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SIGNALS & SYSTEMS

Volume - I : Study Material with Classroom Practice Questions

Classroom Practice solutions
To
Signals & Systems

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Chapter 1 Introduction

Class Room Practice Solutions

01. Ans: (c)

Sol: $x(n) + 2x(-n) = \{-1,-1,3,1,1\}$ max value = 3

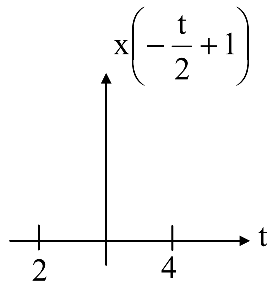
$5x(n)x(n-1) = \{0,5,5,-5,5,0\}$ max value = 5

$x(n)x(-n-1) = \{0,-1,1,1,-1,0\}$ max value = 1

$4x(2n) = \{4,4,-4\}$ max value = 4

02. Ans: (a)

Sol:



Non zero duration = 6

03.

Sol: (a) 0

(b) $(t + \cos\pi t)|_{t=1} = 0$

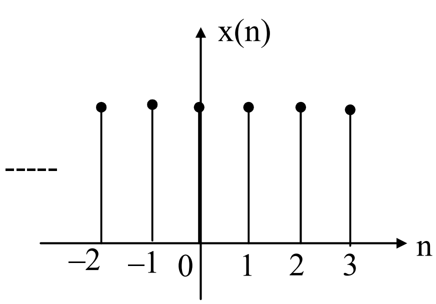
(c) $\cos t.u(t-3)|_{t=0} = 1u(-3) = 0$

(d) $\left.\frac{1}{2}e^{t-2}\right|_{t=2} = \frac{1}{2}$

(e) $t\sin t|_{t=\frac{\pi}{2}} = \frac{\pi}{2}$

04.

Sol: $x(n) = 1 - [\delta(n-4) + \delta(n-5) + \text{-----}]$

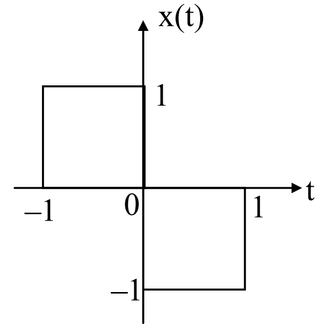


$x(n) = u(-n+3) = u(Mn-n_0)$

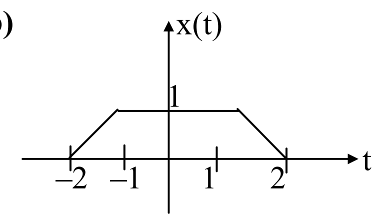
$M = -1$ $n_o = -3$

05...

Sol: (a)



(b)



06.

Sol: (a) as $t \to \infty$, amp $\to 0$ Energy signal

(b) Constant amp – Power signal

(c) Power + energy = Power signal

(d) Periodic signal $\to$ Power signal

(e) as $t \to \infty$, amp$\to\infty$ NENP

(f) as $t \to\infty$, amp$\to \infty$ NENP

07.

Sol:

$$E_{x_1(n)} = \sum_{n=-\infty}^{\infty}|x_1(n)|^2 = \sum_{n=0}^{\infty}\left(\alpha(0.5)^n\right)^2 = \sum_{n=0}^{\infty}\alpha^2(0.25)^n$$

$$= \alpha^2\sum_{n=0}^{\infty}(0.25)^n = \frac{\alpha^2}{1-0.25} = \frac{\alpha^2}{0.75}$$

$$E_{x_2(n)} = \sum_{n=-\infty}^{\infty}|x_2(n)|^2 = 1.5+1.5 = 3$$

Given $E_{x_1(n)} = E_{x_2(n)}$

$$\frac{\alpha^2}{0.75} = 3$$
$$\alpha^2 = 2.25$$
$$\alpha = 1.5$$

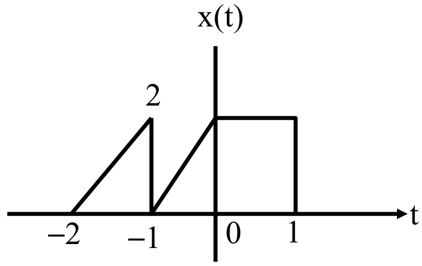
08.

Sol: $x_{oc}(n) = \dfrac{x(n) - x^*(-n)}{2}$

$$= \left[\frac{1+j7}{2}, 0, \frac{-1+j7}{2}\right]$$

09.

Sol:



10.

Sol: (a) $T_1 = \dfrac{1}{9}, T_2 = \dfrac{1}{6}$

$\dfrac{T_1}{T_2} = \dfrac{2}{3}$ LCM = 3

$T_0 = \text{LCM} \times T_1 = 1/3$

(b) $T_1 = \dfrac{15}{11}, T_2 = 15$

$\dfrac{T_1}{T_2} = \dfrac{1}{11}$

LCM = 11

$T_0 = \text{LCM} \times T_1 = 15$

(c) $T_1 = \dfrac{2\pi}{3}, T_2 = \dfrac{2}{5}$

$\dfrac{T_1}{T_2} = \dfrac{5\pi}{3}$ irrational number

So a non-periodic.

(d) $T_0 = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$

(e) It is extending from 0 to ∞
So non-periodic

(f) $x_e(t) = \dfrac{x(t) + x(-t)}{2} = \dfrac{1}{2}\cos 2\pi t$

$T_0 = \dfrac{2\pi}{\omega_0} = \dfrac{2\pi}{2\pi} = 1$

(g) $\dfrac{\omega_0}{2\pi} = \dfrac{5}{6}$ - rational, so periodic

$N_0 = \dfrac{2\pi}{\omega_0} m = \dfrac{6}{5} m$

$N_0 = 6$

(h) $N_1 = 8m \Rightarrow N_1 = 8$
$N_2 = 16m \Rightarrow N_2 = 16$
$N_3 = 4m \Rightarrow N_3 = 4$
$\dfrac{N_1}{N_2} = \dfrac{1}{2}, \dfrac{N_1}{N_3} = 2$
LCM = 2
$N_0 = \text{LCM} \times N_1 = 16.$

(i) $\dfrac{\omega_0}{2\pi} = \dfrac{7}{2}$ - rational, so periodic

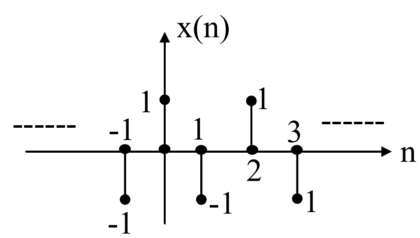
$N_o = \dfrac{2\pi}{\omega_0} m = \dfrac{2}{7} m$

$N_0 = 2$

(j) multiplication of one periodic & non-periodic is non-periodic

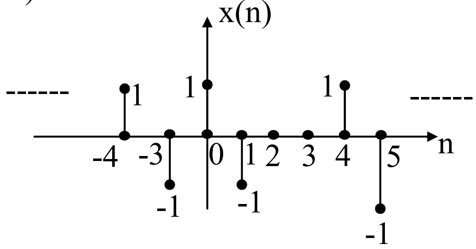
(k) $u(n) + u(-n) = 1 + \delta(n)$ is non-periodic

(l)



$N_0 = 2$

(m)



$N_0=4$

11.

Sol:

(A) $x(nT_s) = 2\cos(150\times\pi\times n\times T_s + 30°)$

$$= 2\cos\left(\frac{3\pi}{4}n + 30^\circ\right)$$

$$\omega_0 = \frac{3\pi}{4}$$

$$N_0 = \frac{2\pi}{\omega_0}m = \frac{8}{3}m$$

$$N_0 = 8$$

(B) Ans: (a)

$$N_1 = \frac{2}{3}m \Rightarrow N_1 = 2$$

$$N_2 = \frac{2}{7}m \Rightarrow N_2 = 2$$

$$N_3 = \frac{20}{25}m \Rightarrow N_3 = 4$$

$$\frac{N_1}{N_2} = 1, \frac{N_1}{N_3} = \frac{1}{2}, \text{ LCM} = 2$$

$$N_0 = \text{LCM} \times N_1 = 4$$

$$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$x(n) = \cos(6\omega_0 n) + \sin(14\omega_0 n) + \cos(5\omega_0 n)$$

so 14^{th} harmonic

12.

Sol: (a) $[x_1(t)+x_2(t)][x_1(t-2)+x_2(t-2)]$

$$\neq x_1(t)x_1(t-2)+x_2(t)x_2(t-2)$$

is non linear

(b) $\sin[x_1(t)+x_2(t)] \neq \sin[x_1(t)]+\sin[x_2(t)]$

is non linear

(c) $\frac{d}{dt}[\alpha x_1(t)+\beta x_2(t)] = \frac{\alpha dx_1(t)}{dt}+\frac{\beta dx_2(t)}{dt}$

is linear

(d) $2[x_1(t)+x_2(t)]+3 \neq 2[x_1(t)+x_2(t)]+6$

is non linear

(e) $\int_{\infty}^{t}[\alpha x_1(t)+\beta x_2(t)]dt$

$$= \alpha\int_{\infty}^{t} x_1(t)dt + \beta\int_{\infty}^{t} x_2(t)dt \text{ is linear}$$

(f) $[x_1(t)+x_2(t)]^2 \neq x_1^2(t)+x_2^2(t)$

is non linear

(g) $[\alpha x_1(t)+\beta x_2(t)]\cos\omega_0 t$

$= \alpha x_1(t)\cos\omega_0 t + \beta x_2(t)\cos\omega_0 t$ is linear

(h) $\log[x_1(n)+x_2(n)] \neq \log[x_1(n)]+\log[x_2(n)]$

is non linear

(i) $|x_1(n)+x_2(n)| \neq |x_1(n)|+|x_2(n)|$

is non linear

(j) $\alpha^* x^*(n) \neq \alpha x^*(n)$ is non linear

(k) non linear

(l) $\frac{x_1(n)+x_2(n)}{x_1(n-1)+x_2(n-1)} \neq \frac{x_1(n)}{x_1(n-1)}+\frac{x_2(n)}{x_2(n-1)}$

is non linear

(m) linear

(n) $e^{x_1(n)+x_2(n)} \neq e^{x_1(n)}+e^{x_2(n)}$ is non linear

13.

Sol: (a) $tx(t-t_o)+3 \neq (t-t_o)x(t-t_o)+3$
time variant

(b) $e^{x(t-t_o)} = e^{x(t-t_o)}$ time invariant

(c) $x(t-t_0)\cos 3t \neq x(t-t_0)\cos 3(t-t_0)$
time variant

(d) sin [x(t–t_0)] = sin[x(t–t_0)] time invariant

(e) $\frac{d[x(t-t_0)]}{d(t-t_0)} = \frac{dx(t-t_0)}{dt-dt_0} = \frac{d}{dt}[x(t-t_0)]$
time invariant

(f) $x^2(t-t_0) = x^2(t-t_0)$ time invariant

(g) $x(2t - t_0) \neq x(2t - 2t_0)$ time variant

(h) $2^{x(n-n_0)}x(n-n_0) = 2^{x(n-n_0)}x(n-n_0)$
time invariant

(i) time variant

(j) time variant
(k) all coefficients are constant
– time invariant

14.

Sol: $x_2(t) = x_1(t) - x_1(t-2)$
$y_2(t) = y_1(t) - y_1(t-2)$
$x_3(t) = x_1(t+1) + x_1(t)$
$y_3(t) = y_1(t+1) + y_1(t)$

15.

Sol: (a) Preset output depends on present input-causal
(b) preset output depends on present input-causal
(c) preset output depends on present input-causal
(d) preset output depends on future input-non causal
(e) preset output depends on present input-causal
(f) preset output depends on present input-causal
(g) $n > n_0$ causal, $n < n_0$ non-causal
(h) non - causal
(i) $y(0) = \sum_{k=-\infty}^{0} x(k)$ present output depends on present input - causal
(j) $y(-1) = \sum_{k=0}^{-1} x(k)$ future input non causal
(k) non-causal for any value of 'm'
(l) $\alpha = 1$ causal, $\alpha \neq 1$ non causal
(m) causal
(n) non causal

16.

Sol: (a) present output depends on present input -static
(b) present output depends on present input -static
(c) present output depends on present input -static
(d) present output depends on present input -static
(e) y(1) = x(3) present output depends on future input -dynamic
(f) dynamic
(g) present output depends on past input - dynamic

17.

Sol: (a) linear, time variant, dynamic

(b) linear, time invariant, dynamic

(c) linear, time invariant, dynamic

(d) non linear, time variant, dynamic

18.
Sol: (a) linear, time invariant, dynamic (a→2)

(b) non linear, time variant, static (b→5)

(c) linear, time variant, dynamic (c→1)

(d)nonlinear, time invariant, dynamic(d→4)

19.
Sol: (a) y(t) = u(t).u(t) = u(t) - stable

(b) y(t) = cos3t u(t) ⇒ $-1 < y(t) < 1$ stable

(c) y(t) = u(t–3) stable

(d) $y(t) = \frac{du(t)}{dt} = \delta(t)$ unstable

(e) $y(t) = \int_{-\infty}^{t} u(\tau)d\tau \Rightarrow r(t)$ is unstable

(f) sin (finite) = finite. stable

(g) y(t) = tu(t) = r(t) unstable

(h) $y(n) = e^{finite}$ = finite stable

(i) y(n) = u(3n) bounded stable

(j) x(n) = 1 ⇒ $y(n) = n - n_0 + 1 \Rightarrow y(\infty) = \infty$
⇒ unstable

20.
Sol: (a) non invertible

(b) non invertible

(c) invertible

(d) non invertible

(e) non invertible

(f) non invertible

(g) non invertible

(h) invertible

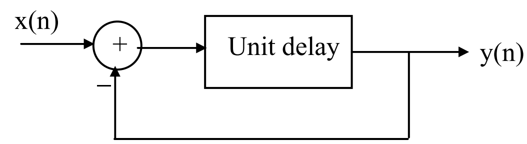
21. Ans: (b)
Sol: $y(t) = x^2(t) * h(t)$

h(t) = 0, t<0 causal

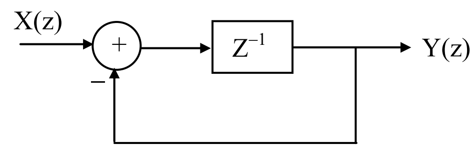
Square term - non linear

no time varying term- time invariant

22.
Sol: Given



Convert to Z-domain



$$\frac{Y(z)}{X(z)} = \frac{z^{-1}}{1+z^{-1}} = \frac{1}{z+1}$$

(i) x(n) = δ (n);

$$\Rightarrow Y(z) = \frac{1}{z+1}X(z)$$

$$Y(z) = \frac{1}{z+1}1 = \frac{1}{z+1}$$

$$Y(z) = z^{-1}\frac{z}{z+1}$$

Taking inverse Z – transform

$y(n) = (-1)^{n-1}$ u (n–1)

if n = 0, 1, 2, 3…….

Then y(n) = [0, 1, –1, 1, –1…….]

(ii) x(n) = u(n);

$$\Rightarrow Y(z) = \frac{1}{z+1}X(Z)$$

$$Y(z) = \frac{1}{z+1}\frac{z}{z-1}$$

$$\frac{Y(z)}{z} = \frac{1}{(z+1)(z-1)}$$

$$= \frac{A}{z+1} + \frac{B}{z-1}$$

$$= \frac{-\frac{1}{2}}{z+1} + \frac{\frac{1}{2}}{z-1}$$

$$Y(z) = -\frac{1}{2}\frac{z}{z+1} + \frac{1}{2}\frac{z}{z-1}$$

$$y(n) = -\frac{1}{2}(-1)^n u(n) + \frac{1}{2}u(n)$$

23. Ans: (b)

Sol: Constant added - non linear (A-true)

Time varying term - time variant(R-true)

24. Ans: (d)

Sol: (S-I): y(n) = 2 x(n) + 4 x(n − 1)

If x(n) is bounded, y(n) is bounded.

∴ Stable. (S-I) is false.

(S-II): h(n) = 2 δ(n) + 4 δ(n − 1)

$$h(n) = \{ \underset{\uparrow}{2}, 4 \}$$

Impulse response h(n) has only two finite nonzero samples. This is the condition for stability.

∴ (S-II) is True.

25. Ans: (a)

Sol: A system is memory less if output, y(t) depends only on x(t) and not on past or future values of input, x(t).

A system is causal if the output, y(t) at any time depends only on values of input, x(t) at that time and in the past.

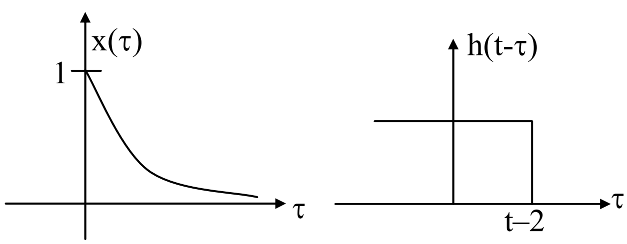
Both (S-I) and (S-II) are true and (S-II) is the correct explanation of (S-I).

Chapter 2 LTI (LSI) Systems

Class Room Practice Solutions

01.

Sol: $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$



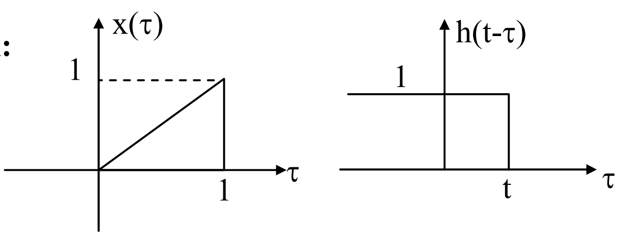
Case (i) t–2 <0 y(t) = 0 ,t<2

Case (ii) t–2>0 $y(t) = \int_{0}^{t-2} e^{-3\tau}d\tau = \frac{1-e^{-3(t+2)}}{3}$,t >2

$$y(t) = \frac{1-e^{-3(t+2)}}{3}u(t-2)$$

02.

Sol:



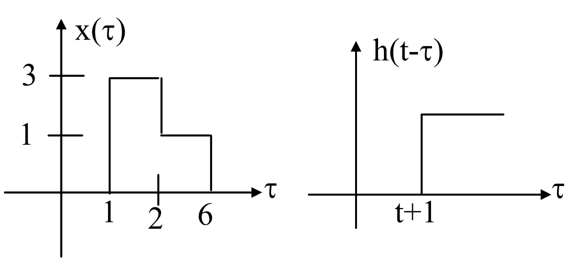
Case (i) t < 0 y(t) = 0

Case (ii) 0<t<1 $y(t) = \int_{0}^{t} \tau d\tau = \frac{t^2}{2}$

Case (iii) t>1 $y(t) = \int_{0}^{1} \tau d\tau = \frac{1}{2}$

03.

Sol:



$y(4) = \int_{6}^{5} 1d\tau = 1$

$y\left(\frac{1}{2}\right) = \int_{1.5}^{6} x(\tau)h\left(\frac{1}{2}-\tau\right)d\tau = \frac{3}{2}+4 = 5.5$

04. Ans: (b)

Sol: $s(t) = \int_{-\infty}^{t} h(\tau)d\tau = u(t-1)+u(t-3)$

s(2) =1

05.

Sol: Assume $-\tau+a = \lambda \Rightarrow -d\tau = d\lambda$

$z(t) = \int_{-\infty}^{\infty} x(\lambda)h(t+a-\lambda)d\lambda = y(t+a)$

06.

Sol: (a) x(t – 7 + 5) = x(t – 2)

(b) $x(t) * \frac{1}{|a|}\delta\left(t+\frac{b}{a}\right) = \frac{1}{|a|}x\left(t+\frac{b}{a}\right)$

(c) $x(t) * [2\delta(t+3)+2\delta(t-3)]$

$= 2x(t+3)+2x(t-3)$

07.

Sol:

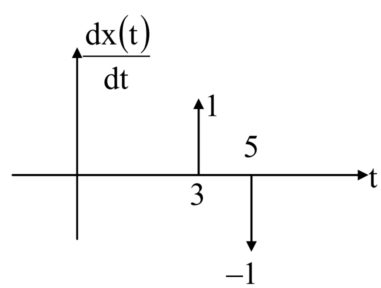
(a) $e^{-1}u(1)\delta(t-1) = e^{-1}\delta(t-1)$

(b) $e^{-t}\big|_{t=1} = e^{-1}$

(c) $e^{-(t-1)}u(t-1)$

08.

Sol:



$$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$$

$$\frac{dx(t)}{dt} * h(t) = h(t-3) - h(t-5)$$

09.

