MECHANICAL ENGINEERING

GATE | PSUs

STRENGTH OF MATERIALS

Volume - I : Study Material with Classroom Practice Questions

Strength of Materials

Solutions for Vol – I _ Classroom Practice Questions

Chapter- 1
Simple Stresses and Strains

1.1 Fundamental, Mechanical Properties of Materials, Stress Strain Diagram

01. **Ans: (b)**

Sol: **Ductility:** A material which undergoes considerable deformation without rupture (large plastic zone)

Brittleness: Failure without warning (No plastic zone) i.e. no plastic deformation

Tenacity: High tensile strength

Creep: Material continues to deform with time under sustain loading

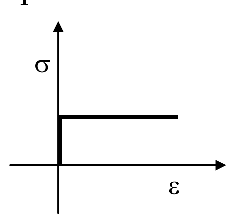
Plasticity: Material continues to deform without any further increase in stress.

Endurance limit: Material has high probability of not failing under Reversal of stress of magnitude below this level.

Fatigue: Decreased Resistance of material to repeated reversal of stresses

02. **Ans: (a)**

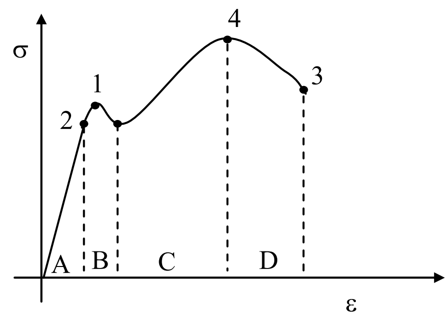
Sol: For rigid plastic material



03. **Ans: (a)**

04. **Ans: (b)**

Sol:



05. **Ans: (b)**

06. **Ans: (a)**

Sol: **Strain hardening: -** increase in strength after plastic zone by rearrangement of molecules in material Visco-elastic material exhibits a mixture of creep + elastic after effects at room temperature. Thus their behavior is time dependant

07. **Ans: (a)**

08. **Ans: (a)**

09. **Ans: (a)**

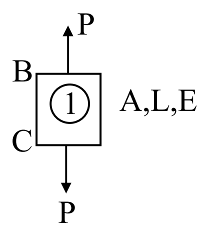
Sol: Addition of carbon will increase strength, thereby ductility will decrease.

1.2 Elastic Constants and Their Relationships

01. **Ans (c)**

Sol: Poisson's ratio $= \dfrac{\text{lateral strain}}{\text{linear strain}} = \dfrac{\Delta D/D}{\Delta L/L}$

$$\mu = \frac{\Delta D/8}{\dfrac{PL}{AE}/L}$$



$$\mu = \frac{\Delta D}{8}\frac{AE}{P}$$

$$0.25 = \frac{\Delta D}{8}\frac{\frac{\pi}{4}(8)^2\times 10^6}{50000}$$

$\Delta D = 1.98 \times 10^{-3} = 0.00198 \quad \cong 0.002$ cm

02. **Ans: (c)**

Sol: Bulk modulus $= \dfrac{\delta P}{\delta V/V}$

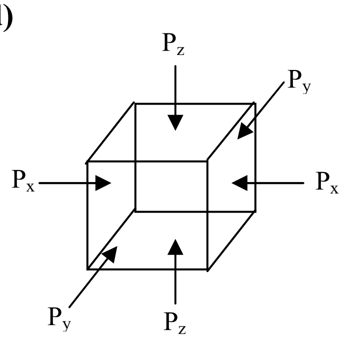
$$\Rightarrow 2.5\times10^5 = \frac{200\times 20}{\delta V}$$

$$\Rightarrow \delta V = 0.016 \text{ m}^3$$

1.3 Linear and Volumetric Charges of Bodies

01. **Ans: (d)**

Sol:



Let $P_y = P_z = P$

$\varepsilon_y = 0$,

$\varepsilon_z = 0$

$$\varepsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu.\frac{\sigma_x}{E}$$

$$0 = \frac{(-P)}{E} - \mu\frac{(-P)}{E} - \mu\frac{(P_x)}{E}$$

$$P = \frac{\mu.P_x}{(1-\mu)}$$

02. **Ans: (a)**

Sol: $\sigma_c = 4\tau$ ---- (given)

Punching force = shear resistance of plate

$\sigma\,\{c/s \text{ area}\} = \tau\,(\text{surface Area})$

$$4\times\tau\times\frac{\pi.D^2}{4} = \tau(\pi.D.t)$$

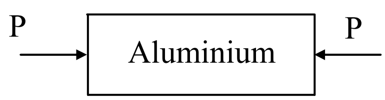
D = t = 10 mm

03. **Ans: (d)**

Sol:

$\sigma_s = 140 \text{ MPa} = \dfrac{P_s}{A_s}$

$P_S = \dfrac{140 \times 500}{3} \approx 23{,}300 \text{ N}$



$\sigma_{Al} = 90 \text{ MPa} = \dfrac{P_{A\ell}}{A_{A\ell}}$

$P_{Al} = 90 \times 400 = 36{,}000 \text{ N}$

$\sigma_B = 100 \text{ MPa} = \dfrac{P_B}{A_B}$

$P_B = \dfrac{100 \times 200}{2} = 10{,}000 \text{ N}$

$P_s = 22{,}300 \text{ N}$, $P_{AL} = 36{,}000 \text{ N}$

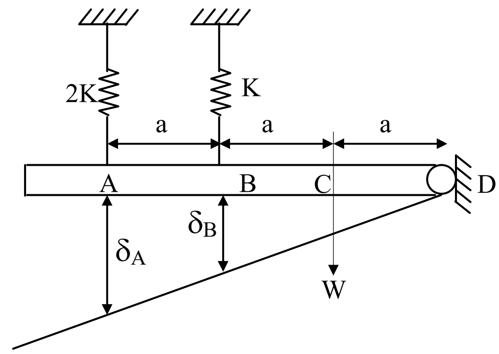
$P_B = 10{,}000 \text{ N}$

Take minimum value

$\therefore\ P = 10{,}000 \text{ N}$

04. **Ans: (c)**

Sol:



From similar triangle

$\dfrac{3a}{\delta_A} = \dfrac{2a}{\delta_B}$

$3\delta_B = 2\delta_A$ ……. (1)

