt}\right|_{max} = 2k$

$\frac{f_{min}}{f_{max}} = \frac{50k + 5k}{50k + 10k} = \frac{45k}{60k} = 0.75$

End of Solution

07. he figure shows the high-frequency C-V curve of a MOS capacitor (at T = 300K) with Φ_{ms} = 0 V and no oxide charges. The flat-band, inversion, and accumulation conditions are represented, respectively, by the points



(a) Q, R, P
(b) Q, P, R
(c) R, P, Q
(d) P, Q, R

07. Ans: (a)

Sol: Given C-V characteristics of MOS capacitor with p-type substrate for high frequencies.

Point-P possible in accumulation mode

Point-Q possible in flat band mode

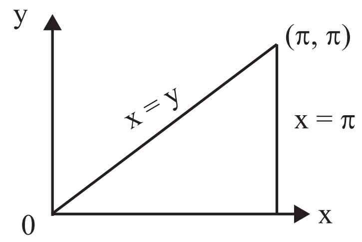
Point-R possible in inversion mode

So, option (1) is correct

08. The value of the integral $\int_0^{\pi}\int_y^{\pi}\frac{\sin x}{x}dx\,dy$, is equal to _______.

08. Ans: 2

Sol: $\int_{y=0}^{\pi}\left[\int_{x=y}^{\pi}\frac{\sin x}{x}dx\right]dy$



Change the order of integration, then

x: 0 to π

y: 0 to x

$\int_{x=0}^{\pi}\left[\int_{y=0}^{y=x}\frac{\sin x}{x}dy\right]dx$

$= \int_{x=0}^{\pi}\frac{\sin x}{x}.x\,dx = -\cos x\,|_0^{\pi}$

$= -(-1-1)$

$=2$

End of Solution

09. The number of distinct eigenvalues of the matrix $A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ is equal to _______.

09. Ans: 3

Sol:

Given that $A = \begin{bmatrix} 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

The given matrix is an upper triangular matrix. It's eigen values are Just diagonal elements only.

∴ Eigen values are 2, 1, 3, 2

∴ Number of distinct eigen values = 3

End of Solution

10. A standard CMOS inverter is designed with equal rise and fall times ($\beta_n = \beta_p$). If the width of the pMOS transistor in the inverter is increased. What would be the effect on the LOW noise margin ($N\,M_L$) and the HIGH noise marging $N\,M_H$?

(a) No change in the noise margins.

(b) $N\,M_L$ decreases and $N\,M_H$ increases.

(c) $N\,M_L$ increases and $N\,M_H$ decreases.

(d) Both $N\,M_L$ and $N\,M_H$ increase.

10. Ans: (c)

Sol:

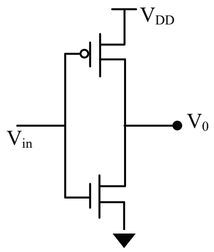


fig: CMOS inverter

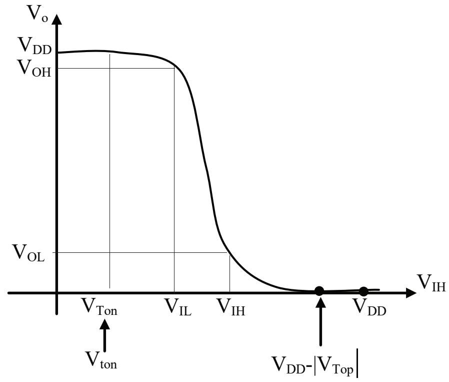


fig: VTC or the CMOS inverter

The behavior of the CMOS inverter for static conditions of operation is described by the voltage transfer characteristics (VTC) and for dynamic operation condition is described by the time response during switching conditions.

$$V_{OH} = V_{DD}$$

$$V_{IL} = \frac{2V_o - |V_{Top}| - V_{DD} + k_r V_{Ton}}{1 + k_r}$$

$$V_o = V_{in} - V_{Top} + \sqrt{(V_{in} - V_{DD} - V_{Top})^2 + k_r (V_{in} - V_{top})^2}$$

$$k_r = \frac{\beta_n}{\beta_p} = \frac{\mu_n c_{ox}\left(\frac{W}{L}\right)_n}{\mu_p c_{ox}\left(\frac{W}{L}\right)_p}$$

$$V_{IH} = \frac{V_{DD} + V_{Top} + k_r (2V_o + V_{Top})}{1 + k_r}$$

$$V_o = V_{in} - V_{Ton} + \sqrt{(V_{in} - V_{Ton})^2 + \frac{1}{k_r}[V_{in} - V_{DD} - V_{Top}]^2}$$

$$V_{OL} = 0$$

$$NM_L = V_{IL} - V_{OL}$$

$$NM_H = V_{OH} - V_{IH}$$

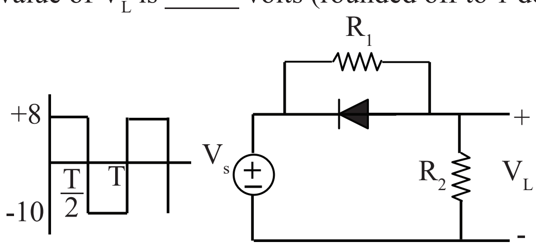
$$\therefore W_p\uparrow \rightarrow NM_L\uparrow$$

$$W_p\uparrow \rightarrow NM_H\downarrow$$

End of Solution

11. In the circuit shown, V_s is a square wave of period T with maximum and minimum values of 8 V and −10V, respectively. Assume that the diode is ideal and $R_1 = R_2 = 50\Omega$.

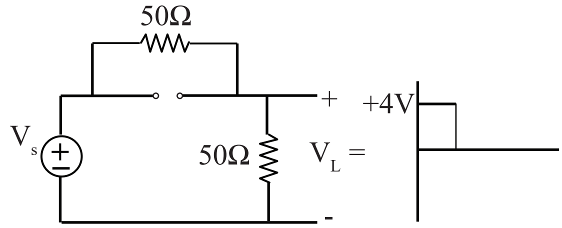
The average value of V_L is _____ volts (rounded off to 1 decimal place).



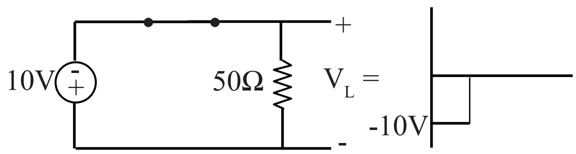
11. Ans: -3

Sol:

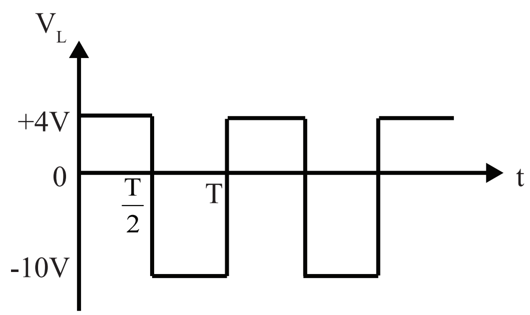
$0 < t < \frac{T}{2}$



$\frac{T}{2} < t < T$



So, output waveform is



$$V_{L\,(Avg)} = \frac{1}{T}\left[\int_0^{T/2} 4\,dt + \int_{T/2}^{T} (-10)\,dt\right]$$

$$= \frac{1}{T}\left[4\left(\frac{T}{2} - 0\right) - 10\left(T - \frac{T}{2}\right)\right]$$

$$= \frac{1}{T}\cdot\frac{T}{2}[4 - 10]$$

$$V_{L\,(Avg)} = -3\,\text{Volts}$$

End of Solution

12. The value of the contour integral $\frac{1}{2\pi j}\oint\left(z + \frac{1}{z}\right)^2 dz$ evaluated over the unit circle $|z| = 1$ is ________ .

12. Ans: 0

Sol: Given $\frac{1}{2\pi J}\oint_C\left(z + \frac{1}{z}\right)^2 dz$ C is $|z| = 1$

$$\oint_C\left(z + \frac{1}{z}\right)^2 dz = \oint_C \frac{(z^2 + 1)^2}{z^2} dz$$

z = 0 is singular point lies inside of the curve C, |z| = 1

By Cauchy's Integral formula

$$\oint_C \frac{(z^2+1)^2}{z^{1+1}} = \frac{1}{2\pi J}\frac{f^1(z)}{1!}\Big|_{at\ z=0}$$

Here $f(z) = (z^2 + 1)^2$

$f^1(z) = 2(z^2 + 1)\,2z \Rightarrow f^1(0) = 0$

$$\therefore \frac{1}{2\pi J}\oint_C \frac{(z^2+1)^2}{z^2}dz = \frac{1}{2\pi J}\times 0 = 0$$

End of Solution

13. Consider the signal $f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi t}{3}\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$, where t is in seconds. Its fundamental time period, in seconds, is _______.

13. Ans: 12

Sol: $f(t) = 1 + 2\cos(\pi t) + 3\sin\left(\frac{2\pi}{3}t\right) + 4\cos\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$

$$\omega_0 = \text{GCD}\left(\pi, \frac{2\pi}{3}, \frac{\pi}{2}\right)$$

$$= \text{GCD}\left(\frac{6\pi}{6}, \frac{4\pi}{6}, \frac{3\pi}{6}\right)$$

$$= \frac{\pi}{6}$$

Time period $T = \frac{2\pi}{\omega_0} = 12\text{secs}$

Alternate method

(i) $\omega_1 = \pi$ $\qquad$ $\omega_2 = \frac{2\pi}{3}$ $\qquad$ $\omega_3 = \frac{\pi}{2}$

$\frac{2\pi}{T_1} = \pi \Rightarrow T_1 = 2$ $\qquad$ $\frac{2\pi}{T_2} = \frac{2\pi}{3} \Rightarrow T_2 = 3$ $\qquad$ $\frac{2\pi}{T_3} = \frac{\pi}{2} \Rightarrow T_3 = 4$

(ii) $\frac{T_1}{T_2} = \frac{2}{3}$

$\frac{T_1}{T_3} = \frac{2}{4} = \frac{1}{2}$

(iii) L.C.M of denominators of step (ii)

= L.C.M (3, 2) = 6

(iv) $T = (\text{L. C. M})\ T_1 = (2)\ (6) = 12$ secs

14. Let Z be an exponential random variable with mean 1. That is, the cumulative distribution function of Z is given by

$$F_z(x) = \begin{cases} 1 - e^{-x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Then Pr (Z > 2 |Z > 1), rounded off to two decimal places, is equal to ______ .

