

# **MECHANICAL ENGINEERING**

IM & OR

**Volume-1: Study Material with Classroom Practice Questions** 

# IM & OR

# Solutions for Volume – I\_ Classroom Practice Questions

# Chapter- 01 PERT & CPM

01. Ans: (a)

**Sol:** CPM deals with deterministic time durations.

02. Ans: (a)

Sol: Critical Path:

- It is a longest path consumes maximum amount of resources
- It is the minimum time required to complete the project

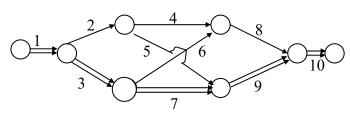
03. Ans: (a)

04. Ans: (a)

**Sol:** Gantt chart indicates comparison of actual progress with the scheduled progress.

05. Ans: (c)

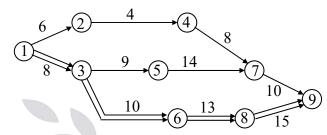
Sol:



Critical path = 1 + 3 + 7 + 9 + 10 = 30 days

06. Ans: (c)

Sol:



Critical path 
$$(1-3-6-8-9) = 8 + 10 + 13 + 15$$
  
= 46 days

07. Ans: (b)

Sol: Rules for drawing Network diagram:

- Each activity is represented by one and only one arrow in the network.
- No two activities can be identified by the same end events.
- Precedence relationships among all activities must always be maintained.
- No dangling is permitted in a network.
- No Looping (or Cycling) is permitted.

08. Ans: (b)

**Sol: Activity:** Resource consuming and well-defined work element.

**Event:** Each event is represented as a node in a network diagram and it does not consume any time or resource.

Dummy Activity: An activity does not consume any kind of resource but merely

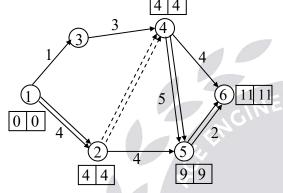


depicts the technological dependence is called a dummy activity.

Float: Permissible delay period for the activity.

## 09. Ans: (b)

Sol:



10. Ans: (a)

#### 11. Ans: (b)

Sol:

- Beta Distribution is used to decide the expected duration of an activity.
- The expected duration of the project can be described by Normal distribution.

# 12. Ans: (b)

Sol: 
$$T_0 = 8 \text{ min}$$
,  $T_m = 10 \text{ min}$ ,  $T_p = 14 \text{ min}$ , 
$$T_e = \frac{T_o + 4T_m + T_p}{6}$$
$$= \frac{8 + 4 \times 10 + 14}{6} = \frac{62}{6} = 10.33 \text{ min}$$

13. Ans: (a)

Sol: Take 4-3, 
$$T_e = 6$$
 days  
Critical path = 1-2-4-3  
= 5 + 14 + 4 = 23 days  
 $\sigma_{\text{critical path}} = \sqrt{V_{1-2} + V_{2-4} + V_{4-3}}$   
=  $\sqrt{2^2 + 2 \ 8^2 + 2^2} = 3 \ 979$ 

$$z = \frac{Due \, date - critical \, path \, duration}{\sigma_{critical \, path}}$$

$$z = \frac{27 - 23}{3.979} = 1.005$$

$$\therefore P(z) = 0.841$$

Sol: D = 36 days, 
$$V = 4$$
 days
$$Z = \frac{36 - 36}{\sqrt{4}} = 0$$

$$\Rightarrow P(z) = 50\%$$

Sol: 
$$\sigma_{cp} = \sqrt{V_{a-b} + V_{b-c} + V_{c-d} + V_{d-e}}$$
  
=  $\sqrt{4 + 16 + 4 + 1} = 5$ 

17. Ans: (a)

**Sol:** The latest that an activity can start from the beginning of the project without causing a delay in the completion of the entire project. It is the maximum time up to which an activity can be delayed to start without effecting the project completion duration time. (LST = LFT - duration).



18. Ans: (c)

**Sol:** The earliest expected completion time,

Critical path: A-B-C-D-F-E-H

$$\Rightarrow$$
 5 + 4 + 8 + 5 + 8 = 30 days

19. Ans: (d)

**Sol:** Critical path:

$$1-3-4-6 = 20 \text{ days}$$

$$z = \frac{24 - 20}{\sqrt{4}} = \frac{4}{2} = 2$$

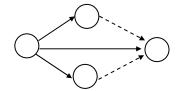
$$\Rightarrow$$
 P(z) = 97.7%

20. Ans: (d)

Sol: Variance = 
$$\left(\frac{t_p - t_o}{6}\right)^2$$
  
=  $\left(\frac{22 - 10}{6}\right)^2$  =  $\left(\frac{22 - 10}{6}\right)^2$ 

- 21. Ans: (a)
- 22. Ans: (b)
- Ans: (a) 23.
- Ans: (b) 24.
- 25. Ans: (c)

Sol:



- 26. Ans: (c)
- 27. Ans: (b)
- **28.** Ans: (d)

Sol:

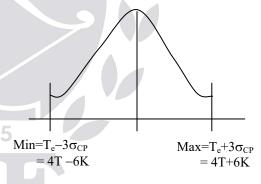
Given each activity having time mean duration 'T' and standard deviation 'K'.

Total time estimate  $T_e = 4T$ 

Variance of the path

$$(\Sigma \text{var})_{\text{CP}} = R^2 + R^2 + R^2 + R^2$$
  
=  $4R^2$ 

Standard deviation of  $CP = \sqrt{\sum (var)_{CP}}$ 



$$\sigma_{CP} = \sqrt{4K^2}$$

$$\sigma_{CP} = \pm 2K$$

Range of overall project duration likely to be in 4T + 6K and 4T - 6K

i.e., 
$$4T \pm 6K$$

Since



# Common solutions for Q.29 & Q.30

29. Ans: (b)

**30.** Ans: (b)

Sol:

| Paths           | Duration   |
|-----------------|------------|
| 1-2-4-5 = (AEF) | 8+9+6=23   |
| 1-2-3-4-5=(ADF) | 8+9+6=23   |
| 1-3-4-5 (BDF)   | 6+9+6 = 21 |
| 1-4-5 (CF)      | 16+6=22    |

- : Highest time taken paths are AEF and ADF
  - :. Critical path's are AEF and ADF Critical paths are '2'.

Possible cases to crash

A by 1 day that cost = 80

F by 1 day that cost = 130

E and D by 1 day that cost = 20 + 40 = 60

#### 31. Ans: (c)

### 32. Ans: (c)

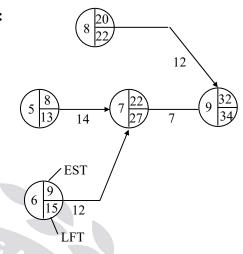
Sol:

| Path | Duration |
|------|----------|
| AB   | 7+5=12   |
| CD   | 6+6 = 12 |
| EF   | 8+4 = 12 |

Three critical paths, number of activities to be crashed are 3.

## 33. Ans: (c)

Sol:

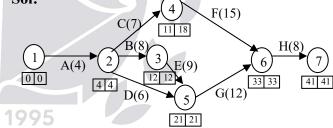


$$(Total Float)_{6-7} = 27 - 9 - 12 = 6$$
  
(Free float)<sub>6-7</sub> = 28 - 9 - 12 = 1

#### 34. Ans: (a-7, b-41)

Sol:

Since



Path duration  
1-2-4-6-7 = 
$$4 + 7 + 15 + 8 = 34$$
  
1-2-3-5-6-7 =  $4 + 8 + 9 + 12 + 8$   
= 41 (days) (critical path)  
1-2-5-6-7 =  $4 + 6 + 12 + 8 = 30$ 

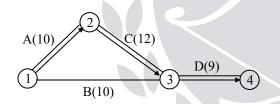
$$|+$$
 TF+7



**35.** Ans: 31 days

Sol:

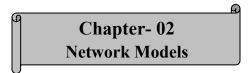
| Activity | Time estimated                         | Standard<br>deviation           |
|----------|--|---------------------------------|
|          | $T_e = \frac{T_o + 4T_m + T_p}{6}$     | $\sigma = \frac{T_p - T_o}{6}$  |
| A        | $\frac{5+4\times10+15}{6} = 10$        | $\frac{15-5}{6} = \frac{5}{3}$  |
| В        | $\frac{2+4\times5+8}{6} = 5$           | $\frac{8-2}{6} = 1$             |
| C        | $\frac{10 + 4 \times 12 + 14}{6} = 12$ | $\frac{14-10}{6} = \frac{2}{3}$ |
| D        | $\frac{6+4\times8+16}{6}=9$            | $\frac{16-6}{6} = \frac{5}{3}$  |



## Critical path:

$$1-2-3-4 = 10 + 12 + 9 = 31$$
 days

$$\begin{split} \sigma_{cp} &= \sqrt{V_{1-2} + V_{2-3} + V_{3-4}} \\ &= \sqrt{\left(\frac{5}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{5}{3}\right)^2} = \sqrt{6} \end{split}$$



Ans: (c) 01.

Sol:

 $d_{ii} \rightarrow$  "Distance from any node i to next node j"

"Denotes shortest path from node P to any node j".

 $d_{ii} = d_{OG}$  (Adjacent nodes)

 $d_{ij} = d_{RG}$  (Adjacent from node R to G)

 $S_i = S_Q$  (Shortest path from node P to node Q)

 $S_j = S_R$  (Shortest path from node P to node R)

We can go from P to G via Q or via R.

