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ENGINEERING MECHANICS

Volume-1 : Study Material with Classroom Practice Questions

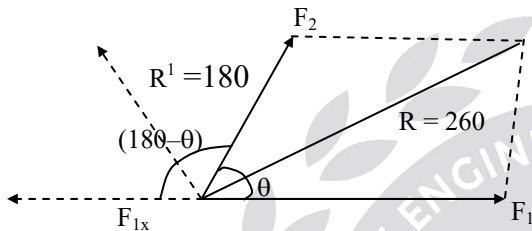
Engineering Mechanics

Solutions for Vol - I_ Classroom Practice Questions

Chapter- 1 Force and Moment Systems

01. Ans: (b)

Sol:



Assume $F_1 = 2F_2$ ($F_1 > F_2$)

$$F_{1x} = 2F_2$$

$$R = \sqrt{F_1^2 + F_2^2 + 4F_2^2 \cos \theta}$$

$$260 = \sqrt{4F_2^2 + F_2^2 + 4F_2^2 \cos \theta}$$

$$260^2 = 5F_2^2 + 4F_2^2 \cos \theta \quad \text{----- (1)}$$

$$R^1 = \sqrt{F_{1x}^2 + F_2^2 + 2F_{1x}F_2 \cos \theta}$$

$$180 = \sqrt{4F_2^2 + F_2^2 + 2 \cdot F_2 \cdot F_2 \cos(180 - \theta)}$$

$$180^2 = 5F_2^2 - 4F_2^2 \cos \theta \quad \text{----- (2)}$$

$$260^2 = 5F_2^2 + 4F_2^2 \cos \theta$$

$$180^2 = 5F_2^2 - 4F_2^2 \cos \theta$$

$$260^2 + 180^2 = 10F_2^2$$

$$\Rightarrow F_2 = 100\text{N},$$

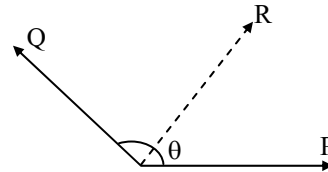
$$260^2 = 5(100)^2 + 4(100)^2 \cos \theta$$

$$\Rightarrow \theta = 63.89$$

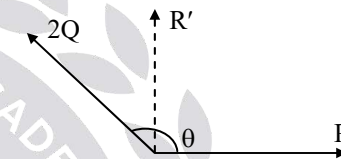
Where θ angle between two forces.

02. Ans: (b)

Sol: Let the angle between the forces be θ



Where, R is the resultant of the two forces.



If Q is doubled i.e., 2Q then resultant (R') is perpendicular to P.

$$\tan 90 = \frac{2Q \sin \theta}{P + 2Q \cos \theta}$$

$$\Rightarrow P + 2Q \cos \theta = 0$$

$$P = -2Q \cos \theta \quad \text{-----(i)}$$

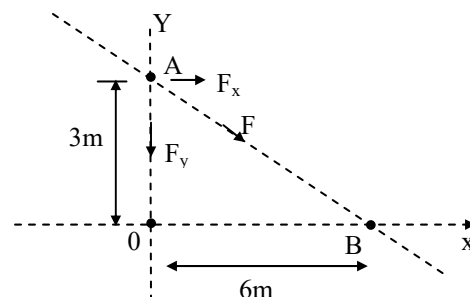
$$\text{Also, } R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$R = Q \text{ [using eq.(i)]}$$

03. Ans: (b)

Sol: Since moment of F about point A is zero.

\therefore F passes through point A,





$$M_0^F = 180\text{N} - m$$

$$M_B^F = 90\text{N} - m$$

$$M_A^F = 0$$

$$M_0^F = 180 = F_x \times 3 + F_y \times 0$$

$$F_x = 60\text{N} \dots\dots (1)$$

$$M_B^F = F_x \times 3 - F_y \times 6 = -90$$

$$60 \times 3 - 6F_y = -90$$

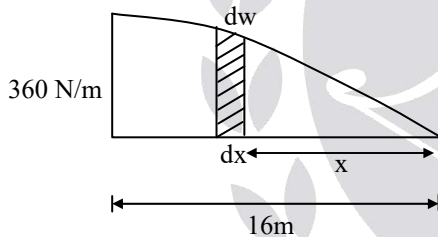
$$\Rightarrow F_y = \frac{270}{6}$$

$$F_y = 45\text{N}$$

$$\therefore F = \sqrt{F_x^2 + F_y^2} = \sqrt{60^2 + 45^2} = 75$$

04. Ans: (a)

Sol:



$$\int_0^w dw = \int_0^{16} w dx$$

$$w = \int_0^{16} 90\sqrt{x} dx = 90 \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_0^{16}$$

$$= 90 \times \frac{2}{3} \left[x^{3/2} \right]_0^{16} = 60 (16)^{3/2}$$

$$w = 3840\text{N}$$

The moment due to average force should be equal to the variable force

$$R \times d = \Sigma dw \times x$$

$$3840 \times d = \int_0^{16} 90\sqrt{x} \cdot dx \cdot x$$

$$= 90 \int_0^{16} x^{1.5} dx$$

$$3840d = 90 \left[\frac{x^{2.5}}{2.5} \right]_0^{16}$$

$$\Rightarrow d = 9.6\text{m}$$

05. Ans: (c)

Sol: Moment about 'O'

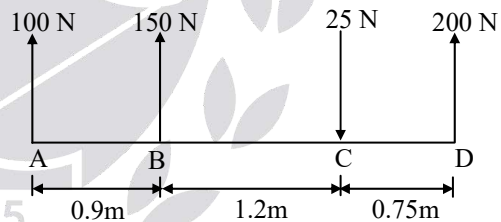
$$M_0 = 100 \sin 60 \times 3$$

$$= 300 \times \frac{\sqrt{3}}{2} = 150\sqrt{3}$$

$$= 259.8 \approx 260\text{N}$$

06. Ans: (a)

Sol:



$$F_R = \Sigma F_y$$

$$F_R = 100 + 150 - 25 + 200 \text{ (upward force)}$$

Positive downward force negative)

$$R = 425\text{N}$$

For equilibrium

$$\Sigma M_A = 0 \text{ (since } R = \text{resultant)}$$

Let R is acting at a distance of 'd'

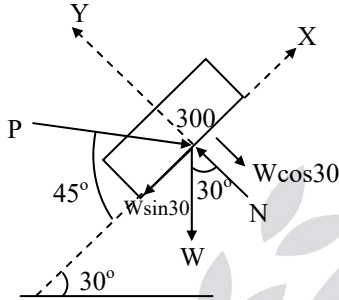
$$425 \times d = 150 \times 0.9 + 25 \times 2.1 - 200 \times 2.85$$

$$\Rightarrow d = 1.535\text{m (from A)}$$

Chapter- 2
Equilibrium of Force Systems

01. Ans: (d)

Sol:



Resolve the forces along the inclined surface

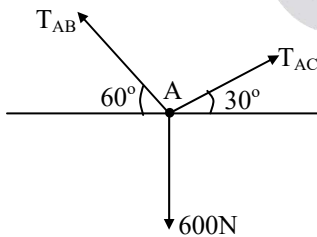
$$\sum F_x = 0$$

$$P \cos 45 - W \sin 30 = 0$$

$$P = \frac{300 \sin 30}{\cos 45} \Rightarrow P = 212.13 \text{ N}$$

02. Ans: (a)

Sol:



$$T_{AB} \cos 60^\circ = T_{AC} \cos 30^\circ$$

$$T_{AB} = \sqrt{3} T_{AC}$$

$$T_{AB} \sin 60^\circ + T_{AC} \sin 30^\circ = 600 \text{ N}$$

$$\frac{3}{2} T_{AC} + \frac{1}{2} T_{AC} = 600$$

$$\Rightarrow T_{AB} = 520 \text{ N}; \quad T_{AC} = 300 \text{ N}$$

03. Ans: (c)

Sol:

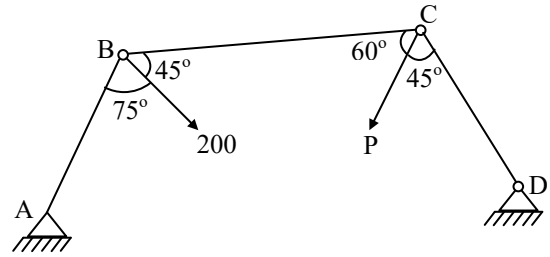


Fig: Free body diagram at 'B'

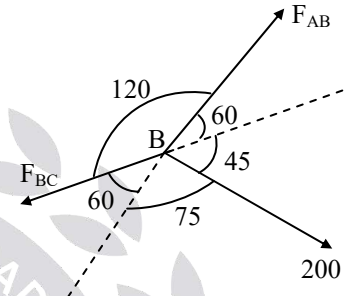
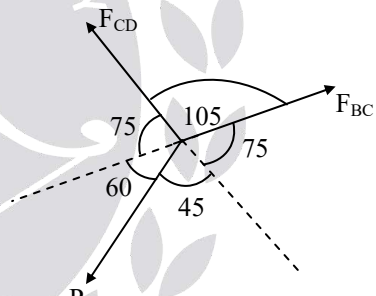


Fig: Free body diagram at 'C'



For Equilibrium of Point 'B'

$$\frac{F_{AB}}{\sin(60 + 75)} = \frac{F_{BC}}{\sin(60 + 45)} = \frac{200}{\sin(120)}$$

$$F_{BC} = 223.07 \text{ N}$$

From Sine rule at "C".

$$\frac{F_{CD}}{\sin(75 + 45)} = \frac{F_{BC}}{\sin(60 + 75)} = \frac{P}{\sin 105}$$

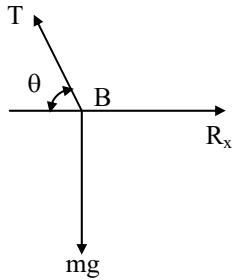
$$P = \frac{223.07 \times \sin 105}{\sin 135}$$

$$P = 304.71 \text{ N}$$



04. Ans: (d)

Sol:



$$\tan\theta = \frac{125}{275} \Rightarrow \theta = 24.45^\circ$$

$$T \sin\theta = mg.$$

$$T \sin 24.45 = (35 \times 9.81)$$

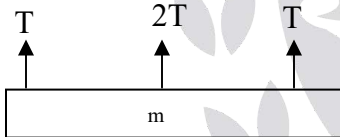
$$T = 829.5 \text{ N}$$

$$R_x = T \cos 24.45 = 755.4 \text{ N}$$

$$R_y = 0$$

05. Ans: (c)

Sol:



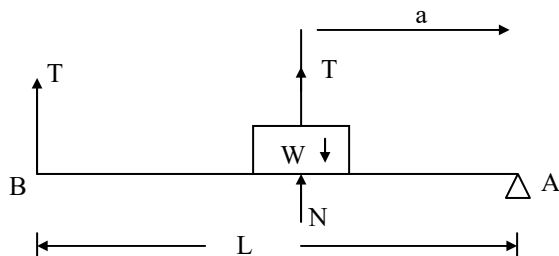
$$T + 2T + T = mg$$

$$4T = mg$$

$$m = 4T/g$$

06. Ans: (b)

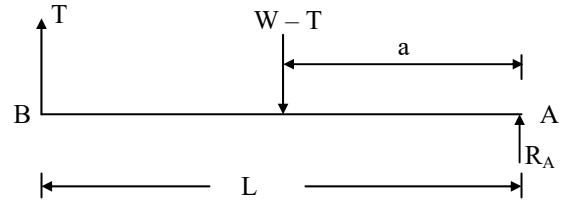
Sol:



For body, $\sum F_y = 0$

$$N - W + T = 0$$

$$\Rightarrow N = W - T$$



$\sum F_y = 0$ for entire system

$$R_A + T - (W - T) = 0$$

$$R_A = W - 2T \quad \text{----- (1)}$$

For equilibrium

$$\sum M_A = 0$$

$$T \times L = (W - T) a$$

$$TL = Wa - Ta$$

$$TL + Ta = Wa$$

$$T(L + a) = Wa$$

$$\Rightarrow T = \frac{Wa}{L + a}$$

T substitute in equation (1)

$$R_A = W - 2 \left(\frac{Wa}{L + a} \right)$$

$$= \frac{W(L + a) - 2Wa}{L + a}$$

$$= \frac{WL + Wa - 2Wa}{L + a}$$

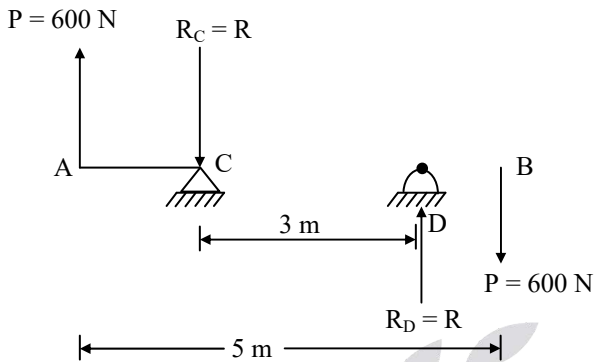
$$= \frac{WL - Wa}{L + a}$$

$$R_A = \frac{W(L - a)}{L + a}$$



07. Ans: (c)

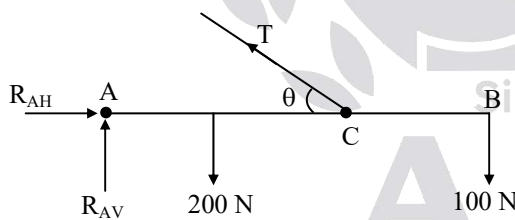
Sol:



$$\begin{aligned} \sum F_y = 0 \\ 600 - R_C + R_D - 600 = 0 \\ \Rightarrow R_C = R_D = R \\ \sum M = 0 \\ 600 \times 5 = R \times 3 \\ \Rightarrow R = 1000 \text{ N} = R_C = R_D \end{aligned}$$

