



# **ELECTRICAL ENGINEERING**



**GATE | PSUs**

**SIGNALS  
AND  
SYSTEMS**

**Volume - I : Study Material with Classroom Practice Questions**

# Signals & Systems

## 1. Introduction

**01. Ans: (c)**

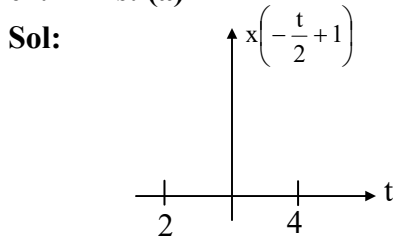
**Sol:**  $x(n) + 2x(-n) = \{-1, -1, 3, 1, 1\}$  max value = 3

$5x(n)x(n-1) = \{0, 5, 5, -5, 5, 0\}$  max value = 5

$x(n)x(-n-1) = \{0, -1, 1, 1, -1, 0\}$  max value = 1

$4x(2n) = \{4, 4, -4\}$  max value = 4

**02. Ans: (a)**



Non zero duration = 6

**03.**

**Sol:** (a) 0

(b)  $(t + \cos \pi t)|_{t=1} = 0$

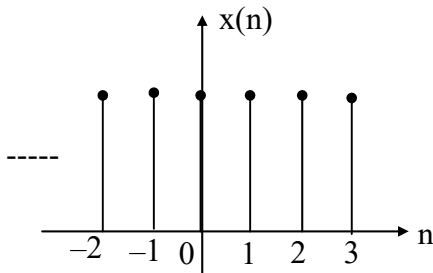
(c)  $\cos t \cdot u(t-3)|_{t=0} = 1u(-3) = 0$

(d)  $\frac{1}{2}e^{t-2}\bigg|_{t=2} = \frac{1}{2}$

(e)  $t \sin t \bigg|_{t=\frac{\pi}{2}} = \frac{\pi}{2}$

**04.**

**Sol:**  $x(n) = 1 - [\delta(n-4) + \delta(n-5) + \dots]$



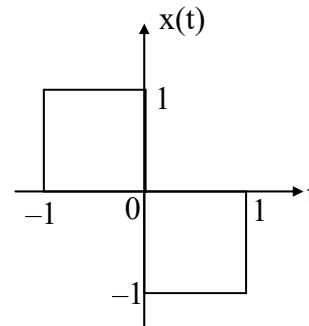
$x(n) = u(-n+3) = u(Mn-n_0)$

$M = -1$

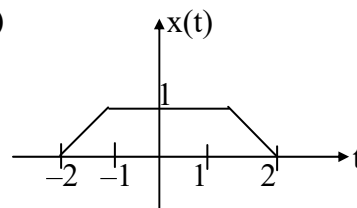
$n_0 = -3$

**05.**

**Sol:** (a)



(b)



**06.**

**Sol:** (a) as  $t \rightarrow \infty$ , amp  $\rightarrow 0$  Energy signal

(b) Constant amp – Power signal

(c) Power + energy = Power signal

(d) Periodic signal  $\rightarrow$  Power signal

(e) as  $t \rightarrow \infty$ , amp  $\rightarrow \infty$  NENP

(f) as  $t \rightarrow \infty$ , amp  $\rightarrow \infty$  NENP

**07.**

**Sol:**  $E_{x_1(n)} = \sum_{n=-\infty}^{\infty} |x_1(n)|^2 = \sum_{n=0}^{\infty} (\alpha(0.5)^n)^2$

$$= \sum_{n=0}^{\infty} \alpha^2 (0.25)^n$$



$$= \alpha^2 \sum_{n=0}^{\infty} (0.25)^n = \frac{\alpha^2}{1-0.25} = \frac{\alpha^2}{0.75}$$

$$E_{x_2(n)} = \sum_{n=-\infty}^{\infty} |x_2(n)|^2 = 1.5 + 1.5 = 3$$

$$\text{Given } E_{x_1(n)} = E_{x_2(n)}$$

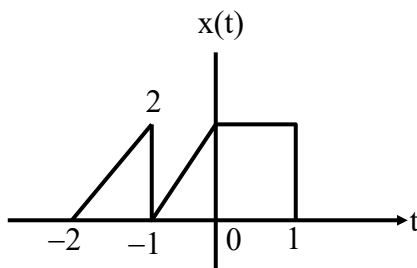
$$\frac{\alpha^2}{0.75} = 3 \Rightarrow \alpha^2 = 2.25 \Rightarrow \alpha = 1.5$$

08.

$$\begin{aligned} \text{Sol: } x_{oc}(n) &= \frac{x(n) - x^*(-n)}{2} \\ &= \left[ \frac{1+j7}{2}, 0, \frac{-1+j7}{2} \right] \end{aligned}$$

09.

Sol:



10.

$$\text{Sol: (a) } T_1 = \frac{1}{9}, T_2 = \frac{1}{6}$$

$$\frac{T_1}{T_2} = \frac{2}{3} \text{ LCM} = 3$$

$$T_0 = \text{LCM} \times T_1 = 1/3$$

$$\text{(b) } T_1 = \frac{15}{11}, T_2 = 15$$

$$\frac{T_1}{T_2} = \frac{1}{11}$$

$$\text{LCM} = 11$$

$$T_0 = \text{LCM} \times T_1 = 15$$

$$\text{(c) } T_1 = \frac{2\pi}{3}, T_2 = \frac{2}{5}$$

$$\frac{T_1}{T_2} = \frac{5\pi}{3} \text{ irrational number}$$

So a non-periodic.

$$\text{(d) } T_0 = \frac{2\pi}{10} = \frac{\pi}{5}$$

(e) It is extending from 0 to  $\infty$

So non-periodic

$$\text{(f) } x_c(t) = \frac{x(t) + x(-t)}{2} = \frac{1}{2} \cos 2\pi t$$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{2\pi} = 1$$

$$\text{(g) } \frac{\omega_0}{2\pi} = \frac{5}{6} - \text{rational, so periodic}$$

$$N_0 = \frac{2\pi}{\omega_0} m = \frac{6}{5} m$$

$$N_0 = 6$$

$$\text{(h) } N_1 = 8m \Rightarrow N_1 = 8$$

$$N_2 = 16m \Rightarrow N_2 = 16$$

$$N_3 = 4m \Rightarrow N_3 = 4$$

$$\frac{N_1}{N_2} = \frac{1}{2}, \frac{N_1}{N_3} = 2$$

$$\text{LCM} = 2$$

$$N_0 = \text{LCM} \times N_1 = 16.$$

$$\text{(i) } \frac{\omega_0}{2\pi} = \frac{7}{2} - \text{rational, so periodic}$$

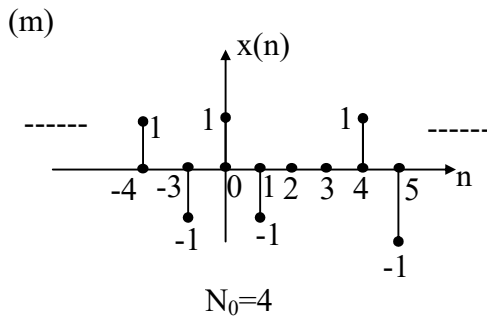
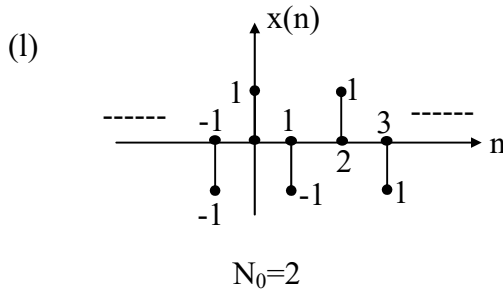
$$N_0 = \frac{2\pi}{\omega_0} m = \frac{2}{7} m$$

$$N_0 = 2$$

(j) multiplication of one periodic & non-periodic is non-periodic



(k)  $u(n) + u(-n) = 1 + \delta(n)$  is non-periodic



11.

**Sol: (a)**  $x(nT_s) = 2\cos(150\pi \times n \times T_s + 30^\circ)$

$$= 2\cos\left(\frac{3\pi}{4}n + 30^\circ\right)$$

$$\omega_0 = \frac{3\pi}{4}$$

$$N_0 = \frac{2\pi}{\omega_0} m = \frac{8}{3}m$$

$$N_0 = 8$$

**(b) Ans: (a)**

$$N_1 = \frac{2}{3}m \Rightarrow N_1 = 2$$

$$N_2 = \frac{2}{7}m \Rightarrow N_2 = 2$$

$$N_3 = \frac{20}{25}m \Rightarrow N_3 = 4$$

$$\frac{N_1}{N_2} = 1, \frac{N_1}{N_3} = \frac{1}{2}, \text{LCM} = 2$$

$$N_0 = \text{LCM} \times N_1 = 4$$

$$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$x(n) = \cos(6\omega_0 n) + \sin(14\omega_0 n) + \cos(5\omega_0 n)$$

So 14<sup>th</sup> harmonic

12.

**Sol: (a)**  $[x_1(t) + x_2(t)][x_1(t-2) + x_2(t-2)]$

$$\neq x_1(t)x_1(t-2) + x_2(t)x_2(t-2)$$

is non linear

$$(b) \sin[x_1(t) + x_2(t)] \neq \sin[x_1(t)] + \sin[x_2(t)]$$

is non linear

$$(c) \frac{d}{dt} [\alpha x_1(t) + \beta x_2(t)] = \frac{\alpha dx_1(t)}{dt} + \frac{\beta dx_2(t)}{dt}$$

is linear

$$(d) 2[x_1(t) + x_2(t)] + 3 \neq 2[x_1(t) + x_2(t)] + 6$$

is non linear

$$(e) \int_{-\infty}^t [\alpha x_1(t) + \beta x_2(t)] dt$$

$$= \alpha \int_{-\infty}^t x_1(t) dt + \beta \int_{-\infty}^t x_2(t) dt \text{ is linear}$$

$$(f) [x_1(t) + x_2(t)]^2 \neq x_1^2(t) + x_2^2(t)$$

is non linear

$$(g) [\alpha x_1(t) + \beta x_2(t)] \cos \omega_0 t$$

$$= \alpha x_1(t) \cos \omega_0 t + \beta x_2(t) \cos \omega_0 t \text{ is linear}$$

$$(h) \log[x_1(n) + x_2(n)] \neq \log[x_1(n)] + \log[x_2(n)]$$

is non linear

$$(i) |x_1(n) + x_2(n)| \neq |x_1(n)| + |x_2(n)|$$

is non linear



(j)  $\alpha * x^*(n) \neq \alpha x^*(n)$  is non linear

(k) non linear

$$(l) \frac{x_1(n) + x_2(n)}{x_1(n-1) + x_2(n-1)} \neq \frac{x_1(n)}{x_1(n-1)} + \frac{x_2(n)}{x_2(n-1)}$$

is non linear

(m) linear

(n)  $e^{x_1(n)+x_2(n)} \neq e^{x_1(n)} + e^{x_2(n)}$  is non linear

**13.**

**Sol:** (a)  $tx(t-t_0) + 3 \neq (t-t_0)x(t-t_0) + 3$

time variant

(b)  $e^{x(t-t_0)} = e^{x(t-t_0)}$  time invariant

(c)  $x(t-t_0)\cos 3t \neq x(t-t_0)\cos 3(t-t_0)$

time variant

(d)  $\sin [x(t-t_0)] = \sin [x(t-t_0)]$  time invariant

$$(e) \frac{d[x(t-t_0)]}{d(t-t_0)} = \frac{dx(t-t_0)}{dt-dt_0} = \frac{d}{dt}[x(t-t_0)]$$

time invariant

(f)  $x^2(t-t_0) = x^2(t-t_0)$  time invariant

(g)  $x(2t-t_0) \neq x(2t-2t_0)$  time variant

(h)  $2^{x(n-n_0)}x(n-n_0) = 2^{x(n-n_0)}x(n-n_0)$

time invariant

(i) time variant

(j) time variant

(k) all coefficients are constant  
– time invariant

**14.**

**Sol:**  $x_2(t) = x_1(t) - x_1(t-2)$

$$y_2(t) = y_1(t) - y_1(t-2)$$

$$x_3(t) = x_1(t+1) + x_1(t)$$

$$y_3(t) = y_1(t+1) + y_1(t)$$

**15.**

**Sol:** (a) Present output depends on present input-  
causal

(b) present output depends on present input-  
causal

(c) present output depends on present input-  
causal

(d) present output depends on future input-  
non causal

(e) present output depends on present input-  
causal

(f) present output depends on present input-  
causal

(g)  $n > n_0$  causal,  $n < n_0$  non-causal

(h) non - causal

(i)  $y(0) = \sum_{k=-\infty}^0 x(k)$  present output depends  
on present input - causal

(j)  $y(-1) = \sum_{k=0}^{-1} x(k)$  future input non causal

(k) non-causal for any value of 'm'

(l)  $\alpha = 1$  causal,  $\alpha \neq 1$  non causal

(m) causal

(n) non causal

**16.**

**Sol:** (a) present output depends on present input  
–static

(b) present output depends on present input  
–static

(c) present output depends on present input  
–static

(d) present output depends on present input  
–static

(e)  $y(1) = x(3)$  present output depends on



- future input -dynamic  
(f) dynamic  
(g) present output depends on past input  
- dynamic

17.

- Sol:** (a) linear, time variant, dynamic  
(b) linear, time invariant, dynamic  
(c) linear, time invariant, dynamic  
(d) non linear, time variant, dynamic

18.

- Sol:** (a) linear, time invariant, dynamic (a→2)  
(b) non linear, time variant, static (b→5)  
(c) linear, time variant, dynamic (c→1)  
(d) nonlinear, time invariant, dynamic (d→4)

19.

- Sol:** (a)  $y(t) = u(t) \cdot u(t) = u(t)$  - stable  
(b)  $y(t) = \cos 3t u(t) \Rightarrow -1 < y(t) < 1$  stable  
(c)  $y(t) = u(t-3)$  stable  
(d)  $y(t) = \frac{du(t)}{dt} = \delta(t)$  unstable  
(e)  $y(t) = \int_{-\infty}^t u(\tau) d\tau \Rightarrow r(t)$  is unstable  
(f)  $\sin(\text{finite}) = \text{finite}$ . stable  
(g)  $y(t) = tu(t) = r(t)$  unstable  
(h)  $y(n) = e^{\text{finite}} = \text{finite}$  stable  
(i)  $y(n) = u(3n)$  bounded stable  
(j)  $x(n) = 1 \Rightarrow y(n) = n - n_0 + 1 \Rightarrow y(\infty) = \infty$   
 $\Rightarrow$  unstable

20.

- Sol:** (a) non invertible  
(b) non invertible

- (c) invertible  
(d) non invertible  
(e) non invertible  
(f) non invertible  
(g) non invertible  
(h) invertible

21. **Ans: (b)**

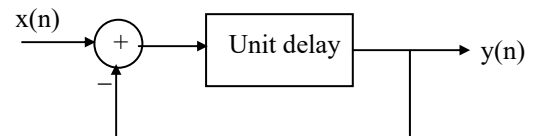
- Sol:**  $y(t) = x^2(t) * h(t)$   
 $h(t) = 0, t < 0$  causal  
Square term - non linear  
no time varying term- time invariant

22. **Ans: (b)**

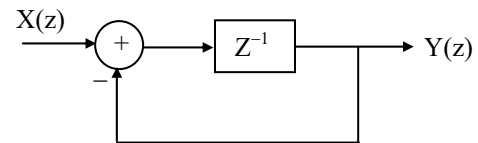
- Sol:** Constant added - non linear (A-true)  
Time varying term - time variant (R-true)

23.

**Sol:** Given



Convert to Z-domain



$$\frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 + z^{-1}} = \frac{1}{z + 1}$$

- (i)  $x(n) = \delta(n)$ ;



$$\Rightarrow Y(z) = \frac{1}{z+1} X(z)$$

$$Y(z) = \frac{1}{z+1} 1 = \frac{1}{z+1}$$

$$Y(z) = z^{-1} \frac{z}{z+1}$$

Taking inverse Z – transform

$$y(n) = (-1)^{n-1} u(n-1)$$

if  $n = 0, 1, 2, 3, \dots$

Then  $y(n) = [0, 1, -1, 1, -1, \dots]$

(ii)  $x(n) = u(n)$ ;

$$\Rightarrow Y(z) = \frac{1}{z+1} X(z)$$

$$Y(z) = \frac{1}{z+1} \frac{z}{z-1}$$

$$\frac{Y(z)}{z} = \frac{1}{(z+1)(z-1)}$$

$$= \frac{A}{z+1} + \frac{B}{z-1}$$

$$= \frac{-\frac{1}{2}}{z+1} + \frac{\frac{1}{2}}{z-1}$$

$$Y(z) = -\frac{1}{2} \frac{z}{z+1} + \frac{1}{2} \frac{z}{z-1}$$

$$y(n) = -\frac{1}{2} (-1)^n u(n) + \frac{1}{2} u(n)$$

**24. Ans: (d)**

**Sol:** (S-I):  $y(n) = 2x(n) + 4x(n-1)$

If  $x(n)$  is bounded,  $y(n)$  is bounded.

$\therefore$  Stable. (S-I) is false.

(S-II):  $h(n) = 2\delta(n) + 4\delta(n-1)$

$$h(n) = \{2, 4\}$$

↑

Impulse response  $h(n)$  has only two finite nonzero samples. This is the condition for stability.

$\therefore$  (S-II) is True.

**25. Ans: (a)**

**Sol:** A system is memory less if output,  $y(t)$  depends only on  $x(t)$  and not on past or future values of input,  $x(t)$ .