14. Ans: 0.3679

Sol: Given that Z is exponential distribution with cumulative function $F_z(x) = 1 - e^{-x} \quad x \geq 0$
$\quad\quad 0 \quad x < 0$

We know that probability density function

$$f_z(x) = F'_z(x) = e^{-x} \quad x \geq 0$$

$$0 \quad x < 0$$

$$P(z > 2/z > 1) = \frac{P((z > 2) \cap (z > 1))}{P(z > 1)}$$

$$= \frac{\int_2^\infty e^{-x} dx}{\int_1^\infty e^{-x} dx}$$

$$= \frac{(-e^{-x})_2^\infty}{(-e^{-x})_1^\infty}$$

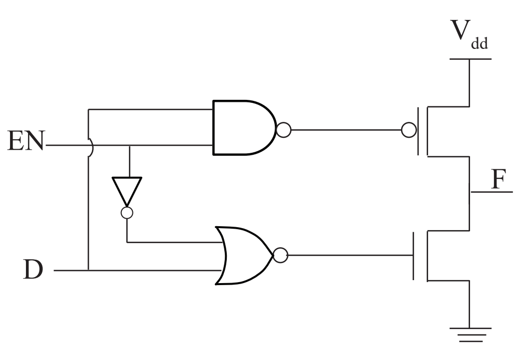
$$= \frac{-(e^{-\infty} - e^{-2})}{-(e^{-\infty} - e^{-1})} = \frac{e^{-2}}{e^{-1}}$$

$$= e^{-1}$$

$$= 0.3679$$

End of Solution

15. In the circuit shown, what are the values of F for EN = 0 and EN = 1, respectively?



(a) Hi-Z and D

(b) 0 and D

(c) Hi-Z and $\overline{D}$

(d) 0 and 1

15. Ans: (a)

Sol: Case-1: When enable = 0, both MOSFETs are off, hence F = Hi-Z

Case-2: When enable = 1, inputs of both MOSFETs is $\overline{D}$ i.e.,

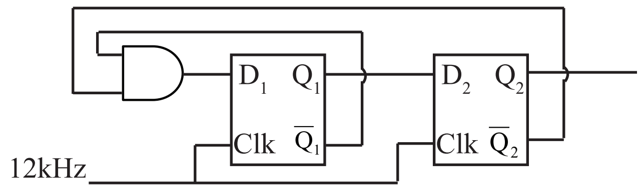
→ If D = 0 is ⇒ n-channel MOSFET is on, p-channel MOSFET is off so F = 0,

→ If D = 1 ⇒ PMOS-on, NMOS-off. thus, $F = V_{DD}$ = logic 1

i.e, when E = 1 ⇒ F = D

End of Solution

16. In the circuit shown, the clock frequency, i.e., the frequency of the CLK signal, is 12 kHz. The frequency of the signal at Q_2 is _____ kHz.



16. Ans: 4

Sol: Given $D_1 = \overline{Q}_1\overline{Q}_2$, $D_2 = Q_1$

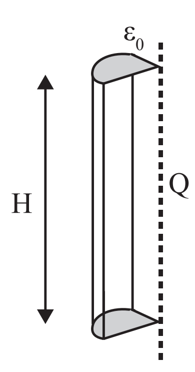
CLK	Present state		Flipflops inputs		Next state	
	Q_1	Q_2	D_1	D_2	Q_1	Q_2
	0	0	1	0	1	0
	1	0	0	1	0	1
	0	1	0	0	0	0

So, the circuit counts 00,10,01,........

Hence, the frequency at Q_2 is $\frac{f_{CLK}}{3} = \frac{12}{3} = 4\text{kHz}$

End of Solution

17. What is the electric flux $\left(\int \vec{E}.d\hat{a}\right)$ through a quarter-cylinder of height H (as shown in the figure) due to an infinitely long line charge along the axis of the cylinder with a charge density of Q ?



(a) $\frac{4H}{Q\varepsilon_0}$ (b) $\frac{HQ}{4\varepsilon_0}$ (c) $\frac{H\varepsilon_0}{4Q}$ (d) $\frac{HQ}{\varepsilon_0}$

17. Ans: (b)

Sol: Electric flux $= \int_S \vec{E}.d\vec{a}$

we know electric field due to line charge

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\hat{a}_\rho$$

Given $\rho_L = Q$

$$\vec{E} = \frac{Q}{2\pi\varepsilon_0\rho}\hat{a}_\rho$$

So

$$\int_S \vec{E}.d\vec{a} = \int_S \frac{Q}{2\pi\varepsilon_0\rho}\hat{a}_\rho.d\vec{a}$$

$$= \frac{Q}{2\pi\varepsilon_0\rho}\int_S da$$

$$= \frac{Q}{2\pi\varepsilon_0\rho} \text{ (surface area of the given portion)}$$

$$\int_S \vec{E}.d\vec{a} = \frac{Q}{2\pi\varepsilon_0\rho}\left(\frac{2\pi\rho H}{4}\right)$$

$$= \frac{QH}{4\varepsilon_0}.$$

End of Solution

18. Which one of the following functions is analytic over the entire complex plane?

(a) $e^{1/z}$ (b) ln(z)

(c) $\frac{1}{1-z}$ (d) cos(z)

18. Ans: (d)

Sol: (a) $e^{1/z}$ is NOT analytic at z = 0

(b) ℓnz is NOT analytic in Domain $D = \{z / x \le 0 , y = 0\}$

(c) $\frac{1}{1-z}$ is NOT analytic at z = 1

$\therefore$ cosz is analytic every where in the complex plane.

End of Solution

19. A linear Hamming code is used to map 4-bit messages to 7-bit codewords. The encoder mapping is linear. If the message 0001 is mapped to the codeword 0000111, and the message 0011 is mapped to the codeword 1100110, then the message 0010 is mapped to

(a) 1100001

(b) 1111000

(c) 0010011

(d) 1111111

19. Ans: (a)

Sol: A code is said to be linear if the algebraec sum of two codes is also another code

0001 $\rightarrow$ 0000111

<u>0011</u> $\rightarrow$ <u>1100110</u>

<u>0010</u> <u>1100001</u>

So, 0010 is mapped into 1100001

20 In the table shown, List-I and List II, respectively, contain terms appearing on the left-hand side and the right-hand side of Maxwell's equations (in their standard form). Match the left-hand side with the corresponding right-hand side.

List I		List II	
1	$\nabla . D$	P	0
2	$\nabla \times E$	Q	ρ
3	$\nabla . B$	R	$-\frac{\partial B}{\partial t}$
4	$\nabla \times H$	S	$J+\frac{\partial D}{\partial t}$

(a) 1 − Q, 2 − R, 3 − P, 4 − S
(b) 1 − Q, 2 − S, 3 − P, 4 − R
(c) 1 − P, 2 − R, 3 − Q, 4 − S
(d) 1 − R, 2 − Q, 3 − S, 4 − P

20. Ans: (a)

Sol: 1. $\nabla.\vec{D} = \rho$ (Q) $\rightarrow$ Gauss's Law

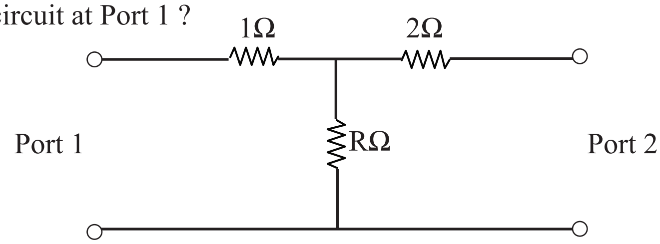
2. $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$ (R) $\rightarrow$ Faraday's Law

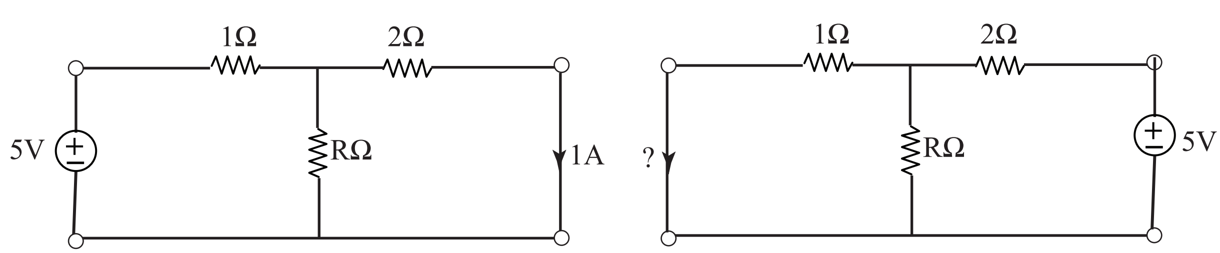
3. $\nabla.\vec{B} = 0$ (P) $\rightarrow$ Gauss's Law for magnetic fields

4. $\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t}$ (S) $\rightarrow$ Ampere's Law

End of Solution

21. Consider the two-port resistive network shown in the figure. When an excitation of 5 V is applied across Port 1, and Port 2 is shorted, the current through the short circuit at Port 2 is measured to be 1 A (sec(a) in the figure). Now, if an excitation of 5 V is applied across Port 2, and Port 1 is shorted (sec(b) in the figure). What is the current through the short circuit at Port 1 ?





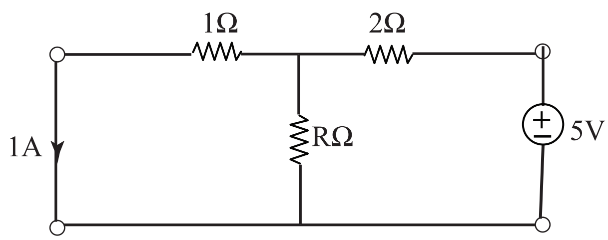
(a) (b)

(a) 1 A (b) 0.5 A (c) 2.5 A (d) 2 A

21. Ans: (a)

Sol: This is direct application of reciprocity theorem - which states, in any linear passive bilateral network excited by a single source. The ratio of response to excitation remains constant even if the position of source and Load are interchanged.

