



CIVIL ENGINEERING



GATE | PSUs

**STRENGTH
OF
MATERIALS**

Volume - I : Study Material with Classroom Practice Questions

Strength of Materials

Solutions for Volume : I Classroom Practice Questions

1.1 Fundamental, Mechanical Properties of Materials, Stress Strain Diagram

01. Ans: (b)

Sol:

Ductility: A material which undergoes considerable deformation without rupture (large plastic zone)

Brittleness: Failure without warning (No plastic zone) i.e. no plastic deformation

Tenacity: High tensile strength

Creep: Material continues to deform with time under sustain loading

Plasticity: Material continues to deform without any further increase in stress.

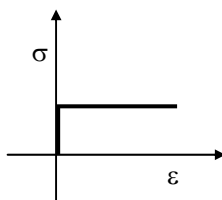
Endurance limit: Material has high probability of not failing under Reversal of stress of magnitude below this level.

Fatigue: Decreased Resistance of material to repeated reversal of stresses

02. Ans: (a)

Sol:

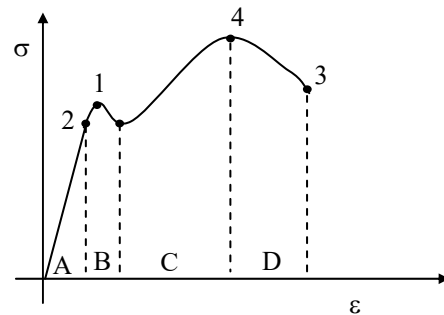
For rigid plastic material



03. Ans: (a)

04. Ans: (b)

Sol:



05. Ans: (b)

06. Ans: (a)

Sol:

Strain hardening: - increase in strength after plastic zone by rearrangement of molecules in material. Visco-elastic material exhibits a mixture of creep + elastic after effects at room temperature. Thus their behavior is time dependant

07. Ans: (a)

08. Ans: (a)

09. Ans: (a)

Sol:

Addition of carbon will increase strength, thereby ductility will decrease.



1.2 Elastic Constants and Their Relationships

01. Ans (c)

Sol:

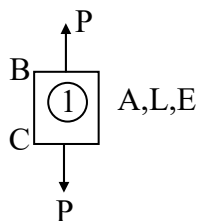
$$\text{Poisson's ratio} = \frac{\text{lateral strain}}{\text{linear strain}} = \frac{\Delta D/D}{\Delta L/L}$$

$$\mu = \frac{\frac{\Delta D}{8}}{\frac{PL}{AE/L}}$$

$$\mu = \frac{\Delta D}{8} \frac{AE}{P}$$

$$0.25 = \frac{\Delta D}{8} \frac{\frac{\pi}{4}(8)^2 \times 10^6}{50000}$$

$$\Delta D = 1.98 \times 10^{-3} = 0.00198 \cong 0.002 \text{ cm}$$



02. Ans: (c)

Sol:

$$\text{Bulk modulus} = \frac{\delta P}{\delta V/V}$$

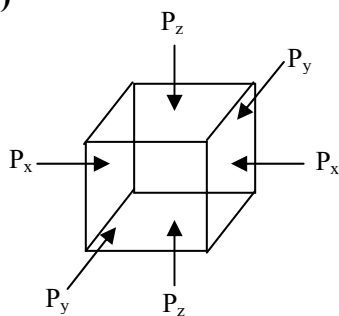
$$\Rightarrow 2.5 \times 10^5 = \frac{200 \times 20}{\delta V}$$

$$\Rightarrow \delta V = 0.016 \text{ m}^3$$

1.3 Linear and Volumetric Changes of Bodies

01. Ans: (d)

Sol:



$$\text{Let } P_y = P_z = P$$

$$\epsilon_y = 0,$$

$$\epsilon_z = 0$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_z}{E} - \mu \frac{\sigma_x}{E}$$

$$0 = \frac{(-P)}{E} - \mu \frac{(-P)}{E} - \mu \frac{(P_x)}{E}$$

$$P = \frac{\mu \cdot P_x}{(1 - \mu)}$$



02. Ans: (a)

Sol:

$$\sigma_c = 4\tau \text{ ---- (given)}$$

Punching force = shear resistance of plate

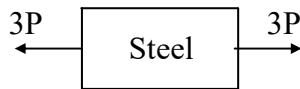
$$\sigma \{c/s \text{ area}\} = \tau (\text{surface Area})$$

$$4 \times \tau \times \frac{\pi \cdot D^2}{4} = \tau (\pi \cdot D \cdot t)$$

$$D = t = 10 \text{ mm}$$

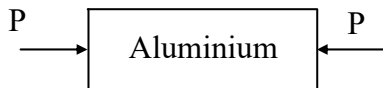
03. Ans: (d)

Sol:



$$\sigma_s = 140 \text{ MPa} = \frac{P_s}{A_s}$$

$$P_s = \frac{140 \times 500}{3} \approx 23,300 \text{ N}$$



$$\sigma_{Al} = 90 \text{ MPa} = \frac{P_{Al}}{A_{Al}}$$

$$P_{Al} = 90 \times 400 = 36,000 \text{ N}$$



$$\sigma_B = 100 \text{ MPa} = \frac{P_B}{A_B}$$

$$P_B = \frac{100 \times 200}{2} = 10,000 \text{ N}$$

$$P_s = 22,300 \text{ N}, \quad P_{Al} = 36,000 \text{ N}$$

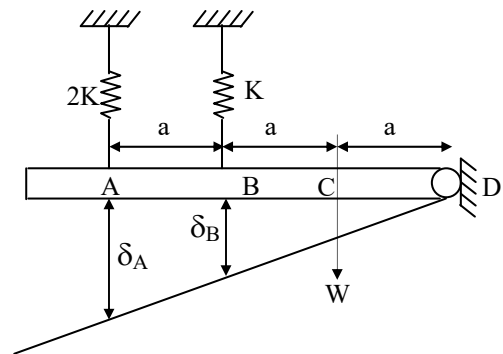
$$P_B = 10,000 \text{ N}$$

Take minimum value

$$\therefore P = 10,000 \text{ N}$$

04. Ans: (c)

Sol:



From similar triangle

$$\frac{3a}{\delta_A} = \frac{2a}{\delta_B}$$

$$3\delta_B = 2\delta_A \text{ (1)}$$

$$\text{Stiffness } K = \frac{W}{\delta}$$

$$\therefore K_A = \frac{W_A}{\delta_A} \Rightarrow \delta_A = \frac{W_A}{2K}$$

$$\text{Similarly } \delta_B = \frac{W_B}{K}$$

$$\text{From equation (1)} \quad 3 \times \frac{W_B}{K} = 2 \times \frac{W_A}{2K}$$

$$\therefore \frac{W_A}{W_B} = 3$$

05. Ans: (d)



1.4 Thermal Stresses

Common Data for Question Nos. 01 & 02

01. Ans: (b)

Sol: Free expansion = Expansion prevented

$$[\ell \alpha t]_s + [\ell \alpha t]_{Al} = \left[\frac{P\ell}{AE} \right]_s + \left[\frac{P\ell}{AE} \right]_{Al}$$

$$11 \times 10^{-6} \times 20 + 24 \times 10^{-6} \times 20$$

$$= \frac{P}{100 \times 10^3 \times 200} + \frac{P}{200 \times 10^3 \times 70}$$

$$\Rightarrow P = 5.76 \text{ kN}$$

02. Ans: (b)

Sol: $\sigma_s = \frac{P_s}{A_s} = \frac{5.76 \times 10^3}{100} = 57.65 \text{ MPa}$

$$\sigma_{Al} = \frac{P}{A_{al}} = \frac{5.76 \times 10^3}{200} = 28.82 \text{ MPa}$$

