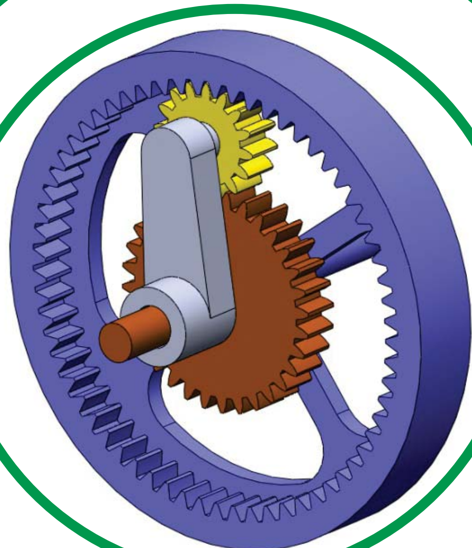
MECHANICAL ENGINEERING



GATE | PSUs

THEORY OF MACHINES
&
VIBRATIONS

Volume - I : Study Material with Classroom Practice Questions

Theory of Machines & Vibrations

Solutions for Vol – I _ Classroom Practice Questions

Chapter- 1
Analysis of Planar Mechanisms

01. Ans: (c)

Sol: It is the failure of Gruebler's equation of DOF because it does not consider the shape and dimensions of the mechanism.

02. Ans: (c)

03. Ans: (b)

Sol: Gruebler's Criterion

$DOF = 3(N-1) - 2P_1$

N = Number of links,

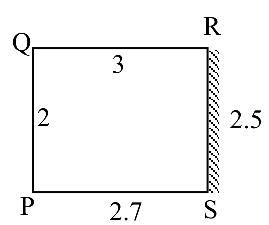
P_1 = Number of rotary joints

Given

$\therefore$ DOF = $3(8-1) - (2\times 10) = 21 - 20 = 1$

04. Ans: (c)

Sol:



The given dimensions of the linkage satisfies Grashof's condition to get double rocker. We need to fix the link opposite to the shortest link. So by fixing link 'RS' we get double rocker.

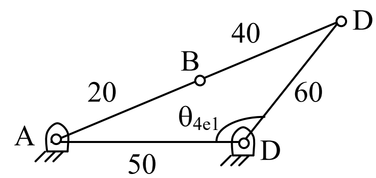
05. Ans: (a)

06. Ans: (d)

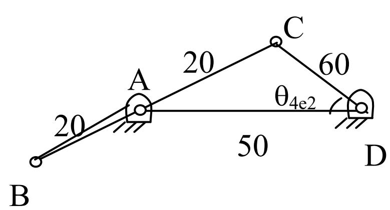
Sol: At toggle position velocity ratio is 'zero' so mechanical advantage is '∞'.

07. Ans: (d)

Sol: The two extreme positions of crank rocker mechanisms are shown below figure.



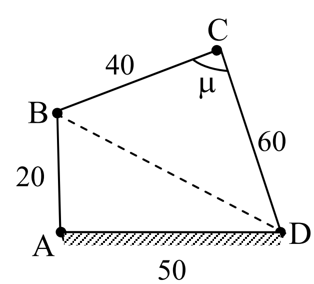
$$\theta_{4e_1} = \cos^{-1}\left(\frac{50^2+60^2-60^2}{2\times 50\times 60}\right) = 65.37^0$$



$$\theta_{4e_2} = \cos^{-1}\left(\frac{60^2+50^2-20^2}{2\times 60\times 50}\right) = 18.19^o$$

08. Ans: (a)

Sol:



Where, μ = Transmission angle

$$BD = \sqrt{20^2 + 50^2} = 53.85\,cm$$

By cosine rule

$$\cos\mu = \frac{BC^2 + CD^2 - BD^2}{2BC \times CD}$$

$$= \frac{40^2 + 60^2 - 53.85^2}{2 \times 40 \times 60} = 0.479$$

$\mu = 61.37°$

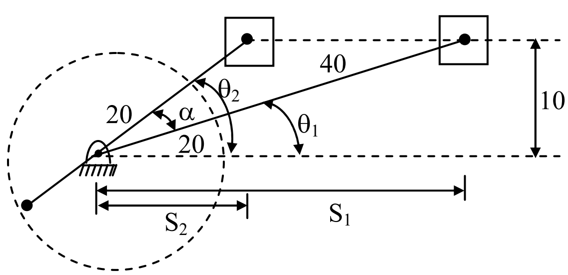
09. Ans: (c)

Sol: Two extreme positions are as shown in figure below.

Let r = radius of crank = 20 cm

l= length of connecting rod = 40 cm

h = 10 cm



Stroke = $S_1 – S_2$

$$S_1 = \sqrt{(\ell + r)^2 - h^2} = \sqrt{60^2 - 10^2} = 59.16cm$$

$$S_2 = \sqrt{(\ell - r)^2 - h^2} = \sqrt{20^2 - 10^2} = 17.32cm$$

Stroke = $S_1 – S_2$ = 59.16 – 17.32 = 41.84 cm

10. Ans: (b)

Sol: $\theta_1 = \sin^{-1}\left(\frac{h}{\ell + r}\right) = \sin^{-1}\left(\frac{10}{60}\right) = 9.55°$

$$\theta_2 = \sin^{-1}\left(\frac{h}{\ell - r}\right) = \sin^{-1}\left(\frac{10}{20}\right) = 30°$$

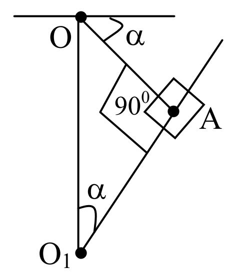
$\alpha = \theta_2 - \theta_1 = 20.41°$

Quick return ratio

$$(QRR) = \frac{180 + \alpha}{180 - \alpha} = 1.2558$$

11. Ans: (c)

Sol:



OO_1 = 40 cm , OA = 20 cm

$$\sin\alpha = \frac{OA}{OO_1} = \frac{20}{40} = \frac{1}{2}$$

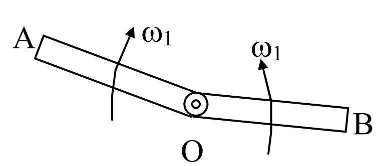
$\Rightarrow \alpha = 30^0$

$$QRR = \frac{180 + 2\alpha}{180 - 2\alpha} = \frac{180 + 60}{180 - 60}$$

$\Rightarrow$ QRR = 2

12. Ans: (c)

Sol: The rubbing velocity is defined as the algebraic sum between the angular velocities of the two links which are connected by pin joints, multiplied by the radius of pin.



Rubbing velocity at point O

V_o= radius × relative radial velocity

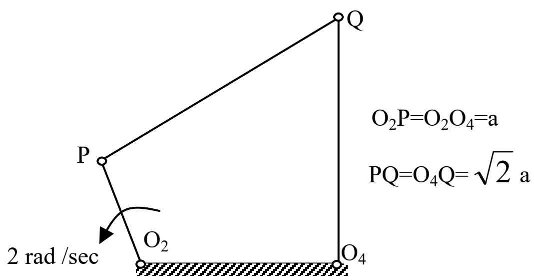
$= (\omega_1 \pm \omega_2)\, r$

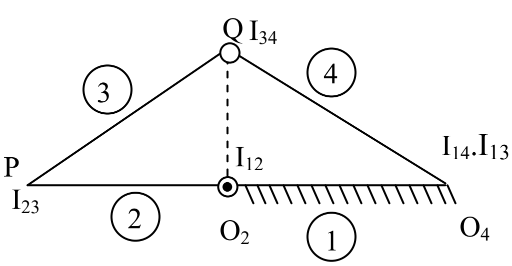
$V_o = (\omega_1 + \omega_2)\, r$

($\because$ the links OA & OB are moving in opposite direction)

13. Ans: (c)

Sol: $\angle O_4O_2P = 180^o$ sketch the position diagram for the given input angle and identify the Instantaneous Centers.





I_{13} is obtained by joining $I_{12}\, I_{23}$ and $I_{14}\, I_3$

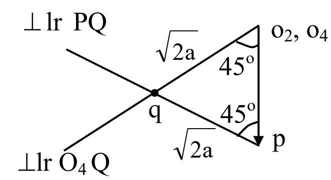
$$\frac{\omega_3}{\omega_2} = \frac{I_{12}\, I_{23}}{I_{13}\, I_{23}} = \frac{a}{2a}$$

$$\frac{\omega_3}{2} = \frac{1}{2}$$

$\omega_3 = 1$ rad /sec

Alternate Method:

The position diagram is isosceles right angle triangle and the velocity triangle is similar to the position diagram.



Velocity (Diagram)

$V_{qp} = \omega_3 l_3 \Rightarrow \sqrt{2}a = \omega_3 \times \sqrt{2}a$

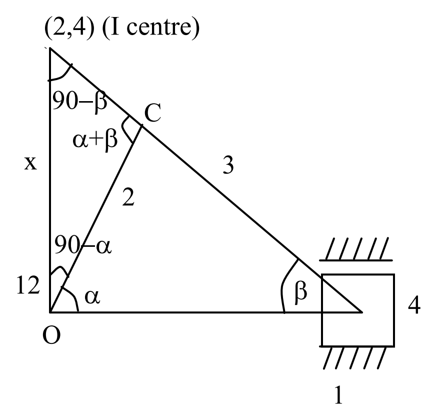
$\omega_3 = 1$

$V_q = l_4\, \omega_4 \Rightarrow \sqrt{2}a = \sqrt{2}a\, \omega_4$

$\Rightarrow \omega_4 = 1$rad/sec

14. Ans: (b)

Sol:



OC = r

Velocity of slider $V_S = (12 - 24) \times \omega_2$

$= x\, \omega_2$

$$\frac{x}{\sin(\alpha+\beta)} = \frac{r}{\sin(90-\beta)}$$

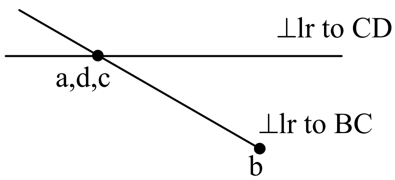
$$x = \frac{r \sin(\alpha+\beta)}{\sin(90-\beta)}$$

$V_S = r\,\omega_2 \sin(\alpha + \beta) \times \sec\beta$

$= V_C \sin(\alpha + \beta) \times \sec\beta$

15. Ans: (a)

Sol:



Velocity diagram

$V_C = 0 = dc \times \omega_{CD}$

$\therefore\ \omega_{CD} = 0$

Note: If input and coupler links are collinear, then output angular velocity will be zero.

16. Ans: (c)

Sol: In a four bar mechanism when input link and output links are parallel then coupler velocity(ω_3) is zero.

$\Rightarrow l_2\,\omega_2 = l_4\,\omega_4$

$l_4 = 2l_2$ (Given)

$\Rightarrow \omega_4 = \omega_2 / 2 = 2/2 = 1$ rad/s

ω_2, ω_4 = angular velocity of input and output link respectively.

Fixed links have zero velocity.

At joint 1, relative velocity between fixed link and input link = 2–0 = 2

Rubbing velocity at joint 1 = Relative velocity × radius of pin = 2×10 = 20 cm/s

At joint 2, rubbing velocity = $(\omega_2+\omega_3) \times r$

$= (2+0)\times 10 = 20$ cm/s

+ve sign means ω_2 and ω_3 are moving in opposite directions.

At joint 3, rubbing velocity $=(\omega_4+\omega_3)\times r$

$= (1+0) \times 10 = 10$ cm/s

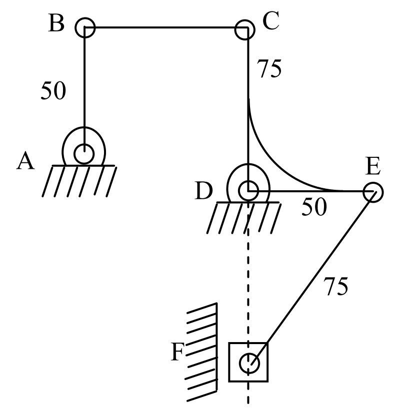
At joint 4, rubbing velocity

$= (\omega_4 - 0) \times r$

$= (1 - 0) \times 10 = 10$ cm/s

17. Ans: (a)

Sol:



Considering the four bar mechanism ABCD, $l_2 || l_4$

$$\therefore \ell_2\omega_2 = \ell_4\omega_4 \Rightarrow \omega_4 = \frac{50\times 3}{75} = 2\,\text{rad/sec}$$

CDE being a ternary link angular velocity of DE is same as that of the link DC (ω_4).

For the slider crank mechanism DEF, crank is perpendicular to the axis of the slider.

$\therefore$ Slider velocity = DE× ω_4

$= 50\times 2 = 100$cm/sec (upward)

18. Ans: (a)

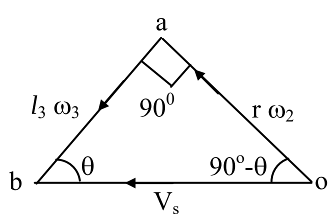
Sol: Here as angular velocity of the connecting rod is zero so crank is perpendicular to the line of stroke.

V_s = velocity of slider = $r\omega_2$

$2 = 1 \times \omega_2 \Rightarrow \omega_2 = 2$ rad/sec

19. Ans: (d)

Sol:



Here the crank is perpendicular to connecting rod

Velocity of rubbing = $(\omega_2 + \omega_3) \times r$

Where, r = radius of crank pin

From the velocity diagram V_{AB} = ab = ?

oa = $\omega_2 \times r = 10 \times 0.3 = 3$ m/sec

Δoab is right angle Δ.

$$\tan\theta = \frac{oa}{ab} = \frac{40}{30} \Rightarrow \theta = 53.13°$$

$$\tan\theta = \frac{r\omega_2}{\ell\omega_3}$$

Where, $n = \frac{\ell}{r}$

$$\omega_3 = \frac{\omega_2}{n^2} = \frac{10}{\left(\frac{4}{3}\right)^2} = \frac{90}{16} = 5.625(CW)$$

$V_{rb} = (\omega_2 + \omega_3) \times r$

$= (10 + 5.625) \times 2.5 = 39$ cm/s

20. Ans: (d)

Sol: As for the given dimensions the mechanism is in a right angle triangle configuration and the crank AB is perpendicular to the lever CD. The velocity of B is along CD only which is purely sliding component

∴ Velocity of the slider

$$= AB \times \omega_{AB} = 10 \times 250 = 2.5\,m/sec$$

Common data Question 21, 22 & 23

21. Ans: (d)

22. Ans : (a)

23. Ans: (c)

Sol: Considering the four bar mechanism ABCD, $AB||DC$

$\therefore AB \times \omega_2 = DC \times \omega_4$

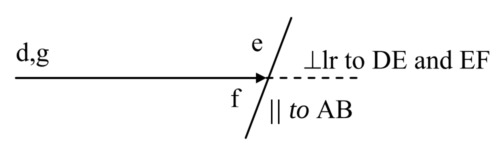
$$\Rightarrow \omega_4 = \frac{30 \times 6}{90} = 2\,rad/sec$$

CDE being a ternary link angular velocity of DE is same as that of the link CE.

For the slider crank mechanism DEF, crank is perpendicular to the axis of the slider.

∴ Slider velocity = DE× ω_4

= 30×2 = 60cm/sec.



(Velocity diagram)

From the above velocity diagram

$V_{ef} = 0$, $\omega_{ef} = 0$

24. Ans : (a)

Sol: $QRR = \dfrac{180+2\alpha}{180-2\alpha} = \dfrac{2}{1} \Rightarrow \alpha = 30^\circ$

$$\sin\alpha = \frac{OS}{OP} \Rightarrow OS = \frac{OP}{2} = 250\text{mm}$$

25. Ans: (b)

Sol: Maximum speed during forward stroke occurs when PQ is perpendicular to the line of stroke of the tool i. e. PQ, OS & OQ are in straight line

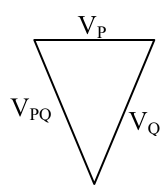
$$\Rightarrow V = 250\times 2 = 750\times\omega_{PQ}$$

$$\Rightarrow \omega_{PQ} = \frac{2}{3}$$

26. Ans: (d)

Sol:

$V_Q = V_P + V_{PQ}$

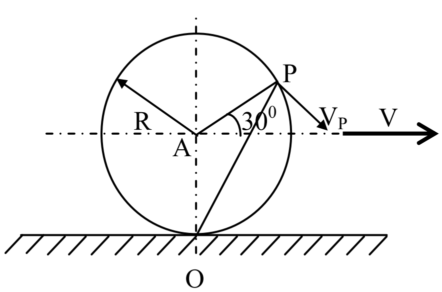


27. Ans: (a)

Sol: For rigid thin disc rolling on plane without slip. The 'I' centre lies on the point of contact.