So, $\boxed{\frac{i}{V} = \text{Constant}}$



So, answer is 1 Amp

End of Solution

22. Radiation resistance of a small dipole current element of length ℓ at a frequency of 3 GHz is 3 ohms. If the length is changed by 1%, then the percentage change in the radiation resistance, rounded off to two decimal places, is ________ %

22. Ans: 2

Sol: Given f = 3GHz

$$R_{rad} = 3\Omega$$

$$\frac{\Delta\ell}{\ell} \times 100 = 1$$

$$\frac{\Delta R}{R} \times 100 = ?$$

$$R_{rad} = 80\pi^2 \left(\frac{\ell}{\lambda}\right)^2$$

$$R_{rad} \ \alpha \ \ell^2$$

$$\frac{\Delta R_{rad}}{R_{rad}} \times 100 = 2\frac{\Delta\ell}{\ell} \times 100$$

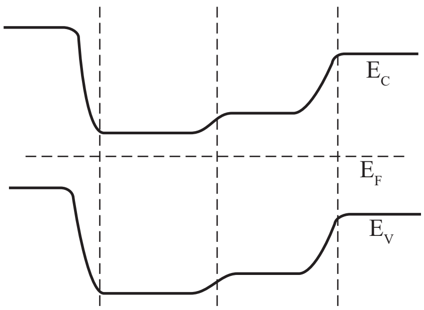
$$= 2(1)$$

$$\frac{\Delta R_{rad}}{R_{rad}} \times 100 = 2\%$$

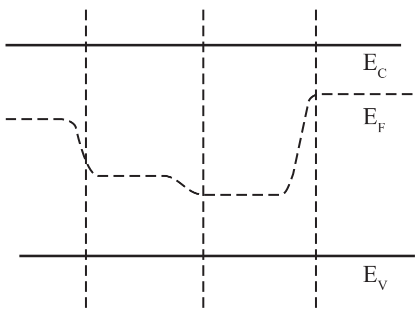
23. Which one of the following options describes correctly the equilibrium band diagram at T = 300 K of a Silicon pnn $^{+}$p^{++} configuration shown in the figure?

p	n	n^+	p^{++}

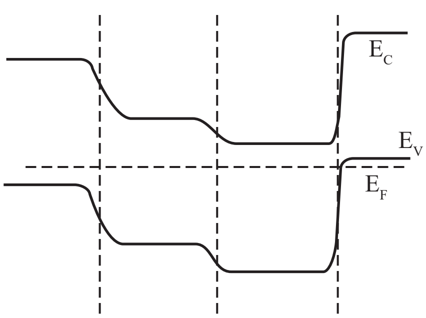
(a)



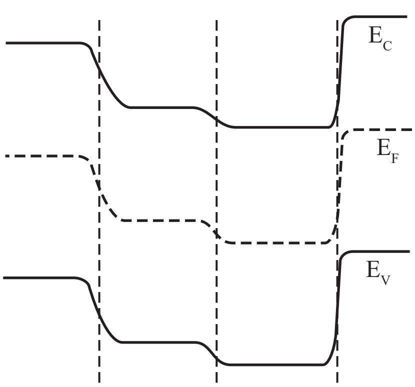
(b)



(c)



(d)



23. Ans: (c)

Sol: → The device is not biased hence fermi level should be constant.

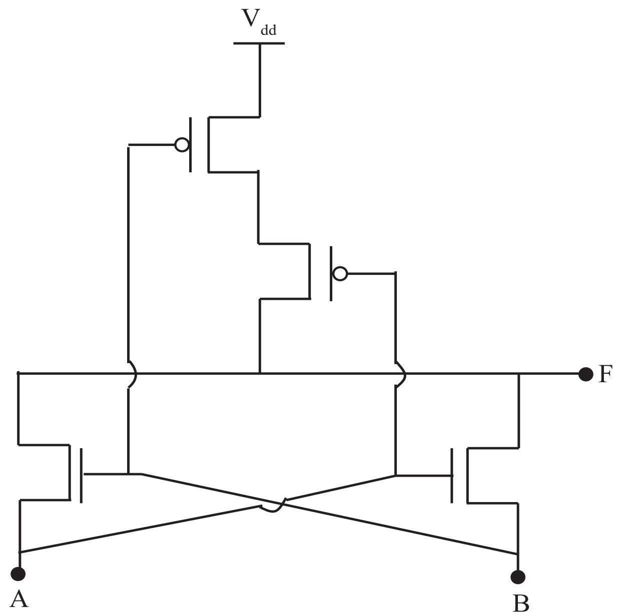
→ In P type fermi level should be closer to Ev.

→ In N type fermi level should be closer to Ec.

→ In P^{++} fermi level penetrates into valence band.

End of Solution

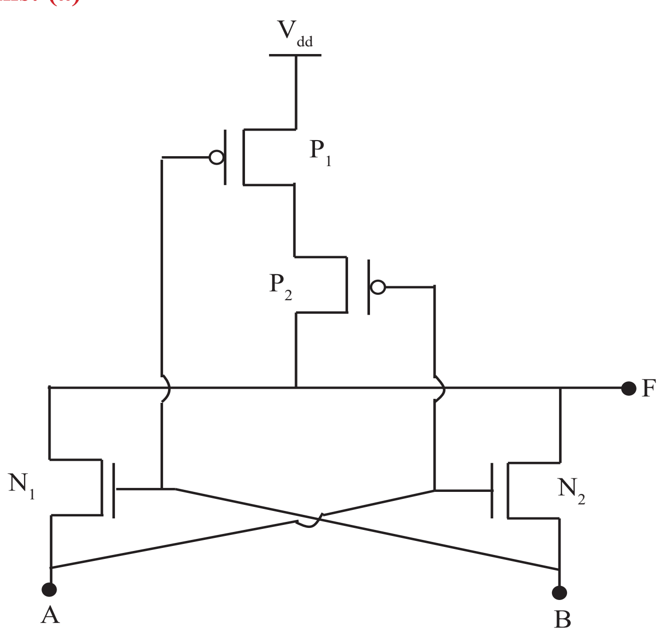
24. In the circuit shown, A and B are the inputs and F is the output. What is the functionality of the circuit?



(a) XNOR (b) SRAM Cell (c) Latch (d) XOR

24. Ans: (a)

Sol:



A	B	N_1	N_2	P_1	P_2	F(Output)
0	0	OFF	OFF	ON	ON	1
1	0	ON	OFF	OFF	ON	0
0	1	OFF	ON	ON	OFF	0
1	1	ON	ON	OFF	OFF	1

∴ Given diagram is XNOR.

End of Solution

25. If X and Y are random variables such that E[2X+Y] = 0 and E[X + 2Y] = 33, then E[X] + E[Y] = _______ .

25. Ans: 11

Sol: Given X,Y are Random variables

E(2X + Y) = 0 and E(X+2Y) = 33

⇒ 2E(X) + E(Y) = 0.........(1)

⇒ E(X) + 2 E(Y) = 33......(2)

By solving (1) and (2) E(Y) = 22

E(X) = –11

∴E(X) + E(Y) = 22 – 11 = 11

End of Solution

26. The dispersion equation of a waveguide, which relates the wavenumber k to the frequency ω, is

$k(\omega) = (1/c)\sqrt{\omega^2 - \omega_0^2}$

where the speed of light c = 3 × 10^8 m/s, and ω_0 is a constant. If the group velocity is 2 × 10^8 m/s, then the phase velocity is

(a) 2 × 10^8 m/s

(b) 4.5 × 10^8 m/s

(c) 3 × 10^8 m/s

(d) 1.5 × 10^8 m/s

26. Ans: (b)

Sol: Given

wave number $\beta = k(\omega) = \frac{1}{c}\sqrt{\omega^2 - \omega_0^2}$

Group velocity (v_g) = 2 ×10^8 m/s

Phase velocity $(v_P) = \frac{\omega}{\beta}$

$$v_P = \frac{\omega c}{\sqrt{\omega^2 - \omega_0^2}}$$

we know

$$v_g = \frac{d\omega}{d\beta} = \frac{1}{\left(\frac{d\beta}{d\omega}\right)}$$

$$\beta = \frac{1}{c}\sqrt{\omega^2 - \omega_0^2}$$

$$\frac{d\beta}{d\omega} = \frac{1}{c}\frac{1}{2\sqrt{\omega^2 - \omega_0^2}}2\omega$$

$$\frac{d\beta}{d\omega} = \frac{\omega}{c\sqrt{\omega^2 - \omega_0^2}}$$

$$\frac{1}{v_g} = \frac{\omega}{c\sqrt{\omega^2 - \omega_0^2}}$$

$$\sqrt{\omega^2 - \omega_0^2} = \frac{\omega v_g}{c}$$

$$= \frac{\omega.2\times 10^8}{3\times 10^8}$$

$$\sqrt{\omega^2 - \omega_0^2} = \frac{2\omega}{3}$$

$$\text{so, } v_p = \frac{\omega c}{\sqrt{\omega^2 - \omega_0^2}} = \frac{\omega c}{\frac{2\omega}{3.}}$$

$$v_p = \frac{3}{2}c = \frac{3}{2}(3\times 10^8) = 4.5\times 10^8 \text{m/s.}$$

(or)

we know the relation

$$v_p\, v_g = c^2$$

$$v_p = \frac{c^2}{v_g}$$

$$v_p = \frac{(3\times 10^8)^2}{2\times 10^8}$$

$v_p = 4.5\times10^8$ m/s.

End of Solution

27. It is desired to find three-tap causal filter which gives zero signal as an output to an input of the form $x(n) = c_1 \exp\left(-\frac{j\pi n}{2}\right) + c_2 \exp\left(\frac{j\pi n}{2}\right)$

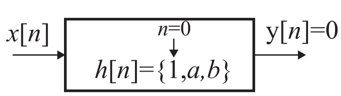
Where c_1 and c_2 are arbitrary real numbers. The desired three-tap filter is given by

$$h[0] = 1, \quad h[1] = a, \quad h[2] = b$$

And

$$h[n] = 0 \text{ for } n < 0 \text{ or } n > 2.$$

What are the values of the filter taps a and b if the output is y[n] = 0 for all n, when x[n] is as given above?



