ESE- 2019 (Prelims) - Offline Test Series Test-18

GENERAL STUDIES AND ENGINEERING APTITUDE

SUBJECT: ENGINEERING MATHEMATICS AND NUMERICAL ANALYSIS SOLUTIONS

01. Ans: (b)

Sol: If A is a square of order 'n' then $|adj(A_{n\times n})| = |A|^{n-1}$

$\Rightarrow |adj(A_{3\times3})| = |A|^{3-1}$

$\Rightarrow |B| = |A|^2$

$$\Rightarrow \begin{vmatrix} 6 & 1 & -5 \\ -2 & -5 & 4 \\ -3 & 3 & x \end{vmatrix} = (-7)^2$$

$\Rightarrow 6(-5x - 12) - (-2x + 12) - 5(-6 - 15) = 49$

$\Rightarrow -28x - 28 = 0$

$\therefore$ x = –1

02. Ans: (a)

Sol: Given that $\lambda_1 = 2$, $\lambda_2 = -4$ and $\lambda_3 = 3$ are the eigen values of A.

$\Rightarrow (\lambda - 2)(\lambda + 4)(\lambda - 3) = 0$

$\Rightarrow \lambda^3 - \lambda^2 - 14\lambda + 24 = 0$

By Cayley-Hamilton theorem, we have $A^3 - A^2 - 14A + 24I = 0$

Multiplying above equation with A^{-1} on both sides, we get

i.e., $A^{-1}(A^3 - A^2 - 14A + 24I) = A^{-1}0$ $\Rightarrow A^2 - A - 14I + 24A^{-1} = 0$

$$\therefore A^{-1} = \frac{1}{24}\left[14I + A - A^2\right]$$

03. Ans: (c)

Sol: Let AX = O be the given homogenous systems such that $A = [X_1 \ \ X_2 \ \ X_3 \ \ X_4]$.

But given that X_1, X_2, X_3 and X_4 are linearly dependent vectors.

$\Rightarrow \rho(A) < 4$

$\Rightarrow \rho(A) <$ number of variables (= 4).

$\therefore$ The given system will have infinitely many solutions.

04. Ans: (c)

Sol: Let $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$ be the given characteristic equation of a matrix $A_{3\times3}$ in AX = O.

Then by comparing the given characteristic equation with general characteristic equation

$\lambda^3 - [tr(A)]\lambda^2 + (11)\lambda - |A| = 0$, we get

$|A| = 6$

If 'A' is an n×n matrix then $adj(adj(A_{n\times n})) = |A|^{n-2}A$

$\Rightarrow adj(adj(A_{3\times3})) = |A|^{3-2}A$

$\therefore adj(adj(A)) = |A|A = 6A$

05. Ans: (d)

Sol: Given that $A^{-1} = \begin{bmatrix} -5 & 1 & -2 & -9 \\ -2 & 1 & -1 & -5 \\ 8 & -2 & 3 & 15 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

$\Rightarrow A^{-1}$ exists.

Consider the given system AX = B

$\Rightarrow A^{-1}(AX) = A^{-1}B$

$\Rightarrow IX = A^{-1}B$

$\therefore X = \begin{bmatrix} -5 & 1 & -2 & -9 \\ -2 & 1 & -1 & -5 \\ 8 & -2 & 3 & 15 \\ 0 & 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -15 \\ -8 \\ 25 \\ -1 \end{bmatrix}$ is a required solution of the given system.

06. Ans: (a)

Sol: The matrix A has only one independent row.

Therefore, rank of A = number of linearly independent rows of A = 1.

07. Ans: (a)

Sol: Given that $A = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}$

$\Rightarrow \lambda = 3, 4, 4$ are eigen value of $A_{3\times3}$ matrix.

Here corresponding to non-repeated eigen value $\lambda = 3$, one independent eigen vector exists.

Now, the number of linearly independent eigen vectors corresponding to two times repeated eigen value $\lambda = 4$ is given by $p = n - r$, where n = number of variables in eigen vector (or) order of the matrix = 3 and $r = \rho(A - \lambda I)$.

Consider $A - \lambda I = \begin{bmatrix} 3-\lambda & 0 & 2 \\ 0 & 4-\lambda & 1 \\ 0 & 0 & 4-\lambda \end{bmatrix}$

$\Rightarrow \quad A - 4I = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ for $\lambda = 4$

$\Rightarrow \quad \rho(A - 4I) = 2$

$\Rightarrow$ Corresponding to $\lambda = 4$, the number of linearly independent eigen vectors = $n - r = 3 - 2 = 1$

Since $\lambda = 3$ is a distinct eigen value, there exists only one independent eigen vector corresponding to $\lambda = 3$.

Hence, the number of linearly independent eigen vectors of the matrix A = 2.