08. Ans: (a)

Sol: F.B.D



$$\begin{aligned} \sum M_A = 0 \\ \tan \theta = \frac{8}{4} \\ \theta = 63.43 \\ T \sin \theta \times 4 (\curvearrowright) - 200 \times 2 (\curvearrowright) - 100 \times 6 (\curvearrowright) = 0 \\ \Rightarrow T = 279.5 \text{ N} \\ \text{Now, } \sum F_x = 0, \end{aligned}$$

$$R_{AH} - T \cos \theta = 0$$

$$R_{AH} = 125 \text{ N}$$

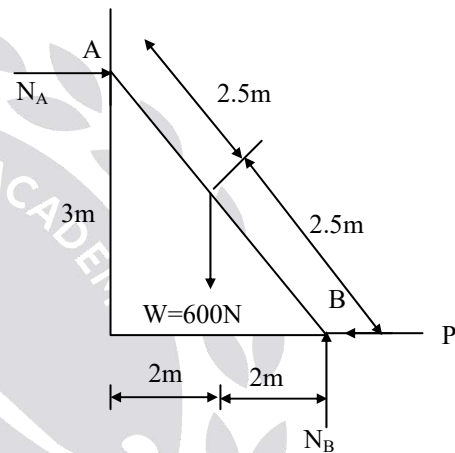
$$\sum F_y = 0$$

$$R_{AV} - 200 - 100 + T \sin \theta = 0$$

$$\Rightarrow R_{VA} = 50 \text{ N}$$

09. Ans: 400 N

Sol:



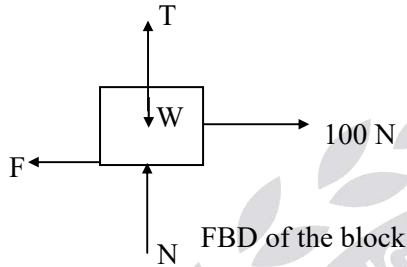
$$\begin{aligned} \sum F_y = 0 \\ N_B - W = 0 \\ N_B = 600 \text{ N} \\ \sum M_A = 0 \\ P \times 3 + W \times 2 - N_B \times 4 = 0 \\ P = \frac{4N_B - 2W}{3} \\ P = \frac{4 \times 600 - 2 \times 600}{3} = 400 \text{ N} \end{aligned}$$



Chapter- 3
Friction

01. Ans: (c)

Sol: The FBD of the above block shown



$$\Sigma Y = 0 \Rightarrow N + T - W = 0$$

$$N = W - T = 981 - T$$

$$F = \mu N = 0.2(981 - T)$$

$$\Sigma X = 0 \Rightarrow 100 - F = 0$$

$$F = 100 = 0.2(981 - T)$$

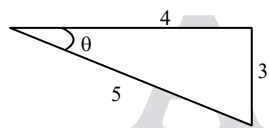
$$\Rightarrow T = 481 \text{ N}$$

02. Ans: (c)

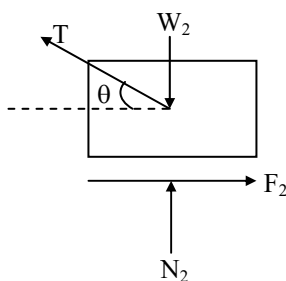
Sol: Given $\tan \theta = \frac{3}{4}$

$$\sin \theta = \frac{3}{5}$$

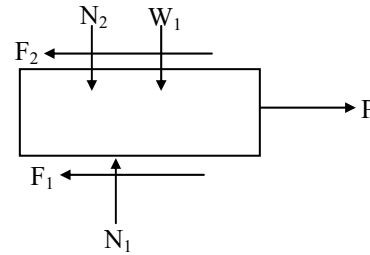
$$\cos \theta = \frac{4}{5}$$



Free body diagram for block (2)



Free body diagram for block (1)



From FBD of block (2)

$$\Sigma F_x = 0$$

$$F_2 = T \cos \theta$$

$$F_2 = \frac{4}{5} T = 0.8T \text{ ----- (1)}$$

$$\Sigma F_y = 0$$

$$N_2 + T \sin \theta - W_2 = 0$$

$$N_2 = W_2 - T \sin \theta$$

$$N_2 = 50 - 0.6T$$

$$\text{But } F_2 = \mu N_2$$

$$\Rightarrow F_2 = 0.3(50 - 0.6T)$$

$$F_2 = 15 - 0.18T \text{ ----- (2)}$$

From (1) & (2)

$$0.8T = 15 - 0.18T$$

$$\Rightarrow 0.98T = 15$$

$$\Rightarrow T = 15.31 \text{ N}$$

$$\therefore N_2 = 50 - 0.6T$$

$$= 50 - 0.6(15.31) = 40.81 \text{ N}$$

$$F_2 = \mu N_2 = 0.3 \times 40.81 = 12.24 \text{ N}$$

From FBD of block (1)

$$\Sigma F_y = 0$$

$$N_1 - N_2 - W_1 = 0$$

$$N_1 = N_2 + W_1 = 40.81 + 200 = 240.81 \text{ N}$$

$$F_1 = \mu N_1 \Rightarrow F_1 = 0.3 \times 240.81$$

$$F_1 = 72.24 \text{ N}$$



$$\Sigma F_x = 0$$

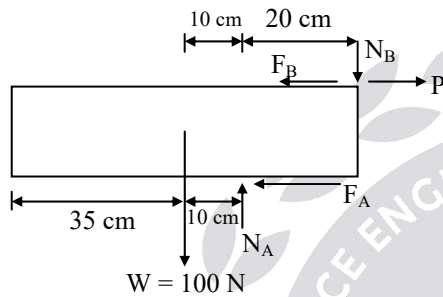
$$P - F_1 - F_2 = 0$$

$$P = F_1 + F_2 = 72.24 + 12.24$$

$$P = 84.48 \text{ N}$$

03. Ans: (b)

Sol: Free Body Diagram



$$F_A = \mu N_A = \frac{1}{3} N_A$$

$$F_B = \mu N_B = \frac{1}{3} N_B$$

$$\Sigma M_B = 0$$

$$-100 \times 30 (\cup) + (N_A \times 20) (\cup) + (F_A \times 12) (\cup) = 0$$

$$-3000 + N_A \times 20 + \frac{1}{3} N_A \times 12 = 0$$

$$\Rightarrow N_A = 125 \text{ N}$$

$$\Sigma F_y = 0$$

$$N_A - N_B - 100 = 0$$

$$\Rightarrow N_B = 25 \text{ N}$$

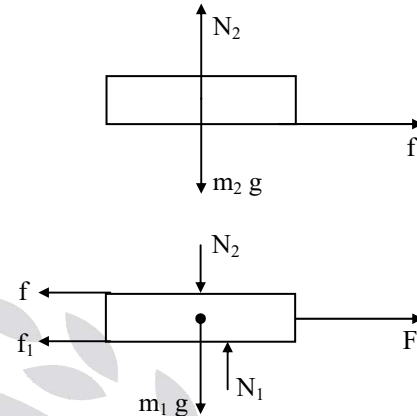
$$\Sigma F_x = 0$$

$$P = F_A + F_B = \frac{1}{3} (N_A + N_B)$$

$$= \frac{1}{3} (125 + 25) = 50 \text{ N}$$

04. Ans: (d)

Sol: F.B.D of both the books are shown below.



where, f is the friction between the two books.

f_1 is the friction between the lower book and ground.

Now, maximum possible acceleration of upper book.

$$a_{\max} = \frac{f_{\max}}{m_2} = \frac{\mu m_2 g}{m_2} = \mu \times g$$

$$= 0.3 \times 9.81 = 2.943 \text{ m/s}^2$$

For slip to occur, acceleration (a_1) of lower book. i.e,

$$a_1 \geq a_{\max}$$

$$\frac{F - f - f_1}{m_1} \geq 2.943$$

$$F - 2.943 - 0.3 \times 2 \times 9.81 \geq 2.943$$

$$[\because f = f_{\max} = 2.943 \text{ and}$$

$$f_1 = \mu \times (m_1 + m_2) g = 0.3 \times 2 \times 9.81]$$

$$F \geq 11.77 \text{ N}$$

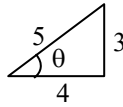
$$F_{\min} = 11.77 \text{ N}$$



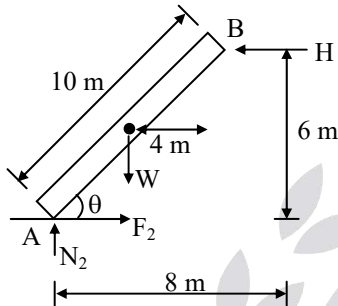
05. Ans: (d)

Sol: $\tan\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{3}{5}$

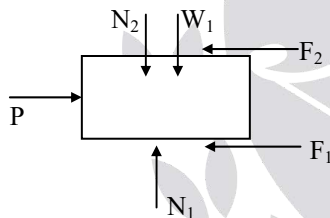
$\cos\theta = \frac{4}{5}$



FBD for bar AB (2)



FBD for block (1)



Given $W = 280 \text{ N}$, $W_1 = 400 \text{ N}$

Now, $\Sigma M_B = 0$

$-W \times 4 (\cup) + N_2 \times 8 (\cup) - F_2 \times 6 (\cup) = 0$

$-280 \times 4 + N_2 \times 8 - \mu N_2 \times 6 = 0$

$\Rightarrow N_2 = 200 \text{ N}$

But, $F_2 = \mu N_2 = 0.4 \times 200 = 80 \text{ N}$

From FBD of block (1)

$\Sigma F_y = 0$

$N_1 - N_2 - W_1 = 0$

$N_1 = N_2 + W_1$

$= 200 + 400$

$N_1 = 600 \text{ N}$

But, $F_1 = \mu N_1 = 0.4 \times 600$

$F_1 = 240 \text{ N}$

$\Sigma F_x = 0$

$P = F_1 + F_2 = 240 + 80$

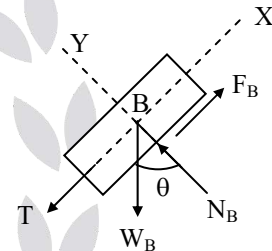
$P = 320 \text{ N}$

06. Ans: (a)

Sol: Given, $W_A = 200 \text{ N}$, $\mu_A = 0.2$

$W_B = 300 \text{ N}$, $\mu_B = 0.5$

FBD for block 'B'.