P to G via Q

$$S_G = S_Q + d_{QG}$$

P to G via R.

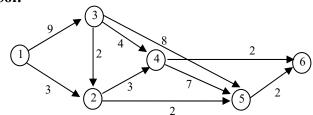
$$S_G = S_R + d_{RG}$$

Optimum answer is minimum above two answers.

$$S_G = MIN [S_Q + d_{QG}; S_R + d_{RG}]$$

**02.** Ans: (c)

Sol:



**Since 1995** 



| Path        | Cost           |
|-------------|----------------|
| 1-3-4-6     | 9+4+2 = 15     |
| 1-3-2-4-6   | 9+2+3+2 = 16   |
| 1-3-4-5-6   | 9+4+7+2 = 22   |
| 1-3-2-5-6   | 9+2+2+2 =15    |
| 1-3-2-4-5-6 | 9+2+3+7+2 = 23 |
| 1-2-4-6     | 3+3+2 = 8      |
| 1-2-5-6     | 3+2+2 = 7      |
| 1-2-4-5-6   | 3+3+7+2 = 15   |
| 1-3-5-6     | 9+8+2 = 19     |

From the given statement, we got shortest path (least total cost) is 1-2-5-6 and a path which does not have 1-2, 2-5, 5-6 activities should be considered.

The next path which does not have the above activities is 1-3-4-6=15

and 
$$1-3-2-4-6=16$$
.

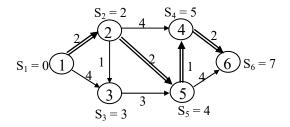
... In this second least total cost is 15.

# 03. Ans: 7

## Sol:

| Path        | Arc length |
|-------------|------------|
| 1-2-4-6     | 8          |
| 1-2-5-4-6   | 7          |
| 1-2-5-6     | 8          |
| 1-2-3-5-4-6 | 9          |
| 1-3-5-4-6   | 10         |
| 1-3-5-6     | 11         |

Shortest path length from node 1 to node 6 is 7.





# Chapter- 03 **Linear Programming**

01. Ans: (d)

Sol: A restriction on the resources available to a firm (stated in the form of an inequality or an equation) is called constraint.

**02.** Ans: (d)

03. Ans: (c)

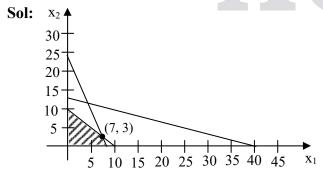
04. Ans: (d)

Sol: The theory of LP states that the optimal solution must lie at one of the corner points.

Ans: (b) **05.** 

**Sol:** The feasible region of a linear programming problem is convex. The value of the decision variables, which maximize or minimize the objective function, is located on the extreme point of the convex set formed by the feasible solutions.

**06.** Ans: (a)



 $Z(7,3) = 2 \times 7 + 5 \times 3 = 29$ 

07. Ans: (a)

**Sol:**  $Z_{max} = x + 2y$ , Subjected to

$$4y - 4x \ge -1 \dots (1)$$

$$5x + y \ge -10 \dots (2)$$

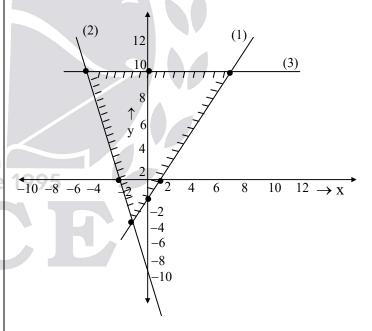
$$y \le 10 \dots (3)$$

x and y are unrestricted in sign

$$(1) \Rightarrow \frac{x}{\left(\frac{1}{4}\right)} + \frac{y}{\left(\frac{-1}{4}\right)} \le 1$$

$$(2) \Rightarrow \frac{x}{(-2)} + \frac{y}{(-10)} \le 1$$

$$(3) \Rightarrow \frac{y}{10} \le 1$$



Only one value gives max value, then solution is unique.



08. Ans: (b)

**Sol:**  $Z_{max} = 3x_1 + 2x_2$ 

Subjected to

$$4x_1+x_2 \le 60$$
 .....(1)

$$8x_1+x_2 \le 90$$
 .....(2)

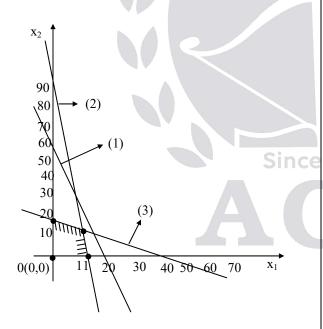
$$2x_1+5x_2 \le 80$$
 ......(3)

$$x_1, x_2 \ge 0$$

$$(1) \Longrightarrow \frac{x_1}{15} + \frac{x_2}{60} \le 1$$

$$(2) \Rightarrow \frac{x_1}{11.25} + \frac{x_2}{90} \le 1$$

$$(3) \Rightarrow \frac{x_1}{40} + \frac{x_2}{16} \le 1$$



From the above graph the No. of corner points for feasible solutions are 4

09. Ans: (c)

**Sol:** Let, P type toys produced = x,

Q type toys produced = y

|                 | P | Q |      |
|-----------------|---|---|------|
| Time            | 1 | 2 | 2000 |
| Raw material    | 1 | 1 | 1500 |
| Electric switch | - | 1 | 600  |
| Profit          | 3 | 5 |      |
|                 | X | y |      |

$$Z_{\text{max}} = 3x + 5y$$

$$x + 2y \le 2000$$
;  $\frac{x}{2000} + \frac{y}{1000} \le 1$ 

$$x + y \le 1500$$
 ;  $\frac{x}{1500} + \frac{y}{1500} \le 1$ 

$$y \le 600$$
 ;  $\frac{y}{600} \le 1$ 

$$x, y \ge 0$$

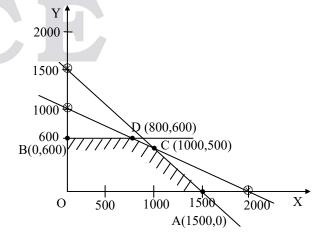
$$Z_{max} = 3x + 5y$$

$$Z_A = 3 \times 1500 + 5 \times 0 = 4500$$

$$Z_B = 3 \times 0 + 5 \times 600 = 3000$$

$$Z_C = 3 \times 1000 + 5 \times 500 = 5500$$

$$Z_D = 3 \times 800 + 5 \times 600 = 5400$$





C does not exist in answer.

Hence,  $Z_{\text{max}}$  is at D, i.e.,  $Z_{\text{max}}$  @ D = 5400

# 10. Ans: (c)

**Sol:** 
$$Z_{max} = x_1 + 1.5 x_2$$

Subject to

$$2x_1+3x_2 \le 6$$
 ----- (1)

$$x_1 + 2x_2 \le 4$$
 ----- (2)

$$x_1, x_2 \ge 0$$

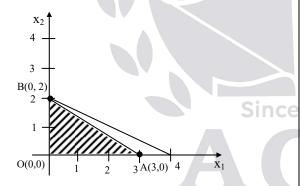
$$\frac{\mathbf{x}_1}{3} + \frac{\mathbf{x}_2}{2} \le 1$$

$$\frac{\mathbf{x}_1}{4} + \frac{\mathbf{x}_2}{2} \le 1$$

Let, "c" in the intersection of (1) and (2)

Solve (1) & (2) for 'c'.

It follows, 
$$x_1 = \frac{12}{5}$$
;  $x_2 = \frac{2}{5}$ 



$$Z_{\text{max}} = x_1 + 1.5x_2$$

$$Z_0 = 0$$

$$Z_A = 3 + 1.5 \times 0 = 3$$

$$Z_B = 3 \times 0 + 1.5 \times 2 = 3$$

Problem is having multiple solutions and it is Optimal at (A) and (B).

# 11. Ans: (a)

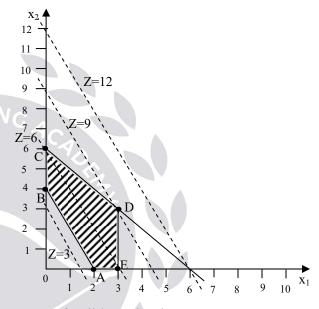
**Sol:** 
$$Z_{\text{max}} = 2x_1 + x_2$$

