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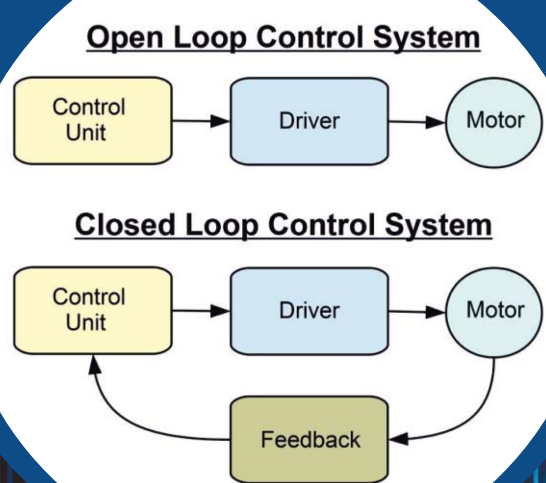
# ELECTRONICS & TELECOMMUNICATION ENGINEERING

## CONTROL SYSTEMS

Volume - 1 : Study Material with Classroom Practice Questions



Chapter

# 1 Basics of Control Systems

## (Solutions for Vol-1_Classroom Practice Questions)

**01. Ans: (c)**

**Sol:** $2\dfrac{d^2y(t)}{dt^2}+3\dfrac{dy(t)}{dt}+4y(t) = r(t) + 2r(t-1)$

Apply LT on both sides

$2s^2\, Y(s)+3sY(s)+4Y(s) = R(s)+2e^{-s}R(s)$

$Y(s)(2s^2 + 3s+4) = R(s)(1+2e^{-s})$

$$\frac{Y(s)}{R(s)} = \frac{1+2e^{-s}}{2s^2+3s+4}$$

**02. Ans: (b)**

**Sol:** $I.R = 2.e^{-2t}u(t)$

Output response $c(t) = (1-e^{-2t})\, u(t)$

Input response $r(t) = ?$

$$T.F = \frac{C(s)}{R(s)}$$

$$T.F = L(I.R) = \frac{2}{s+2}$$

$$R(s) = \frac{C(s)}{T.F} = \frac{\frac{1}{s}-\frac{1}{s+2}}{\frac{2}{s+2}} = \frac{1}{s}$$

$$R(s) = \frac{1}{s}$$

$r(t) = u(t)$

**03. Ans: (b)**

**Sol:** Unit impulse response of unit-feedback control system is given

$c(t) = t.e^{-t}$

$T.F = L(I.R)$

$$= \frac{1}{(s+1)^2}$$

$$\text{Open Loop T.F} = \frac{\text{Closed Loop T.F}}{1-\text{Closed Loop T.F}}$$

$$= \frac{\frac{1}{(s+1)^2}}{1-\frac{1}{(s+1)^2}} = \frac{1}{s^2+2s}$$

**04. Ans: (a)**

**Sol:** G changes by 10%

$$\Rightarrow \frac{\Delta G}{G}\times 100 = 10\%$$

$C_1 = 10\%$ [$\because$ open loop] whose sensitivity is 100%]

%G change = 10%

$$\frac{\%\text{ of change in M}}{\%\text{ of change in G}} = \frac{1}{1+GH}$$

$$\%\text{ of change in M} = \frac{10\%}{1+(10)1} = 1\%$$

% change in $C_2$ by 1%

**05.**

**Sol:** $M = C/R$

$$\frac{C}{R} = M = \frac{GK}{1+GH}$$

$$S_K^M = \frac{\partial M}{\partial K}\times\frac{K}{M} = 1$$

[$\because$ K is not in the loop $\Rightarrow$ sensitivity is 100%]

$$S_H^M = \frac{\partial M}{\partial H}\times\frac{H}{M} = \frac{\partial}{\partial H}\left(\frac{GK}{1+GH}\right)\frac{H}{M}$$

$$= \left(\frac{GK(-G)}{(1+GH)^2}\right)\left[\frac{H}{\frac{GK}{1+GH}}\right]$$

$$S_H^M = \frac{-GH}{(1+GH)}$$

**06.**

**Sol:** Given data

$G = 2 \times 10^3, \partial G = 100$

$$\% \text{ change in } G = \frac{\partial G}{G} \times 100 = 5\%$$

$$\% \text{ change in } M = 0.5\%$$

$$\frac{\% \text{ of change in } M}{\% \text{ of change in } G} = \frac{1}{1+GH}$$

$$\frac{0.5\%}{5\%} = \frac{1}{1+2\times10^3 H}$$

$$1+2\times10^3 H = 10$$

$$H = 4.5\times10^{-3}$$

**07. Ans: (b)**

**Sol:** $K = \frac{\text{output}}{\text{input}} = \frac{c(t)}{r(t)} = \frac{mm}{{}^0c}$

**08. Ans: (d)**

**Sol:** Introducing negative feedback in an amplifier results, increases bandwidth.

# Chapter 2 Signal Flow Graph & Block Diagram
## (Solutions for Vol-1_Classroom Practice Questions)

**01. Ans: (d)**

**Sol:** No. of loops = 3

Loop1: $-G_1G_3G_4H_1H_2H_3$

Loop2: $-G_3G_4H_1H_2$

Loop3: $-G_4H_1$

No. of Forward paths = 3

Forward Path1: $G_1G_3G_4$

Forward Path 2: $G_2G_3G_4$

Forward Path 3: $G_2G_4$

$$= \frac{G_1G_3G_4 + G_2G_3G_4 + G_2G_4}{1 + G_1G_3G_4H_1H_2H_3 + G_3G_4H_1H_2 + G_4H_1}$$

**02. Ans: (a)**

**Sol:** Number of forward paths = 2

Number of loops = 3

$$\frac{Y(s)}{R(s)} = \frac{\frac{1}{s}\times\frac{1}{s}\times\frac{1}{s}[1-0]+\frac{1}{s}}{1-\left[\frac{1}{s}\times(-1)\left(\frac{1}{s}\right)(-1)+\frac{1}{s}\times\frac{1}{s}(-1)+\left(\frac{1}{s}\times\frac{1}{s}(-1)\right)\right]}$$

$$= \frac{\frac{1}{s^3}+\frac{1}{s}}{1-\left[\frac{1}{s^2}-\frac{1}{s^2}-\frac{1}{s^2}\right]} = \frac{\frac{1+s^2}{s^3}}{1+\frac{1}{s^2}} = \frac{\frac{1+s^2}{s^3}}{\frac{s^2+1}{s^2}}$$

$$= \frac{1+s^2}{s}\times\frac{1}{s^2+1} = \frac{1}{s}$$

**03.**

**Sol:** Number of forward paths = 2

Number of loops = 5,

Two non touching loops = 4

$$TF = \frac{24[1-(-0.5)]+10[1-(-3)]}{1-[-24-3-4+(5\times2\times(-1)+(-0.5))]+[30+1.5+2]+\left(\left(\frac{-1}{2}\right)\times(-24)\right)}$$

$$= \frac{76}{88} = \frac{19}{22}$$

**04.**

**Sol:** Number of forward paths = 2

Number of loops = 5

$$T.F = \frac{G_1G_2G_3 + G_1G_4}{1 + G_2G_3H_2 + G_1G_2H_1 + G_1G_2G_3 + G_4H_2 + G_1G_4}$$

**05. Ans: (c)**

**Sol:** From the network

$$V_o(s) = \frac{1}{sC}I(s) \quad ............(1)$$

$$-V_i(s) + RI(s) + V_o(s) = 0$$

$$I(s) = \frac{1}{R}V_i(s) + \left(\frac{-1}{R}\right)V_o(s) .............(2)$$

From SFG

$$V_o(s) = x.I(s) \quad ............(3)$$

$$I(s) = \frac{1}{R}V_i(s) + y\,V_o(s) \quad .......... (4)$$

From equ(1) and (3)

$$x = \frac{1}{sC}$$

From equ(2) and (4)

$$y = -\frac{1}{R}$$

**06. Ans: (a)**

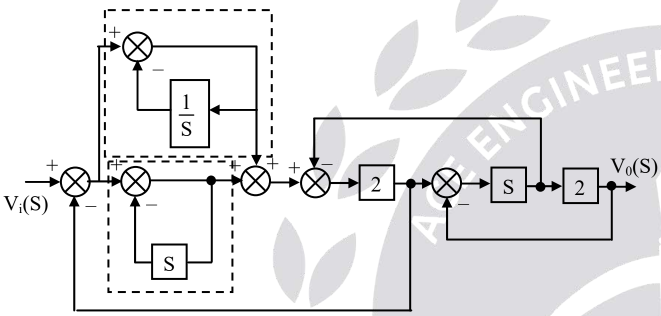
**Sol:** Use gain formula

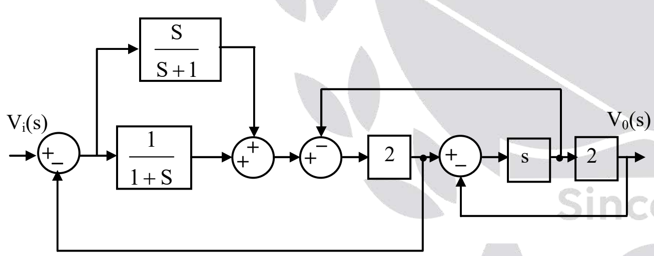
$$\text{transfer function} = \frac{G(s)}{1-\left(G(s)\dfrac{1}{G(s)}+G(s)\right)}$$