28. Ans: (a)

Sol:



Here 'O' is the instantaneous centre

$V_P = \omega \times OP$

$V_A = R\omega$

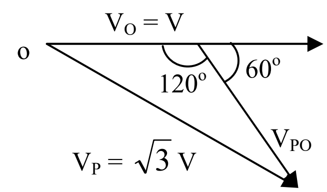
In Δ OAP , $\cos 120^\circ = \dfrac{R^2 + R^2 - OP^2}{2R\times R}$

$$-0.5 = \frac{2R^2 - OP^2}{2R^2}$$

$OP = \sqrt{3}R$

$V_P = \sqrt{3}R\times\omega = \sqrt{3}V$

or

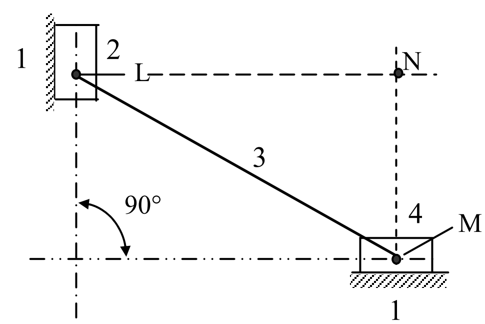


$V_P = \vec{V}_O + \vec{V}_{PO} = \vec{V} + \overrightarrow{OP}\times\omega$

$= \sqrt{V^2 + V^2 + 2V^2\cos 60} \qquad = \sqrt{3}\,V$

29. Ans: (d)

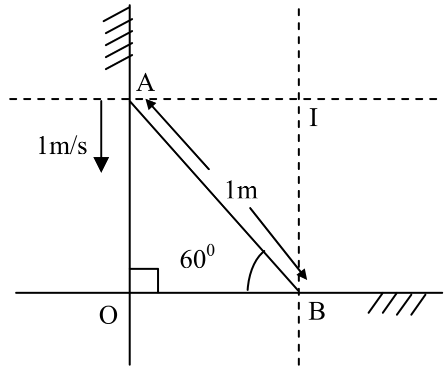
Sol:



By considering the links 1, 2 and 4 as for three centers in line theorem, I_{12}, I_{14} and I_{24} lies on a straight line I_{12} is at infinity along the horizontal direction while I_{14} is at infinity along vertical direction hence I_{24} must be at infinity

30. Ans: (a)

Sol:



$V_a = 1$ m/s

V_a = Velocity along vertical direction

V_b = Velocity along horizontal direction

So instantaneous center of link AB will be perpendicular to A and B respectively i.e at I

$$IA = OB = \cos\theta = 1 \times \cos 60^0 = \frac{1}{2} \text{m}$$

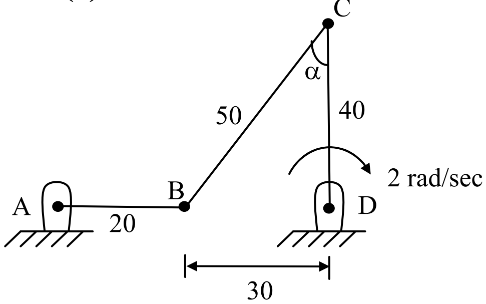
$$IB = OA = \sin\theta = 1 \times \sin 60^0 = \frac{\sqrt{3}}{2} \text{m}$$

$$V_a = \omega \times IA$$

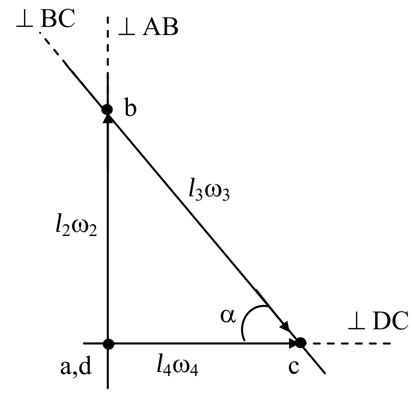
$$\Rightarrow \omega = \frac{V_a}{IA} = \frac{1}{V_2} = 2 \text{ rad/sec}$$

31. Ans: (a)

Sol:



(Position Diagram)



(Velocity Diagram)

Let the angle between BC & CD is α. Same will be the angle between their perpendiculars.

From Velocity Diagram, $\frac{\ell_2\omega_2}{\ell_4\omega_4} = \tan\alpha$

From Position diagram, $\tan\alpha = \frac{30}{40}$

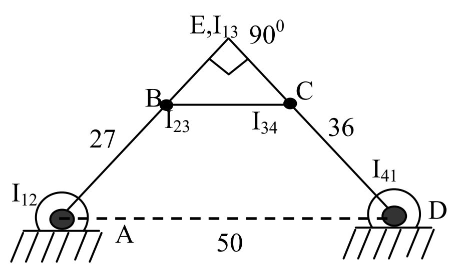
$$\therefore \omega_2 = \omega_4 \times \frac{\ell_4}{\ell_2} \times \tan\alpha = 2 \times \frac{40}{20} \times \frac{30}{40} = 3$$

$\omega_2 = 3$ rad/sec

Note: DC is the rocker (Output link) and AB is the crank (Input link)

32. Ans: (c)

Sol:



I_{13} = Instantaneous center of link 3 with respect to link 1

As AED is a right angle triangle and the sides are being integers so AE = 30 cm and DE = 40 cm

BE = 3 cm and CE = 4 cm

By 'I' center velocity method,

$V_{23} = \omega_2 \times (AB) = \omega_3 \times (BE)$

$$\omega_3 = \frac{1\times 27}{3} = 9\,\text{rad/s}$$

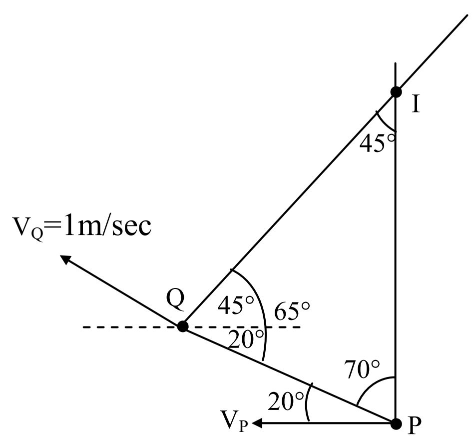
33. Ans: (a)

Sol: Similarly, $V_{34} = \omega_3 \times (EC) = \omega_4 \times (CD)$

$$\omega_4 = \frac{9\times 4}{36} = 1\,\text{rad/s}$$

34. Ans: (d)

Sol: Refer the figure shown below, By knowing the velocity directions instantaneous centre can be located as shown. By knowing velocity (magnitude) of Q we can get the angular velocity of the link, from this we can get the velocity of 'P using sine rule.



'I' is the instantaneous centre.

From sine rule

$$\frac{PQ}{\sin 45} = \frac{IQ}{\sin 70} = \frac{IP}{\sin 65}$$

$$\frac{IP}{IQ} = \frac{\sin 65°}{\sin 70°}$$

$$V_Q = IQ \times \omega = 1$$

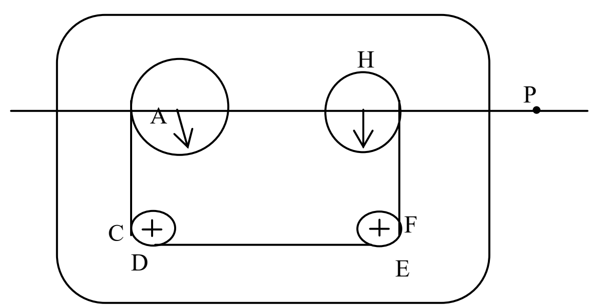
$$\Rightarrow \omega = \frac{V_Q}{IQ}$$

$$V_P = IP \times \omega = \frac{IP}{IQ} \times V_Q = \frac{\sin 65°}{\sin 70°} \times 1$$

$$= 0.9645$$

35. Ans: (c)

Sol: Consider the three bodies the bigger spool (Radius 20), smaller spool (Radius 10) and the frame. They together have three I centers, I centre of big spool with respect to the frame is at its centre A. that of the small spool with respect to the frame is at its centre H. The I centre for the two spools P is to be located.



As for the three centers in line theorem all the three centers should lie on a straight line implies on the line joining of A and H. More over as both the spools are rotating in the same direction, P should lie on the same side of A and H. Also it should be close to the spool running at higher angular velocity. Implies close to H and it is to be on the right of H. Whether P belongs to bigger spool or smaller spool its velocity must be same. As for the radii of the spools and noting that the velocity of the tape is same on both the spools

$\omega_H = 2\omega_A$

$\therefore AP.\omega_A = HP\,\omega_H$ and

$AP = AH + HP \Rightarrow HP = AH$

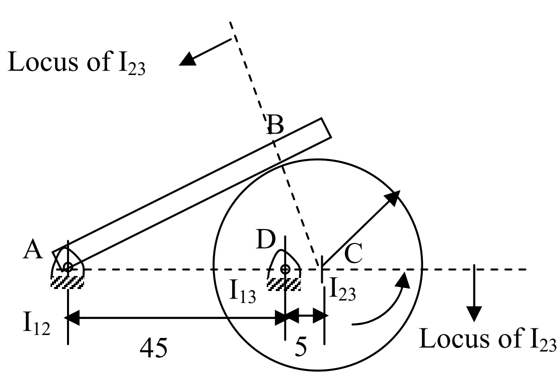
Note:

(i) If two links are rotating in same directions then their Instantaneous centre will never lie in between them. The 'I' center will always close to that link which is having high velocity.

(ii) If two links are rotating in different directions, their 'I' centre will lie in between the line joining the centres of the links.

36. Ans: (b)

Sol: I_{23} should be in the line joining I_{12} and I_{13}. Similarly the link 3 is rolling on link 2.



So the I – Center I_{23} will be on the line perpendicular to the link – 2. (I_{23} lies common normal passing through the contact point)

So the point C is the intersection of these two loci which is the center of the disc.

So $\omega_2(I_{12}, I_{23}) = \omega_3(I_{13}, I_{23})$

$\Rightarrow \omega_2 \times 50 = 1 \times 5$

$\Rightarrow \omega_2 = 0.1$ rad/sec

37. Ans: 20

Sol: Velocity of $P = r\omega = 10\ m/\text{sec}$

$$\omega = \frac{V_p}{R} = \frac{10}{R}$$

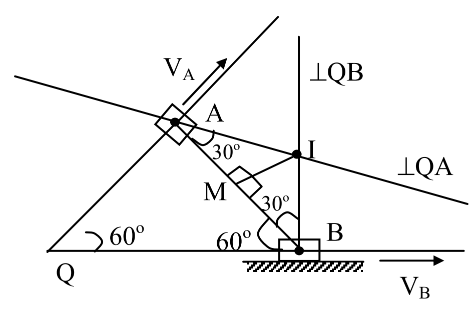
Velocity of $Q = 2R\omega$

$$= 2R \times \frac{10}{R} = 20\ m/\text{sec}$$

38. Ans: 1 (range 0.95 to 1.05)

Sol: Locate the I-centre for the link AB as shown in fig. M is the mid point of AB

Given, $V_A = 2$m/sec



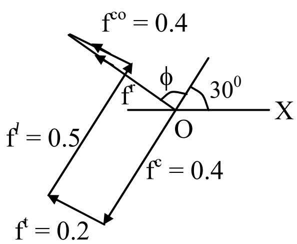
$$V_A = IA.\omega \Rightarrow \omega = \frac{V_A}{IA}$$

$$V_M = IM.\omega = IM\frac{V_A}{IA} = \frac{IM}{IA}.V_A$$

$$= \sin 30^\circ.V_A = \frac{1}{2}.2 = 1\,m/sec$$

39. Ans: (a) & 40. Ans: (b)

Sol:



Centripetal acceleration,

$f^c = r\omega^2 = 0.4$ m/s^2 acts towards the centre

Tangential acceleration, $f^t = r\alpha$

$= 0.2$ m/s^2 acts perpendicular to the link in the direction of angular acceleration.

Linear deceleration = 0.5 m/s^2 acts opposite to velocity of slider

As the link is rotating and sliding so coriolis component of acceleration acts

$f^{co} = 2V\omega = 2 \times 0.2 \times 1 = 0.4$ m/s^2

To get the direction of coriolis acceleration, rotate the velocity vector by 90^0 in the direction of ω.

Resultant acceleration

$$= \sqrt{0.6^2 + 0.1^2} = 0.608\,m/sec^2$$

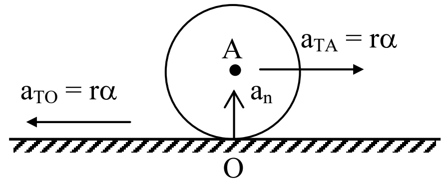
$$\phi = \tan^{-1}\left(\frac{0.6}{0.1}\right) = 80.5$$

Angle of Resultant vector with reference to

OX = 30 +ϕ =30 + 80.5 = 110.53^0

41. Ans: (d)

Sol:

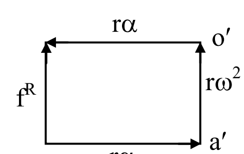


Acceleration at point 'O'

$$a_{\vec{o}} = a_{\vec{TO}} + a_{\vec{TA}} + a_{\vec{n}}$$

$a_{\vec{TO}}$ and $a_{\vec{TA}}$ are linear accelerations with same magnitude and opposite in direction.

$$\Rightarrow a_{\vec{O}} = a_{\vec{n}} = \frac{V^2}{r} = r\omega^2$$

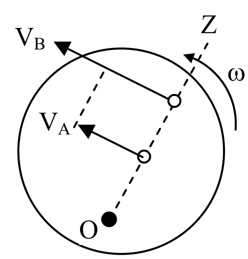


(Acceleration diagram)

Resultant acceleration, $f^R = r\,\omega^2$

42. Ans: (c)

Sol:



$V_B = OB \times \omega$

$V_A = OA \times \omega$

$V_{BA} = V_B - V_A = (OB - OA) \times \omega$

$= \omega\,(r_B - r_A)$

and direction of motion point 'B'.

43. Ans: (d)

Sol: As uniform angular velocity is given,

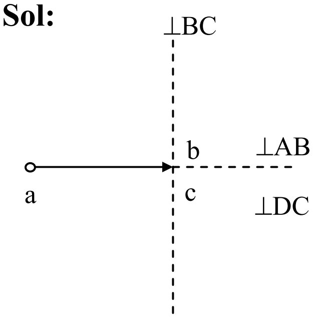
Tangential acceleration, $\alpha = 0$

Centripetal acceleration,

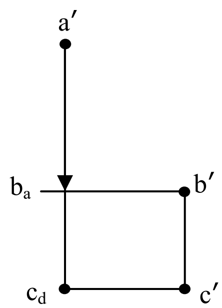
$$f_{BA} = (r_B^2 - r_A^2) \times \omega \quad \text{from Z to 'O'.}$$

44. Ans: (a)

Sol:



Velocity Diagram



Acceleration Diagram

From velocity Diagram, $V_C = V_B$

$l_4\omega_4 = l_2\omega_2$

$25 \times \omega_4 = 50 \times 0.2$

$\Rightarrow \omega_4 = 0.4$ rad/sec

From Acceleration Diagram,

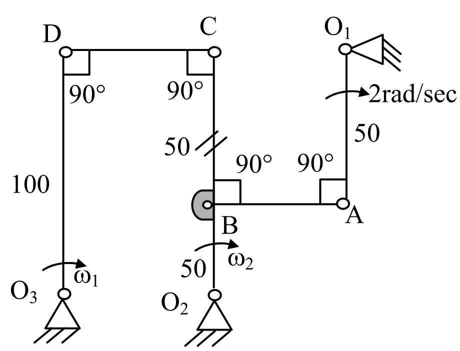
$l_4\alpha_4 = l_2\alpha_2$

$25 \times \alpha_4 = 50 \times 0.1$

$\Rightarrow \alpha_4 = 0.2$ rad/sec^2

45. Ans: (d)

Sol:



As links O_1A and O_2B are parallel then

$V_A = V_B$

$\Rightarrow 50 \times 2 = 50 \times \omega_2$

$\Rightarrow \omega_2 = 2$ rad/sec

As a O_2 C and O_3D are parallel links then

$V_C = V_D$

$\Rightarrow 100 \times 2 = 100 \times \omega_1$

$\Rightarrow \omega_1 = 2$ rad/sec

$V_D = r\omega_1$

$= 100 \times 2 = 200$ mm/sec

$\alpha = 0$ (given), so tangential acceleration a^t

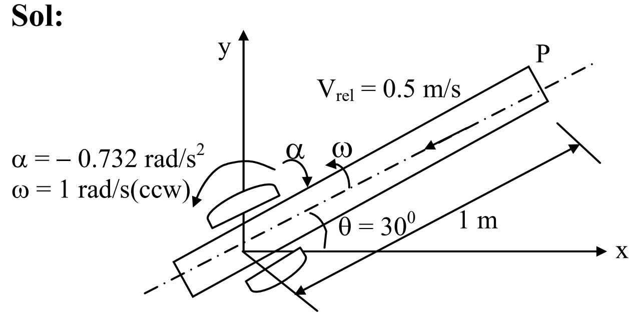
$= r\alpha = 0$

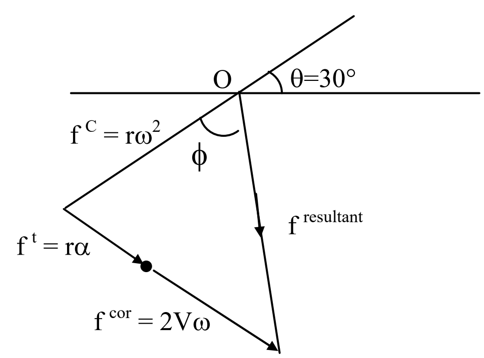
Centripetal acceleration, $f^c = r\omega_1^2$

$= 100 \times (2)^2 = 400$ mm/sec^2

46.