Subjected 
$$x_1 + x_2 \le 6$$

$$x_1 \le 3$$

$$2x_1 + x_2 \ge 4$$

$$x_1, x_2 \ge 0$$



But feasible region is ABCDEA

$$(:: x_1, x_2 > 0)$$

D can be obtained by solving

$$x_1 \le 3 \& x_1 + x_2 \le 6$$

$$\Rightarrow$$
 x<sub>1</sub> = 3 and x<sub>2</sub> = 3 and D (3,3)

| A(2,0)  | $2 \times 2 + 1 \times 0 = 4$ |
|---------|-------------------------------|
| B (0,4) | $0 \times 2 + 1 \times 4 = 4$ |
| C(0,6)  | $0 \times 2 + 1 \times 6 = 6$ |
| E(3,0)  | $3 \times 2 + 0 \times 1 = 6$ |
| D(3,3)  | $3 \times 2 + 1 \times 3 = 9$ |
|         | B (0,4)<br>C(0,6)<br>E(3,0)   |

$$Z_{\text{max}} = 9 \text{ at D } (3,3)$$



12. Ans: (a)

**13.** Ans: (b)

14. Ans: (d)

15. Ans: (a)

**Sol:** 
$$Z_{\text{max}} = 4 x_1 + 6 x_2 + x_3$$

s.t

$$2 x_1 - x_2 + 3x_3 \le 5$$

$$x_1, x_2, x_3 \ge 0$$

$$2 x_1 - x_2 + 3x_3 + s_1 = 5$$

$$Z_{\text{max}} = 4x_1 + 6x_2 + x_3 + 0 \ s_1$$

| $c_j \rightarrow$  | 4              | 6                     | 1  | 0              |                | min       |
|--|----------------|-----------------------|----|----------------|----------------|-----------|
| $\begin{array}{c} c_j \rightarrow \\ s \ v \downarrow \end{array}$ | $\mathbf{x}_1$ | <b>X</b> <sub>2</sub> | X3 | $\mathbf{s}_1$ | $\mathrm{B}_0$ | Ratio     |
| 0 s <sub>1</sub>   | 2              | -1                    | 3  | 15             | 5              | <b>-5</b> |
| $Z_{j}$  | 0              | 0                     | 0  | 0              | 0              |           |
| $ \begin{array}{c c} z_j \\ c_j - z_j \end{array} $                | 4              | (6)                   | 1  | 0              |                |           |
|  |                | 1                     | 0  |                |                |           |
|  |                | EV                    |    |                |                |           |

Entering vector exists but leaving vector doesn't exist as minimum ratio column is having negative values. It is a case of unbounded solution space and unbounded optimal solution to problem.

16. Ans: (d)

**Sol:** Number of zeros in Z row = 4Number of basic variable = 3

> As the number of zeros in Z row is greater than number of basic variable so it has multiple optimal solutions.

17. Ans: (b)

Sol: Solution is optimal; but Number of zeros are greater than the number of basic Variables in  $C_i - Z_i$  (net evaluation row) hence multiple optimal solutions.

Ans: (b) **18.** 

Sol: If all the elements in the objective row are non-negative incase of maximization, then the solution is said to be optimal.

Here, the solution is optimal,  $Z_{max} = 1350$ .

19. Ans: (a)

Sol:

- A tie for leaving variable in simplex procedure implies degeneracy.
  - If in a basic feasible solution, one of the basic variables takes on a zero value then it is case of degenerate solution

### **Common Data**

20. Ans: (d) 21. Ans: (a) 22. Ans: (a)

Sol: As the No. of zeros greater than No. of basic variables hence it is a case of multiple solutions or alternate optimal solution exists.



| Basic                 | $\mathbf{x}_1$ | x <sub>1</sub> x <sub>2</sub> |   | $S_2$ | $S_3$ | RHS |
|-----------------------|----------------|-------------------------------|---|-------|-------|-----|
| Z                     | 0              | 0                             | 0 | 2     | 0     | 48  |
| $s_1$                 | 0              | 5/3                           | 1 | -2/3  | 0     | 14  |
| <b>S</b> 3            | 0              | -1/3                          | 0 | 1/3   | 1     | 5   |
| <b>X</b> <sub>1</sub> | 1              | 2/3                           | 0 | 1/3   | 0     | 8   |

From the table gives the optimum  $x_2 = 0$ ,  $x_1 = 8$ ,  $Z_{max} = 48$ 

Look at the coefficient of the non basic variable in the z-equation of iterations. The coefficient of non basic x2 is zero, indicating that  $x_2$  can enter the basic solution without changing the value of Z, but causing a change in the values of the variables.

Alternate optimal solution:

Here  $x_2$  is the entering variable.

| Row            | Basic          | $\mathbf{x}_1$ | <b>X</b> <sub>2</sub> | $S_1$ | $S_2$ | $S_3$ | RHS | Ratio        |                   |
|----------------|----------------|----------------|-----------------------|-------|-------|-------|-----|--------------|-------------------|
| R <sub>1</sub> | Z              | 0              | 0                     | 0     | 2     | 0     | 48  | 0.           |                   |
| R <sub>2</sub> | $\mathbf{s}_1$ | 0              | 5/3                   | 1     | -2/3  | 0     | 14  | 14/(5/3)=8.4 | →Leaving variable |
| R <sub>3</sub> | S <sub>3</sub> | 0              | -1/3                  | 0     | 1/3   | 1     | 5   | 4            |                   |
| R <sub>4</sub> | $\mathbf{x}_1$ | 1              | 2/3                   | 0     | 1/3   | 0     | 8   | 8/(2/3)=12   |                   |

Entering variable

| Row                                   | Basic                 | <b>X</b> <sub>1</sub> | <b>X</b> 2 | S <sub>1</sub> | S <sub>2</sub> | S <sub>3</sub> | RHS  |
|---------------------------------------|-----------------------|-----------------------|------------|----------------|----------------|----------------|------|
| $R_1$                                 | Z                     | 0                     | 0          | 0              | 2              | 0              | 48   |
| $R_2' = \frac{R_2}{\left(5/3\right)}$ | <b>x</b> <sub>2</sub> | gind                  | le (       | 3/5 5          | -2/5           | 0              | 42/5 |
| $R_3' = R_3' + \frac{R_2'}{3}$        | <b>S</b> 3            | 0                     | 0          | 1/5            | 1/5            | 1              | 39/5 |
| $R_4' = R_4 - \frac{2}{3}R_2'$        | <b>X</b> <sub>1</sub> | 1                     | 0          | -3/5           | 3/5            | 0              | 12/5 |

In the above table 
$$x_1 = \frac{12}{5}$$
,  $x_2 = \frac{42}{5}$ ,  $x_3 = \frac{39}{5}$ 

23. Ans: (c)

24. Ans: (c)

25. Ans: (a)



26. Ans: (c)

**Sol:** 
$$Z_{min} = 10x_1 + x_2 + 5x_3 + 0S_1$$

Dual, 
$$W_{min} = 50y_1$$

subjected to

$$5y_1 \le 10$$
,  $y_1 \le 2$ ,  $W_{max} = 100$ 

$$3y_1 \le 5$$
,  $y_1 \le 5/3$ ,  $W_{max} = 250/3$ 

$$y_1, y_2 \ge 0$$

$$\Rightarrow$$
 Z<sub>max</sub> = 250 / 3

# **Common Data for Questions**

27. Ans: (c)

**Sol:** Given, 
$$Z_{\text{max}} = 5x_1 + 10x_2 + 8x_3$$

Subjected to

$$3x_1 + 5x_2 + 2x_3 \le 60 \rightarrow Material$$

$$4x_1 + 4x_2 + 4x_3 \le 72 \rightarrow$$
 Machine hours

$$2x_1 + 4x_2 + 5x_3 \le 100 \rightarrow \text{Labour hours}$$

$$x_1, x_2, x_3 \ge 0$$

$$3x_1 + 5x_2 + 2x_3 + s_1 = 60$$

$$4x_1 + 4x_2 + 4x_3 + s_2 = 73$$

$$2x_1 + 4x_2 + 5x_3 + s_3 = 100$$

$$Z_{max} = 5x_1 + 10x_2 + 8x_3 + 0s_1 + 0s_2 + 0s_3$$

| $C_{j}$     | $\rightarrow$         | 5              | 1<br>0                | 8      | 0              | 0                | 0   | $B_0$ | Min   |
|-------------|-----------------------|----------------|-----------------------|--------|----------------|------------------|-----|-------|-------|
| С           | S<br>V                | $\mathbf{x}_1$ | <b>x</b> <sub>2</sub> | X<br>3 | $\mathbf{s}_1$ | $s_2$            | S 3 |       | Ratio |
| 10          | <b>x</b> <sub>2</sub> | $\frac{1}{3}$  | 1                     | 0      | $\frac{1}{3}$  | $\frac{-1}{6}$   | 0   | 8     |       |
| 8           | X <sub>3</sub>        | $\frac{2}{3}$  | 0                     | 1      | $\frac{-1}{3}$ | $\frac{5}{12}$   | 0   | 10    |       |
| 0           | $S_3$                 | $\frac{-8}{3}$ | 0                     | 0      | 1/3            | $\frac{-17}{12}$ | 1   | 18    |       |
| $Z_{\rm j}$ |                       | $\frac{26}{3}$ | 1 0                   | 8      | $\frac{2}{3}$  | $\frac{5}{3}$    | 0   | 160   |       |

| C <sub>j</sub>            | -Z <sub>j</sub>            | $\frac{-11}{3}$ | 0 | 0 | $\frac{-2}{3}$ | $\frac{-5}{3}$ | 0 |                   |                            |
|---------------------------|----------------------------|-----------------|---|---|----------------|----------------|---|-------------------|----------------------------|
| $\frac{\mathrm{C_{j}}}{}$ | $\frac{-Z_{j}}{C_{2}}$     | -11             | 0 | 0 | -2             | 10             | 0 | LL=2<br>UL=1<br>0 | 10-2=<br>8<br>10+10<br>=20 |
| C <sub>j</sub>            | $\frac{-Z_{j}}{\zeta_{3}}$ | $\frac{-11}{2}$ | 0 | 0 | 2              | -4             | 0 | LL=4<br>UL=2      | 8-4=4<br>8+2=1<br>0        |

In  $C_j$  – $Z_j$  row all elements are negatives or zeros, hence the solution is optimal and unique.