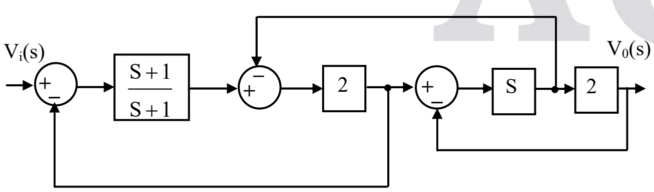
$$= \frac{G(s)}{1-1-G(s)} = -1$$

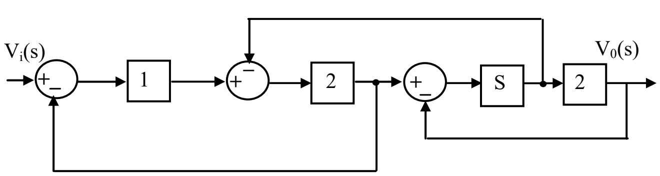
**07.**

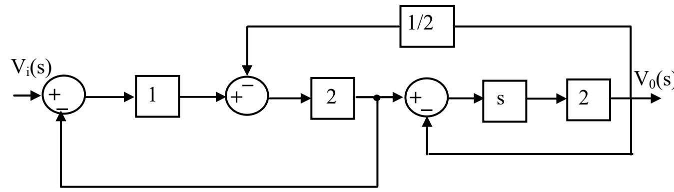
**Sol:**

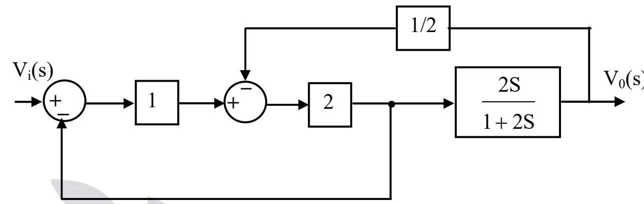


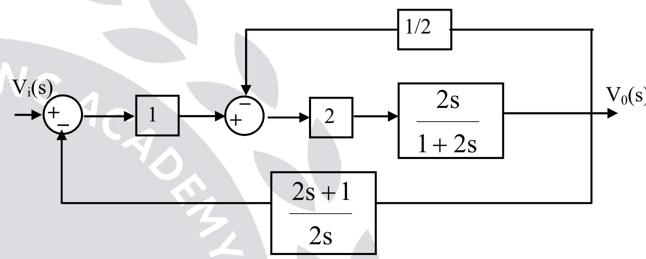



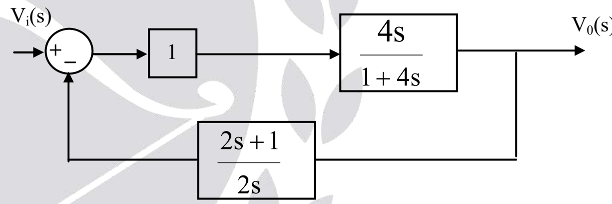




$$\Rightarrow \frac{V_0(s)}{V_i(s)} = \frac{\dfrac{4s}{1+4s}}{1+\dfrac{2(2s+1)}{1+4s}} = \frac{4s}{8s+3}$$

**08.**

**Sol:** Apply Mason's Gain formula

$$M = \frac{Y_{out}}{Y_{in}} = \frac{\sum_{k=1}^{N} M_k \Delta_k}{\Delta}$$

**No. of forward paths = 2**

First forward path gain = $G_1G_2G_3G_4$

Second forward path gain = $G_5G_6G_7G_8$

**No. of loops = 4**

First loop gain = $-G_2H_2$

Second loop gain = – $G_6H_6$

Third loop gain = – $G_3H_3$

Fourth loop gain = – $G_7H_7$

**Non touching loops = 4**

Loop gains $\rightarrow G_2H_2G_6H_6$

$\rightarrow G_2H_2G_7H_7$

$\rightarrow G_6H_6G_7H_7$

$\rightarrow G_2H_2G_3H_3$

Transfer function =

$$\frac{G_1G_2G_3G_4(1+G_6H_6+G_7H_7)+G_5G_6G_7G_8(1+G_2H_2+G_3H_3)}{1+G_2H_2+G_3 \cdot H_3+G_6H_6+G_7H_7+G_2H_2G_6H_6+G_2H_2G_7H_7+G_3H_3G_6H_6+G_3H_3G_7H_7}$$

# Chapter 3 Time Response Analysis

## (Solutions for Vol-1_Classroom Practice Questions)

**01. Ans: (a)**

**Sol:** $\dfrac{C(s)}{R(s)} = \dfrac{1}{1+sT}$, $R(s) = \dfrac{8}{s}$

$$C(s) = \frac{8}{s(1+sT)} \Rightarrow c(t) = 8\left(1-e^{-t/T}\right)$$

$$3.6 = 8\left(1-e^{\frac{-0.32}{T}}\right)$$

$$0.45 = 1 - e^{\frac{-0.32}{T}}$$

$$0.55 = e^{\frac{-0.32}{T}}$$

$$-0.59 = \frac{-0.32}{T}$$

$T = 0.535$ sec

**02. Ans: (c)**

**Sol:** $\cos\phi = \xi$

$\cos 60 = 0.5$

$\cos 45 = 0.707$

Poles left side $0.5 \le \xi \le 0.707$

Poles right side $-0.707 \le \xi \le -0.5$

$\therefore 0.5 \le |\xi| \le 0.707$

3 rad/s $\le \omega_n \le$ 5 rad/s

**03. Ans: (c)**

**Sol:** For R-L-C circuit:

$$T.F = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{1}{Cs} I(s)$$

$$= \frac{1}{Cs}\frac{V_i(s)}{R + Ls + \frac{1}{Cs}}$$

$$T.F = \frac{V_o(s)}{V_i(s)} = \frac{1}{RCs + LCs^2 + 1}$$

$$= \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad 2\xi\omega_n = \frac{R}{L}$$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\xi = \frac{10}{2}\sqrt{\frac{10\times10^{-6}}{1\times10^{-3}}} = 0.5$$

$$M.P = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$= 16.3\% \approx 16\%$$

**04. Ans: (b)**

**Sol:** $$TF = \frac{8/s(s+2)}{1-\left(\frac{-8\,as}{s(s+2)} - \frac{8}{s(s+2)}\right)}$$

$$= \frac{8}{s(s+2) + 8as + 8}$$

$$= \frac{8}{s^2 + 2s + 8as + 8}$$

$$= \frac{8}{s^2 + (2+8a)s + 8}$$

$$\omega_n^2 = 8 \Rightarrow \omega_n = 2\sqrt{2}$$

$2\xi\omega_n = 2 + 8a$

$$\xi = \frac{1+4a}{2\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1+4a}{2\sqrt{2}} \Rightarrow a = 0.25$$

**05. Ans: 4 sec**

**Sol:** $$\text{T.F} = \frac{100}{(s+1)(s+100)}$$

$$= \frac{100}{s^2+101s+100}$$

$\omega_n^2 = 100$

$\omega_n = 10$

$2\xi\omega_n = 101$

$$\xi = \frac{101}{20}$$

$\xi > 1 \rightarrow$system is over damped i.e., roots are real & unequal.

Using dominate pole concept,

$$\text{T.F} = \frac{100}{100(s+1)} = \frac{1}{s+1}, \text{ Here } \tau = 1 \text{ sec}$$

$\therefore$ Setting time for 2% criterion = $4\tau$

= 4 sec

**06.**

**Sol:** $$M_p = \frac{C(t_p)-C(\infty)}{C(\infty)}$$

$$= \frac{1.254-1.04}{1.04} = 0.2$$

$$\xi = \sqrt{\frac{(\ln M_P)^2}{(\ln M_P)^2+\pi^2}}$$

$M_p = 0.2$ ; $\xi = 0.46$

**07. Ans: (d)**

**Sol:** Given data: $\omega_n = 2, \zeta = 0.5$

Steady state gain =1

$$\text{OLTF} = \frac{K_1}{s^2+as+2} \text{ and } H(s) = K_2$$

$$\text{CLTF} = \frac{G(s)}{1+G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1}{s^2+as+2+K_1K_2}$$

DC or steady state gain from the TF

$$\frac{K_1}{2+K_1K_2} = 1$$

$K_1(1 - K_2) = 2$ ……… (1)

CE is $s^2 + as + 2 + K_1K_2 = 0$

$\omega_n = \sqrt{2+K_1K_2} = 2$

$4 = (2+K_1K_2)$

$K_1K_2 = 2$ ………. (2)

Solving equations (1) & (2) we get

$K_1 = 4, \quad K_2 = 0.5$

$2\zeta\,\omega_n = a$

$$2\times\frac{1}{2}\times 2 = a$$

$a = 2$

**08. Ans: A – T, B – S, C- P, D – R, E – Q**

**Sol:**

(A) If the poles are real & left side of s-plane, the step response approaches a steady state value without oscillations.

(B) If the poles are complex & left side of s-plane, the step response approaches a steady state value with the damped oscillations.

(C) If poles are non-repeated on the jω axis, the step response will have fixed amplitude oscillations.

(D) If the poles are complex & right side of s-plane, response goes to '∞' with damped oscillations.

(E) If the poles are real & right side of s-plane, the step response goes to '∞' without any oscillations.