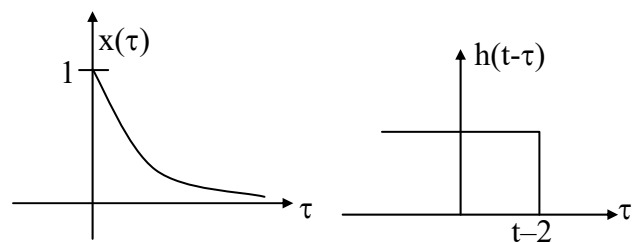
A system is causal if the output,  $y(t)$  at any time depends only on values of input,  $x(t)$  at that time and in the past.

Both (S-I) and (S-II) are true and (S-II) is the correct explanation of (S-I).

## 2. LTI (LSI) Systems

**01.**

$$\text{Sol: } y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$



Case (i)  $t-2 < 0$   $y(t) = 0, t < 2$

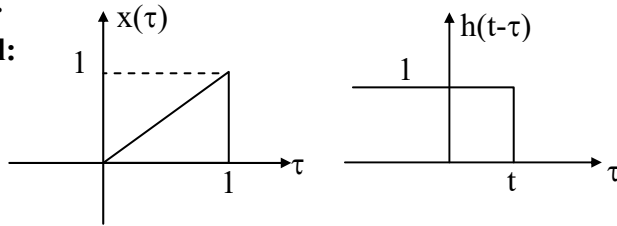
Case (ii)  $t-2 > 0$   $y(t) = \int_0^{t-2} e^{-3\tau} d\tau = \frac{1-e^{-3(t+2)}}{3}, t > 2$

$$y(t) = \frac{1-e^{-3(t+2)}}{3} u(t-2)$$



02.

Sol:



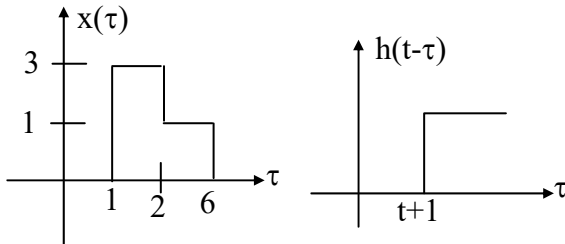
Case (i)  $t < 0$   $y(t) = 0$

Case (ii)  $0 < t < 1$   $y(t) = \int_0^t \tau d\tau = \frac{t^2}{2}$

Case (iii)  $t > 1$   $y(t) = \int_0^1 \tau d\tau = \frac{1}{2}$

03.

Sol:



$$y(4) = \int_6^5 1 d\tau = 1$$

$$y\left(\frac{1}{2}\right) = \int_{1.5}^6 x(\tau) h\left(\frac{1}{2} - \tau\right) d\tau = \frac{3}{2} + 4 = 5.5$$

04. Ans: (b)

Sol:  $s(t) = \int_{-\infty}^t h(\tau) d\tau = u(t-1) + u(t-3)$

$s(2) = 1$

05.

Sol: Assume  $-\tau + a = \lambda \Rightarrow -d\tau = d\lambda$

$$z(t) = \int_{-\infty}^{\infty} x(\lambda) h(t+a-\lambda) d\lambda = y(t+a)$$

06.

Sol: (a)  $x(t-7+5) = x(t-2)$

(b)  $x(t) * \frac{1}{|a|} \delta\left(t + \frac{b}{a}\right) = \frac{1}{|a|} x\left(t + \frac{b}{a}\right)$

(c)  $x(t) * [2\delta(t+3) + 2\delta(t-3)]$   
 $= 2x(t+3) + 2x(t-3)$

07.

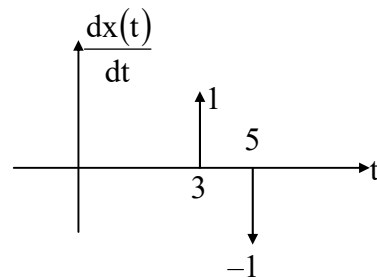
Sol: (a)  $e^{-1} u(1) \delta(t-1) = e^{-1} \delta(t-1)$

(b)  $e^{-t} \Big|_{t=1} = e^{-1}$

(c)  $e^{-(t-1)} u(t-1)$

08.

Sol:



$$\frac{dx(t)}{dt} = \delta(t-3) - \delta(t-5)$$

$$\frac{dx(t)}{dt} * h(t) = h(t-3) - h(t-5)$$

09.

Sol: (a)  $A_x A_h = A_y$ ,  $\int_{-\infty}^{\infty} \delta(\alpha t) dt = \frac{1}{\alpha}$

$$\frac{1}{\alpha} \cdot \frac{1}{\alpha} = \frac{A}{\alpha} \Rightarrow A = \frac{1}{\alpha}$$

(b)  $\frac{1}{\alpha} \cdot \frac{1}{\alpha} = \frac{A}{\alpha}$ ,  $\int_{-\infty}^{\infty} \text{sinc}(\alpha t) dt = \frac{1}{\alpha}$

$$A = \frac{1}{\alpha}$$



$$(c) 1 \times 1 = A\sqrt{2} \quad \int_{-\infty}^{\infty} e^{-at^2} dt = 1$$

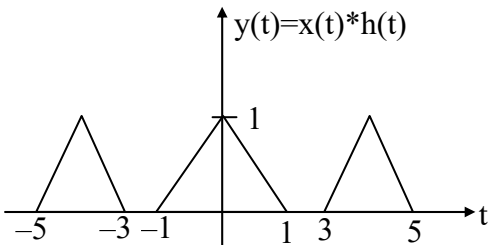
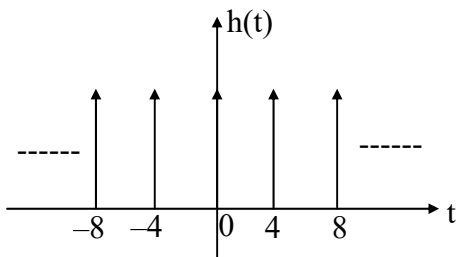
$$A = \frac{1}{\sqrt{2}}$$

$$(d) \pi \times \pi = 2A\pi \quad \int_{-\infty}^{\infty} \frac{1}{1+t^2} dt = \pi$$

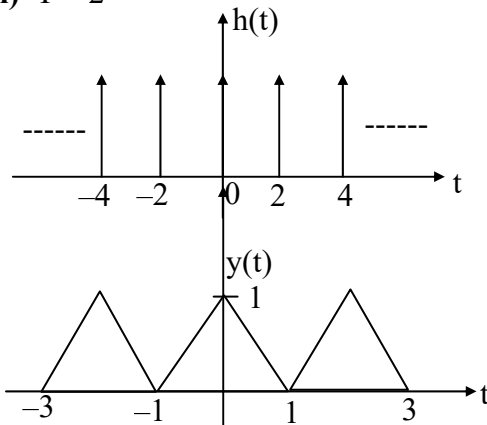
$$A = \frac{\pi}{2}$$

10.

Sol: (i)  $T = 4$

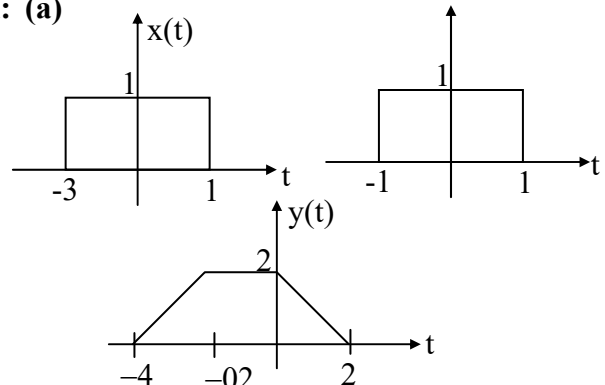


(ii)  $T = 2$



11.

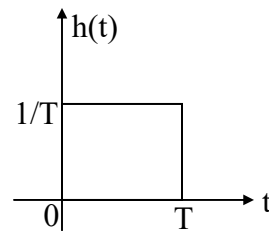
Sol: (a)



(b) Ans: (c)

$$\begin{aligned} tu(t) * u(t-1) &\leftrightarrow \frac{1}{s^2} \frac{e^{-s}}{s} \\ &= \frac{e^{-s}}{s^2} \leftrightarrow \frac{1}{2} (t-1)^2 u(t-1) \end{aligned}$$

(c)



$$h(t) = \frac{1}{T} [u(t) - u(t-T)]$$

$$x(t) = u(t)$$

$$y(t) = x(t) * h(t) = \frac{1}{T} [r(t) - r(t-T)]$$

12. Ans: (a)

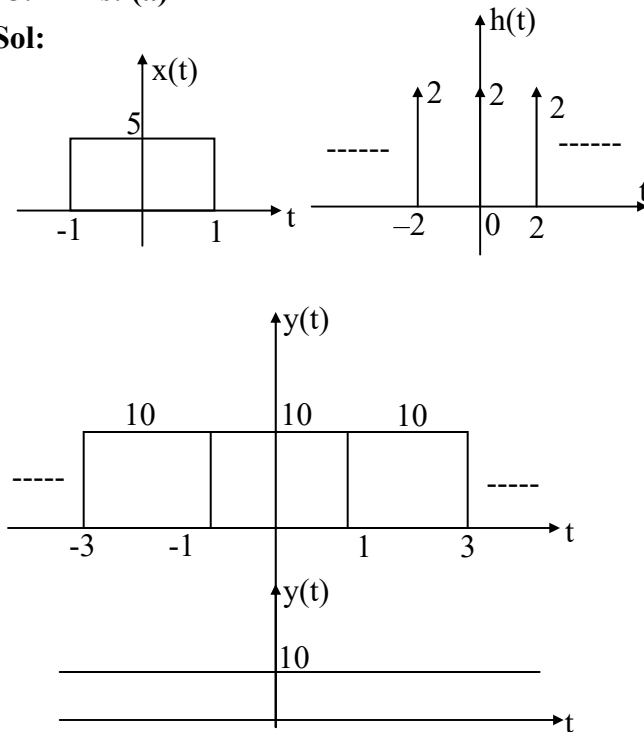
Sol: To get three discontinuities in  $y(t)$  both rectangular pulse must be same width. To get equal width  $h(t) = x(t)$ , it is possible only

$$\alpha = 1$$



13. Ans: (a)

Sol:



14. Ans: (d)

$$\begin{aligned}\text{Sol: } x(t) * h(-t) &= \int_{-\infty}^{\infty} x(\tau) h(-(t-\tau)) d\tau \\ &= \int_{-\infty}^{\infty} x(\tau) h(\tau-t) d\tau\end{aligned}$$

15.

$$\begin{aligned}\text{Sol: } y(n) &= \dots + x(-2)g(n+4) + x(-1)g(n+2) \\ &+ x(0)g(n) + x(1)g(n-2) + x(2)g(n-4) + \dots\end{aligned}$$

$$\begin{aligned}x(n) &= \delta(n-2) = 1 & n=2 \\ &= 0 & \text{otherwise}\end{aligned}$$

$$y(n) = g(n-4)$$

16.

$$\text{Sol: } y(n) = x(n) * h(n)$$

$$= 2(0.5)^n u(n) + (0.5)^{n-3} u(n-3)$$

$$y(1) = 1, y(4) = 5/8$$

17. Ans: (a)

$$\text{Sol: } y(n) = [a, b, c, d, a, b, c, d, \dots N \text{ times}]$$

$y(n)$  is a periodic function with periodic '4'.

$$\text{So } h(n) \text{ must be } h(n) = \sum_{i=0}^{N-1} \delta(n-4i)$$

18. Ans: (b)

$$\text{Sol: } \int_{-\infty}^{\infty} |h(t)| dt = \int_0^{\infty} e^{\alpha t} dt + \int_{-\infty}^0 e^{\beta t} dt < \infty$$

only when  $\alpha < 0, \beta > 0$

19.

$$\text{Sol: (a) } h(n) = \alpha^n u(n) + \beta \alpha^{n-1} u(n-1)$$

$$\text{(b) } h(n) = 0 \quad n < 0 \text{ causal}$$

System stable for any value of ' $\beta$ '

except  $\beta \neq \infty$  and  $|\alpha| < 1$ , except  $\alpha = 0$

20.

$$\text{Sol: (a) } \left(\frac{1}{5}\right)^n u(n) - A \left(\frac{1}{5}\right)^{n-1} u(n-1) = \delta(n)$$

$$\text{When } n=1, A = 1/5$$

$$\text{(b) } H(z) = \frac{1}{1 - \frac{1}{5} z^{-1}}$$

$$H_{\text{inv}}(z) = 1 - \frac{1}{5} z^{-1}$$

$$g(n) = \delta(n) - \frac{1}{5} \delta(n-1)$$

21.

$$\text{Sol: } h_1(n) = \delta(n) - \frac{1}{2} \delta(n-1)$$



$$h_1(n) * h_2(n) = \left(\frac{1}{2}\right)^n u(n) - \frac{1}{2} \left(\frac{1}{2}\right)^{n-1} u(n-1)$$

$$= \left(\frac{1}{2}\right)^n \delta(n) = \delta(n)$$

22.

**Sol:** 1. False

2.  $h(t) = e^{2t}u(t-1)$  is causal, un stable - false

3.  $h(t) = \sin \omega_0 t$ ,  $\int_{-\infty}^{\infty} |h(t)| dt = \int_{-\infty}^{\infty} |\sin \omega_0 t| dt = \infty$   
unstable - true

4.  $y(t) = x(t-2) \rightarrow$  causal-- false

$x(t) = y(t+2) \rightarrow$  non causal ----false

23.

**Sol:** (a)  $h(t) = e^{-2(t-1)}u(t-1)$

(b)  $h(t) = 0 \quad t < 0$  causal

$\int_1^{\infty} e^{-2(t-1)} dt < \infty$  is stable

24. **Ans: (a)**

**Sol:**  $s(t) = u(t) - e^{-\alpha t}u(t)$

$$h(t) = \frac{ds(t)}{dt} = \delta(t) - [e^{-\alpha t}\delta(t) - \alpha e^{-\alpha t}u(t)]$$

$$= \alpha e^{-\alpha t}u(t)$$

26.

**Sol:**  $s(n) = \sum_{k=-\infty}^n h(k) = \sum_{k=-\infty}^n \left(\frac{1}{2}\right)^k u(k)$

$$= \sum_{k=0}^n \left(\frac{1}{2}\right)^k \quad n \geq 0$$

$$= 0 \quad n < 0$$

$$s(n) = 2 \left[ 1 - \left(\frac{1}{2}\right)^{n+1} \right] u(n)$$

27.

**Sol:**  $x(n) = u(n)$ ,  $y(n) = \delta(n)$

$$u(n) - u(n-1) = \delta(n)$$

$$y(n) = x(n) - x(n-1)$$

$$x(n) = nu(n)$$

$$y(n) = nu(n) - nu(n-1) + u(n-1)$$

$$= n\delta(n) + n(n-1)$$

$$= u(n-1)$$

28.