Sol: (a) $A_x\, A_h = A_y$, $\int_{-\infty}^{\infty} \delta(\alpha t)dt = \frac{1}{\alpha}$

$$\frac{1}{\alpha}.\frac{1}{\alpha} = \frac{A}{\alpha}$$

$$A = \frac{1}{\alpha}$$

(b) $\frac{1}{\alpha}.\frac{1}{\alpha} = \frac{A}{\alpha}$, $\int_{-\infty}^{\infty} \sin c(\alpha t)\, dt = \frac{1}{\alpha}$

$$A = \frac{1}{\alpha}$$

(c) $1 \times 1 = A\sqrt{2}$ $\int_{-\infty}^{\infty} e^{-at^2} dt = 1$

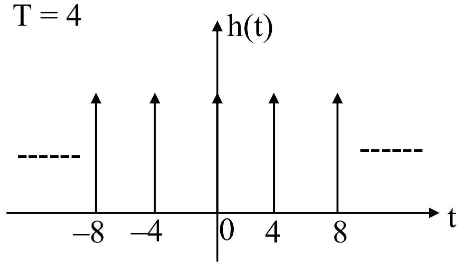
$$A = \frac{1}{\sqrt{2}}$$

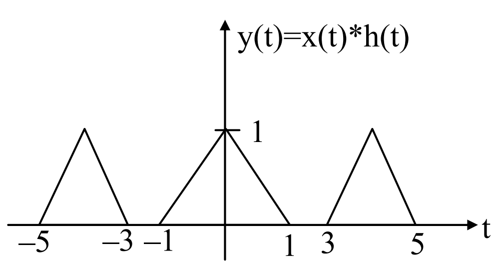
(d) $\pi \times \pi = 2A\pi$ $\int_{-\infty}^{\infty} \frac{1}{1+t^2} dt = \pi$

$$A = \frac{\pi}{2}$$

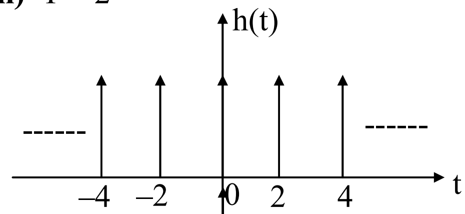
10.

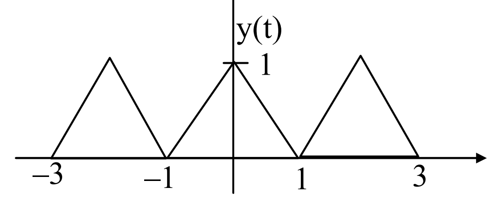
Sol: **(i)** T = 4





(ii) T = 2

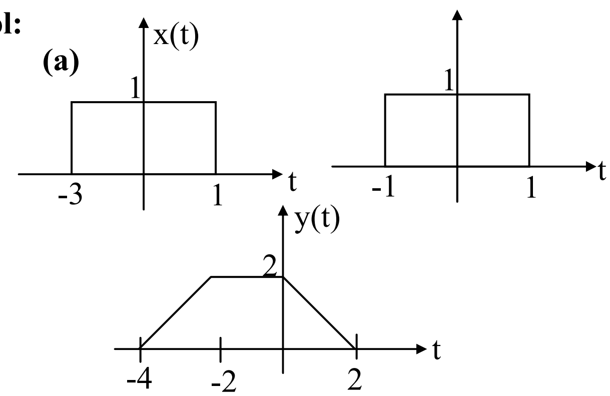




11.

Sol:

(a)

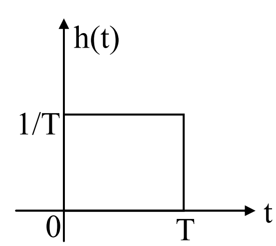


(b) Ans: (c)

$$tu(t) * u(t-1) \leftrightarrow \frac{1}{s^2}\frac{e^{-s}}{s}$$

$$= \frac{e^{-s}}{s^2} \leftrightarrow \frac{1}{2}(t-1)^2 u(t-1)$$

(c)



$$h(t) = \frac{1}{T}[u(t) - u(t-T)]$$

x(t) = u(t)

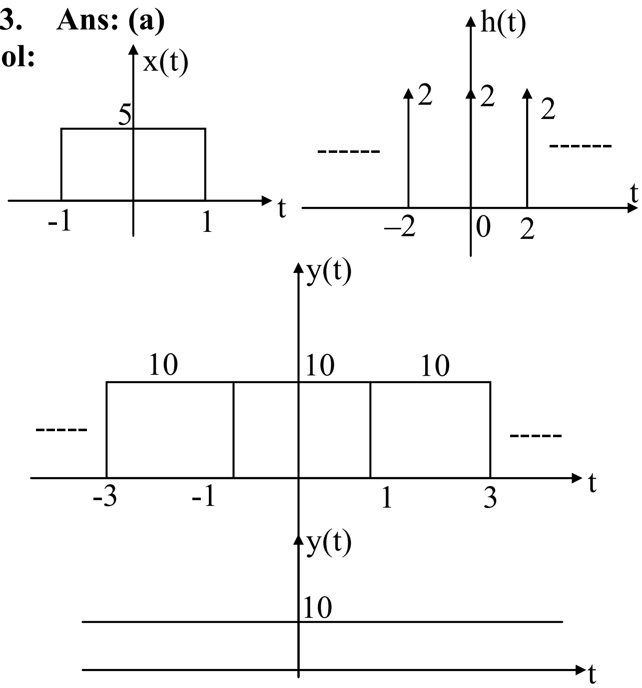
$$y(t) = x(t) * h(t) = \frac{1}{T}[r(t) - r(t - T)]$$

12. Ans: (a)

Sol: To get three discontinuities in y(t) both rectangular pause must be same width. To get equal width h(t) = x(t). it is possible only
$\alpha = 1$

13. Ans: (a)

Sol:



14. Ans: (d)

Sol: $x(t) * h(-t) = \int_{-\infty}^{\infty} x(\tau)h(-(t-\tau))d\tau$

$$= \int_{-\infty}^{\infty} x(\tau)h(\tau - t)d\tau$$

15.

Sol: $y(n) = ---+ x(-2)g(n+4) + x(-1)g(n+2)$
$+ x(0)g(n) + x(1)g(n-2) + x(2)g(n-4) + ---$

x(n) = δ (n–2) = 1   n = 2
= 0   otherwise

y(n) = g(n – 4)

16.

Sol: y(n) = x(n)*h(n)
$= 2(0.5)^n u(n) + (0.5)^{n-3} u(n-3)$
y(1) = 1, y(4) = 5/8

17. Ans: (a)

Sol: y(n) = [a, b, c, d, a, b, c, d---- N times]

y(n) is a periodic function with periodic '4'.

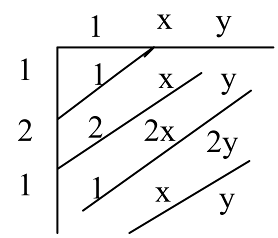
So h(n) must be $h(n) = \sum_{i=0}^{N-1} \delta(n - 4i)$

18. Ans: 31

Sol: x(n) = {1,2,1}

h(n) = {1,x,y}

y(n) = x (n) * h(n)



y(n) = { 1, 2+x, 2x + y + 1, x + 2y, y}
y(1) = 3 = 2+x ⇒ x = 1
y(2) = 4 = 2x+y+1 ⇒ y = 1
y(n) = {1, 3, 4, 3, 1}
10 y (3) + y(4) = 10×3 + 1 = 31

19. Ans: (d)

Sol: $\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=0}^{\infty} a^n + \sum_{n=-\infty}^{-1} b^n < \infty$

only when |a| < 1, |b|>1

20. Ans: (b)

Sol: $\int_{-\infty}^{\infty} |h(t)dt| = \int_{0}^{\infty} e^{\alpha t}dt + \int_{-\infty}^{0} e^{\beta t}dt < \infty$ only when

$\alpha < 0, \beta > 0$

21.

Sol: **(a)** $h(n) = \alpha^n u(n) + \beta\, \alpha^{n-1} u(n-1)$

(b) h(n) = 0 n<0 causal

System stable for any value of ‘β’

except $\beta \neq \infty$ and $|\alpha| < 1$, except $\alpha = 0$

22.

Sol: **(a)** $\left(\frac{1}{5}\right)^n u(n) - A\left(\frac{1}{5}\right)^{n-1} u(n-1) = \delta(n)$

When n =1, A = 1/5

(b) $H(z) = \dfrac{1}{1-\frac{1}{5}z^{-1}}$

$H_{inv}(z) = 1 - \frac{1}{5}z^{-1}$

$g(n) = \delta(n) - \frac{1}{5}\delta(n-1)$

23.

Sol: $h_1(n) = \delta(n) - \frac{1}{2}\delta(n-1)$

$$h_1(n) * h_2(n) = \left(\frac{1}{2}\right)^n u(n) - \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} .u(n-1)$$

$$= \left(\frac{1}{2}\right)^n \delta(n) = \delta(n)$$

24.

Sol: 1. False

2. $h(t) = e^{2t}u(t-1)$ is causal, un stable - false

3. $h(t) = \sin\omega_0 t,\ \int_{-\infty}^{\infty}|h(t)|dt = \int_{-\infty}^{\infty}|\sin\omega_0 t|dt = \infty$

unstable - true

4. y(t) = x(t–2) → causal-- false

x(t) = y(t+2) → non causal ----false

25. Ans: (a)

Sol: $s(t) = u(t) - e^{-\alpha t}u(t)$

$$h(t) = \frac{ds(t)}{dt} = \delta(t) - \left[e^{-\alpha t}\delta(t) - \alpha e^{-\alpha t}u(t)\right]$$

$$= \alpha e^{-\alpha t}u(t)$$

26.

Sol: $s(n) = \sum_{k=-\infty}^{n} h(k) = \sum_{k=-\infty}^{n}\left(\frac{1}{2}\right)^k u(k)$

$$= \sum_{k=0}^{n}\left(\frac{1}{2}\right)^k \quad n \geq 0$$

$$= 0 \quad n < 0$$

$$s(n) = 2\left[1 - \left(\frac{1}{2}\right)^{n+1}\right]u(n)$$

27.

Sol: x(n) = u(n), y (n) = δ(n)

u(n) – u(n–1) = δ(n)

y(n) = x(n) – x(n–1)

x(n) = nu(n)

y(n) = nu(n) – nu(n–1) + u(n–1)

= nδ(n) + n(n–1)

= u(n–1)

28.

Sol: $h_c(t) = h_1(t) * h_2(t)$

$$\int_{-\infty}^{t} h_c(\tau)d\tau = \int_{-\infty}^{t} h_1(\tau)\,d\tau * h_2(\tau)$$

$$= h_1(\tau) * \int_{-\infty}^{t} h_2(\tau)d\tau$$

$s_c(t) = s'(t) * s_2(t)$

$= s_1(t) * s'_2(t)$

$s_c(t) \neq s_1(t) * s_2(t)$

Chapter 3 Fourier Series

Class Room Practice Solutions

01. Ans :Zero

Sol: $T_1 = \frac{\pi}{2}, T_2 = \frac{\pi}{6}$

$\frac{T_1}{T_2} = 3$, $T_0 =$ LCM $\times T_1 = \frac{\pi}{2}$

$\omega_0 = 4$

$x(t) = 3\sin(\omega_0 t + 30°) - 4\cos(3\omega_0 t - 60°)$

second harmonic amplitude = 0

02. Ans: (d)

Sol: (a) periodic – exists

(b) periodic – exists

(c) periodic – exists

(d) non-periodic – not exist

03.

Sol:

(P) **Ans: (b)**
Hidden symmetry a_0, b_n exists

(Q) **Ans: (b)**
Half wave symmetry a_n, b_n exists with odd harmonics

(R) **Ans: (b)**
Odd symmetry & HWS $\rightarrow$ sine terms with odd 'n'

(S) **Ans: (c)**
Even and odd HWS $\rightarrow$ a_0, cosine with odd 'n'

(T) **Ans: (d)**
$a_0 = 0$ (because average value = 0)
Even & HWS as cosine with odd 'n'

04. Ans: (b)

Sol: $f_1 = 5$Hz, $f_2 = 15$Hz

$p = \frac{(4)^2}{2} = 8$ Watts

05. Ans: (b)

Sol: $x(t) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - - -$

$= \tan^{-1}(1) = \frac{\pi}{4}$

06. Ans: (c)

Sol: $\omega = \frac{2\pi}{T}(2k)$, k = 1, 2, ……

The above frequency terms are absent. The above frequency contains even harmonics and also gives that sin terms are absent. only cosine terms are present Finally odd harmonics with cosine terms are present so, x(t) it is a even and halfwave so,

$x(t) = x(T-t)$ even

$x(t) = -x(t-T/2)$ halfwave

07.

(A) Ans: (a)

Sol: $T_1 = 1$, $T_2 = 10\pi$, $T_3 = 8\pi$, $T_4 = \frac{20}{3}\pi$

$T_0 = 40\pi$

$\omega_0 = \frac{2\pi}{T_0} = 0.05 \text{rad/sec}$

08. Ans: (a)

Sol: $\text{Average value} = \frac{\frac{1}{2}(2)(1) + (1)(1) + (1)(3)}{6} = \frac{5}{6}$

09. Ans: (a)

Sol: a_0 = Average value = 0

10. Ans: (d)

Sol: $T_0 = 4$msec $f_0 = \frac{1}{T_0} = 250$Hz

$5 f_0 = 1250$ Hz

11. Ans: (b)

Sol: Odd + HWS $\rightarrow$ sine terms with odd harmonics

12. Ans: (d)

Sol: $a_0 = \frac{1}{2}\int_{-1}^{1} e^{-t}dt = \frac{e - e^{-1}}{2}$

13. Ans: (c)

Sol: Average value $= \frac{1}{2\pi}\int_0^{\pi} 10\sin t\,dt = \frac{10}{\pi}$

$$a_1 = \frac{2}{2\pi}\int_0^{\pi} 10\sin t\cos t\,dt = 0$$

$$b_1 = \frac{2}{2\pi}\int_0^{\pi} 10\sin t\sin t\,dt = 5$$

$$d_1 = \sqrt{a_1^2 + b_1^2} = 5$$

14. Ans: (d)

Sol: $\omega_0 = \pi$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n\cos(n\pi t) + b_n\sin(n\pi t)$$

$x(t) = A\cos(\pi t)$

$$A = a_1 = \frac{-4}{\pi^2}$$

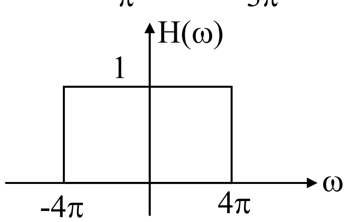
15.

Sol: $a_0 = 5$

$$b_n = \int_0^1 10\sin n\pi t\ dt = \frac{10(1-(-1)^n)}{n\pi}$$

$a_n = 0$

$$x(t) = 5 + \frac{20}{\pi}\sin\pi t + \frac{20}{3\pi}\sin 3\pi t + ---$$



$$y(t) = 5 + \frac{20}{\pi}\sin\pi t + \frac{20}{3\pi}\sin 3\pi t \ -----$$

16.

Sol: $\omega_0 = \frac{\pi}{3}$

$$x(t) = 2 + \cos(2\omega_0 t) + 4\sin(5\omega_0 t)$$

$C_0 = 2$, $C_2 = 1/2$, $C_{-2} = \frac{1}{2}$ $C_5 = \frac{4}{2j}$ $C_{-5} = \frac{-4}{2j}$

17.

Sol: $C_n = \int_0^1 t\,e^{-jn\omega_0 t}dt = \int_0^1 te^{-jn2\pi t}dt = \frac{j}{2n\pi}$

$C_0 = 1/2$

$$a_n = c_n + c_{-n} = 0$$

$$b_n = j(c_n - c_{-n}) = \frac{-1}{4\pi}$$

18.

Sol: (i) $y(t) \Rightarrow d_n = e^{-jn\omega_0}c_n = e^{-jn\pi}c_n = c_n(-1)^n$

(ii) $f(t) = x(t) - y(t)$

$$d_n = c_n - (-1)^n c_n = c_n\left[1 - (-1)^n\right]$$

(iii) $g(t) = x(t) + y(t)$

$$d_n = c_n[1 + (-1)^n]$$

19. Ans: (b)

Sol: $d_n = e^{-jn\omega_0 t_0}c_n + e^{jn\omega_0 t_0}c_n = 2\cos(n\omega_0 t_0)c_n$

$$t_0 = \frac{T}{4}$$

$$d_n = 2c_n\cos\left(\frac{n\pi}{2}\right)$$

$d_n = 0$ for odd harmonics

20.

Sol: $y(t) = \frac{dx(t)}{dt}$

$d_n = jn\omega_0 c_n$

$$c_n = \frac{d_n}{jn\omega_0}$$

$$d_n = \frac{1}{T}\int_{-T/2}^{T/2}\left(\delta(t + d/2) - \delta(t - d/2)\right)e^{-jn\omega_0 t}dt$$

$$= \frac{2j}{T}\sin\left(\frac{n\omega_0 d}{2}\right)$$

$$C_0 = \frac{d}{T}$$

21. Ans: (a)

Sol: $C_n = \frac{1}{T}\int_{-\tau/2}^{\tau/2} Ae^{-jn\omega_0 t}dt = \frac{2A}{n\omega_0 T}\sin\left(\frac{n\times 2\pi\times\tau}{2T}\right)$

T = 10sec, τ = 0.2 sec

$C_n = \frac{2A}{n\omega_0 T}\sin\left(\frac{n\pi}{50}\right)$

n = 50 then $C_n = 0$

22. Ans: (c)

23.

Sol: (a) $x(3t) \rightarrow C_n$, $\omega_0 = 3\omega_0$

(b) $\frac{dx(t)}{dt} \rightarrow jn\omega_0 C_n$

(c) $x(t-1) \rightarrow e^{-jn\omega_0}.C_n$

(d) $\text{Re}[x(t)] \rightarrow \frac{c_n + c_{-n}^*}{2} = \text{Even}(C_n)$

(e) $\frac{C_{n-4} + C_{n+4}}{2}$

24. Ans: 0.45W

Sol $C_0 = a_0 = \frac{1}{2}$ $C_n = \frac{a_n - jb_n}{2} = \frac{-j}{n\pi}(\text{odd}n)$

$a_n = 0, b_n = \frac{20}{n\pi}(\text{odd}n)$

Power up to second harmonic

$= \sum_{n=-2}^{2}|C_n|^2 = 0.45\,\text{W}$

25.

Sol: (a) All periodic signals are power signals. For power signal E = ∞ [given is false]

(b) $C_0 = j2$ (average value) [given is false]

(c) $\frac{j}{T}\int_0^T x_I(t)dt = j2$

$\frac{1}{T}\int_0^T x_I(t)dt = 2$ is possible only when $x_I(t)$ is constant. So given is correct

(d) $C_0 = \frac{1}{T}\int_0^T x_R(t)dt + \frac{j}{T}\int_0^T x_I(t)dt$

$= 0 + j2$

$\frac{1}{T}\int_0^T x_R(t)dt = 0$ only when $x_R(t)$ is odd

given is in correct

26.

Sol:

(a) Power $= \frac{1}{T}\int_{-\infty}^{\infty}|x(t)|^2 dt = \sum_{n=-\infty}^{\infty}|C_n|^2$

$P = \sum_{x=-4}^{4}|C_n|^2$

$= (0.5)^2+(1)^2+(2)^2+(4)^2+(2)^2+(1)^2+(0.5)^2$

= 26.5 Watts

(b) $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

$= C_{-4}e^{-j4\omega_0 t} + C_{-3}e^{-j3\omega_0 t}e^{-\frac{j\pi}{2}} + C_{-2}e^{-j2\omega_0 t}e^{-\frac{j\pi}{4}} + C_{-1}e^{-j\omega_0 t}$

$+ C_0 + C_1 e^{j\omega_0 t} + C_2 e^{j2\omega_0 t}e^{\frac{j\pi}{4}} + C_3 e^{j3\omega_0 t}e^{\frac{j\pi}{2}} + C_4 e^{j4\omega_0 t}$

$= 0.5e^{-j4\omega_0 t} + 1e^{-j3\omega_0 t - \frac{\pi}{2}}$

$+ 2e^{-j2\omega_0 t - \frac{\pi}{4}}\ 0.5e^{j4\omega_0 t} + 1e^{j3\omega_0 t + \frac{\pi}{2}} + 2e^{j2\omega_0 t + \frac{\pi}{4}}$

$= (0.5)\left[e^{-j4\omega_0 t} + e^{j4\omega_0 t}\right] + 2\left[e^{-j2\omega_0 t - \frac{\pi}{4}} + e^{j2\omega_0 t + \frac{\pi}{4}}\right]$

$\left[e^{-j3\omega_0 t - \frac{\pi}{2}} + e^{j3\omega_0 t + \frac{\pi}{2}}\right] + 4$

$\Rightarrow x(t) = \cos 4\omega_0 t + 4\cos\left(2\omega_0 t + \frac{\pi}{4}\right)$

$+ 2\cos\left(3\omega_0 t + \frac{\pi}{2}\right) + 4x^*(-t)$

$= x(t)$

So even symmetry

(d) $f_0 = 10$ Hz

$\omega_0 = 2\pi\, f_0 = 20\,\pi$ rad

$$x(t) = \cos(80\pi t) + 4\cos\left(40\pi t + \frac{\pi}{4}\right) + 2\cos\left(60\pi t + \frac{\pi}{2}\right) + 4$$

Cut off frequency = 25 Hz
= 50 π rad

So output of the filter is

$$y(t) = 4\cos\left(40\pi t + \frac{\pi}{4}\right) + 4$$

27. A-2, B-1, C-3, D-4

28. Ans: (b)

Sol: Frequency constant (S_1)_LTI, frequency not constant (S_2) - not an LTI

29. Ans: (d)

30. Ans: (d)

Sol: For a real valued periodic function f(t) of frequency f_0

$C_n = C^*_{-n}$

A is False but R is True because the discrete magnitude spectrum of real function f(t) is even and phase spectrum is odd.