Stiffness $K = \dfrac{W}{\delta}$

$\therefore K_A = \dfrac{W_A}{\delta_A} \Rightarrow \delta_A = \dfrac{W_A}{2K}$

Similarly $\delta_B = \dfrac{W_B}{K}$

From equation (1) $3 \times \dfrac{W_B}{K} = 2 \times \dfrac{W_A}{2K}$

$\therefore \dfrac{W_A}{W_B} = 3$

05. Ans: (d)

1.4 Thermal Stresses

Common Data for Question Nos. 01 & 02

01. **Ans: (b)**

Sol: Free expansion = Expansion prevented

$$[\ell\alpha t]_s + [\ell\alpha t]_{Al} = \left[\frac{P\ell}{AE}\right]_s + \left[\frac{P\ell}{AE}\right]_{AL}$$

$$11\times10^{-6}\times20 + 24\times10^{-6}\times20$$

$$= \frac{P}{100\times10^3\times200} + \frac{P}{200\times10^3\times70}$$

$\Rightarrow$ P = 5.76 kN

02. **Ans: (b)**

Sol: $\sigma_s = \frac{P_s}{A_s} = \frac{5.76\times10^3}{100} = 57.65$ MPa

$$\sigma_{Al} = \frac{P}{A_{al}} = \frac{5.76\times10^3}{200} = 28.82 \text{ MPa}$$

Common Data for Question Nos.03 & 04

03. **Ans: (d)**

Sol: $\Delta_s + \Delta_{gm} = -\alpha_s t L + \alpha_{gm} tL$

$$\frac{P\times L}{100\times200\times10^3} + \frac{P\times L}{200\times100\times10^3}$$

$$= -6\times10^{-6}\times200\times L + 10\times10^{-6}\times200\times L$$

$$\frac{2\times P}{200\times100\times10^3} = 8\times10^{-4}$$

P = 8 kN

04. **Ans: (b)**

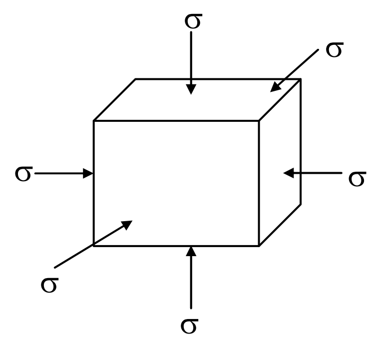
Sol: Stress in steel $\frac{P_s}{A_s} = \frac{8\times10^3}{100} = 80$ MPa

Stress in Gunmetal = $\frac{P_{gm}}{A_{gm}}$

$$= \frac{8\times10^3}{200} = 40\text{MPa}$$

05. **Ans: (a)**

Sol:



Strain in X-direction due to temperature,

$$\varepsilon_t = \alpha(\Delta T)$$

Strain in X-direction due to volumetric stress

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_x = \frac{-\sigma}{E}(1-2\nu)$$

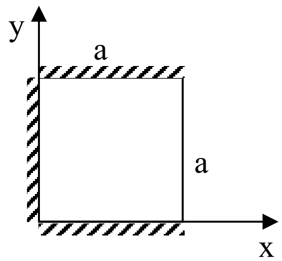
$$-\sigma = \frac{(\varepsilon_x)(E)}{1-2\nu}$$

$$-\sigma = \frac{\alpha(\Delta T)E}{(1-2\nu)}$$

$$\therefore \sigma = \frac{-E\alpha(\Delta T)}{1-2\nu}$$

06. Ans: (b)

Sol:



Free expansion in x direction is a $\propto$ t

Free expansion in y direction is $\alpha \propto t$

Since there is restriction in y direction expansion doesn't take place. So in lateral direction this increase in expansion due to restriction in $\mu a \propto t$

$\therefore$ Total expansion in x direction is

$$= \alpha\,\alpha\,t + \mu\,a\,\alpha\,t$$

$$= a\,\alpha\,t\,(1+v)$$

Chapter- 2
Complex Stresses and Strains

01. **Ans: (b)**

Sol: Maximum principal stress $\sigma_1 = 18$

Minimum principal stress $\sigma_2 = -8$

Maximum shear stress $= \dfrac{\sigma_1 - \sigma_2}{2} = 13$

Normal stress on Maximum shear stress plane

$$= \frac{\sigma_1 + \sigma_2}{2} = \frac{18 + (-8)}{2} = 5$$

02. **Ans: (b)**

Sol: Radius of Mohr's circle $\tau_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$

$$20 = \frac{\sigma_1 - 10}{2}$$

$$\sigma_1 = 50\ \text{N/mm}^2$$

03. **Ans: (b)**

Sol: Long dam $\rightarrow$ plane strain member

$$\varepsilon_z = 0 = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E}$$

$$\sigma_x = 150\,\text{MPa},\ \ \sigma_y = -300\,\text{MPa}, \mu = 0.3$$

$$\therefore\ 0 = \sigma_z - 0.3\times150 + 0.3\times300$$

$$\Rightarrow\ \sigma_z = 45\,\text{MPa}$$

Common Data for Question Nos. 04 & 05

04. **Ans: (a)**

Sol: $\sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$$= \frac{65 + (-13)}{2} \pm \sqrt{\left(\frac{65+13}{2}\right)^2 + 20^2}$$

$$\therefore \sigma_1 = 70\,\text{MPa}, \quad \sigma_2 = -18\,\text{MPa}$$

05. **Ans: (a)**

Sol: $\varepsilon_1 = \dfrac{\sigma_1}{E} - \mu\dfrac{\sigma_2}{E}$;

$$\varepsilon_1 = \frac{\text{Change in diameter}}{\text{Original diameter}}$$

$$\varepsilon_1 = \frac{70 + 0.3\times18}{2\times10^5} = 3.77\times10^{-4}$$

$$\text{Change in diameter} = 3.77\times10^{-4}\times300$$

$$= 0.1131\,\text{mm}$$

$\therefore$ Length of major axes of ellipse

= 300 + change in diameter

= 300.113 mm

Similarly length of minor axes

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E}$$

$$\varepsilon_2 = \frac{\delta D}{300} = \frac{-18 - 0.3\times70}{2\times10^5} = -1.95\times10^{-4}$$

$$\delta D = -1.95\times10^{-4}\times300 = -0.0585\,\text{mm}$$

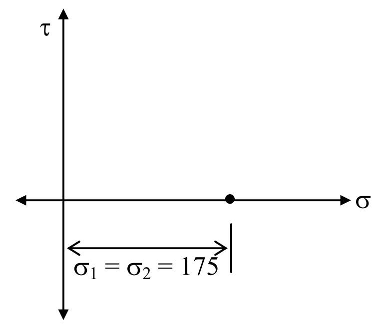
$\therefore$ Minor axis length, = 300 – 0.0585