**Sol:**  $h_c(t) = h_1(t) * h_2(t)$

$$\int_{-\infty}^t h_c(\tau) d\tau = \int_{-\infty}^t h_1(\tau) d\tau * h_2(\tau)$$

$$= h_1(\tau) * \int_{-\infty}^t h_2(\tau) d\tau$$

$$s_c(t) = s'(t) * s_2(t)$$

$$= s_1(t) * s'_2(t)$$

$$s_c(t) \neq s_1(t) * s_2(t)$$

0

### 3. Fourier series

01. **Ans :Zero**

**Sol:**  $T_1 = \frac{\pi}{2}$ ,  $T_2 = \frac{\pi}{6}$

$$\frac{T_1}{T_2} = 3, T_0 = \text{LCM} \times T_1 = \frac{\pi}{2}$$

$$\omega_0 = 4$$

$$x(t) = 3\sin(\omega_0 t + 30^\circ) - 4\cos(3\omega_0 t - 60^\circ)$$

second harmonic amplitude = 0



**02. Ans: (d)**

**Sol:** (a) periodic – exists  
(b) periodic – exists  
(c) periodic – exists  
(d) non-periodic – not exist

**03.**

**Sol: (P) Ans: (b)**

Hidden symmetry  $a_0, b_n$  exists

**(Q) Ans: (b)**

Half wave symmetry  $a_n, b_n$  exists with odd harmonics

**(R) Ans: (b)**

Odd symmetry & HWS  $\rightarrow$  sine terms with odd 'n'

**(S) Ans: (c)**

Even and odd HWS  $\rightarrow a_0$ , cosine with odd 'n'

**(T) Ans: (d)**

$a_0 = 0$  (because average value = 0)

Even & HWS as cosine with odd 'n'

**04. Ans: (b)**

**Sol:**  $f_1 = 5\text{Hz}, f_2 = 15\text{Hz}$

$$p = \frac{(4)^2}{2} = 8 \text{ Watts}$$

**05. Ans: (b)**

**Sol:**  $x(t) = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

**06. Ans: (c)**

**Sol:**  $\omega = \frac{2\pi}{T}(2k), k = 1, 2, \dots$

The above frequency terms are absent.  
The above frequency contains even harmonics and also gives that sin terms are absent. only cosine terms are present  
Finally odd harmonics with cosine terms are present so,  $x(t)$  it is a even and halfwave so,  
 $x(t) = x(T-t)$  even  
 $x(t) = -x(t-T/2)$  halfwave

**07.**

**Sol: (A) Ans: (a)**

$$T_1 = 1, T_2 = 10\pi, T_3 = 8\pi, T_4 = \frac{20}{3}\pi$$

$$T_0 = 40\pi$$

$$\omega_0 = \frac{2\pi}{T_0} = 0.05 \text{ rad/sec}$$

**(B) Ans: (a)**

$$\omega_1 = 100, \quad \omega_2 = 300, \quad \omega_3 = 500$$

$$\omega = \text{G.C.D} \{ 100, 300, 500 \}$$

$$\omega = 100$$

**08. Ans: (a)**

**Sol:** Average value =  $\frac{\frac{1}{2}(2)(1) + (1)(1) + (1)(3)}{6} = \frac{5}{6}$

**09. Ans: (a)**

**Sol:**  $a_0 = \text{Average value} = 0$

**10. Ans: (d)**

**Sol:**  $T_0 = 4\text{msec}$   $f_0 = \frac{1}{T_0} = 250\text{Hz}$

$$5 f_0 = 1250 \text{ Hz}$$



**11. Ans: (b)**

**Sol:** Odd + HWS  $\rightarrow$  sine terms with odd harmonics

**12. Ans: (d)**

**Sol:**  $a_0 = \frac{1}{2} \int_{-1}^1 e^{-t} dt = \frac{e - e^{-1}}{2}$

**13. Ans: (c)**

**Sol:** Average value  $= \frac{1}{2\pi} \int_0^\pi 10 \sin t dt = \frac{10}{\pi}$

$$a_1 = \frac{2}{2\pi} \int_0^\pi 10 \sin t \cos t dt = 0$$

$$b_1 = \frac{2}{2\pi} \int_0^\pi 10 \sin t \sin t dt = 5$$

$$d_1 = \sqrt{a_1^2 + b_1^2} = 5$$

**14. Ans: (d)**

**Sol:**  $\omega_0 = \pi$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi t) + b_n \sin(n\pi t)$$

$$x(t) = A \cos(\pi t)$$

$$A = a_1 = \frac{-4}{\pi^2}$$

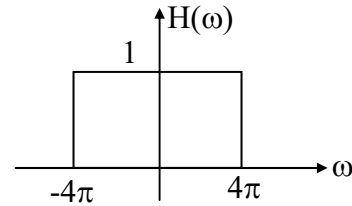
**15.**

**Sol:**  $a_0 = 5$

$$b_n = \int_0^1 10 \sin n\pi t dt = \frac{10(1 - (-1)^n)}{n\pi}$$

$$a_n = 0$$

$$x(t) = 5 + \frac{20}{\pi} \sin \pi t + \frac{20}{3\pi} \sin 3\pi t + \dots$$



$$y(t) = 5 + \frac{20}{\pi} \sin \pi t + \frac{20}{3\pi} \sin 3\pi t$$

**16.**

**Sol:**  $\omega_0 = \frac{\pi}{3}$

$$x(t) = 2 + \cos(2\omega_0 t) + 4 \sin(5\omega_0 t)$$

$$C_0 = 2, C_2 = 1/2, C_{-2} = \frac{1}{2}$$

$$C_5 = \frac{4}{2j}, C_{-5} = \frac{-4}{2j}$$

**17.**

**Sol:**  $C_n = \int_0^1 t e^{-jn\omega_0 t} dt = \int_0^1 t e^{-jn2\pi t} dt = \frac{j}{2n\pi}$

$$C_0 = 1/2$$

$$a_n = c_n + c_{-n} = 0$$

$$b_n = j(c_n - c_{-n}) = \frac{-1}{4\pi}$$

**18.**

**Sol:** (i)  $y(t) \Rightarrow d_n = e^{-jn\omega_0} c_n = e^{-jn\pi} c_n = c_n (-1)^n$

(ii)  $f(t) = x(t) - y(t)$

$$d_n = c_n - (-1)^n c_n = c_n [1 - (-1)^n]$$

(iii)  $g(t) = x(t) + y(t)$

$$d_n = c_n [1 + (-1)^n]$$



**19. Ans: (b)**

**Sol:**  $d_n = e^{-jn\omega_0 t_0} c_n + e^{jn\omega_0 t_0} c_n = 2 \cos(n\omega_0 t_0) c_n$

$$t_0 = \frac{T}{4}$$

$$d_n = 2c_n \cos\left(\frac{n\pi}{2}\right)$$

$$d_n = 0 \text{ for odd harmonics}$$

**20.**

**Sol:**  $y(t) = \frac{dx(t)}{dt}$

$$d_n = jn\omega_0 c_n$$

$$c_n = \frac{d_n}{jn\omega_0}$$

$$d_n = \frac{1}{T} \int_{-T/2}^{T/2} (\delta(t+d/2) - \delta(t-d/2)) e^{-jn\omega_0 t} dt$$

$$= \frac{2j}{T} \sin\left(\frac{n\omega_0 d}{2}\right)$$

$$C_0 = \frac{d}{T}$$

**21. Ans: (a)**

**Sol:**  $C_n = \frac{1}{T} \int_{-\tau/2}^{\tau/2} A e^{-jn\omega_0 t} dt = \frac{2A}{n\omega_0 T} \sin\left(\frac{n \times 2\pi \times \tau}{2T}\right)$

$$T = 10 \text{ sec}, \tau = 0.2 \text{ sec}$$

$$C_n = \frac{2A}{n\omega_0 T} \sin\left(\frac{n\pi}{50}\right)$$

$$n = 50 \text{ then } C_n = 0$$

**22. Ans: (c)**

**23.**

**Sol:** (a)  $x(3t) \rightarrow C_n, \omega_0 = 3\omega_0$

$$(b) \frac{dx(t)}{dt} \rightarrow jn\omega_0 C_n$$

$$(c) x(t-1) \rightarrow e^{-jn\omega_0} C_n$$

$$(d) \text{Re}[x(t)] \rightarrow \frac{C_n + C_n^*}{2} = \text{Even}(C_n)$$

$$(e) \frac{C_{n-4} + C_{n+4}}{2}$$

**24. Ans: 0.45W**

**Sol**  $C_0 = a_0 = \frac{1}{2} \quad C_n = \frac{a_n - jb_n}{2} = \frac{-j}{n\pi} (\text{oddn})$

$$a_n = 0, b_n = \frac{20}{n\pi} (\text{oddn})$$

Power up to second harmonic

$$= \sum_{n=-2}^2 |C_n|^2 = 0.45 \text{ W}$$

**25.**

**Sol:** (a) All periodic signals are power signals.

For power signal  $E = \infty$  [given is false]

(b)  $C_0 = j2$  (average value) [given is false]

$$(c) \frac{j}{T} \int_0^T x_1(t) dt = j2$$

$$\frac{1}{T} \int_0^T x_1(t) dt = 2 \text{ is possible only when}$$

$x_1(t)$  is constant. So given is correct

$$(d) C_0 = \frac{1}{T} \int_0^T x_R(t) dt + \frac{j}{T} \int_0^T x_I(t) dt$$

$$= 0 + j2$$

$$\frac{1}{T} \int_0^T x_R(t) dt = 0 \text{ only when } x_R(t) \text{ is odd}$$

given is in correct



26.

**Sol:** (a) Power =  $\frac{1}{T} \int_{-\infty}^{\infty} |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |C_n|^2$

$$P = \sum_{n=-4}^4 |C_n|^2$$

$$= (0.5)^2 + (1)^2 + (2)^2 + (4)^2 + (2)^2 + (1)^2 + (0.5)^2$$

$$= 26.5 \text{ Watts}$$

(b)  $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$

$$= C_{-4} e^{-j4\omega_0 t} + C_{-3} e^{-j3\omega_0 t} e^{-\frac{j\pi}{2}} + C_{-2} e^{-j2\omega_0 t} e^{-\frac{j\pi}{4}} + C_{-1} e^{-j\omega_0 t}$$

$$+ C_0 + C_1 e^{j\omega_0 t} + C_2 e^{j2\omega_0 t} e^{\frac{j\pi}{4}} + C_3 e^{j3\omega_0 t} e^{\frac{j\pi}{2}} + C_4 e^{j4\omega_0 t}$$

$$= 0.5 e^{-j4\omega_0 t} + 1 e^{-j3\omega_0 t - \frac{\pi}{2}}$$

$$+ 2 e^{-j2\omega_0 t - \frac{\pi}{4}}$$

$$0.5 e^{j4\omega_0 t} + 1 e^{j3\omega_0 t + \frac{\pi}{2}} + 2 e^{j2\omega_0 t + \frac{\pi}{4}}$$

$$= (0.5) [e^{-j4\omega_0 t} + e^{j4\omega_0 t}] + 2 \left[ e^{-j2\omega_0 t - \frac{\pi}{4}} + e^{j2\omega_0 t + \frac{\pi}{4}} \right]$$

$$\left[ e^{-j3\omega_0 t - \frac{\pi}{2}} + e^{j3\omega_0 t + \frac{\pi}{2}} \right] + 4$$

$$\Rightarrow x(t) = \cos 4\omega_0 t + 4 \cos \left( 2\omega_0 t + \frac{\pi}{4} \right)$$

$$+ 2 \cos \left( 3\omega_0 t + \frac{\pi}{2} \right) + 4x^*(-t)$$

$$= x(t)$$

So even symmetry

(d)  $f_0 = 10 \text{ Hz}$

$$\omega_0 = 2\pi f_0 = 20 \pi \text{ rad}$$

$$x(t) = \cos(80\pi t) + 4 \cos \left( 40\pi t + \frac{\pi}{4} \right)$$

$$+ 2 \cos \left( 60\pi t + \frac{\pi}{2} \right) + 4$$

$$\text{Cut off frequency} = 25 \text{ Hz} = 50 \pi \text{ rad}$$

So output of the filter is

$$y(t) = 4 \cos \left( 40\pi t + \frac{\pi}{4} \right) + 4$$

27. A-2, B-1, C-3, D-4

28. **Ans: (b)**

**Sol:** Frequency constant ( $S_1$ )\_LTI, frequency not constant ( $S_2$ ) - not an LTI

29. **Ans: (d)**

30. **Ans: (d)**

**Sol:** For a real valued periodic function  $f(t)$  of frequency  $f_0$

$$C_n = C_{-n}^*$$

A is False but R is True because the discrete magnitude spectrum of real function  $f(t)$  is even and phase spectrum is odd.

#### 4. Fourier Transform

01.

**Sol:**  $X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$

Unit: volt-sec (or) volt/Hz

02.

**Sol:** (a)  $X(0) = \int_{-\infty}^{\infty} x(t) dt = \text{area}$

$$= (4 \times 2) - \left( \frac{1}{2} \times 1 \times 2 \right) = 7$$

(b)  $2\pi x(0) = 4\pi$



03.

**Sol:** (i)  $x(t) = e^{-at}u(t) + e^{at}u(-t)$

$$X(\omega) = \frac{1}{a + j\omega} + \frac{1}{a - j\omega} = \frac{2a}{a^2 + \omega^2}$$

$$(ii) e^{-at}u(t) - e^{at}u(-t) \leftrightarrow \frac{-2j\omega}{a^2 + \omega^2}$$

As  $a \rightarrow 0$

$$u(t) - u(-t) \leftrightarrow \frac{2}{j\omega}$$

$$\text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$$

04.

**Sol:**  $G(\omega) = 1 + \frac{12}{\omega^2 + 9}$

$$g(t) = \delta(t) + 2e^{-3|t|}$$

05. **Ans: Zero**

**Sol:**  $x(t) = \text{rect}(t/2), \quad X(\omega) = 2\text{sinc}(\omega)$

$$y(t) = x(t) + x(t/2), \quad Y(\omega) = X(\omega) + 2X(2\omega)$$

$$y(\omega) = \frac{2\sin \omega}{\omega} + \frac{4\sin 2\omega}{\omega}$$

$$f = 1 \Rightarrow \omega = 2\pi, \quad y(2\pi) = 0$$

06. **Ans: (d)**

**Sol:**  $Y(\omega) = 3X(2\omega)$

$$y(t) = 3/2 x(t/2)$$

07.

**Sol:** i)  $1 \leftrightarrow 2\pi\delta(\omega)$

$$ii) \frac{1}{a + jt} \leftrightarrow 2\pi e^{a\omega} \cdot u(-\omega)$$

$$iii) \frac{2a}{a^2 + t^2} \leftrightarrow 2\pi e^{-a|\omega|} \cdot u(-\omega)$$

$$iv) \frac{1}{\pi t} \leftrightarrow -j \text{sgn}(\omega)$$

08.

**Sol:**  $x_1(t) = \text{rect}\left(\frac{t}{1}\right) \quad X_1(f) = \text{sinc}(f)$

$$x(t) = x\left(t - \frac{1}{2}\right) \quad X(f) = e^{-j\pi f} X(f)$$

$$\text{FT}[x(t) + x(-t)] = X(f) + X(-f) \\ = 2\cos(\pi f) \cdot \text{sinc}(f)$$

09.

**Sol:**  $u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$

$$\frac{1}{jt} + \pi\delta(t) \leftrightarrow 2\pi u(-\omega)$$

$$\frac{1}{2}\delta(t) - \frac{1}{j2\pi t} \leftrightarrow u(\omega)$$

10.

**Sol:**

$$i) x(t) = e^{-3(t-1)}u(t-1)e^{-3}$$

$$X(\omega) = e^{-j\omega}e^{-3} \frac{1}{3 + j\omega}$$

$$ii) \pi\left(\frac{t}{2}\right) \leftrightarrow 2\text{sinc}(\omega)$$

$$\pi\left(\frac{t-1}{2}\right) \leftrightarrow 2e^{-j\omega}\text{sinc}(\omega)$$

$$iii) e^{-2|t|} \leftrightarrow \frac{4}{4 + \omega^2}$$

$$e^{-2|t-2|} \leftrightarrow \frac{4e^{-2j\omega}}{4 + \omega^2}$$

11.

**Sol:** (a)  $F_1(\omega) = F(-\omega)$



(b)  $F_2(\omega) = e^{-j\omega}F(\omega) + e^{-j\omega}.F(-\omega)$

(c)  $f_3(t) = f(t-1) + f(-t-1)$

$F_3(\omega) = e^{-j\omega}F(\omega) + e^{j\omega}.F(-\omega)$

(d)  $f_4(t) = f(t-1/2) + f(-t-1/2)$

$F_4(\omega) = e^{-\frac{j\omega}{2}}F(\omega) + e^{\frac{j\omega}{2}}.F(-\omega)$

(e)  $f_5(t) = \frac{3}{2}f\left(\frac{t}{2}-1\right)$

$F_5(\omega) = 3e^{-2j\omega}F(2\omega)$

**12. Ans: (a)**

**Sol:**  $g(t) = x(t-3) - x(-t+2)$

$G(f) = e^{-j6\pi f}X(f) - e^{-j4\pi f}X(-f)$

**13.**

**Sol:** i)  $\cos \omega_0 t = \frac{1}{2} \left[ e^{j\omega_0 t} + e^{-j\omega_0 t} \right] \leftrightarrow \pi \left[ \delta(\omega + \omega_0) + \delta(\omega - \omega_0) \right]$

ii)  $\sin \omega_0 t \leftrightarrow \frac{\pi}{j} \left[ \delta(\omega - \omega_0) - \delta(\omega + \omega_0) \right]$

iii)  $e^{-at} \sin \omega_c t u(t) \leftrightarrow \frac{1}{2j} \left[ \frac{1}{a + j(\omega - \omega_c)} - \frac{1}{a + j(\omega + \omega_c)} \right]$

iv)  $\text{Arect}\left(\frac{t}{T}\right) \cos \omega_0 t = AT \left[ \text{sa}\left[\frac{\omega + \omega_0}{2}\right]T + \text{sa}\left[\frac{\omega - \omega_0}{2}\right]T \right]$

**14.**

**Sol:**  $\text{sinc}(t) \leftrightarrow \text{rect}(f)$

$\text{sinc}(t) \cos(10\pi t) \leftrightarrow \frac{1}{2} [\text{rect}(f-5) + \text{rect}(f+5)]$

**15.**

**Sol: (a)**  $\frac{1}{4} e^{-\frac{3}{4}t} x(t/4) \leftrightarrow X(4\omega + 3)$

**(b) Ans: (a)**

$X(\omega) = 2\pi\delta(\omega) + \pi[\delta(\omega - 4\pi) + \delta(\omega + 4\pi)]$

$x(t) = 1 + \cos(4\pi t)$

**16. Ans: (d)**

**Sol:**  $X(f) = \delta(f - f_0)$

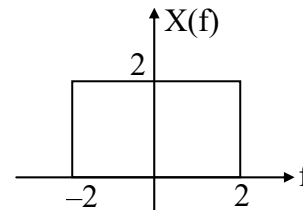
$x(t) = e^{j2\pi f_0 t}$

$x(t) \Big|_{t=\frac{1}{8f_0}} = e^{\frac{j\pi}{4}}$

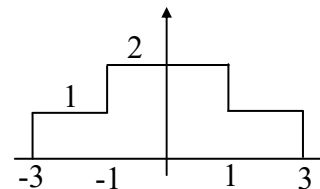
$\angle x(t) = \frac{\pi}{4}$

**17. Ans: (b)**

**Sol:**



$x(t) \cos 2\pi t \leftrightarrow \frac{1}{2} [X(f-1) + X(f+1)]$



**18. Ans: (d)**

**Sol:** Output of multiplier

$= \frac{1}{2} x(t) \cos(2\omega_c t + \theta) + \frac{1}{2} x(t) \cos \theta$

Output of the filter is  $= \frac{1}{2} x(t) \cos \theta \times 2$   
 $= x(t) \cos \theta$



19.