08. Ans: (c)

Sol: Let $I = \int_{0}^{\frac{\pi}{2}} \int_{0}^{\pi} \sin(x+y)\, dx\, dy$

Then $I = \int_{y=0}^{\frac{\pi}{2}} \left[\int_{x=0}^{\pi} \sin(x+y)\, dx \right] dy$

$\Rightarrow \quad I = \int_{y=0}^{\frac{\pi}{2}} \left[-\cos(x+y)\right]_0^{\pi}\, dy$

$\Rightarrow \quad I = \int_{y=0}^{\frac{\pi}{2}} \left[\cos(0+y) - \cos(\pi+y)\right] dy$

$\Rightarrow \quad I = \int_{y=0}^{\frac{\pi}{2}} \left[\cos(y) + \cos(y)\right]\, dy$

$\Rightarrow \quad I = \int_{y=0}^{\frac{\pi}{2}} 2\cos(y)\, dy$

$\Rightarrow \quad I = 2\left[\sin(y)\right]_0^{\frac{\pi}{2}}$

$\Rightarrow \quad I = 2\left[\sin\left(\frac{\pi}{2}\right) - \sin(0)\right]$

$\therefore \quad I = 2$

09. Ans: (b)

Sol: Given $f(x, y) = x^2 + y^2 + 6x - 12$

$\Rightarrow \; p = \frac{\partial f}{\partial x} = 2x + 6, \quad q = \frac{\partial f}{\partial y} = 2y$ and $r = \frac{\partial^2 f}{\partial x^2} = 2, \quad s = \frac{\partial^2 f}{\partial x\, \partial y} = 0, \quad t = \frac{\partial^2 f}{\partial y^2} = 2$

Consider $p = 0$ and $q = 0$

$\Rightarrow 2x + 6 = 0$ and $2y = 0$

$\Rightarrow x = -3$ and $y = 0$

$\therefore$ (x, y) = (–3, 0) is a stationary point.

At (–3, 0), r = 2, s = 0 & t = 2

Now, $rt - s^2 = (2)(2) - (0)^2 = 4 > 0$

and r = 2 > 0

$\Rightarrow$ The function will have a minimum at (–3, 0).

$\therefore$ The minimum value of f(x, y) at (–3, 0) is

$f(-3, 0) = (-3)^2 + (0)^2 + 6(-3) - 12 = -21$

10. Ans: (a)

Sol: Given $u(x, y) = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$

$\Rightarrow$ $u(x, y) = y^0 \sin^{-1}\left(\frac{x}{y}\right) + x^0 \tan^{-1}\left(\frac{y}{x}\right)$ is a homogeneous function with degree n = 0.

$\therefore$ By Euler's theorem of homogeneous functions, we have

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n.u = 0.u = 0$$

11. Ans: (c)

Sol: Let f(x) = log (1 + x) & a = 0.

Then the coefficient of $(x - a)^n$ in the Taylor series expansion of f(x) about x = a is given by

$$a_n = \frac{f^{(n)}(a)}{n!}$$

$$\Rightarrow a_3 = \frac{f'''(0)}{3!}$$

Now, f(x) = log (1 + x)

$$\Rightarrow f'(x) = \frac{1}{1+x}$$

$$\Rightarrow f''(x) = \frac{-1}{(1+x)^2}$$

$\Rightarrow f''' = \dfrac{(-1)(-2)}{(1+x)^3} = \dfrac{2}{(1+x)^3}$

$\therefore\ a_3 = \dfrac{f'''(0)}{3!} = \dfrac{\dfrac{2}{(1+0)^3}}{3!} = \dfrac{1}{3}$ which is the coefficient of x^3 in the Taylor series expansion of f(x) around x = 0.

12. Ans: (c)

Sol: Given $f(x) = \dfrac{x^3}{3} - 3x$ in [0, 3]

$\Rightarrow$ f(x) is a polynomial function.

We know that, every polynomial function f(x) is continuous and differentiable for all the values of 'x'.

$\therefore$ The given function f(x) is

(i) continuous in [0, 3]

and (ii) differentiable in (0, 3)

Here, f(0) = f(3) = 0.