= 299.94 mm

Common Data for Question Nos. 06 & 07

06. **Ans: (b)**

Sol:



07. **Ans: (d)**

08. **Ans: (c)**

Sol: $\sigma_2 = 0$(Given)

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$\frac{\sigma_x + \sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}}$$

$$\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2_{xy}$$

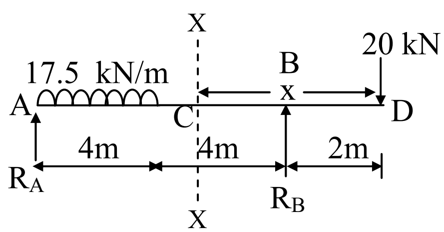
$$\tau^2_{xy} = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2$$

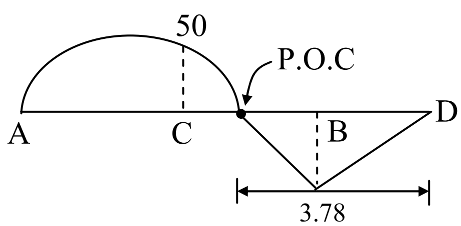
$$\tau^2_{xy} = \sigma_x.\sigma_y \Rightarrow \tau_{xy} = \sqrt{\sigma_x.\sigma_y}$$

Chapter- 3
Shear Force and Bending Moment

01. **Ans: (b)**

Sol: Contra flexure is the point where BM is becoming zero.





$\Sigma M_A = 0$

$$17.5\times4\times\frac{4}{2}+20\times10-R_B\times8=0$$

$R_B = 42.5$ kN

$M_x = -20x + R_B(x - 2)$

For bending moment be zero $M_x = 0$

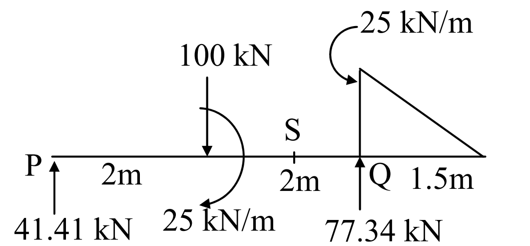
$-20x + 42.5(x - 2) = 0$

$x = 3.78$m From right ie. D

Common Data for Question Nos. 02 & 03

02. **Ans: (b)**

Sol:



Take $\Sigma M_P = 0$

$$\frac{1}{2}\times25\times1.5\times\left(\frac{1.5}{3}+4\right)-\left(R_Q\times4\right)+100\times2+25=0$$

$R_Q = 77.34$ kN

$\Sigma V = 0$

$$R_P + R_Q = 100+\frac{1}{2}\times25\times1.5=118.75 \text{ kN}$$

$\therefore R_p = 41.41$ kN

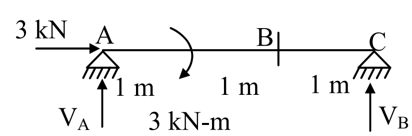
S. F. at P = 41.41 kN

03. **Ans: (c)**

Sol: $M_S = R_P(3)+25-100\times1 = 49.2$ kN-m

04. **Ans: (c)**

Sol:



$-V_B \times 3 + 3 = 0$

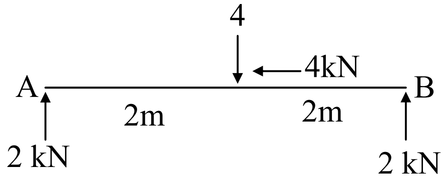
$V_C = 1$kN

$\therefore$ Bending moment at B

$\Rightarrow V_C \times 1 = 1$ kNm

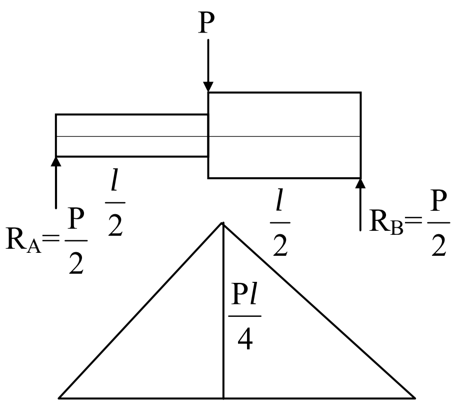
05 **Ans: (a)**

Sol:



06. **Ans: (c)**

Sol:



BMD Diagram

(BM) at $\frac{l}{2}$ from left is $\frac{Pl}{4}$

The given beam is statically determinate structure. Therefore equilibrium equations are sufficient to analyze the problem.

In statically determinate structure the BMD, SFD and Axial force are not affected by section (I), material (E), thermal changes.

07. Ans: (a)

Sol: As the given support is hinge, for different set of loads in different direction beam will experience only axial load.

Chapter- 4
Centre of Gravity & Moment of Inertia

01. **Ans: (a)**

Sol: $\bar{y} = \dfrac{E_1 y_1 + E_2 y_2}{E_1 + E_2}$

$$\Rightarrow \bar{y} = \frac{2E_2\left(h + \frac{h}{2}\right) + E_2 \times \frac{h}{2}}{2E_2 + E_2} \qquad (\because E_1 = 2E_2)$$

$$\bar{y} = 1.167h \text{ from base}$$

02. **Ans: (b)**

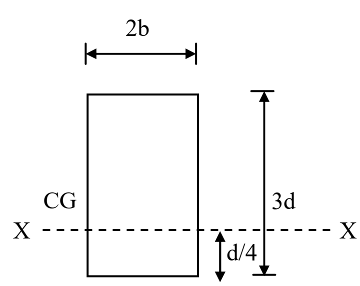
Sol: $\bar{y} = \dfrac{A_1E_1Y_1 + A_2E_2Y_2}{A_1E_1 + A_2E_2}$

$$= \frac{1.5a \times 3a^2 \times E_1 + 1.5a \times 6a^2 \times 2E_1}{3a^2E_1 + 6a^2(2E_1)}$$

$$= \frac{22.5a^3E_1}{15a^2E_1} = 1.5a$$

03. **Ans: 13.875 bd^3**

Sol:



$y = \frac{5}{4}d$

$$\text{M.I about CG} = I_{CG} = \frac{2b(3d)^3}{12} = \frac{9}{2}bd^3$$

$$\text{M.I about } X - X\,|_{\text{at } d/4 \text{ distance}} = I_G + Ay^2$$

$$= \frac{9}{2}bd^3 + 6bd\left(\frac{5}{4}\right)^2 d^2$$

$$= \frac{111}{8}bd^3 = 13.875bd^3$$

04. Ans: 4.38×10^6 mm^4

Sol:

$$I_x = \frac{60\times120^3}{12} - 2\left(\frac{30\times45^3}{12} + 30\times45\times37.5^2\right)$$

$$= 4.38\times10^6 \text{ mm}^4$$

05. Ans: I_x = 152146 mm^4, I_y = 45801.34 mm^4

Sol:

$$I_x = \frac{30\times40^3}{12} - \frac{\pi\times20^4}{64} = 152146\,\text{mm}^4$$

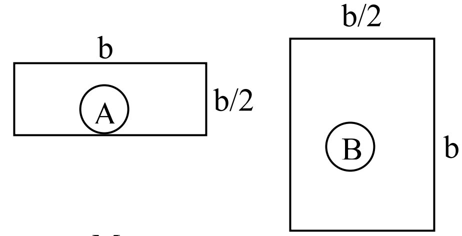
$$I_y = \frac{40\times30^3}{12} - \left(\frac{\pi\times20^4}{64} + 2\left(\frac{\pi}{2}\times10^2\times\left(15 - \frac{4\times10}{3\pi}\right)^2\right)\right)$$

$$= 45801.34 \text{ mm}^4$$

Chapter- 5
Theory of Simple Bending

01. **Ans: (b)**

Sol:



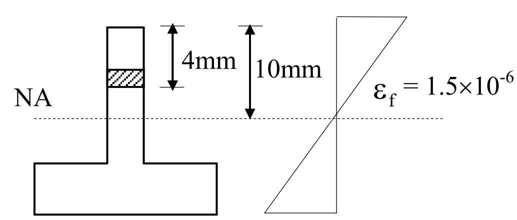
$$\sigma = \frac{M}{Z}$$

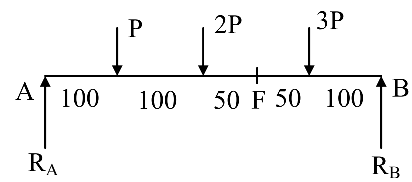
$$\sigma \propto \frac{1}{Z}\;(\because M \text{ is same})$$

$$\therefore \frac{\sigma_A}{\sigma_B} = \frac{Z_B}{Z_A} = \frac{\dfrac{\left(\frac{b}{2}\times b^2\right)}{6}}{\dfrac{b\times\left(\frac{b}{2}\right)^2}{6}} = 2$$

02. **Ans: (b)**

Sol:





$\therefore \sum M_A = 0$

$P \times 100 + 2P \times 200 + 3P \times 300 = R_B \times 400$

$\therefore R_B = 3.5P,\ R_A = 2.5P$

Take moments about F and moment at F

$M_F = R_B \times 150 - 3P \times 50 = 375P$

$$\frac{M_F}{I} = \frac{\sigma_b}{y_F}$$

$$\frac{375P}{2176} = \frac{(1.5\times10^{-6}\times200\times10^{3})}{6}$$

$\therefore$ P = 0.29 N

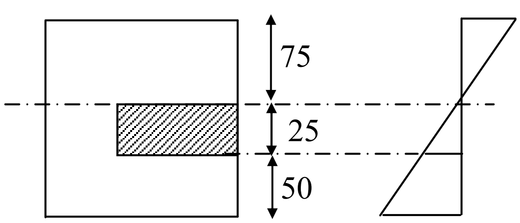
03. **Ans: (b)**

Sol: $\dfrac{E}{R} = \dfrac{\sigma_b}{y_{max}} \Rightarrow \dfrac{2\times10^5}{250} = \dfrac{\sigma_b}{(0.5/2)}$

σ_b = 200 MPa

04. **Ans: (c)**

Sol:



Force on the hatched area

= Avg. Stress × hatched Area

$$\frac{M}{I} = \frac{f}{y}$$

$$\therefore \frac{16\times10^6}{\frac{100\times150^3}{12}} = \frac{f}{25} \Rightarrow f = 14.22 \text{ MPa}$$

$\therefore$ Force on hatched area

= Average stress × hatched area

$$= \left(\frac{0+14.22}{2}\right)(25\times50) = 8.9 \text{ kN}$$

05. **Ans: (c)**

Sol: $\dfrac{f_{Tensile}}{y_{top}} = \dfrac{M}{I}$

$$\Rightarrow f_{Tensile} = \frac{0.3\times3\times10^6}{3\times10^6}\times70$$

(maximum bending stress will be at top fibre so y_1=70mm)

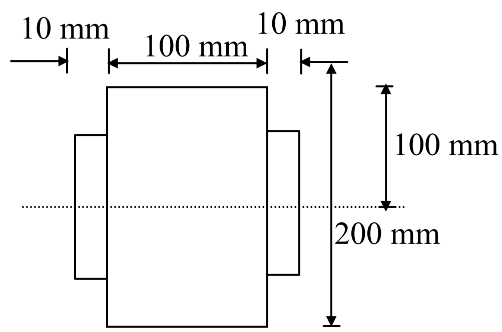
$\therefore f_{Tensile} = 21 \text{ kN/m}^2$

06. **Ans: (c)**

Common Data for Question Nos. 07 & 08

07. **Ans: 80 MPa**

Sol:



Maximum stress in timber = 8 MPa

Modular ratio, m = 20

Stress in timber in steel level,

$100 \rightarrow 8$

$50 \rightarrow f_w$

f_w = 4MPa

Maximum stress developed in steel is = $m\cdot f_w$

= 20×4 = 80 MPa

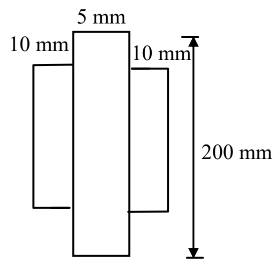
Convert whole structure as a steel structure by using modular ratio.