Sol:





Acceleration diagram

Radial relative acceleration, $f^{linear} = 0$

Centripetal acceleration, $f^c = r\omega^2$

$=1\times1^2 = 1\ m/s^2$ (acts towards the center)

Tangential acceleration, $f^t = r\alpha$

$= 1\times0.732 = 0.732\ m/sec^2$

Coriolis acceleration, $f^{cor} = 2V\omega$

$= 2 \times 0.5 \times 1 = 1\ m/sec^2$

Resultant acceleration, $f^r = \sqrt{1^2 + (1+0.732)^2}$

$= 2\ m/sec^2$

$$\phi = \tan^{-1}\left(\frac{1.732}{1}\right) = 60°$$

$\theta_{reference} = 30 + 180 + 60 = 270^0$

47.

Sol: $x_B = P$, $y_B = P\tan\theta$

$$V_X = \frac{d}{dt}(X_B) = 0$$

$$V_y = \frac{d}{dt}(y_B) = P\sec^2\theta \times \frac{d\theta}{dt} = \frac{P\omega}{\cos^2\theta}$$

Acceleration along X direction

$$a_X = \frac{d}{dt}V_x = 0$$

Acceleration along Y direction

$$a_y = \frac{d}{dt}(V_y)$$

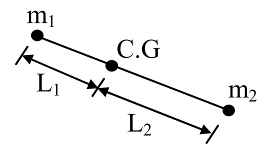
$$= P\omega(-2\cos^{-3}\theta)(-\sin\theta)\omega$$

$$= 2P\omega^2\frac{\sin\theta}{\cos^3\theta}$$

48. Ans: (d)

49. Ans: (a)

Sol:



$$m_1 = \frac{mL_2}{L_1+L_2} = \frac{100\times60}{100} = 60kg$$

$$m_2 = \frac{mL_1}{L_1+L_2} = \frac{100\times40}{100} = 40kg$$

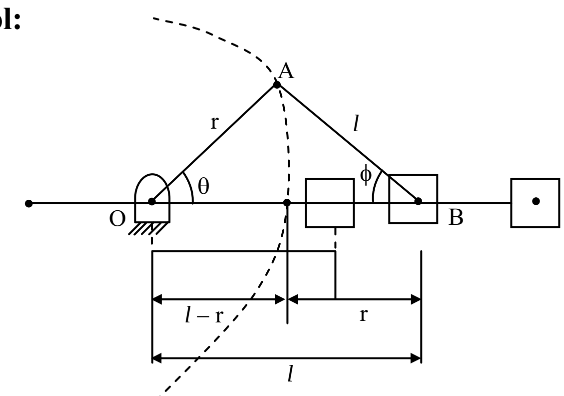
$$I = m_1L_1^2 + m_2L_2^2$$

$$= 60 \times 40^2 + 40\times 60^2$$

$$= 240000\ kg\ cm^2 = 24\ kg\ m^2$$

50. Ans: (b) & 51. Ans: (a)

Sol:



$F_P = 2$ kN

$l = 80$ cm = 0.8 m

r = 20 cm = 0.2m

From the triangle

OAB

$$\cos\phi = \frac{\ell^2 + \ell^2 - r^2}{2\ell^2}$$

$$= \frac{2\times 80^2 - 20^2}{2\times 80^2} \Rightarrow \phi = 14.36$$

$$\cos\theta = \frac{20^2 + 80^2 - 80^2}{2\times 20\times 80} \Rightarrow \theta = 82.82$$

Thrust connecting rod

$$F_T = \frac{F_P}{\cos\phi} = \frac{2}{\cos 14.36} = 2.065\,kN$$

Turning moment,

$$T = F_T \times r = \frac{F_P}{\cos\phi}(\sin(\theta+\phi))\times r$$

$$= \frac{2}{\cos 14.36}\times \sin(14.36+82.82)\times 0.2$$

$= 0.409$ kN – m

52. Ans: (b)

Sol: Calculate AB that will be equal to 260 mm

L = 260 mm, P = 160 mm

S = 60 mm, Q = 240 mm

L+S = 320

P+Q = 400

∴ L+S < P+Q

It is a Grashof's chain

Link adjacent to the shortest link is fixed

∴ Crank – Rocker Mechanism.

53. Ans: (b)

Sol: $O_2A \parallel O_4B$

Then linear velocity is same at A and B.

$\therefore \omega_2 \times O_2A = \omega_4 \times O_4B$

$\therefore 8\times 60 = \omega_4 \times 160$

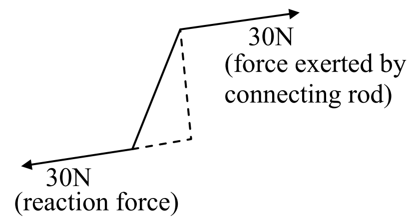
$\omega_4 = 3$ rad/sec

54. Ans: (b)

Sol: As the plane is horizontal ⇒ mg = 0

$\alpha = 0 \Rightarrow I\alpha = 0$, $\tau = 0$ (driving torque)

As the link O_2A is balanced so that its centre of mass falls at O_2 ⇒ centrifugal force = 0



For the given data the only force acting on the link is 30N at A along AB hence the reaction at joint O_2 is 30N

55. Ans: (d)

Sol: $$I\frac{d^2\theta}{dt^2} = T + f(\sin\theta, \cos\theta)$$

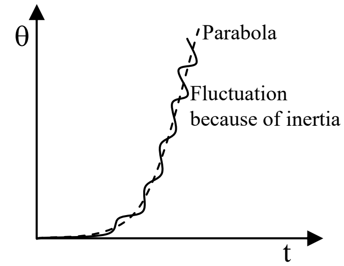
Where 'T' is applied torque, f is inertia torque which is function of sinθ & cosθ

$$\frac{d\theta}{dt} = \frac{T}{I}t + f'(\sin\theta,\cos\theta) + c_1$$

$$\theta = \frac{T}{I}t^2 + c_1 t + f''(\sin\theta,\cos\theta)$$

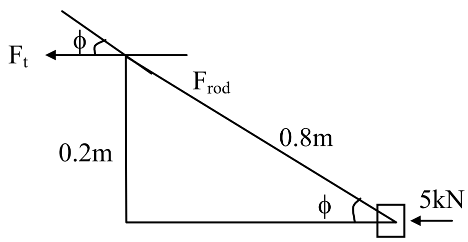
θ is fluctuating on parabola

and @ t = 0 , θ = 0 , $\dot{\theta}(\text{slope}) = 0$ (because it starts from rest)



56. Ans: 1 (range 0.9 to 1.1)

Sol:



Given $F_p = 5kN$

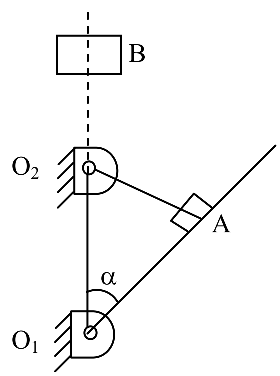
$$F_{rod} = \frac{F_p}{\cos\phi}, F_t = F_{rod}\cos\phi$$

$\therefore F_t = 5kN$

Turning moment = $F_t.r = 5\times0.2 = 1kN\text{-}m$

57. Ans: (a)

Sol :



N = 120 rpm ,

$$\omega_2 = \frac{2\pi N}{60} = 4\pi\,rad/s$$

$\therefore l_1 = O_1 O_2 = 50$ cm

QRR = 1: 2 = $\frac{1}{2}$

$$\frac{1}{2} = \frac{180-2\alpha}{180+2\alpha} \Rightarrow 180 + 2\alpha = 360 - 4\alpha$$

$\Rightarrow 6\alpha = 180°$

$\Rightarrow \alpha = 30°$

$$\sin 30 = \frac{O_2A}{O_1O_2}$$

$$\Rightarrow \frac{1}{2} = \frac{O_2A}{50} \Rightarrow O_2\,A = 25\text{ cm}$$

$\therefore\ l_2 = 25$ cm

At the position given above ($O_1\ O_2$ B) the tool post attains the maximum velocity.

At that given instant

$l_2\ \omega_2 = l_4\ \omega_4$ & velocity of slider is zero.

$l_4 = O_1B = l_1 + l_2 = 50 + 25 = 75$ cm

$\Rightarrow 25 \times 4\pi = 75 \times \omega_4$

$$\omega_4 = \frac{100\pi}{75} = \frac{4\pi}{3} = 4.19\,rad/s$$

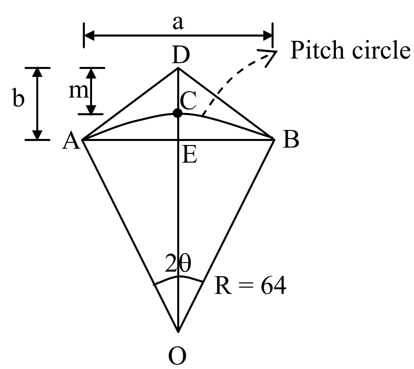
ω_4 = angular velocity of slotted lever.

Chapter- 2
Gear and Gear Trains

01. Ans (a)

02. Ans: (d)

Sol: Angle made by 32 teeth + 32 tooth space = 360°.



$$2\theta = \frac{360}{64} = 5.625$$

$$\theta = 2.8125$$

$$R = \frac{mT}{2} = \frac{4\times 32}{2} = 64\text{mm}$$

$a = R\sin\theta \times 2$

$= 64\times\sin(2.81)\times 2 = 6.28$

OE = Rcosθ = 64×cos(2.8125) = 63.9 mm

b = addendum+ CE = module +(OC – OE)

= 4 + (64– 63.9) = 4.1

03. Ans: (c)

Sol: Helix angle = 90 – 22.5 = 67.5°

04. Ans: (a)

05. Ans: Decreases , Increases

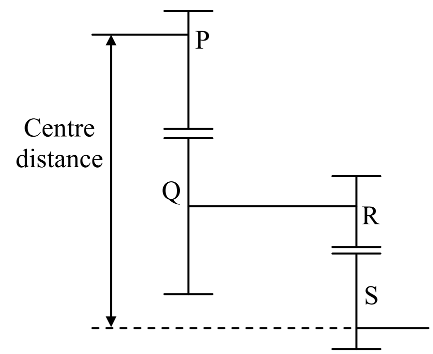
06. Ans: (b)

07. Ans (b)

Sol: For two gears are to be meshed, they should have same module and same pressure angle.

08. Ans: (b)

Sol:



Given $T_p = 20$, $T_Q = 40$, $T_R = 15$, $T_S = 20$

Dia of Q = 2 × Dia of R

$m_Q.T_Q = 2m_R.T_R$

Given, module of R = m_R = 2mm

$$\Rightarrow m_Q = 2\, m_R \frac{T_R}{T_Q} = 2\times 2\times\frac{15}{40} = 1.5\,mm$$

$m_P = m_Q = 2\text{mm}$

$m_S = m_R = 1.5$ mm

$$\text{Radius} = \text{module} \times \frac{\text{No.of teeth}}{2}$$

Centre distance between P and S is given by

$R_P + R_Q + R_R + R_T$

$$= m_P\frac{T_P}{2} + m_Q\frac{T_Q}{2} + m_R\frac{T_R}{2} + m_S\frac{T_S}{2}$$

$$= 1.5\left[\frac{40+20}{2}\right] + 2\left[\frac{15+20}{2}\right]$$

= 45 + 35 = 80 mm

09. Ans: (a)

Sol: $\frac{N_5}{N_2} = \frac{-T_2}{T_3} \times \frac{-T_4}{T_5} = \frac{20}{40} \times \frac{15}{30} = \frac{1}{4}$

$N_5 = \frac{N_2}{4} = \frac{1200}{4} = 300\,\text{rpm}$ in the same direction as that of gear 2 i.e, CCW

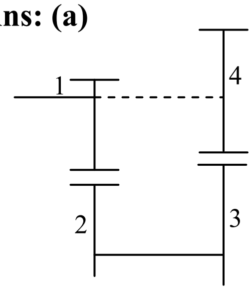
10. Ans: (c)

Sol: $\frac{N_2}{N_6} = \frac{N_3 N_5 N_6}{N_2 N_4 N_5} = \frac{N_3 N_6}{N_2 N_4}$

Wheel 5 is the only Idler gear as the number of teeth on wheel '5' does not appear in the velocity ratio.

11. Ans: (a)

Sol:



$Z_1 = 16$, $Z_3 = 15$, $Z_2 = ?$, $Z_4 = ?$

First stage gear ratio, $G_1 = 4$,

Second stage gear ratio, $G_2 = 3$,

$m_{12} = 3$, $m_{34} = 4$

$Z_2 = 16 \times 4 = 64$

$Z_4 = 15 \times 3 = 45$

12. Ans: (b)

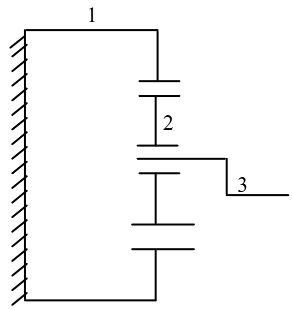
Sol: Centre distance

$= \frac{m_{12}}{2} \times (Z_1 + Z_2) = \frac{m_{34}}{2} \times (Z_3 + Z_4)$

$= \frac{4}{2} \times (15 + 45) = 120\text{mm}$

13. Ans: 5rpm (CCW)

Sol:



$T_1 = 104$, $N_1 = 0$,

$T_2 = 96$, $N_a = 60\text{rpm(CW+ve)}$, $N_2 = ?$

$\frac{N_2 - N_a}{N_1 - N_a} = \frac{T_1}{T_2} = \frac{104}{96}$

$\frac{N_2 - 60}{0 - 60} = \frac{104}{96}$

$N_2 = 60\left[1 - \frac{104}{96}\right] = \frac{-60 \times 8}{96} = -5$ rpm CW

= 5 rpm in CCW

14. Ans: (a)

Sol: By Analytical Approach

$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = \frac{-T_2}{T_1} \times \frac{-T_4}{T_3} = \frac{45}{15} \times \frac{40}{20}$

$\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$

15. Ans: (d)

Sol: Data given:

$\omega_1 = 60$ rpm (CW, +ve)

$\omega_4 = -120$ rpm [2 times speed of gear -1]

We have, $\frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6$

$\Rightarrow \frac{60 - \omega_5}{-120 - \omega_5} = 6$, simplifying

$60 - \omega_5 = -720 - 6\omega_5$

$\omega_5 = -156$ rpm CW

$\omega_5 = 156$ rpm. CCW

16. Ans: (c)

Sol: $\omega_2 = 100$ rad/sec(CW+ve),

$\omega_{arm} = 80$ rad/s (CCW) = –80 rad/sec

$$\frac{\omega_5 - \omega_a}{\omega_2 - \omega_a} = \frac{-T_2}{T_3} \times \frac{T_4}{T_5}$$

$$\frac{\omega_5 - (-80)}{100 - (-80)} = \frac{-20}{24} \times \frac{32}{80} = -\frac{1}{3}$$

$\Rightarrow \omega_5 = -140$ CW = 140 CCW

17. Ans (c)

18. Ans : (d)

Sol: $$\frac{N_A}{N_D} = \frac{T_D}{T_C} \times \frac{T_B}{T_A}$$

$$\Rightarrow \frac{N_A}{100} = \frac{T_D}{50} \times \frac{25}{50} = \frac{T_D}{100}$$

$\Rightarrow N_A = T_D$

From given option (d) is correct.