$\Sigma F_y = 0$

$N_B = W_B \cos\theta$

$N_B = 300 \cos\theta$

But, $F_B = \mu N_B = 0.5 \times 300 \cos\theta$

$= 150 \cos\theta$

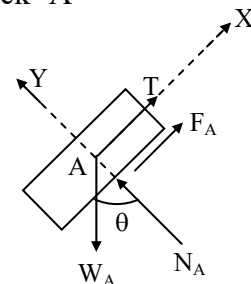
$\Sigma F_x = 0$

$T + W_B \sin\theta - F_B = 0$

$T = F_B - W_B \sin\theta$

$T = 150 \cos\theta - 300 \sin\theta \text{ ----- (1)}$

FBD for block 'A'





$$\Sigma F_y = 0$$

$$N_A - W_A \cos\theta = 0$$

$$N_A = 200 \cos\theta$$

$$F_A = \mu N_A = 0.2 \times 200 \cos\theta$$

$$\text{But, } F_A = 40 \cos\theta$$

$$\Sigma F_x = 0$$

$$T + F_A - W_A \sin\theta = 0$$

$$T = W_A \sin\theta - F_A$$

$$T = 200 \sin\theta - 40 \cos\theta$$

But from equation (1)

$$T = 150 \cos\theta - 300 \sin\theta$$

$$\therefore 150 \cos\theta - 300 \sin\theta = 200 \sin\theta - 40 \cos\theta$$

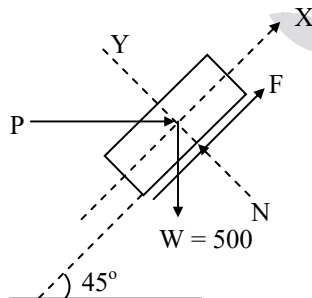
$$190 \cos\theta = 500 \sin\theta$$

$$\tan\theta = \frac{190}{500}$$

$$\Rightarrow \theta = 20.8^\circ$$

07. Ans: (d)

Sol: FBD for the block



$$\Sigma F_y = 0$$

$$N - W \sin 45 - P \sin 45 = 0$$

$$N = \frac{500}{\sqrt{2}} + \frac{P}{\sqrt{2}}$$

$$\text{But, } F = \mu N = 0.25 \left(\frac{500}{\sqrt{2}} + \frac{P}{\sqrt{2}} \right)$$

$$\Sigma F_x = 0$$

$$P \cos 45 + F - W \sin 45 = 0$$

$$P \cos 45 + 0.25 \left(\frac{500}{\sqrt{2}} + \frac{P}{\sqrt{2}} \right) - 500 \times \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow P = 300 \text{ N}$$

08. Ans: (a)

Sol: FBD of block

$$F_1 = \mu N_1$$

$$F_2 = \mu N_2$$

$$\Sigma F_x = 0$$

$$N_2 - F_1 = 0$$

$$\Rightarrow N_2 = F_1 \quad (\because F_1 = \mu N_1)$$

$$N_2 = \mu N_1$$

$$\Sigma F_y = 0$$

$$N_1 + F_2 - W = 0$$

$$N_1 + \mu N_2 - W = 0$$

$$N_1 + \mu^2 N_1 - W = 0 \quad (\because N_2 = \mu N_1)$$

$$N_1 (1 + \mu^2) = W$$

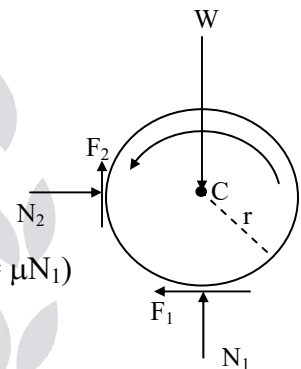
$$N_1 = \frac{W}{1 + \mu^2}$$

$$N_2 = \frac{\mu W}{1 + \mu^2}$$

$$\text{Couple} = (F_1 + F_2) \times r$$

$$= \mu r (N_1 + N_2)$$

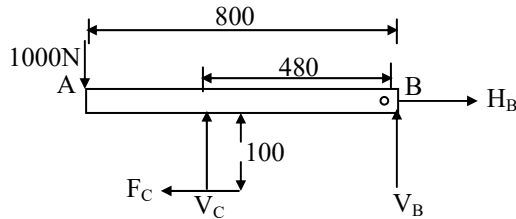
$$= \frac{\mu r \times W (1 + \mu)}{1 + \mu^2} \quad (\because \mu = f)$$



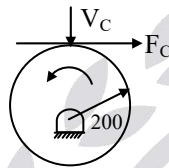


09. Ans: 64 N-m

Sol: FBD of shoe bar :



FBD of Drum Brake :



$$\sum M_B = 0$$

$$V_C \times 480 + F_C \times 100 - 1000 \times 800 = 0$$

$$F_C = \mu V_C = 0.2 V_C$$

$$480 V_C + 0.2 V_C \times 100 = 800000$$

$$500 V_C = 800000$$

$$V_C = 1600 \text{ N}$$

$$F_C = 0.2 V_C = 0.2 \times 1600 = 320 \text{ N}$$

$$M = 0.2 \times F_C = 0.2 \times 320 = 64 \text{ N-m}$$

10. Ans: (a)

Sol: $\beta = 2\theta$

$$\cos\theta = \frac{6}{12}$$

$$\Rightarrow \theta = 60$$

$$\beta = 360 - 2\theta$$

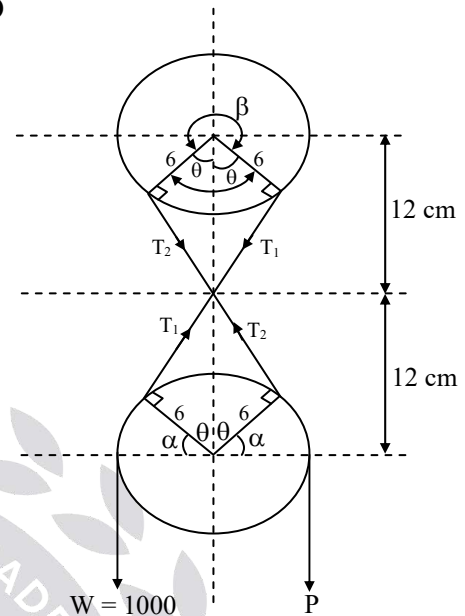
$$\beta = 240 = \frac{4\pi}{3}$$

$$2\alpha + 2\theta = 180$$

$$2\alpha = 180 - 120$$

$$\alpha = 30 = \frac{\pi}{6}$$

FBD



(When W moves upwards)

For P_{\min} calculation,

$$W > T_1$$

$$\frac{W}{T_1} = e^{\mu\alpha}$$

$$T_1 = \frac{1000}{e^{\frac{\pi}{6} \times \frac{1}{3}}} = 846.48 \text{ N}$$

$$\frac{T_1}{T_2} = e^{\mu\beta}$$

$$T_2 = \frac{846.48}{e^{\frac{1}{3} \times \frac{4\pi}{3}}} = 223.12 \text{ N}$$

$$\frac{T_2}{P_{\min}} = e^{\mu\alpha}$$

$$\Rightarrow P_{\min} = \frac{223.12}{e^{\frac{1}{3} \times \frac{\pi}{6}}}$$

$$P_{\min} = 188.86 \text{ N} \approx 189 \text{ N}$$

For P_{\max} calculation

$$\frac{T_1}{W} = e^{\mu\alpha}$$



$$T_1 = 1000 \times e^{\frac{1}{\pi} \times \frac{\pi}{6}}$$

$$T_1 = 1181.36 \text{ N}$$

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$T_2 = 1181.36 \times e^{\frac{1}{\pi} \times \frac{4\pi}{3}} = 4481.65 \text{ N}$$

$$\frac{P_{\max}}{T_2} = e^{\mu\alpha}$$

$$P_{\max} = 4481.68 \times e^{\frac{1}{\pi} \times \frac{\pi}{6}}$$

$$P_{\max} = 5300 \text{ N}$$

11. Ans: (b)

Sol: Given $\mu = 0.2$, $\tan\theta = \frac{3}{4}$

$$\Rightarrow \cos\theta = \frac{4}{5}$$

$$\sin\theta = \frac{3}{5}$$

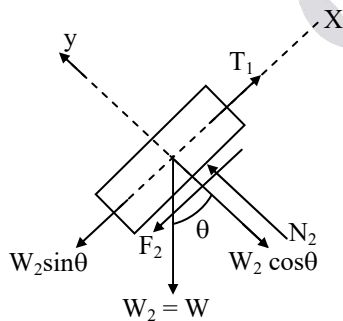


Fig: FBD (1)

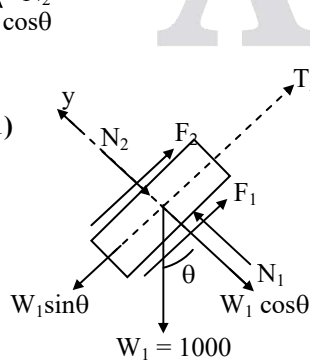


Fig: FBD (2)

From FBD (1)

$$\Sigma F_y = 0$$

$$N_2 - W_2 \cos\theta = 0$$

$$N_2 = W_2 \cos\theta = W \times 0.8$$

$$N_2 = 0.8 W$$

$$\therefore F_2 = \mu N_2 = 0.2 \times 0.8 W$$

$$F_2 = 0.16 W$$

$$\Sigma F_x = 0$$

$$T_1 - W_2 \sin\theta - F_2 = 0$$

$$T_1 = F_2 + W_2 \sin\theta = 0.16 W + 0.6 W$$

$$T_1 = 0.76 W$$

From FBD (2)

$$\Sigma F_y = 0$$

$$N_2 + W_1 \cos\theta = N_1$$

$$N_1 = N_2 + W_1 \cos\theta$$

$$N_1 = 0.8 W + 1000 \times \frac{4}{5}$$

$$N_1 = 0.8 W + 800$$

$$F_1 = \mu N_1 = 0.2 (0.8 W + 800) \\ = 0.16 W + 160$$

$$\frac{T_2}{T_1} = e^{\mu\beta}$$

$$T_2 = T_1 e^{\mu\beta} = 0.76 W e^{0.2 \times \pi}$$

$$T_2 = 1.42 W$$

$$\Sigma F_x = 0$$

$$T_2 + F_1 + F_2 = W_1 \sin\theta$$

$$1.42 W + 0.16 W + 160 + 0.16 W = 1000 \times \frac{3}{5}$$

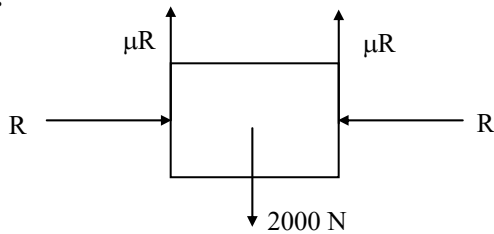
$$1.74 W = 440$$

$$\Rightarrow W = 252.87 \text{ N}$$



12. Ans: (d)

Sol:



At equilibrium

$$2\mu R = 2000$$

$$\Rightarrow R = \frac{2000}{2 \times 0.1} = 10,000 \text{ N}$$

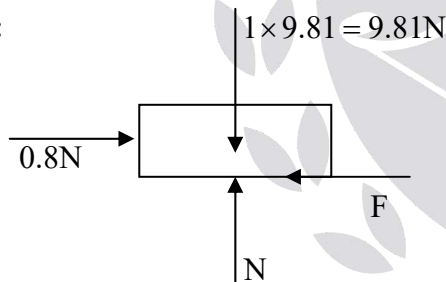
Taking moment about pin

$$10,000 \times 150 = F \times 300$$

$$\Rightarrow F = 5000 \text{ N}$$

13. Ans: (b)

Sol:



$$\Sigma Y = 0$$

$$\Rightarrow N = 9.81 \text{ N}$$

$$F_s = \mu N = 0.1 \times 9.81 = 0.98 \text{ N}$$

The External force applied = 0.8 N < F_s

\Rightarrow Frictional force = External applied
force = 0.8 N

14. Ans: (b)

Sol:

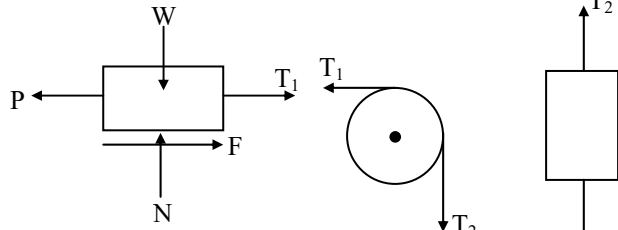


Fig: FBD (1)

Fig: FBD (2) Fig: FBD (3)

From FBD (3)

$$\Sigma F_y = 0$$

$$T_2 - 200 = 0$$

$$\Rightarrow T_2 = 200$$

From FBD (2)

$$\frac{T_1}{T_2} = e^{\mu\beta}$$

$$T_1 = T_2 e^{\mu\beta} = 200 \times e^{0.3 \times \frac{\pi}{2}}$$

$$T_1 = 320.39 \text{ N}$$

From FBD (1)

$$\Sigma F_y = 0$$

$$N - W = 0$$

$$N = 1000 \text{ N}$$

$$F = \mu N$$

$$= 0.3 \times 1000$$

$$F = 300 \text{ N}$$

$$\Sigma F_x = 0, T_1 + F - P = 0$$

$$320.39 + 300 = P$$

$$\Rightarrow P = 620.39$$

$$\Rightarrow P = 620.4 \text{ N}$$



Chapter- 4
Kinematics of Particle Rectilinear
and Curvilinear Motion

01. Ans: (d)

Sol: $x = 2t^3 + t^2 + 2t$

$$V = \frac{dx}{dt} = 6t^2 + 2t + 2$$

$$a = \frac{dv}{dt} = 12t + 2$$

At $t = 0 \Rightarrow V = 2$ and $a = 2$

02. Ans: (a)

Sol: $V = kx^3 - 4x^2 + 6x$

$$V_{\text{at } x=2 \text{ if } k=1} = 2^3 - 4(2)^2 + 6(2) = 4$$

$$a = \frac{dV}{dt} = k \cdot 3x^2 \frac{dx}{dt} - 8x \frac{dx}{dt} + 6 \frac{dx}{dt}$$

$$\begin{aligned} a &= 3x^2(V) - 8x(V) + 6(V) \\ &= 3(2)^2 \times 4 - (8 \times 2 \times 4) + 6(4) \\ &= 8 \text{ m/s}^2 \end{aligned}$$