08. Ans: 8 kN-m

Sol: Moment of inertia,

$$I = \frac{2\times10\times100^3}{12} + \frac{5\times200^3}{12} = 5\times10^6\ \text{mm}^4$$

$f_s = 8\times20 = 160$ MPa



Bending equation

$$\frac{M}{I} = \frac{f_s}{y}$$

$$M = \frac{5\times10^6\times160}{\frac{200}{2}}$$

$$= \frac{5\times10^6\times160\times2}{200} = 8\ \text{kN-m}$$

09. Ans: 2.43 mm

Sol: From figure $A_1B_1 = l = 3$ m (given)

$$AB = \left(R - \frac{h}{2}\right)\alpha = l - l\alpha t_1 \text{ ------ (1)}$$

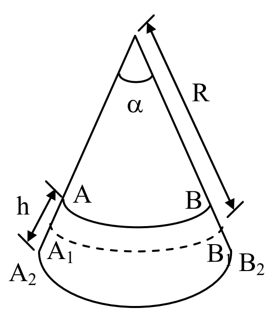
$$A_2B_2 = \left(R + \frac{h}{2}\right)\alpha = l + l\alpha t_2 \text{ ------ (2)}$$

Subtracting above two equations (2) – (1)

$h(\alpha) = l\alpha(t_2 - t_1)$

but $A_1B_1 = l = R\alpha$

$$\Rightarrow \alpha = \frac{l}{R}$$



$$\therefore h\left(\frac{l}{R}\right) = l\alpha(\Delta T)$$

$$R = \frac{h}{\alpha(\Delta T)}$$

$$= \frac{250}{(1.5\times10^{-5})(72-36)}$$

$R = 462.9$ m

From geometry of circles

$(2R-\delta)\delta = \frac{L}{2}\cdot\frac{L}{2}$ {ref. figure in question No. 02}

$$2R\cdot\delta - \delta^2 = \frac{L^2}{4} \text{ (neglect } \delta^2)$$

$$\delta = \frac{L^2}{8R} = \frac{3^2}{8\times462.9} = 2.43\ \text{mm}$$

Shortcut:

Deflection is due to differential temperature of bottom and top ($\Delta T = 72^o - 36^o = 36^o$). Bottom temperature being more, the beam deflects down.

As derived in the Q2 (2 marks)

$$\delta = \frac{\alpha(\Delta T)\ell^2}{8h} = \frac{1.5\times10^{-5}\times36\times3000^2}{8\times250}$$

$= 2.43$ mm (downward)

Chapter- 6
Shear Stress Distribution in Beams

01. **Ans: (a)**

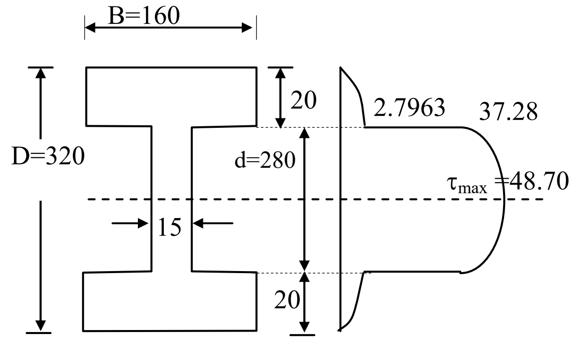
Sol: $\tau_{max} = \frac{3}{2} \times \tau_{avg} = \frac{3}{2} \times \frac{f}{b.d}$

$$3 = \frac{3}{2} \times \frac{50 \times 10^3}{100 \times d}$$

$\therefore d = 250 \text{ mm} = 25 \text{ cm}$

Common Data for Q. 02, 03 and 04:

Sol:



All dimensions are in mm

02. **Ans: 37.3**

Bending moment (M) = 100 kN-m,

Shear Force (SF) = f = 200 kN

$$I = \frac{160 \times 320^3}{12} - \frac{145 \times 280^3}{12}$$

$= 171.65 \times 10^6 \text{ mm}^4$

$$\tau_{\text{at interface of flange \& web}} = \frac{FA\bar{y}}{Ib}$$

$$= \frac{200 \times 10^3}{171.65 \times 10^6 \times 15} \times (160 \times 20 \times 150)$$

$= 37.28$ MPa

03. **Ans: 48.7**

$$\tau_{max} = \frac{FA\bar{y}}{Ib}$$

$$= \frac{200 \times 10^3}{171.65 \times 10^6 \times 15} \times (140 \times 15 \times 70 + 160 \times 20 \times 150)$$

$= 48.70$ MPa

04. **Ans: 3.5**

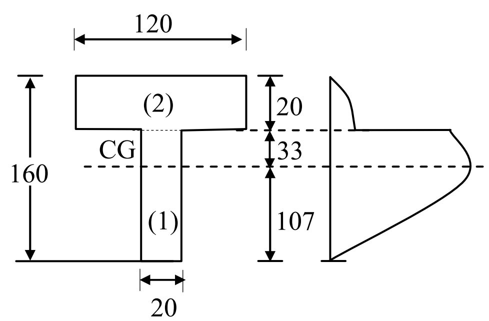
$\tau_{\text{in flange just above web}}$

$$= \frac{200 \times 10^3}{171.65 \times 10^6 \times 160} \times 160 \times 20 \times 150$$

$= 3.5$ MPa

05. **Ans: 61.43 MPa**

Sol:



All dimensions are in mm

$I_{NA} = 13 \times 10^6 \text{ mm}^4$

$y_{CG} = 107$ mm from base

$$\tau_{max} = \frac{FA\bar{y}}{Ib}$$

$A\bar{y} = (120 \times 20 \times 43) + (33 \times 20 \times 16.5)$

$= 114090 \text{ mm}^3$

$$\tau_{max} = \frac{140 \times 10^3 \times 114090}{13 \times 10^6 \times 20} = 61.43 \text{ MPa}$$

Chapter- 7
Torsion

01. **Ans: (a)**

Sol: Twisting moment = 1 × 0.5

= 0.5 kN-m

02. **Ans: (d)**

Sol: $\dfrac{(\text{strength})_{\text{solid}}}{(\text{strength})_{\text{hollow}}} = \dfrac{1}{1-K^4}$

$= \dfrac{1}{1-\left(\frac{1}{2}\right)^4} = \dfrac{16}{15}$

Common Data for Question Nos. 03 & 04

03. **Ans: (a)**

Sol: $(P)_{AB} = 30$ kW

$(P)_{BC} = 45$ kW

$P_{AB} = \dfrac{2\pi NT_{AB}}{60} \Rightarrow T_{AB} = 1.43$ kN-m

$P_{BC} = \dfrac{2\pi NT_{BC}}{60} \Rightarrow T_{BC} = 2.14$ kN-m

$\Rightarrow \dfrac{T}{J} = \dfrac{\tau}{R}$

$\tau_{AB} = \dfrac{T_{AB}}{Z_P} = \dfrac{1.43\times10^6}{\frac{\pi}{16}\times(50)^3} = 58.26$ MPa

$\tau_{BC} = \dfrac{T_{BC}}{Z_P} = \dfrac{2.14\times10^6}{\frac{\pi}{16}\times75^3}$