Common Data for Question Nos. 03 & 04

03. Ans: (d)

Sol: $\Delta_s + \Delta_{gm} = -\alpha_s t L + \alpha_{gm} t L$

$$\frac{P \times L}{100 \times 200 \times 10^3} + \frac{P \times L}{200 \times 100 \times 10^3}$$

$$= -6 \times 10^{-6} \times 200 \times L + 10 \times 10^{-6} \times 200 \times L$$

$$\frac{2 \times P}{200 \times 100 \times 10^3} = 8 \times 10^{-4}$$

$$P = 8 \text{ kN}$$

04. Ans: (b)

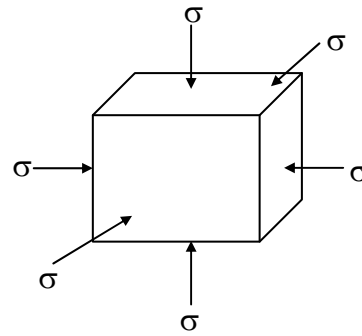
Sol: Stress in steel $\frac{P_s}{A_s} = \frac{8 \times 10^3}{100} = 80 \text{ MPa}$

$$\text{Stress in Gunmetal} = \frac{P_{gm}}{A_{gm}}$$

$$= \frac{8 \times 10^3}{200} = 40 \text{ MPa}$$

05. Ans: (a)

Sol:



Strain in X-direction due to temperature,

$$\epsilon_t = \alpha (\Delta T)$$

Strain in X-direction due to volumetric stress

$$\epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}$$

$$\epsilon_x = \frac{-\sigma}{E} (1 - 2\nu)$$

$$-\sigma = \frac{(\epsilon_x)(E)}{1 - 2\nu}$$

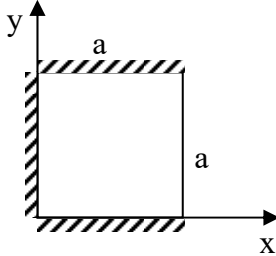
$$-\sigma = \frac{\alpha(\Delta T)E}{(1 - 2\nu)}$$

$$\therefore \sigma = \frac{-E\alpha(\Delta T)}{1 - 2\nu}$$



06. Ans: (b)

Sol:



Free expansion in x direction is $a \propto t$

Free expansion in y direction is $a \propto t$

Since there is restriction in y direction expansion doesn't take place. So in lateral direction this increase in expansion due to restriction in $\mu a \propto t$

$\therefore$ Total expansion in x direction is

$$= \alpha a t + \mu a \alpha t$$

$$= a \alpha t (1 + \mu)$$

Chapter- 2 Complex Stresses and Strains

01. Ans: (b)

Sol: Maximum principal stress $\sigma_1 = 18$

Minimum principal stress $\sigma_2 = -8$

$$\text{Maximum shear stress} = \frac{\sigma_1 - \sigma_2}{2} = 13$$

Normal stress on Maximum shear stress plane

$$= \frac{\sigma_1 + \sigma_2}{2} = \frac{18 + (-8)}{2} = 5$$

02. Ans: (b)

Sol: Radius of Mohr's circle $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$

$$20 = \frac{\sigma_1 - 10}{2}$$

$$\sigma_1 = 50 \text{ N/mm}^2$$

03. Ans: (b)

Sol: Long dam $\rightarrow$ plane strain member

$$\epsilon_z = 0 = \frac{\sigma_z}{E} - \frac{\mu \sigma_x}{E} - \frac{\mu \sigma_y}{E}$$

$$\sigma_x = 150 \text{ MPa}, \sigma_y = -300 \text{ MPa}, \mu = 0.3$$

$$\therefore 0 = \sigma_z - 0.3 \times 150 + 0.3 \times 300$$

$$\Rightarrow \sigma_z = 45 \text{ MPa}$$



Common Data for Question Nos. 04 & 05

04. Ans: (a)

$$\begin{aligned}\text{Sol: } \sigma_1 &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{65 + (-13)}{2} \pm \sqrt{\left(\frac{65 + 13}{2}\right)^2 + 20^2} \\ \therefore \sigma_1 &= 70 \text{ MPa}, \sigma_2 = -18 \text{ MPa}\end{aligned}$$

05. Ans: (a)

$$\text{Sol: } \varepsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} ;$$

$$\varepsilon_1 = \frac{\text{Change in diameter}}{\text{Original diameter}}$$

$$\varepsilon_1 = \frac{70 + 0.3 \times 18}{2 \times 10^5} = 3.77 \times 10^{-4}$$

$$\begin{aligned}\text{Change in diameter} &= 3.77 \times 10^{-4} \times 300 \\ &= 0.1131 \text{ mm}\end{aligned}$$

$$\begin{aligned}\therefore \text{Length of major axes of ellipse} \\ &= 300 + \text{change in diameter} \\ &= 300.113 \text{ mm}\end{aligned}$$

Similarly length of minor axes

$$\varepsilon_2 = \frac{\sigma_2}{E} - \frac{\mu \sigma_1}{E}$$

$$\varepsilon_2 = \frac{\delta D}{300} = \frac{-18 - 0.3 \times 70}{2 \times 10^5} = -1.95 \times 10^{-4}$$

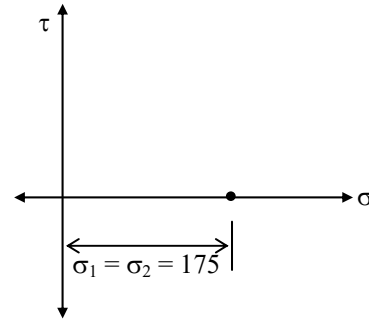
$$\delta D = -1.95 \times 10^{-4} \times 300 = -0.0585 \text{ mm}$$

$$\begin{aligned}\therefore \text{Minor axis length,} &= 300 - 0.0585 \\ &= 299.94 \text{ mm}\end{aligned}$$

Common Data for Question Nos. 06 & 07

06. Ans: (b)

Sol:



07. Ans: (d)

08. Ans: (c)

Sol: $\sigma_2 = 0$ (Given)

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\frac{\sigma_x + \sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

$$\tau_{xy}^2 = \left(\frac{\sigma_x + \sigma_y}{2}\right)^2 - \left(\frac{\sigma_x - \sigma_y}{2}\right)^2$$

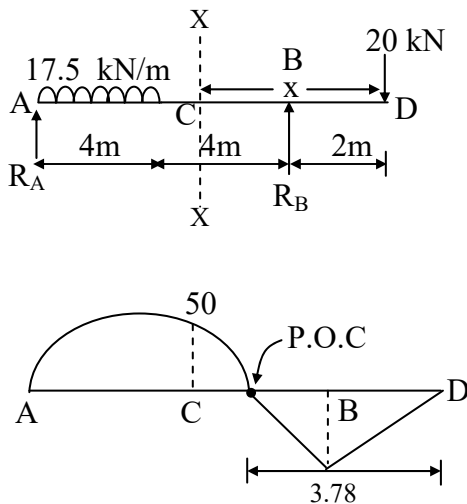
$$\tau_{xy}^2 = \sigma_x \cdot \sigma_y \Rightarrow \tau_{xy} = \sqrt{\sigma_x \cdot \sigma_y}$$



Chapter- 3 Shear Force and Bending Moment

01. Ans: (b)

Sol: Contra flexure is the point where BM is becoming zero.