Basic variables are:

$$x_2 = 8$$
,  $x_3 = 10$ ,  $s_3 = 18$ 

i.e., production of B = 8 units, C = 10 units

18 labours hours remained unutilized

Non Basic variable

$$x_1 = 0,$$
  $s_1 = 0,$   $s_2 = 0$ 

Resource materials and resource machine hours are fully utilized. In  $(C_j - Z_j)$  row at optimality, the values under  $s_1$ ,  $s_2$  and  $s_3$  columns represents the shadow prices.

So, If 1 kg material increases, contribution increases by  $\frac{2}{3}$ .

If 1 kg material decreases, contribution decreases by  $\frac{2}{3}$ .

If 1 kg material increases, then production B increases by  $\frac{1}{3}$  and production C decreases

by 
$$\frac{1}{3}$$

If m/c hr increases by 1 units, contribution increases by 5/3.



If m/c hr decreases by 1 units, contribution decreases by  $\frac{5}{3}$ 

If m/c hr increases by 1 units, production B decreases by  $\frac{1}{6}$  and production increases by

$$\frac{5}{12}$$

If m/c hr decreases by 1 units, production B increases by  $\frac{1}{6}$  and production C decreases

by 
$$\frac{5}{12}$$

If 1 unit of A produces, contribution decreases by  $\frac{11}{3}$ , production B decreases by  $\frac{1}{3}$ , production C decreases by  $\frac{2}{3}$ .

# 28. Ans: (a)

**Sol:** If 3 kg material increases, contribution increases by  $3 \times \frac{2}{3} = \text{Rs. } 2$ 

# 29. Ans: (a)

**Sol:** Present profit =  $160 \Rightarrow 160 - \frac{5}{3} \times 12 = 140$ /-

# 30. Ans: (b)

**Sol:** New production of B

$$= 8 - \left(12 \times \frac{-1}{6}\right) = 8 + \left(12 \times \frac{1}{6}\right)$$
  
= 8 + 2 = 10 units

# 31. Ans: (c)

**Sol:** If materials are increased by 3kgs then the new production of C is =  $10 + \left(3 \times \frac{-1}{3}\right)$ =  $10 - \left(3 \times \frac{1}{3}\right) = 10 - 1 = 9$ 

# 32. Ans: (a)

**Sol:** If 1 unit of A produces, contribution decreases by  $\frac{11}{3}$ 

# 33. Ans: (a)

**Sol:** If 6 units of A are produced then the new profit is,

$$160 - \left(6 \times \frac{11}{3}\right) = 138$$

# 34. Ans: (a)

**Sol:** Production of B,  $3 \times \frac{1}{3} = 1$ 

Production of C,  $3 \times \frac{2}{3} = 2$ 

# Common data 35 & 36

35. Ans: (b) , 36. Ans: (b)

Sol: Basic variables

$$x_1 = 20$$
,  $x_2 = 10$ 

Non-basic variables

 $s_1 = 0 \Rightarrow$  first constraint is fully consumed.

 $s_2 = 0 \implies second constraint is fully consumed.$ 

 $x_3 = 0$  (unwanted variable)



|                       | $\mathbf{x}_1$ | <b>X</b> 2 | X3 | $s_1$ | $s_2$ | RHS |
|-----------------------|----------------|------------|----|-------|-------|-----|
| z-row                 | 0              | 0          | 2  | 1     | 2     | 110 |
| $\mathbf{x}_1$        | 1              | 0          | 1  | 1     | -1    | 20  |
| <b>X</b> <sub>2</sub> | 0              | 0          | 0  | -1    | 2     | 10  |

|                       | $s_1$ |
|-----------------------|-------|
| z-row                 | 1     |
| <b>X</b> <sub>1</sub> | 1     |
| X <sub>2</sub>        | -1    |

If RHS value of 1<sup>st</sup> constraint increases by 1 unit then

### From the table

z increases by 1 unit, x<sub>1</sub> increases by 1 unit, x<sub>2</sub> decreases by 1 unit,

If RHS value of 2nd constraint increases by 1 unit then

|                       | $\mathbf{s}_2$ |
|-----------------------|----------------|
| z-row                 | 2              |
| <b>x</b> <sub>1</sub> | -1             |
| X <sub>2</sub>        | 2              |

### From the table

z increases by 2 units,  $x_1$  decreases by 1 unit x<sub>2</sub> decreases by 2 units,

If RHS value of 1st constraint decreases by 10 units then z decreases by 10 units,

The new objective value,

$$Z_{max} = 110 - 10 = 100$$

37. Ans: (c)

Sol:

|       | $X_1$ | $X_2$    | $S_1$ | $S_2$ | RHS | Ratio |
|-------|-------|----------|-------|-------|-----|-------|
| z-row | -3    | -5       | 0     | 0     | 0   | 0     |
| $S_1$ | 2     | 1        | 1     | 0     | 2   | 2/1=2 |
| $S_2$ | 3     | 2        | 0     | 1     | 4   | 4/2=2 |
|       |       | <b>↑</b> |       |       |     |       |

Entering variable X<sub>2</sub> Minimum ratio =  $min(2/1, 4/2) = 2^*$ \*Tie w.r.t leaving variables S<sub>1</sub> and S<sub>2</sub> Thus it has degenerate solution.

38. Ans: (d)

Sol:

Since

|                | $X_1$    | $X_2$ | $S_1$ | $S_2$ | RHS |
|----------------|----------|-------|-------|-------|-----|
| z-row          | -2       | -1    | 0     | 0     | 0   |
| S <sub>1</sub> | -2       | 1     | 1     | 0     | 4   |
| S <sub>2</sub> | 0        | 1     | 0     | 1     | 3   |
| all and the    | <b>A</b> | /     |       | 1     |     |

199 Entering variable  $X_1$ 

Ratio =  $Min\{4/-2, 3/0\}$ 

As there is no least positive ratio, there is no leaving variable which results the problem has unbounded solution.



# Chapter-04 **Inventory Control**

01. Ans: (b)

**Sol:** EOQ = 
$$\sqrt{\frac{2AS}{CI}}$$

$$EOQ_1 = \sqrt{2} \times \sqrt{\frac{2AS}{CI}}$$

$$EOQ_1 = \sqrt{2} \times EOQ$$

02. Ans: (c)

**Sol:** EOQ = 
$$\sqrt{\frac{2DC_o}{C_c}}$$

03. Ans: (b)

**Sol:** A = 900 unit

S = 100 per order

CI = 2 per unit per year

$$EOQ = ELS = \sqrt{\frac{2AS}{CI}}$$
$$= \sqrt{\frac{2 \times 900 \times 100}{2}} = 300$$

04. Ans: (c)

**Sol: Inventory carrying cost:** 

It involves the cost of investment in inventories, of storage, of obsolescence, of insurance, of maintaining inventory records, etc.

05. Ans: (b)

**Sol:** At EOQ, Carrying cost = Ordering cost

06. Ans: (d)

Sol: Inventory carrying cost involves the cost of investment in inventories, of storage, of obsolescence, of insurance, of maintaining inventory records, etc.

Ans: (a)

Sol: 
$$A = 800$$
,  $S = 50/-$ ,  $C_s = 2$  per unit = CI 
$$(TIC)_{EOQ} = \sqrt{2ASCI}$$
$$= \sqrt{2 \times 800 \times 50 \times 2} = 400$$

08. Ans: (c)

Sol: 
$$TC(Q_1) = TC(Q_2)$$
  

$$\frac{kd}{Q_1} + \frac{hQ_1}{2} = \frac{kd}{Q_2} + \frac{hQ_2}{2}$$

$$kd\left(\frac{Q_2 - Q_1}{Q_1 Q_2}\right) = \frac{h}{2}(Q_2 - Q_1)$$

$$\frac{2kd}{h} = Q_1Q_2$$

$$(Q^*)^2 = Q_1 \times Q_2$$

$$Q^* = \sqrt{Q_1 \times Q_2} = \sqrt{300 \times 600} = 424.264$$

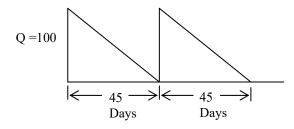
09. Ans: (c)

Sol: 
$$\frac{\text{EOQ}_1}{\text{EOQ}_2} = \sqrt{\left(\frac{2\text{AS}}{\text{CI}}\right)_{\text{A}}} \times \sqrt{\left(\frac{\text{CI}}{2\text{AS}}\right)_{\text{B}}}$$
$$= \sqrt{\left(\frac{2 \times 100 \times 100}{4}\right)} \times \sqrt{\left(\frac{1}{2 \times 400 \times 100}\right)}$$
$$(\text{EOQ})_{\text{A}} : (\text{EOQ})_{\text{B}} = 1:4$$



10. Ans: (d)

**Sol:** (No of orders = 
$$\frac{A}{Q} = \frac{12 \text{ months}}{45 \text{ days}} = \frac{12}{1.5} = 8$$
)



T VC = 
$$\frac{A}{Q}$$
 S +  $\frac{Q}{2}$  CI.  
=  $8 \times 100 + \frac{100}{2} \times 120 = \text{Rs. } 6800$ 