$\tau_{BC} = 25.83$ MPa

∴ Take maximum value of ‘τ’ i.e, 58.26 MPa

04. **Ans: (c)**

Sol: Series

$\theta_{max} = \theta_{AB} + \theta_{BC} = \left(\dfrac{T.L}{CJ}\right)_{AB} + \left(\dfrac{T.L}{CJ}\right)_{BC}$

$= \dfrac{(1.43\times10^6)\times4000}{(8.5\times10^4)\times\left(\frac{\pi}{32}\times50^4\right)} + \dfrac{(2.14\times10^6)\times2000}{(8.5\times10^4)\times\left(\frac{\pi}{32}\times75^4\right)}$

= 0.128 radian

$= 0.128\times\dfrac{180}{\pi} = 7.21 \approx 7.14°$

05. **Ans: 43.27 MPa & 37.5 MPa**

Sol: Given $D_o = 30$ mm , t = 2 mm

∴ $D_i = 30 - 4 = 26$ mm

We know that $\dfrac{\tau}{J} = \dfrac{q}{R}$

$\dfrac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \dfrac{q_{max}}{\left(\frac{30}{2}\right)}$

$q_{max} = 43.279$ N/mm^2

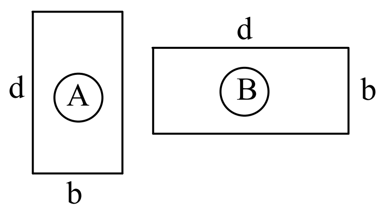
$\dfrac{100\times10^3}{\frac{\pi(30^4-26^4)}{32}} = \dfrac{q_{min}}{\left(\frac{26}{2}\right)}$

$q_{min} = 37.5$ N/mm^2

Chapter- 8
Deflections and Slopes

01. **Ans: (c)**

Sol:



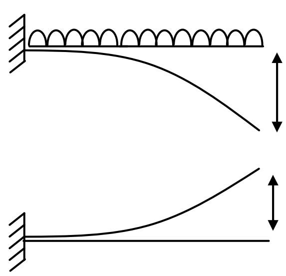
$$y_{max} \propto \frac{1}{I}$$

$$\therefore \frac{y_A}{y_B} = \frac{I_B}{I_A}$$

$$y_B = \frac{y_A \times bd^3/12}{db^3/12} \Rightarrow y_B = \left(\frac{d}{b}\right)^2 y_A$$

02. **Ans: (b)**

Sol: Total load W= wl



$$y_{max} = \frac{W\ell^3}{8EI}$$

$$y_{max} = \frac{W\ell^3}{3EI}$$

$y_{net} = \downarrow y_{udl} - \uparrow y_w$

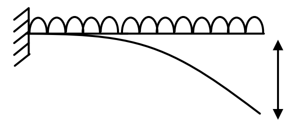
Total Net deflection $= \frac{WL^3}{8E1} - \frac{WL^3}{3EI}$

$$= \frac{-5WL^3}{24EI}$$

(– indicates upward)

03. **Ans: (c)**

Sol:



$$\theta_{max} = \frac{wl^3}{6EI} = 0.02$$

$$y_{max} = \frac{wL^4}{8EI} = 0.018$$

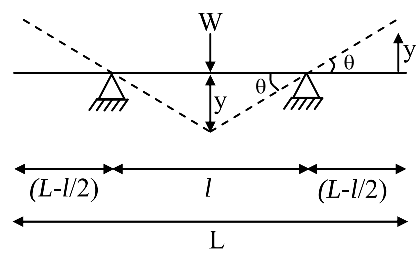
$$= \left(\frac{WL^3}{6EI}\right) \times \frac{L \times 6}{8}$$

$$= \frac{0.02 \times L \times 6}{8} = 0.018$$

L = 1.2 m

04. **Ans: (a)**

Sol:



Conditions given

$$\downarrow y = \frac{wl^3}{48EI}$$

$$\theta = \frac{wl^2}{16EI}$$

$$\tan\theta = \frac{y}{(L-1)/2}$$

θ is small ⇒ tan θ = θ

$$\therefore \theta = \frac{y}{(L-1)/2}$$

$$\therefore y = \theta\left(\frac{L-1}{2}\right)$$

$$\uparrow y = \theta\left(\frac{L-1}{2}\right)$$

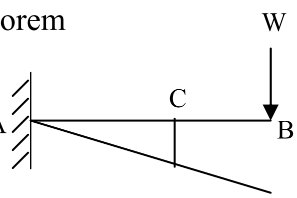
Thus $y\downarrow = y\uparrow$

$$\therefore \frac{w\ell^3}{48EI} = \frac{w\ell^2}{16EI} \times \left(\frac{L-1}{2}\right)$$

$$\frac{1}{L} = \frac{5}{3}$$

05. **Ans: (c)**

Sol: By using Maxwell's law of reciprocals theorem



$\delta_{CB} = \delta_{BC}$

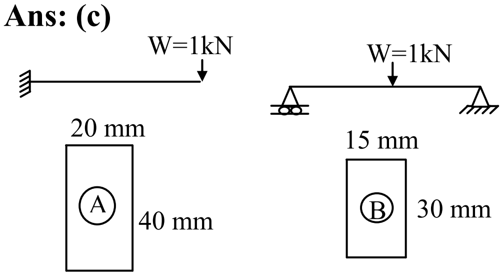
Deflection at 'C' due to unit load at B

= deflection at B due to unit load at C

As the load becomes half deflection becomes half

06. **Ans: (c)**

Sol:

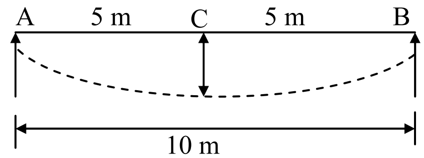


$$y_A = y_B \Rightarrow \left(\frac{wL^3}{3EI}\right)_A = \left(\frac{wL^3}{48EI}\right)_B$$

$$\therefore L_B = 400\text{mm}$$

07. Ans : 0.05

Sol:



$\therefore$ Curvature, $\dfrac{d^2y}{dx^2} = 0.004$

Integrating w.r.t. x,

We get, $\dfrac{dy}{dx} = 0.004x$

$$y = \frac{0.004x^2}{2}$$

$$y = 0.002x^2$$

at mid span, x = 5 m

$\therefore\ y = 0.002\ x^2$

$y = 0.05$ m

Chapter- 9
Thin Cylinders

Common Data for Question Nos. 1 & 2

01. **Ans: (b)**

Sol: $\tau_{max} = \sigma_1 = \dfrac{\sigma_h - 0}{2} = \dfrac{PD}{4t}$

$\therefore \tau_{max} = \dfrac{1.6\times 900}{4\times 12} = 30 \text{ MPa}$

02. **Ans: (b)**

Sol: $\varepsilon_v = \dfrac{\delta V}{V} = 2\varepsilon_h + \varepsilon_l$

$= 2\left[\dfrac{\sigma_h}{E} - \mu\dfrac{\sigma_\ell}{E}\right] + \left[\dfrac{\sigma_\ell}{E} - \dfrac{\mu.\sigma_h}{E}\right]$