**Sol:**  $y(t) = \delta(t+2) + \delta(t-2)$

$$Y(\omega) = e^{2j\omega} + e^{-2j\omega} = 2\cos 2\omega$$

20. **Ans:**  $= \frac{-1}{2\sqrt{\pi}}$

**Sol:**  $x(t) = \frac{1}{2\pi} \left[ \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \right]$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega X(\omega) e^{j\omega t} d\omega$$

$$\begin{aligned} \left. \frac{dx(t)}{dt} \right|_{t=0} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega X(\omega) d\omega \\ &= \frac{1}{2\pi} \left[ \int_{-1}^0 j\omega (-j\sqrt{\pi}) d\omega + \int_0^1 j\omega (j\sqrt{\pi}) d\omega \right] \\ &= \frac{-1}{2\sqrt{\pi}} \end{aligned}$$

21.

**Sol:**  $te^{-a|t|} \leftrightarrow j \frac{d}{d\omega} \left[ \frac{2a}{a^2 + \omega^2} \right] = \frac{-4j\omega}{(a^2 + \omega^2)^2}$

$$te^{-|t|} \leftrightarrow \frac{-4j\omega}{(\omega^2 + 1)^2}$$

Apply duality property

$$\frac{4t}{(t^2 + 1)^2} \leftrightarrow -2\pi j\omega e^{-|\omega|}$$

22.

**Sol:** (i)  $X_1(\omega) = e^{-2j\omega} X(-\omega) + e^{2j\omega} X(-\omega)$

(ii)  $= \frac{1}{3} e^{-2j\omega} X\left(\frac{\omega}{3}\right)$

(iii)  $= (j\omega)^2 e^{-3j\omega} X(\omega)$

(iv)  $= j \frac{d}{d\omega} [j\omega X(\omega)]$

23.

**Sol:**  $x(t) = \text{rect}(t/2)$

$$X(\omega) = \frac{2 \sin \omega}{\omega}$$

(a).  $y_1(t) = x(t-1) \Rightarrow Y_1(\omega) = e^{-j\omega} X(\omega)$

(b).  $\Rightarrow y_2(t) = x(t) * x(t)$

$$\begin{aligned} Y_2(\omega) &= X(\omega) X(\omega) \\ &= \frac{2 \sin \omega}{\omega} \frac{2 \sin \omega}{\omega} \end{aligned}$$

$$Y_2(\omega) = 4 \frac{\sin^2 \omega}{\omega^2}$$

(c).  $y_3(t) = tx(t) \quad Y_3(\omega) = j \frac{d}{d\omega} [X(\omega)]$

(d).  $y_4(t) = x(t) \sin \pi t \leftrightarrow \frac{1}{2j} [X(\omega - \pi) - X(\omega + \pi)]$

(e).  $y_5(t) = \frac{dx(t)}{dt} \leftrightarrow j\omega X(\omega)$

(f).  $y_6(t) = (t+1)x(t) + 2u(t-1)$

(g).  $y_7(t) = y_1\left(\frac{t}{2}\right) \leftrightarrow 2Y_1(2\omega)$

(h).  $\therefore y_8(t) = y_2(2(t+1)) - y_2(2(t-1))$

$$\begin{aligned} Y_8(\omega) &= \frac{1}{2} Y_2\left(\frac{\omega}{2}\right) e^{-j\omega(-1)} - \frac{1}{2} Y_2\left(\frac{\omega}{2}\right) e^{-j\omega(1)} \\ &= \frac{1}{2} Y_2\left(\frac{\omega}{2}\right) e^{j\omega} - \frac{1}{2} Y_2\left(\frac{\omega}{2}\right) e^{-j\omega} \\ &= \frac{1}{2} Y_2\left(\frac{\omega}{2}\right) [e^{j\omega} - e^{-j\omega}] \end{aligned}$$

(i).  $y_9(t) = x\left(\frac{t}{2}\right) - \frac{1}{2} y_2(t)$

$$Y_9(\omega) = 2X(2\omega) - \frac{1}{2} Y_2(\omega)$$

(j).  $z(t) = \frac{1}{2} y_2(2t)$

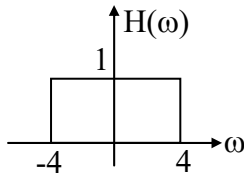


$$y_{10}(t) = z(t+1) + z(t) + z(t-1)$$

$$Y_{10}(\omega) = (1+2\cos\omega) Z(\omega)$$

24. Ans:  $y(t) = \cos 2t$

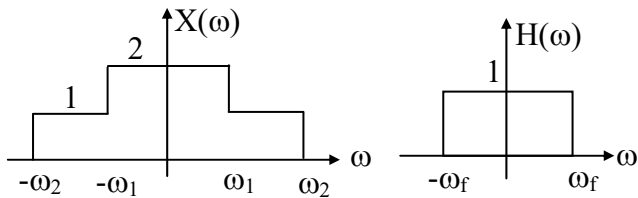
Sol:  $h(t) = \frac{\sin 4t}{\pi t}$   $H(\omega) = \text{rect}\left(\frac{\omega}{8}\right)$



$$y(t) = \cos 2t$$

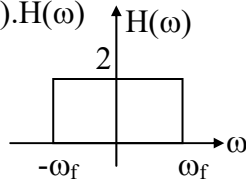
25.

Sol:  $X(\omega) = \text{rect}\left(\frac{\omega}{2\omega_1}\right) + \text{rect}\left(\frac{\omega}{2\omega_2}\right)$

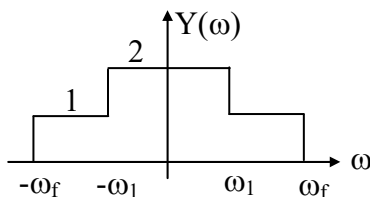


(a).  $0 < \omega_f < \omega_1$   $Y(\omega) = X(\omega).H(\omega)$

$$y(t) = \frac{2 \sin \omega_f t}{\pi t}$$



(b).  $\omega_1 < \omega_f < \omega_2$



$$y(t) = \frac{\sin \omega_1 t}{\pi t} + \frac{\sin \omega_f t}{\pi t}$$

(c).  $\omega_f > \omega_2$

$$y(t) = \frac{\sin \omega_1 t}{\pi t} + \frac{\sin \omega_2 t}{\pi t}$$

26.

Sol: (a)  $X(\omega) = \delta(\omega) + \delta(\omega-5) + \delta(\omega-\pi)$

$$x(t) = 1 + e^{-j5t} + e^{-j\pi t}$$

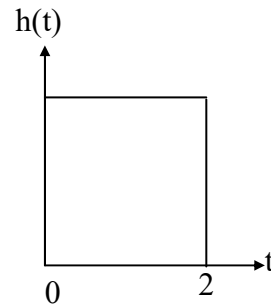
$$e^{-j\pi t} \Rightarrow T_1 = \frac{2\pi}{\pi} = 2$$

$$e^{-j5t} \Rightarrow T_2 = \frac{2\pi}{5} = \frac{2\pi}{5}$$

$$\frac{T_1}{T_2} = \frac{5}{\pi} \text{ is irrational}$$

A periodic

(b)  $h(t) = u(t) - u(t-2)$



$$\Rightarrow h(t) = \text{rect}\left(\frac{t}{2} - 0.5\right)$$

$$\text{rect}(t) \leftrightarrow \frac{\sin \frac{\omega}{2}}{\frac{\omega}{2}}$$

$$\text{rect}\left(\frac{t}{2} - 0.5\right) \leftrightarrow 2e^{-j\omega} \frac{\sin \omega}{\omega}$$



$$\Rightarrow H(\omega) = 2e^{-j\omega} \frac{\sin \omega}{\omega}$$

$$x(t) * h(t) \leftrightarrow H(\omega) X(\omega)$$

$$X(\omega)H(\omega) = [\delta(\omega) + \delta(\omega - 5) + \delta(\omega - \pi)] \left[ 2e^{-j\omega} \frac{\sin \omega}{\omega} \right]$$

$$= \delta(\omega) \underset{x \rightarrow 0}{\text{Lt}} 2e^{-j\omega} \frac{\sin \omega}{\omega} + \delta(\omega - 5) 2e^{-j5} \frac{\sin 5}{5}$$

$$+ \delta(\omega) 2e^{-j\pi} \frac{\sin \pi}{\pi}$$

$$= 2\delta(\omega) + 2e^{-j5} \frac{\sin 5}{5} \delta(\omega - 5) \left[ \underset{x \rightarrow \pi}{\text{Lt}} \frac{\sin x}{x} = 0 \right]$$

$$X(\omega)H(\omega) = 2\delta(\omega) + 2e^{-j5} \frac{\sin 5}{5} \delta(\omega - 5)$$

$$\Rightarrow x(t) * h(t) = 2 + 2e^{-j5} \frac{\sin 5}{5} e^{-j5t}$$

$\Rightarrow$  Periodic

(c). In above problem, convolution of two non periodic signals can be a periodic signal

27.

**Sol:** (a)  $y_1(t) = \text{rect}(t) * \cos \pi t$

$$\text{rect}(t) \leftrightarrow \frac{2}{\omega} \sin \frac{\omega}{2} \left[ \because Y(\omega) = \int_{-\infty}^{\infty} y(t) e^{-j\omega t} dt \right]$$

$$\text{rect}(t) \leftrightarrow \frac{\sin\left(\frac{\omega}{2}\right)}{\left(\frac{\omega}{2}\right)}$$

$$\text{rect}(t) \leftrightarrow \frac{\sin\left(\pi \cdot \frac{\omega}{2\pi}\right)}{\pi \frac{\omega}{2\pi}}$$

$$\text{rect}(t) \leftrightarrow \sin c\left(\frac{\omega}{2\pi}\right)$$

$$\cos \pi \leftrightarrow \pi [\delta(\omega - \pi) + \delta(\omega + \pi)]$$

$$Y_1(\omega) = \sin c\left(\frac{\omega}{2\pi}\right) \times \pi [\delta(\omega - \pi) + \delta(\omega + \pi)]$$

$$Y_1(\omega) = \frac{2}{\omega} \sin \frac{\omega}{2} \times \pi [\delta(\omega - \pi) + \delta(\omega + \pi)]$$

$$= \frac{2}{\omega} \sin \frac{\omega}{2} \times \pi \delta(\omega - \pi) + \frac{2}{\omega} \sin \frac{\omega}{2} \times \pi \delta(\omega + \pi)$$

$$= \frac{2}{\pi} \sin \frac{\pi}{2} \pi \delta(\omega - \pi) + \frac{2}{-\pi} \sin\left(\frac{-\pi}{2}\right) \pi \delta(\omega + \pi)$$

$$= 2 \delta(\omega - \pi) + 2 \delta(\omega + \pi)$$

$$Y_1(\omega) = \frac{2}{\pi} \pi [\delta(\omega - \pi) + \delta(\omega + \pi)]$$

Taking inverse Fourier transform

$$\therefore y_1(t) = \frac{2}{\pi} \cos \pi t$$

(b).  $y_2(t) = \text{rect}(t) * \cos 2\pi t$

Similar to above

$$Y_2(\omega) = \frac{2}{\omega} \sin \frac{\omega}{2} \times \pi [\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$$

$$= \frac{2}{\omega} \sin\left(\frac{\omega}{2}\right) \pi \delta(\omega - 2\pi) + \frac{2}{\omega} \sin\left(\frac{\omega}{2}\right) \pi \delta(\omega + 2\pi)$$

$$= \frac{2}{2\pi} \sin\left(\frac{2\pi}{2}\right) \pi \delta(\omega - 2\pi) + \frac{2}{-2\pi} \sin\left(\frac{-2\pi}{2}\right) \pi \delta(\omega + 2\pi) = 0$$

$$\therefore y_2(t) = 0$$

(c)  $y_3(t) = \sin c(t) * \sin c\left(\frac{t}{2}\right)$

$$\text{rect}(t) \leftrightarrow \sin c\left(\frac{\omega}{2\pi}\right)$$

$$\sin c\left(\frac{t}{2\pi}\right) \leftrightarrow 2\pi \text{rect}(-\omega)$$

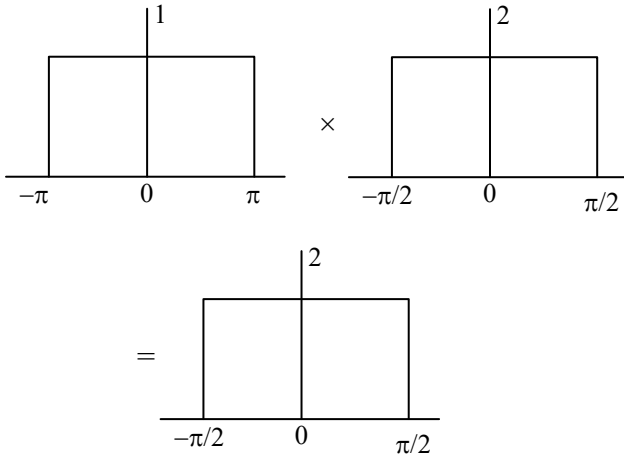
$$\sin c\left(\frac{t}{2\pi}\right) \leftrightarrow 2\pi \text{rect}(\omega)$$

$$\sin c(t) \leftrightarrow \text{rect}\left(\frac{\omega}{2\pi}\right)$$



$$\sin c\left(\frac{t}{2}\right) \leftrightarrow 2 \operatorname{rect}\left(\frac{\omega}{\pi}\right)$$

$$\therefore Y_3(\omega) = \operatorname{rect}\left(\frac{\omega}{2\pi}\right) 2 \operatorname{rect}\left(\frac{\omega}{\pi}\right)$$



$$Y_3(\omega) = 2 \operatorname{rect}\left(\frac{\omega}{\pi}\right)$$

$$Y_3(\omega) \leftrightarrow 2 \operatorname{rect}\left(\frac{\omega}{\pi}\right)$$

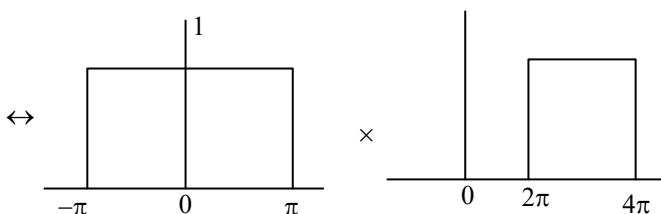
Taking inverse fourier transform

$$y_3(t) = \sin c\left(\frac{t}{2}\right)$$

$$(d). \sin c(t) \leftrightarrow \operatorname{rect}\left(\frac{\omega}{2\pi}\right)$$

$$e^{j3\pi t} \sin c(t) \leftrightarrow \operatorname{rect}\left(\frac{\omega - 3\pi}{2\pi}\right)$$

$$\sin c(t) * e^{j3\pi t} \sin c(t) \leftrightarrow \operatorname{rect}\left(\frac{\omega}{2\pi}\right) \times \operatorname{rect}\left(\frac{\omega - 3\pi}{2\pi}\right)$$



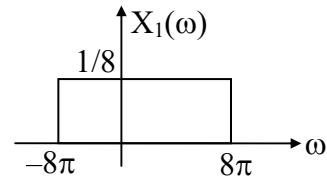
$$\leftrightarrow 0$$

$$\therefore y_4(\omega) = 0$$

$$\Rightarrow y_4(t) = 0$$

**28.**

**Sol:**



$$(a) \sin c(8t) \leftrightarrow$$

$$H(\omega) = 8e^{-j\omega} X_1(\omega) = e^{-j\omega} \quad -8\pi < \omega < 8\pi$$

$$= 0 \quad \text{otherwise}$$

$$Y(\omega) = \pi e^{-j\omega} [\delta(\omega + \pi) + \delta(\omega - \pi)]$$

$$y(t) = \cos \pi(t - 1)$$

**(b) Ans: (d)**

$$G(f) = e^{-\pi f^2} \quad H(f) = e^{-\pi f^2}$$

$$Y(f) = G(f)H(f) = e^{-2\pi f^2}$$

**29. Ans: (c)**

$$\text{Sol: } x(t) = e^{j2\pi t} e^{-\pi t^2}$$

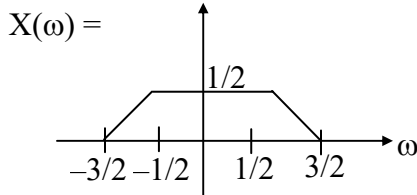
-conjugate even symmetry

**30.**

$$\text{Sol: (a) } Y(\omega) = \frac{1}{2} [X(\omega - \omega_0) + X(\omega + \omega_0)]$$

$$(b) x(t) = \frac{\sin t}{\pi t} \pi \frac{\sin(t/2)}{\pi t}$$

$$X(\omega) = \frac{1}{2\pi} \left[ \operatorname{rect}\left(\frac{\omega}{2}\right) * \pi \operatorname{rect}\left(\frac{\omega}{1}\right) \right]$$

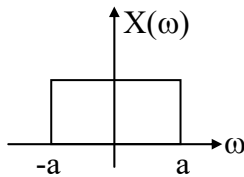


31.

**Sol:**  $\int_{-\infty}^t x(t) dt \leftrightarrow \frac{X(\omega)}{j\omega} + \pi X(0) \delta(\omega)$   
 $\leftrightarrow \frac{\text{rect}(\omega/4\pi)}{j\omega} + \pi \delta(\omega)$

32.

**Sol:**



$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \frac{2a}{\pi} = \frac{a}{\pi}$$

33.

**Sol:**  $E = \frac{1}{2\pi} \left[ \int_{-1}^{-1/2} \pi d\omega + \int_{-1/2}^{1/2} \frac{\pi}{4} d\omega + \int_{1/2}^1 \pi d\omega \right] = \frac{5}{8}$

34.