Now, by Rolle's theorem, $\exists\ c \in (0, 3) \ni f'(c) = 0$

$\Rightarrow \dfrac{3c^2}{3} - 3 = 0$

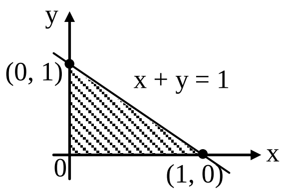
$\Rightarrow c^2 - 3 = 0$

$\Rightarrow c = \pm\sqrt{3}$

$\therefore\ c = \sqrt{3} \in (0, 3)\quad (\because -\sqrt{3} \notin (0, 3))$

13. Ans: (c)

Sol: Given curves are x + y = 1, y = 0 & x = 0



The area of the plane region R bounded by the curve y = f(x), the x-axis from x = a to x = b is given by

$$\text{Area} = \int_{x=a}^{b} f(x)\,dx$$

$$\Rightarrow \text{Area} = \int_{x=0}^{1} (1-x)dx = \left(x - \frac{x^2}{2}\right)_0^1 = \left(1 - \frac{1}{2}\right) - 0 = \frac{1}{2}$$

$$\therefore \text{Area} = \frac{1}{2}$$

14. Ans: (c)

Sol: A differential equation is said to be linear, if the dependent variable and all its derivatives appear in first degree only. Therefore, only the differential equation given in option (c) is linear.

15. Ans: (a)

Sol: Given $x^3 y''' + 3x^2y'' + xy' + y = 0$

$\Rightarrow (x^3D^3 + 3x^2D^2 + xD + 1)y = 0$ .... (1), where $D = \frac{d}{dx}$

Let $x = e^z$ (or) $\log x = z$ and

$xD = \theta$, $x^2D^2 = \theta(\theta - 1)$, $x^3D^3 = \theta(\theta - 1)(\theta - 2)$, where $\theta = \frac{d}{dz}$

Then (1) becomes

$[\theta(\theta - 1)(\theta - 2) + 3\theta(\theta - 1) + \theta + 1]y = 0$

$\Rightarrow (\theta^3 + 1)y = 0$

$\Rightarrow f(\theta)y = 0$ .... (2)

where $f(\theta) = \theta^3 + 1$

Consider auxiliary equation, f(m) = 0

$\Rightarrow$ $m^3 + 1 = 0$

$\Rightarrow$ $(m + 1)(m^2 - m + 1) = 0$

$\Rightarrow$ $m = -1, \dfrac{1 \pm i\sqrt{3}}{2}$

$$\Rightarrow y_c = c_1 e^{-z} + e^{\frac{z}{2}}\left[c_2 \cos\left(\frac{\sqrt{3}}{2}\right)z + c_3 \sin\left(\frac{\sqrt{3}}{2}\right)z\right]$$

$\therefore$ The general solution of (1) is $y = \dfrac{c_1}{x} + x^{\frac{1}{2}}\left[c_2\left(\dfrac{\sqrt{3}}{2}\log x\right) + c_2 \sin\left(\dfrac{\sqrt{3}}{2}\log x\right)\right]$

16. Ans: (c)

Sol: Given $y'' + 2y = x^3 + x^2$

$\Rightarrow$ $f(d)y = Q(x)$ .... (1)

where $f(d) = D^2 + 2$ and $Q(x) = x^3 + x^2$

C.F:

Consider Auxiliary equation, f(m) = 0

$\Rightarrow$ $m^2 + 2 = 0$

$\Rightarrow$ $m = 0 \pm i\sqrt{2}$

$\therefore$ $y_c = c_1 \cos(\sqrt{2})x + c_2 \sin(\sqrt{2})x$

P.I:

$$y_p = \frac{1}{f(D)}Q(x) = \frac{1}{(D^2+2)}(x^3 + x^2)$$

$$\Rightarrow y_p = \frac{1}{2\left(1+\frac{D^2}{2}\right)}(x^2 + x^3) = \frac{1}{2}\left(1+\frac{D^2}{2}\right)^{-1}(x^2 + x^3)$$

$$\Rightarrow y_p = \frac{1}{2}\left[1 - \left(\frac{D^2}{2}\right) + \left(\frac{D^2}{2}\right)^2 - \left(\frac{D^2}{2}\right)^3 + ...\right](x^2 + x^3)$$

$$\Rightarrow \quad y_p = \frac{1}{2}\left[1 - \frac{D^2}{2}\right](x^2 + x^3)$$

$$\therefore \quad y_p = \frac{1}{2}\left[(x^2 + x^3) - \frac{1}{2}(2 + 6x)\right]$$

Hence, the general solution of (1) is $y = y_c + y_p$

i.e., $y = \left[c_1 \cos(\sqrt{2})x + c_2 \sin(\sqrt{2})x\right] + \frac{1}{2}(x^2 + x^3) - \frac{1}{4}(2 + 6x)$

17. Ans: (b)

Sol: Given $f(d)y = Q(x)$ .... (1)

where $f(d) = D^2 - 2D - 1$ and $Q(x) = e^x \cos(x)$

C.F:

Consider Auxiliary equation, $f(m) = 0$

$\Rightarrow m^2 - 2m - 1 = 0$

$\Rightarrow m = 1 \pm \sqrt{2}$

$\therefore \; y_c = e^x\left[c_1 \cosh(\sqrt{2})x + c_2 \sinh(\sqrt{2})x\right]$