03. Ans: (d)

Sol: Given, $a = 6\sqrt{V}$

$$\frac{dV}{dt} = 6\sqrt{V}$$

$$\int \frac{dV}{\sqrt{V}} = \int 6 dt$$

$$2\sqrt{V} = 6t + C_1$$

Given, at $t = 2$ sec, $V = 36$

$$\Rightarrow 2\sqrt{36} = 6(2) + C_1$$

$$\Rightarrow C_1 = 0$$

$$2\sqrt{V} = 6t$$

$$V = 9t^2$$

But $V = \frac{ds}{dt} = 9t^2$

$$\int ds = \int 9t^2 dt$$

$$S = 3t^3 + C_2$$

At, $t = 2$ sec, $S = 30$ m

$$\Rightarrow 30 = 3(2)^3 + C_2$$

$$\Rightarrow C_2 = 6$$

$$\therefore S = 3t^3 + 6$$

At $t = 3$ sec

$$S = 3(3)^3 + 6$$

$$S = 87 \text{ m}$$

04. Ans: (a)

Sol: Given $A = -8S^{-2}$

$$\Rightarrow \frac{dV}{dt} = \frac{d^2s}{dt^2} = -8s^{-2} = a$$

We know that, $\int V dv = \int a ds$

$$\frac{V^2}{2} = \int -8s^{-2} ds$$

$$\frac{V^2}{2} = \frac{8}{S} + C_1$$

Given, at $S = 4$ m, $V = 2$ m/sec

$$\Rightarrow \frac{2^2}{2} = \frac{8}{4} + C_1$$

$$\Rightarrow C_1 = 0$$

$$\therefore \frac{V^2}{2} = \frac{8}{S}$$



$$V = \frac{4}{\sqrt{s}}$$

$$\Rightarrow \frac{ds}{dt} = \frac{4}{\sqrt{s}}$$

$$\Rightarrow \int \sqrt{s} ds = \int 4 dt$$

$$\frac{2}{3}s^{3/2} = 4t + C_2$$

At $t = 1, S = 4$

$$\Rightarrow \frac{2}{3}(4)^{3/2} = 4(1) + C_2$$

$$\Rightarrow C_2 = \frac{16}{3} - 4 = \frac{4}{3}$$

$$\therefore \frac{2}{3}s^{3/2} = 4t + C_2$$

$$\Rightarrow \frac{2}{3}s^{3/2} = 4t + \frac{4}{3}$$

At $t = 2$ sec

$$\frac{2}{3}s^{3/2} = 4(2) + \frac{4}{3}$$

$$\Rightarrow s = 5.808 \text{ m}$$

$$a = \frac{-8}{s^2} = \frac{-8}{5.808^2} = -0.237 \text{ m/sec}^2$$

05. Ans: (c)

Sol: Given, $a = 4t^2 - 2$

$$\frac{dv}{dt} = 4t^2 - 2$$

$$dv = (4t^2 - 2) dt$$

$$v = \frac{4t^3}{3} - 2t + C_1$$

$$\frac{dx}{dt} = \frac{4t^3}{3} - 2t + C_1$$

$$\int dx = \int \left(\frac{4t^3}{3} - 2t + C_1 \right) dt$$

$$x = \frac{4t^4}{3 \times 4} - 2 \cdot \frac{t^2}{2} + C_1 t + C_2$$

$$x = \frac{t^4}{3} - t^2 + C_1 t + C_2$$

Given condition,

At $t = 0, x = -2 \text{ m}$

$$\Rightarrow -2 = C_2$$

At $t = 2, x = -20 \text{ m}$

$$\Rightarrow -20 = \frac{2^4}{3} - 2^2 + 4(2) + (-2)$$

$$\Rightarrow C_1 = \frac{-29}{3}$$

$$\therefore x = \frac{t^4}{3} - t^2 - \frac{29}{3}t - 2$$

\therefore at $t = 4$ sec

$$x = \frac{4^4}{3} - 4^2 - \frac{29}{3}(4) - 2$$

$$= 28.67 \text{ m}$$

06. Ans: (b)

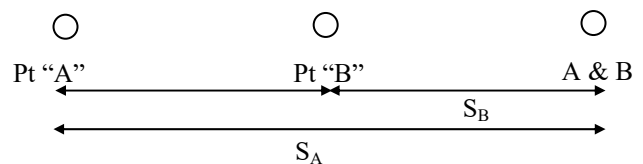
Sol:

$$u_A = 20 \text{ m/sec}$$

$$u_B = 60 \text{ m/sec}$$

$$a_A = 5 \text{ m/sec}^2$$

$$a_B = -3 \text{ m/sec}^2$$



Let S_A be the distance traveled by "A"

Let S_B be the distance traveled by "B"



$$S_A = S_B + 384$$

$$u_A t + \frac{1}{2} a_A t^2 = u_B t + \frac{1}{2} a_B t^2 + 384$$

$$20t + \frac{1}{2} 5t^2 = 60t - \frac{1}{2} 3t^2 + 384$$

$$4t^2 - 40t - 384 = 0$$

$$t = 16 \text{ sec (or) } t = -6 \text{ sec}$$

$$\therefore t = 16 \text{ sec}$$

07. Ans: (b)

Sol: Take, $y = x^2 - 4x + 100$

$$\text{Initial velocity, } V_0 = 4\hat{i} - 16\hat{j}$$

If V_x is constant

$$V_y, a_y \text{ at } x = 16 \text{ m}$$

$$V_x = V_{1x} = \frac{dx}{dt} = 4$$

$$V_y = \frac{dy}{dt} = 2x \frac{dx}{dt} - 4 \frac{dx}{dt}$$

$$(V_y) = 2x(4) - 4(4)$$

$$V_y = 8x - 16$$

$$(V_y)_{\text{at } x=16} = 8(16) - 16 = 112 \text{ m/sec}$$

$$a_y = \frac{dV}{dt} = \frac{d}{dt}(2xV_x - 4V_x)$$

($\because V_x = \text{constant}$)

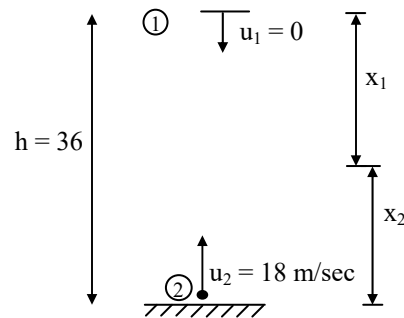
$$= 2V_x \frac{dx}{dt} = 2V_x \cdot V_x$$

$$a_y = 2V_x^2$$

$$(a_y)_{x=16} = 2 \times 4^2 = 32 \text{ m/sec}^2$$

08. Ans: (c)

Sol:



Let at distance of " x_1 " ball (1) crossed ball (2)

$$\therefore x_1 + x_2 = 36$$

$$x_1 = 0(t) + \frac{1}{2}gt^2 \quad (\because s = ut + \frac{1}{2}at^2)$$

$$x_1 = \frac{1}{2}gt^2 \quad \text{----- (1)}$$

$$x_2 = 18(t) - \frac{1}{2}gt^2$$

($\because a = -g$ moving upward)

$$x_1 + x_2 = 36$$

$$\Rightarrow \frac{1}{2}gt^2 + 18t - \frac{1}{2}gt^2 = 36$$

$$\Rightarrow 18t = 36$$

$$\Rightarrow t = 2 \text{ sec}$$

$$\therefore x_1 = \frac{1}{2}(9.81).2^2$$

$$= 19.62 \text{ m (from the top)}$$

$$x_2 = 36 - 19.62$$

$$= 16.38 \text{ m (from the bottom)}$$



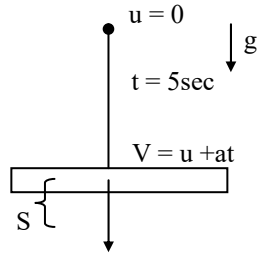
09. Ans: (b)

Sol:

$$V = u + at$$

$$V = 0 + 9.81 (5)$$

$$V = 49.05 \text{ m/sec}$$



V = velocity with which stone strike the glass

Velocity loss = 20% of V

$$= \frac{49.05 \times 20}{100} = 9.81 \text{ m/sec}$$

∴ Initial velocity for further movement in glass = $49.05 - 9.81 = 39.24 \text{ m/sec}$

Distance traveled for 1 sec of time is given by

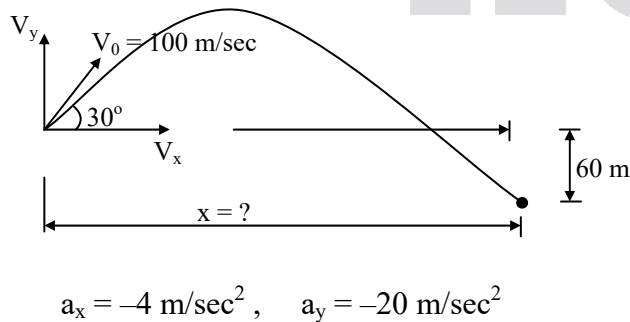
$$S = ut + \frac{1}{2}at^2$$

$$S = 39.24(1) + \frac{1}{2}(9.81)(1)^2$$

$$S = 44.145 \text{ m}$$

10. Ans: (a)

Sol:



$$V_x = V_0 \cos 30 = 100 \times \frac{\sqrt{3}}{2} = 86.6 \text{ m/sec}$$

$$V_y = V_0 \sin 30 = 100 \times \frac{1}{2} = 50 \text{ m/sec}$$

$$y = V_{oy}t + \frac{1}{2}a_y t^2$$

$$-60 = 50t + \frac{1}{2}(-20)t^2$$

$$10t^2 - 50t - 60 = 0$$

$$t = 6 \text{ (or) } -1 \text{ sec}$$

$$\therefore t = 6 \text{ sec}$$

$$x = V_0 t + \frac{1}{2}a_x t^2$$

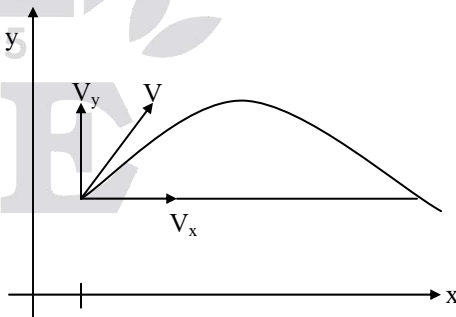
$$x = (86.6 \times 6) + \frac{1}{2}(-4)6^2$$

$$x = 447.6 \text{ m} \approx 448 \text{ m}$$

11. Ans: (a)

Sol: Given, V = 20 m/sec

$$x = 20 \text{ m, } y = 8.0 \text{ m}$$



$$V_x = V \cos \theta, \quad V_y = V \sin \theta$$

$$x = V_x t + \frac{1}{2}at^2 \quad (\because a = 0 \text{ along } x \text{ direction})$$

$$x = V \cos \theta t$$

$$20 = 20 \cos \theta t$$



$$t = \frac{1}{\cos\theta} \text{ ----- (1)}$$

$$y = V_y t - \frac{1}{2}gt^2$$

$$8.0 = V \sin\theta t - \frac{1}{2}gt^2$$

$$8.0 = 20\sin\theta \times \frac{1}{\cos\theta} - \frac{1}{2} \times 9.81 \times \left(\frac{1}{\cos\theta}\right)^2$$

$$8 = 20 \tan\theta - 4.9 \sec^2\theta$$

$$8 = 20\tan\theta - 4.9 (1 + \tan^2\theta)$$

$$4.9 \tan^2\theta - 20 \tan\theta + 12.9 = 0$$

$$\tan\theta_1 = 3.28, \tan\theta_2 = 0.803$$

$$\theta_1 = 73.04; \theta_2 = 38.76$$

12. Ans: (d)

Sol: Range = maximum height

$$\frac{V_0^2 \sin 2\theta}{g} = \frac{V_0^2 \sin^2 \theta}{2g}$$

$$\sin 2\theta = \frac{\sin^2 \theta}{2}$$

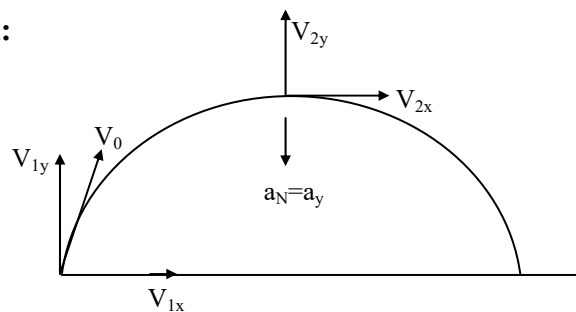
$$\Rightarrow 2\sin\theta \cos\theta = \frac{\sin^2 \theta}{2}$$

$$\Rightarrow \tan\theta = 4$$

$$\therefore \theta = \tan^{-1}(4) = 76^\circ$$

13. Ans: (a)

Sol:



$$V_{1x} = 100 - t^{3/2}$$

$$V_{2y} = 0 \Rightarrow 100 + 10t - 2t^2 = 0$$

$$(t-10)(t+5) = 0$$

$$t = 10 \text{ sec}$$

$$V_{2x} \text{ at } t = 10 \Rightarrow V_{2x} = 100 - 10^{3/2} = 68.37 \text{ m/sec}$$

$$\text{Radius of curvature, } r = \frac{V^2}{a_N}$$

$$\text{Where } a_N = a_y = \left(\frac{dV_y}{dt}\right)_{\text{at } t=10\text{sec}}$$