**09. Ans: (c)**

**Sol:** If R ↑ damping ↑

$$\Rightarrow \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

(i) If R↑, steady state voltage across C will be reduced (wrong)

(Since steady state value does not depend on $\xi$)

If $\xi$ ↑, C (∞) = remain same

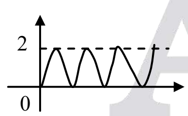
(ii) If $\xi \uparrow$, $\omega_d \downarrow \left(\omega_d = \omega_n\sqrt{1-\xi^2}\right)$

(iii) If $\xi \downarrow$, $t_s \uparrow \Rightarrow 3^{rd}$

Statement is false

(iv) If $\xi = 0$

True




⇒ 2 and 4 are correct

**10.**

**Sol:** (i) Unstable system

∴ error = ∞

(ii) $G(s) = \frac{10(s+1)}{s^2}$

Step → R (s) = $\frac{1}{s}$

$k_p = \infty$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+\infty} = 0$$

Parabolic ⇒ $k_a = 10$

$$e_{ss} = \frac{1}{10} = 0.1$$

**11.**

**Sol:** $G(s) = 10/s^2$ (marginally stable system)

∴ Error can't be determined

**12.**

**Sol:** $e_{ss} = \frac{1}{11}$, $R(s) = \frac{1}{s}$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+k_p} = \frac{1}{11} = \frac{1}{1+10}$$

$$k_p = \underset{s\to 0}{Lt}\ G(s)$$

$$10 = \underset{s\to 0}{Lt}\ G(s)$$

k = 10

$R(s) = \frac{1}{s^2}$ (ramp)

$$e_{ss} = \frac{A}{k_v} = \frac{1}{k_v} = \frac{1}{10}$$

(System is increased by 1)

⇒ $e_{ss} = 0.1$

**13. Ans: (a)**

**Sol:** $T(s) = \frac{(s-2)}{(s-1)\ (s+2)^2}$ (unstable system)

**14. Ans: (b)**

**Sol:** Given data: r(t) = 400tu(t) rad/sec

Steady state error = $10^\circ$

i.e., $e_{ss} = \frac{\pi}{180^\circ}(10^\circ)$ radians

$G(s) = \dfrac{20K}{s(1+0.1s)}$ and $H(s) = 1$

$r(t) = 400tu(t) \Rightarrow 400/s^2$

$\text{Error } (e_{ss}) = \dfrac{A}{K_V} = \dfrac{400}{K_V}$

$K_V = \lim\limits_{s\to 0} s\,G(s)$

$K_V = \lim\limits_{s\to 0} s\dfrac{20K}{s(1+0.1s)}$

$K_V = 20K$

$e_{ss} = \dfrac{400}{20K}$

$e_{ss} = \dfrac{20}{K} = \dfrac{\pi}{18}$

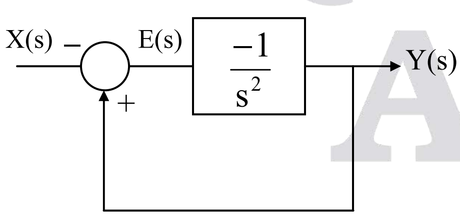
$K = 114.5$

**15. Ans: (d)**

**Sol:** $\dfrac{d^2y}{dt^2} = -e(t)$

$s^2 Y(s) = -E(s)$

$x(t) = t\,u(t) \Rightarrow X(s) = \dfrac{1}{s^2}$




$Y(s) = \dfrac{-1}{s^2} E(s)$

$\dfrac{Y(s)}{E(s)} = \dfrac{-1}{s^2}$

$\dfrac{E(s)}{X(s)} = \dfrac{-1}{1+\dfrac{1}{s^2}}$

$E(s) = \dfrac{-s^2}{1+s^2} X(s)$

$= \dfrac{-s^2}{1+s^2} \times \dfrac{1}{s^2} = \dfrac{-1}{1+s^2}$

$= L^{-1}\left[\dfrac{-1}{1+s^2}\right] = -\sin t$

**16. Ans: (a)**

**Sol:** $e_{ss} = 0.1$ for step input

For pulse input = 10

time = 1 sec

error is function of input

$t \to \infty$ input = 0

$\therefore$ Error = zero

**17. Ans: (c)**

**Sol:** $\dfrac{C(s)}{R(s)} = \dfrac{\dfrac{100}{(s+1)(s+5)}}{1+\dfrac{100\times 0.2}{(s+1)(s+5)}}$

$= \dfrac{100}{(s+1)(s+5)+20}$

$= \dfrac{100}{s^2+6s+5+20}$

$= \dfrac{100}{s^2+6s+25}$

$\omega_n^2 = 25, \omega_n = 5$

$2\xi\omega_n = 6$

$\xi = \dfrac{6}{10} = \dfrac{3}{5}$

$\omega_d = \omega_n\sqrt{1-\xi^2}$

$= 5\sqrt{1-\left(\dfrac{3}{5}\right)^2}$

$= 5 \times \dfrac{4}{5} = 4 \text{ rad/sec}$

**18. Ans: (c)**

**Sol:** $f(t) = \dfrac{Md^2x}{dt^2} + B\dfrac{dx}{dt} + Kx(t)$

Applying Laplace transform on both sides, with zero initial conditions

$F(s) = Ms^2X(s)+BsX(s) + KX(s)$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Characteristic equation is $Ms^2 + Bs+K = 0$

$$s^2 + \frac{B}{M}s + \frac{K}{M} = 0$$

Compare with $s^2 + 2\zeta\,\omega_n s + \omega_n^2 = 0$

$$2\zeta\omega_n = \frac{B}{M}$$

$$\xi = \frac{B}{2\sqrt{MK}} \qquad \omega_n = \sqrt{\frac{K}{M}}$$

Time constant $T = \dfrac{1}{\zeta\omega_n}$

$$= \frac{1}{B}\times 2M$$

$$T = \frac{2M}{B}$$

Hence, statements 2 & 3 are correct

**19. Ans: (c)**

**Sol:** type 1 system has a infinite positional error constant.

**20. Ans: (a)**

**Sol:** Given $G(s) = \dfrac{1}{s(1+s)(s+2)}, H(s) = 1$ .

It is type-I system

Positional error constant $k_p = \underset{s\to 0}{Lt}\; G(s)H(s)$

$$k_p = \underset{s\to 0}{Lt}\; \frac{1}{s(1+s)(s+2)}$$

$$= \infty$$

Steady state error due to step input

$$= \frac{1}{1+k_p} = 0$$

**21.**

**Sol:** Open loop T/F $G(s) = \dfrac{A}{S(S+P)}$

C.L T/F $= \dfrac{A}{S^2 + SP + A}$

$\omega_n = \sqrt{A}$

Setting time $= 4/\xi\omega_n = 4$

$2\xi\omega_n = P \qquad \therefore \dfrac{4}{P/2} = 4$

$\xi\omega_n = P/2 \qquad \Rightarrow P = \dfrac{8}{4} = 2$

$$e^{\frac{-\pi\xi}{\sqrt{1+\xi^2}}} = 0.1 \Rightarrow \frac{\pi\xi}{\sqrt{1-\xi^2}} = \ell n 10$$

$$= 2.3$$

$$\Rightarrow \frac{\xi^2}{1-\xi^2} = 0.5373$$

$$\Rightarrow 1.5373\,\xi^2 = 0.5373$$

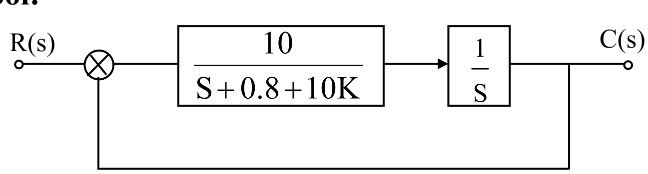
$$\xi = 0.59$$

$\xi\omega_n = 1$

$\Rightarrow \omega_n = 1.694 \Rightarrow A = \omega_n^2 = 2.87$

**22.**

**Sol:**



$$\frac{C(s)}{R(s)} = \frac{10}{s(s+0.8+10K)+10}$$

$$= \frac{10}{s^2+s(0.8+10K)10}$$

$\omega_n = \sqrt{10}$ $\quad 2\xi\omega_n = 0.8 + 10\text{ K}$

$$\Rightarrow 2\times\frac{1}{2}\times\sqrt{10} = 0.8+10K$$

$$\Rightarrow K = 0.236$$

$$t_r = \frac{\pi-\phi}{\omega_d} = \frac{\pi-\cos^{-1}(\xi)}{\omega_n\sqrt{1-\xi^2}}$$

$$= \frac{\pi-\pi/3}{2.88} = 0.74\text{sec}$$

$$t_p = \frac{\pi}{\omega_d} = 1.1\text{sec}$$

$$\%Mp = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = 0.163\times 100 = 16.3\%$$

$$t_s\text{ (for 2\%)} = \frac{4}{\xi\omega_n} = \frac{4}{0.5\times\sqrt{10}} = 2.53\text{sec}$$

# Chapter 4 Stability
## (Solutions for Vol-1_Classroom Practice Questions)

**01.**

**Sol:** CE = $s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$

| | | | | |
|---|---|---|---|---|
| $s^5$ | 1 | 8 | 7 | |
| $s^4$ | 4(1) | 8(2) | 4(1) | |
| $s^3$ | 6(1) | 6(1) | 0 | |
| $s^2$ | 1 | 1 | 0 | $\rightarrow$ Row of AE |
| $s^1$ | 0(2) | 0 | 0 | $\rightarrow$ Row of zero |
| $s^0$ | 1 | | | |

No. of AE roots = 2
No. of sign changes
Below AE = 0
No. of RHP = 0
No. of LHP = 0
No. of $j\omega p$ = 2

No. of CE roots = 5
No. of sign changes
in 1st column = 0
$\therefore$ No .of RHP = 0
No. of $j\omega p$ = 2
$\Rightarrow$ No .of LHP = 3

System is marginally stable.