19. Ans: (c)

Sol: No .of Links, L = 4

No. of class 1 pairs J_1=3

No. of class 2 pairs J_2 =1 (Between gears)

No. of dof = $3(L - 1) - 2J_1 - J_2 = 2$

20. Ans: (c)

Sol: Given $T_2 = 60$ $N_2 = 0$ $T_3 = 20$

$T_4 = 100$ $N_4 = 100$rpm (ccw +ve)

Relative velocity equation

$$\frac{N_4 - N_a}{N_2 - N_a} = -\frac{T_2}{T_4} \Rightarrow \frac{100 - N_a}{0 - N_a} = \frac{-60}{100}$$

$1.6\ N_a = 100$

$$N_a = \frac{100}{1.6} = 62.5 \text{ rpm (ccw)}$$

21. Ans: (*) Change key to Ans: (a)

Sol: r_b = base circle radius,

r_d = dedendum radius

r = pitch circle radius.

For the complete profile to be invoulte,

$r_b = r_d$

$r_d = r - 1$ module

$$r = \frac{mT}{2} = \frac{16 \times 5}{2} = 40 \text{ mm}$$

$\therefore r_b = r_d = 40 - 1 \times 5 = 35$ mm

$r_b = r\cos\phi \Rightarrow \phi \simeq 29^o$

22. Ans: 3.33 N-m

Sol: $$\frac{\omega_s - \omega_a}{\omega_p - \omega_a} = \frac{-Z_p}{Z_s}$$

$$\Rightarrow \frac{0 - 10}{\omega_p - 10} = \frac{-20}{40}$$

$\Rightarrow \omega_p = 30$ rad/sec

By assuming no losses in power transmission

$T_p \times \omega_p + T_s \times \omega_s + T_a \times \omega_a = 0$

$\Rightarrow T_p \times 30 + T_s \times 0 + 5 \times 10 = 0$

$$\Rightarrow T_p = \frac{-50}{30} = -1.67 \text{ N-m}, \ T_p + T_s + T_a = 0$$

$\Rightarrow -1.67 + T_s + 5 = 0$

$\Rightarrow T_s = -3.33$ N-m

Chapter- 3
Fly Wheels

01.

Sol: Given

$P = 80\text{ KW} = 80\times10^3\text{W} = 80{,}000\text{W}$

$\Delta E = 0.9$ Per cycle

$N = 300\,\text{rpm}$

$C_S = 0.02$

$\omega = \frac{2\pi N}{60} = \frac{2\pi\times30}{60} = 31.41\text{rad/s}$

$\rho = 7500\text{ kg/m}^3$

$\sigma_c = 6\text{MN/m}^2$

$\sigma_c = \rho V^2 = \rho R^2\omega^2$

$R = \sqrt{\frac{\sigma_C}{\rho\omega^2}} = \sqrt{\frac{6\times10^6}{7500\times31.41^2}}$

$R = 0.9$ m

$D = 2R = 1.8$m

$N = 300\text{rpm} = 5\text{rps} \rightarrow 0.2$ Sec/rev

1 cycle = 2 revolution $(\because 4\,\text{stroke engine})$

$= 0.4\,\text{sec}$

Energy developed per cycle

$= 0.4\times80 = 32$ kJ

$\Delta E = \text{E per cycle}\times0.9$

$= 32\times10^3\times0.9$

$\Delta E = 28800\,\text{J}$

$\Delta E = I\omega^2 C_S$

$I = \frac{\Delta E}{\omega^2 C_S}$

$I = 1459.58\text{ kg-m}^2$

02.

Sol: Power = 20 kW ,

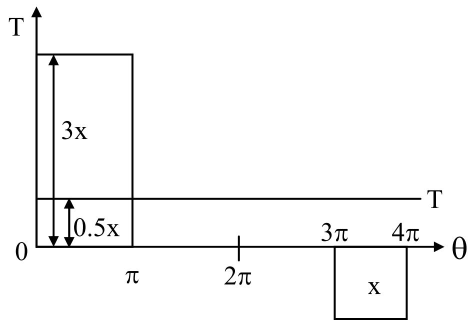
$N = 240\text{ rpm} = \frac{240}{60} = 4\text{rps}$

$= \frac{1}{4}$ sec per rev. = 0.22 sec per rev.

1 cycle = 2 revolution for 4 stroke

So cycle time = $2\times\frac{1}{4} = \frac{2}{4} = 0.5\,\text{sec}$

$C_s = 0.01$



Let torque under the compression stroke = x

Torque under the expansion stroke =3x

Work done per cycle = Net area under the turning moment diagram

= 3xπ – xπ = 2xπ Joule

Mean Torque,

$T_m = \frac{\text{Total work done per cycle}}{\text{Duration of cycle}}$

$= \frac{2x\pi}{4\pi} = 0.5x$

So fluctuation of energy ,

ΔE = 3xπ – 0.5xπ = 2.5 xπ

Power = 20 kW = 20000 kJ/sec

1 cycle time = 0.5 sec

Energy per cycle = 20000 × 0.5 = 10000 kJ

$\therefore\ 2x\pi = 10000$

$\Rightarrow\ x = \dfrac{10000}{2\pi}$ N.m

Fluctuation energy, $\Delta E = 2.5\ x\pi$

$$= 2.5 \times \frac{10000}{2\pi} \times \pi = 12500\text{Nm}$$

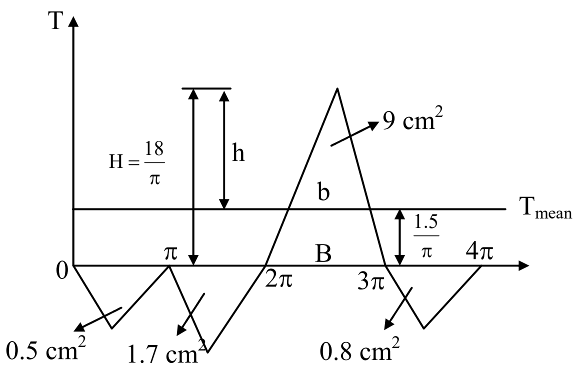
$$\Delta E = I\omega^2 C_s = 12500 = I \times \left(\frac{2\pi N}{60}\right)^2 \times 0.01$$

$$\Rightarrow\ 12500 = I \times \left(\frac{2\times\pi\times 240}{60}\right)^2 \times 0.01$$

$\Rightarrow I = 1978.93$ kg-m^2

03.

Sol:



Given: 1 cm^2 = 1400 J

Assume on x-axis 1 cm = 1 radian and on y-axis 1 cm = 1400 N-m

$a_1 = -0.5$ cm^2

$a_2 = -1.7$ cm^2

$a_3 = 9$ cm^2

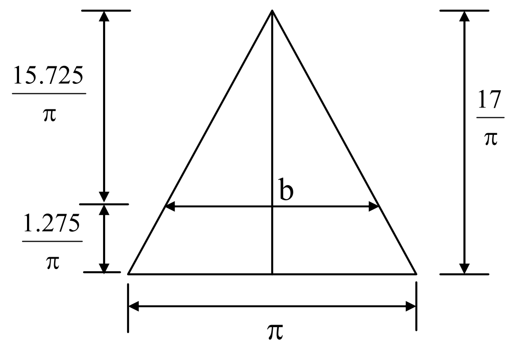
$a_4 = -0.8$ cm^2

Work done per cycle $= -a_1 - a_2 + a_3 - a_4$

$= -0.5 - 1.7 + 9 - 0.8$
$= 6$ cm^2

Mean torque $T_m = \dfrac{\text{Workdone per cycle}}{4\pi}$

$$= \frac{6}{4\pi} = \frac{1.5}{\pi}\text{cm}$$



Area of the triangle (expansion)

$$= \frac{1}{2} \times \pi \times H = 9$$

$H = 18/\pi$

Area above the mean torque line

$$\Delta E = \frac{1}{2} \times b \times h$$

From the similar triangles ,

$$\frac{b}{B} = \frac{h}{H} \Rightarrow b = \frac{16.5}{18} \times \pi$$

$$\Delta E = \frac{1}{2} \times b \times \frac{16.5}{\pi}$$

$$= \frac{1}{2} \times \frac{16.5}{18} \times \frac{16.5}{\pi} = 7.56\ \text{cm}^2$$

$\Delta E = 7.56 \times 1400 = 10587$ N-m

$N_1 = 102$ rpm,

$N_2 = 98$ rpm,

$$\omega_1 = \frac{2\pi N_1}{60} = 10.68\ \text{rad/s}$$

$$\omega_2 = \frac{2\pi N_2}{60} = 10.26\ \text{rad/s}$$

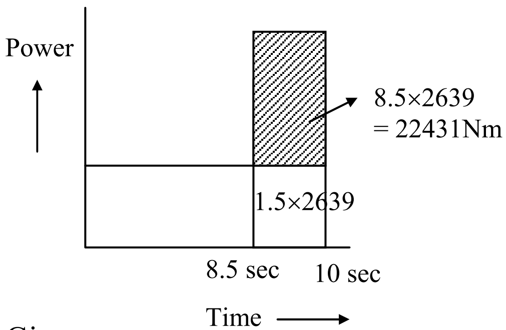
$$\Delta E = \frac{1}{2} \times I \times (\omega_1^2 - \omega_2^2)$$

$$I = \frac{2 \times \Delta E}{(\omega_1^2 - \omega_2^2)} = \frac{2 \times 10587}{10.68^2 - 10.26^2}$$

$I = 26.468$ kgm^2

04.

Sol:



Given:

$d = 40$ mm, $t = 30$ mm

$E_1 = 7$ N-m/mm^2, $S = 100$ mm

$V = 25$ m/s, $V_1 - V_2 = 3\%V$, $C_S = 0.03$

$A = \pi dt = \pi \times 40 \times 30$

$= 3769.9 = 3770$ mm^2

Since the energy required to punch the hole is 7 Nm/mm^2 of sheared area, therefore the Total energy required for punching one hole $= 7 \times \pi dt = 26390$ N-m

Also the time required to punch a hole is 10 sec, therefore power of the motor required $= \frac{26390}{10} = 2639\,\text{Watt}$

The stroke of the punch is 100 mm and it punches one hole in every 10 seconds.

Total punch travel = 200 mm

(up stroke + down stroke)

Velocity of punch = (200/10) = 20 mm/s

Actual punching time = 30/20 = 1.5 sec

Energy supplied by the motor in 1.5 sec is $E_2 = 2639 \times 1.5 = 3958.5 = 3959$ N-m

Energy to be supplied by the flywheel during punching or the maximum fluctuation of energy

$\Delta E = E_1 - E_2$

$= 26390 - 3959 = 22431$ N-m

Coefficient of fluctuation of speed

$$C_S = \frac{V_1 - V_2}{V} = 0.03$$

We know that maximum fluctuation of energy (ΔE)

$22431 = m V^2 C_S = m (25)^2 (0.03)$

$m = 1196$ kg

05.

Sol: Given:

$P = 2$ kW ; $K = 0.5$

$N = 260$ rpm ; $\omega = 27.23$ rad/s

Actual punching time = 1.5 sec

Work done per cycle = 10000 Joule per hole

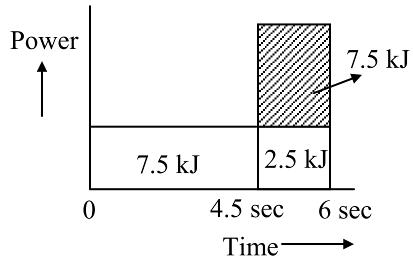
Motor power = 2 kW

$\Delta N = 30$ rpm

$\Delta\omega = 2\pi \times (30/60) = \pi$ rad/sec

600 holes/hr = 10 holes/min ⇒ 6 sec/hole

Cycle time = 6 sec



Energy withdrawn from motor

$= (10000/6) = 1666.67$ J

Energy stored in flywheel

$$= \frac{10000}{6} \times 4.5 = 7.5 \text{ kJ}$$

Fluctuation of Energy $\Delta E = 7500$ J

$$\Delta E = I\,\omega\,\Delta\omega = mk^2\omega\Delta\omega$$

$$m = \frac{\Delta E}{k^2\omega\Delta\omega}$$

Where k = radius of gyration

$$m = \frac{7500}{0.5^2 \times 27.23 \times \pi} = 349.5\,\text{kg}$$

06. Ans: (a)

Sol: $$\frac{\frac{1}{2} \times T_P \times \pi}{4\pi} = 10$$

$T_p = 80$ N-m

07. Ans: (d)

Sol: At A, Energy = E + 100

At B, Energy = E + 100 − 75 = E + 25

At C, Energy = E + 25 + 89 = E + 114

At D, Energy = E + 114 − 77 = E + 37

At E, Energy = E + 37 + 36 = E + 73

At F, Energy = E + 73 − 73 = E

Maximum energy = E + 114 = maximum speed

Minimum energy = E = minimum speed

In flywheel energy is stored in form of kinetic energy.

When flywheel stores energy its kinetic energy increases and when it releases energy its kinetic energy decreases.

08. Ans: (d)

Sol: Work done = −0.5+1−2+25−0.8+0.5

$= 23.2\ \text{cm}^2$

Work done per cycle = 23.2×100 = 2320

$(\because 1\text{cm}^2 = 100\text{N} - \text{m})$

$$T_{mean} = \frac{\text{W.D per cycle}}{4\pi}$$

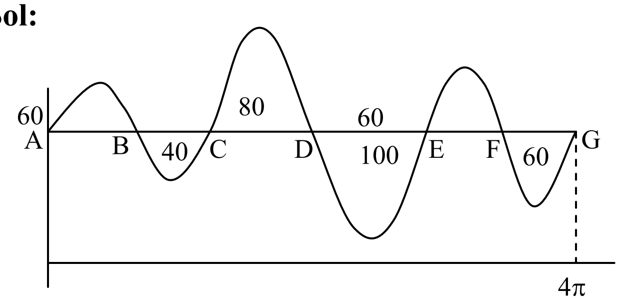
$$= \frac{2320}{4\pi} = \frac{580}{\pi}\ \text{N} - \text{m}$$

Suction = 0 to π, compression = π to 2π

Expansion = 2π to 3π, Exhaust =3 π to 4π

09. Ans: (c)

Sol:



$E_A = E$

$E_B = E + 60$

$E_C = E + 60 - 40 = E + 20$

$E_D = E + 20 + 80 = E + 100 = E_{max}$

$E_E = E + 100 - 100 = E$

$E_F = E + 60$

$E_G = E + 60 - 60 = E_{min}$

10. Ans: (b)

Sol: $I_{disk} = \frac{mr^2}{2}$

$I_1 = \frac{mr_1^2}{2}$, $C_{s1} = 0.04$

$I_2 = 4\times mr_1^2 = 4I_1$

$C_{s2} = \frac{I_1}{I_2} \times C_{s1} = 0.01 \Rightarrow 1\%$ reduce

11. Ans: (d)

12. Ans: (a)

Sol: Let the cycle time = t

Actual punching time = t/4

W = energy developed per cycle

Energy required in actual punching

= 3W/4

During 3t/4 time, energy consumed = W/4

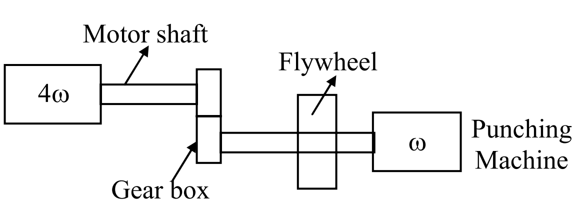
$E_{max} = \frac{3W}{4}$, $E_{min} = \frac{E}{4}$

$\Delta E = E_{max} - E_{min} = \frac{E}{2}$

$\frac{\Delta E}{E} = 0.5$

13. Ans: (c)

Sol:



$C_s = 0.032$

Gear ratio = 4

$I\omega'^2 \times C_s' = I\omega^2 C_s$

$C_s' = C_s\left(\frac{\omega}{\omega'}\right)^2 = \frac{C_s \times \omega^2}{16\,\omega^2} = \frac{C_s}{16}$

= 0.0032 / 16= 0.002

(by taking moment of Inertia, I = constant).