P.I:

$Q(x) = e^x \cos(x) = e^x . v(x)$

Now, $y_p = \frac{1}{f(D)}\left[e^x . \cos(x)\right] = e^x\left[\frac{1}{f(D+1)} \cos(x)\right]$

$$\Rightarrow \quad y_p = e^x\left[\frac{1}{(D+1)^2 - 2(D+1) - 1} \cos(x)\right] = e^x\left[\frac{1}{D^2 - 2}\cos(x)\right]$$

$$\Rightarrow \quad y_p = e^x\left[\frac{1}{-1-2}\cos(x)\right] \quad (\because D^2 = -a^2 = -1)$$

$$\therefore \quad y_p = \frac{e^x \cos(x)}{-3}$$

Hence, the general solution of (1) is $y = y_c + y_p$

i.e., $y = e^x\left[c_1 \cosh(\sqrt{2})x + c_2 \sinh(\sqrt{2})x\right] - \frac{e^x \cos(x)}{3}$

18. Ans: (d)

Sol: Given f(d)y = Q(x) .... (1)

where f(d) = $D^3 - 3D^2 + 4D - 2$ and Q(x) = cos(x) = k cos(ax + b)

Now, $y_p = \frac{1}{f(D)} Q(x) = \frac{1}{D^3 - 3D^2 + 4D - 2} \cos(x)$

$\Rightarrow \quad y_p = \frac{1}{D.D^2 - 3D^2 + 4D - 2} \cos(x)$

$\Rightarrow \quad y_p = \frac{1}{D(-1) - 3(-1) + 4D - 2} \cos(x) \quad (\because D^2 = -a^2 = -1)$

$\Rightarrow \quad y_p = \frac{1}{3D+1} \cos(x) = \frac{1}{1+3D} \times \frac{1-3D}{1-3D} \cos(x)$

$\Rightarrow \quad y_p = \frac{1-3D}{1-9D^2} \cos(x) = \frac{1-3D}{1-9(-1)} \cos(x)$

$\Rightarrow \quad y_p = \frac{\cos(x)}{10} - \frac{3}{10} D(\cos x)$

$\therefore \quad y_p = \frac{\cos(x)}{10} + \frac{3}{10} \sin(x)$ is a particular integral.

19. Ans: (b)

Sol: Given $\frac{dr}{d\theta} = r\tan\theta - \frac{1}{\cos\theta}$

$\Rightarrow \quad \frac{dr}{d\theta} + (-\tan\theta) r = -\sec\theta$ .... (1) $\left(\because \frac{dr}{d\theta} + P(\theta)\, r = Q(\theta)\right)$

Here, $P(\theta) = -\tan\theta$

Now, I.F = $e^{\int P(\theta)\, d\theta}$

$\Rightarrow$ I.F = $e^{\int -\tan\theta\, d\theta}$

$\Rightarrow$ I.F = $e^{\log(\cos\theta)}$

$\therefore$ I.F = $\cos\theta$

20. Ans: (c)

Sol: Given $x\,dy - y\,dx = 3x^2(x^2 + y^2)dx$ .... (1)

$$\Rightarrow \frac{x\,dy - y\,dx}{x^2 + y^2} = 3x^2\,dx$$

$$\Rightarrow d\left[\tan^{-1}\left(\frac{y}{x}\right)\right] = 3x^2\,dx$$

$$\Rightarrow \int 1\,d\left[\tan^{-1}\left(\frac{y}{x}\right)\right] = \int 3x^2\,dx + c$$

$\therefore \tan^{-1}\left(\frac{y}{x}\right) = x^3 + c$ is a general solution of (1)

21. Ans: (a)

Sol: The condition for an Exact differential equation $M\,dx + N\,dy = 0$ is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Here, the condition of the exactness satisfies only for option (a).

$\therefore$ Option (a) is correct.

22. Ans: (d)

Sol: Let $u + iv = f(z) = (x^2 - y + x - x^3) + i(x - y)$

Then $u = x^2 - y + x - x^3$ and $v = x - y$

$\Rightarrow$ $u_x = 2x + 1 - 3x^2$, $u_y = -1$, $v_x = 1$ and $v_y = -1$

Here, $v_x = -u_y$ at every point

but $u_x = v_y$ satisfy only at two points $\left(\frac{2 \pm \sqrt{28}}{6}, y\right)$.

$\therefore$ The given function f(z) is not analytic at any point.