$$\Sigma M_A = 0$$

$$17.5 \times 4 \times \frac{4}{2} + 20 \times 10 - R_B \times 8 = 0$$

$$R_B = 42.5 \text{ kN}$$

$$M_x = -20x + R_B(x - 2)$$

For bending moment be zero $M_x = 0$

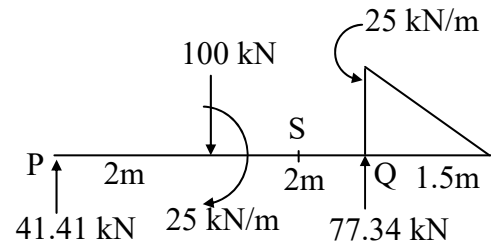
$$-20x + 42.5(x - 2) = 0$$

$$x = 3.78 \text{m From right ie. D}$$

Common Data for Question Nos. 02 & 03

02. Ans: (b)

Sol:



$$\text{Take } \Sigma M_P = 0$$

$$\frac{1}{2} \times 25 \times 1.5 \times \left(\frac{1.5}{3} + 4 \right) - (R_Q \times 4) + 100 \times 2 + 25 = 0$$

$$R_Q = 77.34 \text{ kN}$$

$$\Sigma V = 0$$

$$R_P + R_Q = 100 + \frac{1}{2} \times 25 \times 1.5 = 118.75 \text{ kN}$$

$$\therefore R_P = 41.41 \text{ kN}$$

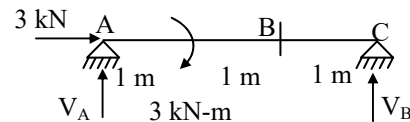
$$\text{S. F. at P} = 41.41 \text{ kN}$$

03. Ans: (c)

$$\text{Sol: } M_S = R_P(3) + 25 - 100 \times 1 = 49.2 \text{ kN-m}$$

04. Ans: (c)

Sol:



$$-V_B \times 3 + 3 = 0$$

$$V_C = 1 \text{ kN}$$

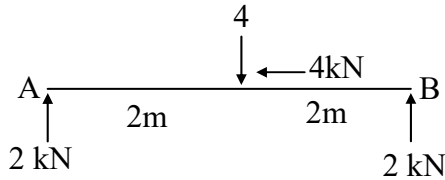
$\therefore$ Bending moment at B

$$\Rightarrow V_C \times 1 = 1 \text{ kNm}$$



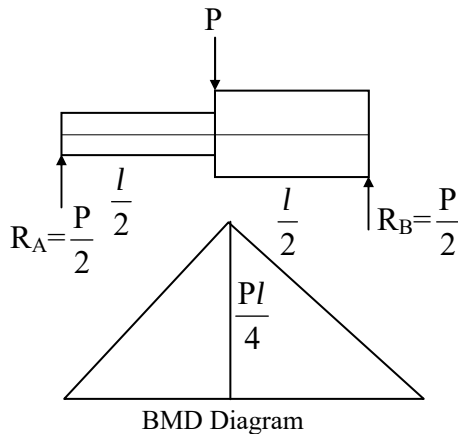
05. Ans: (a)

Sol:



06. Ans: (c)

Sol:



(BM) at $\frac{l}{2}$ from left is $\frac{Pl}{4}$

The given beam is statically determinate structure. Therefore equilibrium equations are sufficient to analyze the problem.

In statically determinate structure the BMD, SFD and Axial force are not affected by section (I), material (E), thermal changes.

07. Ans: (a)

Sol: As the given support is hinge, for different set of loads in different direction beam will experience only axial load.

Chapter- 4 Centre of Gravity & Moment of Inertia

01. Ans: (a)

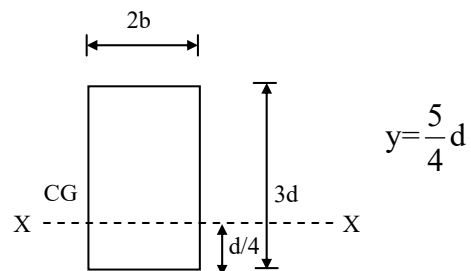
$$\begin{aligned} \text{Sol: } \bar{y} &= \frac{E_1 y_1 + E_2 y_2}{E_1 + E_2} \\ \Rightarrow \bar{y} &= \frac{2E_2 \left(h + \frac{h}{2} \right) + E_2 \times \frac{h}{2}}{2E_2 + E_2} \quad (\because E_1 = 2E_2) \\ \bar{y} &= 1.167h \text{ from base} \end{aligned}$$

02. Ans: (b)

$$\begin{aligned} \text{Sol: } \bar{y} &= \frac{A_1 E_1 Y_1 + A_2 E_2 Y_2}{A_1 E_1 + A_2 E_2} \\ &= \frac{1.5a \times 3a^2 \times E_1 + 1.5a \times 6a^2 \times 2E_1}{3a^2 E_1 + 6a^2 (2E_1)} \\ &= \frac{22.5a^3 E_1}{15a^2 E_1} = 1.5a \end{aligned}$$

03. Ans: $13.875 bd^3$

Sol:



$$\text{M.I about CG} = I_{CG} = \frac{2b(3d)^3}{12} = \frac{9}{2} bd^3$$



$$\begin{aligned} \text{M.I about } X-X \Big|_{\text{at } d/4 \text{ distance}} &= I_G + Ay^2 \\ &= \frac{9}{2}bd^3 + 6bd\left(\frac{5}{4}\right)^2 d^2 \\ &= \frac{111}{8}bd^3 = 13.875bd^3 \end{aligned}$$

04. Ans: $4.38 \times 10^6 \text{ mm}^4$

Sol:

$$\begin{aligned} I_x &= \frac{60 \times 120^3}{12} - 2 \left(\frac{30 \times 45^3}{12} + 30 \times 45 \times 37.5^2 \right) \\ &= 4.38 \times 10^6 \text{ mm}^4 \end{aligned}$$

05. Ans: $I_x = 152146 \text{ mm}^4$, $I_y = 45801.34 \text{ mm}^4$

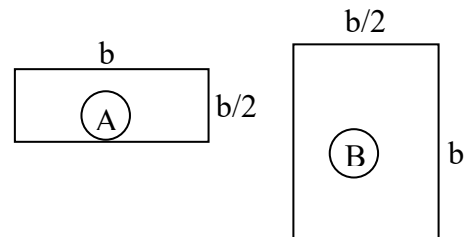
Sol:

$$\begin{aligned} I_x &= \frac{30 \times 40^3}{12} - \frac{\pi \times 20^4}{64} = 152146 \text{ mm}^4 \\ I_y &= \frac{40 \times 30^3}{12} - \left(\frac{\pi \times 20^4}{64} + 2 \left(\frac{\pi}{2} \times 10^2 \times \left(15 - \frac{4 \times 10}{3\pi} \right)^2 \right) \right) \\ &= 45801.34 \text{ mm}^4 \end{aligned}$$

Chapter- 5 Theory of Simple Bending

01. Ans: (b)

Sol:



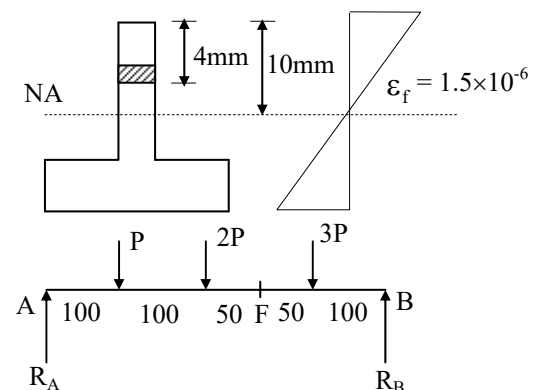
$$\sigma = \frac{M}{Z}$$

$$\sigma \propto \frac{1}{Z} (\because M \text{ is same})$$

$$\therefore \frac{\sigma_A}{\sigma_B} = \frac{Z_B}{Z_A} = \frac{\frac{\left(\frac{b}{2} \times b^2\right)}{6}}{b \times \frac{\left(\frac{b}{2}\right)^2}{6}} = 2$$