Chapter **4**

Fourier Transform

Class Room Practice Solutions

01.

Sol: $X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$

Unit: volt-sec (or) volt/Hz

02.
Sol:

(a) $X(0) = \int_{-\infty}^{\infty} x(t) dt = \text{area}$

$= (4\times 2) - \left(\frac{1}{2}\times 1\times 2\right) = 7$

(b) $2\pi x(0) = 4\pi$

03.
Sol:

(i) $x(t) = e^{-at}u(t) + e^{at}u(-t)$

$X(\omega) = \frac{1}{a+j\omega} + \frac{1}{a-j\omega} = \frac{2a}{a^2+\omega^2}$

(ii) $e^{-at}u(t) - e^{at}u(-t) \leftrightarrow \frac{-2j\omega}{a^2+\omega^2}$

As $a \to 0$

$u(t) - u(-t) \leftrightarrow \frac{2}{j\omega}$

$\text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$

04.

Sol: $G(\omega) = 1 + \frac{12}{\omega^2+9}$

$g(t) = \delta(t) + 2e^{-3|t|}$

05. Ans: Zero

Sol: $x(t) = \text{rect}(t/2)$, $X(\omega) = 2\text{sa}(\omega)$

$y(t) = x(t) + x(t/2)$, $Y(\omega) = X(\omega) + 2X(2\omega)$

$Y(\omega) = \frac{2\sin\omega}{\omega} + \frac{4\sin 2\omega}{\omega}$

$f = 1 \Rightarrow \omega = 2\pi$, $y(2\pi) = 0$

06. Ans: (d)

Sol: $Y(\omega) = 3X(2\omega)$

$y(t) = 3/2\ x(t/2)$

07.

Sol: i) $1 \leftrightarrow 2\pi\delta(\omega)$

ii) $\frac{1}{a+jt} \leftrightarrow 2\pi e^{a\omega}.u(-\omega)$

iii) $\frac{2a}{a^2+t^2} \leftrightarrow 2\pi e^{-a|\omega|}.u(-\omega)$

iv) $\frac{1}{\pi t} \leftrightarrow -j\ \text{sgn}(\omega)$

08.

Sol: $x_1(t) = \text{rect}\left(\frac{t}{1}\right)$ $X_1(f) = \text{sin c}(f)$

$x(t) = x\left(t - \frac{1}{2}\right)$ $X(f) = e^{-j\pi f}X(f)$

$FT[x(t) + x(-t)] = X(f) + X(-f)$

$= 2\cos(\pi f).\ \text{sinc}(f)$

09.

Sol: $u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$

$\frac{1}{jt} + \pi\delta(t) \leftrightarrow 2\pi u(-\omega)$

$\frac{1}{2}\delta(t) - \frac{1}{j2\pi t} \leftrightarrow u(\omega)$

10.
Sol:

i) $x(t) = e^{-3(t-1)}u(t-1)e^{-3}$

$$X(\omega) = e^{-j\omega}e^{-3}\frac{1}{3+j\omega}$$

ii) $\pi\left(\frac{t}{2}\right) \leftrightarrow 2sa(\omega)$

$\pi\left(\frac{t-1}{2}\right) \leftrightarrow 2e^{-j\omega}sa(\omega)$

iii) $e^{-2|t|} \leftrightarrow \frac{4}{4+\omega^2}$

$e^{-2|t-2|} \leftrightarrow \frac{4e^{-2j\omega}}{4+\omega^2}$

11.
Sol:

(a) $f_1(t) = f(t-1/2) + f(-t-1/2)$

$F_1(\omega) = e^{-\frac{j\omega}{2}}F(\omega) + e^{\frac{j\omega}{2}}.F(-\omega)$

(b) $f_2(t) = \frac{3}{2}f\left(\frac{t}{2}-1\right)$

$F_2(\omega) = 3e^{-2j\omega}F(2\omega)$

12. Ans: (a)
Sol: $g(t) = x(t-3) - x(-t+2)$

$G(f) = e^{-j6\pi f}X(f) - e^{-j4\pi f}X(-f)$

13.
Sol:

i) $\cos\omega_0 t = \frac{1}{2}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] \leftrightarrow \pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$

ii) $\sin\omega_0 t \leftrightarrow \frac{\pi}{j}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$

iii) $e^{-at}\sin\omega_c t u(t) \leftrightarrow \frac{1}{2j}\left[\frac{1}{a+j(\omega-\omega_c)} - \frac{1}{a+j(\omega+\omega_c)}\right]$

iv) $A\,rect\left(\frac{t}{T}\right)\cos\omega_0 t = AT\left[sa\left[\frac{\omega+\omega_0}{2}\right]T + sa\left[\frac{\omega-\omega_0}{2}\right]T\right]$

14.
Sol: $Sinc(t) \leftrightarrow rect(f)$

$Sin\,c(t)\cos(10\pi t) \leftrightarrow \frac{1}{2}[rect(f-5) + rect(f+5)]$

15.
Sol: (a) $\frac{1}{4}e^{-j\frac{3}{4}t}x(t/4) \leftrightarrow X(4\omega+3)$

(b) Ans: (a)

$X(\omega) = 2\pi\delta(\omega) + \pi[\delta(\omega-4\pi) + \delta(\omega+4\pi)]$

$x(t) = 1 + \cos(4\pi t)$

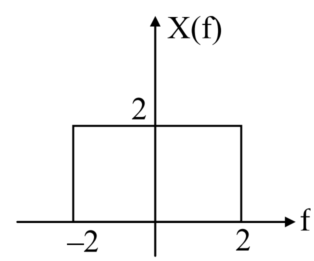
16. Ans: (d)
Sol: $X(f) = \delta(f-f_0)$

$x(t) = e^{j2\pi f_0 t}$

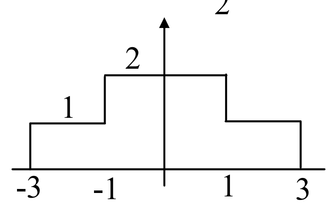
$x(t)\big|_{t=\frac{1}{8f_0}} = e^{\frac{j\pi}{4}}$

$\angle x(t) = \frac{\pi}{4}$

17. Ans: (b)
Sol:



$x(t)\cos 2\pi t \leftrightarrow \frac{1}{2}[X(f-1) + X(f+1)]$



18. Ans: (d)
Sol: Output of multiplier

$= \frac{1}{2}x(t)\cos(2\omega_c t+\theta) + \frac{1}{2}x(t)\cos\theta$

Output of the filter is $= \frac{1}{2}x(t)\cos\theta \times 2$

$= x(t)\cos\theta$

19.
Sol: $y(t) = \delta(t+2) + \delta(t-2)$

$Y(\omega) = e^{2j\omega} + e^{-2j\omega} = 2\cos2\omega$

20. Ans: $= \dfrac{-1}{2\sqrt{\pi}}$

Sol: $x(t) = \dfrac{1}{2\pi}\left[\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega\right]$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} j\omega X(\omega).e^{j\omega t}d\omega$$

$$\left.\frac{dx(t)}{dt}\right|_{t=0} = \frac{1}{2\pi}\int_{-\infty}^{\infty} j\omega X(\omega)d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-1}^{0} j\omega(-j\sqrt{\pi})d\omega + \int_{0}^{1} j\omega(j\sqrt{\pi})d\omega\right]$$

$$= \frac{-1}{2\sqrt{\pi}}$$

21.

Sol: $te^{-a|t|} \leftrightarrow j\dfrac{d}{d\omega}\left[\dfrac{2a}{a^2+\omega^2}\right] = \dfrac{-4ja\omega}{(a^2+\omega^2)^2}$

$$te^{-|t|} \leftrightarrow \frac{-4j\omega}{(\omega^2+1)^2}$$

Apply duality property

$$\frac{4t}{(t^2+1)^2} \leftrightarrow -2\pi j\omega.e^{-|\omega|}$$

22.

Sol:

(i) $X_1(\omega) = e^{-2j\omega}X(-\omega) + e^{2j\omega}X(-\omega)$

(ii) $= \dfrac{1}{3}e^{-2j\omega}X\left(\dfrac{\omega}{3}\right)$

(iii) $= (j\omega)^2 e^{-3j\omega}.X(\omega)$

(iv) $= j\dfrac{d}{d\omega}[j\omega X(\omega)]$

23.

Sol: x(t) = rect (t/2)

$$X(\omega) = \frac{2\sin\omega}{\omega}$$

(a). $y_1(t) = x(t-1) \Rightarrow Y_1(\omega) = e^{-j\omega}X(\omega)$

(b). $\Rightarrow y_2(t) = x(t) * x(t)$

$$Y_2(\omega) = X(\omega)\,X(\omega) = \frac{2\sin\omega}{\omega}\frac{2\sin\omega}{\omega}$$

$$Y_2(\omega) = 4\frac{\sin^2\omega}{\omega^2}$$

(c). $y_3(t) = tx(t) \quad Y_3(\omega) = j\dfrac{d}{d\omega}[x(\omega)]$

(d). $y_4(t) = x(t)\sin\pi t \leftrightarrow \dfrac{1}{2j}[X(\omega-\pi) - X(\omega+\pi)]$

(e). $y_5(t) = \dfrac{dx(t)}{dt} \leftrightarrow j\omega x(\omega)$

(f). $y_6(t) = (t+1)\,x(t) + 2u(t-1)$

(g). $y_7(t) = y_1\left(\dfrac{t}{2}\right) \leftrightarrow 2Y_1(2\omega)$

(h). $y_8(t) = y_2(2(t+1)) - y_2(2(t-1))$

$$Y_8(\omega) = \frac{1}{2}Y_2\left(\frac{\omega}{2}\right)e^{-j\omega(-1)} - \frac{1}{2}Y_2\left(\frac{\omega}{2}\right)e^{-j\omega(1)}$$

$$= \frac{1}{2}Y_2\left(\frac{\omega}{2}\right)e^{j\omega} - \frac{1}{2}Y_2\left(\frac{\omega}{2}\right)e^{-j\omega}$$

$$= \frac{1}{2}Y_2\left(\frac{\omega}{2}\right)\left[e^{j\omega} - e^{-j\omega}\right]$$

(i). $y_9(t) = x\left(\dfrac{t}{2}\right) - \dfrac{1}{2}y_2(t)$

$$Y_9(\omega) = 2X(2\omega) - \frac{1}{2}Y_2(\omega)$$

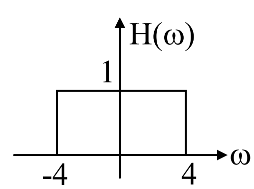
(j). $z(t) = \dfrac{1}{2}y_2(2t)$

$y_{10}(t) = z(t+1) + z(t) + z(t-1)$

$Y_{10}(\omega) = (1+2\cos\omega)\,Z(\omega)$

24. Ans: y(t) = cos 2t

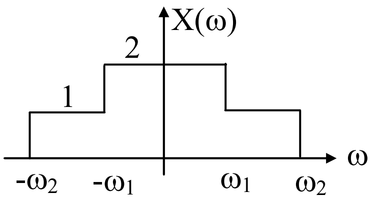
Sol: $h(t) = \dfrac{\sin 4t}{\pi t} \quad H(\omega) = \text{rect}\left(\dfrac{\omega}{8}\right)$

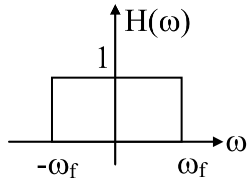


y(t) = cos 2t

25.

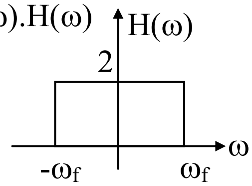
Sol: $X(\omega) = \text{rect}\left(\frac{\omega}{2\omega_1}\right) + \text{rect}\left(\frac{\omega}{2\omega_2}\right)$



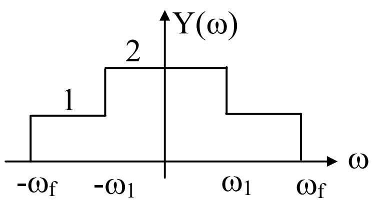


(a). $0 < \omega_f < \omega_1$ $\quad Y(\omega) = X(\omega).H(\omega)$

$$y(t) = \frac{2\sin\omega_f t}{\pi t}$$



(b). $\omega_1 < \omega_f < \omega_2$



$$y(t) = \frac{\sin\omega_1 t}{\pi t} + \frac{\sin\omega_f t}{\pi t}$$

(c). $\omega_f > \omega_2$ $\quad y(t) = \frac{\sin\omega_1 t}{\pi t} + \frac{\sin\omega_2 t}{\pi t}$

26.

Sol:

(a). $X(\omega) = \delta(\omega) + \delta(\omega-5) + \delta(\omega-\pi)$

$x(t) = 1 + e^{-j5t} + e^{-j\pi t}$

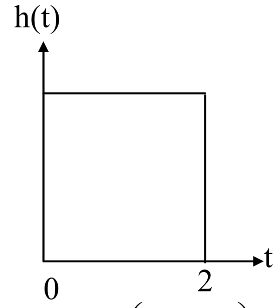
$$e^{-j\pi t} \Rightarrow T_1 = \frac{2\pi}{\pi} = 2$$

$$e^{-j5t} \Rightarrow T_2 = \frac{2\pi}{5} = \frac{2\pi}{5}$$

$\frac{T_1}{T_2} = \frac{5}{\pi}$ is irrational

A periodic

(b). $h(t) = u(t) - u(t-2)$



$$\Rightarrow h(t) = \text{rect}\left(\frac{t}{2} - 0.5\right)$$

$$\text{rect}(t) \leftrightarrow \frac{\sin\frac{\omega}{2}}{\frac{\omega}{2}}$$

$$\text{rect}\left(\frac{t}{2} - 0.5\right) \leftrightarrow 2e^{-j\omega}\frac{\sin\omega}{\omega}$$

$$\Rightarrow H(\omega) = 2e^{-j\omega}\frac{\sin\omega}{\omega}$$

$x(t) * h(t) \leftrightarrow H(\omega)\, X(\omega)$

$$X(\omega)H(\omega) = [\delta(\omega) + \delta(\omega-5) + \delta(\omega-\pi)]\left[2e^{-j\omega}\frac{\sin\omega}{\omega}\right]$$

$$= \delta(\omega)\,\underset{x\to 0}{\text{Lt}}\, 2e^{-j\omega}\frac{\sin\omega}{\omega} + \delta(\omega-5)2e^{-j5}\frac{\sin 5}{5}$$

$$+ \delta(\omega)2e^{-j\pi}\frac{\sin\pi}{\pi}$$

$$= 2\delta(\omega) + 2e^{-j5}\frac{\sin 5}{5}\delta(\omega-5)\left[\underset{x\to\pi}{\text{Lt}}\frac{\sin x}{x} = 0\right]$$

$$X(\omega)H(\omega) = 2\delta(\omega) + 2e^{-j5}\frac{\sin 5}{5}\delta(\omega-5)$$

$$\Rightarrow x(t) * h(t) = 2 + 2e^{-j5}\frac{\sin 5}{5}e^{-j5t}$$

$\Rightarrow$ Periodic

(c). In above problem, convolution of two non periodic signals can be a periodic signal

27.

Sol:

(a). $y_1(t) = \text{rect}(t) * \cos\pi t$

$$\text{rect}(t) \leftrightarrow \frac{2}{\omega}\sin\frac{\omega}{2} \quad \left[\because Y(\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t}dt\right]$$

$$\text{rect}(t) \leftrightarrow \frac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}$$

$$\text{rect}(t) \leftrightarrow \frac{\sin\left(\pi.\frac{\omega}{2\pi}\right)}{\pi\frac{\omega}{2\pi}}$$

$$\text{rect}(t) \leftrightarrow \sin c\left(\frac{\omega}{2\pi}\right)$$

$$\cos\pi \leftrightarrow \pi\ [\ \delta(\omega-\pi) + \delta\ (\omega + \pi)]$$

$$Y_1(\omega) = \sin c\left(\frac{\omega}{2\pi}\right) \times \pi[\delta(\omega-\pi)+\delta(\omega+\pi)]$$

$$Y_1(\omega) = \frac{2}{\omega}\sin\frac{\omega}{2} \times \pi[\delta(\omega-\pi)+\delta(\omega+\pi)]$$

$$= \frac{2}{\omega}\sin\frac{\omega}{2} \times \pi\delta(\omega-\pi) + \frac{2}{\omega}\sin\frac{\omega}{2} \times \pi\delta(\omega+\pi)$$

$$= \frac{2}{\pi}\sin\frac{\pi}{2}\pi\delta(\omega-\pi) + \frac{2}{-\pi}\sin\left(\frac{-\pi}{2}\right)\pi\delta(\omega+\pi)$$

$$= 2\ \delta\ (\omega-\pi) + 2\delta\ (\omega + \pi)$$

$$Y_1(\omega) = \frac{2}{\pi}\pi[\delta(\omega-\pi)+\delta(\omega+\pi)]$$

Taking inverse fourier transform

$$\therefore\ y_1(t) = \frac{2}{\pi}\cos\pi t$$

(b). $y_2(t) = \text{rect}(t) * \cos 2\pi t$

Similar to above

$$Y_2(\omega) = \frac{2}{\omega}\sin\frac{\omega}{2} \times \pi[\delta(\omega-2\pi)+\delta(\omega+2\pi)]$$

$$= \frac{2}{\omega}\sin\left(\frac{\omega}{2}\right)\pi\delta(\omega-2\pi) + \frac{2}{\omega}\sin\left(\frac{\omega}{2}\right)\pi\delta(\omega+2\pi)$$

$$= \frac{2}{2\pi}\sin\left(\frac{2\pi}{2}\right)\pi\delta(\omega-2\pi) + \frac{2}{-2\pi}\sin\left(\frac{-2\pi}{2}\right).\pi\delta(\omega+2\pi) = 0$$

$$\therefore\ y_2(t) = 0$$

(c). $y_3(t) = \sin c(t) * \text{sinc}\left(\frac{t}{2}\right)$

$$\text{rect}\ (t) \leftrightarrow \sin c\ \left(\frac{\omega}{2\pi}\right)$$

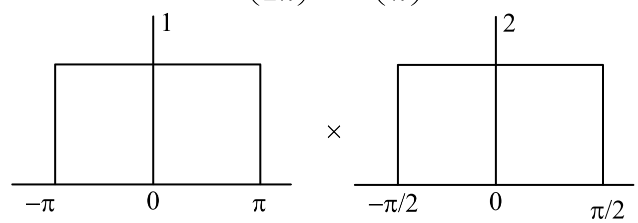
$$\text{sinc}\left(\frac{t}{2\pi}\right) \leftrightarrow 2\pi\,\text{rect}(-\omega)$$

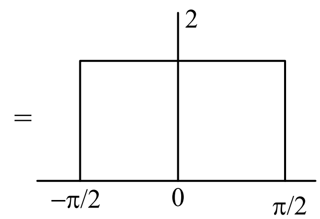
$$\sin c\left(\frac{t}{2\pi}\right) \leftrightarrow 2\pi\,\text{rect}(\omega)$$

$$\sin c(t) \leftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right)$$

$$\sin c\left(\frac{t}{2}\right) \leftrightarrow 2\,\text{rect}\left(\frac{\omega}{\pi}\right)$$

$$\therefore\ Y_3(\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)2\,\text{rect}\left(\frac{\omega}{\pi}\right)$$





$$Y_3(\omega) = 2\,\text{rect}\left(\frac{\omega}{\pi}\right)$$

$$Y_3(\omega) \leftrightarrow 2\,\text{rect}\left(\frac{\omega}{\pi}\right)$$

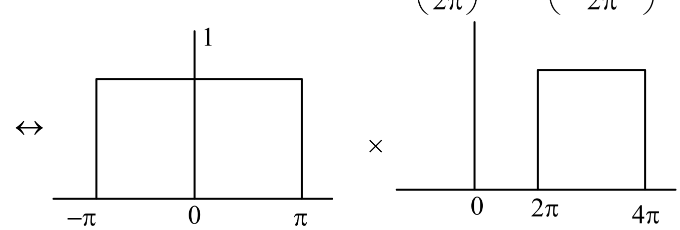
Taking inverse fourier transform

$$y_3(t) = \sin c\left(\frac{t}{2}\right)$$

(d). $\text{sinc}(t) \leftrightarrow \text{rect}\ \left(\frac{\omega}{2\pi}\right)$

$$e^{j3\pi t}\sin c(t) \leftrightarrow \text{rect}\left(\frac{\omega-3\pi}{2\pi}\right)$$

$$\sin c(t) * e^{j3\pi t}\sin c(t) \leftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right) \times \text{rect}\left(\frac{\omega-3\pi}{2\pi}\right)$$



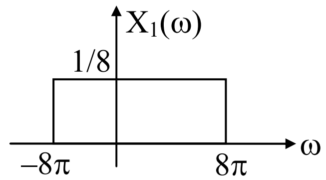
$\leftrightarrow 0$

$\therefore\ Y_4(\omega) = 0$

$\Rightarrow y_4(t) = 0$

28.