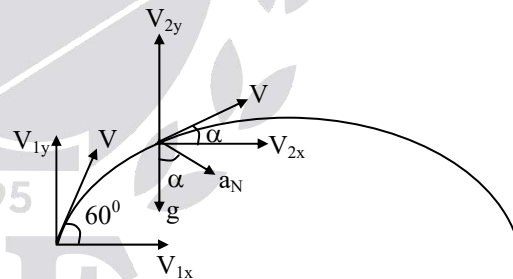
$$= (10 - 4t)_{t=10}$$

$$a_N = -30 \text{ m/sec}^2$$

$$r = \frac{V_{2x}^2}{a_N} = \frac{68.37^2}{30} = 155.8 \text{ m}$$

14. Ans: (a)

Sol:



Given, $v = 100 \text{ m/sec}$

$$v_{1x} = v \cos 60^\circ$$

$$= 100 \times 1/2$$

$$v_{1x} = 50 \text{ m/sec}$$

$$v_{1y} = v \sin 60$$

$$= 100 \times \frac{\sqrt{3}}{2}$$

$$v_{1y} = 86.6 \text{ m/sec}$$



$$v_{2y} = v_{1y} - gt \quad (\text{use } V = u + at)$$

$$= 86.6 - 9.8(1)$$

$$v_{2y} = 76.8 \text{ m/sec}$$

$$v_{2x} = v_{1x} = 50 \text{ m/sec}$$

$$v_{at \ t=1} = \sqrt{v_{2x}^2 + v_{2y}^2}$$

$$= \sqrt{50^2 + 76.8^2}$$

$$= 91.6 \text{ m/sec.}$$

$$\alpha = \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{76.8}{50} \right)$$

$$\alpha = 56.9$$

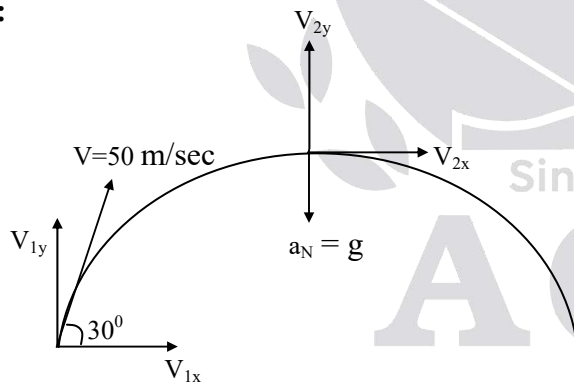
$$a_N = g \cos \alpha = 9.81 \times \cos 56.9$$

$$= 5.35 \text{ m/sec}^2$$

$$r = \frac{V^2}{a_N} = \frac{91.6^2}{5.35} = 1568.62 \text{ m}$$

15. Ans: (d)

Sol:



$$v_{1x} = v \cos 30 = 43.3 \text{ m/sec}$$

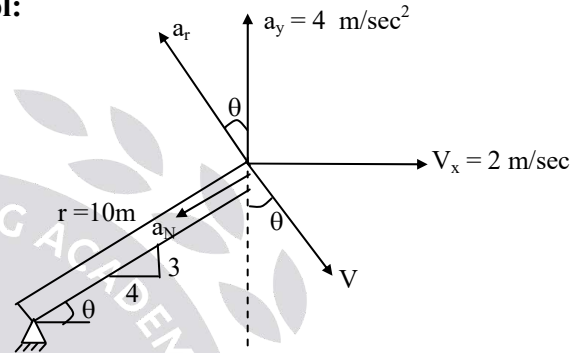
$$a_N = g = a$$

$$r = \frac{V_{1x}^2}{a_N} = \frac{43.3^2}{9.81} = 191.13 \text{ m}$$

Chapter- 5 Kinematics of Rigid Bodies Fixed Axis Rotation and General Plane Motion

01. Ans: (a)

Sol:



$$\tan \theta = \frac{3}{4}$$

$$\theta = \tan^{-1} 3/4 = 36.6^\circ$$

$$a_y = a_T \cos \theta - a_N \sin \theta$$

Note: Velocity will always act in the tangential direction

$$V_x = V \sin \theta$$

$$V = \frac{2}{\sin 36.6} = 3.33 \text{ m/sec}$$

$$\therefore a_N = \frac{V^2}{r} = \frac{3.33^2}{10}$$

$$a_N = 1.111 \text{ m/sec}^2$$

$$a_y = a_T \cos \theta - a_N \sin \theta$$

$$4 = a_T \cos 36.6 - 1.111 \sin 36.6$$

$$\Rightarrow a_T = 5.83 \text{ m/sec}^2$$

$$a_T = r\alpha$$

$$\alpha = \frac{a_T}{r} = \frac{5.83}{10} = 0.583 \text{ rad/sec}^2$$



02. Ans: (c)

Sol: Given $\omega = 4\sqrt{t}$

$$\theta = 2 \text{ radians at } t = 1 \text{ sec}$$

$$\theta = ? \alpha = ? \text{ at } t = 3 \text{ sec}$$

$$\omega = \frac{d\theta}{dt} \Rightarrow \int d\theta = \int \omega dt$$

$$\theta = \int 4\sqrt{t} dt$$

$$\theta = \frac{8}{3}t^{3/2} + c \dots (1)$$

From given condition, at $t = 1, \theta = 2 \text{ rad}$

$$(1) \Rightarrow 2 = \frac{8}{3}(1)^{3/2} + c_1 \Rightarrow c_1 = \frac{-2}{3}$$

$$\therefore \theta = \frac{8}{3}t^{3/2} - \frac{2}{3}$$

$$\text{At } t = 3 \text{ sec, } \theta = \frac{8}{3}(3)^{3/2} - \frac{2}{3}$$

$$\theta_{t=3} = 13.18 \text{ rad}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d(4\sqrt{t})}{dt} = \frac{2}{\sqrt{t}}$$

$$\alpha_{t=3} = \frac{2}{\sqrt{3}} = 1.15 \text{ rad/sec}^2$$

03. Ans: (b)

Sol: $r = 2 \text{ cm, } \omega = 3 \text{ rad/sec, } a = 30 \text{ cm/s}^2$

$$a_N = r\omega^2 = 2(3)^2 = 18 \text{ cm/sec}^2$$

$$\text{Since total acceleration } a = \sqrt{a_T^2 + a_N^2}$$

$$\Rightarrow a^2 = a_T^2 + a_N^2$$

$$30^2 = a_T^2 + 18^2$$

$$a_T = 24 \text{ cm/sec}^2$$

$$a_T = r\alpha = 24$$

$$\alpha = \frac{24}{2} = 12 \text{ rad/sec}^2$$

04. Ans: (d)

Sol: Given angular acceleration, $\alpha = \pi \text{ rad/sec}^2$

Angular displacement in time t_1 and t_2

$$= \pi \text{ rad} = \theta_2 - \theta_1$$

$$\omega_{t_2} = 2\pi \text{ rad/sec}$$

$$\omega_{t_1} = ?$$

$$\omega_{t_1}^2 - \omega_0^2 = 2\alpha\theta_1$$

$$\omega_{t_2}^2 - \omega_0^2 = 2\alpha\theta_2$$

$$\omega_{t_2}^2 - \omega_{t_1}^2 = 2\alpha(\theta_2 - \theta_1)$$

$$4\pi^2 - \omega_{t_1}^2 = 2\pi^2$$

$$\omega_{t_1}^2 = 2\pi^2$$

$$\omega_{t_1} = \pi\sqrt{2}$$

05. Ans: (c)

Sol: Given retardation

$$\alpha = -3t^2$$

$$\frac{d\omega}{dt} = -3t^2$$

$$\int d\omega = \int -3t^2 dt$$

$$\omega = -t^3 + c_1$$

From given condition at $t = 0,$

$$\omega = 27 \text{ rad/sec}$$

$$27 = -0^3 + c_1$$

$$\Rightarrow c_1 = 27$$

$$\therefore \omega = -t^3 + 27$$

Wheel stops at $\omega = 0,$

$$\Rightarrow 0 = -t^3 + 27$$

$$\Rightarrow t = 3 \text{ sec}$$



06. Ans: (c)

Sol: angular speed, $\omega = 5 \text{ rev/sec}$
 $= 5 \times 2\pi \text{ rad/sec}$
 $\omega = 10\pi \text{ rad/sec}$

Radius, $r = 0.1 \text{ m}$

If ω is constant, $d\omega = 0$

$\Rightarrow \alpha = 0 \Rightarrow a_T = 0$ (since $a_T = r\alpha$)

Since $a_T = 0$

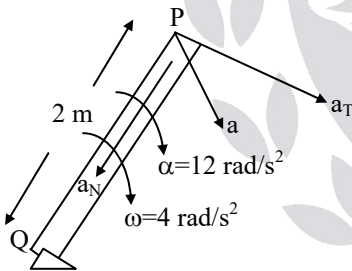
$$a = \sqrt{a_N^2 + a_T^2}$$

$$a = a_N = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

$$= 0.1 \times (10\pi)^2 = 10\pi^2 \text{ m/sec}^2$$

07. Ans: $a = 40 \text{ m/s}^2$

Sol:



Tangential acceleration

$$a_T = r \alpha = 2 \times 12 = 24 \text{ m/s}^2$$

Normal acceleration, $a_N = r \omega^2$

$$= 2 \times 4^2 = 32 \text{ m/s}^2$$

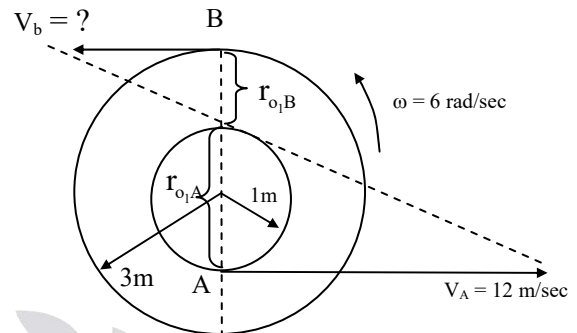
The resultant acceleration

$$a = \sqrt{a_T^2 + a_N^2}$$

$$= \sqrt{24^2 + 32^2} = 40 \text{ m/s}^2$$

08. Ans: (b)

Sol:



$$V_A = r_{o_1A} \times \omega$$

$$\Rightarrow 12 = r_{o_1A} \times 6$$

$$r_{o_1A} = 2 \text{ m}$$

$$4 = 2 + r_{o_1B}$$

$$r_{o_1B} = 2 \text{ m}$$

$$\therefore V_B = r_{o_1B} \times \omega = 2 \times 6$$

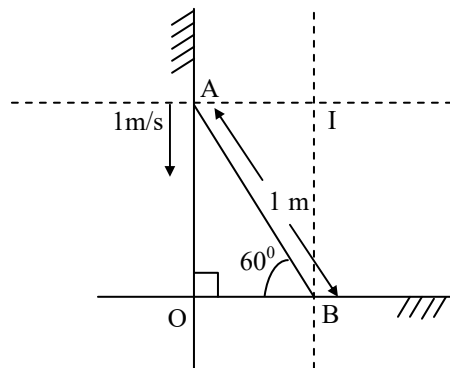
$$V_B = 12 \text{ m/sec}$$

09. Ans: (a)

Sol: Instantaneous centre will have zero velocity because the instantaneous centre is the point of contact between the object and the floor.