02. Ans: (b)

Sol:



$$\therefore \sum M_A = 0$$

$$P \times 100 + 2P \times 200 + 3P \times 300 = R_B \times 400$$



$$\therefore R_B = 3.5P, R_A = 2.5P$$

Take moments about F and moment at F

$$M_F = R_B \times 150 - 3P \times 50 = 375P$$

$$\frac{M_F}{I} = \frac{\sigma_b}{y_F}$$

$$\frac{375P}{2176} = \frac{(1.5 \times 10^{-6} \times 200 \times 10^3)}{6}$$

$$\therefore P = 0.29 \text{ N}$$

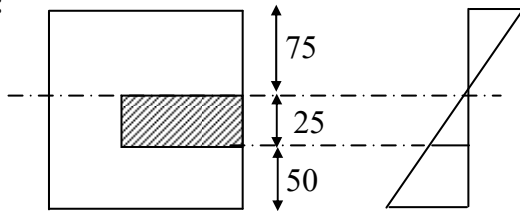
03. Ans: (b)

$$\text{Sol: } \frac{E}{R} = \frac{\sigma_b}{y_{\max}} \Rightarrow \frac{2 \times 10^5}{250} = \frac{\sigma_b}{(0.5/2)}$$

$$\sigma_b = 200 \text{ MPa}$$

04. Ans: (c)

Sol:



Force on the hatched area

$$= \text{Avg. Stress} \times \text{hatched Area}$$

$$\frac{M}{I} = \frac{f}{y}$$

$$\therefore \frac{16 \times 10^6}{\frac{100 \times 150^3}{12}} = \frac{f}{25} \Rightarrow f = 14.22 \text{ MPa}$$

$\therefore$ Force on hatched area

$$= \text{Average stress} \times \text{hatched area}$$

$$= \left(\frac{0 + 14.22}{2} \right) (25 \times 50) = 8.9 \text{ kN}$$

05. Ans: (c)

$$\text{Sol: } \frac{f_{\text{Tensile}}}{y_{\text{top}}} = \frac{M}{I}$$

$$\Rightarrow f_{\text{Tensile}} = \frac{0.3 \times 3 \times 10^6}{3 \times 10^6} \times 70$$

(maximum bending stress will be at top fibre

so $y_1 = 70 \text{ mm}$)

$$\therefore f_{\text{Tensile}} = 21 \text{ kN/m}^2$$

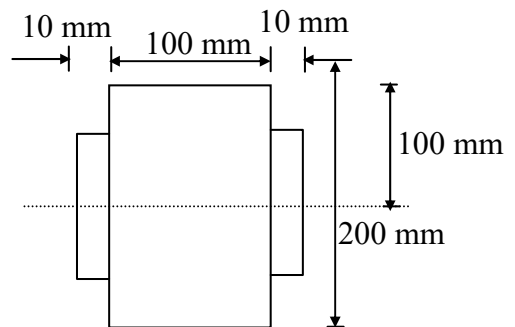
06. Ans: (c)

Sol: There will be no shear force therefore no shear stresses. And point P is above N.A therefore compressive stresses

Common Data for Question Nos. 07 & 08

07. Ans: 80

Sol:



Maximum stress in timber = 8 MPa

Modular ratio, $m = 20$

Stress in timber in steel level,

$$100 \rightarrow 8$$

$$50 \rightarrow f_w$$

$$f_w = 4 \text{ MPa}$$

Maximum stress developed in steel is $= m \cdot f_w$

$$= 20 \times 4 = 80 \text{ MPa}$$

Convert whole structure as a steel structure by using modular ratio.



08. Ans: 8

Sol: Moment of inertia,

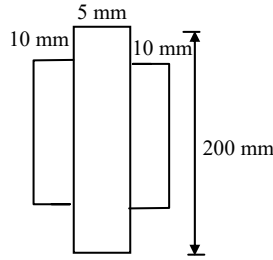
$$I = \frac{2 \times 10 \times 100^3}{12} + \frac{5 \times 200^3}{12} = 5 \times 10^6 \text{ mm}^4$$

$$f_s = 8 \times 20 = 160 \text{ MPa}$$

Bending equation

$$\frac{M}{I} = \frac{f_s}{y}$$

$$M = \frac{5 \times 10^6 \times 160}{\frac{200}{2}} = \frac{5 \times 10^6 \times 160 \times 2}{200} = 8 \text{ kN-m}$$



09. Ans: 2.43

Sol: From figure $A_1B_1 = l = 3 \text{ m}$ (given)

$$AB = \left(R - \frac{h}{2} \right) \alpha = l - l\alpha t_1 \text{ ----- (1)}$$

$$A_2B_2 = \left(R + \frac{h}{2} \right) \alpha = l + l\alpha t_2 \text{ ----- (2)}$$

Subtracting above two equations (2) – (1)

$$h(\alpha) = l\alpha(t_2 - t_1)$$

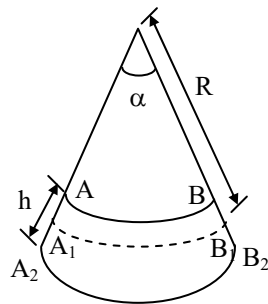
$$\text{but } A_1B_1 = l = R\alpha$$

$$\Rightarrow \alpha = \frac{l}{R}$$

$$\therefore h \left(\frac{l}{R} \right) = l\alpha(\Delta T)$$

$$R = \frac{h}{\alpha(\Delta T)} = \frac{250}{(1.5 \times 10^{-5})(72 - 36)}$$

$$R = 462.9 \text{ m}$$



From geometry of circles

$$(2R - \delta)\delta = \frac{L}{2} \cdot \frac{L}{2} \text{ \{ref. figure in question No. 02\}}$$

$$2R \cdot \delta - \delta^2 = \frac{L^2}{4} \text{ (neglect } \delta^2)$$

$$\delta = \frac{L^2}{8R} = \frac{3^2}{8 \times 462.9} = 2.43 \text{ mm}$$

Shortcut:

Deflection is due to differential temperature of bottom and top ($\Delta T = 72^\circ - 36^\circ = 36^\circ$). Bottom temperature being more, the beam deflects down.

As derived in the Q2 (2 marks)

$$\delta = \frac{\alpha(\Delta T)\ell^2}{8h} = \frac{1.5 \times 10^{-5} \times 36 \times 3000^2}{8 \times 250} = 2.43 \text{ mm (downward)}$$



Chapter- 6 Shear Stress Distribution in Beams

01. Ans: (a)

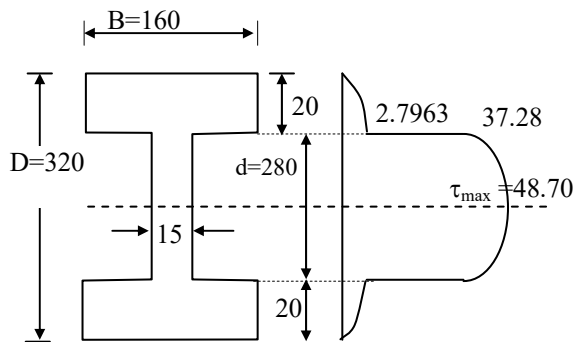
$$\text{Sol: } \tau_{\max} = \frac{3}{2} \times \tau_{\text{avg}} = \frac{3}{2} \times \frac{f}{b.d}$$

$$3 = \frac{3}{2} \times \frac{50 \times 10^3}{100 \times d}$$

$$\therefore d = 250 \text{ mm} = 25 \text{ cm}$$

Common Data for Q. 02, 03 and 04:

Sol:



All dimensions are in mm

02. Ans: 37.3

Bending moment (M) = 100 kN-m,

Shear Force (SF) = f = 200 kN

$$I = \frac{160 \times 320^3}{12} - \frac{145 \times 280^3}{12}$$

$$= 171.65 \times 10^6 \text{ mm}^4$$

$$\tau_{\text{at interface of flange \& web}} = \frac{FA\bar{y}}{Ib}$$

$$= \frac{200 \times 10^3}{171.65 \times 10^6 \times 15} \times (160 \times 20 \times 150)$$

$$= 37.28 \text{ MPa}$$

03. Ans: 48.7

$$\tau_{\max} = \frac{FA\bar{y}}{Ib}$$

$$= \frac{200 \times 10^3}{171.65 \times 10^6 \times 15} \times (140 \times 15 \times 70 + 160 \times 20 \times 150)$$

$$= 48.70 \text{ MPa}$$

04. Ans: 3.5

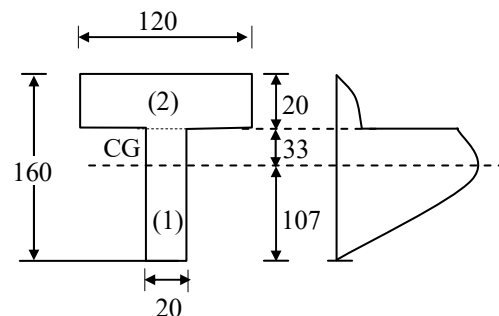
$\tau_{\text{in flange just above web}}$

$$= \frac{200 \times 10^3}{171.65 \times 10^6 \times 160} \times 160 \times 20 \times 150$$

$$= 3.5 \text{ MPa}$$

05. Ans: 61.43 MPa

Sol:



All dimensions are in mm

$$I_{NA} = 13 \times 10^6 \text{ mm}^4$$

$y_{CG} = 107 \text{ mm}$ from base

$$\tau_{\max} = \frac{FA\bar{y}}{Ib}$$

$$A\bar{y} = (120 \times 20 \times 43) + (33 \times 20 \times 16.5)$$

$$= 114090 \text{ mm}^3$$

$$\tau_{\max} = \frac{140 \times 10^3 \times 114090}{13 \times 10^6 \times 20} = 61.43 \text{ MPa}$$



Chapter- 7 Torsion

01. Ans: (a)

Sol: Twisting moment = 1×0.5
= 0.5 kN-m

02. Ans: (d)

$$\text{Sol: } \frac{(\text{strength})_{\text{solid}}}{(\text{strength})_{\text{hollow}}} = \frac{1}{1 - K^4}$$

$$= \frac{1}{1 - \left(\frac{1}{2}\right)^4} = \frac{16}{15}$$

Common Data for Question Nos. 03 & 04

03. Ans: (a)

Sol: $(P)_{AB} = 30 \text{ kW}$

$(P)_{BC} = 45 \text{ kW}$

$$P_{AB} = \frac{2\pi N T_{AB}}{60} \Rightarrow T_{AB} = 1.43 \text{ kN-m}$$

$$P_{BC} = \frac{2\pi N T_{BC}}{60} \Rightarrow T_{BC} = 2.14 \text{ kN-m}$$

$$\Rightarrow \frac{T}{J} = \frac{\tau}{R}$$

$$\tau_{AB} = \frac{T_{AB}}{Z_P} = \frac{1.43 \times 10^6}{\frac{\pi}{16} \times (50)^3} = 58.26 \text{ MPa}$$

$$\tau_{BC} = \frac{T_{BC}}{Z_P} = \frac{2.14 \times 10^6}{\frac{\pi}{16} \times 75^3}$$

$$\tau_{BC} = 25.83 \text{ MPa}$$

$\therefore$ Take maximum value of ' τ ' i.e, 58.26 MPa

04. Ans: (c)

Sol: Series

$$\theta_{\max} = \theta_{AB} + \theta_{BC} = \left(\frac{T.L}{CJ}\right)_{AB} + \left(\frac{T.L}{CJ}\right)_{BC}$$

$$= \frac{(1.43 \times 10^6) \times 4000}{(8.5 \times 10^4) \times \left(\frac{\pi}{32} \times 50^4\right)} + \frac{(2.14 \times 10^6) \times 2000}{(8.5 \times 10^4) \times \left(\frac{\pi}{32} \times 75^4\right)}$$

$$= 0.128 \text{ radian}$$

$$= 0.128 \times \frac{180}{\pi} = 7.21 \approx 7.14^\circ$$

05. Ans: 43.27 & 37.5

Sol: Given $D_o = 30 \text{ mm}$, $t = 2 \text{ mm}$

$$\therefore D_i = 30 - 4 = 26 \text{ mm}$$

$$\text{We know that } \frac{\tau}{J} = \frac{q}{R}$$

$$\frac{100 \times 10^3}{\pi(30^4 - 26^4)} = \frac{q_{\max}}{\left(\frac{30}{2}\right)}$$

$$q_{\max} = 43.279 \text{ N/mm}^2$$

$$\frac{100 \times 10^3}{\pi(30^4 - 26^4)} = \frac{q_{\min}}{\left(\frac{26}{2}\right)}$$

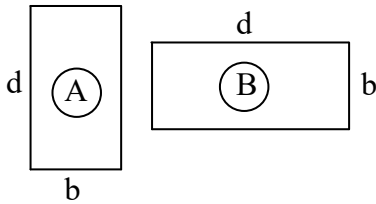
$$q_{\min} = 37.5 \text{ N/mm}^2$$



Chapter- 8 Deflections and Slopes

01. Ans: (c)

Sol:



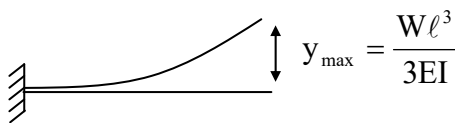
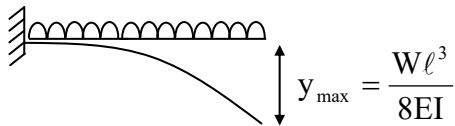
$$y_{\max} \propto \frac{1}{I}$$

$$\therefore \frac{y_A}{y_B} = \frac{I_B}{I_A}$$

$$y_B = \frac{y_A \times bd^3/12}{db^3/12} \Rightarrow y_B = \left(\frac{d}{b}\right)^2 y_A$$

02. Ans: (b)

Sol: Total load $W = wl$



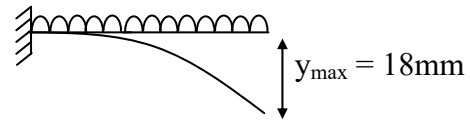
$$y_{\text{net}} = \downarrow y_{\text{udl}} - \uparrow y_w$$

$$\begin{aligned} \text{Total Net deflection} &= \frac{WL^3}{8EI} - \frac{WL^3}{3EI} \\ &= \frac{-5WL^3}{24EI} \end{aligned}$$

(- indicates upward)

03. Ans: (c)

Sol:



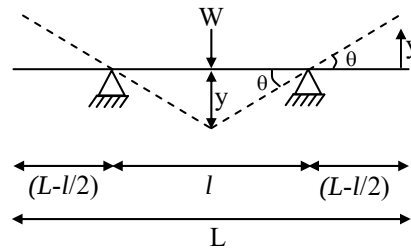
$$\theta_{\max} = \frac{wl^3}{6EI} = 0.02$$

$$\begin{aligned} y_{\max} &= \frac{wL^4}{8EI} = 0.018 \\ &= \left(\frac{WL^3}{6EI}\right) \times \frac{L \times 6}{8} \\ &= \frac{0.02 \times L \times 6}{8} = 0.018 \end{aligned}$$

$$L = 1.2 \text{ m}$$

04. Ans: (a)

Sol:



Conditions given

$$\downarrow y = \frac{wl^3}{48EI}$$

$$\theta = \frac{wl^2}{16EI}$$

$$\tan \theta = \frac{y}{(L-l)/2}$$

$$\theta \text{ is small} \Rightarrow \tan \theta = \theta$$

$$\therefore \theta = \frac{y}{(L-l)/2}$$



$$\therefore y = \theta \left(\frac{L-1}{2} \right)$$

$$\uparrow y = \theta \left(\frac{L-1}{2} \right)$$

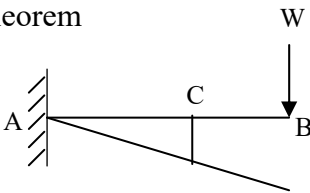
Thus $y \downarrow = y \uparrow$

$$\therefore \frac{w\ell^3}{48EI} = \frac{w\ell^2}{16EI} \times \left(\frac{L-1}{2} \right)$$

$$\frac{1}{L} = \frac{5}{3}$$

05. Ans: (c)

Sol: By using Maxwell's law of reciprocals theorem



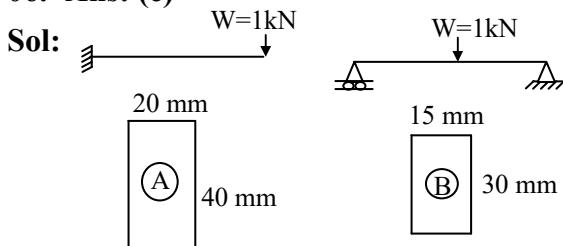
$$\delta_{CB} = \delta_{BC}$$

Deflection at 'C' due to unit load at B

= deflection @ B due to unit load at C

As the load becomes half deflection becomes half

06. Ans: (c)

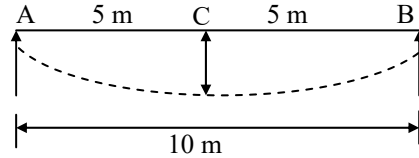


$$y_A = y_B \Rightarrow \left(\frac{WL^3}{3EI} \right)_A = \left(\frac{WL^3}{48EI} \right)_B$$

$$\therefore L_B = 400\text{mm}$$

07. Ans : 0.05

Sol:



$$\therefore \text{Curvature, } \frac{d^2y}{dx^2} = 0.004$$

Integrating w.r.t. x,

$$\text{We get, } \frac{dy}{dx} = 0.004x$$

$$y = \frac{0.004x^2}{2}$$

$$y = 0.002x^2$$

@ mid span, $x = 5 \text{ m}$

$$\therefore y = 0.002x^2$$

$$y = 0.05 \text{ m}$$

Chapter- 9 Thin Cylinders

Common Data for Question Nos. 1 & 2

01. Ans: (b)

$$\text{Sol: } \tau_{\max} = \sigma_l = \frac{\sigma_h - 0}{2} = \frac{PD}{4t}$$

$$\therefore \tau_{\max} = \frac{1.6 \times 900}{4 \times 12} = 30 \text{ MPa}$$



02. Ans: (b)

$$\text{Sol: } \varepsilon_v = \frac{\delta V}{V} = 2\varepsilon_h + \varepsilon_l$$

$$\begin{aligned} &= 2 \left[\frac{\sigma_h}{E} - \mu \frac{\sigma_l}{E} \right] + \left[\frac{\sigma_l}{E} - \mu \frac{\sigma_h}{E} \right] \\ &= 2 \left[\frac{60}{2 \times 10^5} - \frac{0.3 \times 30}{2 \times 10^5} \right] + \left[\frac{30}{2 \times 10^5} - \frac{0.3 \times 60}{2 \times 10^5} \right] \\ &= 5.7 \times 10^{-4} \end{aligned}$$

$$\therefore \delta V = 5.7 \times 10^{-4} \times \frac{\pi(900)^2}{4} \times 2000$$

$$\therefore \delta V = 725.23 \text{ cm}^3$$

Common Data for Question Nos. 03 & 04

03. Ans: 1.25 & 2.5

$$\text{Sol: } R = 0.5 \text{ m, } D = 1 \text{ m, } t = 1 \text{ mm}$$

$$P = \rho gh$$

$$\begin{aligned} \text{At } 0.5 \text{ m depth } P &= \gamma h (10 \times 10^3) \times 0.5 \\ &= 5000 \text{ N/m}^2 \\ &= (5 \times 10^{-3}) \text{ MPa} \end{aligned}$$

$$\begin{aligned} \text{Hoop stress, } \sigma_h = \sigma_1 &= \frac{PD}{2t} \\ &= \frac{5 \times 10^{-3} \times 1000}{2 \times 1} = 2.5 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \text{Longitudinal stress, } \sigma_l = \sigma_2 &= \frac{PD}{4t} \\ &= \frac{5 \times 10^{-3} \times 1000}{4 \times 1} = 1.25 \text{ MPa} \end{aligned}$$

04. Ans: (2.125 × 10⁻⁵ & 5 × 10⁻⁶)

$$\text{Sol: } E = 100 \text{ GPa, } \mu = 0.3$$

$$\begin{aligned} \varepsilon_h &= \frac{\sigma_h}{E} - \mu \left(\frac{\sigma_l}{E} \right) \\ &= \frac{2.5}{100 \times 10^3} - 0.3 \left(\frac{1.25}{100 \times 10^3} \right) = 2.125 \times 10^{-5} \end{aligned}$$

$$\begin{aligned} \varepsilon_l &= \frac{\sigma_l}{E} - \mu \frac{\sigma_h}{E} \\ &= \frac{1}{E} [\sigma_l - \mu \sigma_h] \\ &= \frac{1}{100 \times 10^3} [1.25 - 0.3 \times 2.5] = 5 \times 10^{-6} \end{aligned}$$

Chapter- 10 Columns and Struts

01. Ans: (c)

$$\text{Sol: } P_e = \frac{\pi^2 \times EI}{l_e^2}$$

$$\text{For a given system, } l_e = \frac{l}{2}$$

$$\therefore P_e = \frac{4\pi^2 \times EI}{l^2}$$

02. Ans: (b)

$$\text{Sol: } \frac{P_1}{P_2} = \frac{l_{2e}^2}{l_{1e}^2}$$

$$\frac{P_1}{P_2} = \frac{l^2}{(2l)^2}$$

$$P_1 : P_2 = 1 : 4$$



03. Ans: 4

Sol: $P = \frac{\pi^2}{l^2} EI$

$\therefore P \propto I$

$$\frac{P}{P_o} = \frac{I_{\text{bonded}}}{I_{\text{loose}}} = \frac{\left[\frac{b(2t)^3}{12} \right]}{2 \left[\frac{bt^3}{12} \right]} = 4$$

04. Ans: (c)

Sol: Euler's theory is applicable for axially loaded columns

$$F \cos 45 = \frac{\pi^2}{l^2} EI$$

There fore, $F = \frac{\pi^2 EI}{\sqrt{2} L^2}$

05. Ans: (a)

Sol: Buckling load $P_e = \frac{\pi^2 EI}{L_c^2}$

Here $L_c = L = 3 \text{ m}$,

$\alpha = 12 \times 10^{-6} / ^\circ\text{C}$, $d = 50 \text{ mm} = 0.05 \text{ m}$

$$\frac{P_e L}{AE} = L \alpha \Delta T$$

$$\frac{\pi^2 EI \times L}{L^2 \times AE} = L \alpha \Delta T$$

$$\frac{\pi^2 \times E \times \frac{\pi}{64} \times d^4 \times L}{L^2 \times \frac{\pi}{4} d^2 \times E} = L \alpha \Delta T$$

$$\Rightarrow \Delta T = \frac{\pi^2 \times d^2}{16 \times L^2 \times \alpha} = \frac{\pi^2 \times (0.05)^2}{16 \times 3^2 \times 12 \times 10^{-6}} = 14.278$$