(a) a = 1, b = 1
(b) a = –1, b = 1
(c) a = 0, b = 1
(d) a = 0, b = –1

27. Ans: (c)

Sol: Given $x(n) = c_1 \exp\left(-\frac{j\pi n}{2}\right) + c_2 \exp\left(\frac{j\pi n}{2}\right)$

and h(0) = 1; h(1) = a; h(2) = b

$H(e^{j\omega}) = 1 + ae^{-j\omega} + be^{-j2\omega}$

Given that y(n) = 0 for all n

From the given options by trail & errors of we make a = 0 & b = 1

Due to $c_1 e^{-\frac{j\pi n}{2}} \Rightarrow \text{output} = 1 + 0.e^{+\frac{j\pi}{2}} + e^{-j2\left(-\frac{\pi}{2}\right)} = 1 - 1 = 0$

$c_2 e^{\frac{j\pi n}{2}} \Rightarrow \text{output} = 1 + 0.e^{-\frac{j\pi}{2}} + e^{-j2\left(\frac{\pi}{2}\right)} = 1 - 1 = 0$

End of Solution

28. Let the state-space representation of an LTI system be $\dot{x}(t) = A\,x(t) + B\,u(t)$, $y(t) = C\,x(t) + d\,u(t)$ where A,B,C are matrices, d is a scalar, u(t) is the input to the system and y(t) is its output. Let $B = [0 \;\; 0 \;\; 1]^T$ and d = 0. which one of the following options for A and C will ensure that the transfer function of this LTI system is $H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1}$?

(a) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$ and $C = [0 \;\; 0 \;\; 1]$

(b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ and $C = [0 \;\; 0 \;\; 1]$

(c) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -1 \end{bmatrix}$ and $C = [1 \;\; 0 \;\; 0]$

(d) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ and $C = [1 \;\; 0 \;\; 0]$

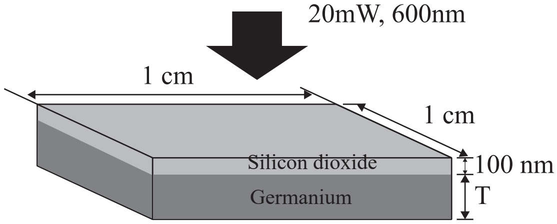
28. Ans: (d)

Sol: $H(s) = \frac{1}{s^3 + 3s^2 + 2s + 1}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [u]$$

$$[Y] = [1 \;\; 0 \;\; 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

29. A germanium sample of dimensions 1 cm × 1 cm is illuminated with a 20 mW, 600 nm laser light source as shown in the figure. The illuminated sample surface has a 100 nm of loss-less Silicon dioxide layer that reflects one-fourth of the incident light. From germanium interface, one-third is absorbed in the germanium layer, and one-third is transmitted through the other isde of the sample. If the absorption coefficient of germanium at 600 nm is 3×10^4 cm–1 and the bandgap is 0.66 eV, the thickness of the germanium layer, rounded off to 3 decimal places, is _____ μm.



29. Ans: 0.231

Sol: $1 - e^{-\alpha x} = 0.5$

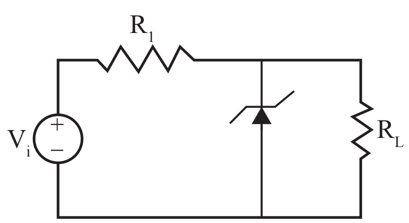
$e^{-\alpha x} = 0.5$

$$x = \frac{-\ln(0.5)}{\alpha} = \frac{-\ln(0.5)}{3 \times 10^4} \text{cm}$$

$= 0.231$ mm

End of Solution

30. In the circuit shown, the breakdown voltage and the maximum current of the Zener diode are 20 V and 60 mA. respectively. The values of R_1 and R_L are 200 Ω and 1 kΩ, respectively. What is the range of V_i that will maintain the Zener diode in the 'on' state?



(a) 18 V to 24 V (b) 22 V to 34 V (c) 24 V to 36 V (d) 20 V to 28 V

30. Ans: (c)

Sol: $$\frac{V_{I,min} - 20}{0.2k} = \frac{20}{1k}$$

$V_{I,min} = 24V$

$$\frac{V_{I,max} - 20}{0.2k} = 60mA + 20mA$$

$V_{I,max} = 36V$

31. Let h[n] be a length-7 discrete-time finite impulse response filter, given by

$$h[0] = 4,\quad h[1] = 3,\quad h[2] = 2,\quad h[3] = 1,$$
$$h[-1] = -3,\quad h[-2] = -2,\quad h[-3] = -1,$$

and h[n] is zero for $|n| \geq 4$. A length-3 finite impulse response approximation g[n] of h[n] has to be obtained such that

$$E(h,g) = \int_{-\pi}^{\pi} \left|H(e^{j\omega}) - G(e^{j\omega})\right|^2 d\omega$$

is minimized, where $H(e^{j\omega})$ and $G(e^{j\omega})$ are the discrete-time Fourier transforms of h[n] and g[n], respectively. For the filter that minimizes E(h, g), the value of 10g[-1] + g[1], rounded off to 2 decimal places, is ______.

31. Ans: - 27

Sol: To minimize energy in the error signal, there are different approaches like, Prony's method, pade approximation. As g(n) length is three samples. Assume samples as g(-1), g(0), and g(1). We can minimize E(h, g) by making h(n) = g(n) using rectangular window & Parseval's theorem of DTFT.

Based on which 10g(-1) + g(1) = 10(-3) + 3 = -27 .

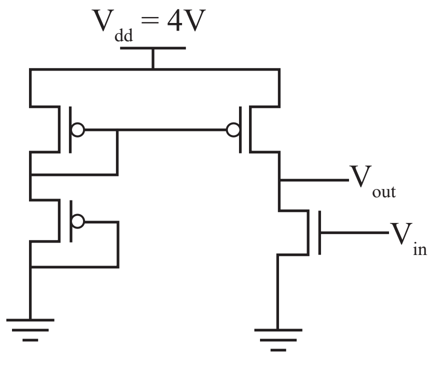
End of Solution

32. In the circuit shown, the threshold voltages of the pMOS ($|V_{tp}|$) and nMOS (V_{tn}) transistors are both equal to 1 V. All the transistors have the same output resistance r_{ds} of 6 MΩ. The other parameters are listed below:

$$\mu_n C_{ox} = 60\mu A/V^2; \left(\frac{W}{L}\right)_{nMOS} = 5$$

$$\mu_n C_{ox} = 30\ \mu A/V^2; \left(\frac{W}{L}\right)_{pMOS} = 10$$

μ_n and μ_p are the carrier mobilities, and C_{ox} is the oxide capacitance per unit area. Ignoring the effect of channel length modulation and body bias, the gain of the circuit is _____ (rounded off to 1 decimal place).



32. Ans: -900

Sol: $I_{DC} = \frac{1}{2}\mu_P C_{OX}\left(\frac{W}{L}\right)_P (V_{SGP} - |V_{TP}|)^2 = \frac{1}{2} \times 30\frac{\mu A}{V^2} \times 10 \times (2-1)^2 V^2$

$= 150\mu A$

$g_{mn} = \sqrt{2I_{DC}\mu_n C_{OX}\left(\frac{W}{L}\right)_N} = \sqrt{2 \times 150\mu A \times 60\frac{\mu A}{V^2} \times 5}$

$= 300\mu S$

$A_v = -g_m(r_{ds} \| r_{ds}) = -300 \times 3 = -900$

33. Consider a causal second-order system with the transfer function

$$G(s) = \frac{1}{1+2s+s^2}$$

with a unit-step $R(s) = \frac{1}{s}$ as an input. Let C(s) be the corresponding output. The time taken by the sytem output c(t) to reach 94% of its steady-state value $\lim_{t\to\infty} c(t)$, rounded off to two decimal places, is

(a) 4.50 (b) 3.89 (c) 2.81 (d) 5.25

33. Ans: (a)

Sol: $G(s) = \frac{1}{1+2s+s^2}$

$G(s) = \frac{1}{(s+1)^2}$

$G(s) = \frac{1}{s(s+1)^2} = \frac{1}{s} - \frac{1}{(s+1)^2} - \frac{1}{s+1}$

$C(t) = \left(1 - te^{-t} - e^{-t}\right)u(t)$

$\Rightarrow$ 94% of ss value = 0.94

$0.94 = (1 - te^{-t} - e^{-t})$

$0.06 = e^{-t}(t+1)$

$\Rightarrow$ By option verification method

Let t = 4.5

$e^{-t}(t+1) = e^{-4.5}(1 + 4.5) = 0.06109 \approx 0.06$

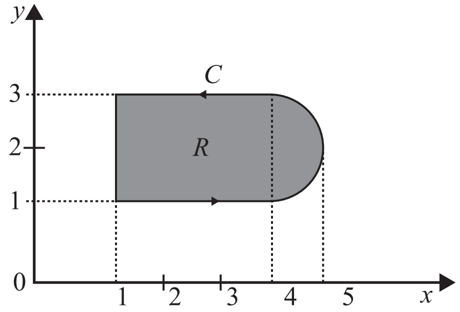
Thus option 4.5 is the Answer

End of Solution

34. Consider the line integral

$$\int_C (xdy - ydx)$$

the integral being taken in a counterclockwise direction over the closed curve C that forms the boundary of the region R shown in the figure below. The region R is the area enclosed by the union of a 2 × 3 rectangle and a semi-circle of radius 1. The line integral evaluates to



(a) $6 + \pi/2$ (b) $8 + \pi$ (c) $12 + \pi$ (d) $16 + 2\pi$

34. Ans: (c)

Sol: By Greens theorem

$$\int_C \underbrace{xdy}_{N} - \underbrace{ydx}_{M} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)dxdy$$

$$\Rightarrow \int_C (xdy - ydx) = \iint_R (1+1)dxdy$$

$$= \int\int 2dxdy$$

$$\Rightarrow \int_C (xdy - ydx) = 2 \times \text{Area of the region}$$

$$= 2(\text{Area of a rectangle} + \text{Area of a semi circle})$$

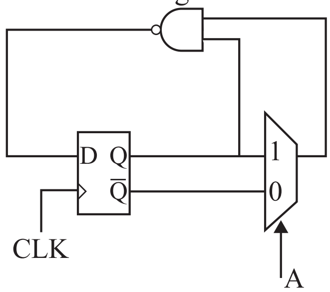
$$= 2\left[2\times 3 + \frac{\pi(1)^2}{2}\right] = 2\left(6 + \frac{\pi}{2}\right)$$