23. Ans: (d)

Sol: A function u(x, y) is said to be a Harmonic function if $u_{xx} + u_{yy} = 0$ (or) $\nabla^2 u = 0$, where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

Let $u = x^2 + y^2$

Then $u_x = 2x$ and $u_y = 2y$

$\Rightarrow$ $u_{xx} = 2$ and $u_{yy} = 2$

Consider $u_{xx} + u_{yy} = 2 + 2 = 4$

$\Rightarrow$ $u_{xx} + u_{yy} \neq 0$

$\therefore$ $u = x^2 + y^2$ is not a harmonic function.

Hence, option (d) is correct.

24. Ans: (a)

Sol: Given $u = x^4 - 6x^2y^2 + y^4$

$\Rightarrow$ $u_x = 4x^3 - 12xy^2$ and $u_y = 4y^3 - 12x^2y$

Consider $dv = \left(\frac{\partial v}{\partial x}\right)dx + \left(\frac{\partial v}{\partial y}\right)dy$

$$\Rightarrow \quad dv = \left(-\frac{\partial u}{\partial y}\right)dx + \left(\frac{\partial u}{\partial x}\right)dy$$

$$\Rightarrow \quad dv = (12x^2y - 4y^3)\,dx + (4x^3 - 12xy^2)\,dy$$

Integrate first term w.r.t 'x' by treating 'y' as constant on R.H.S and consider the terms without 'x' in the 2nd term on R.H.S and integrate w.r.t 'y'.

i.e., $\int dv = \int (12x^2y - 4y^3)\,dx + \int (0-0)dy + c$

$$\Rightarrow \quad v(x, y) = 12\frac{x^3}{3}y - 4xy^3 + c$$

$\therefore$ $v(x, y) = 4x^3y - 4xy^3 + c$ is a required imaginary part of analytic function f(z).

25. Ans: (c)

Sol: Given $v = v(x, y) = 3x^2y + 6xy - y^3$

$\Rightarrow \quad v_x = 6xy + 6y \quad$ and $v_y = 3x^2 + 6x - 3y^2$

Consider $du = \left(\frac{\partial u}{\partial x}\right) dx + \left(\frac{\partial u}{\partial y}\right) dy$

$$\Rightarrow \qquad du = \left(\frac{\partial v}{\partial y}\right) dx + \left(\frac{-\partial v}{\partial x}\right) dy$$

$$\Rightarrow \qquad du = (3x^2 + 6x - 3y^2)\, dx + (-6xy - 6y)\, dy$$

Integrate first term w.r.t 'x' by treating 'y' as constant on R.H.S and consider the terms without 'x' in the 2nd term on R.H.S and integrate w.r.t 'y'.

i.e., $\int du = \int (3x^2 + 6x - 3y^2)\, dx + \int (0 - 6y) dy + c$

$$\Rightarrow \quad u(x, y) = 3\frac{x^3}{3} + 6\frac{x^2}{2} - 3xy^2 - 6\frac{y^2}{2} + c$$

$\therefore \quad u(x, y) = x^3 + 3x^2 - 3xy^2 - 3y^2 + c$

Consider $f(z) = u(x, y) + i\, v(x, y)$

$\Rightarrow f(z) = (x^3 + 3x^2 - 3xy^2 - 3y^2 + c) + i(3x^2y + 6xy - y^3)$

$\therefore\ f(z) = z^3 + 3z^2 + c$ (i.e. for $x = z$ and $y = 0$) is a required analytic function.

26. Ans: (c)

Sol: Let $f(z) = \frac{1}{z^2 - 4} = \frac{1}{(z-2)(z+2)}$

Then the singular points of f(z) are given by $z^2 - 4 = 0$.

i.e., $z = 2, -2$ are singular points.

Here, C: $x^2 + y^2 = 9$ (or) $|z| = 3$

but $\quad |z| = |2| = 2 < 3 \quad$ and $\quad |z| = |-2| = 2 < 3$

$\Rightarrow$ both singular points $z = 2$ and $z = -2$ lie inside the circle 'C'.

So, we evaluate the given integral by Cauchy's integral formula

$$\oint_c f(z)\, dz = \oint_c \frac{\phi(z)}{[z - z_0]}\, dz = 2\pi i\, \phi(z_0).$$

Now, $f(z) = \dfrac{1}{(z-2)(z+2)} = \dfrac{1}{4(z-2)} - \dfrac{1}{4(z+2)}$

$\Rightarrow \quad \oint_c f(z)\, dz = \dfrac{1}{4}\oint_c \dfrac{1}{(z-2)}\, dz - \dfrac{1}{4}\oint_c \dfrac{1}{[z-(-2)]}\, dz$

$\Rightarrow \quad \oint_c f(z)\, dz = \dfrac{1}{4} 2\pi i\ \phi(2) - \dfrac{1}{4}\ 2\pi i\ \phi(-2)$

$\therefore \quad \oint_c f(z)\, dz = \dfrac{1}{4}(2\pi i) - \dfrac{1}{4}(2\pi i) = 0$

27. Ans: (c)

Sol: Let f(z) = cosh(z) and $z_0 = \pi i$

Then the coefficient of $(z - z_0)^n$ in the Taylor series expansion of f(z) about $z = z_0$ is given by $a_n = \dfrac{f^{(n)}(z_0)}{n\,!}$.