Thus, if the flywheel is shifted from machine shaft to motor shaft when the fluctuation of energy (ΔE) is same, then coefficient of fluctuation of speed decreases by 0.2% times.

14. Ans: 0.5625

Sol: The flywheel is considered as two parts $\frac{m}{2}$ as rim type with Radius R and $\frac{m}{2}$ as disk type with Radius $\frac{R}{2}$

$I_{Rim} = \frac{m}{2}R^2$,

$I_{disk} = \frac{1}{2}\times\frac{m}{2}\times\left(\frac{R}{2}\right)^2 = \frac{mR^2}{16}$

$I = \frac{mR^2}{2} + \frac{mR^2}{16} = \frac{9}{16}mR^2$

$= 0.5625\ mR^2$

$\therefore\ \alpha = 0.5625$

15. Ans: 31.42

Sol: From the T - θ diagram energy is stored into flywheel during $\frac{\pi}{2}$ to π

$$\Delta E = \frac{\pi}{2} \times 3000 \,N - m$$

From ω-θ diagram,

ω_{max} = 20 rad/sec

ω_{min} = 10 rad/sec

$$\Delta E = \frac{1}{2} I\left(\omega_{max}^2 - \omega_{min}^2\right)$$

$$\Rightarrow I = \frac{2\Delta E}{\omega_{max}^2 - \omega_{min}^2}$$

$$I = \frac{2 \times \frac{\pi}{2} \times 3000}{20^2 - 10^2} = 31.42\,kgm^2$$

16. Ans: 104.71

Sol: N = 100 rpm

$$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} (10000 + 1000 \sin 2\theta - 1200 \cos 2\theta) d\theta$$

$$= \frac{1}{\pi}\left[10000\theta - 500 \cos 2\theta - 600 \sin 2\theta\right]_0^{\pi}$$

= 10000 Nm

$$Power = \frac{2\pi NT}{60}$$

$$= \frac{2 \times \pi \times 100 \times 10000}{60} = 104719.75 \text{ W}$$

P = 104.719 kW

Chapter- 4
Governor

01. Ans: (a) **02. Ans: (d)**

03. Ans: (b)

Sol: $$h = \frac{g}{m\omega^2}\left[m + \frac{(M)(1+k)}{2}\right]$$

K = 1

$$\frac{0.50}{\sqrt{2}} = \frac{g}{\omega^2}(21) \Rightarrow \omega = 24 \text{ rad/sec}$$

04. Ans: (a)

Sol: $$mr\omega^2 = \frac{r}{h}\left(mg + \frac{Mg(1+k)}{2}\right)$$

k = 1

$$\omega^2 = \frac{9.8}{2 \times 0.2}(10 + 2)$$

ω = 17.15 rad/sec

05. Ans: (a)

Sol: $$mr\omega^2 a = \frac{1}{2} \times 200 \times \delta \times a$$

$$\delta = \frac{1 \times 20^2 \times 0.25 \times 2}{200}$$

= 0.5 × 2 = 1 cm

06. Ans: (a)

Sol: $$mr\omega^2 \times a = \left(\frac{F_s}{2}\right) \times a$$

$F_s = 2mr\omega^2$

$= 2 \times 1 \times 0.4 \times (20)^2 = 320$ N

07. Ans: (c) **08. Ans: (c)** **09. Ans: (b)**

10. Ans: (a)

Sol: $r_1 = 50$ cm , $F_1 = 600$ N

$F = a + rb$

$600 = a + 50\,b$

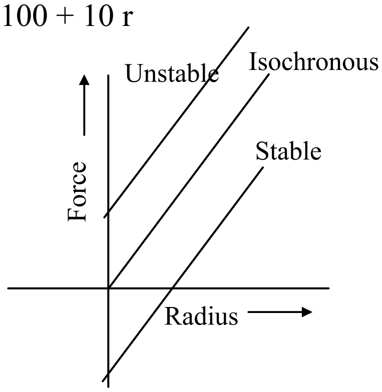
$700 = a + 60\,b$

$10\,b = 100$

$b = 10$ N/cm

$a = 100$ N

$F = 100 + 10\,r$

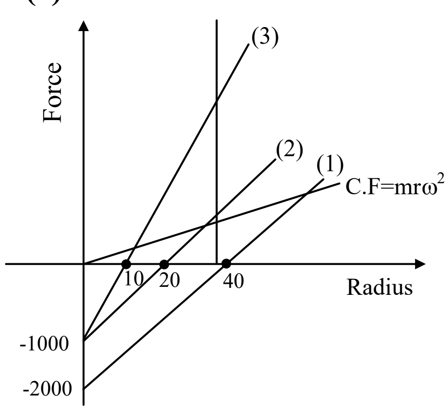


This is unstable governor. It can be isochronous if its initial compression is reduced by 100 N.

11. Ans: (d) **12. Ans: (d)**

13. Ans: (a)

Sol:



At radius, $r_1 = F_1 < F_2 < F_3$

∴ As Controlling force is less suitable 1 is for low speed and 2 for high speed ad 3 is for still high speed.

(1) is active after 40 cm

(2) is active after 20 cm

(3) is active after 10 cm

At given radius above 20

$F_3 > F_2$

$mr\omega_3^2 > mr\omega_2^2$

$\omega_3 > \omega_2$

14. Ans: (b)

15. Ans: (c) **16. Ans: (c)**

17. Ans: (b)

Sol: F = 14 N , r = 2cm

F = 38 N, r = 6cm

$14 = 2a + b$

$38 = 6a + b$

$4a = 24$

$\Rightarrow a = 6$

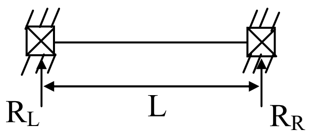
$b = 2$

⇒ unstable governor

Chapter- 5
Balancing

01. Ans: (c)

Sol:



(a) Static Balance

At High speed R_L and R_R at '2L' apart i.e.,

(b) Dynamic balance

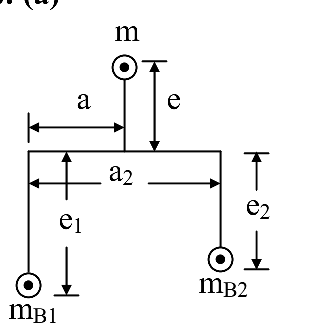
$\therefore\ R_L' \times 2L$ = Unbalanced couple

$\therefore\ R_L' \times 2L = R_L\ L$

$$\therefore\ R_L' = \frac{R_L}{2}$$

02. Ans: (a)

Sol:



$m\omega^2 ea = m_{B_1}\,\omega^2 e_1 + m_{B_2}\,\omega^2 e_2$

couple about the plane of B

$mea = m_{B_2}\, e_2 a_2$

$9\times(0.5)\times(0.5) = m_{B_2}\times(0.5)\times(1.5)$

$m_{B_2} = 3$ kg , $m_{B_1} = 6$ kg

03. Ans: (c)

04. Ans: (a)

Sol: Dynamic force $= \dfrac{W}{g} e\omega^2$

Couple $= \dfrac{W}{g} e\omega^2\, a$

Reaction on each bearing $= \pm\dfrac{W}{g} e\omega^2 \dfrac{a}{l}$

Total reaction on bearing

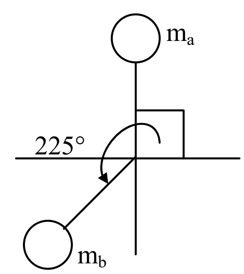
$$= \left(\frac{W}{g} e\omega^2 \frac{a}{l}\right) - \left(\frac{W}{g} e\omega^2 \frac{a}{l}\right) = 0$$

05. Ans: (b)

06. Ans: (a)

07. Ans: (b)

Sol:



$m_a = 5$ kg, $r_a = 20$ cm

$m_b = 6$ kg, $r_b = 20$ cm

$m_c = ?$, $r_c = 20$ cm

$m_d = ?,\ \theta_c = ?\ ,\ \theta_d = ?$

Take reference plane as 'C'

For complete balancing

$\sum mr = 0$ & $\sum mrl = 0$

$2m_d\cos\theta_d - 9\sqrt{2} = 0$

$\Rightarrow m_d\cos\theta_d = 9\sqrt{2}$

$2m_d\sin\theta_d - 5 - 9\sqrt{2} = 0$

$m_d\sin\theta_d = = \frac{1}{2}\left(5+9\sqrt{2}\right)$

$m_d = \sqrt{\left(\frac{9}{\sqrt{2}}\right)^2 + \left[\frac{1}{2}\left(5+9\sqrt{2}\right)\right]^2} = 10.91\text{kg}$

$\theta_d = \tan^{-1}\left[\frac{\frac{1}{2}\left(5+9\sqrt{2}\right)}{\frac{9}{\sqrt{2}}}\right] = 54.31^0$

$= 90 - 54.31 = 35.68$ w.r.t 'A'

$m_c\cos\theta_c + m_d\cos\theta_d - 3\sqrt{2} = 0$

$\Rightarrow m_c\cos\theta_c + 10.91\cos 54.31 - 3\sqrt{2} = 0$

$m_c\cos\theta_c = -2.122$

$m_c\sin\theta_c + m_d\sin\theta_d - 3\sqrt{2} + 5 = 0$

$m_c\sin\theta_c + 10.91\sin 54.31 - 3\sqrt{2} + 5 = 0$

$m_c\sin\theta_c = -9.618$

$m_c = \sqrt{(-2.122)^2 + (-9.618)^2} = 9.85\text{kg}$

$\tan\theta_c = \frac{-9.618}{-2.122}$

$\theta_c = 257.56$ or $257.56 - 90$ w.r.t 'A'

$= 167.56$

S.No	m	(r×20)cm	(l×20)cm	θ	mrcosθ	mrsinθ	mrlcosθ	mrlsinθ
A	5	1	–1	90	0	5	0	–5
B	6	1	3	225	$-3\sqrt{2}$	$-3\sqrt{2}$	$-9\sqrt{2}$	$-9\sqrt{2}$
C	m_c	1	0	θ_c	$m_c\cos\theta_c$	$m_c\sin\theta_c$	0	0
D	m_d	1	2	θ_d	$m_d\cos\theta_d$	$m_d\sin\theta_d$	$2m_d\cos\theta_d$	$2m_d\sin\theta_d$

Common data Q. 08 & 09

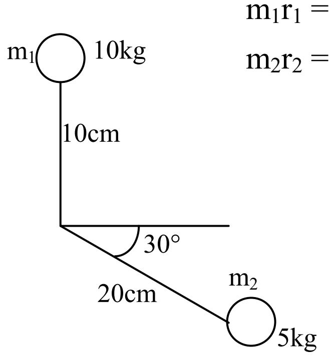
08. Ans: (a)

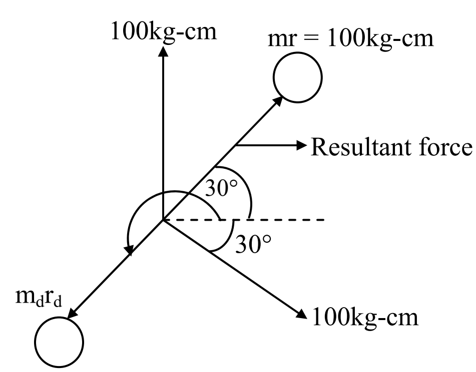
Sol: m_1 = kg , m_2 = 5kg , r_1 = 10cm

r_2 = 20cm, m_d = ?, r_d = 10cm

m_1r_1 = 100 kg cm

m_2r_2 = 100kg cm





Keep the balancing mass m_d at exactly opposite to the resultant force

$\therefore m_d r_d = 100$kg-cm

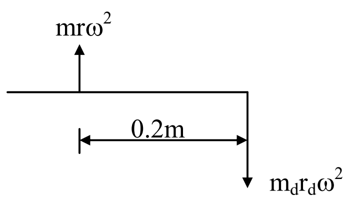
$\Rightarrow m_d \times 10 = 100$ kg-cm

$m_d = 10$kg cm

$\theta_d = 180 + 30 = 210$

09. Ans: (d)

Sol:



mr = 100kg-cm = 1kgm

$$N = 600 \text{ rpm} \Rightarrow \omega = \frac{2\pi N}{60} = 20\pi \text{rad/s}$$

Couple 'C' = $mr\omega^2 \times 0.2 = 1\times(20\pi)^2\times 0.2$

$= 789.56$Nm

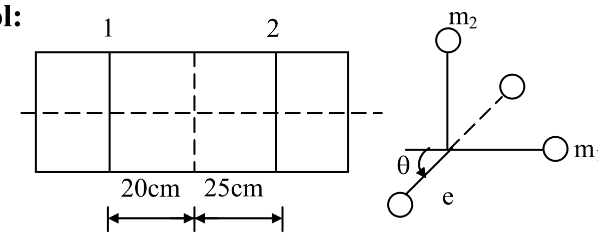
Reaction on the bearing

$$= \frac{\text{couple}}{\text{distance between bearing}}$$

$$= \frac{789.56}{0.4} = 1973.92\text{N}$$

10. Ans: (a)

Sol:



$r_1 = 10$cm, $r_2 = 10$cm, $m_1 = 52$ kg

$m_2 = 75$kg, $\theta_1 = 0$ (Reference)

$\theta_2 = 90°$, m = 2000kg , e = ?, θ=?

$me\cos\theta = m_1 r_1 = 520$

$me\sin\theta = m_2 r_2 = 750$

$$me = \sqrt{(m_1 r_1)^2 + (m_2 r_2)^2} = \sqrt{520^2 + 750^2}$$
$$= 913 \text{ kg} - \text{cm}$$

$$e = \left(\frac{913}{2000}\right) = 0.456\text{cm}$$

$$\theta = \tan^{-1}\left(\frac{m_2 r_2}{m_1 r_1}\right) = \tan^{-1}\left(\frac{75}{52}\right) = 55.26^0$$

$= 180 + 55.26 = 235.26°$

w.r.t mass '1'.

11. Ans: (a)

Sol:

Plane	m (kg)	r (m)	L (m) (reference Plane A)	θ	F_x (mrcosθ)	F_y (mrsinθ)	C_x (mr*l*cosθ)	C_y (mr*l*sinθ)
D	2 kg.m		0.3	0	2	0	0.6	0
A	$-m_a$	0.5m	0	θ_a	$-0.5m_a\cos\theta_a$	$-0.5m_a\sin\theta_a$	0	0
B	$-m_b$	0.5m	0.5	θ_b	$-0.5m_b\cos\theta_b$	$-0.5m_b\sin\theta_b$	$-\frac{m_b}{4}\cos\theta_b$	$-\frac{m_b}{4}\sin\theta_b$

$C_x = 0 \Rightarrow \dfrac{m_b \cos\theta_b}{4} = 0.6$

$C_y = 0 \Rightarrow \dfrac{m_b \sin\theta_b}{4} = 0$

$\Rightarrow m_b = 2.4\text{kg}, \quad \theta_b = 0$

$\Sigma F_x = 0$

$\Rightarrow 2 - 0.5\, m_a \cos\theta_a - 0.5\, m_b \cos\theta_b = 0$

$\Rightarrow \dfrac{m_a}{2}\cos\theta_a = 0.8$

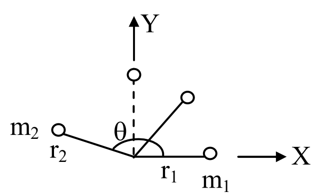
$\Sigma F_y = 0 \Rightarrow \dfrac{m_a}{2}\sin\theta_a = 0$

$\therefore\ \theta_a = 0°,\ m_a = 1.6$ kg

(Note: mass is to be removed so that is taken as –ve).