**Sol:**  $E_{x(t)} = 1/4$

$$|X(\omega)|^2 = \frac{1}{4 + \omega^2}$$

$$S_{yy}(\omega) = |X(\omega)|^2 |H(\omega)|^2 = \frac{1}{4 + \omega^2}, -\omega_c < \omega < \omega_c$$

$$E_{y(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{yy}(\omega) d\omega \Rightarrow \frac{1}{8} = \frac{1}{2\pi} \frac{1}{2} \tan^{-1} \left( \frac{\omega}{2} \right) \Big|_{-\omega_c}^{\omega_c}$$

$$\omega_c = 2 \text{ rad/sec}$$

35.

**Sol:**  $e^{-2|t|} \leftrightarrow \frac{4}{\omega^2 + 4}$

$$\int_{-\infty}^{\infty} \frac{8}{(\omega^2 + 4)^2} d\omega = 2 \int_{-\infty}^{\infty} \left( \frac{4}{\omega^2 + 4} \right)^2 d\omega$$

$$= \frac{1}{2} (2\pi) \int_{-\infty}^{\infty} |e^{-2|t|}|^2 dt$$

$$= \frac{\pi}{2}$$

36. **Ans:**  $B = \frac{2.302}{a}$

**Sol:**  $g(t) = \frac{2a}{a^2 + t^2}$

We know  $e^{-a|t|} \leftrightarrow \frac{2a}{a^2 + \omega^2}$

By duality property  $\frac{2a}{a^2 + t^2} \leftrightarrow e^{-a|\omega|}$

Given  $\int_{-B}^B |e^{-a|\omega|}|^2 d\omega = 0.99 \int_{-\infty}^{\infty} |e^{-a|\omega|}|^2 d\omega$

$$\Rightarrow \int_{-B}^0 e^{2a\omega} d\omega + \int_0^B e^{-2a\omega} d\omega = 0.99 \left[ \int_{-\infty}^0 e^{2a\omega} d\omega + \int_0^{\infty} e^{-2a\omega} d\omega \right]$$

$$\Rightarrow \left[ \frac{e^{2a\omega}}{2a} \right]_{-B}^0 + \left[ \frac{e^{-2a\omega}}{-2a} \right]_0^B = 0.99 \left[ \left[ \frac{e^{2a\omega}}{2a} \right]_{-\infty}^0 + \left[ \frac{e^{-2a\omega}}{-2a} \right]_0^{\infty} \right]$$

$$\Rightarrow \frac{1}{2a} [1 - e^{-2aB}] - \frac{1}{2a} [e^{-2aB} - 1] = \frac{0.99}{2a} [1 + 1]$$

$$\Rightarrow 2 - 2e^{-2aB} = 2 \times 0.99$$

$$\Rightarrow 1 - e^{-2aB} = 0.99$$

$$\Rightarrow 0.01 = e^{-2aB}$$

$$\Rightarrow \ln(100) = 2aB$$

$$\Rightarrow B = \frac{\ln(100)}{2a} = \frac{4.605}{2a} = \frac{2.302}{a}$$



**37. Ans: (a)**

$$\text{Sol: } E = \int_{-\infty}^{\infty} |X_1(f)|^2 df = \frac{2}{3} \times 10^{-8}$$

**38. Ans: (c)**

$$\text{Sol: } \angle H(\omega) = \frac{-\omega}{60} \quad -30\pi < \omega < 30\pi$$

$$\omega_0 = 10\pi \quad |H(10\pi)| = 2, \quad \angle H(10\pi) = \frac{-\pi}{6}$$

$$\omega_0 = 26\pi \quad |H(26\pi)| = 1, \quad \angle H(26\pi) = \frac{-13\pi}{30}$$

$$y(t) = 4 \cos\left(10\pi t - \frac{\pi}{6}\right) + \sin\left(26\pi t - \frac{13\pi}{30}\right)$$

**39.**

$$\text{Sol: } \theta(\omega) = -\omega t_0$$

$$t_p(\omega) = \frac{-\theta(\omega)}{\omega} = t_0$$

$$t_g(\omega) = \frac{-d\theta(\omega)}{d\omega} = t_0$$

Both are constant

**40.**

**Sol: (i) Ans: (c)**

$$H(f) = \frac{1}{1 + j2\pi fRC}$$

$$|H(f)| = \frac{1}{\sqrt{1 + 4\pi^2 f^2 R^2 C^2}}$$

$$|H(f_1)| \geq 0.95$$

$$f_1 = 52.2 \text{ Hz}$$

**(ii) Ans: (a)**

$$\theta(f) = -\tan^{-1}(2\pi fRC)$$

$$t_g(f) = \frac{-d\theta(f)}{df} = \frac{1}{2\pi} \left[ \frac{2\pi RC}{1 + (2\pi fRC)^2} \right]$$

$$t_g(100) = 0.71 \text{ msec}$$

**41. Ans: (c)**

$$\text{Sol: } y(t) = \frac{1}{100} \cos(100(t - 10^{-8})) \cos(10^6(t - 1.56 \times 10^{-6}))$$

$$t_g = 10^{-8}, t_p = 1.56 \times 10^{-6}$$

**42.**

**Sol:** 20 to 30 kHz no distortion

**43.**

**Sol:** For  $-200 < \omega < 200$ , there is no amplitude distortion.

And For  $-100 < \omega < 100$ , there is no phase distortion

$$x_1(t)$$

$$\omega = 20 \text{ and } \omega = 60$$

So no phase distortion and no amplitude distortion.

$$x_2(t)$$

$$\omega = 20, \quad \omega = 140$$

Amplitude distortion, do not occurs.

Phase distortion occurs.

$$[\because \omega = 140]$$

$$x_3(t)$$

$$\omega = 20, \quad \omega = 220,$$

Phase distortion and amplitude distortion occurs

$$[\because \omega = 220]$$

**44.**

$$\text{Sol: (a) } H(\omega) = \frac{1}{1 - \omega^2 + \sqrt{2}j\omega}$$

$$|H(\omega)| = \frac{1}{\sqrt{(1 - \omega^2)^2 + (\sqrt{2}\omega)^2}}$$



$$|H(\omega)|_{\omega=0} = \frac{1}{\sqrt{1+0}} = 1$$

$$(b) A = \frac{A_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$|H(\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + (\sqrt{2}\omega)^2}} = \frac{1}{\sqrt{1+\omega^4}}$$

$$\text{At } \omega = \omega_c \quad |H(\omega)| = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{1+\omega_c^4}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = 1 \text{ rad/sec}$$

$$\text{Given } |H(\omega)| = 0.01$$

$$|H(\omega_1)|^2 = \left(\frac{1}{100}\right)^2 \frac{1}{1+\omega_1^4} = \frac{1}{10000}$$

$$\omega_1 \approx 10 \text{ rad/sec}$$

(c)

$$\begin{array}{ccc} A \cos(\omega_0 t) & \boxed{H(\omega)} & A |H(\omega_0)| \cos(\omega_0 t + \angle H(\omega_0)) \\ A \sin(\omega_0 t) & & A |H(\omega_0)| \sin(\omega_0 t + \angle H(\omega_0)) \end{array}$$

45.

$$\text{Sol: } R_{xx}(\tau) = \frac{A^2}{2} \cos(\omega_0 \tau) = 18 \cos(6\pi \tau)$$

$$\text{Power} = R_{xx}(0) = 18$$

46.

$$\text{Sol: } r_{xx}(\tau) = x(t) * x(-t) = e^{-3t} u(t) * e^{3t} u(-t)$$

$$r_{xx}(\tau) \xleftrightarrow{\text{F.T}} S_{xx}(\omega) = \frac{1}{9+\omega^2} \Rightarrow r_{xx}(\tau) = \frac{1}{6} e^{-3|\tau|}$$

47.

Sol:

$$(a) |H(\omega)|^2 = \frac{1}{1+\omega^2}, \quad |X(\omega)|^2 = \frac{1}{4+\omega^2}$$

$$S_{YY}(\omega) = |X(\omega)|^2 |H(\omega)|^2$$

$$(b) y(t) = x(t) * h(t) = [e^{-t} - e^{-2t}] u(t)$$

$$E_{y(t)} = \int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{1}{12}$$

$$E_{x(t)} = \frac{1}{4}$$

$$E_{y(t)} = \frac{1}{3} E_{x(t)}$$

48.

Sol: i) Ans: (b)

$$x(t) = e^{-8t} u(t) * e^{-8t} u(t) = \frac{1}{16} e^{-8|t|}$$

$$x\left(\frac{1}{16}\right) = \frac{1}{16\sqrt{e}}$$

ii) Ans: (c)

$$S_{GG}(\omega) = |G(\omega)|^2 = \frac{1}{64+\omega^2}$$

$$S_{GG}(0) = \frac{1}{64}$$

iii) Ans: (b)

$$y(\tau) = e^{-8t} u(t) * e^{8t} u(-t)$$

$$y(\tau) = \frac{1}{16} e^{-8|\tau|}$$

$$y(0) = \frac{1}{16}$$

49.

$$\text{Sol: } r_{xy}(\tau) = x(t) * y(-t) = e^{-t} u(t) * e^{3t} u(-t)$$

$$r_{xy}(\tau) \leftrightarrow \frac{1}{1+j\omega} \frac{1}{3-j\omega} = \frac{1/2}{1+j\omega} + \frac{1/2}{3-j\omega}$$



50.

**Sol:** Given  $x(t) = \text{sinc } 10t$

$$\text{Sinc } t \leftrightarrow \text{rect} \left( \frac{\omega}{2\pi} \right)$$

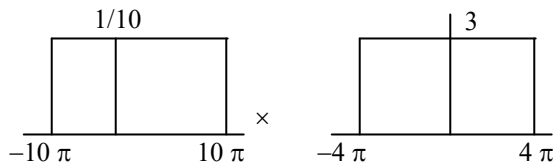
$$\text{sinc}(10t) \leftrightarrow \frac{1}{10} \text{rect} \left( \frac{\omega}{20\pi} \right)$$

$$X(\omega) = \frac{1}{10} \text{rect} \left( \frac{\omega}{20\pi} \right)$$

$$H(\omega) = 3 \text{rect} \left( \frac{\omega}{8\pi} \right) e^{-j2\omega}$$

$$\therefore Y(\omega) = X(\omega) H(\omega)$$

$$= \frac{1}{10} \text{rect} \left( \frac{\omega}{20\pi} \right) 3 \text{rect} \left( \frac{\omega}{8\pi} \right) e^{-j2\omega}$$



$$= \frac{3}{10} \text{rect} \left( \frac{\omega}{8\pi} \right) e^{-j2\omega}$$

$\therefore$  output energy

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |Y(\omega)|^2 d\omega$$

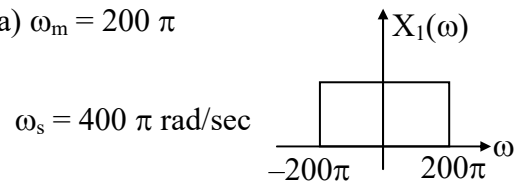
$$= \frac{1}{2\pi} \int_{-4\pi}^{4\pi} \frac{9}{100} d\omega$$

$$= \frac{1}{2\pi} \cdot \frac{9}{100} \times 8\pi$$

$$\text{Output energy} = \frac{36}{100} \text{ J}$$

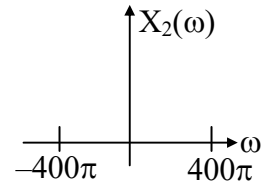
51.

**Sol:** (a)  $\omega_m = 200 \pi$



(b)  $\omega_m = 400 \pi$

$\omega_s = 800 \pi \text{ rad/sec}$



$$(c) x_3(t) = \frac{5}{2} [\cos(500\pi t) + \cos(3000\pi t)]$$

$$\omega_m = 5000 \pi$$

$$\omega_s = 10,000 \pi \text{ rad/sec}$$

$$(d) X_4(\omega) = \frac{1}{6 + j\omega} \cdot \text{rect} \left( \frac{\omega}{2a} \right)$$

$$\omega_m = a$$

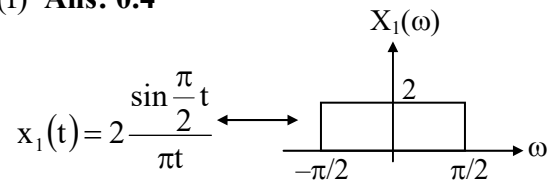
$$f_m = \frac{a}{2\pi}$$

$$f_s = 2f_m = \frac{a}{\pi} \text{ Hz}$$

$$(e) \omega_m = 120 \pi, f_m = 60 \text{ Hz}$$

$$(f_s) = 2f_m = 120 \text{ Hz}$$

(f) **Ans: 0.4**



$$\sum_{n=-\infty}^{+\infty} \delta(t - nT_s) \leftrightarrow f_s \sum_{n=-\infty}^{+\infty} \delta(\omega - n\omega_s)$$



$$\sum_{n=-\infty}^{+\infty} \delta(t-10n) \leftrightarrow \frac{1}{10} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - n \frac{\pi}{5}\right)$$

$$x_1(t) * \sum_{n=-\infty}^{\infty} \delta(t-10n) \leftrightarrow X_1(\omega) \frac{1}{10} \sum_{n=-\infty}^{+\infty} \delta\left(\omega - n \frac{\pi}{5}\right)$$

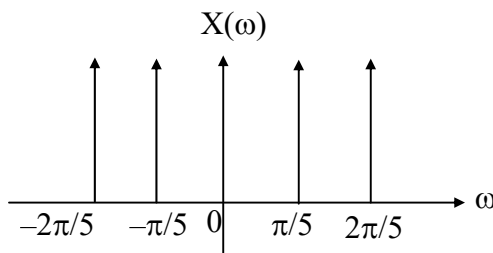
$$X(\omega) = \frac{1}{10} \sum_{n=-\infty}^{+\infty} X_1\left(\frac{n\pi}{5}\right) \delta\left(\omega - n \frac{\pi}{5}\right)$$

$$X(\omega) = \frac{1}{10} \left[ \dots + X_1(0)\delta(\omega) + X_1\left(\frac{\pi}{5}\right)\delta\left(\omega - \frac{\pi}{5}\right) + \right.$$

$$\left. X_1\left(\frac{2\pi}{5}\right)\delta\left(\omega - \frac{2\pi}{5}\right) + X_1\left(\frac{3\pi}{5}\right)\delta\left(\omega - \frac{3\pi}{5}\right) + \dots \right]$$

$$X_1\left(\frac{\pi}{5}\right) = 2, X_1\left(\frac{2\pi}{5}\right) = 2,$$

$$X_1\left(\frac{3\pi}{5}\right) = X_1\left(\frac{4\pi}{5}\right) = \dots = 0$$



The maximum frequency in above signal is

$$\omega_m = 2\pi/5$$

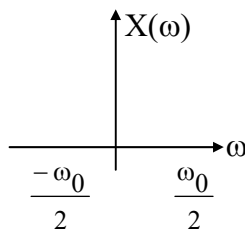
$$2\pi f_m = 2\pi/5$$

$$f_m = 1/5$$

$$\text{Nyquist rate} = 2f_m = 2/5 = 0.4$$

**52.**

**Sol:**

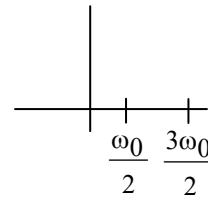


(a)  $X(\omega) + e^{-j\omega} X(\omega)$  no change in frequency axis  $(\omega_s)_{\min} = 2\omega_m = \omega_0$

(b)  $\frac{dx(t)}{dt} \leftrightarrow j\omega X(\omega)$   $\omega_s = \omega_0$

(c)  $x(3t) \leftrightarrow \frac{1}{3} X\left(\frac{\omega}{3}\right)$

(d)  $\frac{1}{2} X(\omega + \omega_0) + \frac{1}{2} X(\omega - \omega_0)$



$$\omega_s = 2 \times \frac{3\omega_0}{2} = 3\omega_0$$

**53.**

**Sol:** (a)  $x_1(2t) \leftrightarrow \frac{1}{2} X_1\left(\frac{\omega}{2}\right)$   $f_m = 4k$

$$f_s = 8k$$

(b)  $x_2(t-3) \leftrightarrow e^{-3j\omega} X_2(\omega)$   $f_m = 3k$

$$f_s = 6k$$

(c)  $X_1(\omega) + X_2(\omega)$   $f_m = 3k$

$$f_s = 6k$$

(d)  $X_1(\omega) * X_2(\omega)$   $f_m = 5k$

$$f_s = 10k$$

(e)  $X_1(\omega) \cdot X_2(\omega)$   $f_m = 2k$

$$f_s = 4k$$

(f)  $\frac{1}{2} [X_1(\omega + 1000\pi) + X_1(\omega - 1000\pi)]$

$$f_m = 250 \text{ Hz}$$

$$(f_s)_{\min} = 2f_m = 5k$$



**54. Ans: (a)**

**Sol:**  $f_m = 200\text{Hz}$ ,  $f_s = 300\text{Hz}$

The frequency in sampled signals are = 200, 100, 500, 400, 800. Cutoff frequency of filter is 100 Hz.