10. Ans: (a)

Sol:





$$V_a = 1 \text{ m/s}$$

V_a = along vertical

V_b = along horizontal

So instantaneous center of V_a and V_b will be perpendicular to A and B respectively

$$IA = OB = l \times \cos \theta = 1 \times \cos 60^\circ = \frac{1}{2} m$$

$$IB = OA = l \times \sin \theta = 1 \times \sin 60^\circ = \frac{\sqrt{3}}{2} m$$

$$V_a = \omega \times IA$$

$$\Rightarrow \omega = \frac{V_a}{IA} = 2 \text{ rad/sec}$$

'I' is the instantaneous centre.

From sine rule

$$\frac{PQ}{\sin 45^\circ} = \frac{IQ}{\sin 70^\circ} = \frac{IP}{\sin 65^\circ}$$

$$\frac{IP}{IQ} = \frac{\sin 65^\circ}{\sin 70^\circ}$$

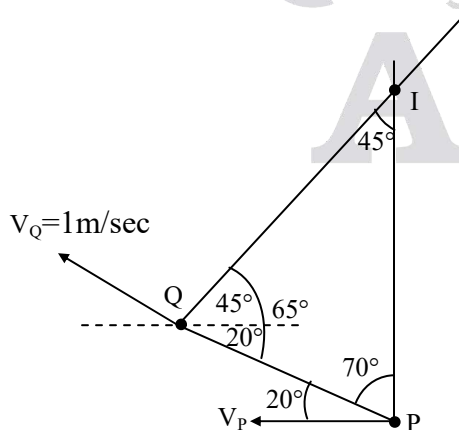
$$V_Q = IQ \times \omega = 1$$

$$\Rightarrow \omega = \frac{V_Q}{IQ}$$

$$V_P = IP \times \omega = \frac{IP}{IQ} \times V_Q = \frac{\sin 65^\circ}{\sin 70^\circ} \times 1 = 0.9645$$

11. Ans: (d)

Sol: Refer the figure shown below, by knowing the velocity directions instantaneous centre can be located as shown. By knowing velocity (magnitude) of Q we can get the angular velocity of the link, from this we can get the velocity of 'P' using sine rule.

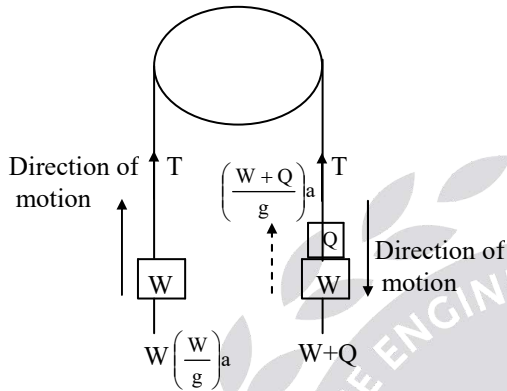




Chapter- 6
Kinetics of Particle and Rigid Bodies

01. Ans: (a)

Sol:



For the left cord,

$$\Sigma F_y = 0$$

$$T = \left(\frac{W}{g}\right)a + W \dots\dots\dots(1)$$

For the right cord

$$\Sigma F_y = 0$$

$$T + \left(\frac{W+Q}{g}\right)a = (W+Q) \dots\dots(2)$$

From (1) & (2)

$$\left(\frac{W}{g}\right)a + W = W+Q - \left(\frac{W+Q}{g}\right)a$$

$$\left(\frac{W}{g}\right)a + W = W + Q - \left(\frac{W}{g}\right)a - \left(\frac{Q}{g}\right)a$$

$$Q - \frac{Qa}{g} = \frac{2Wa}{g}$$

$$Q \left(\frac{g-a}{g}\right) = \frac{2Wa}{g} \Rightarrow Q = \frac{2Wa}{g-a}$$

02. Ans: (b)

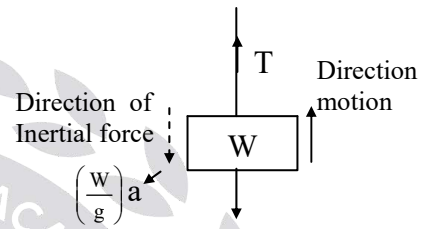
Sol: $u = 0, \quad v = 1.828 \text{ m/sec}, \quad S = 1.825 \text{ m},$

$$v^2 - u^2 = 2as$$

$$1.828^2 - 0 = 2a \times 1.828$$

$$a = \frac{1.828}{2}$$

$$a = 0.914 \text{ m/sec}^2$$



For equilibrium, $\Sigma F_y = 0$

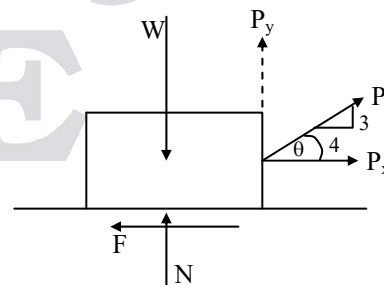
$$T = W + \left(\frac{W}{g}\right)a$$

$$= 4448 + \frac{4448}{9.81} \times 0.194$$

$$T = 4862.42 \text{ N}$$

03. Ans: (a)

Sol:



$$\tan \theta = \frac{3}{4}$$

$$\theta = \tan^{-1}(3/4) = 36.86$$

$$(F_{\text{net}})_x = ma$$



$$P_x - F = \left(\frac{W}{g}\right)a$$

$$P \cos 36.86 - F = \left(\frac{W}{g}\right)a$$

$$0.8P - F = \left(\frac{2224}{g}\right)(0.2g)$$

$$0.8P - F = 444.8$$

$$0.8P - F = 444.8 + F$$

$$P = 556 + 1.25F \dots\dots(1)$$

$$\Sigma F_y = 0$$

$$N + P_y - W = 0$$

$$N = W - P_y \text{ (since } \mu = \frac{F}{N} \text{)}$$

$$F = \mu N$$

$$F = \mu (W - P_y)$$

$$= 0.2(2224 - P \sin 36.86)$$

$$F = 444.8 - 0.12P \dots\dots(2)$$

From (1) & (2)

$$P = 556 + 1.25(444.8 - 0.12P)$$

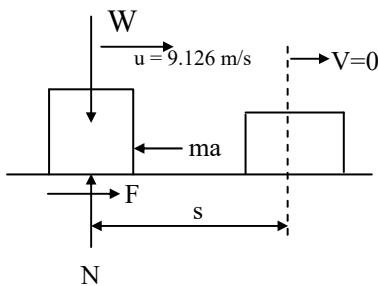
$$1.15P = 1112$$

$$P = 966.95$$

$$P = 967 \text{ N}$$

04. Ans: (d)

Sol:



From static equilibrium condition

$$\Sigma F_y = 0$$

$$N - W = 0$$

$$N = W = 44.48 \text{ N}$$

From dynamic equilibrium condition

$$\Sigma F_x = 0$$

$$F = ma$$

$$\mu N = \frac{W}{g} a$$

$$\mu = \frac{a}{g}$$

$$a = \mu g \dots\dots(1)$$

$$\text{Since } v^2 - u^2 = 2as$$

$$0 - (9.126)^2 = 2(-a) \times 13.689$$

$$a = 3.042 \dots\dots(2)$$

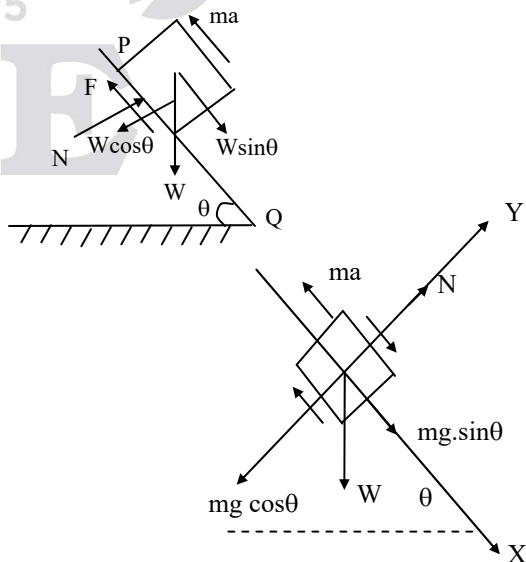
From (1) & (2)

$$3.042 = \mu(9.81)$$

$$\Rightarrow \mu = 0.31$$

05. Ans: (a)

Sol:





$$\Sigma F_y = 0 \text{ (static equilibrium)}$$

$$N - W \cos \theta = 0$$

$$N = W \cos \theta = mg \cos \theta$$

$$\text{Since } F = \mu N = \mu mg \cos \theta \dots (1)$$

$$\Sigma F_x = 0 \text{ (Dynamic equilibrium)}$$

$$F + ma - W \sin \theta = 0$$

$$F = -ma + mg \sin \theta$$

$$F = mg \sin \theta - ma \dots (2)$$

From (1) & (2)

$$\mu mg \cos \theta = mg \sin \theta - ma$$

$$\Rightarrow a = g \sin \theta - \mu g \cos \theta$$

$$\Rightarrow a = g \cos \theta (\tan \theta - \mu)$$

Given PQ = s

$$s = ut + \frac{1}{2} at^2$$

$$s = 0(t) + \frac{1}{2} at^2 \Rightarrow t = \sqrt{\frac{2s}{a}}$$

$$= \sqrt{\frac{2s}{g \cos \theta (\tan \theta - \mu)}}$$

$$T_A = 2T_B \dots (1)$$

Work done by A & B equal

$$T_A S_A = T_B S_B$$

$$2T_B S_A = T_B S_B$$

$$2S_A = S_B$$

$$2a_A = a_B \dots (2)$$

For 'B' body

$$T_B = m_B a_B + m_B g \dots (3)$$

For 'A' body

$$T_A = m_A g - m_A a_A \dots (4)$$

(2), (3) & (4) sub in (1)

$$m_A g - m_A a_A = 2(m_B (2a_A) + m_B g)$$

$$m_A g - m_A a_A = 4m_B a_A + 2m_B g$$

$$m_A a_A + 4m_B a_A = m_A g - 2m_B g$$

$$a_A = \frac{m_A g - 2m_B g}{m_A + 4m_B}$$

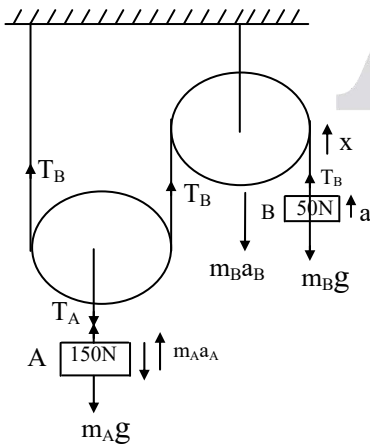
$$= \frac{150 - 2(50)}{150 + 4(50)}$$

$$= \frac{150 - 100}{150 + 200} = \frac{50}{350} = \frac{1}{7} = 0.142$$

$$= \frac{50}{15 + 20} = \frac{50}{35} = 1.42$$

06. Ans: (a)

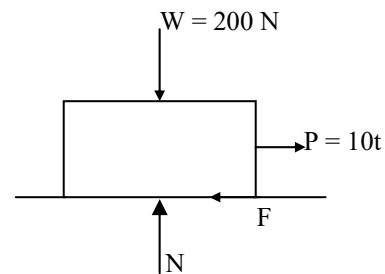
Sol:



07. Ans: 4.905 m/s

Sol: $\mu_S = 0.4$; $\mu_K = 0.2$

FBD of the block





W.r.t free body diagram of the block:

$$F_S = \mu_S N ;$$

$$F_K = \mu_K N$$

$$\Sigma F_y = 0$$

$$N - W = 0$$

$$N = W = 200 \text{ N}$$

Limiting friction or static friction

$$(F_S) = 0.4 \times 200 = 80 \text{ N}$$

Kinetic Friction

$$(F_K) = 0.2 \times 200 = 40 \text{ N}$$

The block starts moving only when the force, P exceeds static friction, F_S

Thus, under static equilibrium

$$\Rightarrow \Sigma F_x = 0$$

$$\Rightarrow P - F_S = 0 \Rightarrow 10t = 80$$

$$t = \frac{80}{10} = 8 \text{ sec}$$

\therefore The block starts moving only when $t > 8$ seconds

During 8 seconds to 10 seconds of time:

According to Newton's second law of motion

Force = mass \times acceleration

$$(P - F_K) = m \times \frac{dv}{dt} \Rightarrow (10t - 40) = \frac{200}{9.81} \times \frac{dv}{dt}$$