Consider f(z) = cosh(z)

$\Rightarrow \quad f'(z) = \sinh(z)$

$\Rightarrow \quad f''(z) = \cosh(z)$

$\Rightarrow \quad f'''(z) = \sinh(z)$

Now, the coefficient of $(z - \pi i)^3$ is $a_3 = \dfrac{f'''(\pi i)}{3!}$.

$\Rightarrow \quad a_3 = \dfrac{\sinh(\pi i)}{3!}$

$\therefore \quad a_3 = \dfrac{i\sin(\pi)}{6} = 0.$

28. Ans: (d)

Sol: Given $f(z)=\dfrac{1}{(z-1)(z-2)}$ in $1 < |z| < 2$

$\Rightarrow \quad f(z)=\dfrac{1}{z-2}-\dfrac{1}{z-1}$

$\because \quad 1 < |z| < 2$

$\Rightarrow \quad 1 < |z|$ and $|z| < 2$

$\therefore \quad \left|\dfrac{1}{z}\right|<1$ and $\left|\dfrac{z}{2}\right|<1$

Now, f(z) = $\dfrac{1}{z-2}-\dfrac{1}{z-1} = \dfrac{1}{-2\left(1-\dfrac{z}{2}\right)}-\dfrac{1}{z\left(1-\dfrac{1}{z}\right)}$

$\Rightarrow \quad f(z)=\dfrac{1}{-2}\left[1-\left(\dfrac{z}{2}\right)\right]^{-1}-\dfrac{1}{z}\left[1-\left(\dfrac{1}{z}\right)\right]^{-1}$

$\Rightarrow \quad f(z)=\dfrac{1}{-2}\left[1+\left(\dfrac{z}{2}\right)+\left(\dfrac{z}{2}\right)^2+\ldots\ldots\right]-\dfrac{1}{z}\left[1+\left(\dfrac{1}{z}\right)+\left(\dfrac{1}{z}\right)^2+\ldots\ldots\right]$

$\Rightarrow \quad f(z)=\left[\dfrac{-1}{2}-\dfrac{z}{4}-\dfrac{z^2}{8}-\ldots.\right]+\left[\dfrac{-1}{z}-\dfrac{1}{z^2}-\dfrac{1}{z^3}-\ldots..\right]$, which is a Laurent series expansion of f(z) about z = 0 in $1 < |z| < 2$.

$\therefore$ The coefficient of $\dfrac{1}{z^2}$ is –1.

29. Ans: (c)

Sol: Let $f(x) = x^3 - 8x - 40 = 0$ and $[a, b] = [x_0, x_1] = [4, 5]$

Then $f(x_0) = f(4) = 64 - 32 - 40 = -8 < 0$

$f(x_1) = f(5) = 125 - 40 - 40 = 45 > 0$

Now, the iterative formula of Regula-Falsi method is

$$x_{n+1} = \frac{x_{n-1}.f(x_n) - x_n.f(x_{n-1})}{f(x_n) - f(x_{n-1})}, \ n = 1, 2, 3.......$$

$$\Rightarrow \ x_2 = \frac{x_0.\ f(x_1) - x_1.f(x_0)}{f(x_1) - f(x_0)}, \text{ for } n = 1$$

$$\Rightarrow \ x_1 = \frac{(4)(45) - (5)(-8)}{45 - (-8)}$$

$$\therefore \quad x_1 = \frac{180 + 40}{45 + 8} = \frac{220}{53} = 4.151 \text{ is a required first approximation of the equation } f(x) = 0$$

30. Ans: (b)

Sol: Simpson's rule gives the exactly value of the integral if f(x) is a polynomial of degree ≤ 3.

$$\int_0^2 (1 + 5x - 100x^2)dx = \left(x + \frac{5x^2}{2} - \frac{100x^3}{3}\right)_0^2$$

$$= 2 + \frac{20}{2} - \frac{800}{3}$$

$$= 12 - \frac{800}{3}$$

$$= \frac{36 - 800}{3}$$

$$= -\frac{764}{3}$$

31. Ans: (b)

Sol: Let $f(x) = x^3 + x + 12 = 0$ and $[a, b] = [-3, -2]$.