Sol:



(a). sinc(8t) $\leftrightarrow$

$$H(\omega) = 8e^{-j\omega}X_1(\omega) = e^{-j\omega} \quad -8\pi < \omega < 8\pi$$
$$= 0 \qquad \text{otherwise}$$

$$Y(\omega) = \pi e^{-j\omega}[\delta(\omega+\pi) + \delta(\omega-\pi)]$$

$y(t) = \cos\pi(t-1)$

(b). **Ans: (d)**

$$G(f) = e^{-\pi f^2} \quad H(f) = e^{-\pi f^2}$$
$$Y(f) = G(f)H(f) = e^{-2\pi f^2}$$

29. Ans: (c)

Sol: $x(t) = e^{j2\pi t}e^{-\pi t^2}$

-conjugate even symmetry

30.

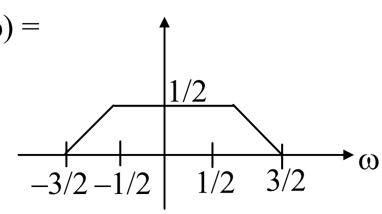
Sol:

(a). $Y(\omega) = \frac{1}{2}[X(\omega-\omega_0) + X(\omega+\omega_0)]$

(b). $x(t) = \frac{\sin t}{\pi t}\pi\frac{\sin(t/2)}{\pi t}$

$$X(\omega) = \frac{1}{2\pi}\left[\text{rect}\left(\frac{\omega}{2}\right) * \pi\text{rect}\left(\frac{\omega}{1}\right)\right]$$

X(ω) =



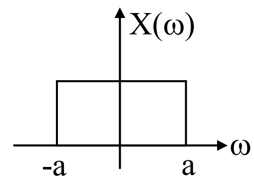
31.

Sol: $\int_{-\infty}^{t} x(t)dt \leftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$

$$\leftrightarrow \frac{\text{rect}(\omega/4\pi)}{j\omega} + \pi\delta(\omega)$$

32.

Sol:



$$E = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega = \frac{2a}{\pi} = \frac{a}{\pi}$$

33.

Sol: $E = \frac{1}{2\pi}\left[\int_{-1}^{-1/2}\pi d\omega + \int_{-1/2}^{1/2}\frac{\pi}{4}d\omega + \int_{1/2}^{1}\pi d\omega\right] = \frac{5}{8}$

34.

Sol: $E_{x(t)} = 1/4$

$$|X(\omega)|^2 = \frac{1}{4+\omega^2}$$

$$S_{YY}(\omega) = |X(\omega)|^2\,|H(\omega)|^2 = \frac{1}{4+\omega^2}, -\omega_c < \omega < \omega_c$$

$$E_{y(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}S_{yy}(\omega)d\omega \Rightarrow \frac{1}{8} = \frac{1}{2\pi}\frac{1}{2}\tan^{-1}\left(\frac{\omega}{2}\right)\Bigg|_{-\omega_c}^{\omega_c}$$

$\omega_c = 2$ rad/sec

35.

Sol: $e^{-2|t|} \leftrightarrow \frac{4}{\omega^2+4}$

$$\int_{-\infty}^{\infty}\frac{8}{(\omega^2+4)^2}d\omega = 2\int_{-\infty}^{\infty}\left(\frac{4}{\omega^2+4}\right)^2 d\omega$$

$$= \frac{1}{2}(2\pi)\int_{-\infty}^{\infty}\left|e^{-2|t|}\right|^2 dt$$

$$= \frac{\pi}{2}$$

36. Ans: $B = \frac{2.302}{a}$

Sol: $g(t) = \frac{2a}{a^2+t^2}$

We know $e^{-a|t|} \leftrightarrow \frac{2a}{a^2+\omega^2}$

By duality property $\frac{2a}{a^2+t^2} \leftrightarrow e^{-a|\omega|}$

Given $\int_{-B}^{B}\left|e^{-a|\omega|}\right|^2 d\omega = 0.99\int_{-\infty}^{\infty}\left|e^{-a|\omega|}\right|^2 d\omega$

$\Rightarrow \int_{-B}^{0} e^{2a\omega}d\omega + \int_{0}^{B} e^{-2a\omega}d\omega = 0.99\left[\int_{-\infty}^{0} e^{2a\omega}d\omega + \int_{0}^{\infty} e^{-2a\omega}d\omega\right]$

$\Rightarrow \left.\frac{e^{2a\omega}}{2a}\right]_{-B}^{0} + \left.\frac{e^{-2a\omega}}{-2a}\right]_{0}^{B} = 0.99\left[\left.\frac{e^{2a\omega}}{2a}\right]_{-\infty}^{0} + \left.\frac{e^{-2a\omega}}{-2a}\right]_{0}^{\infty}\right]$

$\Rightarrow \frac{1}{2a}\left[1-e^{-2aB}\right] - \frac{1}{2a}\left[e^{-2aB}-1\right] = \frac{0.99}{2a}[1+1]$

$\Rightarrow 2 - 2e^{-2aB} = 2\times 0.99$

$\Rightarrow 1 - e^{-2aB} = 0.99$

$\Rightarrow 0.01 = e^{-2aB}$

$\Rightarrow \ln(100) = 2aB$

$\Rightarrow B = \frac{\ell n(100)}{2a} = \frac{4.605}{2a} = \frac{2.302}{a}$

37. Ans: (a)

Sol: $E = \int_{-\infty}^{\infty}|X_1(f)|^2 df = \frac{2}{3}\times 10^{-8}$

38. Ans: (c)

Sol: $\angle H(\omega) = \frac{-\omega}{60} \quad -30\pi < \omega < 30\pi$

$\omega_0 = 10\pi \quad |H(10\pi)| = 2, \ \angle H(10\pi) = \frac{-\pi}{6}$

$\omega_0 = 26\pi \quad |H(26\pi)| = 1, \ \angle H(26\pi) = \frac{-13\pi}{30}$

$y(t) = 4\cos\left(10\pi t - \frac{\pi}{6}\right) + \sin\left(26\pi t - \frac{13\pi}{30}\right)$

39.

Sol: $\theta(\omega) = -\omega t_0$

$t_p(\omega) = \frac{-\theta(\omega)}{\omega} = t_0$

$t_g(\omega) = \frac{-d\theta(\omega)}{d\omega} = t_0$

Both are constant

40.

Sol:

(i) Ans: (c)

$H(f) = \frac{1}{1+j2\pi fRC}$

$|H(f)| = \frac{1}{\sqrt{1+4\pi^2 f^2 R^2 C^2}}$

$|H(f_1)| \geq 0.95$

$f_1 = 52.2$ Hz

(ii) Ans: (a)

$\theta(f) = -\tan^{-1}(2\pi fRC)$

$t_g(f) = \frac{-d\theta(f)}{df} = \frac{1}{2\pi}\left[\frac{2\pi RC}{1+(2\pi fRC)^2}\right]$

$t_g(100) = 0.71$ msec

41. Ans: (c)

Sol: $y(t) = \frac{1}{100}\cos\left(100\left(t-10^{-8}\right)\right)\cos\left(10^6\left(t-1.56\times10^{-6}\right)\right)$

$t_g = 10^{-8}, t_p = 1.56\times10^{-6}$

42.

Sol: 20 to 30 kHz no distortion

43.

Sol:

(a) $H(\omega) = \frac{1}{1-\omega^2+\sqrt{2}j\omega}$

$|H(\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2+(\sqrt{2}\omega)^2}}$

$|H(\omega)|_{\omega=0} = \frac{1}{\sqrt{1+0}} = 1$

(b) $A = \frac{A_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$$|H(\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + (\sqrt{2}\omega)^2}} = \frac{1}{\sqrt{1+\omega^4}}$$

At $\omega = \omega_c$ $|H(\omega)| = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{1+\omega_c^4}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = 1\text{rad/sec}$$

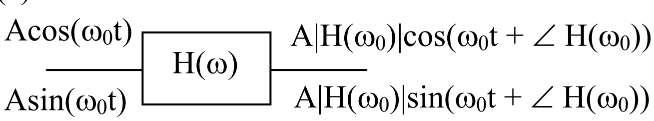
Given $|H(\omega)| = 0.01$

$$|H(\omega_1)|^2 = \left(\frac{1}{100}\right)^2$$

$$\frac{1}{1+\omega_1^4} = \frac{1}{10000}$$

$\omega_1 \approx 10$ rad/secs

(c)



44.

Sol: For $-200 < \omega < 200$, there is no amplitude distortion.

And For $-100 < \omega < 100$, there is no phase distortion

$x_1(t)$

$\omega = 20$ and $\omega = 60$

So no phase distortion and no amplitude distortion.

$x_2(t)$

$\omega = 20$, $\omega = 140$

Amplitude distortion, do not occurs.

Phase distortion occurs.

[$\because \omega = 140$]

$x_3(t)$

$\omega = 20$, $\omega = 220$,

Phase distortion and amplitude distortion occurs

[$\because \omega = 220$]

45.

Sol: $R_{xx}(\tau) = \frac{A^2}{2}\cos(\omega_0\tau) = 18\cos(6\pi\tau)$

Power = $R_{xx}(0) = 18$

46.

Sol: $r_{xx}(\tau) = x(t) * x(-t) = e^{-3t}u(t) * e^{3t}.u(-t)$

$$r_{xx}(\tau) \overset{F.T}{\leftrightarrow} S_{XX}(\omega) = \frac{1}{9+\omega^2} \quad \Rightarrow r_{xx}(\tau) = \frac{1}{6}e^{-3|\tau|}$$

47.

Sol:

(a) $|H(\omega)|^2 = \frac{1}{1+\omega^2}$, $|X(\omega)|^2 = \frac{1}{4+\omega^2}$

$S_{YY}(\omega) = |X(\omega)|^2 |H(\omega)|^2$

(b) $y(t) = x(t) * h(t) = [e^{-t} - e^{-2t}]u(t)$

$$E_{y(t)} = \int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{12}$$

$$E_{x(t)} = \frac{1}{4}$$

$$E_{y(t)} = \frac{1}{3}E_{x(t)}$$

48.

Sol:

i) Ans: (b)

$$x(t) = e^{-8t}u(t) * e^{-8t}u(t) = \frac{1}{16}e^{-8|t|}$$

$$x\left(\frac{1}{16}\right) = \frac{1}{16\sqrt{e}}$$

ii) Ans: (c)

$$S_{GG}(\omega) = |G(\omega)|^2 = \frac{1}{64+\omega^2}$$

$$S_{GG}(0) = \frac{1}{64}$$

iii) Ans: (b)

$$y(\tau) = e^{-8t}u(t) * e^{8t}u(-t)$$

$$y(\tau) = \frac{1}{16}e^{-8|\tau|}$$

$$y(0) = \frac{1}{16}$$

49.

Sol: $r_{xy}(\tau) = x(t) * y(-t) = e^{-t}u(t) * e^{3t}u(-t)$

$$r_{xy}(\tau) \leftrightarrow \frac{1}{1+j\omega}\frac{1}{3-j\omega} = \frac{1/2}{1+j\omega} + \frac{1/2}{3-j\omega}$$

$$r_{xy}(\tau) = \frac{1}{2}e^{-\tau}u(\tau) + \frac{1}{2}e^{3\tau}u(-\tau)$$

50.

Sol: Given x(t) = sinc 10t

$$\text{Sinc } t \leftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right)$$

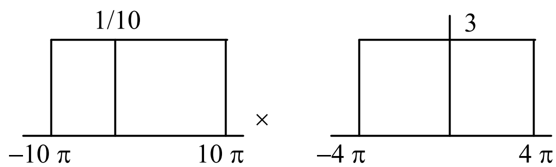
$$\sin c(10t) \leftrightarrow \frac{1}{10}\text{rect}\left(\frac{\omega}{20\pi}\right)$$

$$X(\omega) = \frac{1}{10}\text{rect}\left(\frac{\omega}{20\pi}\right)$$

$$H(\omega) = 3\text{rect}\left(\frac{\omega}{8\pi}\right)e^{-j2\omega}$$

$\therefore$ Y(ω) = X(ω) H (ω)

$$= \frac{1}{10}\text{rect}\left(\frac{\omega}{20\pi}\right)3\,\text{rect}\left(\frac{\omega}{8\pi}\right)e^{-j2\omega}$$



$$= \frac{3}{10}\text{rect}\left(\frac{\omega}{8\pi}\right)e^{-j2\omega}$$

$\therefore$ output energy

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}|Y(\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-4\pi}^{4\pi}\frac{9}{100}$$

$$= \frac{1}{2\pi}\cdot\frac{9}{100}\times 8\pi$$

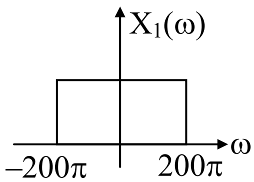
Output energy = $\frac{36}{100}$ J

51.

Sol:

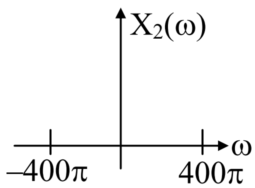
(a). $\omega_m = 200\,\pi$

$\omega_s = 400\,\pi$ rad/sec



(b). $\omega_m = 400\,\pi$

$\omega_s = 800\,\pi$ rad/sec



(c). $x_3(t) = \frac{5}{2}[\cos(500\pi t) + \cos(3000\pi t)]$

$\omega_m = 5000\,\pi$

$\omega_s = 10{,}000\,\pi$ rad/sec

(d). $X_4(\omega) = \frac{1}{6+j\omega}.\text{rect}\left(\frac{\omega}{2a}\right)$

$\omega_m = a$

$f_m = \frac{a}{2\pi}$

$f_s = 2f_m = \frac{a}{\pi}$ Hz

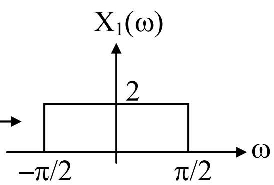
(e). $\omega_m = 120\,\pi$, $f_m = 60$Hz

$(f_s) = 2f_m = 120$ Hz

(f) **Ans: 0.4**

Sol:

$$x_1(t) = 2\frac{\sin\frac{\pi}{2}t}{\pi t} \longleftrightarrow$$



$$\sum_{n=-\infty}^{+\infty}\delta(t - nT_s) \leftrightarrow f_s\sum_{n=-\infty}^{+\infty}\delta(\omega - n\omega_s)$$

$$\sum_{n=-\infty}^{+\infty}\delta(t - 10n) \leftrightarrow \frac{1}{10}\sum_{n=-\infty}^{+\infty}\delta\left(\omega - n\frac{\pi}{5}\right)$$

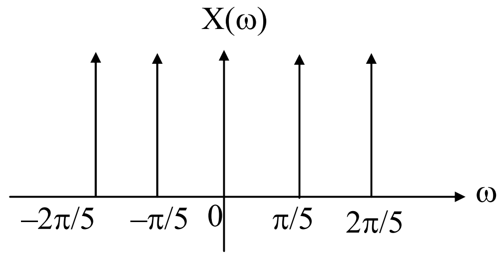
$$x_1(t) * \sum_{n=-\infty}^{\infty}\delta(t - 10n) \leftrightarrow X_1(\omega)\frac{1}{10}\sum_{n=-\infty}^{+\infty}\delta\left(\omega - n\frac{\pi}{5}\right)$$

$$X(\omega)=\frac{1}{10}\sum_{n=-\infty}^{+\infty}X_1\left(\frac{n\pi}{5}\right)\delta\left(\omega-n\frac{\pi}{5}\right)$$

$$X(\omega)=\frac{1}{10}\left[---+X_1(0)\delta(\omega)+X_1\left(\frac{\pi}{5}\right)\delta\left(\omega-\frac{\pi}{5}\right)+X_1\left(\frac{2\pi}{5}\right)\delta\left(\omega-\frac{2\pi}{5}\right)+X_1\left(\frac{3\pi}{5}\right)\delta\left(\omega-\frac{3\pi}{5}\right)+----\right]$$

$$X_1\left(\frac{\pi}{5}\right)=2, X_1\left(\frac{2\pi}{5}\right)=2,$$

$$X_1\left(\frac{3\pi}{5}\right)=X_1\left(\frac{4\pi}{5}\right)=----=0$$



The maximum frequency in above signal is

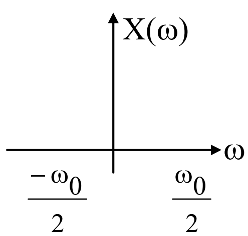
$\omega_m = 2\pi/5$

$2\pi f_m = 2\pi/5$

$f_m = 1/5$

Nyquist rate = $2f_m = 2/5 = 0.4$

52.
Sol:



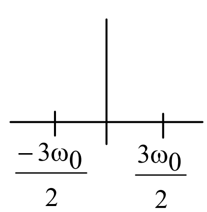
(a). $X(\omega)+e^{-j\omega}.X(\omega)$ no change in frequency axis

$(\omega_s)_{min} = 2\omega_m = \omega_0$

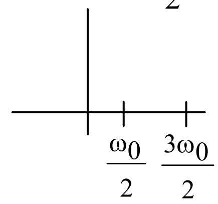
(b). $\frac{dx(t)}{dt}\leftrightarrow j\omega.X(\omega)$ $\omega_s = \omega_0$

(c). $x(3t)\leftrightarrow\frac{1}{3}.X\left(\frac{\omega}{3}\right)$

$$\omega_S = 2\times\frac{3\omega_0}{2}=3\omega_0$$



(d). $\frac{1}{2}X(\omega+\omega_0)+\frac{1}{2}X(\omega+\omega_0)$



$$\omega_S = 2\times\frac{3\omega_0}{2}=3\omega_0$$

53.
Sol:

(a) $x_1(2t)\leftrightarrow\frac{1}{2}X_1\left(\frac{\omega}{2}\right)$ $f_m = 4k$

$f_s = 8k$

(b) $x_2(t-3)\leftrightarrow e^{-3j\omega}.X_2(\omega)$ $f_m = 3k$

$f_s = 6k$

(c) $X_1(\omega)+X_2(\omega)$ $f_m = 3k$

$f_s = 6k$

(d) $X_1(\omega)*X_2(\omega)$ $f_m = 5k$

$f_s = 10k$

(e) $X_1(\omega).X_2(\omega)$ $f_m = 2k$

$f_s = 4k$

(f) $\frac{1}{2}[X_1(\omega+1000\pi)+X_1(\omega-1000\pi)]$

$f_m = 250Hz$

$(f_s)_{min} = 2f_m = 5k$

54. Ans: (a)

Sol: $f_m = 200Hz$, $f_s = 300Hz$

The frequency in sampled signals are = 200, 100, 500, 400, 800. Cutoff frequency of filter is 100 Hz.

Output frequency = 100 Hz

55. Ans: (b)

Sol: If $f_s = f_m$ -The spectrum is constant spectrum

56. Ans: (a)

Sol: $f_m < f_c < f_s - f_m \Rightarrow 5 < f_c < 9$

57. Ans: (c)

Sol: $f_m = 100$, $f_s - f_m = 150$

$f_s = 250$

$$T_s = \frac{1}{f_s} = 4\text{m sec}$$

58. Ans: (d)

Sol: $$f_s = \frac{1}{T_0} = \frac{1}{10^{-3}} = 10^3 = 1\text{KHz}$$

$$C_n = \frac{1}{T_0}\int_{\frac{-T_0}{6}}^{\frac{T_0}{6}} 3.e^{-jn\omega_0 t}dt = \frac{\sin\left(\frac{n\pi}{3}\right)}{n\pi}$$

$\therefore$ $C_n = 0$ for n = 3, 6, 9 …..

$C_n \neq 0$ for n = 0, 1, 2, 4, 6, 7, 8, 10……

$\therefore$ $\pm f \pm 3f_s$, $+ f \pm 6 f_s$ ….

Are not present in signal

$\pm$ 400 $\pm$ 3 (1000) = $\pm$3.4 K, $\pm$ 2.6 K

So options with 3.4 K and 2.6 K are wrong

So (c) and (a) are wrong.

3.6 K is out of the given range [2.5 to 3.5]

So (B) is wrong

So (D) is correct.