11. Ans: (b)

Sol: Average inventory

$$= \frac{Q}{2} = \frac{6000}{2} = 3000 \text{ per year}$$
$$= 250 \text{ per month}$$

12. Ans: (b)

**Sol:** 
$$P = 1000$$
,  $r = 500$ ,  $Q = 1000$   
 $I_{max} = \frac{1000}{1000} (1000 - 500) = 500$ 

13. Ans: (c)

**Sol:** D = 1000 units,  $C_0$  = Rs.100/order,  $C_c$  = 100/unit/year,  $C_s$  = 400/unit/year

$$Q_{\text{max}} = \text{EOQ}_{\text{s}} \times \frac{C_{\text{s}}}{C_{\text{c}} + C_{\text{s}}}$$

$$= \sqrt{\frac{2DC_{\text{o}}}{C_{\text{c}}}} \sqrt{\frac{C_{\text{c}} + C_{\text{s}}}{C_{\text{s}}}} \times \left(\frac{C_{\text{s}}}{C_{\text{c}} + C_{\text{s}}}\right)$$

$$= 40 \text{ units}$$

14. Ans: (d)

**Sol:** Re-order level =  $1.25[\Sigma x p(x)]$ 

$$= 1.25 [80 \times 0.2 + 100 \times 0.25 + 120 \times 0.3 + 140 \times 0.25]$$

= 140 units

| Demand                 | 80   | 100  | 120  | 140  |
|------------------------|------|------|------|------|
| Probability            | 0.20 | 0.25 | 0.30 | 0.25 |
| Cumulative probability | 0.2  | 0.45 | 0.75 | 1.0  |
| (Service level)        | 0.2  | 0.73 | 0.75 | 1.0  |

Service Level = 100 %

17. Ans: (d)

**Sol:** C – Class means these class items will have very less consumption values. – least consumption values

B 
$$\rightarrow 300 \times 0.15 = 45$$
  
F  $\rightarrow 300 \times 0.1 = 30$   
C  $\rightarrow 2 \times 200 = 400$   
E  $\rightarrow 5 \times 0.3 = 1.5$   
J  $\rightarrow 5 \times 0.2 = 1.0$   
G  $\rightarrow 10 \times 0.05 = 0.5$   
H  $\rightarrow 7 \times 0.1 = 0.7$ 

.. G, H items are classified as C class items because they are having least consumption values.

18. Ans: (b)

**Sol:** In ABC analysis:

Category "A" = Low safety stock

Category "B" = Medium safety stock

Category "C" = High safety stock

**Since 1995** 



# Chapter-05 **Forecasting**

01. Ans: (d)

**02.** Ans: (d)

Sol:

- A simple moving average is a method of computing the average of a specified number of the most recent data values in a series.
- This method assigns equal weight to all observations in the average.
- Greater smoothing effect could be obtained by including more observations in the moving average.

03. Ans: (a)

**Sol:** 3 period moving avg = 
$$\frac{100 + 99 + 101}{3}$$
 = 100

4 period moving average

$$=\frac{102+100+99+101}{4}=100.5$$

5 period moving average

$$=\frac{99+102+100+99+101}{5}=100.2$$

Arithmetic Mean

$$= \frac{101+99+102+100+99+101}{6}$$
$$= 100.33$$

04. Ans: (a)

Sol: 
$$D_t = 100$$
 units ,  $F_t = 105$  units 
$$\alpha = 0.2$$
 
$$F_{t+1} = 105 + 0.2 \ (100 - 105) = 104$$

05. Ans: (c)

**Sol:** 
$$D_t = 105$$
,  $F_t = 97$ ,  $\alpha = 0.4$   $F_{t+1} = 97 + 0.4 (105 - 97) = 100.2$ 

06. Ans: (c)

**Sol:** 
$$F_{t+1} = F_t + a (X_t - F_t)$$

07. Ans: (c)

Sol: Another form of weighted moving average is the exponential smoothed average. This method keeps a running average of demand and adjusts if for each period in proportion to the difference between the latest actual demand and the latest value of the forecast.

08. Ans: (a)

09. Ans: (b)

Sol:

1995

| Period | $\mathbf{D_{i}}$ | Fi                | $(D_i - F_i)^2$    |
|--------|------------------|-------------------|--------------------|
| 14     | 100              | 75                | 625                |
| 15     | 100              | 87.5              | 156.25             |
| 16.    | 100              | 93.75             | 39.0625            |
|        |                  | $\Sigma(D_i - 1)$ | $(F_i)^2 = 820.31$ |

$$F_{15} = F_{14} + \alpha (D_{14} - F_{14})$$
$$= 75 + 0.5(100 - 75) = 87.5$$



$$F_{16} = F_{15} + \alpha (D_{15} - F_{15})$$

$$= 87.5 + 0.5(100 - 87.5) = 93.75$$

Mean square error (MSE) = 
$$\frac{\sum (D_i - F_i)^2}{n}$$
  
=  $\frac{820.31}{3}$  = 273.13

# 10. Ans: (a)

Sol:

| Period | Di | Fi   | $ (D_i-F_i) $ |
|--------|----|------|---------------|
| 1      | 10 | 9.8  | 0.2           |
| 2      | 13 | 12.7 | 0.3           |
| 3      | 15 | 15.6 | 0.6           |
| 4      | 18 | 18.5 | 0.5           |
| 5      | 22 | 21.4 | 0.6           |

$$\Sigma |D_i - F_i| = 2.2$$

# 11. Ans: (d)

Sol:

 $m_1$  = moving average periods give forecast  $F_1(t)$ 

 $m_2$  = moving average periods give forecast  $F_2(t)$ 

$$m_1 > m_2$$

F<sub>1</sub>(t) is a stable forecast has less variability.F<sub>2</sub>(t) is a sensitive (inflationary) forecast and has high variability.

# 12. Ans: (d)

**Sol:** Following are the purposes of long term forecasting:

- To plan for the new unit of production.
- To plan for the long-term financial requirement.

- To make the proper arrangement for training the personal.
- Budgetary allegations are not done in the beginning of a project. So, deciding the purchase program is not the purpose of long term forecasting.

## 13. Ans: (d)

Sol:

- Time horizon is less for a new product and keeps increasing as the product ages. So, statement (I) is correct.
- Judgemental techniques apply statistical method like random sampling to a small population and extrapolate it on a larger scale. So, statement (II) is correct.
- Low values of smoothing constant result in stable forecast. So statement (3) is correct.

Sol:

(i) 
$$F_7 = \frac{60 + 50 + 40}{3} = 50$$

(ii) 
$$F_7 = \frac{60 \times 0.5 + 50 \times 0.25 + 40 \times 0.25}{0.5 + 0.25 + 0.25} = 52.5$$

(iii) 2 period moving average = 
$$\frac{60+50}{2}$$
 = 55

4 period moving average

$$=\frac{60+50+40+20}{4}=42.5$$

5 period moving average

$$=\frac{60+50+40+20+30}{5}=40$$



# 15. Ans: (114.8 units, 9 periods)

**Sol:** At  $\alpha = 0.2$ 

$$F_{\text{may}} = 100 + 0.2 (200 - 100) = 120$$

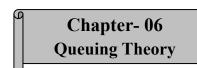
$$F_{june} = 120 + 0.2 (50 - 120) = 106$$

$$F_{\text{july}} = 106 + 0.2 (150 - 106) = 114.8$$

| Time  | Demand | Forecast |
|-------|--------|----------|
| April | 200    | 100      |
| May   | 50     | 120      |
| June  | 150    | 106      |
| July  | -      | 114.8    |

$$\alpha = \frac{2}{n+1}$$

$$n+1=\frac{2}{\alpha} \Rightarrow n=\frac{2}{0.2}-1=9 \text{ period}$$



01. Ans: (a)

**Sol:**  $\lambda = 3$  per day

$$\mu = 6$$
 per day

$$W_{q} = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{3}{6(6-3)} = \frac{1}{6} day$$

02. Ans: (c)

**Sol:**  $\lambda = 0.35 \text{ min}^{-1}$ ,

$$\mu = 0.5 \text{ min}^{-1}$$

$$P_{n} = \left[1 - \frac{\lambda}{\mu}\right] \left[\frac{\lambda}{\mu}\right]^{n}$$
$$= \left[1 - \frac{0.35}{0.5}\right] \left[\frac{0.35}{0.5}\right]^{8} = 0.0173$$

03. Ans: (a)

**Sol:** 
$$\lambda = 10 \text{ hr}^{-1}$$
,

$$\mu = 15 \text{ hr}^{-1}$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{10^2}{15(15 - 10)} = 1.33$$

04. Ans: (b)

**Sol:** 
$$\lambda = 4 \text{ hr}^{-1}$$
,  $\mu = \frac{60}{12} = 5 \text{ hr}^{-1}$ 

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{4^2}{5(5 - 4)} = \frac{16}{5} = 3.2$$



05. Ans: (b)

**Sol:** 
$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\lambda^2}{\mu^2 \left(1 - \frac{\lambda}{\mu}\right)} = \frac{\rho^2}{\left(1 - \rho\right)}$$