Then $f(a) = f(-3) = (-3)^3 + (-3) + 12 = -18 < 0$

and $f(b) = f(-2) = (-2)^3 + (-2) + 12 = 2 > 0$

Step (1):

The formula of first approximation to the real root of f(x) using bisection method is given by

$$x_1 = \frac{a+b}{2} = \frac{-3-2}{2} = -2.5$$

Step (2):

Now, $f(x_1) = f(-2.5) = f\left(-\frac{5}{2}\right) = \left(\frac{-5}{2}\right)^3 + \left(\frac{-5}{2}\right) + 12 = \frac{-49}{8} < 0$

but f(b) = f(–2) = 2 > 0

$\therefore$ The root lies between –2.5 & –2 and it is given by

$$x_2 = \frac{b+x_1}{2}$$

Hence, $x_2 = \frac{-2-2.5}{2} = \frac{-4.5}{2} = -2.25$ is a required 2nd approximation of f(x) = 0.

32. Ans: (c)

Sol: Let $f(x) = e^x - 2x^2 = 0$ and $x_0 = 1$

Then $f'(x) = e^x - 4x$

Step (1):

Now, $f(x_0) = f(1) = e - 2 = 0.7182$

and $f'(x_0) = f'(1) = e - 4 = -1.2817$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{(e-2)}{(e-4)} = 1 - \frac{(0.7182)}{(-1.2817)} = 1 + 0.5603 = 1.5603$$

33. Ans: (d)

Sol: Given $y' = \frac{dy}{dx} = x^2 + y^2$ .... (1) $\left(\because \frac{dy}{dx} = f(x, y)\right)$

with y(0) = 1 .... (2) $(\because y(x_0) = y_0)$

Let $x_0 = 0$, $y_0 = 1$, h = 0.2 and $f(x, y) = x^2 + y^2$.

Then $x_1 = x_0 + h = 0 + 0.2 = 0.2$

Now, the formula of Euler's method is given by

$$y(x_1) \simeq y_0 + h\, f(x_0, y_0)$$

$\therefore$ $y(0.2) \simeq y_1 = 1 + (0.2)\left(x_0^2 + y_0^2\right) = 1 + 0.2 = [\,0 + 1] = 1.2$

34. Ans: (b)

Sol: Given f(x) = x in (–2, 2) = (–*l*, *l*)

$\Rightarrow$ f(x) is an odd function ($\because$ f(–x) = –f(x))

$\Rightarrow$ $a_0 = 0$ and $a_n = 0$

$\Rightarrow$ The fouries series expansion of f(x) contains only sine terms.

Now, the fourier series of given an odd function f(x) is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{\ell}\right) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right)$$

where , $b_n = \dfrac{2}{\ell}\displaystyle\int_0^{\ell} f(x)\sin\left(\frac{n\pi x}{\ell}\right)dx = \dfrac{2}{2}\displaystyle\int_0^{2} x \sin\left(\frac{n\pi x}{2}\right)dx$

$$\Rightarrow \qquad b_n = \left[x.\frac{\cos\left(\frac{n\pi x}{2}\right)}{\left(\frac{-n\pi}{2}\right)} - \frac{\sin\left(\frac{n\pi x}{2}\right)}{\left(\frac{-n^2\pi^2}{2^2}\right)} \right]_0^2$$

$$\Rightarrow \qquad b_n = \left[\frac{-4}{n\pi}\cos(n\pi) - 0\right] - [0 - 0]$$

$$\Rightarrow \qquad b_n = \frac{-4\,(-1)^n}{n\pi} = \frac{(-1)^{n+1}\,4}{n\pi}$$

$\therefore$ The fourier series of f(x) is

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}4}{n\pi}\sin\left(\frac{n\pi x}{2}\right)$$

35. Ans: (d)

Sol: P(Red ball) = $\frac{8}{18} = \frac{4}{9}$

P(Not red ball) = $1 - \frac{4}{9} = \frac{5}{9}$

Hence, option (d) is correct.

36. Ans: (b)

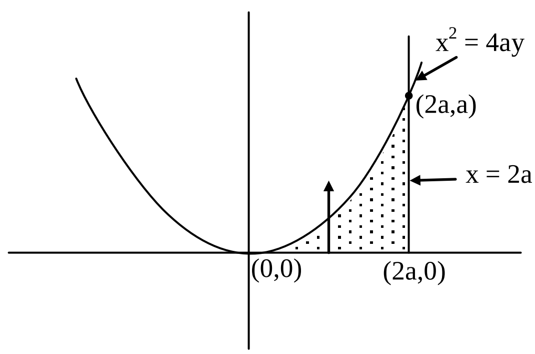
Sol: Total area under normal curve is 1.

The area under normal curve is symmetric about its mean.