12. Ans: (a)

Sol:



$\dfrac{F_x}{\omega^2} = m_1 r_1 + m_2 r_2 \cos\theta$

$= 20 \times 15 + 25 \times 20 \cos 135$

$= -53.55$ gm-cm

$\dfrac{F_y}{\omega^2} = m_2 r_2 \sin\theta_2 = 25\times 20 \sin 135$

$= 353.553$ gm-cm

$m_b r_b = \sqrt{F_x^{\,2} + F_y^{\,2}}$

$\Rightarrow m_b = \dfrac{\sqrt{F_x^{\,2} + F_y^{\,2}}}{r_b}$

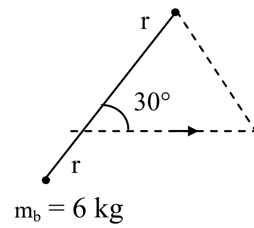
$= \dfrac{\sqrt{(-53.55)^2 + (353.553)^2}}{20}$

$= 17.88$ gm

$\theta_b = \tan^{-1}\dfrac{F_y}{F_x} = \tan^{-1}\left(\dfrac{353.553}{-53.55}\right) = 98.7°$

13. Ans: 30 N

Sol:



Crank radius

= stroke/2 = 0.1 m,

ω = 10 rad/sec

Unbalanced force along perpendicular to the line of stroke = $m_b r\omega^2 \sin 30°$

$= 6 \times (0.1) \times (10)^2 \sin 30° = 30$ N

14. Ans: (b)

15. Ans: (b)

16. Ans: (b)

Sol: m = 10 kg, r = 0.15 m,

c = 0.6, θ = 60°, ω = 4 rad/sec

Residual unbalance along the line of stroke = $(1 - c)\, m r\omega^2 \cos\theta$

$= (1 - 0.6)\times 10 \times 0.15 \times 4^2 \cos 60$

$= 4.8$ N

17. Ans: 2

Sol: By symmetric two system is in dynamic balance when

$mea = m_1 e_1 a_1$

$m_1 = m\dfrac{e}{e_1}.\dfrac{a}{a_1} = 1\times\dfrac{50}{20}\dfrac{2}{2.5} = 2\text{kg}$

Chapter- 6
Cams

01. Ans: (d) **02. Ans: (a)**

03. Ans: (d) **04. Ans: (b)**

05. Ans: (b)

Sol: L = 4 cm , $\phi = 90^o = \pi/2$ radian ,

$\omega = 2$ rad/sec , $\theta = \frac{2}{3}\times 90 = 60°$

$$\frac{\theta}{\phi} = \frac{2}{3}$$

$$s(t) = \frac{L}{2}\left(1-\cos\frac{\pi\theta}{\phi}\right)$$

$$= 2(1-\cos 120) = 3\text{cm}$$

$$V(t) = \frac{L}{2}\times\frac{\pi}{\phi}\times\omega\times\sin\left(\frac{\pi\theta}{\phi}\right)$$

$$= \frac{4}{2}\times 2\times 2\sin(120) = 7\text{cm/s}$$

$$a(t) = \frac{L}{2}\left(\frac{\pi}{\phi}\right)^2\times\omega^2\times\cos\left(\frac{\pi\theta}{\phi}\right)$$

$$= \frac{4}{2}\times 2^2\times 2^2\times\cos(120) = -16\text{cm/sec}^2$$

06. Ans: (a)

Sol: L = 10 cm , $\phi = 180° = \pi$ rad ,

V_{max} = 25 cm/s

$$V_{max} = 25 = \frac{L}{2}\times\frac{\pi}{\phi}\times\omega$$

$$\Rightarrow 25 = \frac{10}{2}\times\frac{\pi}{\pi}\times\omega$$

$\Rightarrow \omega = 5$ rad/sec

$$a_{max} = \frac{L}{2}\times\left(\frac{\pi}{\phi}\right)^2\times\omega^2$$

$$= \frac{10}{2}\times(1)^2\times 5^2 = 125\text{ cm/sec}^2$$

07. Ans: (a)

Sol: $\phi = 90°$, $\frac{\pi}{\phi} = 2$

L = 6 mm , $\omega = 1$ rad/s

When, s = 3mm,

$$s = \frac{L}{2}\left(1-\cos\frac{\pi\theta}{\phi}\right)$$

$$3 = 3\left(1-\cos\frac{\pi\theta}{\phi}\right)$$

$$\cos\frac{\pi\theta}{\left(\frac{\pi}{2}\right)} = 0$$

$\Rightarrow \cos 2\theta = 0$

$\Rightarrow 2\cos^2\theta - 1 = 0$

$\Rightarrow \theta = 45°$

$$\frac{\theta}{\phi} = \frac{1}{2}$$

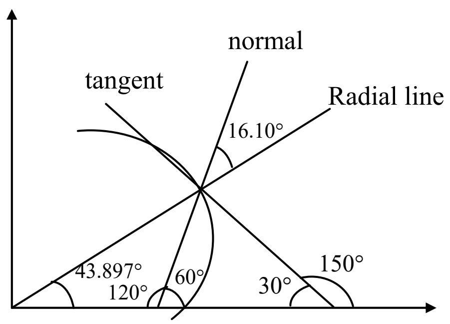
$$V = \frac{L}{2}\frac{\pi}{\phi}\times\omega\times\sin(\pi(1/2))$$

$$= \frac{6}{2}\times 2\times 1\times\sin\left(\frac{\pi}{2}\right) = 6\text{ mm/sec}$$

$$a = \frac{L}{2}\times\left(\frac{\pi}{\phi}\right)^2\times\omega^2\times\cos\left(\frac{\pi\theta}{\phi}\right) = 0$$

08. Ans: (b)

Sol:



$x = 15\cos\theta$,

$y = 10 + 5\sin\theta$

$$\tan\phi = \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\left(\frac{dx}{d\theta}\right)} = \frac{5\cos\theta}{-15\sin\theta}$$

at $\theta = 30°$,

$$\tan\phi = \frac{5\times\frac{\sqrt{3}}{2}}{-15\times\frac{1}{2}} = -\frac{1}{\sqrt{3}} \Rightarrow \phi = 150°$$

$$\tan\theta = \frac{y}{x} = \frac{10+5\sin\theta}{15\cos\theta} = \frac{10+5\sin 30}{15\cos 30}$$

$\theta = 43.897°$

Pressure angle is angle between normal and radial line = 16.10° .

or $x = 15\cos\theta$,

$y = 10 + 5\sin\theta$ at $\theta = 30^o$

$$\left(\frac{x}{15}\right)^2 + \left(\frac{y-10}{5}\right)^2 = 1$$

$$x = \frac{15\sqrt{3}}{2}, \quad y = 125$$

$$\frac{2x}{15^2} + \frac{2(y-10)}{5^2}\cdot\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-x}{(y-10)9} = \frac{-15\sqrt{3}}{2\left(\frac{3}{2}\right)\times 9} = \frac{-1}{\sqrt{3}}$$

$$\tan\theta = \frac{-1}{\sqrt{3}}$$

Then normal makes with x-axis

$\tan^{-1}(\sqrt{3}) = 60^o$

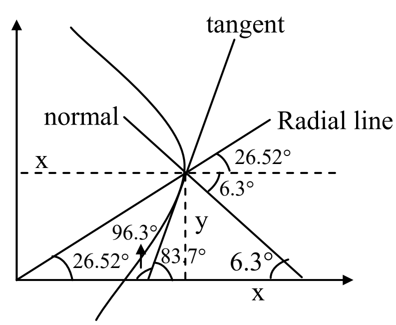
$$\tan\theta = \frac{y}{x} = \frac{10+5\sin\theta}{15\cos\theta} = \frac{10+5\sin 30}{15\cos 30}$$

$\theta = 43.897°$

With follower axis angle made by normal (pressure angle) = 60°–43.897° = 16.10°

09. Ans: (a)

Sol:



Let α be the angle made by the normal to the curve

$$\left(\frac{dy}{dx}\right)_{(4,2)} = 9$$

$$\tan\alpha = \frac{dy}{dx} = 4x - 7$$

At $x = 4$ & $y = 2$, $\alpha = \tan^{-1}(9) = 83.7°$

The normal makes an angle

$$= \tan^{-1}\left(\frac{-1}{9}\right) = 6.3° \text{ with x axis}$$

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.52°$$

Pressure angle is angle between normal and radial line = 26.52 + 6.3 = 32.82°

10. Ans: (b)

Sol: For the highest position the distance between the cam center and follower

= (r+5) mm

For the lowest position it is (r –5) mm

So the distance between the two positions

= (r+5) – (r–5) = 10 mm

Chapter - 7
Gyroscope

01. Ans: (b) **02. Ans: (d)** **03. Ans: (b)**

04. Ans: (b) **05. Ans: (c)** **06. Ans: (d)**

07. Ans:

Sol: N = 300 rpm

$$\omega_s = \frac{2\pi N}{60} = \frac{2\pi\times 3000}{60} = 100\pi \ \hat{i} \text{ rad/s}$$

I = 47.25 kg-m^2

2π rad $\rightarrow$ 17s

$$\omega_p = \left(\frac{2\pi}{17}\right)\hat{j}$$

$$C_g = \vec{\omega}_p \times \vec{H} = I\omega\times\omega_p\left(\hat{i}\times\hat{j}\right)$$

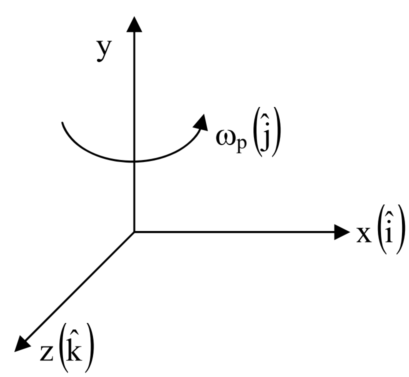
$$= I\omega\times\omega_p\left(\hat{k}\right)$$

$$= 47.25 \times 100\pi \times \frac{2\pi}{17}$$

$$= 5486.33 \text{ N.m}$$

08. Ans:

Sol:



$I = mk^2$, $m = 6000$ kg ,

$k = 0.45$ m ,

$\omega = 2400$ rpm = 80π rad/sec = 251.2 rad/sec

$$\omega_p = \frac{18\times1860}{60\times3600} = 0.155 \text{rad/sec}$$

$$\omega_p = \frac{V}{R}\hat{j}$$

(i) Gyroscope couple

$$C_g = \vec{H}\times\vec{\omega}_p$$

$$= I\omega\times\omega_p\left(\hat{i}\times\hat{j}\right) = mk^2\omega\times\omega_p\times\hat{k}$$

$$= 6000\times0.45^2\times251.2\times0.155$$

$$= 47.3\ \hat{k}\ \text{kN-m}$$

Bow portion is raised.

(ii) Pitching amplitude, A = 7.5°

$\alpha = A\sin\omega t$

$\tau = 18$ sec ,

$$f = \frac{1}{18}\text{Hz}$$

$$\omega = \frac{2\pi}{18}\text{rad/sec}$$

Maximum angular velocity of precession,

$\omega_p = A\,\omega$

$$= 7.5\times\frac{\pi}{180}\times\frac{2\pi}{18} = 0.0457\left(-\hat{k}\right)\ \text{rad/sec}$$

$$\vec{H} = I\omega\,\hat{i} = 6000\times0.45^2\times80\pi\hat{i}$$

$$= 30536.28\ \hat{i}$$

$$C_g = \vec{H}\times\vec{\omega}_p$$

$$= I\omega\times\omega_p\left(\hat{i}\times-\hat{k}\right)$$

$$= 6000\times0.45^2\times80\pi\,\hat{i}\times0.0457\left(-\hat{k}\right)$$

$$= 13.955\ \left(\hat{j}\right)\ \text{kN-m}$$

(as the bow portion is lowered, the ship turns towards left or port side)

Maximum acceleration = $A\,\omega^2$

$$= 7.5\times\frac{\pi}{180}\times\left(\frac{2\pi}{18}\right)^2\ \text{rad/sec}^2$$

$$= 0.016\ \text{rad/sec}^2$$

(iii) $\omega_{rolling}$ = 0.035 rad/sec

$\omega_p = 0$ during rolling

$C_g = \vec{H}\times\vec{\omega}_p = 0$ (No gyroscope effect)

09. Ans: 200 (range 199 to 201)

Sol: R=100m, v = 20m/sec,

$$\omega_p = \frac{V}{R} = 0.2\frac{\text{rad}}{\text{sec}} \qquad \omega_s = 100\text{rad/sec}$$

$I = 10\text{kg-m}^2$

Gyroscopic moment

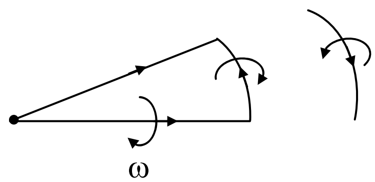
$$= I\omega_s\omega_p = 10\times0.2\times100\text{N-m}$$

$$= 200\text{N-m}$$

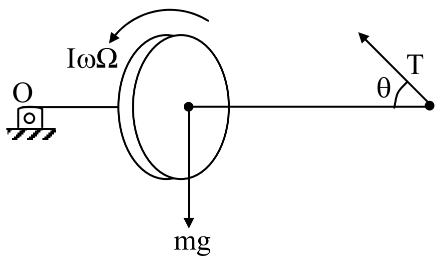
10. Ans:

Sol:

(i)



The gyroscopic couple is = $I\omega\Omega$



$$\sum M_o = 0$$

$$2a.T\sin\theta + I\omega\Omega = mg \times a$$

$$\frac{2a.T.b}{\sqrt{4a^2+b^2}} + \frac{mr^2}{2}\omega\Omega = mg \times a$$

$$T = \frac{\sqrt{4a^2+b^2}}{2ab}\left(mga - \frac{mr^2}{2}\omega\Omega\right)$$

For clockwise rotation of precession

(ii) $\sum M_o = 0$

$$2a.T\sin\theta - I\omega\Omega = mg \times a$$

$$T = \frac{\left(mga + \frac{1}{2}mr^2\omega\Omega\right)\left(b^2+4a^2\right)^{\frac{1}{2}}}{2ab}$$

Chapter - 8
Mechanical Vibrations

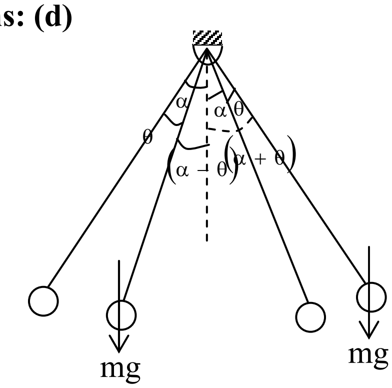
01. Ans: (b)

Sol: $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow 0.5 = 2\pi \times \sqrt{\frac{L}{9.81}}$

$$\Rightarrow L = 62.12 \text{ mm}$$

02. Ans: (d)

Sol:



Let the system is displaced by θ from the equilibrium position. It's position will be as shown in figure.

By considering moment equilibrium about the axis of rotation (Hinge)

$$I\ddot{\theta} + mg\ell\sin(\alpha+\theta) - mg\ell\sin(\alpha-\theta) = 0$$

$$I = m\ell^2 + m\ell^2 = 2m\ell^2$$

After simplification

$$2m\ell^2\ddot{\theta} + 2mg\ell\cos\alpha\,\sin\theta = 0$$

For small oscillations (θ is small) $\sin\theta = \theta$

$$\therefore 2m\ell^2\ddot{\theta} + 2mg\ell\cos\alpha.\theta = 0$$

$$\omega_n = \sqrt{\frac{2mg\ell\cos\alpha}{2m\ell^2}} = \sqrt{\frac{g\cos\alpha}{\ell}}$$

03. Ans: (c)

Sol: Let, V_o is the initial velocity, 'm' is the mass

Equating Impulse = momentum

$$mV_o = 5kN \times 10^{-4}\ sec$$

$$= 5\times10^3 \times 10^{-4} = 0.5\ sec$$

$$\therefore V_0 = \frac{0.5}{m} = 0.5\ m/sec$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{10000}{1}} = 100\ rad/sec$$

When the free vibrations are initiate with initial velocity,

The amplitude

$$X = \frac{V_0}{\omega_n}\ \text{(Initial displacement)}$$

$$\therefore X = \frac{V_0}{\omega_n} = \frac{0.5\times10^3}{100} = 5\ mm$$

04. Ans: (a)

Sol: Note: ω_n depends on mass of the system not on gravity

$$\therefore \omega_n \propto \frac{1}{\sqrt{m}}$$

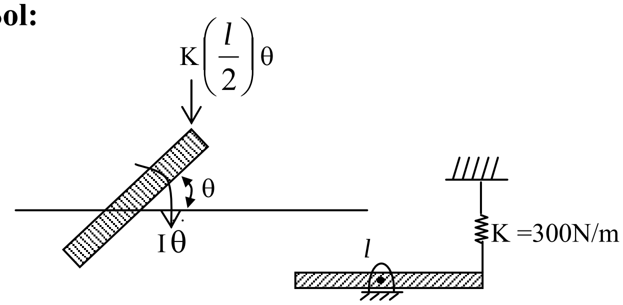
$$\text{If } \omega_n = \sqrt{\frac{g}{\delta}},\ \delta = \frac{mg}{K}$$

$$\therefore \omega_n = \sqrt{\frac{g}{\left(\frac{mg}{K}\right)}} = \sqrt{\frac{K}{m}}$$

$\therefore \omega_n$ is constant every where.