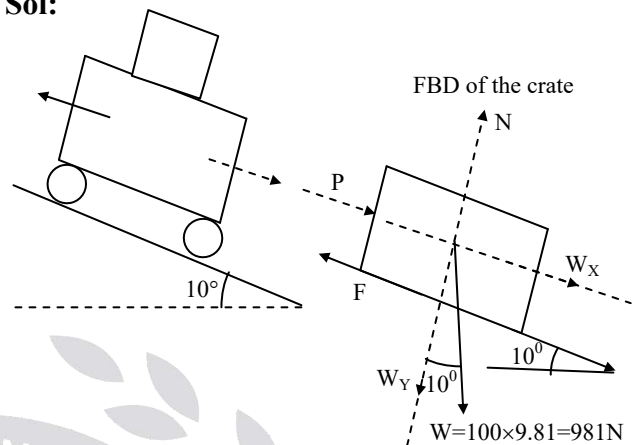
$$\int_8^{10} (10t - 40) dt = \frac{200}{9.81} \int_0^v dv$$

$$[5t^2 - 40t]_8^{10} = 20.387 \times V \Rightarrow (180 - 80) = 20.387 \times V$$

$$\text{Velocity (V)} = 4.905 \text{ m/s}$$

08. Ans: 1.198 m/s²

Sol:



W.r.t. FBD of the crate:

$$W_X = W \sin 10^\circ = 981 \times \sin 10^\circ = 170.34 \text{ N}$$

$$W_Y = W \cos 10^\circ = 981 \times \cos 10^\circ = 966.09 \text{ N}$$

$$\Sigma F_Y = 0 \Rightarrow N - W_Y = 0$$

$$N = W_Y = 966.09 \text{ N};$$

$$F = \mu N = 0.3 \times 966.09 = 289.828 \text{ N}$$

$$\Sigma F_X = 0 \Rightarrow P + W_X - F = 0$$

$$\Rightarrow P + 289.828 - 170.34 = 0$$

$$P = 119.488 \text{ N}$$

$$P = ma = 119.488 \text{ N}$$

$$\Rightarrow a = \frac{119.488}{100} = 1.198 \text{ m/s}^2$$

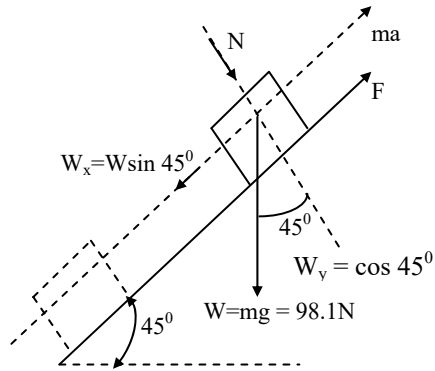
09. Ans: 57.67 m

Sol:

$$W_x = W \sin 45$$

$$= 98.1 \times \sin 45 = 69.367 \text{ N}$$

$$W_y = W \cos 45 = 69.367 \text{ N}$$



$$\Sigma F_Y = 0$$

$$N - W_Y = 0$$

$$N = W_Y = 69.367 \text{ N}$$

$$F = \mu_K N = 0.5 \times 69.367 = 34.683 \text{ N}$$

$$\Sigma F_x = 0 \text{ (Dynamic Equilibrium)}$$

D' Alembert principle)

$$W_x - F - ma = 0$$

$$69.367 - 34.683 - 10 \times a = 0$$

$$a = 3.468 \text{ m/s}^2$$

$$S = ut + \frac{1}{2} at^2$$

$\therefore t$ is unknown we can not use this equation

$$\text{So use } V^2 - u^2 = 2as$$

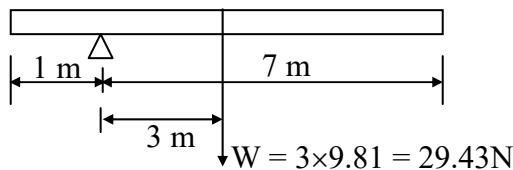
$$V = 20 \text{ m/s}^2; u = 0; a = 3.468 \text{ m/s}^2$$

$$V^2 = 2as$$

$$S = \frac{V^2}{2 \times a} = \frac{20^2}{2 \times 3.468} = 57.67 \text{ m}$$

10. Ans: 2.053 rad/s²

Sol:



$$M = I\alpha$$

$$M = 29.43 \times 3 = 88.29 \text{ N-m}$$

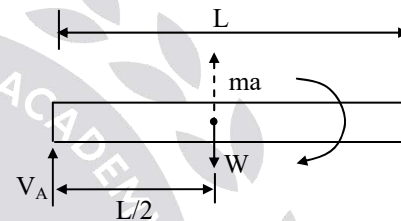
$$I = I_0 + Ad^2 = \frac{m\ell^2}{12} + md^2 = \frac{3 \times 8^2}{12} + 3 \times 3^2$$

$$= 16 + 27 = 43 \text{ kg-m}^2$$

$$\alpha = \frac{M}{I} = \frac{88.29}{43} = 2.053 \text{ rad/s}^2$$

11. Ans: (d)

Sol:



$$\Sigma F_y = 0$$

$$V_A + ma = W$$

$$V_A = m(g - a) \dots (1)$$

$$\text{Where, } a = \frac{L}{2} \alpha$$

$$\text{Since, } M = I\alpha$$

$$W \times \frac{L}{2} = \left(\frac{mL^2}{12} + m \left(\frac{L}{2} \right)^2 \right) \alpha$$

$$mg \times \frac{L}{2} = \frac{4mL^2}{12} \times \frac{2a}{L}$$

$$a = \frac{3}{4} g \dots (2)$$

from (1) & (2)

$$V_A = m \left(g - \frac{3}{4} g \right) = \frac{mg}{4}$$

$$V_A = \frac{W}{4}$$



12. Ans: (d)

Sol: $I = 5\text{kg.m}^2$

$R = 0.25\text{m}$

$F = 8\text{N}$

Mass moment of inertia, $I_x = I_y = \frac{mr^2}{4}$

$I_z = \frac{mr^2}{2}$

$M = I\alpha$

$8 \times 0.25 = 5\alpha$

$\alpha = 0.4$

$\omega^2 - \omega_0^2 = 2\alpha\theta$

$\omega^2 - 0^2 = 2(0.4) \times \pi$ (since for half revolution $\theta = \pi$)

$\omega = 1.58 \text{ rad/sec}$

13. Ans: 4.6 seconds

Sol: $M = 60 \text{ N - m}$

$L = 2\text{m}, \quad \omega_0 = 0,$

$\omega = 200 \text{ rpm} = \frac{200 \times 2\pi}{60}$

$\omega = 20.94 \frac{\text{rad}}{\text{sec}}$

Moment, $M = I\alpha$

$60 = \frac{mL^2}{12} \times \alpha$

$\Rightarrow 60 = \frac{40 \times 2^2}{12} \times \alpha$

$\alpha = 4.5 \text{ rad/sec}^2$

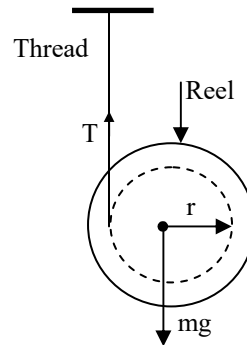
$\omega = \omega_0 + \alpha t$

$20.94 = 4.5t$

$\Rightarrow t = 4.65 \text{ sec}$

14. Ans: (a)

Sol:



a = linear acceleration,

k = radius of gyration

For vertical translation motion

$mg - T = ma$ ----- (1)

For rotational motion

$T \times r = I\alpha$

$Tr = mk^2 \alpha = mk^2 \times \frac{a}{r}$

$\Rightarrow T = \frac{mk^2}{r^2} \times a$ ----- (2)

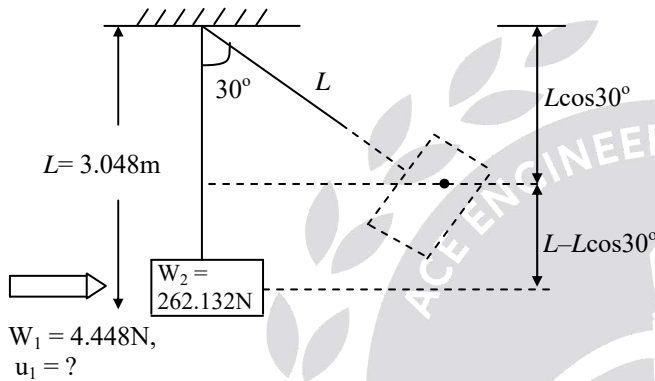
$mg - \frac{mk^2}{r^2} \times a = ma \Rightarrow a = \frac{gr^2}{(k^2 + r^2)}$



Chapter- 7
Work-Energy Principle and Impulse Momentum Equation

01. Ans: (a)

Sol:



The loss of KE of shell converted to do the work in lifting the sand box and shell to a height of " $L - L\cos30^\circ$ "

$$\text{i.e., } Wd = \frac{1}{2} mV^2$$

$$\text{Where } d = L - L\cos30^\circ \\ = 3.048 - 3.048 \times \cos30^\circ = 0.41 \text{ m}$$

$$266.58 \times 0.41 = \frac{1}{2} \left(\frac{266.58}{9.81} \right) \times V^2$$

$$\Rightarrow V = 2.83 \text{ m/sec}$$

Where V is the velocity of block & shell

By momentum equation

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Where $v_1 = v_2 = V$ & $u_1 = ?$, $u_2 = 0$

$$\frac{4.448}{9.81} \times u_1 = \frac{4.448 + 262.132}{9.81} \times 2.83$$

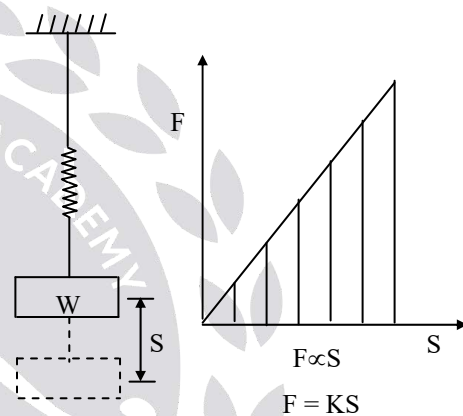
$$\Rightarrow u_1 = 169.6 \text{ m/sec}$$

u_1 & u_2 = Initial velocity of shell and block respectively

V_1 & V_2 = Final velocity of block & shell

02. Ans: (b)

Sol:



Strain energy in spring = Area under the force displacement curve.

$$= \frac{1}{2} F \times s = \frac{1}{2} (ks) \times s = \frac{1}{2} ks^2$$

$$\frac{1}{2} ks^2 = \text{Gain of KE}$$

$$\frac{1}{2} ks^2 = \frac{1}{2} mv^2$$

$$\Rightarrow v^2 = \frac{ks^2}{m} = \frac{ks^2}{w} g$$

$$v = \sqrt{\frac{kg}{w}} \cdot s \quad \left(\because m = \frac{w}{g} \right)$$



03. Ans: (a)

Sol: Given, $m = 2 \text{ kg}$

Position at any time is given as

$$x = t + 5t^2 + 2t^3$$

At $t = 0$, $x = 0$,

At $t = 3 \text{ sec}$,

$$x = 3 + 5(3^2) + 2(3^3) = 102 \text{ m}$$

$$\text{Velocity, } V = \frac{dx}{dt} = 1 + 10t + 6t^2$$

Initial velocity i.e., $t = 0$, is $v_i = 1 \text{ m/s}$

Final velocity i.e., at $t = 3 \text{ sec}$,

$$\text{is } v_f = 1 + 10(3) + 6(3)^2 = 85 \text{ m/s}$$

Work done = change in KE

$$\begin{aligned} &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2} \times 2(85^2 - 1^2) = 7224 \text{ J} \end{aligned}$$

04. Ans: (a)

Sol: Given force $F = e^{-2x}$

$$\text{Work done} = \int_{x_1}^{x_2} F dx$$

$$= \int_{0.2}^{1.5} e^{-2x} dx = \left[\frac{e^{-2x}}{-2} \right]_{0.2}^{1.5} = 0.31 \text{ J}$$

05. Ans: (b)

Sol: $F = 4x - 3x^2$

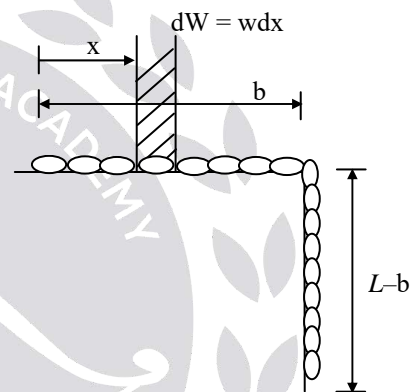
Potential Energy at $x = 1.7 =$ work required to move object from 0 to 1.7m

$$PE = \int_0^{1.7} F dx$$

$$\begin{aligned} &= \int_0^{1.7} (4x - 3x^2) dx \\ &= \left[4\left(\frac{x^2}{2}\right) - 3\left(\frac{x^3}{3}\right) \right]_0^{1.7} \\ &= [2x^2 - x^3]_0^{1.7} \\ &= 2(1.7)^2 - (1.7)^3 = 0.867 \text{ J} \end{aligned}$$

06. Ans: (c)

Sol:



Where $w =$ weight per unit meter

$dw =$ a small work done in moving small elemental “ dx ” of chain through a d/s “ x ”

Work done = change in KE

$$\left(\int_0^b dw \times x \right) + (w(L-b) \times b) = \frac{1}{2} \left(\frac{wL}{g} \right) v^2$$

$$\int_0^b w dx \cdot x + w(L-b)b = \frac{1}{2} \frac{wLv^2}{g}$$

$$\frac{wb^2}{2} + w(L-b)b = \frac{1}{2} \frac{wLv^2}{g}$$

$$\frac{wb^2}{2} + wLb - wb^2 = \frac{1}{2} \frac{wLv^2}{g}$$

$$wLb - \frac{wb^2}{2} = \frac{1}{2} \frac{wLv^2}{g}$$



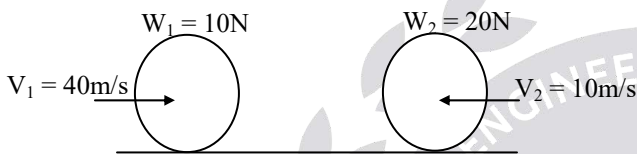
$$b\left(L - \frac{b}{2}\right) = \frac{1}{2} \frac{Lv^2}{g}$$

$$v^2 = 2gb\left(1 - \frac{b}{2L}\right)$$

$$v = \sqrt{gb\left(2 - \frac{b}{L}\right)}$$

07. Ans: (d)

Sol:



$$m_1 = 1\text{ kg}, m_2 = 2\text{ kg}, (\text{since } g = 10\text{ m/sec}^2)$$

Velocities before impact

$$v_1 = 40\text{ m/sec}, v_2 = -10\text{ m/s}$$

Velocities after impact

$$u_1 = ? u_2 = ?$$

Coefficient of restitution $e = 0.6$

From momentum equation

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$\Rightarrow 1(40) + 2(-10) = 1(u_1) + 2(u_2)$$

$$\Rightarrow u_1 + 2u_2 = 20 \dots \dots \dots (1)$$

$$e = \frac{u_2 - u_1}{v_1 - v_2} = \frac{\text{relative velocity of Separation}}{\text{relative velocity of approach}}$$

$$0.6 = \frac{u_2 - u_1}{40 - (-10)}$$

$$\Rightarrow u_2 - u_1 = 30 \dots \dots \dots (2)$$

From 1 & 2

$$u_1 = -13.33\text{ m/sec}$$

$$u_2 = 16.66\text{ m/sec}$$

08. Ans: (b)

Sol: Given, $m_1 = 3\text{ kg}, m_2 = 6\text{ kg}$

Velocities before impact

$$u_1 = 4\text{ m/s}, u_2 = -1\text{ m/s}$$

Velocities after impact

$$v_1 = 0\text{ m/s}, v_2 = ?$$

From momentum equation

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$3(4) + 6(-1) = 3(0) + 6(v_2)$$

$$\Rightarrow 6 = 6v_2$$

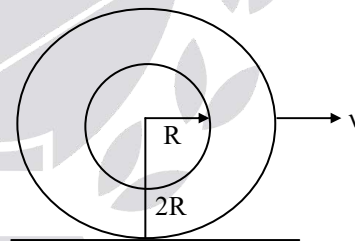
$$\Rightarrow v_2 = 1\text{ m/s}$$

$$\text{Coefficient of restitution, } e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e = \frac{1 - 0}{4 - (-1)} = \frac{1}{5}$$

09. Ans: (c)

Sol:



$$KE = \frac{1}{2} mV^2 + \frac{1}{2} I\omega^2$$

$$\text{Where, } \omega = \frac{V}{2R}$$

$$I = \frac{1}{2} m((2R)^2 + R^2) = \frac{5}{2} mR^2$$

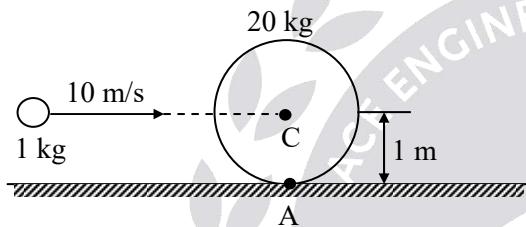
$$\therefore KE = \frac{1}{2} mV^2 + \frac{1}{2} \left(\frac{5}{2} mR^2\right) \left(\frac{V}{2R}\right)^2$$

$$\begin{aligned}
 KE &= \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{5}{2}mR^2\right)\left(\frac{V}{2R}\right)^2 \\
 &= \frac{1}{2}mV^2 + \frac{5}{4}mR^2 \times \frac{V^2}{4R^2} \\
 &= \frac{1}{2}mV^2 + \frac{5}{16}mV^2
 \end{aligned}$$

$$KE = \frac{13mV^2}{16}$$

10. Ans: (a)

Sol:



Method I :

By conservation of linear momentum, we get
 $1 \times 10 = (20 + 1) \times V_{cm}$ (where, V_{cm} = velocity of centre of mass)

$$\Rightarrow V_{cm} = \frac{10}{21} \text{ m/s}$$

Applying angular momentum conservation about an axis passing through the contact point (A) and perpendicular to the plane of paper, we get

$$1 \times 10 \times 1 = I_{cm} \omega + 21 \times \frac{10}{21} \times 1$$

[Angular momentum about any axis passing through A can be written as,

$$\vec{L}_A = \vec{L}_{cm} + m(\vec{r} \times \vec{V}_{cm})]$$

$$\Rightarrow \omega = 0 \text{ rad/sec}$$

Method II :

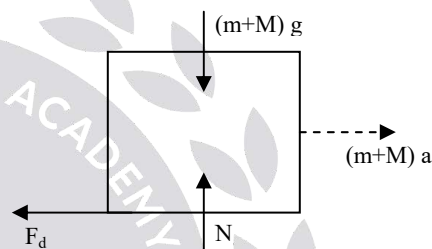
Applying angular momentum conservation about an axis passing through centre of wheel and perpendicular to the plane of paper.

$$\therefore 0 = I_{cm} \omega$$

$$\Rightarrow \omega = 0 \text{ rad/sec}$$

11. Ans: (a)

Sol:



$m_1 = m \rightarrow$ mass of bullet

$m_2 = M \rightarrow$ mass of block

$u_1 = V \rightarrow$ bullet initial velocity

$u_2 = 0 \rightarrow$ block initial velocity

$v_1 = v_2 = v \rightarrow$ velocity of bullet and block after impact.

$$F_d = \mu N$$

$$(M+m)a = \mu(M+m)g$$

$$\Rightarrow a = \mu g$$

From momentum equation

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$mV + m(0) = (m + M)V$$

$$v = \frac{mV}{m + M}$$

Now from $v^2 - u^2 = 2as$

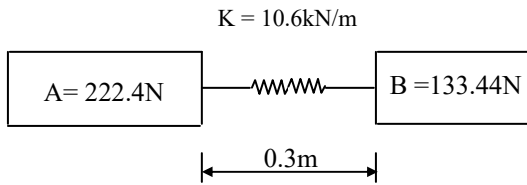
$$0 - \left(\frac{mV}{m + M}\right)^2 = 2\mu g s$$



$$V = \frac{m + M}{m} \sqrt{2\mu g s}$$

12. Ans: (a)

Sol:



$$u_A = 0, \quad u_B = 0$$

From momentum equation

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$0 = 222.4 v_A + 133.44 v_B \dots \dots \dots (1)$$

$$\frac{1}{2} k s^2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$10.6 \times 10^3 \times 0.15^2 = \frac{222.4}{9.81} v_A^2 + \frac{133.44}{9.81} v_B^2 \dots \dots \dots (2)$$

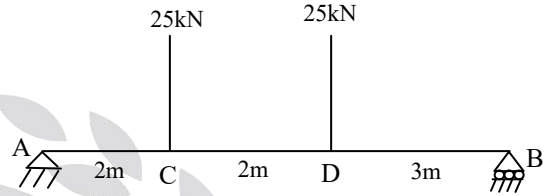
From 1 & 2

$$v_A = -1.98 \text{ m/s}, \quad v_B = 3.3 \text{ m/s}$$

Chapter- 8
Virtual Work

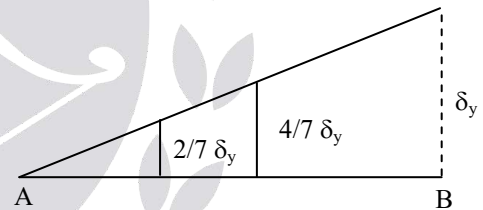
01. Ans: $\frac{200}{7}$ kN, $\frac{150}{7}$ kN

Sol:



Let R_A & R_B be the reactions at support A & B respectively.

Let δ_y displacement be given to the beam at B without giving displacement at 'A'



The corresponding displacement at C & D are $\frac{2}{7} \delta_y$ and $\frac{4}{7} \delta_y$

By virtual work principle,

$$R_A \times 0 - 25 \times \frac{2}{7} \delta_y - 25 \times \frac{4}{7} \delta_y + R_B \times \delta_y = 0$$

$$\Rightarrow \left(\frac{-150}{7} + R_B \right) \delta_y = 0$$

$$\text{Since } \delta_y \neq 0, R_B - \frac{150}{7} = 0$$

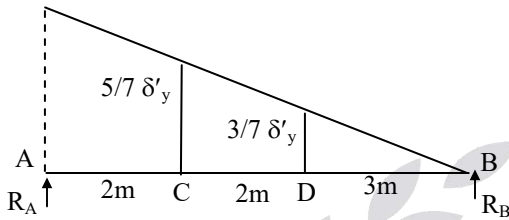
$$R_B = \frac{150}{7} \text{ kN}$$



Now let us give virtual displacement at A as δ'_y ,

Therefore corresponding displacement at C

& D are $\frac{5}{7}\delta'_y$ & $\frac{3}{7}\delta'_y$



\therefore By virtual work principle,

$$R_A \times \delta'_y - 25 \times \frac{5}{7} \delta'_y - 25 \times \frac{3}{7} \delta'_y + R_B \times 0 = 0$$

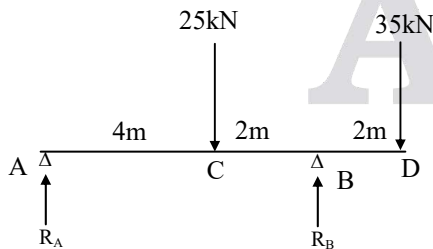
$$\left(R_A - \frac{125}{7} - \frac{75}{7} \right) \delta'_y = 0$$

$$\delta'_y \neq 0, R_A - \frac{200}{7} = 0$$

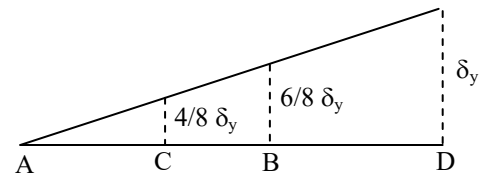
$$R_A = \frac{200}{7} \text{ kN}$$

02. Ans: $-\frac{10}{3} \text{ kN}, \frac{190}{3} \text{ kN}$

Sol:



Let the virtual displacement at D as δ_y , then corresponding displacement at different point as shown below (Assume no displacement at A).



By virtual work principle,

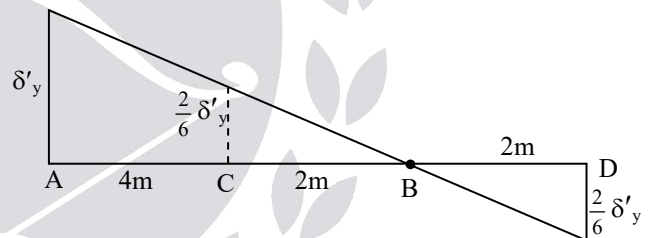
$$R_A \times 0 - 25 \times \frac{4}{8} \delta_y + R_B \times \frac{6}{8} \delta_y - 35 \times \delta_y = 0$$

$$-\frac{25}{2} \delta_y + \frac{3R_B}{4} \delta_y - 35 \delta_y = 0$$

$$\frac{3}{4} R_B = 35 + \frac{25}{2} \quad (\text{since } \delta_y \neq 0)$$

$$R_B = \frac{190}{3}$$

Now, Let the virtual displacement at A as δ'_y



The corresponding displacement at C & D

are $\frac{2}{6}\delta'_y$ and $\frac{2}{6}\delta'_y$

Now by virtual work principle,

$$R_A \times \delta'_y - 25 \times \frac{2}{6} \delta'_y + R_B \times 0 + 35 \times \frac{2}{6} \delta'_y = 0$$

Since $\delta'_y \neq 0$,

$$R_A - \frac{25}{3} + \frac{35}{3} = 0$$

$$R_A = \frac{-10}{3} \text{ kN}$$