(ii) $s^2 + 1 = 0$

$s = \pm 1\, j = \pm j\omega_n$

$\omega_n = 1$ rad/ sec

Oscillating frequency $\omega_n = 1$ rad/sec

**02.**

**Sol:** (i) $s^5 + s^4 + s^3 + s^2 + s + 1 = 0$

| | | | |
|---|---|---|---|
| $+\, s^5$ | 1 | 1 | 1 |
| $+\, s^4$ | 1 | 1 | 1 |
| $+\, s^3$ | 0(2) | 0(1) | 0 |
| $+\, s^2$ | $\frac{1}{2}$ | 1 | |
| (1) $-\, s^1$ | $-3$ | 0 | |
| (2) $+\, s^0$ | 1 | | |

AE (1) = $s^4 + s^2 + 1 = 0$

$$\frac{d(AE)}{ds} = 4s^3 + 2s = 0$$

$\Rightarrow 2s^3 + s = 0$

**AE**

No. of sign changes below AE = 2
No. of AE roots = 4
No .of RHP = 2
No .of LHP = 2
No. of $j\omega p$ = 0

**CE**

No. of sign changes in 1st column = 2
No. of CE roots = 5
No. of RHP = 2
No. of LHP = 3
No. of $j\omega p$ = 0

System is unstable

(ii) $s^6 + 2s^5 + 2s^4 + 0s^3 - s^2 - 2s - 2 = 0$

| | | | | |
|---|---|---|---|---|
| $s^6$ | 1 | 2 | $-1$ | $-2$ |
| $s^5$ | 2(1) | 0 | $-2(-1)$ | 0 |
| $s^4$ | 2(1) | +0 | $-2(-1)$ | 0 |
| $s^3$ | 0(4) | 0 | 0 | 0 |
| $s^2$ | $0(\varepsilon)$ | $-1$ | 0 | 0 |
| $s^1$ | $4/\varepsilon$ | | | |
| $-s^0$ | $-1$ | | | |

$AE = s^4 - 1 = 0$

$$\frac{dAE}{ds} = 4s^3 + 0 = 0$$

| **CE** | **AE** |
|---|---|
| No. of CE roots = 6 | No. of AE roots = 4 |
| No. of sign changes in the 1st column= 1 | No. of sign changes below AE = 1 |
| No .of RHP = 1 | No. of RHP = 1 |
| No .of LHP = 3 | No. of $j\omega p$ = 2 |
| No. of $j\omega p$ = 2 | No. of LHP = 1 |

**03.**

**Sol:** $CE = s^3 + 20s^2 + 16s + 16K = 0$

$$\begin{array}{c|cc} s^3 & 1 & 16 \\ s^2 & 20 & 16K \\ s^1 & \frac{20(16)-16K}{20} & 0 \\ s^0 & 16K & \end{array}$$

(i) For stability $\dfrac{20(16)-16K}{20} > 0$

$\Rightarrow 20(16) - 16K > 0$

$\Rightarrow K < 20$ and $16K > 0 \Rightarrow K > 0$

Range of K for stability $0 < K < 20$

(ii) For the system to oscillate with $\omega_n$ it must be marginally stable

i.e., $s^1$ row should be 0

$s^2$ row should be AE

$$\boxed{\therefore \text{A.E roots} = \pm j\omega_n}$$

$\therefore s^1$ row $\Rightarrow 20(16) - 16K = 0$

$\Rightarrow K = 20$

AE is $20s^2 + 16K = 0$

$20s^2 + 16(20) = 0$

$\Rightarrow s = \pm j4$

$\omega_n = 4$ rad/sec

**04.**

**Sol:** $CE = 1 + \dfrac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$

$s^3 + as^2 + (K+2)s + K + 1 = 0$

$s^3 + as^2 + (K+2)s + (K+1) = 0$

$$\begin{array}{c|cc} s^3 & 1 & K+2 \\ s^2 & a & K+1 \\ s^1 & \frac{a(K+2)-(K+1)}{a} & 0 \\ s^0 & K+1 & \end{array}$$

Given,

$\omega_n = 2$

$\Rightarrow s^1$ row $= 0$

$s^2$ row is A.E

$a(K+2) - (K+1) = 0$

$$a = \frac{K+1}{K+2}$$

$AE = as^2 + K + 1 = 0$

$$= \frac{K+1}{K+2}s^2 + K + 1 = 0$$

$$(k+1)\left(\frac{s^2}{k+2} + 1\right) = 0$$

$s^2 + k + 2 = 0$

$s = \pm j\sqrt{(k+2)}$

$\omega_n = \sqrt{k+2} = 2$

$k = 2$

$$a = \frac{k+1}{k+2} = \frac{3}{4} = 0.75$$

**05.**

**Sol:** $s^3 + ks^2 + 9s + 18$

$$\begin{array}{c|cc} s^3 & 1 & 9 \\ s^2 & K & 18 \\ s^1 & \dfrac{9K-18}{K} & 0 \\ s^0 & 18 & \end{array}$$

Given that system is marginally stable,
Hence
$s^1$ row = 0

$$\frac{9K-18}{K} = 0$$

$9K = 18 \Rightarrow K = 2$
A.E is $9s^2 + 18 = 0$
$Ks^2 + 18 = 0$,
$2s^2 + 18 = 0$
$2s^2 = -18$
$s = \pm j3$
$\therefore\ \omega_n = 3$ rad/sec.

**06. Ans: (d)**

**Sol:** Given transfer function $G(s) = \dfrac{k}{(s^2+1)^2}$

Characteristic equation $1 - G(s).H(s) = 0$

$$1 - \frac{k}{(s^2+1)^2} = 0$$

$s^4 + 2s^2 + 1 - k = 0$ ….. (1)
RH criteria

| | | | |
|---|---|---|---|
| $s^4$ | 1 | 2 | 1–K |
| $s^3$ | 4 | 4 | |
| $s^2$ | 1 | 1–K | |
| $s^1$ | 4K | | |
| $s^o$ | 1–K | | |

$AE = s^4 + 2s^2 + 1 - K$

$$\frac{d}{ds}(AE) = 4s^3 + 4s$$

$1-K > 0$ no poles are on RHS plane and LHS plane.
All poles are on $j\omega$- axis
$\therefore\ 0 < K < 1$ system marginally stable

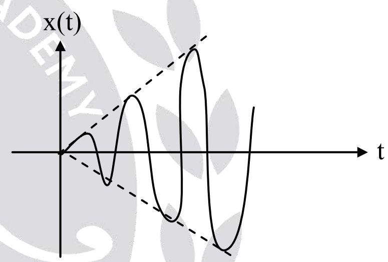
**07. Ans: (d)**

**Sol: Assertion: FALSE**
Let the TF= s. "s" is the differentiator
Impulse response $L^{-1}[TF] = L^{-1}[s] = \delta'(t)$

$$\underset{t\to\infty}{Lt}\ \delta'(t) = 0$$

$\therefore$ It is BIBO stable

**Reason: True**
$x(t) = t \sin t$




$$\underset{t\to\infty}{Lt}\ x(t) = \underset{t\to\infty}{Lt}\ t\sin t \text{ is unbounded}$$

**08. Ans: (a)**

**Sol: Assertion: TRUE**
If feedback is not properly utilized the closed loop system may become unstable.
**Reason: True**
Feedback changes the location of poles

Let $G(s) = \dfrac{-2}{s+1}$ $\quad H(s) = 1$

Open loop pole $s = -1$ (stable)

$$CLTF = \frac{\dfrac{-2}{s+1}}{1+\dfrac{-2}{s+1}} = \frac{-2}{s-1}$$

Closed loop pole is at $s = 1$ (unstable)
$\therefore$After applying the feedback no more system is open loop. It becomes closed loop system. Hence poles are affected.

# Chapter 5 Root Locus Diagram

## (Solutions for Vol-1_Classroom Practice Questions)

**01. Ans: (a)**

**Sol:** $s_1 = -1 + j\sqrt{3}$

$s_2 = -3 - j\sqrt{3}$

$$G(s).H(s) = \frac{K}{(s+2)^3}$$

$s_1 = -1 + j\sqrt{3}$

$$G(s).H(s) = \frac{K}{\left(-1+j\sqrt{3}+2\right)^3}$$

$$= \frac{K}{\left(1+j\sqrt{3}\right)^3}$$

$$= -3\tan^{-1}(\sqrt{3})$$

$$= -180^\circ$$

It is odd multiples of 180°, Hence $s_1$ lies on Root locus

$s_2 = -3 - j\sqrt{3}$

$$G(s).H(s) = \frac{K}{\left(-3-j\sqrt{3}+2\right)^3}$$

$$= \frac{K}{\left(-1-j\sqrt{3}\right)^3}$$

$$= -3\,[180^\circ + 60^\circ] = -720^\circ$$

It is not odd multiples of 180°, Hence $s_2$ is not lies on Root locus.

**02. Ans: (a)**

**Sol:** Over damped – roots are real & unequal $\Rightarrow 0 < k < 4$

(b) k = 4 roots are real & equal

$\Rightarrow$ Critically damped $\xi = 1$

(c) k > 4 $\Rightarrow$ roots are complex

$0 < \xi < 1 \Rightarrow$ under damped

**03. Ans: (a)**

**Sol:** Asymptotes meeting point is nothing but centroid

$$\text{centroid } \sigma = \frac{\sum \text{poles} - \sum \text{zeros}}{p - z}$$

$$= \frac{-3-0}{3-0} = -1$$

centroid = (–1, 0)

**04. Ans: (b)**

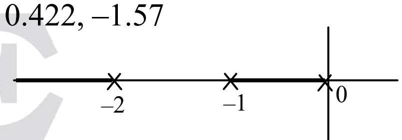
**Sol:** break point = $\dfrac{dK}{ds} = 0$

$$\frac{d}{ds}\left(G_1(s).H_1(s)\right) = 0$$

$$\frac{d}{ds}\left[s(s+1)(s+2)\right] = 0$$

$3s^2 + 6s + 2 = 0$

$s = -0.422, -1.57$




But s = – 1.57 do not lie on root locus

So, s = –0.422 is valid break point.