59. Ans: (c)

Sol: $$y(t) = 5\times10^{-6}x(t)\sum_{n=-\infty}^{\infty}\delta(t - nTs)$$

$$x(t) = 10\cos(8\pi\times10^3 t)\ \&\ T_S = 100\mu\text{sec}$$

$$fs = \frac{1}{100\mu} = 10\text{kHz}$$

$$Y(\omega) = \frac{5\times10^{-6}}{T_s}\sum_{n=-\infty}^{\infty}X(\omega - n\omega s)$$

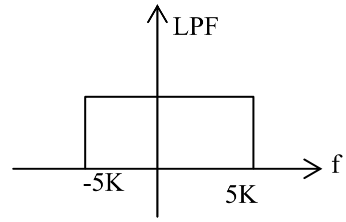
Given $\omega_m = 8\pi\times10^3$

$f_m = 4k$

The output frequencies are $\pm$fm = 4K,-4k

$fs \pm f_m$ = 14K, 6k

$-fs \pm f_m$ = –6K,–14K



Only 4k & – 4k are allowed then the output is

$$o/p = \frac{5\times10^{-6}}{Ts}\times10\cos(8\pi\times10^3 t)$$

$$= \frac{5\times10^{-6}}{100\times10^{-6}}\times10\cos(8\pi\times10^3 t)$$

$$= \frac{5\times10^{-6}}{100\times10^{-6}}\times10\cos(8\pi\times10^3 t)$$

$$o/p = 5\times10^{-1}\times\cos(8\pi\times10^3 t)$$

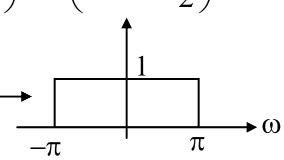
60. Ans: (a)

Sol: $$x(t) = \cos\left(10\pi t + \frac{\pi}{4}\right)$$

f_s = 15 Hz, $\omega_s = 2\pi f_s = 30\ \pi$Hz

$$h(t) = \left(\frac{\sin\pi t}{\pi t}\right).\cos\left(40\pi t - \frac{\pi}{2}\right)$$

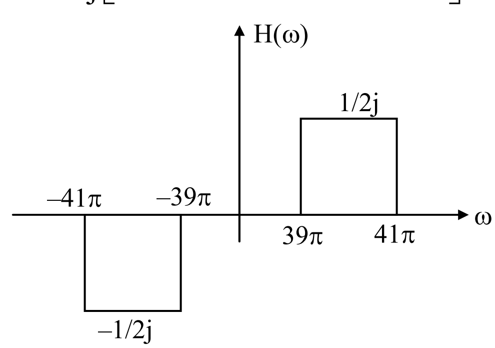
$$\frac{\sin\pi t}{\pi t} \longleftrightarrow$$



$$h(t) = \frac{\sin\pi t}{\pi t}\left[\cos(40\pi t)\cos\frac{\pi}{2} + \sin 40\pi t\ \sin\frac{\pi}{2}\right]$$

$$h(t) = \frac{\sin\pi t}{\pi t}.\sin 40\pi t$$

$$= \frac{1}{2j}\left[\frac{\sin\pi t}{\pi t}.e^{j40\pi t} - \frac{\sin\pi t}{\pi t}.e^{-j40\pi t}\right]$$



$$x(t) = \cos(10\pi t)\cos\frac{\pi}{4} - \sin(10\pi t)\sin\frac{\pi}{4}$$

$$X(\omega) = \frac{1}{\sqrt{2}}\left[\pi(\delta(\omega+10\pi)+\delta(\omega-10\pi))\right]$$

$$-\frac{1}{\sqrt{2}}\left[\frac{\pi}{j}(\delta(\omega-10\pi)-\delta(\omega+10\pi))\right]$$

Sampled signal spectrum

$$X_\delta(\omega) = f_s\sum_{n=-\infty}^{\infty}X(\omega - n\omega_s)$$

n = 0, $\omega_m, -\omega_m = -10\pi, 10\pi$

n = 1, $\omega_s - \omega_m, \omega_s + \omega_m = 20\pi, 40\pi$

n = 2, $2\omega_s - \omega_m, 2\omega_s + \omega_m = 50\pi, 70\pi$

only 40π frequency is allowed output of filter is

$$Y(\omega) = \frac{15}{\sqrt{2}}\left[\frac{-\pi}{2j}\delta(\omega+40\pi)+\frac{\pi}{2j}\delta(\omega-40\pi)\right]$$

$$-\frac{15}{\sqrt{2}}\left[\frac{\pi}{j}\times\frac{1}{2j}\delta(\omega-40\pi)-\frac{\pi}{j}\left(\frac{-1}{2j}\right)\delta(\omega+40\pi)\right]$$

$$= \frac{15}{\sqrt{2}}\left[-\frac{\pi}{2j}\delta(\omega+40\pi)+\frac{\pi}{2j}\delta(\omega-40\pi)\right]$$

$$-\frac{15}{\sqrt{2}}\left[\frac{-\pi}{2}\delta(\omega-40\pi)-\frac{\pi}{2}\delta(\omega+40\pi)\right]$$

$$= \frac{15}{\sqrt{2}}\left[-\frac{\pi}{2j}\delta(\omega+40\pi)+\frac{\pi}{2j}\delta(\omega-40\pi)\right.$$

$$\left.+\frac{\pi}{2}\delta(\omega-40\pi)+\frac{\pi}{2}\delta(\omega+40\pi)\right]$$

$$Y(\omega) = \frac{15}{\sqrt{2}}\left[\frac{\pi}{2}[\delta(\omega+40\pi)+\delta(\omega-40\pi)]\right.$$

$$\left.+\frac{\pi}{2j}[\delta(\omega-40\pi)-\delta(\omega+40\pi)]\right]$$

$$y(t) = \frac{15}{\sqrt{2}}\left[\frac{1}{2}\cos 40\pi t+\frac{1}{2}\sin 40\pi t\right]$$

$$y(t) = \frac{15}{2}\left[\cos 40\pi t\cos\frac{\pi}{4}+\sin 40\pi t\sin\frac{\pi}{4}\right]$$

$$y(t) = \frac{15}{2}\cos\left(40\pi t - \frac{\pi}{4}\right)$$

61. Ans: (c)

Sol: x(t) = m(t) c(t)

Where c(t) is carrier signal and m(t) is a base band signal

and $f_c > f_H$ (where f_c is carrier frequency, f_H is the highest frequency component of m(t))

$$\hat{x}(t) = m(t).\hat{c}(t)$$

Where $\hat{f}(t)$ is Hilbert transform of f(t).

For the above problem c(t) = $\sin\left(\pi t - \frac{\pi}{4}\right)$

and $m(t) = -\sqrt{2}\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)$

Complex envelope

$$= [x(t)+j\hat{x}(t)]e^{-j2\pi f_c t}$$

$$= -\sqrt{2}\left[m(t)\sin\left(\pi t-\frac{\pi}{4}\right)-jm(t)\cos\left(\pi t-\frac{\pi}{4}\right)\right]e^{-j2\pi f_c t}$$

$$= -\sqrt{2}m(t)\left[\cos\left(\pi t-\frac{\pi}{4}\right)+j\sin\left(\pi t-\frac{\pi}{4}\right)\right]e^{-j2\pi f_c t}$$

$$= -\sqrt{2}m(t)e^{+j\left(\pi t-\frac{\pi}{4}\right)}\cdot e^{-j2\pi\left(\frac{1}{2}\right)t}$$

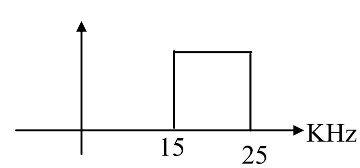
$$= j\sqrt{2}m(t)\,e^{-j\frac{\pi}{4}} = \sqrt{2}m(t)e^{-\frac{j\pi}{4}}$$

$$= \sqrt{2}\left(\frac{\sin(\pi t/5)}{\pi t/5}\right)e^{j\frac{\pi}{4}}$$

62. Ans: 10kHz

Sol: Band Pass sampling theorem its states that we can sample at the rate of $f_s = \frac{2f_H}{K}$ where k is the largest integer not exceeding

$$K = \frac{f_H}{f_H - f_L}$$



For m(t)

$$K = \frac{f_H}{f_H - f_L} = \frac{25}{25-15} = 2.5 \text{ means } 2$$

$$f_s = \frac{2f_H}{K}$$

As m(t) maximum frequency is 5 kHz minimum sampling rate required is 10 kHz.

63. Ans: (c)

64. Ans: (b)

Chapter **5**

Laplace Transforms

Class Room Practice Solutions

01.

Sol: (1) $X_1(s) = \frac{1}{s+1} + \frac{1}{s+3}, \sigma > -1$

(2) $X_2(s) = \frac{1}{s+2} - \frac{1}{s-4}, -2 < \sigma < 4$

(3) no common ROC so no laplace transform for $x_3(t)$.

(4) no common ROC, no laplace transform

(5) no common ROC, no laplace transform

(6) $X_6(s) = \frac{1}{s+1} - \frac{1}{s-1}, \ -1 < \sigma < 1$

02.

Sol: ROC = $(\sigma > -5) \cap (\sigma > \text{Re}(-\beta)) = \sigma > -3$

Imaginary port of 'β' any value, real part of 'β' is 3.

03.

Sol: $\sigma > 2, \sigma < -3, -3 < \sigma < -1, -1 < \sigma < 2$

04.

Sol: $X(s) = \frac{e^{-3s}}{s+1} - \frac{e^{-3s}}{s+2}$

$x(t) = e^{-(t-3)}.u(t-3) - e^{-2(t-3)}.u(t-3)$

05.

Sol:

(a). $x(t) = e^{-5(t-1)}.u(t-1).e^{-5} \leftrightarrow \frac{e^{-s}.e^{-5}}{s+5}, \sigma > -5$

(b). $g(t) = Ae^{-5t}.u(-t-t_0)$

$G(s) = \frac{-A.e^{(s+5)t_0}}{s+5}, \sigma < -5$

$A = -1, t_0 = -1$

06.

Sol: $x(t) = 5r(t) - 5r(t-2) - 15u(t-2) + 5u(t-4)$

$X(s) = \frac{5}{s^2} - \frac{5e^{-2s}}{s^2} - \frac{15e^{-2s}}{s} + \frac{5e^{-4s}}{s}$

07. Ans: (a)

08. Ans: (c)

Sol: $X(s) = \frac{1}{(s+1)(s+3)}$

$G(s) = X(s-2) = \frac{1}{(s-1)(s+1)}$

$G(\omega)$ converges means ROC include $j\omega$ axis

$-1 < \sigma < 1$

09.

Sol: $G(s) = X(s) + \alpha \times (-s)$

$= \frac{\beta s - \beta - \alpha\beta s - \alpha\beta}{s^2 - 1} = \frac{s}{s^2 - 1}$

$\alpha\beta - \beta = -1, -\beta - \alpha\beta = 0$

$\alpha = -1, \beta = ½$

10.

Sol: $\frac{dy(t)}{dt} = -2y(t) + \delta(t) \qquad \frac{dy(t)}{dt} = 2x(t)$

$sY(s) = -2Y(s) + 1$ ----- (1)

$sY(s) = 2X(s)$ ------ (2)

solving (1) and (2)

$Y(s) = \frac{2}{s^2+4}, X(s) = \frac{s}{s^2+4}$

11.

Sol: (a) $X(s) = \frac{-4}{s+2} + \frac{4}{(s+1)^3} - \frac{4}{(s+1)^2} + \frac{4}{s+1}$

$x(t) = -4e^{-2t}.u(t) + 4\frac{t^2}{2}e^{-t}.u(t)$

$-4te^{-t}.u(t) + 4e^{-t}.u(t)$

(b) $X(s) = -\frac{e^{-2s}}{(s+1)^3}$

$$x(t) = -\frac{(t-2)^2}{2}.e^{-(t-2)}.u(t-2)$$

$$\frac{t^2}{2}e^{-t}u(t) \leftrightarrow \frac{1}{(s+1)^3}$$

12.

Sol: $y(t) + y(t) * x(t) = x(t) + \delta(t)$

$Y(s) + Y(s)X(s) = X(s) + 1$

$Y(s) = 1$

$y(t) = \delta(t)$

13.

Sol: $x_1(t-2) \leftrightarrow \frac{e^{-2s}}{s+2}, \sigma > -2$

$$x_2(-t+3) \leftrightarrow \frac{e^{-3s}}{-s+3}, \sigma < 3$$

$$Y(s) = \frac{e^{-2s}}{s+2}.\frac{e^{-3s}}{-s+3}, -2 < \sigma < 3$$

14.

Sol: $sY(s) + 4Y(s) + 3\frac{Y(s)}{s} = X(s)$

$$H(s) = \frac{s}{(s+1)(s+3)} = \frac{\frac{-1}{2}}{s+1} + \frac{\frac{3}{2}}{s+3}$$

$$h(t) = \frac{-1}{2}e^{-t}.u(t) + \frac{3}{2}e^{-3t}.u(t)$$

$$X(s) = \frac{1}{s} + 1 = \frac{s+1}{s}$$

$$Y(s) = X(s).H(s) = \frac{1}{s+3}$$

$$y(t) = e^{-3t}.u(t)$$

15. Ans: (d)

Sol: $X(s) = \frac{1}{s+2} + e^{-6s}, H(s) = \frac{1}{s}$

$$Y(s) = X(s).H(s) = \frac{1}{s(s+2)} + \frac{e^{-6s}}{s}$$

$$y(t) = \frac{1}{2}\left[u(t) - e^{-2t}.u(t)\right] + u(t-6)$$

16. Ans: (b)

Sol: $F_2(s) = e^{-s\tau}.F_1(s)$

$$G(s) = \frac{e^{-s\tau}.F_1(s).F_1^*(s)}{|F_1(s)^2|} = \frac{e^{-s\tau}|F_1(s)|^2}{|F_1(s)|^2}$$

$G(s) = e^{-s\tau}$

$g(t) = \delta(t-\tau)$

17. Ans: (b)

Sol: $\frac{V(s)}{X(s)} = \frac{1}{s+1}$ $\qquad \frac{Y(s)}{V(s)} = \frac{1}{s+1}$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}.\frac{1}{s+1} = \frac{1}{(s+1)^2}$$

$h(t) = t\, e^{-t}.u(t)$

18.

Sol: **(a)** $\frac{Y(s)}{X(s)} = \frac{1}{s}$ given statement is false

(b) $x(t) = u(t)$
$y(t) = r(t)$ is unbounded
given statement is false

(c) $x(t) = u(-t)$
$y(t) = \infty$ is unbounded
given statement is false

(d) Given true

19.

Sol: $s^2Y(s) + \alpha sY(s) + \alpha^2Y(s) = X(s)$

$$H(s) = \frac{1}{s^2 + \alpha s + \alpha^2}$$

$$G(s) = \frac{\alpha^2}{s}H(s) + sH(s) + \alpha H(s)$$

$$G(s) = \left[\frac{\alpha^2 + s^2 + s\alpha}{s}\right]\left[\frac{1}{s^2 + \alpha s + \alpha^2}\right] = \frac{1}{s}$$

Number of poles = 1.

20. Ans: (d)

21.

Sol: (a). $x(0) = \underset{s\to\infty}{Lt}\, sX(s) = 2$

$x(\infty) = \underset{s\to 0}{Lt}\, sX(s) = 0$

(b). $X(s) = \frac{4s+5}{2s+1}$ improper function

$X(s) = 2 + \frac{3}{2s+1} = \frac{3}{2s+1}$

neglect the constant '2' in the above function.

$x(0) = \underset{s\to\infty}{Wt}\, s.\frac{3}{2s+1} = \frac{3}{2}$

$x(\infty) = \underset{s\to 0}{Lt}\, sX(x) = \underset{s\to 0}{Lt}\, \frac{4s^2+5s}{2s+1} = 0$

(c). x(0) = 0

Final value theorem not applicable

(d) x(0) = 0

$x(\infty) = -1$

22.

Sol: $H(s) = \frac{k(s+1)}{(s+2)(s+4)}$ $\quad X(s) = \frac{1}{s}$

$Y(s) = H(s).X(s) = \frac{k(s+1)}{s(s+2)(s+4)}$

$y(\infty) = \underset{s\to 0}{Lt}\, sY(s) = \frac{k}{8} = 1 \Rightarrow k = 8$

$H(s) = \frac{-4}{s+2} + \frac{12}{s+4}$

$h(t) = -4e^{-2t}u(t) + 12e^{-4t}.u(t)$

23.

Sol: $H(j\omega) = \frac{j\omega - 2}{(j\omega)^2 + 4j\omega + 4}$

$x(t) = 8\cos 2t,\ \omega_0 = 2$

$H(j\omega_0) = \frac{j-1}{4j} = \frac{1}{4} + \frac{1}{4}j$

$|H(\omega_0)| = \frac{1}{2\sqrt{2}}, \angle H(\omega_0) = \frac{\pi}{4}$

$y(t) = \frac{8}{2\sqrt{2}}\cos\left(2t + \frac{\pi}{4}\right) = 2\sqrt{2}\cos\left(2t + \frac{\pi}{4}\right)$

24. Ans: (a)

Sol: $H(j\omega) = \frac{-\omega^2 + 1}{-\omega^2 + 2j\omega + 1}$

$\omega_0 = 1$ rad/sec

$H(\omega_0) = 0$

$y(t) = 0$ for all ω_s

25. Ans: (d)

Sol: $H(s) = \frac{2}{s^2 - s - 2}$ $\quad X(s) = \frac{1}{s}$

$Y(s) = X(s).H(s) = \frac{2}{s(s+1)(s-2)}$

S = 2 pole lies right side of s-plane

$y(\infty) = \infty$ unbounded

26. Ans: (d)

27.

Sol: Given $X(s) = \frac{s+2}{s-2}$

$y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$

$Y(s) = \frac{2}{3}.\frac{1}{s-2} + \frac{1}{3}e^{-t}u(t)$

$Y(s) = \frac{2}{3}.\frac{1}{s-2} + \frac{1}{3}\frac{1}{s+1}$

$\Downarrow \qquad \Downarrow$

$\sigma < 2 \qquad \sigma > -1$

(a). $\therefore\ H(s)=\dfrac{Y(s)}{X(s)}$

$$=\frac{\frac{1}{3}\left[\frac{2(s+1)+s-2}{(s-2)(s+1)}\right]}{\left[\frac{s+2}{s-2}\right]}\quad \begin{array}{c}\sigma<2,\sigma>-1,\sigma>0\\ \Downarrow\\ \sigma>-1\end{array}$$

$$=\frac{1}{3}\frac{3s}{(s+1)(s+2)}$$

$$=\frac{s}{(s+1)(s+2)},\sigma>-1$$

(b). The input is $e^{3t}\ \forall t$

$\therefore$ the output $= H(3)\times$ input

$$=\frac{3}{4\times5}e^{3t}$$

$$y(t)=\frac{3}{20}e^{3t}$$

28.

Sol: $H(s)=\dfrac{s^2+s-2}{s+3}$

$$H_{inv}(s)=\frac{1}{H(s)}=\frac{s+3}{(s+2)(s-1)}$$

$\sigma>+1$ causal unstable

Does not exist in this case a causal & stable system

29. Ans: (c)

Sol:

(a) A system to be stable & causal all the poles of the system should lie in the left half of s-plane.

(b) Any system property like causality, stability doesn't depend on the location of zero's. It depends only on poles location.

(c) There is no necessity that the poles lie within |s| = 1

All the roots of characteristic equation means all the poles of the system should lie in left half of s-plane.

30. Ans: (a)

Sol: $Y(s)=\dfrac{1}{s+2},H(s)=\dfrac{s-1}{s+1}$

$$X(s)=\frac{Y(s)}{H(s)}=\frac{s+1}{(s-1)(s+2)}=\frac{2/3}{s-1}+\frac{1/3}{s+2}$$

Stable input $-2<\sigma<1$

$$x(t)=-\frac{2}{3}e^{t}u(-t)+\frac{1}{3}e^{-2t}.u(t)$$

31. Ans: (d)

Sol: A solution to the differential equation with the input set to zero is often referred to as the natural response of the system.

32. Ans: (a)

Sol: input, $x(t)=e^{s_0 t}$

$$\Rightarrow X(s)=\frac{1}{s-s_0}$$

Output, Y(s) = X(s) H(s)

$$=\frac{H(s)}{s-s_0}$$

On partial fraction expansion,

$$Y(s)=\frac{H(s_0)}{s-s_0}$$

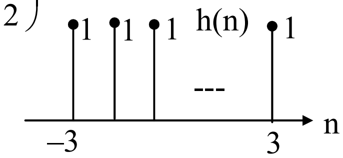
So, $y(t)=H(s_0)e^{s_0 t}$

Chapter 6 DTFT

Class Room Practice Solutions

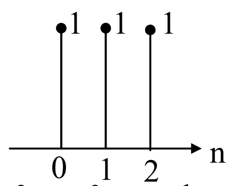
01.
Sol:

(a) $H(\omega)=\dfrac{\sin\left(\frac{7\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$



h(n) ≠ 0 n < 0 – non – causal

(b)



h(n) = 0 n < 0 causal

(c) $h(n) = \delta(n-3) + \delta(n+2)$ - non causal

02.