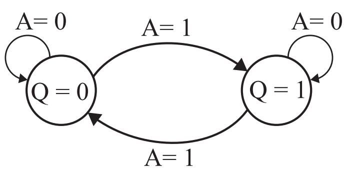
$$= 2\frac{(12+\pi)}{2}$$

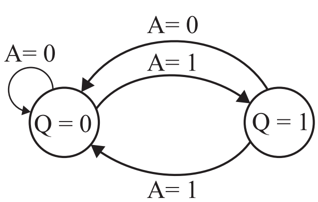
$$= 12 + \pi$$

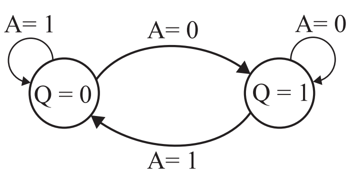
End of Solution

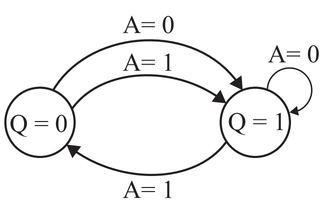
35. The state transition diagram for the circuit shown is



(a)


(b)


(c)


(d)


35. Ans: (d)

Sol: Q(t + 1) = D.........(1)

$D = \overline{(\bar{A}\,\bar{Q} + AQ)(Q)}$ ....(2)

Substitute (2) in (1)

$Q(t+1) = \overline{A.Q}$

So, if $A = 0 \Rightarrow Q(t+1) = 1$

$A = A \Rightarrow Q(t+1) = \bar{Q}$

If $A = 0,\ D = \overline{Q.\bar{Q}} = \bar{0} = 1$

If $A = 1,\ D = \overline{QQ} = \bar{Q}$

36. In an ideal pn junction with an ideality factor of 1 at T = 300 K, the magnitude of the reverse-bias voltage required to reach 75% of its reverse saturation current, rounded off to 2 decimal places, is _____mV.
[$k = 1.38 \times 10^{-23}$ JK^{-1}, $h = 6.625 \times 10^{-34}$ J-s, $q = 1.602 \times 10^{-19}$C]

36. Ans: 35.87

Sol: $I_R = 0.75\ I_S$

$I_D = -0.75\ I_S$

$I_S\ (e^{V_D/V_T} - 1) = -0.75\ I_S$

$e^{V_D/V_T} = 0.25$

$V_D = V_T \ln(0.25)$

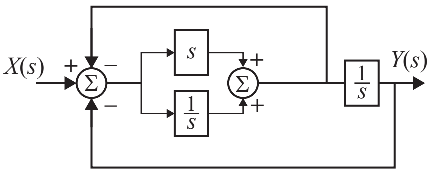
$V_R = -\ V_T \ln(0.25)$

$$= -\frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times -1.386$$

= 35.87 mV

End of Solution

37. The block diagram of a system is illustrated in the figure shown, where X(s) is the input and Y(s) is the output.
The transfer function $H(s) = \frac{Y(s)}{X(s)}$ is



(a) $H(s) = \frac{s+1}{s^2+s+1}$

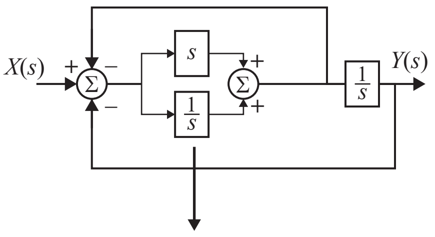
(b) $H(s) = \frac{s^2+1}{s^3+2s^2+s+1}$

(c) $H(s) = \frac{s^2+1}{2s^2+1}$

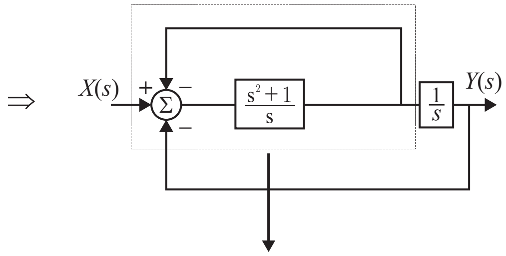
(d) $H(s) = \frac{s^2+1}{s^3+s^2+s+1}$

37. Ans: (b)

Sol:



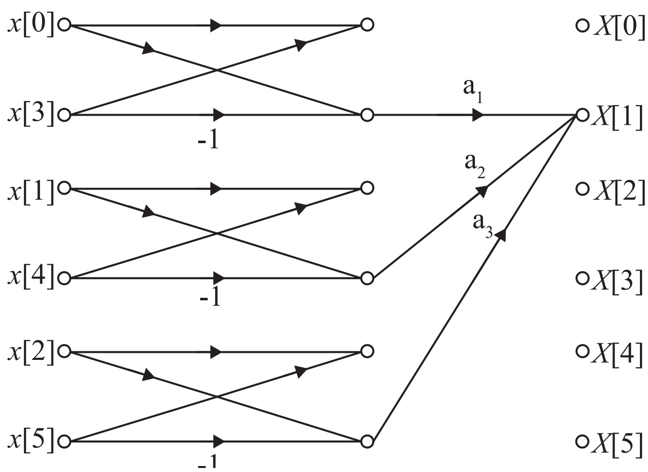
$$\left(s + \frac{1}{s}\right) = \left(\frac{s^2+1}{s}\right)$$



$$\frac{Y(s)}{X(s)} = \frac{(s^2+1)}{s(s^2+s+1)+s^2+1} = \frac{s^2+1}{s^3+2s^2+s+1}$$

End of Solution

38. Consider a six-point decimation-in-time Fast Fourier Transform (FFT) algorithm, for which the signal-flow graph corresponding to X[1] is shown in the figure. Let $W_6 = \exp\left(-\frac{j2\pi}{6}\right)$. In the figure, what should be the values of the coefficients a_1, a_2, a_3 in terms of W_6 so that X[1] is obtained correctly?



(a) $a_1 = 1, a_2 = W_6 . a_3 = W_6^2$

(b) $a_1 = -1, a_2 = W_6^2, a_3 = W_6$

(c) $a_1 = 1, a_2 = W_6^2, a_3 = W_6$

(d) $a_1 = -1, a_2 = W_6, a_3 = W_6^2$

38. Ans: (a)

Sol: $X(k) = \sum_{n=0}^{N-1=5} x(n)\, W_6^{kn}$

$$= x(0) + x(1)W_6^{k} + x(2)W_6^{2k} + x(3)W_6^{3k} + x(4)W_6^{4k} + x(5)W_6^{5k}$$

$$X(1) = x(0) + x(1)W_6^{1} + x(2)W_6^{2} + x(3)W_6^{3} + x(4)W_6^{4} + x(5)W_6^{5}$$

Based on symmetry

$W_N^{k+\frac{N}{2}} = -W_N^k \qquad W_6^3 = -W_6^0 = -1$

$W_6^4 = -W_6^1$

$W_6^5 = -W_6^2$

From the SFG

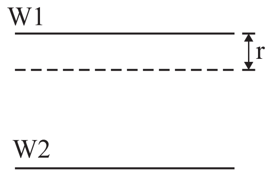
We can say $a_1 = 1$

$a_2 = W_6$

$a_3 = W_6^2$

End of Solution

39. Two identical copper wires W1 and W2, placed in parallel as shown in the figure, carry currents I and 2I, respectively, in opposite directions. If the two wires are separated by a distance of 4r, then the magnitude of the magnetic field $\vec{B}$ between the wires at a distance r from W1 is



(a) $\frac{\mu_0 I}{6\pi r}$

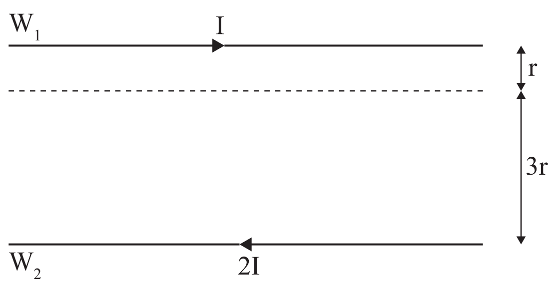
(b) $\frac{\mu_0^2 I^2}{2\pi r^2}$

(c) $\frac{6\mu_0 I}{5\pi r}$

(d) $\frac{5\mu_0 I}{6\pi r}$

39. Ans: (d)

Sol:



$\vec{B} = \vec{B}_1 + \vec{B}_2$

$\vec{B}_1 = \frac{\mu_0 I}{2\pi r} \otimes$

$\vec{B}_2 = \frac{\mu_0 2I}{2\pi(3r)} \otimes$

$\vec{B} = \vec{B}_1 + \vec{B}_2$

$\vec{B} = \frac{\mu_0 I}{2\pi r}\left[1 + \frac{2}{3}\right] \otimes$

$\vec{B} = \frac{5\mu_o I}{6\pi r} \otimes$

End of Solution

40. Consider a differentiable function f(x) on the set of real numbers such that f(-1) = 0 and $|f'(x)| \le 2$. Given these conditions, which one of the following inequalities is necessarily true for all x $\in$[-2, 2]?