$= 2\left[\dfrac{60}{2\times 10^5} - \dfrac{0.3\times 30}{2\times 10^5}\right] + \left[\dfrac{30}{2\times 10^5} - \dfrac{0.3\times 60}{2\times 10^5}\right]$

$= 5.7 \times 10^{-4}$

$\therefore \delta V = 5.7\times 10^{-4} \times \dfrac{\pi(900)^2}{4} \times 2000$

$\therefore \delta V = 725.23 \text{ cm}^3$

Common Data for Question Nos. 03 & 04

03. **Ans:1.25 MPa & 2.5 MPa**

Sol: R = 0.5 m, D = 1m, t = 1mm

$P = \rho g h$

At 0.5 m depth $P = \gamma h\ (10\times 10^3)\times 0.5$

$= 5000 \text{ N/m}^2$

$= (5\times 10^{-3}) \text{ MPa}$

Hoop stress, $\sigma_h = \sigma_1 = \dfrac{PD}{2t}$

$= \dfrac{5\times 10^{-3}\times 1000}{2\times 1} = 2.5\text{MPa}$

Longitudinal stress, $\sigma_\ell = \sigma_2 = \dfrac{PD}{4t}$

$= \dfrac{5\times 10^{-3}\times 1000}{4\times 1} = 1.25\text{MPa}$

04. **Ans: (2.125×10^{-5} & 5×10^{-6})**

Sol: E = 100 GPa , $\mu = 0.3$

$\varepsilon_h = \dfrac{\sigma_h}{E} - \mu\left(\dfrac{\sigma_\ell}{E}\right)$

$= \dfrac{2.5}{100\times 10^3} - 0.3\left(\dfrac{1.25}{100\times 10^3}\right) = 2.125\times 10^{-5}$

$\varepsilon_l = \dfrac{\sigma_l}{E} - \mu.\dfrac{\sigma_h}{E}$

$= \dfrac{1}{E}\left[\sigma_l - \mu\sigma_h\right]$

$= \dfrac{1}{100\times 10^3}\left[1.25 - 0.3\times 2.5\right] = 5\times 10^{-6}$

Chapter- 10
Columns and Struts

01. **Ans: (c)**

Sol: $P_e = \dfrac{\pi^2 \times EI}{l_e^2}$

For a given system , $l_e = \dfrac{l}{2}$

$\therefore P_e = \dfrac{4\pi^2 \times EI}{l^2}$

02. **Ans: (b)**

Sol: $\dfrac{P_1}{P_2} = \dfrac{l_{2e}^2}{l_{1e}^2}$

$\dfrac{P_1}{P_2} = \dfrac{l^2}{(2l)^2}$

$P_1 : P_2 = 1 : 4$

03. **Ans: 4**

Sol: $P = \dfrac{\pi^2}{l^2}$ EI

$\therefore P \propto I$

$$\frac{P}{P_o} = \frac{I_{\text{bonded}}}{I_{\text{loose}}} = \frac{\left[\dfrac{b(2t)^3}{12}\right]}{2\left[\dfrac{bt^3}{12}\right]} = 4$$

04. **Ans**: **(c)**

Sol: Euler's theory is applicable for axially loaded columns

$F\cos 45 = \dfrac{\pi^2}{l^2}$ EI

There fore, $F = \dfrac{\pi^2 EI}{\sqrt{2}\, L^2}$

05. **Ans: (a)**

Sol: Buckling load $P_e = \dfrac{\pi^2 EI}{L_C^2}$

Here $L_C = L = 3$ m ,

$\alpha = 12 \times 10^{-6}$ /°C, d = 50 mm = 0.05 m

$$\frac{P_e L}{AE} = L\alpha\Delta T$$

$$\frac{\pi^2 EI \times L}{L^2 \times AE} = L\alpha\Delta T$$

$$\frac{\pi^2 \times E \times \frac{\pi}{64} \times d^4 \times L}{L^2 \times \frac{\pi}{4} d^2 \times E} = L\alpha\Delta T$$

$$\Rightarrow \Delta T = \frac{\pi^2 \times d^2}{16 \times L^2 \times \alpha} = \frac{\pi^2 \times (0.05)^2}{16 \times 3^2 \times 12 \times 10^{-6}}$$

$= 14.278$

Chapter- 11
Strain Energy Resilience

01. **Ans: (d)**

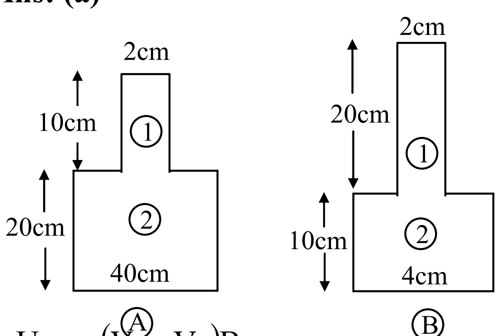
02. Ans: (b)

Sol: $U = \dfrac{P^2 \ell}{AE} \Rightarrow U \propto \ell$

$$\frac{U_1}{U_2} = \frac{\ell_A}{\ell_B} = 3:1$$

03. **Ans: (a)**

Sol:



$$\frac{U_B}{U_A} = \frac{(V_1 + V_2)B}{(V_1 + V_2)A}$$

$$\therefore \frac{U_B}{U_A} = \frac{\left[\dfrac{\sigma_1^2}{2E} \times V_1 + \dfrac{\sigma_2^2}{2E} \times V_2\right]_B}{\left[\dfrac{\sigma_1^2}{2E} \times V_1 + \dfrac{\sigma_2^2}{2E} \times V_2\right]_A}$$

$$= \frac{\left[\dfrac{P^2}{A_1^2} \times A_1 \times L_1 + \dfrac{P^2 \times A_2 \times L_2}{A_2^2}\right]}{\left[\dfrac{P^2 \times A_1 \times L_1}{A_1^2} + \dfrac{P^2 \times A_2 \times L_2}{A_2^2}\right]_A}$$

$$\therefore \frac{U_B}{U_A} = \frac{\left[\dfrac{L_1}{A_1} + \dfrac{L_2}{A_2}\right]_B}{\left[\dfrac{L_1}{A_1} + \dfrac{L_2}{A_2}\right]_A} = \frac{7.165}{4.77} = \frac{3}{2}$$