Sol: (a) $Y\left(e^{j\omega}\right)=\dfrac{1}{1-\frac{1}{4}e^{-j\omega}}$

$$Y\left(e^{j0}\right)=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$$

(b) $X\left(e^{j\omega}\right)=\displaystyle\sum_{n=-\infty}^{\infty}x(n)e^{-j\omega n}$

$\omega = \pi$

$$X\left(e^{j\pi}\right)=\sum_{n=-\infty}^{\infty}x(n)(-1)^n=\cos^3(3\pi)=-1$$

(c) $H\left(e^{j\omega}\right)=1+2e^{-j\omega}+3e^{-2j\omega}+4e^{-3j\omega}$

DC gain $H(e^{j\omega}) = 1+2+3+4 = 10$

03.
Sol:

(i) $X(e^{j\omega})=1+e^{j\omega}+e^{-j\omega}+\frac{3}{2}[1+\cos 2\omega]$

$$X(e^{j\omega})=1+e^{-j\omega}+e^{j\omega}+\frac{3}{2}\left[1+\frac{e^{2j\omega}+e^{-2j\omega}}{2}\right]$$

$$X(e^{j\omega})=1+e^{-j\omega}+e^{j\omega}+\frac{3}{2}+\frac{3}{4}e^{2j\omega}+\frac{3}{4}e^{-2j\omega}$$

$$X(e^{j\omega})=\sum_{n=-\infty}^{\infty}x(n)e^{-j\omega n}$$

$x(0)=1+\frac{3}{2}=\frac{5}{2}$, $x(1) = 1$, $x(-1) = 1$,

$x(2) = \frac{3}{4}$, $x(-2) = \frac{3}{4}$

$$x(n)=\left[\frac{3}{4},1,\frac{5}{2},1,\frac{3}{4}\right]$$
$$\uparrow$$

(ii) $x(n) = 2\delta(n + 3) - 3\delta(n - 3)$

$X(e^{j\omega}) = 2e^{3j\omega} - 3\,e^{-3j\omega} = 2[e^{3j\omega} - e^{-3j\omega}] - e^{-3j\omega}$

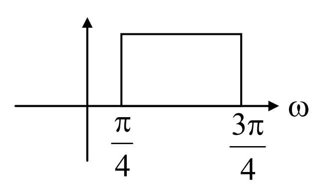
$X(e^{j\omega}) = 4j\sin(3\omega) - e^{-3j\omega}$

Given $X(e^{j\omega}) = a\sin(b\omega) + ce^{jd\omega}$

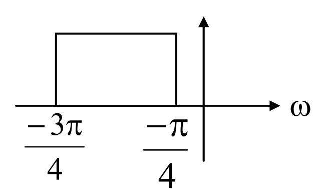
$a = 4j$, $b = 3$, $c = -1$, $d = -3$

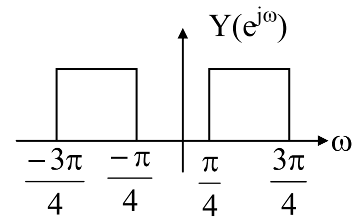
04.

Sol: $\dfrac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}.e^{j\frac{\pi}{2}n} \leftrightarrow$



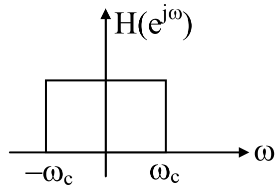
$\dfrac{\sin\left(\frac{\pi}{4}n\right)}{\pi n}.e^{-j\frac{\pi}{2}n} \leftrightarrow$





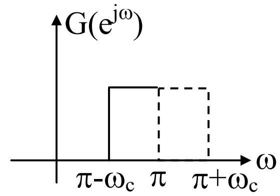
05.

Sol: (a)



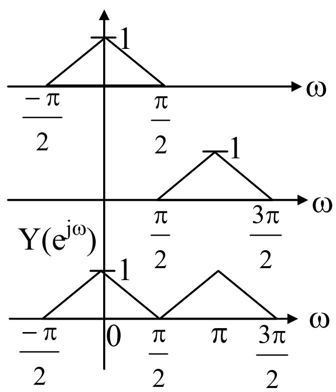
$$g(n) = (-1)^n . h(n)$$

$$G(e^{j\omega}) = H(e^{j(\omega-\pi)})$$



Ideal HPF

(b) $Y(e^{j\omega}) = X(e^{j\omega}) + X(e^{j(\omega-\pi)})$



$Y(e^{j0}) = 1$, $Y(e^{j\pi}) = 1$

06.

Sol: $$\left(\frac{1}{2}\right)^{\frac{n}{10}} . u\left(\frac{n}{10}\right) \leftrightarrow \frac{1}{1-\frac{1}{2}e^{-j10\omega}}$$

07. Ans: (b)

Sol: $x(2n) = [1, \underset{\uparrow}{3}, 1]$

FT [x(2n)] = $3 + 2\cos\omega$

08.

Sol: $$na^n . u(n) \leftrightarrow j\frac{d}{d\omega}\left[\frac{1}{1-ae^{-j\omega}}\right] = \frac{ae^{-j\omega}}{\left(1-ae^{-j\omega}\right)^2}$$

09.

Sol: $$\sum_{n=0}^{\infty} n(1/2)^n . e^{-j\omega n} = \frac{\frac{1}{2}e^{-j\omega}}{\left(1-\frac{1}{2}e^{-j\omega}\right)^2}$$

Put ω = 0 in above equality

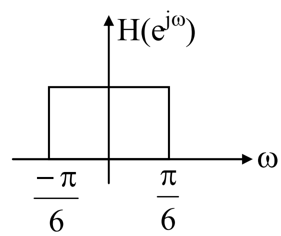
$$\sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = \frac{1/2}{\left(1-\frac{1}{2}\right)^2} = 2$$

10.

Sol: $$ne^{jn\frac{\pi}{8}}\alpha^{n-3} . u(n-3) \leftrightarrow j\frac{d}{d\omega}\left[\frac{e^{-3j(\omega-\pi/8)}}{1-\alpha e^{-j(\omega-\pi/8)}}\right]$$

11.

Sol:



Input signal frequencies are $\frac{\pi}{8}, \frac{\pi}{4}$

Then the output is $y(n) = \sin\left(\frac{\pi}{8}n\right)$

12.

Sol: $x(n) = e^{j\omega_0 n}$, $\quad y(n) = e^{j\omega_0 n} . H(e^{j\omega_0})$

$$H(e^{j\omega}) = 8\sqrt{2}\cos 2\omega - 4\sqrt{2}\cos\omega$$

$$\omega_0 = \frac{\pi}{4}$$

$$H(e^{j\omega_0}) = -4 \qquad y(n) = -4e^{jn\frac{\pi}{4}}$$

13.

Sol: $H(e^{j\omega}) = 2\,\alpha\cos\omega + \beta$

$$H(e^{j\omega})\Big|_{\omega=\frac{2\pi}{3}} = 0 \quad H(e^{j\omega})\Big|_{\omega=\frac{2\pi}{8}} = 1$$

$\alpha = \beta$ $\qquad \alpha\sqrt{2} + \beta = 1$

$$\beta = \frac{1}{1+\sqrt{2}}$$

DC gain = $H(e^{j0}) = 3\alpha = \frac{3}{1+\sqrt{2}}$

14.

Sol: $H\left(e^{j\omega}\right) = \frac{b+e^{-j\omega}}{1-ae^{-j\omega}}$

$|H(e^{j\omega})|^2 = 1 \Rightarrow H(e^{j\omega}).H^*(e^{j\omega}) = 1$

$$\left[\frac{b+e^{-j\omega}}{1-ae^{-j\omega}}\right]\left[\frac{b+e^{j\omega}}{1-ae^{j\omega}}\right] = 1$$

Only when $a = -b$

15. Ans: (a)

Sol: $H\left(e^{j\omega}\right) = 1 + \alpha e^{-j\omega} + \beta e^{-2j\omega}$

$x(n) = 1+4\cos n\pi$

$x_1(n) = 1 \quad \omega = 0$

$|H(e^{j0})| = 1 + \alpha + \beta \quad \angle H(e^{j0}) = 0$

$y_1(n) = 1+\alpha+\beta$

$x_2(n) = 4\cos n\pi \quad \omega = \pi$

$|H(e^{j\pi})| = 1 - \alpha + \beta \quad \angle H(e^{j\pi}) = 0$

$y_2(n) = 4\,(1-\alpha+\beta)\cos n\pi$

$y(n) = (1+\alpha+\beta)+4(1-\alpha+\beta)\cos n\pi$

$y(n) = 4$ only when $\alpha = 2, \beta = 1$

16. Ans: (a)

Sol: $Y\left(e^{j0}\right) = \sum_{n=0}^{2} x(n). \sum_{n=0}^{4} h(n) = 15LB$

17.

Sol: $y(n) = x(n) + 2x\,(n-1) + x(x-2)$

$Y(e^{j\omega}) = X(e^{j\omega})\,[\,1+2e^{-j\omega} + e^{-2j\omega}]$

$H(e^{j\omega}) = [\,1+e^{-j\omega}\,]^2$

$= [\,1+\cos\omega - j\sin\omega]^2$

(a) $|H(e^{j\omega})| = (1+\cos\omega)^2 + \sin^2(\omega)$

$$\angle H(e^{j\omega}) = -2\tan^{-1}\frac{\sin\omega}{1+\cos\omega}$$

$10 \rightarrow \omega = 0 \quad \Rightarrow |H(e^{j\omega})| = 1$

$\angle H(e^{j\omega}) = 0°$

$4\cos\left(\frac{\pi n}{2}+\frac{\pi}{4}\right) \rightarrow \omega = \frac{\pi}{2} \Rightarrow \left|H\left(e^{j\omega}\right)\right| = 2$

$\Rightarrow \angle H(e^{j\omega}) = -90°$

(b) Output of given input $10 + 4\cos\left(\frac{\pi n}{2}+\frac{\pi}{4}\right)$ is

$$10 + 4(2)\cos\left(\frac{\pi n}{4}+\frac{\pi}{4}-\frac{\pi}{2}\right)$$

$$= 10 + 8\cos\left(\frac{\pi n}{4}-\frac{\pi}{4}\right)$$

18. Ans: (b)

Sol: anti symmetric, $k = -2$

$\theta(\omega) = -2\omega$

Slope $= -2$

19. Ans: (b)

Sol: $x(n) = \cos\left(\frac{5\pi}{2}n\right) = \cos\left(\frac{\pi}{2}n\right) \qquad \omega_0 = \frac{\pi}{2}$

$|H(e^{j\omega})| = 1 \quad \angle H\left(e^{j\omega_0}\right) = -\frac{\pi}{8}$

$$y(n) = \cos\left(\frac{n\pi}{2}-\frac{\pi}{8}\right)$$

20. Ans: (a)

Sol: $X(e^{j\omega}) = 2 + 2\cos\omega + e^{-5j\omega} + 2e^{-4j\omega}$

$$X\left(e^{\frac{j\pi}{4}}\right) = \frac{1+j}{\sqrt{2}}$$

$$\angle X\left(e^{j\pi/4}\right) = \tan^{-1}\left(\frac{1/\sqrt{2}}{1/\sqrt{2}}\right) = \frac{\pi}{4}$$

21.

Sol: $H(\omega)=\sum_{n=-\infty}^{\infty} h(n)e^{-j\omega n}=\sum_{n=0}^{2} h(n)e^{-j\omega n}$

$$=\frac{1}{3}+\frac{1}{3}e^{-j\omega}+\frac{1}{3}e^{-2j\omega}$$

$$=\frac{1}{3}e^{-j\omega}\left[e^{j\omega}+e^{-j\omega}\right]+\frac{1}{3}e^{-j\omega}$$

$$=\frac{1}{3}e^{-j\omega}[2\cos\omega]+\frac{1}{3}e^{-j\omega}$$

$$H(\omega)=\frac{2}{3}e^{-j\omega}\cos\omega+\frac{1}{3}e^{-j\omega}$$

$$H(\omega)=\frac{1}{3}e^{-j\omega}[1+2\cos\omega]$$

$$H(\omega)=0\Rightarrow\frac{1}{3}e^{-j\omega}[1+2\cos\omega]=0 \text{ only}$$

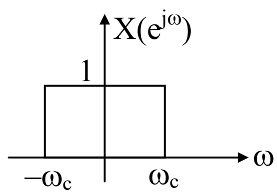
when

$1+2\cos\omega=0$

$$\cos=-\frac{1}{2}=\cos\left(\frac{2\pi}{3}\right)$$

$$\omega=\frac{2\pi}{3}=2.093 \text{ rad}$$

22.
Sol:



$$E=\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1d\omega=\frac{\omega_c}{\pi}$$

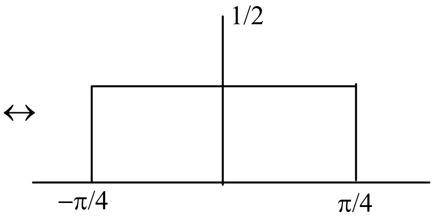
23. Ans: $\frac{1}{40}$

Sol: By plancheral's relation

$$\sum_{n=-\infty}^{\infty} x(n)y(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)Y\left(e^{j\omega}\right)d\omega$$

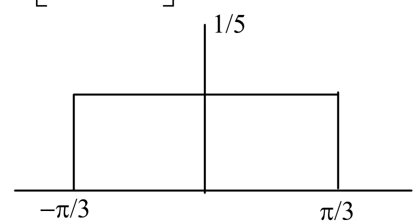
$$x(n)=\frac{\sin\left(\frac{n\pi}{4}\right)}{2\pi n}=\frac{1}{2}\left[\frac{\sin\left(\frac{n\pi}{4}\right)}{\pi n}\right]$$

$$\frac{1}{2}\left[\frac{\sin\left(\frac{n\pi}{4}\right)}{\pi n}\right]\leftrightarrow$$



$$y(n)=\frac{1}{5}\left[\frac{\sin\left(\frac{n\pi}{3}\right)}{\pi n}\right]$$

y(n)



$$\sum_{n=-\infty}^{\infty}\frac{\sin\frac{n\pi}{4}}{2\pi n}\times\frac{\sin\frac{n\pi}{3}}{5\pi n}=\frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{1}{2}\right)\left(\frac{1}{5}\right)d\omega$$

$$=\frac{1}{40}$$

24.
Sol:

(a). $X\left(e^{j0}\right)=\sum_{n=-\infty}^{\infty} x(n)=6$

(b). $X\left(e^{j\pi}\right)=\sum_{n=-\infty}^{\infty}(-1)^n x(n)=2$

(c). $\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)d\omega=2\pi x(0)=4\pi$

(d). $\int_{-\pi}^{\pi} X\left(e^{j\omega}\right)e^{2j\omega}d\omega=2\pi x(2)=0$

(e). $\int_{-\pi}^{\pi}\left|X\left(e^{j\omega}\right)\right|^2 d\omega = 2\pi\left[\sum_{n=-\infty}^{\infty}\left|X(n)\right|^2\right] = 28\pi$

(f). $\int_{-\pi}^{\pi}\left|\frac{d}{d\omega}X\left(e^{j\omega}\right)\right|^2 d\omega = 2\pi\left[\sum_{n=-\infty}^{\infty}\left|nx(1)\right|^2\right]$

$= 158\times 2\pi = 316\pi$

(g). $\angle X\left(e^{j\omega}\right) = -\alpha\omega = -\left(\frac{N-1}{2}\right)\omega = -5\omega$

25. Ans: (d)

Sol: f(n) = h(n) * h(n)

	1	2	2
1	1	2	2
2	2	4	4
2	2	4	4

f(n) = { $\underset{\uparrow}{1}$, 4, 8, 8, 4} ⇒ causal

g(n) = h(n) * h(−n)

h(−n) = { 2 2 $\underset{\uparrow}{1}$ }

↑

h(−n) ranges from n = −2 to n = 0

h(n) ranges from n = 0 to n = 2

∴ g(n) ranges from n = −2 to n = 2

	1	2	2
2	2	4	4
2	2	4	4
1	1	2	2

g(n) = { 2, 6, $\underset{\uparrow}{9}$, 6, 2}

⇒ g(n) is non causal and maximum value is 9.

26.

Sol: $\frac{2\pi\times 5k}{40k} \le \omega \le \frac{2\pi\times 10k}{40k}$

FS = $2f_m$

= 2×20k

= 40k

$+\frac{\pi}{4} \le \omega \le \frac{\pi}{2}$

27. Ans: (a)

Sol: $x(nT_s) = \cos(-20nT_s) = \cos\left(\frac{-20n}{1000}\right)$

$= \cos\left(\frac{n\pi}{4}\right) = \cos\left(\frac{9\pi}{4}\right)$

$\frac{\Omega_0}{1000} = \frac{\pi}{4}$; $\frac{\Omega_0}{1000} = \frac{9\pi}{4}$

$\Omega_0 = 250\pi,\ 2250\pi$

Chapter 7 Z - Transforms

Class Room Practice Solutions

01.

Sol: $X(z)=\frac{z}{z+1}-\frac{z}{z-\alpha}$

ROC = $(|z| > 1) \cap (|z| < |\alpha|) = 1 < |z| < 2$

Only when $\alpha = \pm 2$, 'n_0' any value

02.

Sol: (a) finite duration both sided $0 < |z| < \infty$

(b) finite duration right sided $|z| > 0$

(c) infinite duration right sided $(|z| > 1/2) \cap (|z| > 3/4) = |z| > 3/4$

(d) $(|z|>1/3) \cap (|z|< 3) \cap (|z| > 1/2) = 1/2< |z|<3$

03. Ans: (a)

Sol: ROC = $(|z| > |a|) \cap (|z| < |b^2|)$ common ROC exists only when $|a| < |b^2|$

04. i) Ans: (b)

Sol: ROC = $(|z| > |a|) \cap (|z| > |b|) \cap (|z| < |c|)$
$= |b| < |z| < |c|$

ii) ROC = $(|Z| > |\alpha|) \cap (|Z| < |\beta|)$

$X(z)=\frac{Z}{Z-\alpha}-\frac{Z}{Z-\beta}$

(a) $\alpha > \beta$ no Z.T

(b) $\alpha < \beta$ Z.T is exist

(c) $\alpha = \beta$ no Z.T

05. Ans: (c)

Sol: $X(z)=\frac{-1/2}{1-\frac{1}{2}z^{-1}}+\frac{3/2}{1+\frac{1}{2}z^{-1}}$

$x(n)=-\frac{1}{2}\left(\frac{1}{2}\right)^n u(n)+\frac{3}{2}\left(\frac{-1}{2}\right)^n .u(n)$

$x(2) = 1/4$

06. Ans: (d)

Sol: poles = $j, -j$, zeros = 0,0

$X(z)=\frac{kz^2}{z^2+1}$

$X(1) = 1 \Rightarrow k = 2$

$X(z)=\frac{2z^2}{z^2+1}$

07. Ans: (b)

Sol: $X(z) = \sum_{n=0}^{\infty}\frac{3^n}{2+n}z^{2n}$

$= \frac{1}{2}+z^2+\frac{9}{4}z^4+-----$

$x(n) = \left\{-----,\frac{9}{4},0,1,0,\underset{\uparrow}{\frac{1}{2}}\right\}$

Now consider (a) option

$Y_1(z) = \sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n z^{-n}$

$= 1+\frac{2}{3}z^{-1}+\frac{9}{4}z^{-2}+-----$

$\sum_{n=-\infty}^{\infty}x(n)\,y_1(n) \neq 0$

Now consider option (b)

$Y_2(z) = z^{-1} + 4z^{-3} + ------$

$y_2(n) = \{0, 1, 0, 4, ------\}$

$\sum_{n=-\infty}^{\infty}x(n)\,y_2(n) = 0$

08. Ans: r = –1/2

Sol: $H(z) = \frac{1}{1-\frac{1}{2}z^{-1}}+\frac{r}{1+\frac{1}{4}z^{-1}}$

$= \frac{1+\frac{1}{4}z^{-1}+r(1-\frac{1}{2}z^{-1})}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{4}z^{-1})}$

Consider the numerator

$$1+\frac{1}{4}z^{-1}+r\left(1-\frac{1}{2}z^{-1}\right)$$

$$(1+r)+\left(\frac{1}{4}-\frac{r}{2}\right)z^{-1}$$

$$\text{zero} = \frac{-\left(\frac{1}{4}-\frac{r}{2}\right)}{1+r}$$

If zero = 1

$$\frac{\frac{1}{4}-\frac{r}{2}}{1+r}=1 \Rightarrow \frac{1}{4}-\frac{r}{2}=1+r$$

$$\frac{-3r}{2}=\frac{3}{4} \Rightarrow r=-1/2$$

If zero = –1

$$\frac{\frac{1}{4}-\frac{r}{2}}{1+r}=-1 \Rightarrow \frac{1}{4}-\frac{r}{2}=-1-r$$

$$\frac{r}{2}=\frac{-5}{4} \Rightarrow r=-5/2 \text{ is not valid}$$

Because given as $|r|<1$

09. Ans: (a)

Sol: $H(z)=\dfrac{z^4}{z^4+\frac{1}{4}}$

$H(z)=H(z^{-1})$

h(n) = h(–n)

So h (n) is real for all 'n'

10.

Sol: $(-3)^n.u(n-2) \leftrightarrow \dfrac{9z^{-1}}{z+3}, |z|>3$

$$(-3)^{-n}.u(-n-2) \leftrightarrow \frac{9z}{z^{-1}+3}, |z|<\frac{1}{3}$$

11.