(a) $f(x) \le \frac{1}{2}|x+1|$ (b) $f(x) \le \frac{1}{2}|x|$

(c) $f(x) \le 2|x+1|$ (d) $f(x) \le 2|x|$

40. Ans: (c)

Sol: From the option (3)

if the max of f(x) = 2|x+1| then

$$f(x) = \begin{cases} 2(x+1) \text{if } x+1 \ge 0 \Rightarrow x \ge -1 \\ -2(x+1) \text{if } x+1 < 0 \Rightarrow x < -1 \end{cases}$$

$$f'(x) = \begin{cases} 2 \quad \text{if } \ x \ge -1 \\ -2 \text{ if } \ x < -1 \end{cases}$$

$|f'(x)| \le 2$ and f(−1) = 0

So, option (c) is correct

End of Solution

41. Consider the homogeneous ordinary differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0, \qquad x > 0$$

with y(x) as a general solution. Given that

y(1) = 1 and y(2) = 14

the value of y(1.5), rounded off to two decimal places, is _____

41. Ans: 5.25

Sol: $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$

Given that y(1) = 1

y(2) = 14

Put $x = e^z$ (or) z = ln x

$\theta = \frac{d}{dz}$, $xDy = \theta y$

$x^2D^2y = \theta(\theta - 1)y$

$\therefore$ The given DE is equivalent to

$\theta(\theta - 1) - 3\theta y + 3y = 0$

$\theta^2 y - 4\,\theta y + 3y = 0$

$$\therefore\ \frac{d^2y}{dz^2} - 4\frac{dy}{dz} + 3y = 0$$

Axillary equation $m^2 - 4m + 3 = 0$

$(m - 1)(m - 3) = 0$

$\therefore\ m = 1, 3$

The solution is $y = C_1e^z + C_2e^{3z}$

$\therefore\ y = C_1x + C_2x^3$

$y(1) = 1 \Rightarrow 1 = C_1 + C_2$

$y(2) = 14 \Rightarrow 14 = 2C_1 + 8C_2$

By solving $C_2 = 2, C_1 = -1$

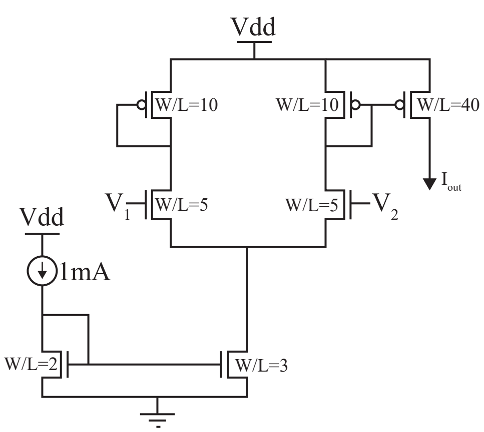
$\therefore\ y = -x + 2x^3$

$\therefore\ y(1.5) = -1.5 + 2(1.5)^3$

$= 5.25$

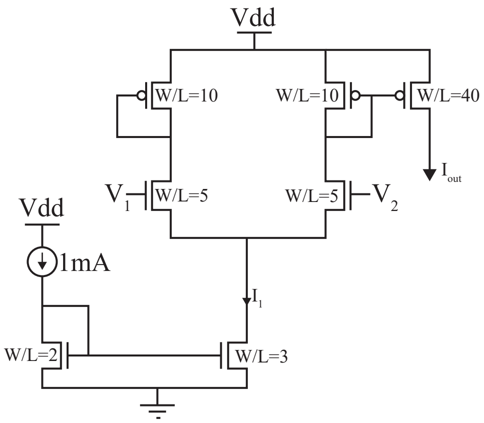
End of Solution

42. In the circuit shown, $V_1 = 0$ and $V_2 = V_{dd}$. The other relevant parameters are mentioned in the figure. Ignoring the effect of channel length modulation and the body effect, the value of I_{out} is _____mA (rounded off to 1 decimal place).



42. Ans: 6

Sol:



$$I_1 = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} \times 1\text{mA} = \frac{3}{2}\text{mA}$$

$$I_{out} = \frac{\left(\frac{W}{L}\right)_4}{\left(\frac{W}{L}\right)_3} \times \frac{3}{2}\text{mA}$$

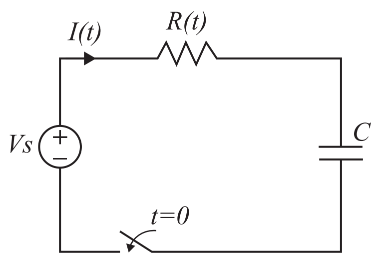
$$= \frac{40}{10} \times \frac{3}{2} = 6\text{mA}$$

End of Solution

43. The RC circuit shown below has a variable resistance R(t) given by the following expression:

$$R(t) = R_0\left(1 - \frac{t}{T}\right) \text{ for } 0 \le t \le T$$

where R_0 = 1Ω, and C = 1 F. We are also given that T = 3 R_0C and the source voltage is V_s = 1V. If the current at time t = 0 is 1 A, then the current I(t), in amperes, at time t = T/2 is _____ (rounded off to 2 decimal places).



43. Ans: 0.1

Sol: In general for RC circuit with state variable voltage across capacitor $V_C(t) = V_S\left[1 - e^{-t/\tau}\right]$

But current $I(t) = i_C(t) = C\frac{dV_C(t)}{dt}$

$$I(t) = C\frac{d}{dt}\left[V_S\left(1 - e^{-t/\tau}\right)\right]$$

$$= CV_S\left[0 - e^{-t/\tau} \times \frac{1}{T}\right]$$

$$I(t) = \frac{V_S}{R(t)} e^{-t/R(t).C}$$

Given $R(t) = R_0\left[1 - \frac{t}{T}\right]$

$$\left.\begin{array}{l} R_0 = 1\Omega \\ T = 3R_0C = 3 \end{array}\right\} R(t) = 1\left[1 - \frac{t}{3}\right]$$

$$I(t) = \frac{1}{\left(1 - \frac{t}{3}\right)} e^{-t/\left(1-\frac{t}{3}\right)}$$

At $t = \frac{T}{2} = \frac{3}{2}$ sec

$$I\left(t = \frac{3}{2}\right) = \frac{1}{\left(1 - \frac{1}{2}\right)} e^{-(3/2)/(1/2)}$$

$I = 2\,e^{-3}A$

$= 0.099 \approx 0.1$ Amps

End of Solution

44. A single bit, equally likely to be 0 and 1, is to be sent across an additive white Gaussian noise (AWGN) channel with power spectral density $N_0/2$. Binary signaling, with $0 \to p(t)$ and $1 \to q(t)$, is used for the transmission, along with an optimal receiver that minimizes the bit-error probability.

Let $\varphi_1(t)$, $\varphi_2(t)$ form an orthonormal signal set.

If we choose $p(t) = \varphi_1(t)$ and $q(t) = -\varphi_1(t)$, we would obtain a certain bit-error probability P_b.

If we keep $p(t) = \varphi_1(t)$, but take $q(t) = \sqrt{E}\,\varphi_2(t)$, for what value of E would we obtain the same bit-error probability P_b?

(a) 3

(b) 2

(c) 1

(d) 0

44. Ans: (a)

Sol: 0 is represented by p(t)

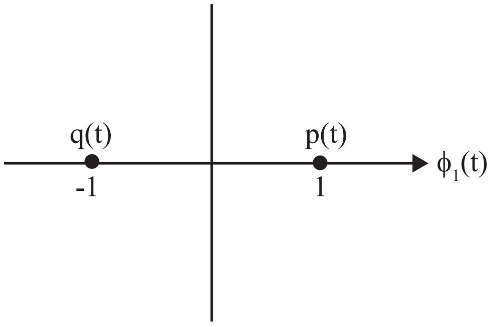
1 is represented by q(t)

$\phi_1(t)$ and $\phi_2(t)$ form are orthonormal signal set

$p(t) = \phi_1(t)$

$q(t) = -\phi_1(t)$

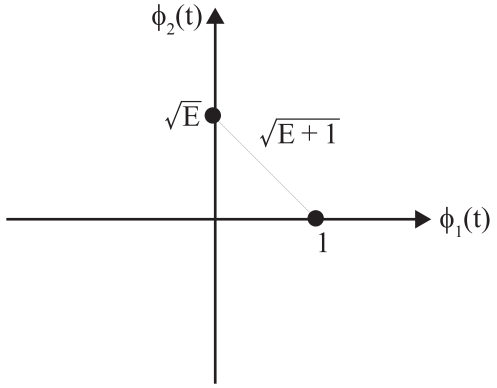
The signal space diagram is



$d_{min} = 2$

If $p(t) = \phi_1(t)$ and $q(t) = \sqrt{E}\,\phi_2(t)$

The signal space diagram is



bit error probability will be same if the d_{min} is same

$\sqrt{1+E} = 2$

$1+E = 4$

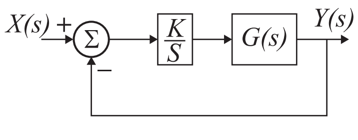
$E = 3$

End of Solution

45. Consider a unity feedback system, as in the figure shown, with an integral compensator $\frac{K}{s}$ and open-loop transfer function

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

where K > 0. The positive value of K for which there are exactly two poles of the unity feedback system on the jω axis is equal to _____ (rounded off to two decimal places).



45. Ans: 6

Sol: $G_C(s) = \dfrac{K}{s(s^2+3s+2)}$

CE $\rightarrow 1 + G_C(s) = 0$

CE $\rightarrow s^3 + 3s^2 + 2s + k = 0$

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 3 & k \\ s^1 & \left(\dfrac{6-k}{3}\right) = 0 & 0 \\ s^0 & k & \end{array}$$

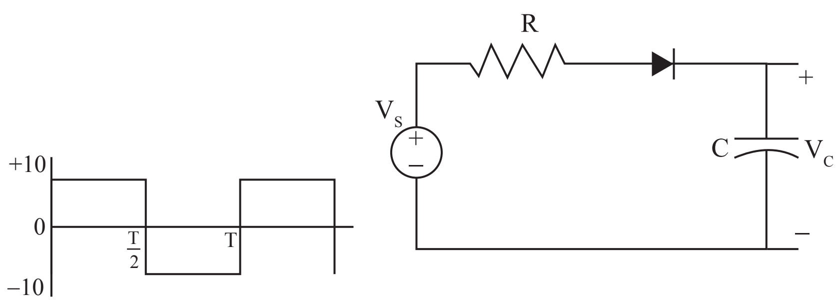
AE $\rightarrow 3s^2 + 6 = 0$

$s = \pm j\sqrt{2}$

$k = 6$

End of Solution

46. In the circuit shown, V_s is a 10 V square wave of period T = 4 ms with R = 500 Ω and C = 10 μF. The capacitor is initially uncharged at t = 0, and the diode is assumed to be ideal. The voltage across the capacitor (V_c) at 3 ms is equal to ____ volts (rounded off to one decimal place).