Output frequency = 100 Hz

**55. Ans: (b)**

**Sol:** If  $f_s = f_m$  -The spectrum is constant spectrum

**56. Ans: (a)**

**Sol:**  $f_m < f_c < f_s - f_m \Rightarrow 5 < f_c < 9$

**57. Ans: (c)**

**Sol:**  $f_m = 100$ ,  $f_s - f_m = 150$

$f_s = 250$

$T_s = \frac{1}{f_s} = 4\text{msec}$

**58. Ans: (d)**

**Sol:**  $f_s = \frac{1}{T_0} = \frac{1}{10^{-3}} = 10^3 = 1\text{KHz}$

$$C_n = \frac{1}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} 3e^{-jn\omega_0 t} dt = \frac{\sin\left(\frac{n\pi}{3}\right)}{n\pi}$$

$\therefore C_n = 0$  for  $n = 3, 6, 9, \dots$

$C_n \neq 0$  for  $n = 0, 1, 2, 4, 6, 7, 8, 10, \dots$

$\therefore \pm f \pm 3f_s, \dots + f \pm 6f_s, \dots$

Are not present in signal

$\pm 400 \pm 3(1000) = \pm 3.4\text{ K}, \pm 2.6\text{ K}$

So options with 3.4 K and 2.6 K are wrong

So (c) and (a) are wrong.

3.6 K is out of the given range [ 2.5 to 3.5]

So (B) is wrong

So (D) is correct.

**59. Ans: (c)**

**Sol:**  $y(t) = 5 \times 10^{-6} x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

$x(t) = 10 \cos(8\pi \times 10^3 t)$  &  $T_s = 100\mu\text{sec}$

$f_s = \frac{1}{100\mu} = 10\text{kHz}$

$Y(\omega) = \frac{5 \times 10^{-6}}{T_s} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$

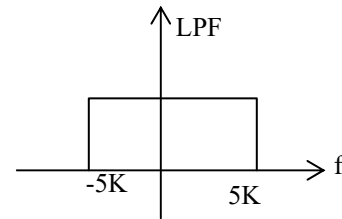
Given  $\omega_m = 8\pi \times 10^3$

$f_m = 4\text{k}$

The output frequencies are  $\pm f_m = 4\text{K}, -4\text{k}$

$f_s \pm f_m = 14\text{K}, 6\text{k}$

$-f_s \pm f_m = -6\text{K}, -14\text{K}$



Only 4k & - 4k are allowed then the output is

$$\begin{aligned} \text{o/p} &= \frac{5 \times 10^{-6}}{T_s} \times 10 \cos(8\pi \times 10^3 t) \\ &= \frac{5 \times 10^{-6}}{100 \times 10^{-6}} \times 10 \cos(8\pi \times 10^3 t) \\ &= \frac{5 \times 10^{-6}}{100 \times 10^{-6}} \times 10 \cos(8\pi \times 10^3 t) \\ \text{o/p} &= 5 \times 10^{-1} \times \cos(8\pi \times 10^3 t) \end{aligned}$$

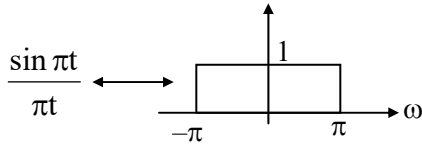
**60. Ans: (a)**

**Sol:**  $x(t) = \cos\left(10\pi t + \frac{\pi}{4}\right)$

$f_s = 15\text{ Hz}$ ,  $\omega_s = 2\pi f_s = 30\pi\text{Hz}$

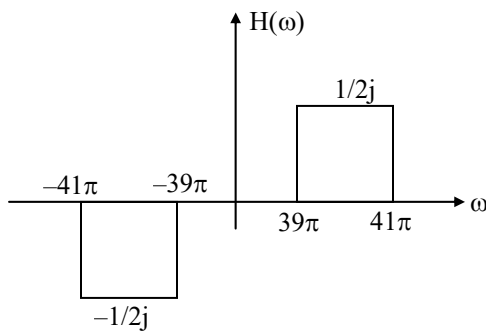


$$h(t) = \left( \frac{\sin \pi t}{\pi t} \right) \cdot \cos \left( 40\pi t - \frac{\pi}{2} \right)$$



$$h(t) = \frac{\sin \pi t}{\pi t} \left[ \cos(40\pi t) \cos \frac{\pi}{2} + \sin 40\pi t \sin \frac{\pi}{2} \right]$$

$$\begin{aligned} h(t) &= \frac{\sin \pi t}{\pi t} \cdot \sin 40\pi t \\ &= \frac{1}{2j} \left[ \frac{\sin \pi t}{\pi t} \cdot e^{j40\pi t} - \frac{\sin \pi t}{\pi t} \cdot e^{-j40\pi t} \right] \end{aligned}$$



$$x(t) = \cos(10\pi t) \cos \frac{\pi}{4} - \sin(10\pi t) \sin \frac{\pi}{4}$$

$$\begin{aligned} X(\omega) &= \frac{1}{\sqrt{2}} [\pi(\delta(\omega + 10\pi) + \delta(\omega - 10\pi))] \\ &\quad - \frac{1}{\sqrt{2}} \left[ \frac{\pi}{j} (\delta(\omega - 10\pi) - \delta(\omega + 10\pi)) \right] \end{aligned}$$

Sampled signal spectrum

$$X_s(\omega) = f_s \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

$$n = 0, \omega_m, -\omega_m = -10\pi, 10\pi$$

$$n = 1, \omega_s - \omega_m, \omega_s + \omega_m = 20\pi, 40\pi$$

$$n = 2, 2\omega_s - \omega_m, 2\omega_s + \omega_m = 50\pi, 70\pi$$

only  $40\pi$  frequency is allowed output of filter is

$$\begin{aligned} Y(\omega) &= \frac{15}{\sqrt{2}} \left[ \frac{-\pi}{2j} \delta(\omega + 40\pi) + \frac{\pi}{2j} \delta(\omega - 40\pi) \right] \\ &\quad - \frac{15}{\sqrt{2}} \left[ \frac{\pi}{j} \times \frac{1}{2j} \delta(\omega - 40\pi) - \frac{\pi}{j} \left( \frac{-1}{2j} \right) \delta(\omega + 40\pi) \right] \\ &= \frac{15}{\sqrt{2}} \left[ -\frac{\pi}{2j} \delta(\omega + 40\pi) + \frac{\pi}{2j} \delta(\omega - 40\pi) \right] \\ &\quad - \frac{15}{\sqrt{2}} \left[ \frac{-\pi}{2} \delta(\omega - 40\pi) - \frac{\pi}{2} \delta(\omega + 40\pi) \right] \\ &= \frac{15}{\sqrt{2}} \left[ -\frac{\pi}{2j} \delta(\omega + 40\pi) + \frac{\pi}{2j} \delta(\omega - 40\pi) \right. \\ &\quad \left. + \frac{\pi}{2} \delta(\omega - 40\pi) + \frac{\pi}{2} \delta(\omega + 40\pi) \right] \end{aligned}$$

$$\begin{aligned} Y(\omega) &= \frac{15}{\sqrt{2}} \left[ \frac{\pi}{2} [\delta(\omega + 40\pi) + \delta(\omega - 40\pi)] \right] \\ &\quad + \frac{\pi}{2j} [\delta(\omega - 40\pi) - \delta(\omega + 40\pi)] \end{aligned}$$

$$y(t) = \frac{15}{\sqrt{2}} \left[ \frac{1}{2} \cos 40\pi t + \frac{1}{2} \sin 40\pi t \right]$$

$$y(t) = \frac{15}{2} \left[ \cos 40\pi t \cos \frac{\pi}{4} + \sin 40\pi t \sin \frac{\pi}{4} \right]$$

$$y(t) = \frac{15}{2} \cos \left( 40\pi t - \frac{\pi}{4} \right)$$

**61. Ans: (c)**

**Sol:**  $x(t) = m(t) c(t)$

Where  $c(t)$  is carrier signal and  $m(t)$  is a base band signal

and  $f_c > f_H$  (where  $f_c$  is carrier frequency,  $f_H$  is the highest frequency component of  $m(t)$ )

$$\hat{x}(t) = m(t) \hat{c}(t)$$

Where  $\hat{f}(t)$  is Hilbert transform of  $f(t)$ .



For the above problem  $c(t) = \sin\left(\pi t - \frac{\pi}{4}\right)$

$$\text{and } m(t) = -\sqrt{2} \left( \frac{\sin(\pi t / 5)}{\pi t / 5} \right)$$

Complex envelope

$$\begin{aligned} &= [x(t) + j\hat{x}(t)]e^{-j2\pi f_c t} \\ &= -\sqrt{2} \left[ m(t) \sin\left(\pi t - \frac{\pi}{4}\right) - jm(t) \cos\left(\pi t - \frac{\pi}{4}\right) \right] e^{-j2\pi f_c t} \\ &= -\sqrt{2} m(t) \left[ \cos\left(\pi t - \frac{\pi}{4}\right) + j \sin\left(\pi t - \frac{\pi}{4}\right) \right] e^{-j2\pi f_c t} \\ &= -\sqrt{2} m(t) e^{+j\left(\pi t - \frac{\pi}{4}\right)} \cdot e^{-j2\pi \left(\frac{1}{2}\right)t} \\ &= j\sqrt{2} m(t) e^{-j\frac{\pi}{4}} = \sqrt{2} m(t) e^{-j\frac{\pi}{4}} \\ &= \sqrt{2} \left( \frac{\sin(\pi t / 5)}{\pi t / 5} \right) e^{j\frac{\pi}{4}} \end{aligned}$$

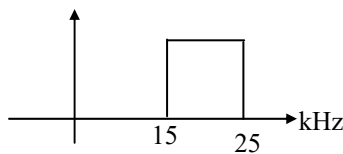
**62. Ans: 10 kHz**

**Sol:** Band Pass sampling theorem its states that

we can sample at the rate of  $f_s = \frac{2f_H}{K}$  where

$K$  is the largest integer not exceeding

$$K = \frac{f_H}{f_H - f_L}$$



For  $m(t)$

$$K = \frac{f_H}{f_H - f_L} = \frac{25}{25 - 15} = 2.5 \text{ means } 2$$

$$f_s = \frac{2f_H}{K}$$

As  $m(t)$  maximum frequency is 5 kHz  
minimum sampling rate required is 10 kHz.

**63. Ans: (c)**

**64. Ans: (b)**

### 5. Laplace Transform

**01.**

**Sol:** (1)  $X_1(s) = \frac{1}{s+1} + \frac{1}{s+3}, \sigma > -1$

(2)  $X_2(s) = \frac{1}{s+2} - \frac{1}{s-4}, -2 < \sigma < 4$

(3) no common ROC so no laplace transform for  $x_3(t)$ .

(4) no common ROC, no laplace transform

(5) no common ROC, no laplace transform

(6)  $X_6(s) = \frac{1}{s+1} - \frac{1}{s-1}, -1 < \sigma < 1$

**02.**

**Sol:**  $\text{ROC} = (\sigma > -5) \cap (\sigma > \text{Re}(-\beta)) = \sigma > -3$

Imaginary part of ' $\beta$ ' any value, real part of ' $\beta$ ' is 3.

**03.**

**Sol:**  $\sigma > 2, \sigma < -3, -3 < \sigma < -1, -1 < \sigma < 2$

**04.**

**Sol:**  $X(s) = \frac{e^{-3s}}{s+1} - \frac{e^{-3s}}{s+2}$

$$x(t) = e^{-(t-3)} \cdot u(t-3) - e^{-2(t-3)} \cdot u(t-3)$$



05.

**Sol:**

$$(a) \quad x(t) = e^{-5(t-1)} \cdot u(t-1) \cdot e^{-5} \leftrightarrow \frac{e^{-s} \cdot e^{-5}}{s+5}, \sigma > -5$$

$$(b) \quad g(t) = A e^{-5t} \cdot u(-t-t_0)$$

$$G(s) = \frac{-A \cdot e^{(s+5)t_0}}{s+5}, \sigma < -5$$

$$A = -1, t_0 = -1$$

06.

**Sol:**  $x(t) = 5r(t) - 5r(t-2) - 15u(t-2) + 5u(t-4)$

$$X(s) = \frac{5}{s^2} - \frac{5e^{-2s}}{s^2} - \frac{15e^{-2s}}{s} + \frac{5e^{-4s}}{s}$$

07. **Ans: (a)**

08. **Ans: (c)**

**Sol:**  $X(s) = \frac{1}{(s+1)(s+3)}$

$$G(s) = X(s-2) = \frac{1}{(s-1)(s+1)}$$

$G(\omega)$  converges means ROC include  $j\omega$  axis  
 $-1 < \sigma < 1$

09.

**Sol:**  $G(s) = X(s) + \alpha x(-s)$

$$= \frac{\beta s - \beta - \alpha \beta s - \alpha \beta}{s^2 - 1} = \frac{s}{s^2 - 1}$$

$$\alpha \beta - \beta = -1, -\beta - \alpha \beta = 0$$

$$\alpha = -1, \beta = \frac{1}{2}$$

10.

**Sol:**  $\frac{dy(t)}{dt} = -2y(t) + \delta(t) \quad \frac{dy(t)}{dt} = 2x(t)$

$$sY(s) = -2Y(s) + 1 \text{ ---- (1)}$$

$$sY(s) = 2X(s) \text{ ----- (2)}$$

solving (1) and (2)

$$Y(s) = \frac{2}{s^2 + 4}, X(s) = \frac{s}{s^2 + 4}$$

11.

**Sol:** (a)  $X(s) = \frac{-4}{s+2} + \frac{4}{(s+1)^3} - \frac{4}{(s+1)^2} + \frac{4}{s+1}$

$$x(t) = -4e^{-2t} \cdot u(t) + 4 \frac{t^2}{2} e^{-t} \cdot u(t) - 4te^{-t} \cdot u(t) + 4e^{-t} \cdot u(t)$$

(b)  $X(s) = -\frac{e^{-2s}}{(s+1)^3}$

$$x(t) = -\frac{(t-2)^2}{2} \cdot e^{-(t-2)} \cdot u(t-2)$$

$$\frac{t^2}{2} e^{-t} u(t) \leftrightarrow \frac{1}{(s+1)^3}$$

12.

**Sol:**  $y(t) + y(t) * x(t) = x(t) + \delta(t)$

$$Y(s) + Y(s)X(s) = X(s) + 1$$

$$Y(s) = 1$$

$$y(t) = \delta(t)$$

13.

**Sol:**  $x_1(t-2) \leftrightarrow \frac{e^{-2s}}{s+2}, \sigma > -2$

$$x_2(-t+3) \leftrightarrow \frac{e^{-3s}}{-s+3}, \sigma < 3$$

$$Y(s) = \frac{e^{-2s}}{s+2} \cdot \frac{e^{-3s}}{-s+3}, -2 < \sigma < 3$$



14.

**Sol:**  $sY(s) + 4Y(s) + 3\frac{Y(s)}{s} = X(s)$

$$H(s) = \frac{s}{(s+1)(s+3)} = \frac{-1}{s+1} + \frac{3}{s+3}$$

$$h(t) = \frac{-1}{2}e^{-t} \cdot u(t) + \frac{3}{2}e^{-3t} \cdot u(t)$$

$$X(s) = \frac{1}{s} + 1 = \frac{s+1}{s}$$

$$Y(s) = X(s)H(s) = \frac{1}{s+3}$$

$$y(t) = e^{-3t} \cdot u(t)$$

15. **Ans: (d)**

**Sol:**  $X(s) = \frac{1}{s+2} + e^{-6s}, H(s) = \frac{1}{s}$

$$Y(s) = X(s)H(s) = \frac{1}{s(s+2)} + \frac{e^{-6s}}{s}$$

$$y(t) = \frac{1}{2}[u(t) - e^{-2t} \cdot u(t)] + u(t-6)$$

16. **Ans: (b)**

**Sol:**  $F_2(s) = e^{-s\tau} \cdot F_1(s)$

$$G(s) = \frac{e^{-s\tau} \cdot F_1(s) \cdot F_1^*(s)}{|F_1(s)|^2} = \frac{e^{-s\tau} |F_1(s)|^2}{|F_1(s)|^2}$$

$$G(s) = e^{-s\tau}$$

$$g(t) = \delta(t-\tau)$$

17. **Ans: (b)**

**Sol:**  $\frac{V(s)}{X(s)} = \frac{1}{s+1} \quad \frac{Y(s)}{V(s)} = \frac{1}{s+1}$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1} \cdot \frac{1}{s+1} = \frac{1}{(s+1)^2}$$

$$h(t) = t e^{-t} \cdot u(t)$$

18.

**Sol:** (a)  $\frac{Y(s)}{X(s)} = \frac{1}{s}$  given statement is false

(b)  $x(t) = u(t)$

$$y(t) = r(t) \text{ is unbounded}$$

given statement is false

(c)  $x(t) = u(-t)$

$$y(t) = \infty \text{ is unbounded}$$

given statement is false

(d) Given true

19.

**Sol:**  $s^2Y(s) + \alpha sY(s) + \alpha^2Y(s) = X(s)$

$$H(s) = \frac{1}{s^2 + \alpha s + \alpha^2}$$

$$G(s) = \frac{\alpha^2}{s} H(s) + sH(s) + \alpha H(s)$$

$$G(s) = \left[ \frac{\alpha^2 + s^2 + s\alpha}{s} \right] \left[ \frac{1}{s^2 + \alpha s + \alpha^2} \right] = \frac{1}{s}$$

Number of poles = 1.

20. **Ans: (d)**

21.

**Sol:**  $H(j\omega) = \frac{j\omega - 2}{(j\omega)^2 + 4j\omega + 4}$

$$x(t) = 8 \cos 2t, \omega_0 = 2$$

$$H(j\omega_0) = \frac{j-1}{4j} = \frac{1}{4} + \frac{1}{4}j$$

$$|H(\omega_0)| = \frac{1}{2\sqrt{2}}, \angle H(\omega_0) = \frac{\pi}{4}$$



$$y(t) = \frac{8}{2\sqrt{2}} \cos\left(2t + \frac{\pi}{4}\right) = 2\sqrt{2} \cos\left(2t + \frac{\pi}{4}\right)$$

22.