Point of intersection wrt jω axis

$$s^3 + 3s^2 + 2s + k = 0$$

| | | |
|---|---|---|
| $s^3$ | 1 | 2 |
| $s^2$ | 3 | k |
| $s^1$ | $\dfrac{6-k}{3}$ | 0 |
| $s^0$ | k | |

As $s^1$ Row = 0

k = 6

$3s^2 + 6 = 0$

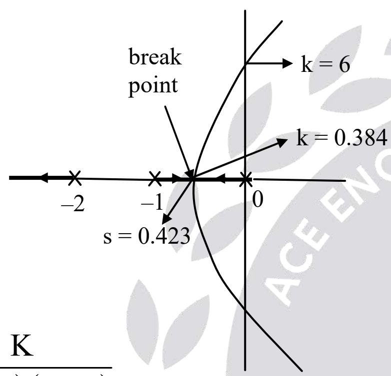
$s^2 = -2$

$s = \pm j\sqrt{2}$

point of inter section: $s = \pm\ j\sqrt{2}$

**05. Ans: (b)**

**Sol:**




$$\frac{K}{s(s+1)(s+2)}$$

substitute s = –0.423 and apply the magnitude criteria.

$$\left|\frac{K}{(-0.423)(-0.423+1)(-0.423+2)}\right| = 1$$

K = 0.354

when the roots are complex conjugate then the system response is under damped.

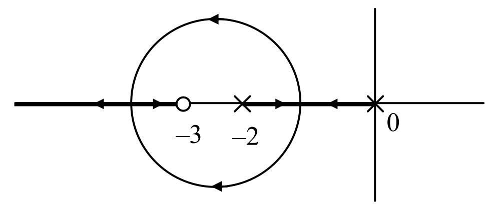
From K > 0.384 to K < 6 roots are complex conjugate then system to be under damped the values of k is 0.384 < K < 6.

**06. Ans: (c)**

**Sol:** If the roots are lies on the real axis then system exhibits the non-oscillatory response. from K ≥ 0 to K ≤ 0.384 roots lies on the real axis. Hence for 0 ≤ K ≤ 0.384 system exhibits the non-oscillatory response.

**07. Ans: (a)**

**Sol:**




$$\frac{d}{ds}[G(s).H(s)] = \frac{d}{ds}\left[\frac{k(s+3)}{s(s+2)}\right]$$

$s^2 + 6s + 6 = 0$

break points – 1.27, – 4.73

$$\text{radius} = \frac{4.73 - 1.27}{2} = 1.73$$

center = (–3, 0)

**08. Ans: (c)**

**Sol:** $G(s).H(s) = \dfrac{K(s+3)}{s(s+2)}$

$$k\Big|_{s=-4} = \left|\frac{(-4)(-4+2)}{(-4+3)}\right|$$

$$= \left|\frac{(-4)(-2)}{(-1)}\right| = 8$$

**09. Ans: (a)**

**Sol:** $s^2-4s+8 = 0 \Rightarrow s = 2\pm2j$ are two zeroes

$s^2+4s+8 = 0 \Rightarrow s = -2\pm2j$ are two poles

$$\phi_A = 180 - \angle GH\Big|_{s=2\pm2j}$$

$$GH = \frac{k[s-(2+2j)][s-(2-2j)]]}{[s-(-2+2j)][s-(-2-2j)]]}$$

$$\angle GH\Big|_{s=2+2j} = \frac{\angle k \angle 4j}{\angle 4 \angle 4+4j}$$

$$= 90^\circ - 45^\circ = 45^\circ$$

$\phi_A = 180^\circ - 45^\circ = \pm 135^\circ$

**10. Ans: (b)**

**Sol:** $s^2-4s+8=0 \Rightarrow s=2\pm 2j$ are two zeroes

$s^2+4s+8=0 \Rightarrow s=-2\pm 2j$ are two poles

$$\phi_d = 180^0 + \angle GH\Big|_{S=-2\pm 2j}$$

$$\angle GH\Big|_{S=-2\pm 2j} = \angle \frac{k[s-(2+2j)][s-(2-2j)]}{[s-(-2+2j)][s-(-2-2j)]}\Bigg|_{S=-2\pm 2j}$$

$$= \frac{\angle k(-4)(-4+4j)}{\angle 4j}$$

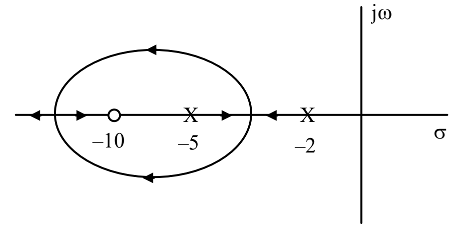
$$= 180^\circ + 180^\circ - 45^\circ - 90^\circ = 225^\circ$$

$$\phi_d = 180^\circ + 225^\circ = 405^\circ$$

$$\therefore \phi_d = \pm 45^\circ$$

**11. Ans: (d)**

**Sol:** Poles s = –2, –5 ; Zero s = –10




$\therefore$ Breakaway point exist between –2 and –5

**12.**

**Sol:** Refer Pg No: 84, Vol-1 Ex: 7

# Chapter 6 Frequency Response Analysis
## (Solutions for Vol-1_Classroom Practice Questions)

**01. Ans: (c)**

**Sol:** $G(s).H(s) = \dfrac{100}{s(s+4)(s+16)}$

Phase crossover frequency ($\omega_{pc}$):

$\angle G(j\omega).H(j\omega)/_{\omega=\omega_{pc}} = -180°$

$-90° - \tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -180°$

$-\tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -90°$

$\tan[\tan^{-1}(\omega_{pc}/4) + \tan^{-1}(\omega_{pc}/16)] = \tan(90°)$

$$\frac{\frac{\omega_{pc}}{4}+\frac{\omega_{pc}}{16}}{1-\frac{\omega_{pc}}{4}.\frac{\omega_{pc}}{16}} = \frac{1}{0}$$

$\omega_{pc}^2 = 16 \times 4 \Rightarrow \omega_{pc} = 8$ rad/sec

**02. Ans: (d)**

**Sol:** $G(s).H(s) = \dfrac{100}{s(s+2)(s+16)}$

Gain margin (G.M) = $\dfrac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$

$$|G(j\omega).H(j\omega)|_{\omega=\omega_{pc}} = \frac{100}{\omega_{pc}\sqrt{\omega_{pc}^2+16}\sqrt{\omega_{pc}^2+16^2}}$$

$$= \frac{5}{64}$$

$$G.M = \frac{64}{5} = 12.8$$

**03. Ans: (c)**

**Sol:** $G(s).H(s) = \dfrac{2e^{-0.5s}}{(s+1)}$

gain crossover frequency,

$\omega_{gc} = |G(j\omega).H(j\omega)|_{\omega=\omega_{gc}} = 1$

$$\frac{2}{\sqrt{\omega_{gc}^2+1}} = 1$$

$\omega_{gc}^2 + 1 = 4 \Rightarrow \omega_{gc} = \sqrt{3}$ rad/sec

**04. Ans: (b)**

**Sol:** $\omega_{gc} = \sqrt{3}$ rad/sec

P.M = $180° + \angle G(j\omega).H(j\omega)/\omega = \omega_{gc}$

$\angle G(j\omega).H(j\omega)/_{\omega=\omega_{gc}} = -0.5\,\omega_{gc} - \tan^{-1}(\omega_{gc})$

$= -109.62°$

P.M = 70.39°

**05. Ans: (a)**

**Sol:** $M_r = 2.5 = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

$$2\xi\sqrt{1-\xi^2} = \frac{1}{2.5}$$

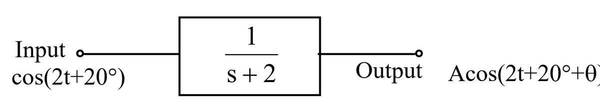
$\xi^4 - \xi^2 + 0.04 = 0$

$\xi^2 = 0.958$ $\quad$ $\xi^2 = 0.0417$

$\xi = 0.204$ $\quad$ ($M_r > 1$)

**06. Ans: (a)**

**Sol:** Closed loop T.F = $\dfrac{1}{s+2}$




$$A = \frac{1}{\sqrt{\omega^2+4}} = \frac{1}{\sqrt{4+4}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

$\phi = -\tan^{-1}\omega/2$

$= -\tan^{-1}2/2$

$\Rightarrow \phi = -\tan^{-1}(1) = -45°$

$$\text{output} = \frac{1}{2\sqrt{2}}\cos\left(2t + 20^\circ - 45^\circ\right)$$

$$= \frac{1}{2\sqrt{2}}\cos(2t - 25^\circ)$$

**07. Ans: (c)**

**Sol:** Initial slope = –40 dB/dec

Two integral terms $\left(\frac{1}{s^2}\right)$

$\therefore$ Part of TF $= G(s)H(s) = \dfrac{K}{s^2}$

at ω = 0.1

change in slope = –20 – (–40)

= 20°

Part of TF = G(s) H(s) $= \dfrac{K\left(1+\dfrac{s}{0.1}\right)}{s^2}$

At ω = 10 slope changed to –60 dB/dec

Change in slope = –60–(–20)

= – 40dB/dec

$$\text{TF }(G(s)H(s)) = \frac{K\left(1+\dfrac{s}{0.1}\right)}{s^2\left(\dfrac{s}{10}+1\right)^2}$$

20 log K− 2 (20 log 0.1) = 20 dB

20 log K = 20–40

20 log K = –20

K = 0.1

$$G(s)H(s) = \frac{(0.1)\left(1+\dfrac{s}{0.1}\right)}{s^2\left(1+\dfrac{s}{10}\right)^2}$$

$$= \frac{(0.1)\times 10^2(s+0.1)}{(0.1)s^2(s+10)^2}$$

$$G(s)H(s) = \frac{100(s+0.1)}{s^2(s+10)^2}$$

**08. Ans: (b)**

**Sol:** $$G(s)H(s) = \frac{Ks}{\left(1+\dfrac{s}{2}\right)\left(1+\dfrac{s}{10}\right)}$$

12 = 20 log K+ 20 log 0.5

12 = 20logK + (−6)