05. Ans: (c)

Sol:



By energy method

$$E = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}Kx^2 = \text{constant}$$

$$E = \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}K\times\left(\frac{\ell}{2}\theta\right)^2 = \text{constant}$$

Differentiating w.r.t 't'

$$\frac{dE}{dt} = I\ddot{\theta} + \frac{K}{2}\times\frac{\ell^2}{4}\times2\theta = 0$$

$$I = \frac{m\ell^2}{12}$$

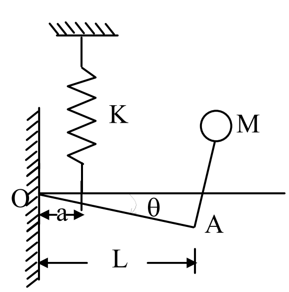
$$\frac{m\ell^2}{12}\ddot{\theta} + \frac{K\ell^2}{4}\theta = 0$$

$$\Rightarrow \ddot{\theta} + \frac{3K}{m}\theta = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{3K}{m}} = 30\ rad/sec$$

06. Ans: (a)

Sol:



Assume that in equilibrium position mass M is vertically above 'A'. Consider the displaced position of the system at any instant as shown above figure.

If Δ_{st} is the static extension of the spring in equilibrium position, its total extension in the displaced position is $(\Delta_{st} + a\theta)$.

From the Newton's second law, we have

$$I_0 \ddot{\theta} = Mg(L + b\theta) - k(\Delta_{st} + a\theta)a \quad ...(1)$$

But in the equilibrium position

$MgL = k\Delta_{st}\, a$

Substituting the value in equation (1), we have $I_0 \ddot{\theta} = (Mgb - ka^2)\theta$

$$\Rightarrow I_0 \ddot{\theta} + (ka^2 - Mgb)\theta = 0$$

$$\omega_n = \sqrt{\frac{ka^2 - Mgb}{I_0}}$$

$$\tau = 2\pi\sqrt{\frac{I_0}{ka^2 - Mgb}}$$

The time period becomes an imaginary quantity if $ka^2 < Mgb$. This makes the system unstable. Thus the system to vibrate the limitation is

$$ka^2 > Mgb$$

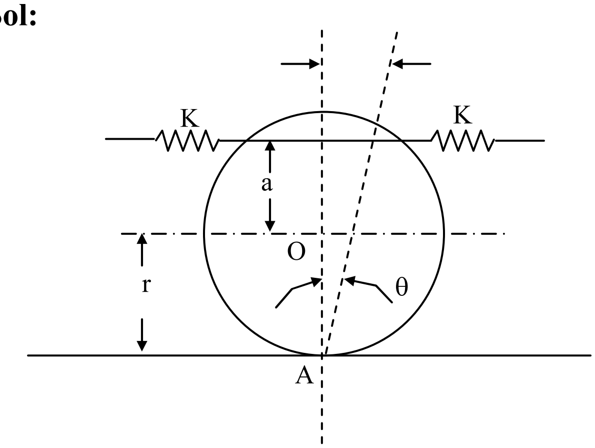
$$b < \frac{ka^2}{Mg}$$

Where W = Mg

07. Ans: (a)

08.

Sol:



$$KE = \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{2}I\dot{\theta}^2$$

$$= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{4}mr^2\dot{\theta}^2 = \frac{3}{4}mr^2\dot{\theta}^2$$

$$PE = \frac{1}{2}Kx^2 + \frac{1}{2}Kx^2 = Kx^2$$

$$x = (r + a)\theta$$

$$\Rightarrow PE = K\{(r + a)\theta\}^2$$

$$\frac{d}{dt}KE + \frac{d}{dt}PE = 0$$

Substituting in the above equation

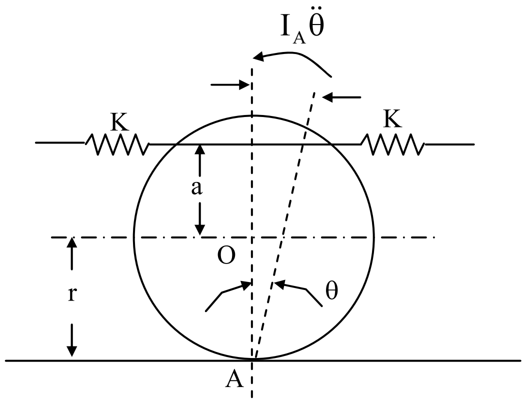
$$\frac{3}{2}mr^2\ddot{\theta} + 2K(r + a)^2\theta = 0$$

Natural frequency

$$f_n = \frac{1}{2\pi}\sqrt{\frac{4K(r + a)^2}{3mr^2}}$$

So $f_n = 47.74$ Hz.

Or



Taking the moment about the instantaneous centre 'A'.

$I_A\ddot{\theta} + 2K(r+a)\,\theta\,(r+a) = 0$

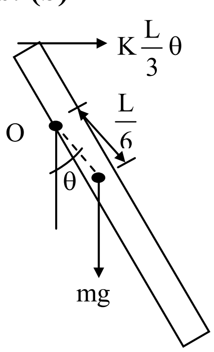
$$I_A = \frac{mr^2}{2} + mr^2 = \frac{3}{2}mr^2$$

$$\frac{3}{2}mr^2\ddot{\theta} + 2k(r+a)^2\theta = 0$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{2k(r+a)^2}{\frac{3}{2}mr^2}} = \sqrt{\frac{4k(r+a)^2}{3mr^2}}$$

09. Ans: (b)

Sol:



By considering the equilibrium about the pivot 'O'

$$I_O\ddot{\theta} + mg\times\frac{L}{6}\sin\theta + K\frac{L}{3}\theta\times\frac{L}{3} = 0$$

$$\frac{mL^2}{9}\ddot{\theta} + \left(mg\times\frac{L}{6} + \frac{KL^2}{9}\right)\theta = 0 \quad (\because \sin\theta \approx \theta)$$

$$\omega_n = \sqrt{\frac{mg\times\frac{L}{6} + \frac{KL^2}{9}}{\frac{mL^2}{9}}} \Rightarrow \omega_n = \sqrt{\frac{3g}{2L} + \frac{K}{m}}$$

10. Ans: (d)

Sol: $X_0 = 10$ cm, $\omega_n = 5$ rad/sec

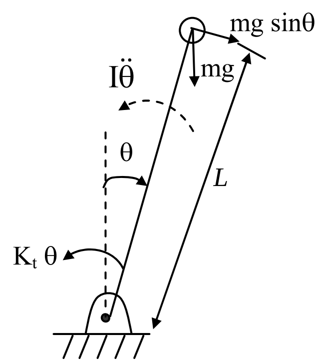
$$X = \sqrt{x_0^2 + \left(\frac{v_0}{\omega_n}\right)^2}$$

If $v_0 = 0$ then $X = x_0$

$\therefore X = x_0 = 10$cm

11. Ans: (c) & 12. Ans: (c)

Sol:



$I = mL^2$

The equation of motion is

$$mL^2\ddot{\theta} + (k_t - mgL)\theta = 0$$

Inertia torque = mL^2

Restoring torque = $k_t - mgL\sin\theta$

$= (k_t - mgL)\theta$

13. Ans: 0.0658 N.m²

Sol: For a cantilever beam stiffness, $K = \frac{3EI}{\ell^3}$

Natural frequency, $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3EI}{m\ell^3}}$

Given f_n= 100 Hz

$\Rightarrow \omega_n = 2\pi f_n = 200\ \pi$

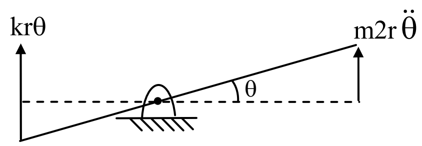
$200\pi = \sqrt{\frac{3EI}{m\ell^3}}$

Flexural Rigidity

$$EI = \frac{(200.\pi)^2.m\ell^3}{3} = 0.0658\ \text{N.m}^2$$

14. Ans: (d)

Sol: Free body diagram



Moment equilibrium about hinge

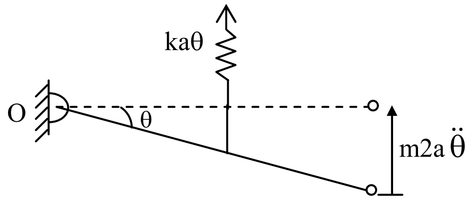
$m2r\ddot{\theta}.2r + k\theta.r = 0$

$4mr^2\ddot{\theta} + kr^2\theta = 0$

$$\omega_n = \sqrt{\frac{kr^2}{4mr^2}} = \sqrt{\frac{k}{4m}} = \sqrt{\frac{400}{4}}$$

15. Ans: (a)

Sol:



By taking the moment about 'O', $\Sigma m_o = 0$

$(m2a\ddot{\theta} \times 2a) + (ka\theta \times a) = 0$

$\Rightarrow 4a^2\,m\,\ddot{\theta} + ka^2\theta = 0$

Where, $m_{eq} = 4a^2m$, $k_{eq} = ka^2$

Natural frequency, $\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}}$

$$= \sqrt{\frac{ka^2}{4a^2m}} = \sqrt{\frac{k}{4m}}\ \frac{\text{rad}}{\text{sec}}$$

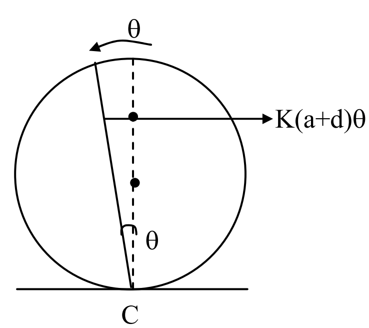
$\because \omega_n = 2\pi f$

$$\Rightarrow f = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \times \sqrt{\frac{k}{4m}}\ \text{Hz}$$

16. Ans: (a)

Sol: Moment equilibrium above instantaneous centre (contact point)

$$-k(a+d)\theta.(a+d) = I_c\ddot{\theta}$$



$I_c = \frac{3}{2}Ma^2$,

$$\omega_a = \sqrt{\frac{k(a+d)^2}{\frac{3}{2}Ma^2}}$$

$$\omega_n = \sqrt{\frac{2k(a+d)^2}{3Ma^2}}$$

17. Ans: 10 (range 9.9 to 10.1)

Sol: $KE = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2$

$m = 5kg, \quad \theta = \frac{x}{r}$

$I = \frac{20\times r^2}{2} = 10r^2$

$KE = \frac{1}{2}5\dot{x}^2 + \frac{1}{2}10r^2.\frac{\dot{x}^2}{r^2} = \frac{1}{2}(15)\dot{x}^2$

$\therefore m_{eq} = 15$

$PE = \frac{1}{2}kx^2$

$\therefore k_{eq} = k = 1500N/m$

Natural frequency

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{1500}{15}} = 10rad/sec$$

18. Ans: (b)

Sol: In damped free vibrations the oscillatory motion becomes non-oscillatory at critical damping.

Hence critical damping is the smallest damping at which no oscillation occurs in free vibration

19. Ans: (a)

Sol: $\omega_n = 50$ rad/sec $= \sqrt{\frac{5}{m}}$

If mass increases by 4 times

$$\omega_{n_1} = \sqrt{\frac{k}{4m}} = \frac{1}{2}\times\sqrt{\frac{k}{m}} = \frac{50}{2} = 25\,rad/sec$$

Damped frequency natural frequency,

$$\omega_d = \sqrt{1-\xi^2}\times\omega_n$$

$$\Rightarrow 20 = \sqrt{1-\xi^2}\times 25 = 0.6 = 60\%$$

20. Ans: (a)

Sol: K_1, K_2 = 16 MN/m

K_3, K_4 = 32 MN/m

$K_{eq} = K_1 + K_2 + K_3 + K_4$

m = 240 kg

$\omega_n = \sqrt{\frac{K_e}{m}}$

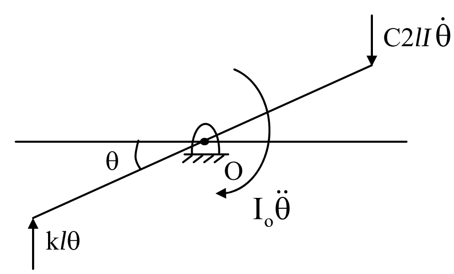
$K_{eq} = ((16\times 2)+(32\times 2))\times 10^6 = 96\times 10^6$ N/m

$\omega_n = \sqrt{\frac{96\times 10^6}{240}} = 632.455$ rad/sec

$N = \frac{\omega_n \times 60}{2\pi} = 6040$ rpm

21. Ans: (a)

Sol:



For slender rod, $I_o = \left[\rho\frac{x^3}{3}\right]_{-\ell}^{2\ell}$

$$= \frac{\rho}{3}\times(8\ell^3+\ell^3) = \frac{9\rho\ell^3}{3} = 3\rho\ell^3 = m\ell^2$$

Where, ρ= m/3*l*

Considering the equilibrium at hinge 'O'.

$I_o\ddot{\theta} + c2l\dot{\theta} \times 2l + kl\theta \times l = 0$

$\Rightarrow ml^2\ddot{\theta} + 4l^2c\,\dot{\theta} + kl^2\theta = 0$

$I_{equivalent} = ml^2,\ C_{eq} = 4l^2c,\ k_{eq} = kl^2$

22. Ans: (b)

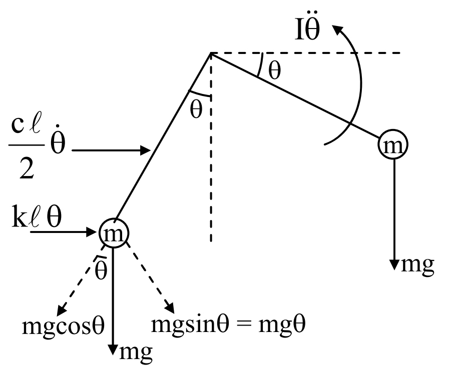
Sol: Damping ratio, $\xi = \dfrac{c}{c_c} = \dfrac{c_{eq}}{2\sqrt{k_{eq}m_{eq}}}$

$$= \frac{4\ell^2 c}{2\times\sqrt{k\ell^2\times m\ell^2}}$$

$$= \frac{4\ell^2 c}{2\times\sqrt{mk\ell^4}} = \frac{2c}{\sqrt{km}}$$

23. Ans: (a)

Sol:



$I = m(2l)^2 + ml^2 = 5ml^2$

The equation motion is

$$\left(m\times(2\ell)^2 + m\ell^2\right)\ddot{\theta} + \frac{c\ell^2}{4}\dot{\theta} + k\ell^2\theta + mg\ell\theta = 0$$

$$= 5m\ell^2\ddot{\theta} + \frac{c\ell^2}{4}\dot{\theta} + k\ell^2\theta + mg\ell\theta = 0$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{k\ell^2 + mg\ell}{5m\ell^2}}$$

$$= \sqrt{\frac{400}{5\times10}} = 3.162\ \text{rad/s}$$

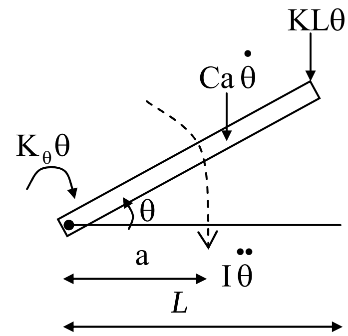
24. Ans: (a)

Sol: $$\xi = \frac{c_{eq}}{2\sqrt{k_{eq}m_{eq}}} = \frac{\left(\frac{c\ell^2}{4}\right)}{2\sqrt{(k\ell^2 + mg\ell)\times 5m\ell^2}}$$

$$= \frac{\frac{400\times1^2}{4}}{2\sqrt{(400\times1^2 + 10\times9.81\times1)\times5\times10\times1^2}} = 0.316$$

25. Ans: (a)

Sol:



By moment equilibrium

$$I\ddot{\theta} + Ca^2\dot{\theta} + KL^2\theta + K_\theta\theta = 0$$

$$\frac{mL^2}{3}\ddot{\theta} + Ca^2\dot{\theta} + \left(KL^2 + K_\theta\right)\theta = 0$$

$$\omega_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{KL^2 + K_\theta}{mL^2/3}}$$

$$\omega_n = \sqrt{\frac{1500}{0.833}} = 42.26\ \text{rad/sec}$$

26. Ans: (c)

Sol: Refer to the above equilibrium equation

$C_{eq} = Ca^2$

$= 500\times0.4^2 = 80\dfrac{N-m-sec}{rad}$

$\Rightarrow C = 80$ Nms/rad

Note: For angular co-ordinate

Unit of Equivalent inertia = $\frac{N-m}{rad/s^2} = kg - m^2$

Unit of equivalent damping coefficient = $\frac{N-m}{rad/s}$

Unit of equivalent stiffness = N-m/rad

27. Ans: (a)

Sol: $\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = 0$

$x(t=0) = X,$

$\dot{x}(t=0) = 0$

$x(t) = Ae^{-\xi\omega_n t}\cos(\omega_d t - \phi)$

After 'n' complete oscillation

$t = n\,\tau_d$

$x(t) = Ae^{-\xi\omega_n n\tau_d}\cos(\omega_d n\tau_d - \phi)$

$\tau_d = \frac{2\pi}{\omega_d}$

$x(t) = Ae^{-\xi\omega_n n\frac{2\pi}{\omega_d}}\cos\left(\omega_d n\frac{2\pi}{\omega_d} - \phi\right)$

$x(t=n\tau_d) = Ae^{-\xi\omega_n n\frac{2\pi}{\omega_n\sqrt{1-\xi^2}}}\cos(-\phi)$

$= Ae^{\frac{-2\pi n\xi}{\sqrt{1-\xi^2}}}\cos(\phi)$

x (t=0) = X

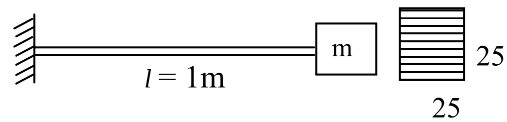
X = A cos (–ϕ) = A cos ϕ

$X(t = n\tau_d) = Xe^{\frac{-2\pi\times n\times\xi}{\sqrt{1-\xi^2}}}$

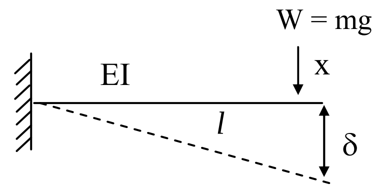
28. Ans: (a)

Sol: Given length of cantilever beam,

l = 1000 mm = 1m, m = 20 kg



Cross section of beam = square



Moment of inertia of the shaft,

$I = \frac{1}{12}bd^3 = \frac{25\times(25)^3}{12} = 3.25\times10^{-8}\,m^4$

$E_{steel} = 200\times10^9$ Pa

Mass, M = 20kg

Stiffness, $K = \frac{3EI}{\ell^3}$

Critical damping coefficient,

$C_C = 2\sqrt{Km} = 1250\ Ns/m$

29. Ans: (1.25)

Sol: Given m = 1 kg , K = 100 N/m

$C = 25\frac{N-sec}{m}$

Critical damping

$C_C = 2\sqrt{Km} = 20\frac{N-sec}{m}$

Damping Ratio $= \frac{C}{C_C} = \frac{25}{20} = 1.25$

30. Ans: (c)

31. Ans: (d)

Sol: x = 10 cm at $\frac{\omega}{\omega_n} = 1$;

$\xi = 0.1$

At resonance $x = \frac{x_0}{2\xi} = 10$ cm

$\Rightarrow x_0 = 2 \times 0.1 \times 10 = 2$ cm

x_0 = static deflection

At $\frac{\omega}{\omega_n} = 0.5$,

$$x = \frac{x_0}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$x = \frac{2}{\sqrt{\left[1-(0.5)^2\right]^2 + (2\times 0.1\times 0.5)^2}} = 2.64\,\text{cm}$$

32. Ans:(a)

Sol: $m\ddot{x} + Kx = F\cos\omega t$

m = ?

K = 3000 N/m,

X = 50 mm = 0.05 m

F = 100 N,

$\omega = 100\,\text{rad/sec}$

$$X = \frac{F}{K - m\omega^2}$$

$$\Rightarrow m = \frac{K}{\omega^2} - \frac{F}{X\omega^2} = 0.1\,\text{kg}$$

33. Ans: (a)

Sol: $\delta = ln\left(\frac{x_1}{x_2}\right) = ln\,2 = 0.693$

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

$$= \frac{0.693}{\sqrt{4\pi^2 + 0.693^2}} = 0.109$$

$c = 2\xi\sqrt{k\,m} = 2\times 0.109 \times \sqrt{100\times 1}$

= 2.19 N-sec/m

34. Ans: (b)

Sol: x_{static} = 3mm , ω = 20 rad/sec

As $\omega > \omega_n$

So, the phase is 180° .

$$-x = \frac{x_{static}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$x = \frac{3}{\sqrt{\left(1-\left(\frac{20}{10}\right)^2\right) + \left(2\times 0.109 \times \frac{20}{10}\right)^2}}$$

= 1 mm opposite to F.

35. Ans: (c)

Sol: At resonance, magnification factor = $\frac{1}{2\xi}$

$$\Rightarrow 20 = \frac{1}{2\xi}$$

$$\Rightarrow \xi = \frac{1}{40} = 0.025$$

36. Ans: (c)

Sol: M = 100 kg, m = 20 kg, e = 0.5 mm

K = 85 kN/m, C = 0 or $\xi = 0$

$\omega = 20\pi$ rad/sec

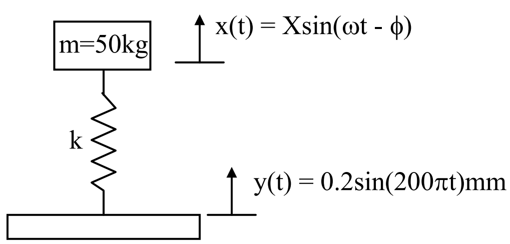
Dynamic amplitude

$$X = \frac{me\omega^2}{\pm\left(k - M\omega^2\right)} = \frac{20 \times 5 \times 10^{-4} \times (20\pi)^2}{\pm\left(8500 - 100 \times (20\pi)^2\right)}$$

$= 1.27 \times 10^{-4}$ m

37. Ans:

Sol:



$\omega = 200\pi$ rad/sec, –X = 0.01 mm

Y = 0.2 mm

$$\frac{X}{Y} = \frac{k}{k - m\omega^2}$$

$$\Rightarrow \frac{-0.01}{0.2} = \frac{k}{k - 50 \times (200\pi)^2}$$

$\Rightarrow$ k = 939.96 kN/m

38. Ans: (b)

Sol: m= 5kg, c = 20 ,

k = 80, F= 8, $\omega = 4$

$$x = \frac{F}{\left(k - m\omega^2\right) + (c\omega)^2}$$

$$= \frac{8}{\sqrt{\left(80 - 5 \times 4^2\right) + (20 \times 4)^2}} = 0.1$$

$$\text{Magnification factor} = \frac{x}{x_{static}}$$

$$x_{static} = \frac{F}{k} = \frac{8}{80} = 0.1$$

$$\text{Magnification factor} = \frac{0.1}{0.1} = 1$$

39. Ans: (c)

Sol: Given m = 250 kg

K = 100, 000 N/m

N = 3600 rpm

$\xi = 0.15$

$$\omega_n = \sqrt{\frac{K}{m}} = 20 \text{ rad /sec}$$

$$\omega = \frac{2\pi \times N}{60} = 377 \text{ rad/sec}$$

$$TR = \frac{\sqrt{1 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}} = 0.0162$$

40. Ans: 10 N.sec/m

Sol: Given systems represented by

$$m\ddot{x} + c\dot{x} + kx = F\cos\omega t$$

$$\text{For which, } X = \frac{F}{\sqrt{\left(K - m\omega^2\right)^2 + (C\omega)^2}}$$

Given K = 6250 N/m, m = 10 kg, F = 10 N

$\omega = 25$ rad/sec, X= 40×10^{-3}

$$\omega_n = \sqrt{\frac{K}{m}} = 25 \text{ rad/sec}$$

$\omega t = 25t \Rightarrow \omega = 25$ rad/sec

$\omega = \omega_n$ or $K = m\omega_n^2$

$$\therefore X = \frac{F}{C\omega} \Rightarrow C = \frac{F}{X\omega}$$

$$= \frac{10}{40\times10^{-3}\times25} = 10\frac{N-sec}{m}$$

41. Ans: (b)

Sol: Transmissibility (T) reduces with increase in damping up to the frequency ratio of $\sqrt{2}$. Beyond $\sqrt{2}$, T increases with increase in damping

42. Ans: (c).

Sol: Because f = 144 Hz execution frequency.

f_{R_n} (Natural frequency) is 128.

$$\frac{\omega}{\omega_{R_n}} = \frac{f}{f_{R_n}} = \frac{144}{128} = 1.125$$

It is close to 1, which ever sample for which $\frac{\omega}{\omega_n}$ close to 1 will have more response, so sample R will show most perceptible to vibration

43. Ans: (b)

Sol: Given Problem of the type

$$m\ddot{x} + c\dot{x} + kx = F\cos\omega t$$

for which, $X = \dfrac{F}{(k-m\omega^2)^2 + (c\omega)^2}$

or $X = \dfrac{F/K}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right) + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$

Given F = 10, $\omega_n = 10\omega$

$k = 150\ N/m$ or $\frac{\omega}{\omega_n} = \frac{1}{10} = 0.1$

$\xi = 0.2$

$$X = \frac{10/150}{\sqrt{(1-0.1)^2 + (2\times0.2\times0.1)^2}}$$

$$= 0.0669 \simeq 0.07$$

44. Ans: 6767.7 N/m

Sol: Given f = 60 Hz, m = 1 kg

$$\omega = 2\pi f = 120\pi rad/sec$$

Transmissibility ratio, TR = 0.05

Damping is negligible, C = 0 , K =?

We know $TR = \dfrac{K}{K - m\omega^2}$ when C = 0

As TR is less than 1 $\Rightarrow \omega/\omega_n >> \sqrt{2}$

TR is negative

$$\therefore \quad -0.05 = \frac{K}{K - m\omega^2}$$

Solving we get K = 6767.7 N/m

45. Ans: 20 (range 19.9 to 20.1)

Sol: k = 10kN/m , F_0 = 100 N , ξ = 0.25

$$X = \frac{(F_0/k)}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

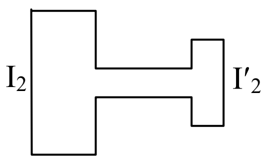
$\frac{\omega}{\omega_n} = 1$ at resonance

$$X = \frac{F_0}{2k\zeta} = \frac{100}{2\times10\times0.25\times10^3} = 20\ mm$$

46. Ans: (b)

Sol: To get the equivalent inertia of disc on 'B' at the speed of shaft 'A' then their kinematic energies will be same.

Shift the disc to shaft "A" end

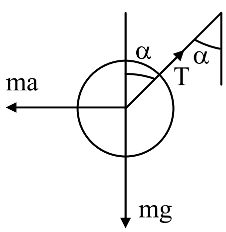


$$\frac{1}{2}\times I'_2\times\omega_A^2=\frac{1}{2}\times I_2\times\omega_B^2$$

$$\Rightarrow I'_2=I_2\times\left(\frac{\omega_B}{\omega_A}\right)^2=I_2 n^2$$

47. Ans: (c)

Sol:



Where, a = acceleration of train

$T\cos\alpha = mg$

$T\sin\alpha = ma$

$$\tan\alpha=\frac{ma}{mg}$$

$a = g\tan\alpha = 9.81\tan(9.81°)$

$= 1.69\ m/s^2$

48. Ans: (a)

49. Ans: (b)

Sol: $e = 2mm = 2\times10^{-3}m$,

$\omega_n = 10$ rad/s,

$N = 300$ rpm

$$\omega=\frac{2\pi N}{60}=10\pi\,rad/sec$$

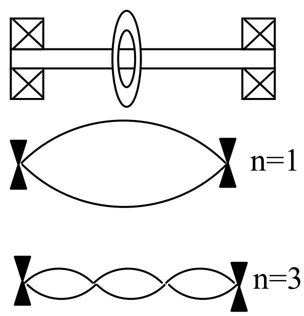
$$X=\frac{me\omega^2}{k-m\omega^2}=\frac{e\omega^2}{\left(\frac{k}{m}\right)^2-\omega^2}=\frac{e\omega^2}{\omega_n^2-\omega^2}$$

$$X=\frac{e\left(\frac{\omega}{\omega_n}\right)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2}=\frac{2\times10^{-3}\times\left(\frac{10\pi}{10}\right)^2}{\pm\left(1-\left(\frac{10\pi}{10}\right)^2\right)}$$

$= 2.25\times10^{-3}\ m = 2.25\ mm$

50. Ans: (a)

Sol: Number of nodes observed at a frequency of 1800 rpm is 2



n-mode number

The whirling frequency of shaft,

$$f=\frac{\pi}{2}\times n^2\sqrt{\frac{gEI}{WL^4}}$$

For 1st mode frequency, $f_1 = \frac{\pi}{2} \times \sqrt{\frac{gEI}{WL^4}}$

$$f_n = n^2 f_1$$

As there are two nodes present in 3rd mode,

$$f_3 = 3^2 f_1 = 1800\,\text{rpm}$$

$$\therefore f_1 = \frac{1800}{9} = 200\,\text{rpm}$$

$\therefore$ The first critical speed of the shaft = 200 rpm

51. Ans: (b)

Sol: Critical or whirling speed

$$\omega_c = \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{\delta}}\ \text{rad/sec}$$

If N_C is the critical or whirling speed in rpm

then $\frac{2\pi N_C}{60} = \sqrt{\frac{g}{\delta}}$

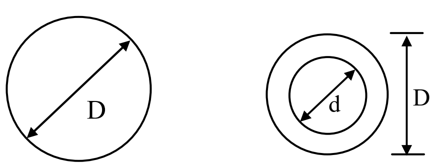
$$\Rightarrow \frac{2\pi N_C}{60} = \sqrt{\frac{9.81\,\text{m/s}^2}{1.8\times10^{-3}\,\text{m}}}$$

$\Rightarrow$ N_C = 705.32 rpm ≈ 705 rpm

52. Ans:

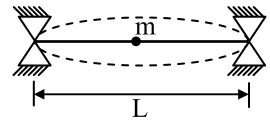
Sol:

Original shaft — Modified shaft



(solid shaft) *d= 0.75 D*

Let us take a simply supported shaft



$$k_b = \text{beam stiffness} = \frac{48EI}{L^3}$$

For a solid shaft, $I_s = \frac{\pi}{64} D^4$

$$k_b = \frac{P}{\delta} = \frac{48EI}{L^3}$$

$$k_b \propto I$$

$$k_b \propto D^4$$

For a hollow shaft :

$$I_{hollow} = \frac{\pi}{64}\left(D^4 - (0.75D)^4\right)$$

$$= \frac{\pi}{64} D^4 \times \left(1 - 0.75^4\right)$$

$$(k_b)_{\text{hollow shaft}} \propto 0.68\,D^4$$

Natural frequency,

$$\omega_n = \sqrt{\frac{k_b}{m_s + m}}$$

where, m_s = mass of shaft, m = point mass

By neglecting mass of point mass

$$\omega_n = \sqrt{\frac{k_b}{m_s}}$$

For solid shaft :

$$m_s = \rho \times \frac{\pi}{4} \times D^2 \times L$$

$(m_s)_{solid\ shaft} \propto D^2$

$(\omega_n)_{solid\ shaft} \propto \sqrt{\frac{D^4}{D^2}}$

$(\omega_n)_{solid\ shaft} \propto D$

For hollow shaft :

$$m_s = \rho \times \frac{\pi}{4} \times \left(D^2 - 0.75^2 D^2\right) \times L$$

$$= \rho \times \frac{\pi}{4} D^2 \times L\left(1 - 0.75^2\right)$$

$(m_s)_{hollow} \propto 0.437\ D^2$

$(\omega_n)_{hollow\ shaft} \propto \sqrt{\frac{0.68\,D^4}{0.437\,D^2}} = \sqrt{\frac{0.68}{0.437}}D$

$\therefore (\omega_n)_{hollow\ shaft} \propto 1.247\ D$

$(\omega_n)_{hollow\ shaft} > (\omega_n)_{solid\ shaft}$

$(\omega_n)_{hollow\ shaft} = 1400 \times 1.247 = 1745$ rpm

As critical speed is 1400 rpm and 1745 > 1400. So there is no whirling of shaft for a hollow shaft. So hollow shaft is saver as its natural frequency is greater than critical speed of solid shafts.