**06.** Ans: (d)

Sol: 
$$\lambda = \frac{1}{4} = 0.25 \text{min}^{-1}$$

$$\mu = \frac{1}{3} = 0.33 \text{ min}^{1}$$

$$\rho = \frac{\lambda}{\mu} = \frac{0.25}{0.33} = 0.75$$

07. Ans: (b)

Sol: 
$$\lambda = \frac{1}{10} = 0.1 \,\text{min}^{-1}$$
  
 $\mu = \frac{1}{4} = 0.25 \,\text{min}^{-1}$   
System busy  $\Rightarrow (\rho) = \frac{\lambda}{\mu} = \frac{0.1}{0.25} = 0.4$ 

08. Ans: (c)

Sol: 
$$\lambda = 4 \text{hr}^{-1}, \mu = 6 \text{ hr}^{-1}$$

$$P(Q_S \ge 2) = \left(\frac{\lambda}{\mu}\right)^2$$

$$= \left(\frac{4}{6}\right)^2 = \frac{4}{9}$$

09. Ans: (c)

# Chapter- 07 **Sequencing & Scheduling**

01. Ans: (a) Sol: SPT rule

| Job | Process time (days) | <b>Completion time</b> |
|-----|---------------------|------------------------|
| 1   | 4                   | 4                      |
| 3   | 5                   | 9                      |
| 5   | 6                   | 15                     |
| 6   | 8                   | 23                     |
| 2   | 9                   | 32                     |
| 4   | 10                  | 42                     |
|     | $\Sigma C_i =$      | 125                    |

Average Flow Time = 
$$\frac{\sum C_i}{n}$$
  
=  $\frac{125}{6}$  = 20.83

02. Ans: (a)

Sol: According to SPT rule total inventory cost is minimum.

Ans: (d)

Since

**Sol:** EDD rule can minimize maximum lateness. The job sequence is R-P-Q-S

04. Ans: (d)

Sol: Johnson's rule:

Optimum job sequence III – I – IV – II Do the job 1<sup>st</sup> if the minimum time happens to be on the machine (M) and do it on the end if .it is on second machine (N). Select either in case of a tie.



05. Ans: (b)

Sol:

| Job | M  |    |     | N  |    |     | Idle |
|-----|----|----|-----|----|----|-----|------|
|     | In | PT | Out | In | PT | Out |      |
| III | 0  | 1  | 1   | 1  | 2  | 3   | -    |
| Ι   | 1  | 3  | 4   | 4  | 6  | 10  | 1    |
| IV  | 4  | 7  | 11  | 11 | 5  | 16  | 1    |
| II  | 11 | 5  | 16  | 16 | 2  | 18  | -    |

Total idle time on machine (N) = 3

06. Ans: (a)

Sol: Optimum sequence of jobs

2 3 1 4

07. Ans: (b)

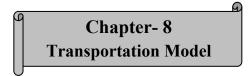
Sol: Optimum sequence is

| R | T | S | Q | U | P |
|---|---|---|---|---|---|

| Job |    | M <sub>1</sub> |     |     | M <sub>2</sub> |     |  |
|-----|----|----------------|-----|-----|----------------|-----|--|
|     | In | PT             | Out | In  | PT             | Out |  |
| R   | 0  | 8              | 8   | 8   | 13             | 215 |  |
| T   | 8  | 11             | 19  | 21  | 14             | 35  |  |
| S   | 19 | 27             | 46  | 46  | 20             | 66  |  |
| Q   | 46 | 32             | 78  | 78  | 19             | 97  |  |
| U   | 78 | 16             | 94  | 97  | 7              | 104 |  |
| P   | 94 | 15             | 109 | 109 | 6              | 115 |  |

The optimal make-span time = 115 days

08. Ans: (c)



01. Ans: (c)

**Sol:** A no. of allocations: m + n - 1

 $\Rightarrow$  5 + 3 -1 = 7

02. Ans: (a)

**Sol:** For degeneracy in transportations, number of allocations < (m + n) - 1

where m = no. of rows,

n = no. of columns

03. Ans: (b)

Sol: In Transportation problem for solving the initial feasible solution for total cost, Vogel's approximation methods are employed for obtaining solutions which are faster than LPP due to the reduced number of equations for solving.

Optimality is reached using MODI/ U-V method or stepping stone method.

04. Ans: (b)

**Sol:** It generates the best initial basic feasible solution. This method is the best choice in order to get an optimal solution within minimum number of iterations.

The Vogel's approximation method is also known as the penalty method.



05. Ans: (a)

**Sol:** No. of allocations = 5

 $\therefore$  no. of allocations = m + n - 1

$$m + n - 1 = 4 + 3 - 1$$

:. It is a degenerate solution

06. Ans: (a)

Sol:

|        | 1  | 2  | 3       | 4  | Supply   |
|--------|----|----|---------|----|----------|
| A      | 10 | 5  | 20      | 10 | 15       |
| В      | 12 | 10 | 9<br>15 | 20 | 25       |
| C      | 5  | 14 | 16      | 5  | 10       |
| Demand | 5  | 15 | 15      | 15 | 50<br>50 |

Evaluation of empty cells:

Cell (A1) Evaluation = 
$$C_{A1}$$
- $C_{A4}$ +  $C_{C4}$ - $C_{C1}$   
=  $10 - 11 + 18 - 5 = 12$ 

Cell (A3) Evaluation = 
$$C_{A3} - C_{A2} + C_{B2} - C_{B3}$$
  
=  $20 - 9 + 7 - 2 = 16$ 

Cell (B4) Evaluation = 
$$20 - 7 + 2 - 11 = 4$$

Cell (C2) Evaluation = 
$$14 - 2 + 11 - 18 = 5$$

Cell (C3) Evaluation = 
$$16-9+7-2-18=5$$

If cell cost evaluation value is '-ve', indicates further unit transportation cost is decreasing and if cost evaluation value is '+ve' indicates further unit transportation cost is increases. If cost evaluation value is zero, unit transportation cost doesn't change.

∴ As for A3 cell cost evaluation is +16, means that, if we transport goods to A3 the unit transportation cost is increased by 16/-.

Common Data for Questions Q07, Q08 & Q09:

07. Ans: (b) 08

08. Ans: (a)

09. Ans: (b)

Sol:

|   |   | 1  | 2  | 3        | 4  |
|---|---|----|----|----------|----|
|   |   | 6  | 1  | 9        | 3  |
|   | A |    |    | 25       | 45 |
| 1 | В | 30 | 5  | 25<br>25 | 8  |
|   | С | 55 | 35 | 4        | 7  |

No. of allocations = 6

$$R + C - 1 = 6$$

As No. of allocations = R + C - 1

Hence the problem is not degeneracy case.

Opportunity cost of cell (i, j) is

$$C_{ij} - (U_i + V_j)$$

If  $C_{ij} - (U_i + V_j) \ge 0 \Rightarrow$  problem is optimal, Empty cell evaluation (or) Opportunity cost of cells:

$$A_1 = -12$$
,  $A_2 = -19$ ,  $B_2 = -8$ 

$$B_4 = 12$$
,  $C_3 = 3$ ,  $C_4 = 12$ 

From the above as A2 has opportunity cost '-19' indicates unit transportation cost is decreased by 19/-

By forming loop A2, A3, B2, B3 it is observed that to transport minimum quantity is 25 among 25, 30, 35.



:. The reduction in the transportation cost is  $25 \times 19 = 475$ 

#### **10.** Ans: (c)

Sol:

|    |    | _ |    | + |    |
|----|----|---|----|---|----|
|    | 10 |   | 14 |   |    |
| 7  |    |   | 12 |   | 16 |
| _5 | 8  | + |    |   |    |

By stepping stone method,

Cell evaluation of B – 1 cell

$$= +7 - 5 + 8 - 10 + 14 - 12$$
  
= 2/-

|         |              |    |                  |    | A            |    |
|---------|--------------|----|------------------|----|--------------|----|
|         |              | 10 | ) – θ            | 20 | $\theta + 0$ |    |
|         | +θ           |    |                  |    |              | 35 |
| -       |              |    |                  |    | <u>5 – θ</u> | 33 |
| ا<br>20 | $\theta - 0$ | 10 | ) + <del>0</del> |    |              |    |

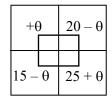
 $\theta = \text{minimum of } |10 - \theta, 5 - \theta, 20 - \theta| = 0$ 

 $\theta = 5$  units

Increase in cost =  $5 \times 2 = 10/$ 

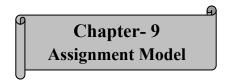
#### Ans: (c) 11.

**Sol:** To find the number units shifted to  $A_2$  cell.



 $\theta$  = minimum value of  $|15-\theta$ ,  $20-\theta| = 0$ 

$$\theta = 15$$
 units



01. Ans: (a)

**Sol:** Let  $C_{ij}$  = unit assignment cost

 $X_{ij}$  = Decision variable (allocation)

$$Minimize \ Z = \sum_{i=1}^{n} \ \sum_{J=1}^{n} C_{ij} X_{ij}$$

Subject to: 
$$\sum_{i=1}^{n} X_{ij} = 1$$

$$\sum_{i=1}^{n} X_{ij} = 1$$

 $X_{ij} = 1$  (when assigned)

 $X_{ij} = 0$  (when not assigned)

- Number of decision variables =  $n^2$  (or)  $m^2$
- Number of basic variables = Number of

assignments

IM & OR

$$= n (or) m$$

Ans: (c) 02.