20 log K = 18 dB = 20 log $2^3$

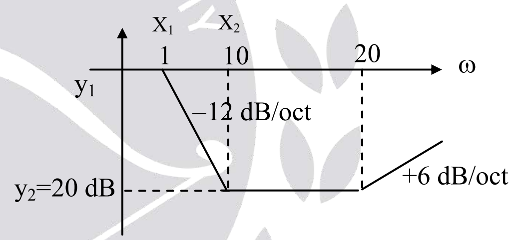
K = 8

$$G(s)H(s) = \frac{8s\times 2\times 10}{(2+s)(10+s)}$$

$$G(s)H(s) = \frac{160s}{(2+s)(10+s)}$$

**09. Ans: (b)**

**Sol:**




$$G(s)H(s) = \frac{K\left(1+\dfrac{s}{10}\right)^2\left(1+\dfrac{s}{20}\right)}{(1+s)^2}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = -40\,\text{dB/dec}$$

$$\frac{20 - y_1}{\log 10 - \log 1} = -40$$

$$y_1 = +60\,\text{dB}\Big|_{\omega\le 1}$$

$\Rightarrow$ 20 log K = 60

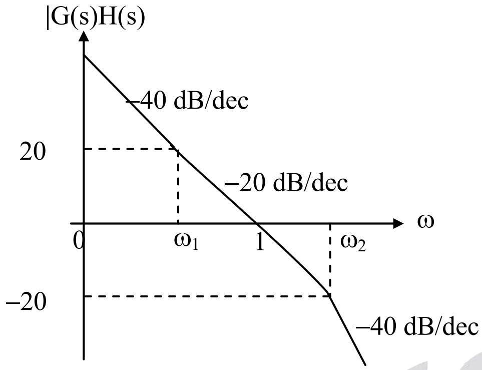
K = $10^3$

$$G(s)H(s) = \frac{10^3(s+10)^2(s+20)}{10^2\times 20\times(s+1)^2}$$

$$= \frac{(s+10)^2(s+20)}{2(s+1)^2}$$

**10. Ans: (d)**

**Sol:**




**$\omega_1$ calculation:**

$$\frac{0-20}{\log 1-\log \omega_1}$$

= –20 dB/dec

$\omega_1 = 0.1$

**$\omega_2$ calculation:**

$$\frac{-20-0}{\log \omega_2-\log 1}$$

= – 20dB/dec

$\omega_2 = 10$

$$G(s)H(s)=\frac{K\left(1+\frac{s}{0.1}\right)}{s^2\left(1+\frac{s}{10}\right)}$$

20logK–2 (20 log 0.1) = 20

20 logK = 20 – 40

K = 0.1

$$G(s)H(s)=\frac{0.1\times\frac{1}{0.1}(0.1+s)}{s^2\frac{1}{10}(10+s)}$$

$$=\frac{10(0.1+s)}{s^2(10+s)}$$

**11.**

**Sol:** $\dfrac{200}{s(s+2)}=\dfrac{100}{s\left(1+\frac{s}{2}\right)}$

$x = -KT \Rightarrow -(100)\times\frac{1}{2} = x = -50$

**12. Ans: (c)**

**Sol:** For stability (–1, j0) should not be enclosed by the polar plot.

For stability

1 > 0.01 K

$\Rightarrow$ K < 100

**13.**

**Sol:** GM = –40 dB

$20\log\frac{1}{a}=-40 \Rightarrow a=10^2$

POI = 100

**14.**

**Sol: (i)** GM = $\frac{1}{0.1}$ = +10 = 20 dB

PM = 180°– 140°= 40°

**(ii)** PM = 180–150°= 30°

GM = $\frac{1}{0}$ = ∞ POI = 0

**(iii)** $\omega_{PC}$ does not exist

GM = $\frac{1}{0}$ = ∞ PM = 180° + 0° = 180°

**(iv)** $\omega_{gc}$ not exist

$\omega_{pc} = \infty$

GM = $\frac{1}{0}$ = ∞

PM = ∞

**(v)** $GM = \frac{1}{0.5} = 2$

$PM = 180 - 90$

$= 90^0$

**15. Ans: (d)**

**Sol:** For stability (–1, j0) should not be enclosed by the polar plot. In figures (1) & (2) (–1, j0) is not enclosed.

$\therefore$ Systems represented by (1) & (2) are stable.

**16. Ans: (b)**

**Sol:** Open loop system is stable, since the open loop poles are lies in the left half of s-plane

$\therefore$ P = 0.

From the plot N = –2 .

No.of encirclements N = P – Z

N = –2, P = 0 (Given)

$\therefore$ N = P – Z

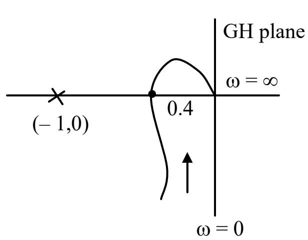
–2 = 0 – Z

Z = 2

Two closed loop poles are lies on RH of s-plane and hence the closed loop system is unstable.

**17. Ans: (c)**

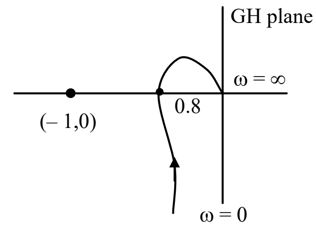
**Sol:**




$\frac{K_c}{K} = 0.4$ When K = 1

Now, K double, $\frac{K_c}{K} = 0.4$

$K_c = 0.4\times2 = 0.8$




even though the value of K is double, the system is stable (negative real axis magnitude is less than one)

Oscillations depends on ‘ ξ’

$\xi \propto \frac{1}{\sqrt{K}}$ as K is increased ξ reduced, then more oscillations.

**18. Ans: (a)**

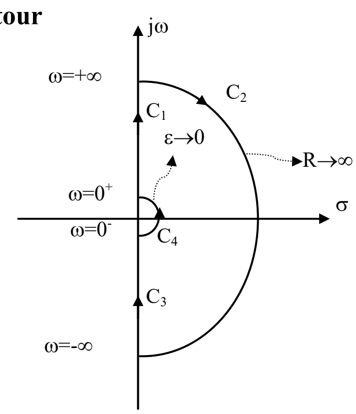
**Sol:** Given system $G(s) = \frac{10(s-12)}{s(s+2)(s+3)}$

It is a non minimum phase system since s = 12 is a zero on the right half of s-plane

**19.**

**Sol:** Given that $G(s)H(s) = \frac{10(s+3)}{s(s-1)}$

**s-plane**
**Nyquist Contour**



- Nyquist plot is the mapping of Nyquist contour(s-plane) into G(s)H(s) plane.
- The Nyquist contour in the s-plane enclosing the entire right half of S-plane is shown figure.

The Nyquist Contour has four sections $C_1$, $C_2$, $C_3$ and $C_4$. These sections are mapped into G(s)H(s) plane

**Mapping of section $C_1$**: It is the positive imaginary axis, therefore sub s = jω, (0≤ω≤∞) in the TF G(s) H(s), which gives the polar plot

$$G(s)H(s) = \frac{10(s+3)}{s(s-1)}$$

Let s = jω

$$G(j\omega)H(j\omega) = \frac{10(j\omega+3)}{j\omega(j\omega-1)}$$

$$G(j\omega)H(j\omega) = \frac{10\sqrt{\omega^2+9}}{\omega\sqrt{\omega^2+1}}\angle\left\{\tan^{-1}\left(\frac{\omega}{3}\right) - [90^0 + 180^0 - \tan^{-1}(\omega)]\right\}$$

At $\omega = 0 \Rightarrow \infty\angle -270^0$

At $\omega = \omega_{pc} = \sqrt{3} \Rightarrow 10\angle -180^0$

At $\omega = \infty \Rightarrow 0\angle -90^0$

point of intersection of the Nyquist plot with respect to negative real axis is calculated below

$$\text{Arg}G(j\omega)H(j\omega) = \arg\frac{10(j\omega+3)}{j\omega(j\omega-1)}$$

$= -180^0$ will give the '$\omega_{pc}$'

Magnitude of G(jω)H(jω) gives the point of intersection

$$\angle\tan^{-1}\left(\frac{\omega}{3}\right) - [90^0 + 180^0 - \tan^{-1}(\omega))$$

$$= -180^0\Big|_{\omega=\omega_{pc}}$$

$$\angle\tan^{-1}\left(\frac{\omega_{pc}}{3}\right) - [90^0 + 180^0 - \tan^{-1}(\omega_{pc})) = -180^0$$

$$\tan^{-1}\left(\frac{\omega_{pc}}{3}\right) + \tan^{-1}(\omega_{pc}) = 90^0$$

Taking "tan" both the sides

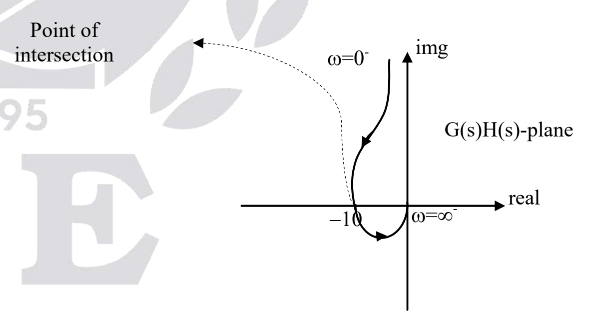
$$\frac{\frac{\omega_{pc}}{3} + \omega_{pc}}{1 - \frac{(\omega_{pc})^2}{3}} = \tan 90^0 = \infty$$

$$1 - \frac{\omega_{pc}^2}{3} = 0$$

$$\omega_{pc} = \sqrt{3}\ \text{rad/sec}$$

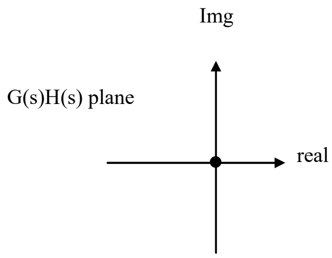
Therefore the point of intersection is

$$|G(j\omega)H(j\omega)| \text{ at } \omega_{pc} = \frac{10\sqrt{\omega_{pc}^2 + 3^2}}{\omega_{pc}\sqrt{1+\omega_{pc}^2}} = 10$$




The mapping of the section $C_1$ is shown figure.