46. Ans: 3.3

Sol: $V_S = 10V$

$V_c(t) = 10V\,(1 - e^{-t/Rc})$

$V_C(t = 2ms) = 10V\left(1 - e^{-\frac{2\times10^{-3}}{500\times10\times10^{-6}}}\right)$

$= 3.3V$

T/2↔T: Diode is OFF

$V_c(t = 3ms) = 3.3V$

HEARTY CONGRATULATIONS TO OUR

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AIR 1  CHIRAG JHA EE

AIR 1  VINAY PRAKASH CE

AIR 1  AMAN JAIN ME

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AIR 2  PUNIT SINGH CE

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AIR 3  RAMESH KAMULLA E&T

AIR 3  SRIJAN VARMA EE

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AIR 3  MAYUR PATIL ME

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AIR 4  AMIT KUMAR ME

AIR 5  NARENDRA KUMAR E&T

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AIR 6  KUMUD JINDAL E&T

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AIR 7  KARTIKEYA DUTTA E&T

AIR 7  TEKCHAND DESHWAL EE

AIR 7  ROHIT KUMAR CE

AIR 8  SURYASH GAUTAM E&T

AIR 8  RAVI TEJA MANNE EE

AIR 8  VIJAYA NANDAN CE

AIR 8  ROHIT BANSAL ME

AIR 9  SHANAVAS CP E&T

AIR 9  SOUVIK DEB ROY EE

AIR 9  ROOPESH MITTAL CE

AIR 10  PRATHAMESH E&T

AIR 10  MILAN KRISHNA EE

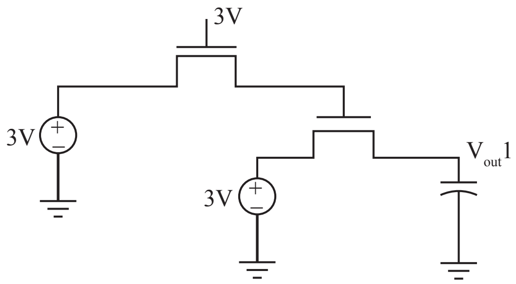
AIR 10  SRICHAND POONIYA CE

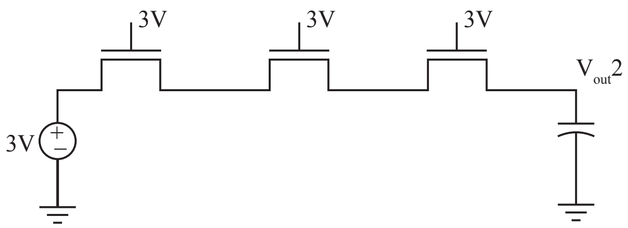
TOTAL SELECTIONS in Top 10	34

	E&T	EE	CE	ME
TOP 10	10	10	8	6

and many more...

47. In the circuits shown the threshold voltage of each nMOS transistor is 0.6 V. Ignoring the effect of channel length modulation and body bias, the values of Vout1 and Vout2, respectively, in volts, are





(a) 1.8 and 2.4
(b) 2.4 and 2.4
(c) 1.8 and 1.2
(d) 2.4 and 1.2

47. Ans: (a)

Sol: Given, $V_{TN} = 0.6V, \lambda = 0$ & $V_{SB} = 0$

$\therefore V_{th} = V_{tho}$ (or) $V_t = V_{to}$

Since, N-MOS, current always flow from drain to source,

$\Rightarrow$from fig (1)

$\therefore V_{G1} = 3V, V_{D1} = 3V, V_{TN} = 0.6V$

Since, to ON N-MOS [E - mode],

$V_{GS} \geq V_{Th}$

Let $V_{GS} = V_{Th} \Rightarrow V_{G1} - V_{s1} = V_{Th}$

$\therefore V_{S1} = V_{G1} - V_{Th} = 2.4V$

Since, $V_{S1} = V_{G2} \Rightarrow V_{G2} = 2.4V,$

& $V_{D2} = 3V$

$\therefore$ Let $V_{GS2} = V_{Th} \Rightarrow V_{G2} - V_{S2} = V_{Th}$

$\therefore V_{S2} = V_{out1} = 2.4 - 0.6$

$= 1.8$ V.

$\therefore V_{out1} = 1.8V.$

Similarly from fig (2)

$V_{G1} = V_{G2} = V_{G3} = 3V$ & $V_{D1} = 3V$

$\therefore V_{S1} = V_{G1} - V_{Th} = 3 - 0.6 = 2.4V$

$\therefore V_{S1} = V_{O2} = 2.4V$

$\therefore V_{S2} = V_{G2} - V_{Th} = 3 - 0.6 = 2.4$ V

& $V_{S3} = V_{G3} - V_{Th} = 2.4V$

$V_{out2} = 2.4V$

$\therefore V_{o1} = 1.8V, V_{o2} = 2.4\ V$

End of Solution

48. A rectangular waveguide of width w and height h has cut-off frequencies for TE_{10} and TE_{11} modes in the ratio 1:2. The aspect ratio w/h, rounded off to two decimal places, is ______

48. Ans: 1.73

Sol: $f_c|_{TE_{10}} = \frac{c}{2}\frac{1}{a}$

$$f_c|_{TE_{11}} = \frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}$$

Given

$$\frac{f_{cTE_{10}}}{f_{cTE_{11}}} = \frac{1}{2}$$

$$f_{cTE_{11}} = 2f_{cTE_{10}}$$

$$\frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = 2\frac{c}{2}\frac{1}{a}.$$

$$\frac{1}{a^2}+\frac{1}{b^2} = \frac{4}{a^2}$$

$$\frac{1}{b^2} = \frac{3}{a^2}$$

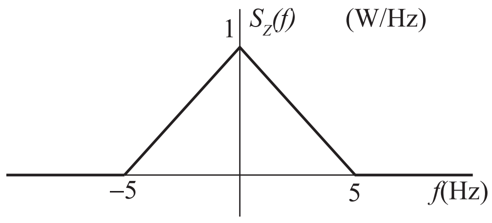
$$\left(\frac{a}{b}\right)^2 = 3$$

$$\frac{a}{b} = \sqrt{3}$$

$$\frac{a}{b} = 1.732$$

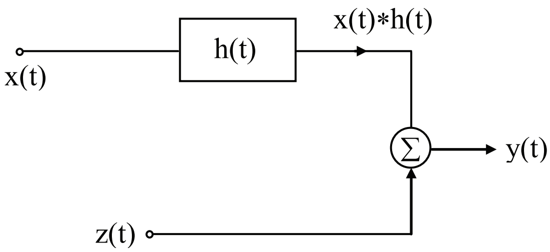
End of Solution

49. Let a random process Y(t) be described as Y(t) = h(t) ∗ X(t) + Z(t), where X(t) is a white noise process with power spectral density $S_x(f) = 5$ W/Hz. The filter h(t) has a magnitude response given by $|H(f)| = 0.5$ for $-5 \le f \le 5$, and zero elsewhere. Z(t) is a stationary random process, uncorrelated with X(t), with power spectral density as shown in the figure. The power in Y(t), in watts, is equal to____ W (rounded off to two decimal places).



49. Ans: 17.5

Sol:



Let us assume $x(t)*h(t) = w(t)$

$$y(t) = w(t) + z(t)$$

$$R_{YY}(\tau) = R_{WW}(\tau) + R_{ZZ}(\tau) + R_{WZ}(\tau) + R_{ZW}(\tau)$$

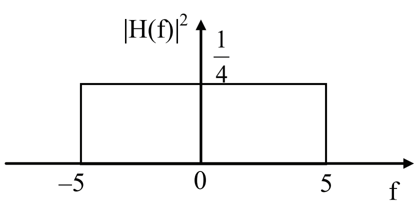
x(t) & z(t) are uncorrelated

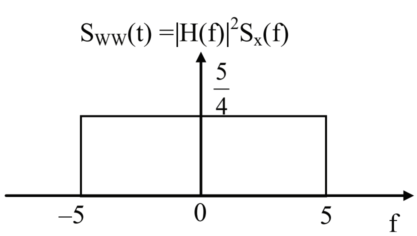
$$R_{wz}(\tau) = R_{zw}(\tau) = 0$$

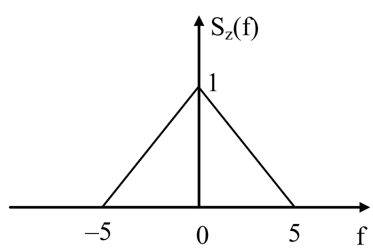
$$R_{YY}(\tau) = R_{ww}(\tau) + R_{zz}(\tau)$$
$$S_{YY}(f) = S_{ww}(f) + S_{zz}(f)$$

The power of y(t) is $P_Y = \int_{-\infty}^{\infty} S_{YY}(f)df = \int_{-\infty}^{\infty} S_{WW}(f)df + \int_{-\infty}^{\infty} S_{ZZ}(f)df$

$$S_{WW}(f) = |H(f)|^2 S_X(f)$$



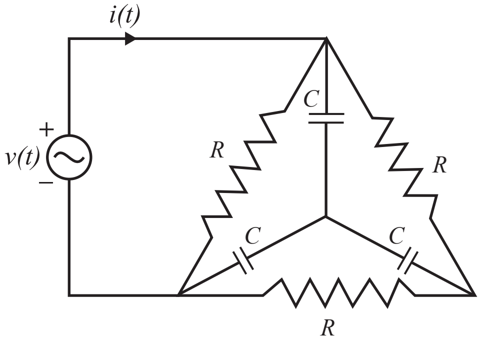




$$P_Y = \frac{5}{4}(10) + \frac{1}{2}(10)(1) = 12.5 + 5 = 17.5 \text{ W}$$

End of Solution

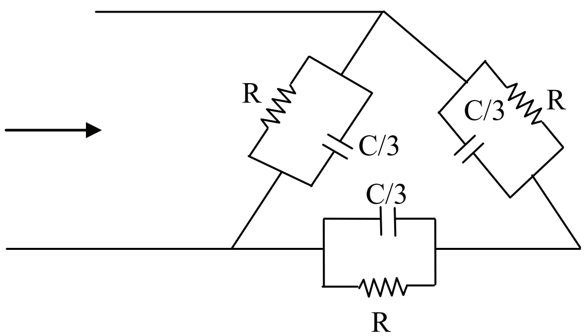
50. In the circuit shown, if $v(t) = 2\sin(1000t)$ volts, $R = 1\ k\Omega$ and $C = 1\ \mu F$, then the steady-state current $i(t)$, in milliAmperes (mA), is



(a) $2\sin(1000t) + 2\cos(1000t)$
(b) $\sin(1000t) + 3\cos(1000t)$
(c) $\sin(1000t) + \cos(1000t)$
(d) $3\sin(1000t) + \cos(1000t)$

50. Ans: (d)

Sol:

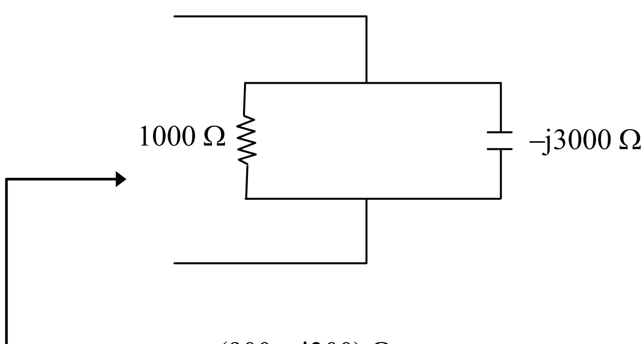


$R = 1000\ \Omega$ $\qquad \frac{C}{3} = \frac{1}{3}\mu F$

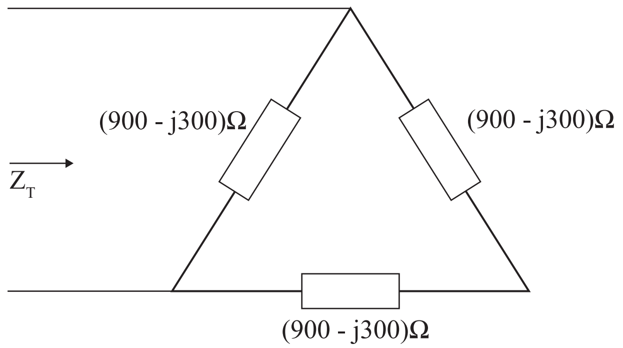
at $\omega = 1000$ rad/sec

$$X_C = \frac{-j}{\omega C} = \frac{-j}{1000 \times \frac{1}{3} \times 10^{-6}}$$

$$X_C = -j3000\Omega$$



$= (900 - j300)\ \Omega$

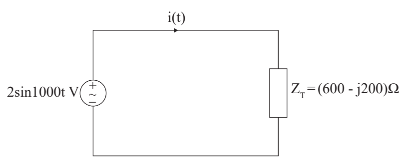


$$Z_T = (900 - j\,300) \parallel (1800 - j600)$$

$$Z_T = \frac{(1440000 - j1080000)}{(2700 - j900)}$$

$$Z_T = (600 - j200)\,\Omega$$

Finally



$$\text{So, } i(t) = \frac{2\sin 1000t}{(600 - j200)} = \frac{2\sin 1000t}{632.455\angle -18.434}$$

So,

$i(t) = 3.162 \sin(1000t + 18.434^o)$ mA

$S(A + B) = \sin A \cos B + \cos A \sin B$

So,

$i(t) = 3.162 (\sin 1000t \ \cos 18.434 + \cos 1000t \sin 18.434)$ mA

$i(t) = (3 \sin 1000t + \cos 1000t)$ mA

End of Solution

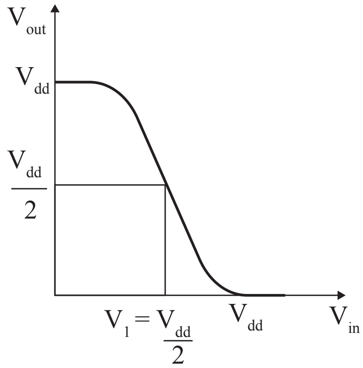
51. A CMOS inverter, designed to have a mid-point voltage V_1 equal to half of V_{dd}, as shown in the figure, has the following parameters:

$V_{dd} = 3$ V

$\mu_n C_{ox} = 100\mu A/V^2; V_{tn} = 0.7V$ for nMOS

$\mu_p C_{ox} = 40\mu A/V^2; |V_{tp}| = 0.9V$ for pMOS

The ratio of $\left(\frac{W}{L}\right)_n$ to $\left(\frac{W}{L}\right)_p$ is equal to _____ (rounded off to 3 decimal places).



51. Ans: 0.225

Sol: $\mu_n C_{ox}\left(\frac{W}{L}\right)_N (1.5 - V_{TN})^2 = \mu_P C_{ox}\left(\frac{W}{L}\right)_P (1.5 - |V_{TP}|)^2$

$$100 \times \left(\frac{W}{L}\right)_N \times 0.8^2 = 40 \times \left(\frac{W}{L}\right)_P \times 0.6^2$$

$$\frac{(W/L)_N}{(W/L)_P} = \frac{9}{16} \times \frac{4}{10} = 0.225$$

End of Solution

52. The quantum efficiency (η) and responsivity (R) at a wavelenth λ (in μm) in a p-i-n photodetector are related by

(a) $R = \frac{\lambda}{\eta \times 1.24}$

(b) $R = \frac{1.24 \times \lambda}{\eta}$

(c) $R = \frac{\eta \times \lambda}{1.24}$

(d) $R = \frac{1.24}{\eta \times \lambda}$

52. Ans: (c)

Sol:

$$R = \frac{\eta q}{hv} = \frac{\eta q \lambda}{hc}$$

$$R = \frac{\eta\lambda}{1.24}$$

End of Solution

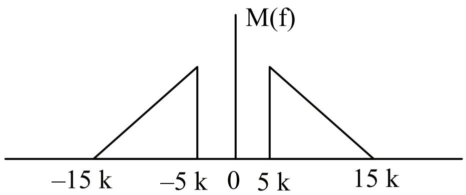
53. A voice signal m(t) is in the frequency range 5 kHz to 15 kHz. The signal is amplitude-modulated to generate an AM signal $f(t) = A(1+m(t)) \cos 2\pi f_c t$, where $f_c = 600$ kHz. The AM signal $f(t)$ is to be digitized and archived. This is done by first sampling $f(t)$ at 1.2 times the Nyquist frequency, and then quantizing each sample using a 256-level quantizer. Finally, each quantized sample is binary coded using K bits, where K is the minimum number of bits required for the encoding. The rate, in Megabits per second (rounded off to 2 decimal places), of

of the resulting stream of coded bits is ____ Mbps.

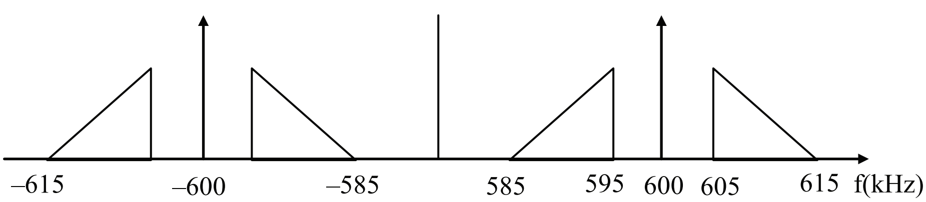
53. Ans:

As there is an ambiquity in the question, one of the question out of the following two cases may be possible

Sol: Spectrum of the voice signal m(t) is



The spectrum of modulated signal is



School of thought 1

The modulated signal is a bandpass signal

so the minimum sampling rate is $f_s = \frac{2f_H}{N}$

$f_H = 615\text{kHz}$ $\qquad N = \text{int}\left(\frac{615}{30}\right) = 20.5 = 20$

$f_s = \frac{2 \times 615}{20}\text{kHz} = 61.5\text{kHz}$

sampling rate = $61.5 \times 1.2\text{k} = 73.8\text{kHz}$

$L = 256 = 2^8$

Minimum number of bits required to encode each sample is '8'.

so the bit rate = $73.8 \times 10^3 \times 8 = 0.5904$ Mbps

school of thought 2

sampling rate = $2 \times 615 \times 10^3 \times 1.2 = 1.476$ MHz

so the bit rate = $1.476 \times 8 = 11.808$ Mbps

54. A random variable X takes values -1 and +1 with probabilities 0.2 and 0.8, respectively. It is transmitted across a channel output is $Y = X + N$. The noise N is independent of X, and is uniformly distributed over the interval [-2,2]. the receiver makes a decision

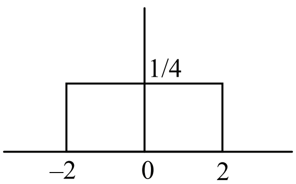
$$\hat{X} = \begin{cases} -1, & \text{if } Y \leq \theta \\ +1, & \text{if } Y > \theta \end{cases}$$

where the threshold $\theta \in [-1,1]$ is chosen so as to minimize the probability of error $\Pr[\hat{X} \neq X]$. The minimum probability of error, rounded off to 1 decimal place, is _____

54. Ans: 0.1

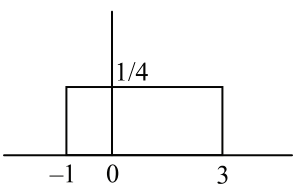
Sol: $P(-1) = 0.2$, $P(1) = 0.8$

the pdf of noise is

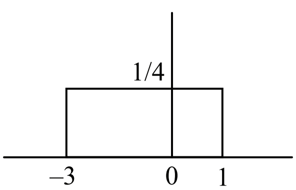


the received signal $Y = X + N$

The pdf if '1' is transmitted



The pdf if '–1' is transmitted



The threshold voltage should lies between –1 and +1

$$P_e = 0.8\int_{-\infty}^{V_{TH}} \frac{1}{4}dz + 0.2\int_{V_{TH}}^{\infty} \frac{1}{4}dz$$

$$= 0.8\int_{-1}^{V_{TH}} \frac{1}{4}dz + 0.2\int_{V_{TH}}^{1} \frac{1}{4}dz$$

$$P_e = 0.2[V_{TH} + 1] + 0.05[1 - V_{TH}]$$

$V_{TH} = -1 \rightarrow P_e = 0.1$

$V_{TH} = 0 \rightarrow P_e = 0.25$

$V_{TH} = 1 \rightarrow P_e = 0.4$

So the minimum probability of error is 0.1

55. Consider a long-channel MOSFET with channel length 1 μm and width 10 μm. The device parameters are acceptor concentration $N_A = 5 \times 10^{16}$ cm^{-3}, electron mobility $\mu_n = 800$ cm^2/V-s, oxide capacitance/area $C_{ox} = 3.45 \times 10^{-7}$F/cm^2, threshold voltage $V_T = 0.7$ V. The drain saturation current (I_{Dsat}) for a gate voltage of 5 V is ____ mA (rounded off to two decimal places). [$\varepsilon_0 = 8.854 \times 10^{-14}$F/cm, $\varepsilon_{si} = 11.9$]

55. Ans: 25.5

Sol: $I_{D,sat} = \frac{1}{2}\mu_n C_{ox} \times \frac{W}{L}(V_{GS} - V_{TH})^2$

$$= \frac{1}{2} \times 800 \times 3.45 \times 10^{-7} \times \frac{10}{1} \times (5 - 0.7)^2$$

$= 25.5$ mA

End of Solution