03. Ans: (a)

04. Ans: (c)

Sol

| l: |   | $S_1$ | $S_2$ | $S_3$ |             | $S_1$    | $S_2$ | $S_3$ |             |
|----|---|-------|-------|-------|-------------|----------|-------|-------|-------------|
|    | P | 110   | 120   | 130   |             | <b>b</b> | 0     | 0     | Column      |
|    | Q | 115   | 140   | 140   |             | 0        | 15    | 5     | Transaction |
|    | R | 125   | 145   | 165   |             | 0        | 10    | 20    |             |
|    | P | 0     | 10    | 20    |             | -5-      | 0     | -0-   |             |
|    | Q | 0     | 25    | 25    | Row         | 0        | 10    | 0     |             |
|    | R | 0     | 20    | 40    | Transaction | -0-      | -5    | 15    |             |
|    |   |       |       |       |             |          |       |       |             |



$$P-S_2 - 120$$

$$Q-S_3 - 140$$

$$R-S_1 - 125$$

**Total = 385** 

# 05. Ans: (1-B, 2-D, 3-C, 4-A)

# Sol: Step-1:

Take the row minimum of subtract it from all elements of corresponding row.

| 1 | 0 | 2 | 3 |
|---|---|---|---|
| 0 | 2 | 2 | 1 |
| 8 | 5 | 0 | 1 |
| 0 | 6 | 2 | 4 |

# Step -2:

Take the column minimum & substract it from all elements of corresponding column.

|   |   | - |   |
|---|---|---|---|
| 1 | 0 | 2 | 2 |
| 0 | 2 | 2 | 0 |
| 8 | 5 | 0 | 0 |
| 0 | 6 | 2 | 3 |

# Step -3:

Select single zero row or column and assign at the all where zero exists. If there is no single zero row or column. Then use straight line method.

$$1 - B: 7$$
  
 $2 - D: 8$   
 $3 - C: 2$ 

$$4 - A : 5$$

Total cost = 22

06. Ans:  $(C_1-J_2, C_2-J_1, C_3-J_4, C_4-J_3)$ 

Sol:

|   | A  | В  | C  | D  |
|---|----|----|----|----|
| 1 | 10 | 5  | 15 | 13 |
| 2 | 3  | 9  | 8  | 18 |
| 3 | 10 | 7  | 2  | 3  |
| 4 | 5  | 11 | 7  | 9  |

**Step - 1:** 

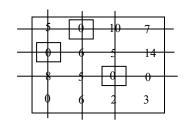
| 5 | 0 | 10 | 8  |
|---|---|----|----|
| 0 | 6 | 5  | 15 |
| 8 | 5 | 0  | 1  |
| 0 | 6 | 2  | 4  |
|   |   |    |    |

Step - 2:

**Since 1995** 

| 5 | 0 | 10 | 7  |
|---|---|----|----|
| 0 | 6 | 5  | 14 |
| 8 | 5 | 0  | 0  |
| 0 | 6 | 2  | 3  |

Step -3:



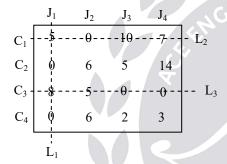


It may be noted there are no remaining zeroes and row – 4 and column – 4 each has no assignment. Thus optimal solution is not reached at this stage. Therefore, proceed to following important steps.

# Step -4:

Draw the minimum number of horizontal and vertical lines necessary to cover all zeroes at least once.

Take the above Table



- Mark row -4 in which there is no (i) assignment
- Mark column 1 which have zeroes in (ii) marked column.
- (iii) Next mark row 2 because this row contains assignment in marked column 1.

No further rows or columns will be required mark during this procedure.

- (iv) Draw the required lines as follows.
  - (a) Draw L<sub>1</sub> through marked column 1
  - (b) Draw L2 and L3 through unmarked row (1 and 3)

## Step - 5:

Select the smallest element (2).

Among all the uncovered elements of the above table and substract this value from all the elements of the matrix not covered by lines and add to every element that lie at the intersection of the lines  $L_1$ ,  $L_2$ , and  $L_3$  and leaving the remaining element unchange.

It may be added that there are no remaining zeroes and every row and column has an assignment.

Since, the no. of assignment = no. of row or column

The solution is optimal

The pattern of assignment at which job has been assigned to each contractor.

| Contractor | Job   | Amount (Rs)×1000         |
|------------|-------|--------------------------|
| $C_1$      | $J_2$ | 5                        |
| $C_2$      | $J_1$ | 3                        |
| $C_3$      | $J_4$ | 3                        |
| $C_4$      | $J_3$ | 7                        |
|            |       | $18 \times 1000 = 18000$ |

Minimum amount = Rs. 18,000/-



# 07. Ans: (A-J<sub>1</sub>, B-J<sub>2</sub>, C-J<sub>4</sub>, D-J<sub>3</sub>, TC=107)

Sol:

|   | Job              | Job                 | Job | Job |             |  |  |  |
|---|------------------|---------------------|-----|-----|-------------|--|--|--|
|   | 1                | 2                   | 3   | 4   |             |  |  |  |
| A | 20               | 36                  | 31  | 27  |             |  |  |  |
| В | 24               | 34                  | 45  | 22  |             |  |  |  |
|   | 22               | 45                  | 38  | 18  |             |  |  |  |
| D | 37               | 40                  | 35  | 28  |             |  |  |  |
|   |                  |                     |     |     |             |  |  |  |
| A | 0                | 16                  | 11  | 7   | Row         |  |  |  |
| В | 2                | 12                  | 23  | 0   | Transaction |  |  |  |
| С | 4                | 27                  | 20  | 0   |             |  |  |  |
| D | 9                | 12                  | 7   | 0   |             |  |  |  |
|   |                  |                     |     |     |             |  |  |  |
| A | 0                | 4                   | 4   | 7   | Column      |  |  |  |
| В | 2                | 0                   | 16  | 0   | Transaction |  |  |  |
| C | 4                | 15                  | 1 3 | 0   |             |  |  |  |
| D | 9                | 0                   | 0   | 0   |             |  |  |  |
|   |                  |                     |     |     |             |  |  |  |
|   | A – .            | $J_1 \rightarrow 2$ |     |     |             |  |  |  |
|   | B                | $J_2 \rightarrow 3$ |     |     |             |  |  |  |
|   | $\mathbf{C}$ – . | $J_4 \rightarrow 1$ |     |     |             |  |  |  |
|   |                  | $J_3 \rightarrow 3$ |     |     |             |  |  |  |
|   |                  | 10                  |     |     |             |  |  |  |

08. Ans: (1-A, 2-C, 3-B, 4-Dummy, TC=35)

**Sol:** Here no. of rows  $\neq$  no. of column

:. The algorithm is not balanced so add one dummy column.

| Operates | Machine |    |    |       |  |  |
|----------|---------|----|----|-------|--|--|
| Operates | A       | В  | C  | Dummy |  |  |
| 1        | 9       | 26 | 15 | 0     |  |  |
| 2        | 13      | 27 | 6  | 0     |  |  |
| 3        | 35      | 20 | 15 | 0     |  |  |
| 4        | 18      | 30 | 20 | 0     |  |  |

| 9  | 26 | 15 | 0 |
|----|----|----|---|
| 13 | 27 | 6  | 0 |
| 35 | 20 | 15 | 0 |
| 18 | 30 | 20 | 0 |

# **Step – 2:**

Here the operator – 4 is assigned to dummy column.

:. He is the idle worker.

Since



# **Chapter-10** PPC & Aggregate Planning

01. Ans: (d) 02. Ans: (b)

03. Ans: (b)

Sol:

| M | Ionths | Month 1 | Month 2  | Month 3 | Unused capacity | Capacity Available |
|---|--------|---------|----------|---------|-----------------|--------------------|
|   | RT     | 90      | 10       | 24      | 10              | 100                |
| 1 | OT     | 24      |          |         |                 | 20                 |
|   | RT     |         | 100      | 22      | 9/              | 100                |
| 2 | ОТ     |         | 20       | 26      |                 | 20                 |
|   | RT     |         | Since 19 | 80      |                 | 80                 |
| 3 | OT     |         |          | 30 24   | 10              | 40                 |
|   | RT     |         |          |         | ,               |                    |
|   | OT     | 90      | 130      | 110     |                 |                    |

Level of planned production in overtimes in 3<sup>rd</sup> period is '30'.