**Sol:** (a).  $x(0) = \lim_{s \rightarrow \infty} sX(s) = 2$

$$x(\infty) = \lim_{s \rightarrow 0} sX(s) = 0$$

(b).  $X(s) = \frac{4s+5}{2s+1}$  improper function

$$X(s) = 2 + \frac{3}{2s+1} = \frac{3}{2s+1}$$

neglect the constant '2' in the above function.

$$x(0) = \lim_{s \rightarrow \infty} \frac{3}{2s+1} = \frac{3}{2}$$

$$x(\infty) = \lim_{s \rightarrow 0} sX(s) = \lim_{s \rightarrow 0} \frac{4s^2 + 5s}{2s+1} = 0$$

(c).  $x(0) = 0$

Final value theorem not applicable

(d)  $x(0) = 0$

$$x(\infty) = -1$$

23.

**Sol:**  $H(s) = \frac{k(s+1)}{(s+2)(s+4)}$   $X(s) = \frac{1}{s}$

$$Y(s) = H(s)X(s) = \frac{k(s+1)}{s(s+2)(s+4)}$$

$$y(\infty) = \lim_{s \rightarrow 0} sY(s) = \frac{k}{8} = 1 \Rightarrow k = 8$$

$$H(s) = \frac{-4}{s+2} + \frac{12}{s+4}$$

$$h(t) = -4e^{-2t}u(t) + 12e^{-4t}u(t)$$

24. **Ans: (a)**

**Sol:**  $H(j\omega) = \frac{-\omega^2 + 1}{-\omega^2 + 2j\omega + 1}$

$$\omega_0 = 1 \text{ rad/sec}$$

$$H(\omega_0) = 0$$

$$y(t) = 0 \text{ for all } \omega_s$$

25. **Ans: (d)**

**Sol:**  $H(s) = \frac{2}{s^2 - s - 2}$   $X(s) = \frac{1}{s}$

$$Y(s) = X(s)H(s) = \frac{2}{s(s+1)(s-2)}$$

S = 2 pole lies right side of s-plane

$$y(\infty) = \infty \text{ unbounded}$$

26. **Ans: (d)**

27.

**Sol:** Given  $X(s) = \frac{s+2}{s-2}$

$$y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$$

$$Y(s) = \frac{2}{3} \cdot \frac{1}{s-2} + \frac{1}{3}e^{-t}u(t)$$

$$Y(s) = \frac{2}{3} \cdot \frac{1}{s-2} + \frac{1}{3} \cdot \frac{1}{s+1}$$

$$\Downarrow \quad \Downarrow$$

$$\sigma < 2 \quad \sigma > -1$$

(a).  $\therefore H(s) = \frac{Y(s)}{X(s)}$

$$= \frac{\frac{1}{3} \left[ \frac{2(s+1) + s-2}{(s-2)(s+1)} \right]}{\left[ \frac{s+2}{s-2} \right]} \quad \begin{matrix} \sigma < 2, \sigma > -1, \sigma > 0 \\ \Downarrow \\ \sigma > -1 \end{matrix}$$



$$= \frac{1}{3} \frac{3s}{(s+1)(s+2)}$$

$$= \frac{s}{(s+1)(s+2)}, \sigma > -1$$

(b). The input is  $e^{3t} \forall t$

$\therefore$  the output =  $H(s) \times \text{input}$

$$= \frac{3}{4 \times 5} e^{3t}$$

$$y(t) = \frac{3}{20} e^{3t}$$

28.

**Sol:**  $H(s) = \frac{s^2 + s - 2}{s + 3}$

$$H_{\text{inv}}(s) = \frac{1}{H(s)} = \frac{s + 3}{(s + 2)(s - 1)}$$

$\sigma > +1$  causal unstable

Does not exist in this case a causal & stable system

29. **Ans: (c)**

**Sol:** (a) A system to be stable & causal all the poles of the system should lie in the left half of s-plane.

(b) Any system property like causality, stability doesn't depend on the location of zero's. It depends only on poles location.

(c) There is no necessity that the poles lie within  $|s| = 1$   
All the roots of characteristic equation means all the poles of the system should lie in left half of s-plane.

30. **Ans: (a)**

**Sol:**  $Y(s) = \frac{1}{s + 2}, H(s) = \frac{s - 1}{s + 1}$

$$X(s) = \frac{Y(s)}{H(s)} = \frac{s + 1}{(s - 1)(s + 2)} = \frac{2/3}{s - 1} + \frac{1/3}{s + 2}$$

Stable input  $-2 < \sigma < 1$

$$x(t) = -\frac{2}{3} e^t u(-t) + \frac{1}{3} e^{-2t} u(t)$$

31. **Ans: (d)**

**Sol:** A solution to the differential equation with the input set to zero is often referred to as the natural response of the system.

32. **Ans: (a)**

**Sol:** input,  $x(t) = e^{s_0 t}$

$$\Rightarrow X(s) = \frac{1}{s - s_0}$$

$$\text{Output, } Y(s) = X(s) H(s) = \frac{H(s)}{s - s_0}$$

On partial fraction expansion,

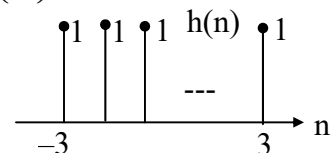
$$Y(s) = \frac{H(s_0)}{s - s_0}$$

$$\text{So, } y(t) = H(s_0) e^{s_0 t}$$

## 6. DT FT

01.

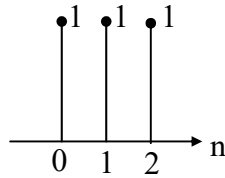
**Sol:** (a)  $H(\omega) = \frac{\sin\left(\frac{7\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$





$h(n) \neq 0 \ n < 0$  – non – causal

(b)



$h(n) = 0 \ n < 0$  causal

(c)  $h(n) = \delta(n-3) + \delta(n+2)$  - non causal

02.

**Sol:** (a)  $Y(e^{j\omega}) = \frac{1}{1 - \frac{1}{4}e^{-j\omega}}$

$$Y(e^{j0}) = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

(b)  $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$   
 $\omega = \pi$

$$X(e^{j\pi}) = \sum_{n=-\infty}^{\infty} x(n)(-1)^n = \cos^3(3\pi) = -1$$

(c)  $H(e^{j\omega}) = 1 + 2e^{-j\omega} + 3e^{-2j\omega} + 4e^{-3j\omega}$

DC gain  $H(e^{j0}) = 1 + 2 + 3 + 4 = 10$

03.

**Sol:**(i)  $X(e^{j\omega}) = 1 + e^{j\omega} + e^{-j\omega} + \frac{3}{2}[1 + \cos 2\omega]$

$$X(e^{j\omega}) = 1 + e^{-j\omega} + e^{j\omega} + \frac{3}{2} \left[ 1 + \frac{e^{2j\omega} + e^{-2j\omega}}{2} \right]$$

$$X(e^{j\omega}) = 1 + e^{-j\omega} + e^{j\omega} + \frac{3}{2} + \frac{3}{4}e^{2j\omega} + \frac{3}{4}e^{-2j\omega}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$x(0) = 1 + \frac{3}{2} = \frac{5}{2}, \quad x(1) = 1, \quad x(-1) = 1,$$

$$x(2) = \frac{3}{4}, \quad x(-2) = \frac{3}{4}$$

$$x(n) = \left[ \frac{3}{4}, 1, \frac{5}{2}, 1, \frac{3}{4} \right]$$

↑

(ii)  $x(n) = 2\delta(n+3) - 3\delta(n-3)$

$$X(e^{j\omega}) = 2e^{3j\omega} - 3e^{-3j\omega} = 2[e^{3j\omega} - e^{-3j\omega}] - e^{-3j\omega}$$

$$X(e^{j\omega}) = 4j\sin(3\omega) - e^{-3j\omega}$$

Given  $X(e^{j\omega}) = a\sin(b\omega) + ce^{jd\omega}$

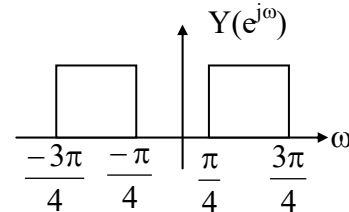
$a = 4j, \quad b = 3, \quad c = -1, \quad d = -3$

04.

**Sol:**

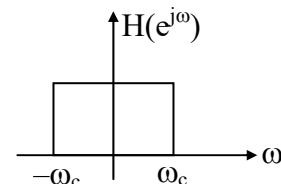
$$\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \cdot e^{j\frac{\pi}{2}n} \leftrightarrow \begin{array}{c} \text{Rectangular pulse from } \frac{\pi}{4} \text{ to } \frac{3\pi}{4} \end{array}$$

$$\frac{\sin\left(\frac{\pi}{4}n\right)}{\pi n} \cdot e^{-j\frac{\pi}{2}n} \leftrightarrow \begin{array}{c} \text{Rectangular pulse from } -\frac{3\pi}{4} \text{ to } -\frac{\pi}{4} \end{array}$$



05.

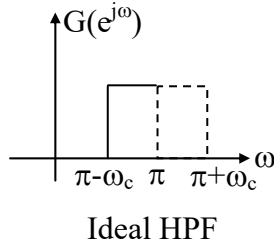
**Sol:** (a)



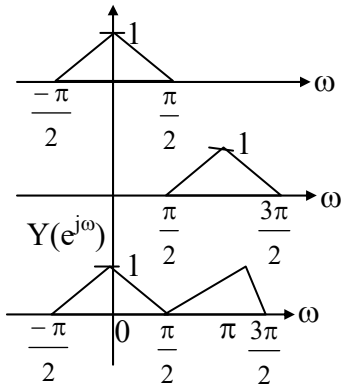


$$g(n) = (-1)^n \cdot h(n)$$

$$G(e^{j\omega}) = H(e^{j(\omega-\pi)})$$



$$(b) Y(e^{j\omega}) = X(e^{j\omega}) + X(e^{j(\omega-\pi)})$$



$$Y(e^{j0}) = 1, Y(e^{j\pi}) = 1$$

06.

$$\text{Sol: } \left(\frac{1}{2}\right)^{\frac{n}{10}} \cdot u\left(\frac{n}{10}\right) \leftrightarrow \frac{1}{1 - \frac{1}{2}e^{-j10\omega}}$$

07. Ans: (b)

$$\text{Sol: } x(2n) = [1, 3, 1]$$

↑

$$\text{FT } [x(2n)] = 3 + 2\cos\omega$$

08.

$$\text{Sol: } na^n \cdot u(n) \leftrightarrow j \frac{d}{d\omega} \left[ \frac{1}{1 - ae^{-j\omega}} \right] = \frac{ae^{-j\omega}}{(1 - ae^{-j\omega})^2}$$

09.

$$\text{Sol: } \sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^n \cdot e^{-j\omega n} = \frac{\frac{1}{2}e^{-j\omega}}{\left(1 - \frac{1}{2}e^{-j\omega}\right)^2}$$

Put  $\omega = 0$  in above equality

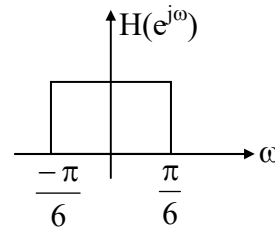
$$\sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^n = \frac{1/2}{\left(1 - \frac{1}{2}\right)^2} = 2$$

10.

$$\text{Sol: } ne^{jn\frac{\pi}{8}} \alpha^{n-3} \cdot u(n-3) \leftrightarrow j \frac{d}{d\omega} \left[ \frac{e^{-3j(\omega-\pi/8)}}{1 - \alpha e^{-j(\omega-\pi/8)}} \right]$$

11.

Sol:



Input signal frequencies are  $\frac{\pi}{8}, \frac{\pi}{4}$

Then the output is  $y(n) = \sin\left(\frac{\pi}{8}n\right)$

12.

$$\text{Sol: } x(n) = e^{j\omega_0 n}, \quad y(n) = e^{j\omega_0 n} \cdot H(e^{j\omega_0})$$

$$H(e^{j\omega}) = 8\sqrt{2} \cos 2\omega - 4\sqrt{2} \cos \omega$$



$$\omega_0 = \frac{\pi}{4}$$

$$H(e^{j\omega_0}) = -4 \quad y(n) = -4e^{jn\frac{\pi}{4}}$$

13.

**Sol:**  $H(e^{j\omega}) = 2\alpha \cos\omega + \beta$

$$H(e^{j\omega})\bigg|_{\omega=\frac{2\pi}{3}} = 0 \quad H(e^{j\omega})\bigg|_{\omega=\frac{2\pi}{8}} = 1$$

$$\alpha = \beta \quad \alpha\sqrt{2} + \beta = 1$$

$$\beta = \frac{1}{1+\sqrt{2}}$$

$$\text{DC gain} = H(e^{j0}) = 3\alpha = \frac{3}{1+\sqrt{2}}$$

14.

**Sol:**  $H(e^{j\omega}) = \frac{b + e^{-j\omega}}{1 - ae^{-j\omega}}$

$$|H(e^{j\omega})|^2 = 1 \Rightarrow H(e^{j\omega}) \cdot H^*(e^{j\omega}) = 1$$

$$\left[ \frac{b + e^{-j\omega}}{1 - ae^{-j\omega}} \right] \left[ \frac{b + e^{j\omega}}{1 - ae^{j\omega}} \right] = 1$$

Only when  $a = -b$

15. **Ans: (a)**

**Sol:**  $H(e^{j\omega}) = 1 + \alpha e^{-j\omega} + \beta e^{-2j\omega}$

$$x(n) = 1 + 4\cos n\pi$$

$$x_1(n) = 1 \quad \omega = 0$$

$$|H(e^{j0})| = 1 + \alpha + \beta \quad \angle H(e^{j0}) = 0$$

$$y_1(n) = 1 + \alpha + \beta$$

$$x_2(n) = 4\cos n\pi \quad \omega = \pi$$

$$|H(e^{j\pi})| = 1 - \alpha + \beta \quad \angle H(e^{j\pi}) = 0$$

$$y_2(n) = 4(1 - \alpha + \beta)\cos n\pi$$

$$y(n) = (1 + \alpha + \beta) + 4(1 - \alpha + \beta)\cos n\pi$$

$$y(n) = 4 \text{ only when } \alpha = 2, \beta = 1$$

16. **Ans: (a)**

**Sol:**  $Y(e^{j0}) = \sum_{n=0}^2 x(n) \cdot \sum_{n=0}^4 h(n) = 15LB$

17.

**Sol:**  $y(n) = x(n) + 2x(n-1) + x(n-2)$

$$Y(e^{j\omega}) = X(e^{j\omega}) [1 + 2e^{-j\omega} + e^{-2j\omega}]$$

$$H(e^{j\omega}) = [1 + e^{-j\omega}]^2$$

$$= [1 + \cos\omega - j\sin\omega]^2$$

(a)  $|H(e^{j\omega})| = (1 + \cos\omega)^2 + \sin^2\omega$

$$\angle H(e^{j\omega}) = -2\tan^{-1} \frac{\sin\omega}{1 + \cos\omega}$$

$$10 \rightarrow \omega = 0 \Rightarrow |H(e^{j0})| = 1$$

$$\angle H(e^{j0}) = 0^\circ$$

$$4\cos\left(\frac{\pi n}{2} + \frac{\pi}{4}\right) \rightarrow \omega = \frac{\pi}{2} \Rightarrow |H(e^{j\omega})| = 2$$

$$\Rightarrow \angle H(e^{j\omega}) = -90^\circ$$

(b) Output of given input

$$10 + 4\cos\left(\frac{\pi n}{2} + \frac{\pi}{4}\right) \text{ is}$$

$$10 + 4(2)\cos\left(\frac{\pi n}{4} + \frac{\pi}{4} - \frac{\pi}{2}\right)$$

$$= 10 + 8\cos\left(\frac{\pi n}{4} - \frac{\pi}{4}\right)$$

18. **Ans: (b)**

**Sol:** anti symmetric,  $k = -2$

$$\theta(\omega) = -2\omega$$

$$\text{Slope} = -2$$



**19. Ans: (b)**

**Sol:**  $x(n) = \cos\left(\frac{5\pi}{2}n\right) = \cos\left(\frac{\pi}{2}n\right)$   $\omega_0 = \frac{\pi}{2}$

$$|H(e^{j\omega})| = 1 \quad \angle H(e^{j\omega_0}) = -\frac{\pi}{8}$$

$$y(n) = \cos\left(\frac{n\pi}{2} - \frac{\pi}{8}\right)$$

**20. Ans: (a)**

**Sol:**  $X(e^{j\omega}) = 2 + 2\cos\omega + e^{-5j\omega} + 2e^{-4j\omega}$

$$X\left(e^{j\frac{\pi}{4}}\right) = \frac{1+j}{\sqrt{2}}$$

$$\angle X(e^{j\pi/4}) = \tan^{-1}\left(\frac{1/\sqrt{2}}{1/\sqrt{2}}\right) = \frac{\pi}{4}$$

**21.**

**Sol:**  $h(n) = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$

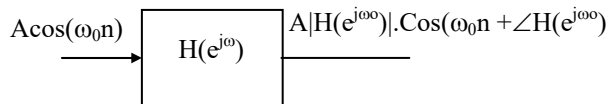
↑

$$(a) H(e^{j\omega}) = \left[\frac{1}{3}e^{j\omega} + \frac{1}{3} + \frac{1}{3}e^{-j\omega}\right]$$

$$= \frac{1}{3}(1 + 2\cos\omega)$$

(b) Phase = 0

(c)



Given  $x(n) = \cos\left(\frac{n\pi}{3}\right)$

$$\omega_0 = \frac{\pi}{3}$$

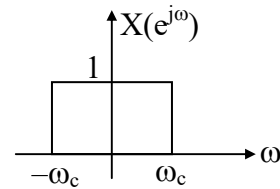
$$H(e^{j\omega}) = \frac{1}{3}(1 + 2\cos\omega) = \frac{1}{3}\left(1 + 2\cos\left(\frac{\pi}{3}\right)\right)$$

$$= \frac{1}{3}(1 + 1) = \frac{2}{3}$$

$$y(n) = \frac{2}{3}\cos\left(\frac{4\pi}{3}\right)$$

**22.**

**Sol:**



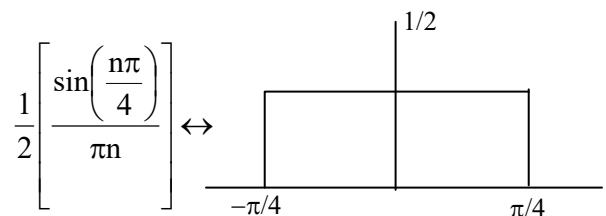
$$E = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} 1 d\omega = \frac{\omega_c}{\pi}$$

**23. Ans:  $\frac{1}{40}$**

**Sol:** By plancheral's relation

$$\sum_{n=-\infty}^{\infty} x(n)y(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})Y(e^{j\omega})d\omega$$

$$x(n) = \frac{\sin\left(\frac{n\pi}{4}\right)}{2\pi n} = \frac{1}{2} \left[ \frac{\sin\left(\frac{n\pi}{4}\right)}{\pi n} \right]$$





$$y(n) = \frac{1}{5} \left[ \frac{\sin\left(\frac{n\pi}{3}\right)}{\pi n} \right]$$

$y(n)$        $-\pi/3$        $\pi/3$

$$\sum_{n=-\infty}^{\infty} \frac{\sin \frac{n\pi}{4}}{2\pi n} \times \frac{\sin \frac{n\pi}{3}}{5\pi n} = \frac{1}{2\pi} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{1}{2}\right) \left(\frac{1}{5}\right) d\omega$$

$$= \frac{1}{40}$$

24.