04. **Ans: (c)**

Sol: A_1 = modulus of resilience

$A_1 + A_2$ = mod of toughness

$$A_1 = \frac{1}{2} \times 0.004 \times 70 \times 10^6 = 14 \times 10^4$$

$$A_2 = \frac{1}{2} \times \left(0.008 \times 50 \times 10^6\right) + \left(0.008 \times 70 \times 10^6\right)$$

$$= 76 \times 10^4$$

$$A_1 + A_2 = (14 + 76) \times 10^4 = 90 \times 10^4$$

05. **Ans: (d)**

Sol: $U = \dfrac{P^2}{2A^2E}.V$

$U \propto P^2$

Due to the application of P_1 & P_2 one after the other

$(U_1 + U_2) \propto P_1^2 + P_2^2$ ……….. (1)

Due to the application of P_1 and P_2 together at the same time.

$U \propto (P_1 + P_2)^2$ ……….........(2)

It is obvious that

$(P_1^2 + P_2^2) < (P_1 + P_2)^2$

$\therefore (U_1 + U_2) < U$

06.

Sol: $U = U_1 + U_2 = \dfrac{T^2L}{2GJ_1} + \dfrac{T^2L}{2GJ_2}$

$$J_1 = \frac{\pi}{32}(50)^4 ; J_2 = \frac{\pi}{32}(26)^4$$

$L = 100\,\text{mm}$

$G = 80 \times 10^3\ \ N/mm^2$ on substitution,

U = 1.5 N-mm

Chapter - 12
Springs

01. Ans: (a)

02. Ans: (b)

Sol: Stiffness of Spring (S)

$$S = \frac{Gd^4}{64nR^3} = \frac{G(2r)^4}{64nR^3}$$

$$S = \frac{Gr^4}{4nR^3}$$

03. Ans: (d)

Sol: $k = \frac{G\,d^4}{64\,R^3 n}$

$k \propto d^4$

Let $d_1 = d$

If d is doubles i.e $d_2 = 2d$

$$\frac{k_1}{k_2} = \frac{d_1^4}{d_2^4} \Rightarrow \frac{k_1}{k_2} = \frac{d^4}{(2d)^4}$$

$k_2 = 16k_1$

04. Ans: (a)

Sol: $\delta = \frac{64WR^3n}{Gd^4}$

$\delta \,\alpha\, R^3$

$$\frac{\delta_1}{\delta_2} = \frac{R_1^3}{R_2^3} = \frac{R_1^3}{\left(\frac{R_1}{2}\right)^3} = 8$$

05. Ans: (a)

Sol: For springs in series: effective stiffness is

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2}$$

Therefore, $K_e = \frac{K_1K_2}{K_1 + K_2}$

06. Ans: (d)

Sol: Deflection of closely coiled spring

$$\delta = \frac{64\,R^3 wn}{G\,d^4}$$

$\therefore\ \delta \,\alpha\, n$

07. Ans: (d)

Sol: For springs connected in series

$$\frac{1}{K_e} = \frac{1}{S} + \frac{1}{2S} \Rightarrow K_e = \frac{2S}{3}$$

For springs connected in parallel

$(K_e) = K_1 + K_2 = S + 2S = 3S$

$$\therefore\ \frac{(K_e)_{series}}{(K_e)_{parallel}} = \frac{2S/3}{3S} = \frac{2}{9}$$

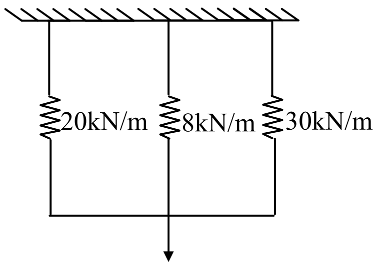
08. Ans: (d)

Sol: When one spring placed in other then those two springs will be in parallel. Hence combined stiffness is given by

$$K_e = K_A + K_B$$

09. Ans: (a)

Sol:



$$\frac{1}{k_e} = \frac{1}{10} + \frac{1}{40}$$

$K_e = 8$

$K_e = 20+8+30$

$= 58$ kN/m

10. Ans: (a)

Sol: $K_1 = K$

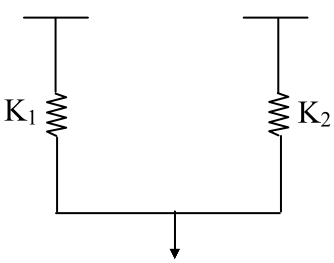
$K_2 = 2K + 2K$

$= 4K$

$$\frac{K_2}{K_1} = \frac{4K}{K} = 4$$

11. Ans: (a)

Sol: Equivalent Load Diagram:



$K_{eq} = K_1 + K_2$

$= 300 + 100$

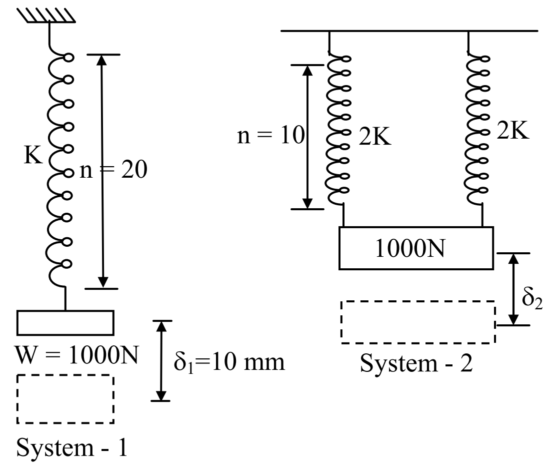
$K_{eq} = 400$ MN/m

$$\delta = \frac{F}{K_{eq}} = \frac{400\text{kN}}{400 \times 10^3 \text{ kN}/\text{m}}$$

$$= \frac{1}{1000}\text{m} = 1\text{mm}$$

12. Ans: (d)

Sol:



From system $\rightarrow (1)$

$$\delta_1 = \frac{1000}{K}$$

$$K = \frac{1000}{10} = 100\text{ N/m}$$

From system $\rightarrow (2)$

$K_{eq} = 2K + 2K$

$= 4K$

$K_{eq} = 4\times 100 = 400$

$$\delta_2 = \frac{1000}{400} = 2.5\text{m}$$