**Mapping of section $C_2$:** It is the radius 'R' semicircle, therefore sub $s = \lim_{R\to\infty} Re^{j\theta}$ (θ is from $90^0$ to $0^0$ to $-90^0$) in the TF G(s)H(s), which merges to the origin in G(s)H(s) plane.

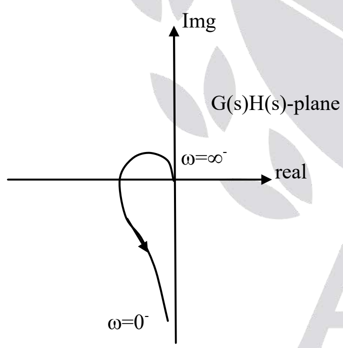



$$G(s)H(s)=\frac{10(s+3)}{s(s-1)}$$

$$G(Re^{j\theta})H(Re^{j\theta})=\frac{2(Re^{j\theta}+3)}{Re^{j\theta}(Re^{j\theta}-1)}\approx 0$$

The plot is shown in figure.

**Mapping of section $C_3$:** It is the negative imaginary axis, therefore sub $s = j\omega$, $(-\infty \le \omega \le 0)$ in the TF G(s)H(s), which gives the mirror image of the polar plot and is symmetrical with respect to the real axis, The plot is shown in figure.




**Mapping of section $C_4$:** It is the radius 'ε' semicircle, therefore sub $s=\lim\limits_{\varepsilon\to 0}\varepsilon e^{j\theta}$ $(-90^o \le \theta \le 90^o)$ in the TF G(s)H(s), which gives clockwise infinite radius semicircle in G(s)H(s) plane.

The plot is shown below

$$G(\varepsilon e^{j\theta})H(\varepsilon e^{j\theta})=\frac{10(\varepsilon e^{j\theta}+3)}{\varepsilon e^{j\theta}(\varepsilon e^{j\theta}-1)}$$

$$G(\varepsilon e^{j\theta})H(\varepsilon e^{j\theta})\approx\frac{10\times 3}{-\varepsilon e^{j\theta}}=\infty\angle 180^0-\theta$$

When, $\theta=-90^0$ $\quad\infty\angle 270^0$

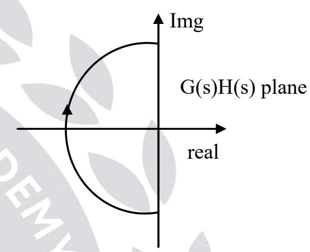
$\theta=-40^0$ $\quad\infty\angle 220^0$

$\theta=0^0$ $\quad\infty\angle 0^0$

$\theta=40^0$ $\quad\infty\angle 140^0$

$\theta=90^0$ $\quad\infty\angle 90^0$

It is clear that the plot is clockwise '∞' radius semicircle centred at the origin



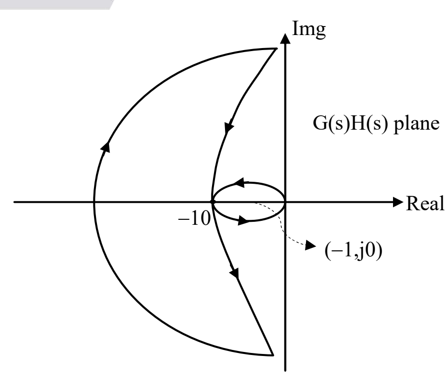

Combining all the above four sections, the Nyquist plot of $G(s)H(s)=\dfrac{10(s+3)}{s(s-1)}$ is shown in figure below

From the plot N=1

Given that P=1

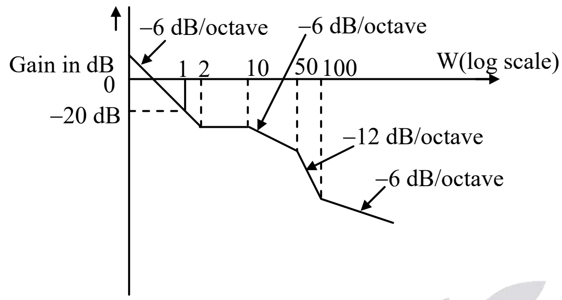
N=P−Z

Z = P − N = 1 − 1= 0, therefore system is stable



**20.**

**Sol:** The given bode plot is shown below.




Initial slope = – 6 db/octave.

i.e. there is one pole at origin (or) one integral term.

portion of transfer function

$$G(s) = \frac{K}{s}$$

At ω = 2 rad/sec, slope is changed to 0dB/octave.

∴ change in slope = present slope – previous slope

= 0 – (–6) = 6 dB/octave

∴ There is a real zero at corner frequency $\omega_1 = 2$.

$$(1+sT_1) = \left(1+\frac{s}{\omega_1}\right) = \left(1+\frac{s}{Z}\right)$$

At ω = 10 rad/sec, slope is changed to –6dB/octave.

∴ change in slope = – 6 – 0

= – 6 dB/octave.

∴ There is a real pole at corner frequency $\omega_2 = 2$.

$$\frac{1}{1+sT_2} = \frac{1}{\left(1+\frac{s}{\omega_2}\right)} = \frac{1}{\left(1+\frac{s}{10}\right)}$$

At ω = 50 rad/sec, slope is changed to –12dB/octave.

∴ change in slope = – 12 – (–6)

= – 6 dB/octave

∴ There is a real pole at corner frequency $\omega_3 = 50$ rad/sec.

$$\frac{1}{1+ST_3} = \frac{1}{\left(1+\frac{S}{\omega_3}\right)} = \frac{1}{\left(1+\frac{S}{50}\right)}$$

At ω = 100 rad/sec, the slope changed to –6 dB/octave.

∴ change in slope = – 6 – (–12)

= 6 dB/octave.

∴ There is a real zero at corner frequency $\omega_4 = 100$ rad/sec.

$$\therefore (1+sT_4) = \left(1+\frac{s}{\omega_4}\right) = \left(1+\frac{s}{100}\right)$$

$$\therefore \text{ Transfer function } = \frac{K\left(1+\frac{s}{2}\right)\left(1+\frac{s}{100}\right)}{s\left(1+\frac{s}{50}\right)\left(1+\frac{s}{10}\right)}$$

$$= \frac{K(s+2)(s+100)}{s(s+50)(s+10)} \frac{\frac{1}{2}\cdot\frac{1}{100}}{\frac{1}{50}\cdot\frac{1}{10}}$$

$$= \frac{2.5K(s+2)(s+100)}{s(s+10)(s+50)}$$

In the given bode plot,

at ω = 1rad/sec, Magnitude = –20dB.

$$-20\text{dB} = 20\log K - 20\log\omega + 20\sqrt{1+\left(\frac{\omega}{2}\right)^2} + 20\sqrt{1+\left(\frac{\omega}{100}\right)^2}$$

$$-20\log\sqrt{1+\left(\frac{\omega}{50}\right)^2} - 20\log\sqrt{1+\left(\frac{\omega}{10}\right)^2}$$

At ω = 1 rad/sec,

–20 = 20logK – 20 logω/ω = 1 rad/sec

[∵ Remaining values eliminated]

– 20 = 20logK

⇒ K = 0.1

∴Transfer function

$$\frac{C(s)}{R(s)} = \frac{0.25(s+2)(s+100)}{s(s+10)(s+50)}$$

# Chapter 7 Controllers & Compensators
## (Solutions for Vol-1_Classroom Practice Questions)

**01. Ans: (a)**

**Sol:** $G_C(s) = (-1)\left(-\frac{Z_2}{Z_1}\right)$

$$= (-1)(-1)\left(\frac{R_2 + \frac{1}{sC}}{R_1}\right)$$

$$G_c(s) = \frac{(100\times10^3) + \frac{1}{s\times10^{-6}}}{10^6}$$

$$G_c(s) = \frac{1+0.1s}{s}$$

**02. Ans: (c)**

**Sol:** CE $\Rightarrow 1 + G_c(s)\,G_p(s) = 0$

$$= 1 + \frac{1+0.1s}{s} \times \frac{1}{(s+1)(1+0.1s)}$$

$$= 1 + \frac{1+0.1s}{s(s+1)(1+0.1s)} = 0$$

$\Rightarrow s^2 + s + 1 = 0 \Rightarrow \omega_n = 1,$

$$e^{\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right]_{\xi=0.5}} = 0.163$$

$M_p = 16.3\%$

**03. Ans: (b)**

**Sol:** T.F $= \frac{k(1+0.3s)}{1+0.17s}$

$T = 0.17$, $aT = 0.3 \Rightarrow a = \frac{0.3}{0.17}$

$C = 1\ \mu F$

$T = \frac{R_1R_2}{R_1+R_2}C$, $a = \frac{R_1+R_2}{R_2}$

$$\frac{R_1R_2}{R_1+R_2} = \frac{0.17}{1\times10^{-6}} = 170000$$

$$\frac{R_1+R_2}{R_2} = 1.764$$

$aT = R_1 C$

$$R_1 = \frac{aT}{C} = \frac{0.3}{C} = (0.3)(10^6)$$

$= 300\ k\Omega$

Bv

$300\ k + R_2 - 1.76\ R_2 = 0$

$$R_2 = \frac{300}{0.70} = 394.736$$

$= 400\ k\Omega$

**04. Ans: (d)**

**Sol:** PD controller improves transient stability and PI controller improves steady state stability. PID controller combines the advantages of the above two controllers.