Chapter- 11 Strain Energy Resilience

01. Ans: (d)

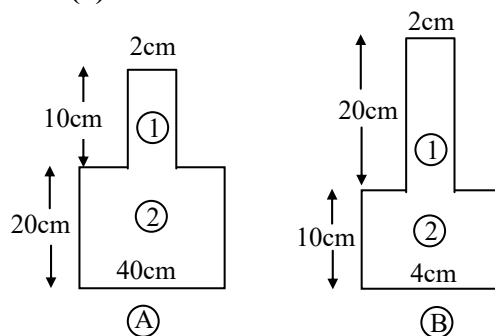
02. Ans: (b)

Sol: $U = \frac{P^2 \ell}{AE} \Rightarrow U \propto \ell$

$$\frac{U_1}{U_2} = \frac{\ell_A}{\ell_B} = 3:1$$

03. Ans: (a)

Sol:



$$\frac{U_B}{U_A} = \frac{(V_1 + V_2)_B}{(V_1 + V_2)_A}$$

$$\therefore \frac{U_B}{U_A} = \frac{\left[\frac{\sigma_1^2}{2E} \times V_1 + \frac{\sigma_2^2}{2E} \times V_2 \right]_B}{\left[\frac{\sigma_1^2}{2E} \times V_1 + \frac{\sigma_2^2}{2E} \times V_2 \right]_A}$$

$$= \frac{\left[\frac{P^2}{A_1^2} \times A_1 \times L_1 + \frac{P^2 \times A_2 \times L_2}{A_2^2} \right]}{\left[\frac{P^2 \times A_1 \times L_1}{A_1^2} + \frac{P^2 \times A_2 \times L_2}{A_2^2} \right]_A}$$



$$\therefore \frac{U_B}{U_A} = \frac{\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} \right]_B}{\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} \right]_A} = \frac{7.165}{4.77} = \frac{3}{2}$$

04. Ans: (c)

Sol: A_1 = modulus of resilience

$A_1 + A_2$ = mod of toughness

$$A_1 = \frac{1}{2} \times 0.004 \times 70 \times 10^6 = 14 \times 10^4$$

$$A_2 = \frac{1}{2} \times (0.008 \times 50 \times 10^6) + (0.008 \times 70 \times 10^6) \\ = 76 \times 10^4$$

$$A_1 + A_2 = (14 + 76) \times 10^4 = 90 \times 10^4$$

05. Ans: (d)

$$\text{Sol: } U = \frac{P^2}{2A^2E} \cdot V$$

$$U \propto P^2$$

Due to the application of P_1 & P_2 one after the other

$$(U_1 + U_2) \propto P_1^2 + P_2^2 \dots\dots\dots (1)$$

Due to the application of P_1 and P_2 together at the same time.

$$U \propto (P_1 + P_2)^2 \dots\dots\dots (2)$$

It is obvious that

$$(P_1^2 + P_2^2) < (P_1 + P_2)^2$$

$$\therefore (U_1 + U_2) < U$$

06. Ans: 1.5

$$\text{Sol: } U = U_1 + U_2 = \frac{T^2 L}{2GJ_1} + \frac{T^2 L}{2GJ_2}$$

$$J_1 = \frac{\pi}{32} (50)^4; J_2 = \frac{\pi}{32} (26)^4$$

$$L = 100 \text{ mm}$$

$$G = 80 \times 10^3 \text{ N/mm}^2 \text{ on substitution,}$$

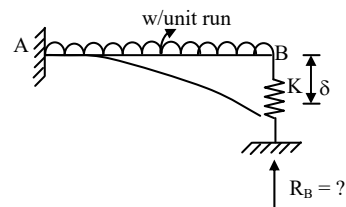
$$U = 1.5 \text{ N-mm}$$

Chapter- 12 Propped and Fixed Beams

Class Room Practice Solutions

01. Ans: (d)

Sol:



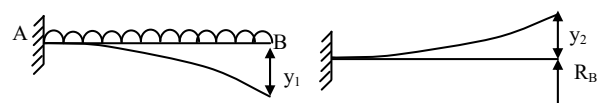
$$K = \text{Stiffness} = \frac{\text{Load}}{\text{deflection}}$$

$$\therefore K = \frac{R_B}{\delta}$$

$\therefore$ Compatibility condition

Deflection @ B = δ

$$\therefore K = \frac{R_B}{\delta} \Rightarrow \delta = \frac{R_B}{K}$$





$$y_1 = \frac{w\ell^4}{8EI} \quad y_2 = \frac{R_B \ell^3}{3EI}$$

$$y_1 = y_2 = \delta$$

$$\therefore \frac{w\ell^4}{8EI} - \frac{R_B \ell^3}{3EI} = \delta$$

$$\frac{w\ell^4}{8EI} - \frac{R_B \ell^3}{3EI} = \frac{R_B}{K}$$

$$\frac{w\ell^4}{8EI} = \frac{R_B}{K} + \frac{R_B \ell^3}{3EI}$$

$$\frac{w\ell^4}{8EI} = R_B \ell^3 \left[\frac{1}{K\ell^3} + \frac{1}{3EI} \right]$$

$$\frac{w\ell^4}{8EI} = R_B \left[\frac{3EI + K\ell^3}{3EI \times K\ell^3} \right] \times \ell^3$$

$$\frac{w\ell}{8EI} = \frac{R_B}{3EI} \left[\frac{3EI + K\ell^3}{K\ell^3} \right]$$

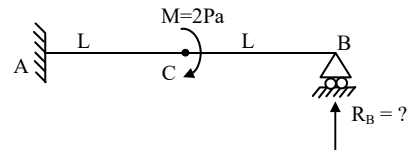
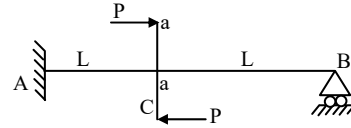
$$\frac{3w\ell}{8} = R_B \left[\frac{3EI + K\ell^3}{K\ell^3} \right]$$

$$\frac{3w\ell}{8} = R_B \left[1 + \frac{3EI}{K\ell^3} \right]$$

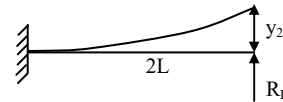
$$R_B = \frac{3w\ell/8}{1 + \frac{3EI}{K\ell^3}}$$

02. Ans: $\frac{9pa}{8L}$

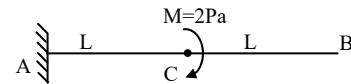
Sol:



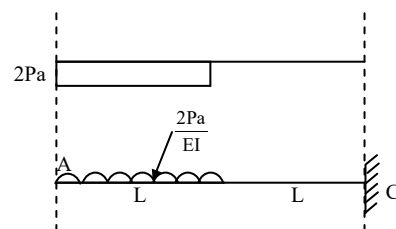
Applying, superposition principle



$$y_1 = \frac{R_B (2L)^3}{3EI} = \frac{8R_B L^3}{3EI}$$



By conjugate beam method



$\therefore y_c = \text{deflection @ C}$

= B.M.D. @ C by conjugate beam

$$y_c = \frac{2Pa}{EI} \times L \times \left[L + \frac{L}{2} \right]$$



$$= \frac{2Pa}{EI} \times L \times \frac{3L}{2}$$

$$= \frac{3PaL^2}{EI}$$

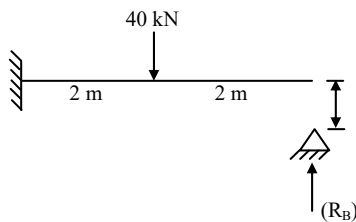
Compatibility Condition (y_B) = 0

$$\therefore y_1 = y_c$$

$$\frac{8R_B L^3}{3EI} = \frac{3PaL^2}{EI}$$

$$R_B = \frac{9Pa}{8L} (\uparrow)$$

03. Ans: 12.51 kN



$$E = 200 \text{ GPa}$$

$$I = 2 \times 10^6 \text{ mm}^4$$

As per compatibility

$$\frac{(R_B)(4000)^3}{3EI} = \frac{(40 \times 10^3)(2000)^3}{3 \times EI} + \frac{40 \times 10^3 \times (2000)^2}{2EI} \times 2000 + 1 \text{ mm}$$

$$\text{Where } EI = 4 \times 10^{11} \text{ N/mm}^2$$

$$\therefore \frac{R_B(4000)^3}{3 \times 4 \times 10^{11}} = \frac{40 \times 10^3 \times (2000)^3}{3 \times 4 \times 10^{11}} + \frac{40 \times 10^3 \times (2000)^2}{2 \times 4 \times 10^{11}} + 1$$

$$R_B = 12.51 \text{ kN}$$

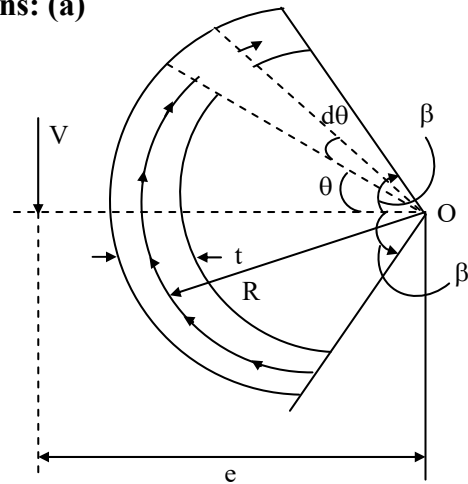
Chapter- 13 Shear Centre

Class Room Practice Solutions

01. Ans: (a)

02. Ans: (a)

Sol:



$$\text{shaded area} = R d\theta \times t$$

Net moment about O = 0

$$\Rightarrow \int_{-\beta}^{\beta} \left(\frac{q_{\theta}}{t} \times t \times R d\theta \right) \times R - ve = 0, \text{ where}$$

$$q_{\theta} = \frac{VA\bar{y}}{I}$$

$$A\bar{y} = \int y dA$$

$$A\bar{y} = \int_{\theta}^{\beta} R \sin \theta (R d\theta) \times t$$

$$A\bar{y} = R^2 t \int_{\theta}^{\beta} \sin \theta d\theta = -R^2 t (\cos \beta - \cos \theta)$$



$$\Rightarrow A\bar{y} = R^2 t (\cos \theta - \cos \beta) \text{-----(1)}$$

$$I = \int y^2 dA = 2 \int_0^\beta R^2 \sin \theta (R d\theta) t$$

$$I = 2R^3 t \int_0^\beta \sin^2 \theta d\theta = R^3 t \int_0^\beta 2 \sin^2 \theta d\theta$$

$$I = R^3 t \int_0^\beta [1 - \cos 2\theta] d\theta$$

$$I = R^3 t \left[\theta - \frac{\sin 2\theta}{2} \right]_0^\beta$$

$$I = R^3 t \left[\beta - \frac{\sin 2\beta}{2} \right]$$

$$I = R^3 t [\beta - \sin \beta \cos \beta] \text{-----(2)}$$

$$\begin{aligned} \Rightarrow q\theta &= \frac{VA\bar{y}}{I} = \frac{VR^2 t (\cos \theta - \cos \beta)}{R^3 t (\beta - \sin \theta \cos \beta)} \\ &= \frac{V(\cos \theta - \cos \beta)}{R(\beta - \sin \theta \cos \theta)} \end{aligned}$$

$$V_e = \frac{\int_{-\beta}^\beta q_\theta R^2 d\theta}{V}$$

$$V_e = \int_{-\beta}^\beta \frac{V(\cos \theta - \cos \beta)}{R(\beta - \sin \beta \cos \beta)} \times \frac{R^2 d\theta}{V}$$

$$\left(\frac{R}{\beta - \sin \beta \cos \beta} \right) \left[(\sin \theta - \theta \cos \beta) \right]_{-\beta}^\beta$$

$$\frac{R}{\beta - \sin \beta \cos \beta} [(\sin \beta - \beta \cos \beta) - (-\sin \beta + \beta \cos \beta)]$$

$$e = \left(\frac{2R}{\beta - \sin \beta \cos \beta} \right) (\sin \beta - \beta \cos \beta) \text{----(3)}$$

If $\theta = \pi$,

$$L = 2R$$

Chapter- 14 Theories of Failure

Class Room Practice Solutions

01. Ans: (d)

Sol: $f = f_y = 2500 \text{ kg/cm}^2$

$$\sigma_1 = 2000 \text{ kg/cm}^2$$

$$\sigma_3 = ?$$

Max shear stress theory (tresca, guest & coulomb)

$$\begin{aligned} \tau_{\max} &= \frac{(\sigma_1 - \sigma_3)}{2} \neq \frac{f_y}{2} \\ &= \frac{2000 - \sigma_3}{2} = \frac{2500}{2} \end{aligned}$$

$$\sigma_3 = -500 \text{ (comp)}$$

02. Ans: (b)

Sol: $D = 100 \text{ cm}$

$$P = 10 \text{ kg/cm}^2$$

$$f = f_y = 2000 \text{ kg/cm}^2$$

$$\text{FOS} = 4 \quad t = ?$$

Mx. P –stress theory

$$\sigma_1 = \sigma_h = \frac{PD}{2t} \neq f_y$$

$$\frac{10 \times 100}{2 \times t} = 2000$$

$$t = 2.5 \text{ mm}$$

$$\text{Safe allowable load} = 2.5 \times \text{F.O.S}$$

$$= 2.5 \times 4 \Rightarrow 10 \text{ mm}$$



03. Ans: (b)

Sol: $\sigma_1 = 1.55$ (T)

$$\sigma_2 = \sigma$$

$$\sigma_3 = -\sigma/2$$

$$f_y = 2000 \text{ kg/cm}^2$$

$$\mu = 0.3$$

In which theory of failure $\sigma = 1000 \text{ kg/cm}^2$

Check

(a) Max principal stress theory

$$\sigma_1 = f_y$$

$$1.5\sigma_1 = 2000$$

$$\sigma_1 = 1333 \text{ kg/cm}^2$$

(b) Max shear stress theory

$$\left(\frac{\sigma_1 - \sigma_3}{2} \right) = \frac{f_y}{2}$$

$$\left(\frac{1.5\sigma + \frac{\sigma}{2}}{2} \right) = \frac{2000}{2}$$

$$\frac{4}{2} = 2000$$

$$\sigma = 1000 \text{ kg/cm}^2$$

04. Ans: (c)

Sol: $\sigma_1 = 800 \text{ kg/cm}^2$

$$\sigma_2 = 400 \text{ kg/cm}^2$$

$$\epsilon_1 \text{ \& } f_y/E$$

$$\frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \frac{\mu \sigma_3}{E} = \frac{f_y}{E}$$

$$\frac{800}{E} - 0.25 \frac{(400)}{E} = \frac{f_y}{E}$$

$$f_y = 800 - 100 \Rightarrow 700 \text{ kg/cm}^2$$