Sol: g(n) = δ(n) –δ(n–6)

$G(z) = 1-z^{-6}$, |z| > 0

12.

Sol: $X(z)=z^2+2z+\dfrac{2z}{z-2}$

$x(n) = \delta(n+2) + 2\delta(n+1) - 2\,(2)^n u(-n-1)$

13.

Sol: $(a)^n x(n) \leftrightarrow X(z/a)$

$$X\left(\frac{Z}{e^{-j\pi/6}}\right) \leftrightarrow \left(e^{-j\frac{\pi}{6}}\right)^n x(n)$$

$$X\left(\frac{Z}{e^{j\pi/6}}\right) \leftrightarrow \left(e^{j\frac{\pi}{6}}\right)^n x(n)$$

$$y(n)=j\left(e^{-jn\frac{\pi}{6}}x(n)-e^{jn\frac{\pi}{6}}x(n)\right)$$

$$y(n)=j\left[-2j\sin\left(\frac{n\pi}{6}\right)\right]x(n)$$

$$y(n)=2x(n)\sin\left(\frac{n\pi}{6}\right)$$

$$X(z)=\frac{1}{z-\frac{3}{4}}$$

$$x(n)=\left(\frac{3}{4}\right)^{n-1}u(n-1)$$

$$y(n)=2\left(\frac{3}{4}\right)^{n-1}u(n-1)\sin\left(\frac{n\pi}{6}\right)$$

14.

Sol: $x(n)=\left(\dfrac{5}{4}\right)^n u(n)+\left(\dfrac{10}{7}\right)^n u(n)$

$$\left(\frac{5}{4}\right)^n u(n) \leftrightarrow \frac{z}{z-\frac{5}{4}}, \quad |z|>5/4$$

$$\left(\frac{7}{10}\right)^n u(n) \leftrightarrow \frac{z}{z-\frac{7}{10}} \quad |z|>\frac{7}{10}$$

$$\left(\frac{7}{10}\right)^{-n} u(-n) \leftrightarrow \frac{z^{-1}}{z^{-1}-\frac{7}{10}} \quad |z^{-1}|>\frac{7}{10}$$

$$\left(\frac{10}{7}\right)^n u(-n) \leftrightarrow \frac{\frac{1}{z}}{\frac{1}{z}-\frac{7}{10}} \quad |z| < \frac{10}{7}$$

$$X(z) = \frac{z}{z-\frac{5}{4}} + \frac{\frac{1}{z}}{\frac{1}{z}-\frac{7}{10}} \quad \text{ROC}$$

$$\left(|z| > \frac{5}{4} \cap |z| < \frac{10}{7}\right)$$

$$\text{ROC} = \frac{5}{4} < |z| < \frac{10}{7}$$

15.

Sol: $X(z) = z^4 + z^2 - 2z + 2 - 3z^{-4}$

$H(z) = 2z^{-3}$

$Y(z) = X(z).H(z) = 2z + 2z^{-1} - 4z^2 + 4z^{-3} - 6z^{-7}$

$y(4) = 0$

16.

Sol: $x_1(n+3) \leftrightarrow \dfrac{z^3}{1-\frac{1}{2}z^{-1}}, |z| > \dfrac{1}{2}$

$x_2(-n+1) \leftrightarrow \dfrac{z^{-1}}{1-\frac{1}{3}z}, |z| < 3$

$$Y(z) = \frac{z^2}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z\right)}, \frac{1}{2} < |z| < 3$$

17.

Sol: $H(z) = \dfrac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)}$

$X(z) = 1 - \frac{1}{3}z^{-1}$

$$Y(z) = H(z).X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$\Rightarrow y(n) = \left(\frac{1}{2}\right)^n .u(n)$$

18. Ans: (a)

Sol: When $\alpha = \beta$ it gives linear phase.

19.

Sol: (1) $x(n) = z_0^n$, $y(n) = z_0^n H(z_0)$

$y(n) = (-2)^n . H(-2) = 0$

$H(-2) = 0$

$$(2)\ H(z) = \frac{Y(z)}{X(z)} = \frac{1 + a.\frac{1}{1-\frac{1}{4}z^{-1}}}{\frac{1}{1-\frac{1}{2}z^{-1}}}$$

(a) $H(-2) = 0$

$a = \dfrac{-9}{8}$

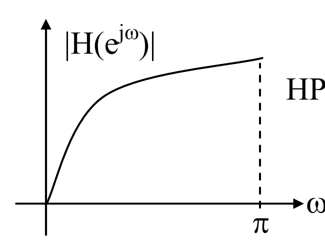
(b) $y(n) = (1)^n . H(1)$

$H(1) = -1/4$

$y(n) = \dfrac{-1}{4}(1)^n$

20. Ans: (a)

Sol: $H(e^{j\omega}) = e^{-2j\omega} - e^{-3j\omega}$



And it is FIR Filter because finite coefficients

21. Ans: (a)

Sol: y(n) = h(n) * g(n)

$Y(e^{j\omega}) = H(e^{j\omega})\, G(e^{j\omega})$

$$\Rightarrow Y(e^{j\omega}) = \frac{G(e^{j\omega})}{\left[1-\frac{1}{2}e^{-j\omega}\right]}$$

$$\Rightarrow G(e^{j\omega}) = Y(e^{j\omega}) - \frac{1}{2}e^{-j\omega}\, Y(e^{j\omega})$$

$$\Rightarrow g(n) = y(n) - \frac{1}{2}y(n-1)$$

Put n = 1

$$\Rightarrow g(1) = y(1) - \frac{1}{2}\, y(0)$$

$$= \frac{1}{2} - \frac{1}{2}$$

g(1) = 0

22. Ans: (c)

Sol: $H(e^{j\omega}) = 1-e^{-6j\omega} = 0$ only when

$6\omega = 2\pi n$ (n = 1)

$$\omega = \frac{\pi}{3}$$

$$\frac{2\pi\times f}{9k} = \frac{\pi}{3}$$

f = 1.5k

23.

Sol: $x(n) = -0.5\,(2)^n\,.\,u(-n-1)$

x(0) = 0

24.

Sol: $x(n) = \begin{cases} 1 & n\,\text{even} \\ 0 & n\,\text{odd} \end{cases}$

$$\Rightarrow X(Z) = 1 + Z^{-2} + Z^{-4} + \ldots.$$

$$= \frac{1}{1-Z^{-2}}$$

$$= \frac{1}{(1-Z^{-1})(1+Z^{-1})}$$

$$x(\infty) = \underset{Z\to1}{Lt}(1-Z^{-1})X(Z)$$

$$= \underset{Z\to1}{Lt}(1-z^{-1})\frac{1}{(1+Z^{-1})(1-Z^{-1})}$$

$$= \frac{1}{2}$$

25.

Sol:

(a) $$h(n) = \frac{\delta(n)+\delta(n-1)+\delta(n-2)}{10}$$

$$H(z) = \frac{1+z^{-1}+z^{-2}}{10} = \frac{z^2+z+1}{10z^2}$$

2 finite poles, 2 finite zeros

(b) Given x(n) = u(n)

$$X(z) = \frac{1}{1-z^{-1}}$$

$$Y(z) = H(z)\,X(z) = \frac{(1+z^{-1}+z^{-2})}{10(1-z^{-1})}$$

$$y(\infty) = \underset{z\to1}{Lt}(1-z^{-1})Y(z)$$

$$= \underset{z\to1}{Lt}(1-z^{-1})\left[\frac{1+z^{-1}+z^{-2}}{10}\right]\left[\frac{1}{1-z^{-1}}\right]$$

$$y(\infty) = \frac{1+1+1}{10} = \frac{3}{10}$$

26. Ans: (a)

Sol: $$H(z) = \frac{6z^{-1}}{1-0.7z^{-1}+0.3z^{-2}}$$

Z = 1 (static gain)

H(1) = 10

27. Ans: (c)

Sol: $Y(z) = H(z).X(z)$

$$= \frac{A}{1-z^{-1}} + \frac{1}{\left(1-\frac{1}{3}z^{-1}\right)(1-z^{-1})}$$

$$y(\infty) = \underset{z\to 1}{Lt}(1-z^{-1})Y(z)$$

$$\Rightarrow A + \frac{3}{2} = 0$$

$$A = \frac{-3}{2}$$

28. Ans: (c)

Sol: $H(z) = \dfrac{\beta z - 2z^2}{2z^2 - \alpha}$

Pole = $\pm\sqrt{\dfrac{\alpha}{2}}$

$\left|\sqrt{\dfrac{\alpha}{2}}\right| < 1 \Rightarrow |\alpha| < 2$, any value of 'β'

29.

Sol:

(a). An LTI system is stable if and only if ROC includes jω axis.
$-0.5 < \text{Re}\{s\} < 2$

(b). For an LTI system to be stable, all the poles must lie left side of the jω axis
S = 2 is the pole in the right half of s-plane.
So it is not possible.

(c). Re {s} –3
Re { s} > 2
$-3 < \text{Re}\{s\} < -0.5$
$-0.5 < \text{Re}\{s\} < 2$ are the four possible ROC's

30. Ans: (d)

Sol: $H(z) = \dfrac{\left(z - \frac{3}{4}e^{j\theta}\right)\left(z - \frac{3}{4}e^{-j\theta}\right)}{z - \frac{4}{3}}$

Numerator order > denominator order so – non causal system & $|z| < \dfrac{4}{3}$ - stable

31. Ans: (d)

Sol: Poles ⇒ $1 - 0.5Z^{-1} = 0 \Rightarrow Z = 0.5$
Zeros ⇒ $1 - 2Z^{-1} = 0 \Rightarrow Z = 2$
It all zeros and poles are inside the unit circle [|Z| = 1] then it is a minimum phase system.
So given system is Non minimum phase system if all poles are inside unit circle then we can say system is causal and stable. So given system is stable.

32. Ans: (a)

Sol: $H(z) = -\dfrac{1}{2} + \dfrac{1}{2}.\dfrac{z}{z-2}$

$$h(n) = \frac{-1}{2}\delta(n) - \frac{1}{2}(2)^n.u(-n-1)$$

33. Ans: (c)

Sol: Poles $z = \pm 2j$
|poles| = 2
ROC = |z| < 2 because system is stable.
In this case system is non-causal

34. Ans: (c)

Sol: $H(z) = \dfrac{z}{z + \frac{1}{2}}$ is a stable system because

pole $z = -\dfrac{1}{2}$ is inside the unit circle.

The poles of H(z) should be inside the unit circle for a stable system.
∴ A is True but R is false.

35. Ans: (c)

Sol: $H(z) = \dfrac{z^2+1}{(z+0.5)(z-0.5)}$

(1) The system is stable because poles $z = \pm 0.5$ are inside the unit circle.

(2) $h(0) = \underset{z\to\infty}{Lt} H(z) = 1$

(3) $\omega = \dfrac{2\pi f}{f_s} = \dfrac{2\pi \times \frac{f_s}{4}}{f_s} = \dfrac{\pi}{2}$

$$H(e^{j\omega}) = \frac{e^{2j\omega}+1}{(e^{j\omega}+0.5)(e^{j\omega}-0.5)} \text{ at } \omega = \frac{\pi}{2} = 0$$

36. Ans: (c)

Sol: A causal LTI system is stable if and only if all of poles of H(z) lie inside the unit circle.

37. Ans: (b)

Sol: In (A) and (R) replace H(s) by H(z) for the question to be meaningful.

$$H(z) = \frac{z^3 - 2z^2 + z}{z^2 + \frac{1}{4}z + \frac{1}{8}} = \frac{N(z)}{D(z)}$$

As N(z) is of higher order than D(z), the system is not causal, as δ(n + 1) is one of the terms in the output for the input δ(n).

If the N(z) is of lower order than the denominator, the system

(i) may be causal or

(ii) may not be causal as it depends upon the ROC of the given H(z).

∴ Both A and R are individually true but R is not the correct explanation of A.

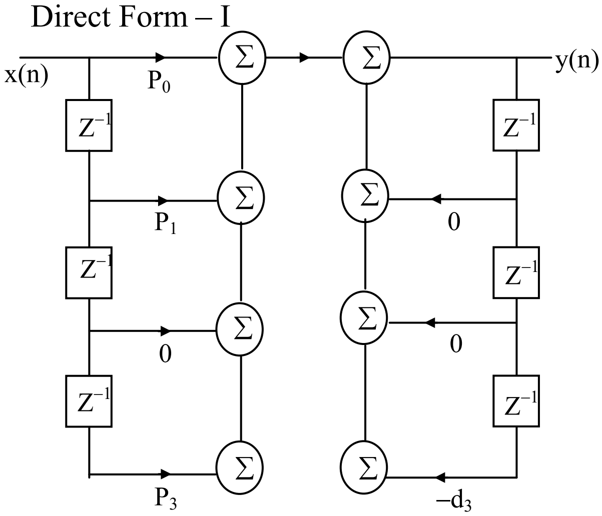
38. Ans: (a)

Sol: Both I& II are true and 'II' is the correct reason for 'I'

39. Ans: (b)

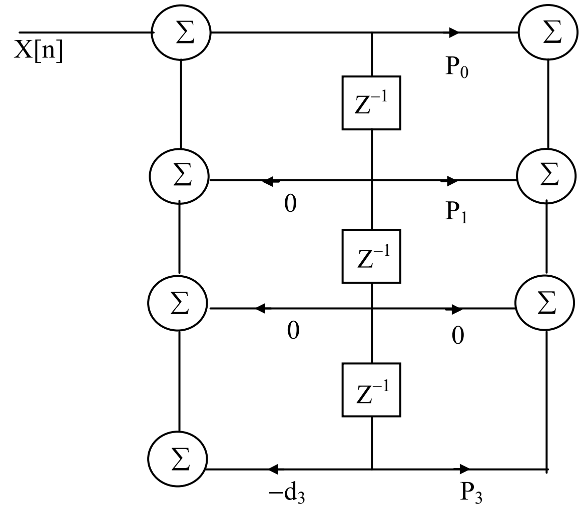
Sol: $H(Z) = \frac{P_0 + P_1 Z^{-1} + P_3 Z^{-3}}{1 + d_3 Z^{-3}}$

Direct Form – I



No. of delays = 6

Direct Form – II



No. of delay's = 3

40.

Sol: $H(z) = z^{-1} = H_1(z)\, H_2(z)$

$$H_2(z) = z^{-1}\left[\frac{1 - 0.6z^{-1}}{1 - 0.4z^{-1}}\right]$$

41. Ans: (a)

Sol: $H(z) = \frac{1}{1 - 0.7z^{-1} + 0.13z^{-2}}$

$a_0 = 1$, $a_1 = 0.7$, $a_2 = -0.13$

42.

Sol: (a) $H(Z) = \frac{1 + 0.5Z^{-1}}{1 + 0.1Z^{-1} - 0.3Z^{-2}}$

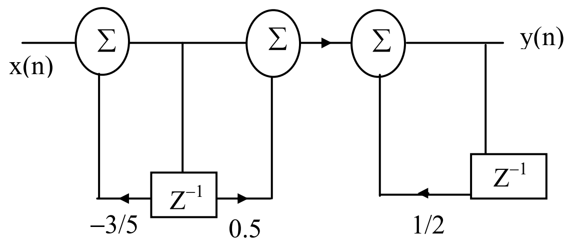
$$H(Z) = \frac{1 + 0.5Z^{-1}}{(-0.5 - 0.3Z^{-1})(-2 + Z^{-1})}$$

Cascaded form:

$$H(Z) = \frac{1 + 0.5Z^{-1}}{(-0.5 - 0.3Z^{-1})} \cdot \frac{1}{(-2 + Z^{-1})}$$

$$H(Z) = \frac{-\frac{1}{2}\left[1 + 0.5Z^{-1}\right]}{\left(1 + \frac{3}{5}Z^{-1}\right)} \frac{-2}{\left(1 - \frac{1}{2}Z^{-1}\right)}$$

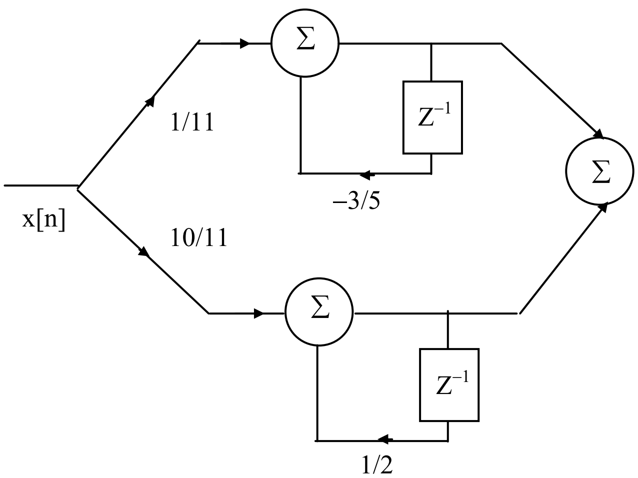
$$= \frac{1+0.5Z^{-1}}{1+\frac{3}{5}Z^{-1}} \cdot \cdot \frac{1}{1-\frac{1}{2}Z^{-1}}$$



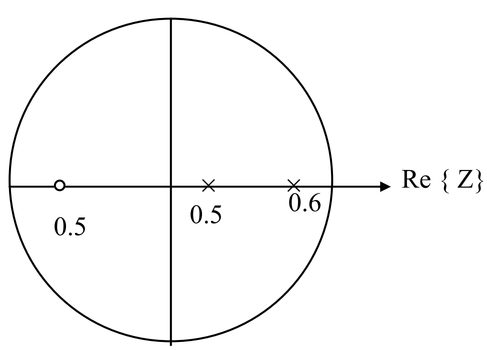
Parallel form:

$$H(Z) = \frac{\left(1+0.5Z^{-1}\right)}{\left(1+\frac{3}{5}Z^{-1}\right)\left(1-\frac{1}{2}Z^{-1}\right)}$$

$$= \frac{1}{11} \cdot \frac{1}{1+\frac{3}{5}Z^{-1}} + \frac{10}{11} \cdot \frac{1}{1-\frac{1}{2}Z^{-1}}$$



(b)



$|Z| = \frac{3}{5}, |Z| = 0.5$ are poles

$|Z| = 0.5$ is zero

43.

Sol: $H(z) = \dfrac{1-\frac{k}{4}z^{-1}}{1+\frac{k}{3}z^{-1}}$

Pole $= \left|\frac{-k}{3}\right| < 1$

$|k| < 3$

44. Ans: (c)

Sol: From signal below graph reduction

$$H(z) = \frac{2+z^{-1}}{1+2z^{-1}}$$

$$= \frac{2z+1}{z+2}$$

45. Ans: (b)

Sol: $H\left(e^{j\omega}\right) = \dfrac{2e^{j\omega}+1}{e^{j\omega}+2}$

$|H(e^{j0})| = 1$

$|H(e^{j\pi/2})| = 1$

$|H(e^{j\pi})| = 1$

So, All pass filter

46. Ans: (a)

Sol: $1 - k[z^{-1} + z^{-2}] = 0$

$z^2 - zk - k = 0$

$$z_{1,2} = \frac{+k \pm \sqrt{k^2+4k}}{2}$$

For causal & stable $|\text{poles}| < 1$

$k = 1 \Rightarrow z_{1,2} = \frac{1 \pm \sqrt{5}}{2} = \frac{1 \pm 2.236}{2}$

(outside the unit circle)

$k = 2 \Rightarrow z_{1,2} = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$

$= 1 \pm 1.732$

outside the unit circle

Here k = [–1, 1/2]

Chapter 8 Digital Filter Design

01.
Sol:

(a) $H(s) = \dfrac{1}{s+2}$

$H(s) = \dfrac{1}{s+a} \Rightarrow H(z) = \dfrac{1}{1-e^{-aT_s}z^{-1}}$

Where $T_S = \dfrac{1}{F_s} = \dfrac{1}{2}$

a = 2

$H(z) = \dfrac{1}{1-e^{-1}z^{-1}} = \dfrac{z}{z-e^{-1}}$

(b) $h(t) = e^{-2t}.u(t)$

$h(nT_s) = e^{-2nTs}\ u(nT_s) = e^{-n}.u\left(\dfrac{n}{2}\right)$

(c) $Y(s) = H(s).X(s) = \dfrac{1}{s(s+2)} = \dfrac{\left(\frac{1}{2}\right)}{s} - \dfrac{\left(\frac{1}{2}\right)}{s+2}$

$y(t) = \dfrac{1}{2}\left[1-e^{-2t}\right]u(t)$

$y(nT_S) = \dfrac{1}{2}\left[1-e^{-n}\right]u\left(\dfrac{n}{2}\right)$

04.