**05.**

**Sol:** For $K_I = 0 \Rightarrow$

$$\frac{C(s)}{R(s)} = \frac{(K_P + K_D s)}{s(s+1) + (K_P + K_D s)}$$

$$= \frac{K_P + K_D s}{s^2 + (1+K_D)s + K_P}$$

$\omega_n = \sqrt{K_P}$

$2\xi\omega_n = 1 + K_D$

$\Rightarrow 2(0.9)\sqrt{K_P} = 1 + K_D$

$\Rightarrow 1.8\sqrt{K_P} = 1 + K_D$ ……… (1)

Dominant time constant $\frac{1}{\xi\omega_n} = 1$

$$\Rightarrow \omega_n = \frac{1}{0.9} = 1.111$$

$$K_P = \omega_n^2 = 1.11^2$$

$$= 1.234$$

From eq. (1),

$$\Rightarrow 1.8 \times \frac{1}{0.9} = 1 + K_D$$

$$\Rightarrow K_D = 1$$

Chapter **8**

# State Space Analysis

## (Solutions for Vol-1_Classroom Practice Questions)

**01. Ans: (a)**

**Sol:** TF $= \dfrac{1}{s^2+5s+6}$

$$= \frac{1}{(s+2)(s+3)}$$

$$= \frac{1}{s+2} + \frac{-1}{s+3}$$

$$\therefore\ A = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$C = [1 \quad 1]$

**02. Ans: (c)**

**Sol:** Given problem is Controllable canonical form.

**(or)**

$$TF = C[sI - A]^{-1}B + D$$

$$= [6\ \ 5\ \ 1]\begin{bmatrix} s & 1 & 0 \\ 0 & s & 1 \\ -5 & -3 & s+6 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$= \frac{3s^2+15s+18}{s^3+6s^2+3s+5}$$

**03. Ans: (d)**

**Sol:** $\dfrac{d^2y}{dt^2} + \dfrac{3dy}{dt} + 2y = u(t)$

$2^{nd}$ order system hence two state variables are chosen

Let $x_1$ (t), $x_2$ (t) are the state variables

CCF − SSR

Let $x_1(t) = y(t)$ ……….. (1)

$x_2(t) = \dot{y}(t)$ ……….. (2)

Differentiating (1)

$\dot{x}_1(t) = \dot{y}(t) = x_2(t)$ ……….. (3)

$\dot{x}_2(t) = \ddot{y}(t) = u(t) - 3y^1(t) - 2y(t)$

$= u(t) - 3x_2(t) - 2x_1(t)$ ……. (4)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B}u(t)$$

From equation 1. The output equation in matrix form

$$y(t) = [1\ \ 0]\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, D = 0$$

**04. Ans: (b)**

**Sol:** OCF − SSR

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u(t)$$

$$y(t) = [0\ 1]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

**05. Ans: (c)**

**Sol:** Normal form − SSR

$$TF = \frac{Y(s)}{G(s)} = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)}$$

⇒ Diagonal canonical form

The eigen values are distinct i.e., − 1 & − 2.

∴ Corresponding normal form is called as diagonal canonical form

DCF − SSR

$$\frac{Y(s)}{U(s)} = \frac{b_1}{s+1} + \frac{b_2}{s+2}$$

$b_1 = 1$, $b_2 = -1$

$$Y(s) = \underbrace{\frac{b_1}{s+1}}_{x_1}U(s) + \underbrace{\frac{b_2}{s+2}}_{x_2}U(s)$$

Let $Y(s) = X_1(s) + X_2(s)$

Where $y(t) = x_1(t) + x_2(t)$ ………. (1)

Where $X_1(s) = \frac{b_1}{s+1}U(s)$

$s X_1(s) + X_1(s) = b_1 U(s)$

Take Laplace Inverse

$\dot{x}_1 + x_1 = b_1 u(t)$ ……… (2)

$X_2(s) = \frac{b_2}{s+2}U(s)$

$s X_2(s) + 2 X_2(s) = b_2 U(s)$

Laplace Inverse

$\dot{x}_2 + 2x_2 = b_2 u(t)$

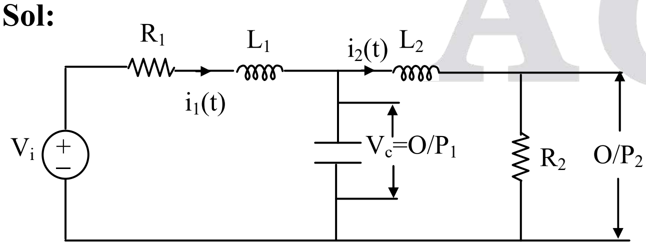
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ -1\end{bmatrix}u(t)$$

From(1) output equation.

$$y(t) = [1\ 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

**06. Ans: (c)**

**Sol:**




$O/P_1 \Rightarrow y_1 = V_c$

$O/P_2 \Rightarrow y_2 = R_2\, i_2$

$$y = \begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & R_2\end{bmatrix}\begin{bmatrix}V_c\\ i_1\\ i_2\end{bmatrix}$$

$y = C X$

$$C = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & R_2\end{bmatrix}$$

**07. Ans: (a)**

**Sol:** T.F $= C[sI-A]^{-1}B + D$

$$= [1\quad 0]\begin{bmatrix}s+4 & 1\\ 3 & s+1\end{bmatrix}^{-1}\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$= [1\quad 0]\frac{1}{s^2+5s+1}\begin{bmatrix}s+1 & -1\\ -3 & s+4\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$= \frac{1}{s^2+5s+1}[1\quad 0]_{1\times 2}\begin{bmatrix}s+1 & -1\\ -3 & s+4\end{bmatrix}_{2\times 2}\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$= \frac{1}{s^2+5s+1}[s+1\quad -1]_{1\times 2}\begin{bmatrix}1\\ 1\end{bmatrix}_{2\times 1}$$

$$= \frac{1}{s^2+5s+1}[s+1-1]$$

$$= \frac{s}{s^2+5s+1}$$

**08. Ans: (c)**

**Sol:** State transition matrix $\phi(t) = L^{-1}[(sI-A)^{-1}]$

$$sI - A = \begin{bmatrix}s+3 & -1\\ 0 & s+2\end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+2)(s+3)}\begin{bmatrix}s+2 & 1\\ 0 & s+3\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{s+3} & \frac{1}{(s+2)(s+3)}\\ 0 & \frac{1}{s+2}\end{bmatrix}$$

$$L^{-1}[[sI - A]^{-1}] = \begin{bmatrix}e^{-3t} & e^{-2t} - e^{-3t}\\ 0 & e^{-2t}\end{bmatrix}$$

**09. Ans: (b)**

**Sol: Controllability**

$$[M] = [B \quad AB \quad A^2B.. \quad A^{n-1}B]$$

$$AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$|M| = -1 \neq 0$ (Controllable)

**Observability**

$$[N] = [C^T \; A^TC^T \; .... \; (A^T)^{n-1}C^T]$$

$$A^TC^T = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$|N| = 0$ (Not observable)

**10. Ans: (c)**

**Sol:** According to Gilberts test the system is controllable and observable.

**11. Ans: (c)**

**Sol:** $$\frac{Y(s)}{U(s)} = \frac{b_1s^2 + b_2s + b_3}{s^3 + a_1s^2 + a_2s + a_3}$$

at node $\dot{x}_1$

$\dot{x}_1 = -a_1x_1 - a_2x_2 - a_3x_3$

at $\dot{x}_2 = x_1$ & $\dot{x}_3 = x_2$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

**12.**

**Sol:** The given state space equations:

$\dot{X} = X_2$

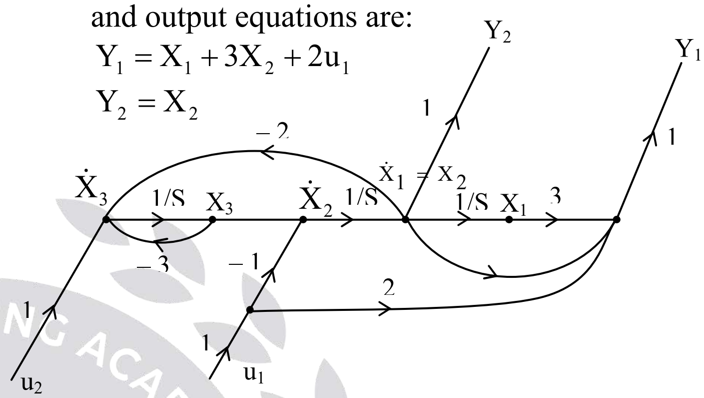
$\dot{X}_2 = X_3 - u_1$

$\dot{X}_3 = -2X_2 - 3X_3 + u_2$

and output equations are:

$Y_1 = X_1 + 3X_2 + 2u_1$

$Y_2 = X_2$




The given state space equations in matrix for

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}_{3\times3} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}_{3\times1} + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 1 \end{bmatrix}_{3\times2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2\times1}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{2\times3} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}_{3\times1} + \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}_{2\times2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2\times1}$$

Where A: State matrix
B: Input matrix
C: Output matrix
D: Transition matrix

Characteristic equation

$|sI - A| = 0$

$$\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} S & -1 & 0 \\ 0 & S & -1 \\ 0 & 2 & S+3 \end{vmatrix} = 0$$

$\Rightarrow s[s(s+3) + 2] + 1(0) = 0$

$\Rightarrow s(s^2 + 3s + 2) = 0$

$\Rightarrow s(s+1)(s+2) = 0$

The roots are 0, –1, –2.