RT = Regular time

OT = Over time



04. Ans: (b)

Sol:

|       | Cumulative | Cumulative | Ir  | iventory  | Cost             |                   |
|-------|------------|------------|-----|-----------|------------------|-------------------|
| Month | Production | Demand     | End | Stock out | End<br>inventory | Stock<br>out cost |
| 1     | 100        | 80         | 20  | -         | 40               | -                 |
| 2     | 180        | 180        | -   | -         | -                | -                 |
| 3     | 250        | 260        | -   | 10        | -                | 100               |
| 4     | 320        | 300        | 20  | -         | 40               | -                 |
|       |            |            |     |           | 80               | 100               |
|       |            | NEER       | INC | Total     | 180              | )                 |

05. Ans: (b) 06. Ans: (d)

07. Ans:

Sol:

|   |                    |                   |          | Total Capacity    |                   |                  |                    |
|---|--------------------|-------------------|----------|-------------------|-------------------|------------------|--------------------|
|   | Supply from        | Period 1          | Period 2 | Period 3          | Period 4          | Un used capacity | Available (supply) |
| В | eginning inventory | 200 0             | 5        | 10                | 15                | 7 7/             | 200                |
| 1 | Regular            | <sup>700</sup> 60 | 65       | 70                | 75                | 0                | 700                |
| 1 | Overtime           | 70                | 75       | ce 8099           | 5 85              | 300              | 300                |
| 2 | Regular            |                   | 500 60   | 65                | <sup>200</sup> 70 | 0                | 700                |
|   | Overtime           | <i>F</i>          | 70       | 75                | 80                | 300              | 300                |
| 3 | Regular            |                   |          | <sup>200</sup> 60 | 500 65            | 0                | 700                |
|   | Overtime           |                   |          | 70                | <sup>200</sup> 75 | 100              | 300                |
| 4 | Regular            |                   |          |                   | <sup>700</sup> 60 | 0                | 700                |
| • | Overtime           |                   |          |                   | <sup>300</sup> 70 | 0                | 300                |
|   |                    | 900               | 500      | 200               | 1900              | 700              | 4200               |

Total cost = 
$$(700 \times 60) + (500 \times 60) + (200 \times 70) + (200 \times 60) + (500 \times 65) + (200 \times 75) + (700 \times 60) + (300 \times 70) = \text{Rs } 2,08,500/-$$





# 08. Ans:

Sol:

| De | emand for Total |                   |         |                   |                   |          | Capacity  |
|----|-----------------|-------------------|---------|-------------------|-------------------|----------|-----------|
|    | ipply from      | Period1           | Period2 | Period3           | Period4           | Unused   | Available |
|    |                 | 1=2               |         |                   |                   | capacity | (supply)  |
| Ве | eginning        | <sup>150</sup> 0  | 2       | 4                 | 6                 | -        | 150       |
|    | Inventory       |                   |         |                   |                   |          |           |
| 1  | Regular         | 900 25            | 27      | 29                | 31                | -        | 900       |
|    | Overtime        | <sup>150</sup> 30 | 32      | 34                | 36                | -        | 150       |
|    |                 |                   |         | DIN               | 91                |          |           |
|    | Subcontract     | <sup>200</sup> 35 | CINEL   | KING              |                   | 100 _    | 300       |
|    |                 | T (V)             |         | $\Lambda$         | AO                |          |           |
| 2  | Regular         | 1/2               | 600 25  | 27                | 29                | -        | 600       |
|    | Overtime        |                   | 125 30  | 32                | 34                | - 60     | 125       |
|    |                 |                   |         |                   |                   |          |           |
|    | Subcontract     |                   | 175 35  | -                 |                   | 125      | 300       |
|    |                 |                   |         |                   |                   |          |           |
| 3  | Regular         |                   |         | <sup>700</sup> 25 | 27                | 1        | 700       |
|    | Overtime        |                   |         | 100 30            | 50 32             | 7/       | 150       |
|    |                 |                   | Cina    | - 4005            |                   |          |           |
|    | Subcontract     |                   | Sinc    | 35                | -                 | 300 _    | 300       |
|    |                 |                   |         |                   |                   |          |           |
| 4  | Regular         |                   |         |                   | 800 25            | -        | 800       |
|    | Overtime        |                   |         |                   | <sup>200</sup> 30 | -        | 200       |
|    | Subcontract     |                   |         |                   | <sup>250</sup> 35 | 50 _     | 300       |
|    | Subcontract     | 1400              | 000     | 900               | 1200+100          | _        |           |
|    |                 | 1400              | 900     | 800               | 1200+100          | 575      | 4975      |
|    |                 |                   |         |                   |                   |          | 4975      |

Total cost =  $(900 \times 25) + (150 \times 30) + (200 \times 35) + (600 \times 25) + (125 \times 30) + (175 \times 35) + (700 \times 25) + (100 \times 30) + (50 \times 32) + (800 \times 25) + (200 \times 30) + (250 \times 35) = \text{Rs } 1,15,725/-$ 

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# Chapter-11 Material Requirement & Planning

01. Ans: (b)

# 02. Ans: (c)

Sol: Based on master production schedule, a material requirements planning system:

- Creates schedules, identifying specific parts and materials required to produce end items.
- Determines exact unit numbers needed.
- Determines the dates when orders for those materials should be released, based on lead times.

03. Ans: (d)

**Sol:** Refer to the solution of Q.No. 02

## 04. Ans: (c)

**Sol:** MRP has three major input components:

- 1. Master production Schedule of end items required. It dictates gross or projected requirements for end items to the MRP system.
- 2. Inventory status file of on-hand and onorder items, lot sizes, lead times etc.
- 3. Bill of materials (BOM) or Product structure file what components and sub assemblies go into each end product.

05. Ans: (c)

06. Ans: (c)

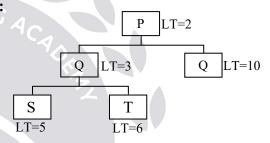
07. Ans: (b)

08. Ans: (b)

09. Ans: (c)

Sol:

Since



Maximum Lead time = 12 weeks



# Chapter- 12 **Break Even Analysis**

# 01. Ans: (c)

**Sol:** Total fixed cost, TFC = Rs 5000/-

Sales price, SP = Rs 30/-

Variable cost, VC = Rs 20/-

Break even production per month,

$$Q^* = {TFC \over SP - VC} = {5000 \over 30 - 20} = 500 \text{ units}$$

## 02. Ans: (a)

**Sol:** Total cost = 20 + 3X ----(1)

Total cost = 50 + X ----(2)

By solving equ. (1) and (2)

$$2X = 30$$

$$\therefore$$
 X = 15 units

When X = 10 units

$$TC_1 = 20 + (3 \times 10) = Rs 50/-$$

$$TC_2 = 50 + (1 \times 10) = Rs 60/-$$

Among both, total cost for process is less So process-1 is choose.

## 03. Ans: (c)

**Sol:** In automated assembly there are less labour, so variable cost is less, but fixed is more because machine usage is more. In job shop production, labour is more but machine is less. So variable cost is more and fixed cost is less.

**Sol:** TC = Total cost

 $TC_A = Total cost for jig-A$ 

 $TC_B = Total \text{ for jig-B}$ 

$$TC_A = TC_B$$

$$800 + 0.1X = 1200 + 0.08X$$

$$0.02X = 400$$

$$\therefore X = \frac{400}{0.02} = \frac{400}{2} \times 100 = 20,000 \text{ units}$$

05. Ans: (d)

**Sol:** Sales price – Total cost = Profit

 $(C_P \times 14000) - (47000 + 14000 \times 15) = 23000$ 

$$\therefore C_P = 20$$

06. Ans: (b)

Ans: (a) **07.** 

08. Ans: (c)

09. Ans: 1500

Since

Sol: 5 X

 $S_2 = 120$  $S_1 = 100$ 

 $F_1 = 20,000$  $F_2 = 8000$ 

 $V_1 = 12$  $V_2 = 40$ 

$$P = q(S - V) - F$$

 $P_1 = q(100 - 12) - 20,000$ 

 $P_2 = q(120 - 40) - 80,000$ 

 $P_1 = P_2$ 

88q - 20,000 = 80q - 80,000

12000 = 8q

 $\Rightarrow$  q = 1500



**10.** Ans: (b)

11. Ans: (c)

**Sol:** At breakeven point

Total cost = Total revenue

$$FC + VC \times Q = SP \times Q$$

$$Q = \frac{FC}{(SP - VC)}$$

$$FC = 1000/-$$

$$VC = 3/-$$

$$SP = 4/-$$

$$Q = \frac{1000}{(4-3)} = 1000 \text{ units}$$

If sales price is increased to 25%

$$SP = 4 + \frac{1}{4} \times 4 = 5/-$$

$$Q^* = \frac{1000}{(5-3)} = 500 \text{ units}$$

:. Breakeven quantity decreases by

$$\frac{100 - 500}{100} \times 100 = 50\%$$

**Ans: 16** 12.

**Sol:** Preparation cost for

Conventional lathe = 30,

CNC lathe = 150

Production time of

Conventional lathe = 30 min,

Variable cost per hour

Conventional lathe = 75 per hour

$$=\frac{75}{60}\times30$$
 per product

CNC lathe = 120 per hour

$$= \frac{120}{60} \times 15 \text{ per product}$$

Total cost for Q products

Conventional lathe = 30 + 37.5 Q

 $CNC \ lathe = 150 + 30 \ Q$ 

At break even quantities

$$(TC)_1 = (TC)_2$$

$$\Rightarrow$$
 30 + 37.5 Q = 150 + 30 Q

$$\Rightarrow$$
 7.5 Q = 120

$$\Rightarrow$$
 Q = 16

: CNC lathe is economical when production per day is above 16.

13. Ans: (d)

Sol:

Since

|              | Standard                                   | Automatic                         |  |  |
|--------------|--|-----------------------------------|--|--|
|              | machine tool                               | machine tool                      |  |  |
| $F_1 = F.C.$ | $\frac{30}{60} \times 200 = \text{Rs.}100$ | $2 \times 800 =$ Rs. $1600 = F_2$ |  |  |
| V.C          | $=\frac{20}{60}\times200$                  | $=\frac{5}{60}\times800$          |  |  |
|              | = Rs. 73.33                                | = Rs. 66.67                       |  |  |

$$q = \frac{1600 - 100}{73.33 - 66.67} = 225 \text{ volts}$$

If greater than 225 units then automatic machine tool is economic.