Sol: $H(s) = \dfrac{1}{s+a} \Rightarrow H(z) = \dfrac{1}{1-e^{-aT_S}z^{-1}}$

$f_s = 200$ Hz , $f_c = 50$ Hz

$\omega_c = \dfrac{2\pi f_c}{f_s} = \dfrac{\pi}{2}$

$H'(s) = H(s)\Big|_{s\to\frac{s}{\omega_c}} = \dfrac{s}{1.57}$

$H'(s) = \dfrac{1.57}{s+1.57}$

$H(z) = \dfrac{1.57}{1-e^{-1.57(1)}z^{-1}} = \dfrac{1.57}{1-0.208z^{-1}}$

If we want to match the gains of H(s) at s = 0 and H(z) at z = 1, the digital transfer function is extra multiplied by

$\dfrac{1}{1.98}\left[H(z)\,|_{z=1} = 1.98\right]$

$H(z) = \dfrac{1.57\left(\dfrac{1}{1.98}\right)}{1-0.208z^{-1}}$

05.
Sol:

(a) $H(z) = H(s)\Big|_{S\to\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$

$T = \dfrac{1}{F_s} = \dfrac{1}{2}$

$H(z) = H(s)\Big|_{S=4\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$

$$H(z) = \frac{3}{\left[4\left[\dfrac{1-z^{-1}}{1+z^{-1}}\right]\right]^2 + 3\left[4\left[\dfrac{1-z^{-1}}{1+z^{-1}}\right]\right] + 3}$$

$$H(z) = \frac{3\left[1+z^{-1}\right]^2}{16\left[1-z^{-1}\right]^2 + 12\left[1-z^{-2}\right] + 3\left[1+z^{-1}\right]^2}$$

(b) Gain of H(s) at ω = 3 is

$H(j\omega) = \dfrac{3}{(j\omega)^2 + 3j\omega + 3}$

$|H(j\omega)| = \dfrac{3}{\sqrt{(3-\omega^2)^2 + (3\omega)^2}}$

$|H(j\omega)|_{\omega=3} = \dfrac{3}{\sqrt{(3-9)^2+(6)^2}} = \dfrac{3}{\sqrt{(6)^2+(6)^2}}$

$= \dfrac{3}{\sqrt{72}} = \dfrac{3}{6\sqrt{2}} = \dfrac{1}{2\sqrt{2}} = 2.828$

Given f = 20 Hz

$\omega = \dfrac{2\pi\times f}{fs} = \dfrac{2\pi\times 20\,kHz}{80\,kHz} = \dfrac{\pi}{2}$

$$H(e^{j\omega}) = \frac{3(1+e^{-j\omega})^2}{16(1-e^{-j\omega})^2 + 12(1-e^{-2j\omega}) + 3(1+e^{-j\omega})^2}$$

$$H(e^{j\omega})\Big|_{\omega=\frac{\pi}{2}} = \frac{3(1-j)^2}{16(1+j)^2+12(2)+3(1-j)^2}$$

$$= \frac{3(-2j)}{16(2j)+24+3(-2j)} = \frac{-6j}{26j+24}$$

$$\left|H(e^{j\frac{\pi}{2}})\right| = \frac{6}{\sqrt{(26)^2+(24)^2}} = \frac{6}{35.38} = 0.169$$

06.

Sol:

(a) $H(s) = \frac{s}{s^2+s+1}$

$$H(j\omega) = \frac{j\omega}{-\omega^2+j\omega+1} = \frac{j\omega}{1-\omega^2+j\omega}$$

$$|H(j\omega)| = \frac{\omega}{\sqrt{(1-\omega^2)^2+\omega^2}}$$

ω	|H(jω)|
0	0
∞	0

Band pass filter

07.

Sol: $\alpha_p = 1$ db , fp = 4 kHz

$\alpha_s = 40$ db , fs = 6 kHz

FS = 24 kHz

Butter worth filter :

(1) order $N \geq \frac{\log\left[\sqrt{\frac{10^{0.1\alpha_S}-1}{10^{0.1\,\alpha_P}-1}}\right]}{\log\left[\frac{\Omega_S}{\Omega_P}\right]}$

$$\omega_p = \frac{2\pi\times f_p}{F_s} = \frac{2\pi\times 4}{24} = \frac{\pi}{3}$$

$$\omega_s = \frac{2\pi\times f_s}{F_s} = \frac{2\pi\times 6}{24} = \frac{\pi}{2}$$

$$\frac{\Omega_S}{\Omega_P} = \frac{\tan\left(\frac{\omega_S}{2}\right)}{\tan\left(\frac{\omega_P}{2}\right)} = \frac{\tan\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{6}\right)} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

$$N \geq \frac{\log\left[\sqrt{\frac{10^{0.1(40)}-1}{10^{0.1(1)}-1}}\right]}{\log(\sqrt{3})} = \frac{\log\left[\sqrt{\frac{10^4-1}{10^{0.1}-1}}\right]}{\log(\sqrt{3})}$$

$$N \geq \frac{\log\left[\sqrt{\frac{9999}{1.258}}\right]}{\log(\sqrt{3})} = \frac{\log[\sqrt{7948.33}]}{\log(\sqrt{3})}$$

$$N \geq \frac{\log[89.15]}{\log(1.732)}$$

$$N \geq \frac{1.950}{0.238}$$

$N \geq 8.19$

$N = 9$

Tchebyshew filter:

$$N \geq \frac{\cosh^{-1}\left[\sqrt{\frac{10^{0.1\alpha_S}-1}{10^{0.1\,\alpha_P}-1}}\right]}{\cosh^{-1}\left[\frac{\Omega_S}{\Omega_P}\right]}$$

$$N \geq \frac{\cosh^{-1}[89.15]}{\cosh^{-1}[1.732]} = \frac{5.183}{1.146}$$

$N \geq 4.52$

$N = 5$

08.

Sol: $\alpha_p = 0.5$ db, fp = 1.2 kHz

$\alpha_s = 40$ db, fs = 2 kHz

FS = 8 kHz

Butter worth filter :

$$\omega_P = \frac{2\pi fp}{Fs} = \frac{2\pi\times 1.2}{8} = \frac{3\pi}{10}$$

$$\omega_S = \frac{2\pi fp}{FS} = \frac{2\pi\times 2}{8} = \frac{\pi}{2}$$

$$N \geq \frac{\log\left[\sqrt{\frac{10^{0.1\alpha_S}-1}{10^{0.1\,\alpha_P}-1}}\right]}{\log\left[\frac{\Omega_S}{\Omega_P}\right]}$$

$$\frac{\Omega_S}{\Omega_P} = \frac{\tan\left(\frac{\omega_P}{2}\right)}{\tan\left(\frac{\omega_S}{2}\right)} = \frac{\tan\left(\frac{3\pi}{20}\right)}{\tan\left(\frac{\pi}{4}\right)} = 0.509$$

$$N \geq \frac{\log\left[\sqrt{\frac{10^{0.1(40)}-1}{10^{0.1(1)}-1}}\right]}{\log(1.964)}$$

$$N \geq \frac{3.949}{0.293}$$

$N \geq 13.47$

$N = 14$

Tchebyshew filter:

$$N \geq \frac{\cosh^{-1}\left[\sqrt{\frac{10^{0.1\alpha_S}-1}{10^{0.1\,\alpha_P}-1}}\right]}{\cosh^{-1}\left[\frac{\Omega_S}{\Omega_P}\right]}$$

$$N \geq \frac{\cosh^{-1}[8911]}{\cosh^{-1}[1.964]} = \frac{9.788}{1.295}$$

$N \geq 7.55$

$N = 8$

09.

Sol:

$\alpha_p = 1$ db , $\omega_p = 0.3\pi$

$\alpha_s = 60$ db , $\omega_s = 0.35\pi$

Butter worth filter :

$$\text{order} \quad N \geq \frac{\cosh^{-1}\left[\frac{10^{0.1\alpha_S}-1}{10^{0.1\,\alpha_P}-1}\right]}{\cosh^{-1}\left[\frac{\Omega_S}{\Omega_P}\right]}$$

$$\frac{\Omega_S}{\Omega_P} = \frac{\tan\left(\frac{0.35\pi}{2}\right)}{\tan\left(\frac{0.3\pi}{2}\right)} = \frac{0.612}{0.509} = 1.202$$

$$N = \frac{\cosh^{-1}\left[\frac{10^6-1}{10^{0.1}-1}\right]}{\cosh^{-1}[1.202]}$$

$$N = \frac{15.85}{0.625} = 25.36$$

$N = 26$

11.

Sol: $z_1 = \frac{1}{2}e^{j\frac{\pi}{3}}$

$z_2 = z_1^* = \frac{1}{2}e^{-j\frac{\pi}{3}}$

$z_3 = z_1^{-1} = 2e^{-j\frac{\pi}{3}}$

$z_4 = \left[z_1^*\right]^{-1} = 2e^{j\frac{\pi}{3}}$

12. Ans: (a)

Sol: $H(z) = [\,1 + 2z^{-1} + 2\,z^{-2}]\,G(z)$

Liner FIR has symmetry (or) anti symmetry

So, $G(z) = 3 + 2z^{-1} + z^{-2}$

$H(z) = [1 + 2z^{-1} + 2\,z^{-2}]\,[3 + 2z^{-1} + z^{-2}]$

$= 3 + 8z^{-1} + 10\,z^{-2} + 8z^{-3} + 3z^{-4}$

13.

Sol: (a) $H(z) = 1 + z^{-2}$

$H(z)\big|_{z=1} = 2$ Band stop filter type – I

$H(z)\big|_{z=-1} = 2$

(b) $H(z) = 1 + 2z^{-1} + 2\,z^{-2} + z^{-3}$

$H(z)\big|_{z=1} = 6$ low pass filter type – II

$H(z)\big|_{z=-1} = 0$

(c) $H(z) = 1 - z^{-2}$

$H(z)\big|_{z=1} = 0$ Band pass filter type – III

$H(z)\big|_{z=-1} = 0$

(d) $H(z) = -1 + 2z^{-1} - 2\,z^{-2} + z^{-3}$

$H(z)\big|_{z=1} = 0$ High pass filter of type-IV

$H(z)\big|_{z=-1} = -6$

14.

Sol: (a) h(n) = [2, –3, 4, 1, 4, –3, 2]

(b) h(n) = [2, –3, 4, 1, 1, 4, –3, 2]

(c) h(n) = [2, –3, 4, 1, 0, 1, 4, 3, – 2]

(d) h(n) = [2, –3, 4, 1, –1, –4, 3, –2]

16.

Sol: $$h_d(n) = \frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} e^{-3j\omega}.e^{j\omega n}d\omega = \frac{\sin\frac{\pi}{4}(n-3)}{\pi(n-3)}$$

n	$h_d(n)$	$\omega(n) = 0.54 - 0.48\cos\left(\frac{2\pi n}{6}\right)$	H(n) = $h_d(n).\omega(n)$
0	0.075	0.08	a = 6×10^{-3}
1	0.159	0.31	b = 0.049
3	1/4	1	c = 0.173
4	0.225	0.77	d = 0.25
5	0.159	0.31	c = 0.173
6	0.075	0.08	b = 0.049 a = 6×10^{-3}

$$H(z) = \sum_{n=0}^{6} h(n)z^{-4}$$
$$= a[1+z^{-6}] + b[z^{-1}+z^{-5}] + c[z^{-2}+z^{-4}] + dz^{-3}$$

Chapter 9 DFT & FFT

Class Room Practice Solutions

01.

Sol: $\Delta F = \frac{F_S}{N} = \frac{10\times 10^3}{1024}$

02.

Sol: $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2+2j \\ 2 \\ -2-2j \end{bmatrix}$

03.

Sol: i) $X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk}$

$X(0) = \sum_{n=0}^{N-1} x(n)$

Given x(n) = –x(N–1–n)

n = 0 ⇒ x(0) = – x(N–1)

n = 1 ⇒ x(1) = – x(N–2)

X(0) = x(0) + x(1) +…..+ x(N–3) + x(N–2) + x(N–1)

From the given condition x(0) and x(N–1) Cancel each other. In the same way x(1) and x(N–2) cancel each other.

So finally all the terms will cancel and becomes zero.

ii) x(n) = x(N–1–n)

$X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}\cdot\frac{N}{2}n}$

$= \sum_{n=0}^{N-1} x(n) e^{j\pi n}$

$= \sum_{n=0}^{N-1} x(n)(-1)^n$

$= x(0) - x(1) + x(2) + ... - x(N-3) + x(N-2) - x(N-1)$

Given condition is x(n) = x(N –1 –n)

n = 0 ⇒ x(0) = x(N–1)

n = 1 ⇒ x(1) = x(N–2)

From given condition, x(0), x(N–1) cancel each other.

x(1), x(N–2) cancel each other. Finally all the terms vanishes and becomes zero.

04.

Sol: **a.** $x([n-2])_4 = [4, 3, 6, 5]$

b. $x([n+1])_4 = [5, 4, 3, 6]$

c. $x([-n])_4 = [6, 3, 4, 5]$

05.

Sol: If x(n) is real X(k) = X*(N–k)

X(5) = X*(3)

X(6) = X*(2)

X(7) = X*(1)

06. Ans: (a)

Sol: [p q r s] = [a b c d] Ⓝ [a b c d]

DFT of [p q r s] = [α β γ δ]. [α β γ δ]

DFT of [p q r s] = $[\alpha^2\ \beta^2\ \gamma^2\ \delta^2]$

07.

Sol: (a) $X(0) = \sum_{n=0}^{5} x(n) = -3$

(b) Nx(0) = 6×1 = 6

(c) $\sum_{n=0}^{5} (-1)^n x(n) = 21$

(d) $N\left[\sum_{n=0}^{5} |x(n)|^2\right] = 546$

(e) Nx(3) = 6 (–4) = –24

08.

Sol: According to given signals we can say

$x_2(n) = x_1(n-4)$

$$X_2(K) = X_1(K)e^{-j}\frac{2\pi}{8}.4K$$

$$X_2(K) = e^{-j\pi K}X_1(K)$$

$$X_2(K) = (-1)^K X_1(K)$$

09.

Sol: $Y(k) = e^{-j\frac{2\pi}{6}4k}$

$y(n) = x((n-4))_6 = \{2, 1, 0, 0, 4, 3\}$

10.

Sol: $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j\frac{2\pi}{N}nk}$, $N = 10$

11.

Sol: (a) $\Delta f = \frac{f_s}{N} = \frac{20\times10^3}{10^3} = 20$

For k = 150, f = 20×150 = 3kHz

For k = 800, f = (16 – 20) kHz = –4 kHz

12. Ans: (a)

Sol: Q (K) – 3 point DFT

$$q(n) = \frac{1}{N}\sum_{K=0}^{N-1} Q(K)e^{\frac{j2\pi nK}{N}}$$

n = 0

$$q(0) = \frac{1}{3}\sum_{K=0}^{2} Q(K) = \frac{Q(0)+Q(1)+Q(2)}{3}$$

Q(0) = X(0), Q(1) = X(2), Q(2) = X(4)

$$Q(0) = X(0) = \sum_{n=0}^{N-1} x(n)$$

$$= \sum_{n=0}^{5} x(n) = 4+3+2+1 = 10$$

$$Q(1) = X(2) = \sum_{n=0}^{5} x(n).e^{\frac{-j2\pi n(2)}{6}}$$

$$= \sum_{n=0}^{5} x(n)e^{\frac{-j2\pi}{3}n}$$

$$= x(0)+x(1)e^{\frac{-j2\pi}{3}}+x(2)e^{\frac{-j4\pi}{3}}+x(3)e^{-j2\pi}$$

$$= 4+3\left[\frac{-1}{2}-j\frac{\sqrt{3}}{2}\right]+2\left[\frac{-1}{2}+\frac{j\sqrt{3}}{2}\right]+1$$

$$= 4-\frac{3}{2}-\frac{j3\sqrt{3}}{2}-1+\frac{2j\sqrt{3}}{2}+1$$

$$Q(1) = \frac{5}{2}-\frac{\sqrt{3}}{2}j$$

$$Q(2) = X(4) = \sum_{n=0}^{5} x(n)e^{\frac{-j2\pi n(4)}{6}}$$

$$= \sum_{n=0}^{5} x(n)e^{\frac{-j4\pi n}{3}}$$

$$Q(2) = x(0)+x(1)e^{\frac{-j4\pi}{3}}+x(2)e^{\frac{-j8\pi}{3}}+x(3)e^{\frac{-j4\pi(3)}{3}}$$

$$= 4+3\left[\frac{-1}{2}+\frac{j\sqrt{3}}{2}\right]+2\left[\frac{-1}{2}-\frac{j\sqrt{3}}{2}\right]+x(3).(1)$$

$$= 4-\frac{3}{2}+\frac{j\sqrt{3}(3)}{2}-1-j\frac{2}{2}\sqrt{3}+1$$

$$= \frac{5}{2}+\frac{\sqrt{3}}{2}j$$

$$q(0) = \frac{10+\frac{5}{2}-\frac{\sqrt{3}}{2}j+\frac{5}{2}+\frac{\sqrt{3}}{2}j}{3} = \frac{15}{3} = 5$$

13.

Sol: $\text{DFT}\left(x^2(n)\right) = \frac{1}{N}[X(k) \circledN X(k)] = \begin{bmatrix} 6 \\ -4j \\ -2 \\ 4j \end{bmatrix}$

14. Ans: (c)

Sol: $(-1)^n.g(n) \leftrightarrow G\left(\left(k-\frac{N}{2}\right)\right)_N$

15. Ans: (a)

Sol: (A) For 8 point DFT , value at n = 9 means value at n = 1

we know

$$x(n)=\frac{1}{N}\sum_{K=0}^{N-1}X(K)e^{j\left(\frac{2\pi}{N}\right)Kn}$$

$$\frac{1}{8}\sum_{K=0}^{7}X(K)e^{j\left(\frac{2\pi}{N}\right)K.1}=x(1)$$

(B) W(K) = X(K) + X(K + 4)

$$W(K) = X(K) + X\left(K+\frac{N}{2}\right)$$

$$w(n)=x(n)+(-1)^n x(n)$$

(C) Y(K) = 2 X(K) K = 0, 2, 4, 6

= 0 K = 1, 3, 5, 7

$\Rightarrow Y(K) = X(K) + (-1)^K X(K)$

$\Rightarrow y(n) = x(n) + x\left(n-\frac{N}{2}\right)$

16. Ans: (a)

Sol: W(k) = X(k).Y(k) = [176, 12+4j, 0, 12-4j]

$$w(2)=\frac{-1}{N}\sum_{k=0}^{3}(-1)^k.W(k)=\frac{152}{4}=38$$

17. Ans: (c)

Sol: $1 \leftrightarrow N\delta(k)$

$$(-1)^n \leftrightarrow N\delta\left(k-\frac{N}{2}\right)$$

18.

Sol: $f_m = 100$ Hz

$f_s = 200$ Hz

$\Delta f \le 0.5$ Hz

(a) DFT $\Delta f = \frac{f_s}{N}$

$$N=\frac{f_s}{\Delta f}=\frac{200}{0.5}=400$$

(b) radix – 2FFT

$N = 2^9 = 512$ samples (at N = 400)

$$\Delta f=\frac{200}{512}=0.39\text{ Hz}$$

19.

Sol:

$f_1 = 25$, $f_2 = 100$, $f_s = 800$Hz

(a) N = 100 samples

$$\Delta f=\frac{f_s}{N}=\frac{800}{8}=8\text{Hz}$$

25Hz corresponding to $\frac{25}{8} = 3.125$

100 Hz corresponding to $\frac{100}{8} = 12.5$

Both frequencies are not relating.

(b) N = 128

$$\Delta f=\frac{800}{128}=6.25\text{Hz}$$

$25\text{Hz} \rightarrow \frac{25}{6.25} = 4\text{V}$

$100\text{ Hz} \rightarrow \frac{100}{6.25} = 16\text{V}$

20.

Sol: X(K) = [1,–2,1–j,j2,0,---]

(a) $X(K) = X^*(N-K)$

$X(5) = X^*(8-5) = X^*(3) = -j2$

$X(6) = X^*(2) = 1+j$

$X(7) = X^*(1) = -2$

(b) $y(n) = (-1)^n x(n)$

Y(K) = X(K–4) last four sample will shifted to beginning

(c) $g(n) = x\left(\frac{n}{2}\right)$

Zero interpolation in time domain corresponds to replication of the DFT spectrum

21. Ans: 6

Sol: Interpolation in time domain equal to replication in frequency domain.

$x_1(n) = x\left(\frac{n}{3}\right)$

$X_1(k)$ = [12, 2j, 0, –2j, 12, 2j, 0, -2j, 12, 2j, 0, –2j]

$X_1(8) = 12$, $X_1(11) = -2j$

$$\left|\frac{X_1(8)}{X_1(11)}\right| = \left|\frac{12}{-2j}\right| = 6$$

22.

Sol:

(a) $t = 1\mu s$

N = 1024, total time to perform multiplication using DFT directly

$= (1024)^2 \times 1\mu s = 1.05$ sec

(b) by FFT, $T = \left[\frac{N}{2}\log_2 N\right]1\mu s$

$= \left[\frac{1024}{2}\log_2 1024\right]1\mu s$

= 5.12 msec

23.

Sol: $f_s = 10$ kHz, N = 1024, $\Delta f = \frac{f_s}{N}$

Over all time required for processing the entire data $= \frac{N}{f_s} = \frac{1024}{10\times 10^3}$ = 102.4 msec

Complex multiplications = 4 times real multiplications