P(X ≤ 0) = Area under normal curve to the left of the mean = 0.5

37. Ans: (d)

Sol: Area = $\iint dx\,dy$



$$= \int_{x=0}^{2a} \int_{y=0}^{\frac{x^2}{4a}} dy\,dx$$

$$= \int_0^{2a} \frac{x^2}{4a} dx = \left(\frac{x^3}{12a}\right)_0^{2a}$$

$$= \frac{8a^3}{12a} = \frac{2}{3}a^2$$

38. Ans: (a)

Sol: $\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}.........\frac{2}{3}$ if n is odd

$$\int_0^{\frac{\pi}{2}} \sin^7 x\,dx = \frac{6}{7}\times\frac{4}{5}\times\frac{2}{3} = \frac{16}{35}$$

39. Ans: (a)

Sol: $\int_{(0,0)}^{(1,1)} \left[\left(3x^2+4xy+y^2\right)dx+\left(2x^2+2xy\right)dy\right]$

$$= \iint_R \left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right) dx\,dy \qquad \text{(By Green's Theorem)}$$

$$= \iint_R \left[(4x+2y)-(4x+2y)\right]dx\,dy$$

$$= 0$$

40. Ans: (d)

Sol: $\text{div}\,\vec{F} = 2x\,\vec{i}+4y\,\vec{j}+6z\,\vec{k} = 12$

By Gauss-divergence theorem, we have

$$\iint_S \vec{F}.\hat{n}\,ds = \iiint_V \text{div}\,\vec{F}\,dx\,dy\,dy$$

$$= \iiint_V 12\,dx\,dy\,dy$$

$= 12$(volume of the sphere)

$$= (12)\times\frac{4}{3}\pi(1)^3$$

$$= 16\pi$$

$= -16\pi$ ($\because \hat{n}$ is inward unit normal vector to S)

41. Ans: (d)

Sol: Combined mean $= \dfrac{n\times 2\bar{x}+2n\bar{x}}{n+2n}$

$$= \frac{2n\bar{x}+2n\bar{x}}{3n} = \frac{4n\bar{x}}{3n}$$

$$= \frac{4}{3}\bar{x}$$

42. Ans: (d)

Sol: Two digit number N = {10, 11, ..., 99}

Total numbers = 90

$P(25 > N > 85) = P(N < 25) + P(N > 85)$

$$= \frac{15}{90} + \frac{14}{90} = \frac{29}{90}$$

43. Ans: (b)

Sol: Since $P(A) + P(B) = 1$

$$P(A) + 3P(A) = 1$$

$$4P(A) = 1 \Rightarrow P(A) = \frac{1}{4}$$

$$\Rightarrow P(B) = \frac{3}{4}$$

$$\therefore\ P(\overline{B}) = 1 - P(B)$$

$$= 1 - \frac{3}{4} = \frac{1}{4}$$

44. Ans: (d)

Sol: Given $B \subset A \Rightarrow A \cap B = B$

$$P(A|B) + P(A^C|B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A^C \cap B)}{P(B)} = \frac{P(A \cap B) + P(B) - P(A \cap B)}{P(B)} = 1$$

45. Ans: (c)

Sol: P(Defective relay) = $0.5 \times 0.01 + 0.3 \times 0.02 + 0.2 \times 0.02 = 0.015$

Required Probability = $P(P_3|D) = \frac{0.2 \times 0.02}{0.015} = 0.266$

46. Ans: (c)

Sol: $P(\text{Alive}) = P(W).P(A|W) + P(W^C)\ .\ P(A|W^C) = 0.9 \times 0.85 + 0.1 \times 0.2 = 0.785$

47. Ans: (c)

Sol: Given np = 6 ; npq = 4

$6q = 4 \Rightarrow q = \frac{4}{6} = \frac{2}{3} \Rightarrow p = \frac{1}{3}$

And also np = 6

$\frac{n}{3} = 6 \Rightarrow n = 18$

$P(X = 0) = q^n = \left(\frac{2}{3}\right)^{18}$

48. Ans: (c)

Sol: Given $f(x) = \frac{10}{x^2}$, $x > 10$

$P(X > 20) = \int_{20}^{\infty} f(x)\,dx = \left.\frac{-10}{x}\right]_{20}^{\infty} = \frac{1}{2}$

49. Ans: (b)

Sol: $V(X) = E(X^2) - (E(X))^2$

$\lambda = 6 - \lambda^2 \Rightarrow \lambda^2 + \lambda - 6 = 0$

$\lambda = 2$

Then $P(X \le 1.2) = P(X = 0) + P(X = 1) = e^{-2} + 2e^{-2} = 3e^{-2}$

50. Ans: (b)

Sol: P(X > 40, 000)

$z = \frac{40{,}000 - 34{,}000}{4000} = 1.5$

P(X > 40, 000) = P(z > 1.5) = 0.5 – 0.4332 = 0.0668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