**Sol:** (a).  $X(e^{j0}) = \sum_{n=-\infty}^{\infty} x(n) = 6$

(b).  $X(e^{j\pi}) = \sum_{n=-\infty}^{\infty} (-1)^n x(n) = 2$

(c).  $\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega = 2\pi x(0) = 4\pi$

(d).  $\int_{-\pi}^{\pi} X(e^{j\omega}) e^{2j\omega} d\omega = 2\pi x(2) = 0$

(e).  $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 2\pi \left[ \sum_{n=-\infty}^{\infty} |x(n)|^2 \right] = 28\pi$

(f).  $\int_{-\pi}^{\pi} \left| \frac{d}{d\omega} X(e^{j\omega}) \right|^2 d\omega = 2\pi \left[ \sum_{n=-\infty}^{\infty} |nx(1)|^2 \right]$   
 $= 158 \times 2\pi = 316\pi$

(g).  $\angle X(e^{j\omega}) = -\alpha\omega = -\left(\frac{N-1}{2}\right)\omega = -5\omega$

25. **Ans: (d)**

**Sol:**  $f(x) = h(n) * h(n)$

|   | 1 | 2 | 2 |
|---|---|---|---|
| 1 | 1 | 2 | 2 |
| 2 | 2 | 4 | 4 |
| 2 | 2 | 4 | 4 |

$f(n) = \{1, 4, 8, 8, 4\} \Rightarrow \text{causal}$

↑

$g(n) = h(n) * h(-n)$

$h(-n) = \{2, 2, 1\}$

↑

$h(-n)$  ranges from  $n = -2$  to  $n = 0$

$h(n)$  ranges from  $n = 0$  to  $n = 2$

$\therefore g(n)$  ranges from  $n = -2$  to  $n = 2$

|   | 1 | 2 | 2 |
|---|---|---|---|
| 2 | 2 | 4 | 4 |
| 2 | 2 | 4 | 4 |
| 1 | 1 | 2 | 2 |

$g(n) = \{2, 6, 9, 6, 2\}$

↑

$\Rightarrow g(n)$  is non causal and maximum value is 9.

26.

**Sol:**  $\frac{2\pi \times 5k}{40k} \leq \omega \leq \frac{2\pi \times 10k}{40k}$

$FS = 2f_m$

$= 2 \times 20k$

$= 40k$

$+\frac{\pi}{4} \leq \omega \leq \frac{\pi}{2}$



27. Ans: (a)

$$\text{Sol: } x(nT_s) = \cos(-20nT_s) = \cos\left(\frac{-20n}{1000}\right)$$

$$= \cos\left(\frac{n\pi}{4}\right) = \cos\left(\frac{9\pi}{4}\right)$$

$$\frac{\Omega_0}{1000} = \frac{\pi}{4} ; \quad \frac{\Omega_0}{1000} = \frac{9\pi}{4}$$

$$\Omega_0 = 250\pi, 2250\pi$$

### 7. Z-Transform

01.

$$\text{Sol: } X(z) = \frac{z}{z+1} - \frac{z}{z-\alpha}$$

$$\text{ROC} = (|z| > 1) \cap (|z| < |\alpha|) = 1 < |z| < 2$$

Only when  $\alpha = \pm 2$ , ' $n_0$ ' any value

02.

Sol: (a) finite duration both sided  $0 < |z| < \infty$

(b) finite duration right sided  $|z| > 0$

(c) infinite duration right sided  $(|z| > 1/2) \cap$

$$(|z| > 3/4) = |z| > 3/4$$

(d)  $(|z| > 1/3) \cap (|z| < 3) \cap (|z| > 1/2) = 1/2 < |z| < 3$

03. Ans: (a)

Sol:  $\text{ROC} = (|z| > |a|) \cap (|z| < |b|^2)$  common ROC exists only when  $|a| < |b|^2$

04. i) Ans: (b)

$$\text{Sol: } \text{ROC} = (|z| > |a|) \cap (|z| > |b|) \cap (|z| < |c|) \\ = |b| < |z| < |c|$$

ii)  $\text{ROC} = (|z| > |\alpha|) \cap (|z| < |\beta|)$

$$X(z) = \frac{z}{z-\alpha} - \frac{z}{z-\beta}$$

(a)  $\alpha > \beta$  no Z.T

(b)  $\alpha < \beta$  Z.T is exist

(c)  $\alpha = \beta$  no Z.T

05. Ans: (c)

$$\text{Sol: } X(z) = \frac{-1/2}{1 - \frac{1}{2}z^{-1}} + \frac{3/2}{1 + \frac{1}{2}z^{-1}}$$

$$x(n) = -\frac{1}{2}\left(\frac{1}{2}\right)^n u(n) + \frac{3}{2}\left(\frac{-1}{2}\right)^n \cdot u(n)$$

$$x(2) = \frac{1}{4}$$

06. Ans: (d)

Sol: poles =  $j, -j$ , zeros =  $0, 0$

$$X(z) = \frac{kz^2}{z^2 + 1}$$

$$X(1) = 1 \Rightarrow k = 2$$

$$X(z) = \frac{2z^2}{z^2 + 1}$$

07. Ans: (b)

$$\text{Sol: } X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n} \\ = \frac{1}{2} + z^2 + \frac{9}{4}z^4 + \dots$$

$$x(n) = \left\{ \begin{matrix} \dots, \frac{9}{4}, 0, 1, 0, \frac{1}{2} \\ \uparrow \end{matrix} \right\}$$

Now consider (a) option

$$Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n} \\ = 1 + \frac{2}{3}z^{-1} + \frac{9}{4}z^{-2} + \dots$$



$$\sum_{n=-\infty}^{\infty} x(n) y_1(n) \neq 0$$

Now consider option (b)

$$Y_2(z) = z^{-1} + 4z^{-3} + \dots$$

$$y_2(n) = \{0, 1, 0, 4, \dots\}$$

$$\sum_{n=-\infty}^{\infty} x(n) y_2(n) = 0$$

**08. Ans:  $r = -1/2$**

$$\begin{aligned} \text{Sol: } H(z) &= \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{r}{1 + \frac{1}{4}z^{-1}} \\ &= \frac{1 + \frac{1}{4}z^{-1} + r(1 - \frac{1}{2}z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{4}z^{-1})} \end{aligned}$$

Consider the numerator

$$1 + \frac{1}{4}z^{-1} + r\left(1 - \frac{1}{2}z^{-1}\right)$$

$$(1+r) + \left(\frac{1}{4} - \frac{r}{2}\right)z^{-1}$$

$$\text{zero} = \frac{-\left(\frac{1}{4} - \frac{r}{2}\right)}{1+r}$$

**If zero = 1**

$$\frac{\frac{1}{4} - \frac{r}{2}}{1+r} = 1 \Rightarrow \frac{1}{4} - \frac{r}{2} = 1+r$$

$$\frac{-3r}{2} = \frac{3}{4} \Rightarrow r = -1/2$$

**If zero = -1**

$$\frac{\frac{1}{4} - \frac{r}{2}}{1+r} = -1 \Rightarrow \frac{1}{4} - \frac{r}{2} = -1-r$$

$$\frac{r}{2} = \frac{-5}{4} \Rightarrow r = -5/2 \text{ is not valid}$$

Because given as  $|r| < 1$

**09. Ans: (a)**

$$\text{Sol: } H(z) = \frac{z^4}{z^4 + \frac{1}{4}}$$

$$H(z) = H(z^{-1})$$

$$h(n) = h(-n)$$

So  $h(n)$  is real for all 'n'

**10.**

$$\text{Sol: } (-3)^n \cdot u(n-2) \leftrightarrow \frac{9z^{-1}}{z+3}, |z| > 3$$

$$(-3)^{-n} \cdot u(-n-2) \leftrightarrow \frac{9z}{z^{-1}+3}, |z| < \frac{1}{3}$$

**11.**

$$\text{Sol: } g(n) = \delta(n) - \delta(n-6)$$

$$G(z) = 1 - z^{-6}, |z| > 0$$

**12.**

$$\text{Sol: } X(z) = z^2 + 2z + \frac{2z}{z-2}$$

$$x(n) = \delta(n+2) + 2\delta(n+1) - 2(2)^n u(-n-1)$$

**13.**

$$\text{Sol: } X(z) = \frac{(z+j)(z-j)}{z-1/2} = \frac{z^2+1}{z-\frac{1}{2}}$$

$$Y(z) = X\left(\frac{z}{1/2}\right) = X(2z) = \frac{4z^2+1}{2z-\frac{1}{2}}$$



14.

$$\text{Sol: } (2)^n \cdot u(-n) \leftrightarrow \frac{1}{1 - \frac{1}{2}z}, |z| < 2$$

15.

$$\begin{aligned} \text{Sol: } X(z) &= z^4 + z^2 - 2z + 2 - 3z^{-4} \\ H(z) &= 2z^{-3} \\ Y(z) &= X(z)H(z) = 2z + 2z^{-1} - 4z^2 + 4z^{-3} - 6z^{-7} \\ y(4) &= 0 \end{aligned}$$

16.

$$\begin{aligned} \text{Sol: } x_1(n+3) &\leftrightarrow \frac{z^3}{1 - \frac{1}{2}z^{-1}}, |z| > \frac{1}{2} \\ x_2(-n+1) &\leftrightarrow \frac{z^{-1}}{1 - \frac{1}{3}z}, |z| < 3 \\ Y(z) &= \frac{z^2}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z\right)}, \frac{1}{2} < |z| < 3 \end{aligned}$$

17.

$$\begin{aligned} \text{Sol: } H(z) &= \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)}, x(z) \\ &= 1 - \frac{1}{3}z^{-1} \\ Y(z) &= H(z)X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \\ \Rightarrow y(n) &= \left(\frac{1}{2}\right)^n \cdot u(n) \end{aligned}$$

18. Ans: (a)

Sol: when  $\alpha = \beta$  it gives linear phase.

19.

Sol: (1)  $x(n) = z_0^n$ ,  $y(n) = z_0^n H(z_0)$

$$y(n) = (-2)^n \cdot H(-2) = 0$$

$$H(-2) = 0$$

$$(2) H(z) = \frac{Y(z)}{X(z)} = \frac{1 + a \cdot \frac{1}{1 - \frac{1}{4}z^{-1}}}{\frac{1}{1 - \frac{1}{2}z^{-1}}}$$

$$(a) H(-2) = 0$$

$$a = \frac{-9}{8}$$

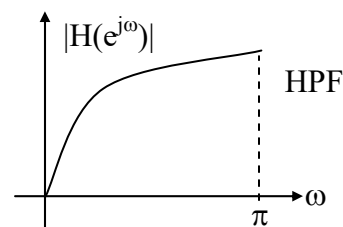
$$(b) y(n) = (1)^n \cdot H(1)$$

$$H(1) = -1/4$$

$$y(n) = \frac{-1}{4}(1)^n$$

20. Ans: (a)

$$\text{Sol: } H(e^{j\omega}) = e^{-2j\omega} - e^{-3j\omega}$$



And it is FIR Filter because finite coefficients

21. Ans: (a)

$$\text{Sol: } y(n) = h(n) * g(n)$$

$$Y(e^{j\omega}) = H(e^{j\omega}) G(e^{j\omega})$$



$$\Rightarrow Y(e^{j\omega}) = \frac{G(e^{j\omega})}{\left[1 - \frac{1}{2}e^{-j\omega}\right]}$$

$$\Rightarrow G(e^{j\omega}) = Y(e^{j\omega}) - \frac{1}{2}e^{-j\omega} Y(e^{j\omega})$$

$$\Rightarrow g(n) = y(n) - \frac{1}{2}y(n-1)$$

Put  $n = 1$

$$\Rightarrow g(1) = y(1) - \frac{1}{2}y(0)$$

$$= \frac{1}{2} - \frac{1}{2}$$

$$g(1) = 0$$

**22. Ans: (c)**

**Sol:**  $H(e^{j\omega}) = 1 - e^{-6j\omega} = 0$  only when

$$6\omega = 2\pi n \quad (n = 1)$$

$$\omega = \frac{\pi}{3}$$

$$\frac{2\pi \times f}{9k} = \frac{\pi}{3}$$

$$f = 1.5k$$

**23.**

**Sol:**  $x(n) = -0.5(2)^n \cdot u(-n-1)$

$$x(0) = 0$$

**24.**

$$\text{Sol: } x(n) = \begin{cases} 1 & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

$$\Rightarrow X(Z) = 1 + Z^{-2} + Z^{-4} + \dots$$

$$= \frac{1}{1 - Z^{-2}}$$

$$= \frac{1}{(1 - Z^{-1})(1 + Z^{-1})}$$

$$\begin{aligned} x(\infty) &= \lim_{Z \rightarrow 1} (1 - Z^{-1})X(Z) \\ &= \lim_{Z \rightarrow 1} (1 - Z^{-1}) \frac{1}{(1 + Z^{-1})(1 - Z^{-1})} \\ &= \frac{1}{2} \end{aligned}$$

**25. Ans: (a)**

$$\text{Sol: } H(z) = -\frac{1}{2} + \frac{1}{2} \cdot \frac{z}{z-2}$$

$$h(n) = \frac{-1}{2}\delta(n) - \frac{1}{2}(2)^n \cdot u(-n-1)$$

**26. Ans: (a)**

$$\text{Sol: } H(z) = \frac{6z^{-1}}{1 - 0.7z^{-1} + 0.3z^{-2}}$$

$$Z = 1 \text{ (static gain)}$$

$$H(1) = 10$$

**27. Ans: (c)**

$$\text{Sol: } Y(z) = H(z)X(z)$$

$$= \frac{A}{1 - z^{-1}} + \frac{1}{\left(1 - \frac{1}{3}z^{-1}\right)(1 - z^{-1})}$$

$$y(\infty) = \lim_{z \rightarrow 1} (1 - z^{-1})Y(z)$$

$$\Rightarrow A + \frac{3}{2} = 0$$

$$A = -\frac{3}{2}$$

**28. Ans: (c)**

$$\text{Sol: } H(z) = \frac{\beta z - 2z^2}{2z^2 - \alpha}$$

$$\text{Pole} = \pm \sqrt{\frac{\alpha}{2}}$$



$$\left| \sqrt{\frac{\alpha}{2}} \right| < 1 \Rightarrow |\alpha| < 2, \text{ any value of } \beta$$

29.

**Sol:** (a) An LTI system is stable if and only if ROC includes  $j\omega$  axis.

$$-0.5 < \operatorname{Re} \{s\} < 2$$

(b) For an LTI system to be stable, all the poles must lie left side of the  $j\omega$  axis  
 $S = 2$  is the pole in the right half of  $s$ -plane.

So it is not possible.

(c)  $\operatorname{Re} \{s\} < -3$

$$\operatorname{Re} \{s\} > 2$$

$$-3 < \operatorname{Re} \{s\} < -0.5$$

$-0.5 < \operatorname{Re} \{s\} < 2$  are the four possible ROC's

30. **Ans: (d)**

$$\text{Sol: } H(z) = \frac{\left(z - \frac{3}{4}e^{j0}\right)\left(z - \frac{3}{4}e^{-j0}\right)}{z - \frac{4}{3}}$$

Numerator order > denominator order

so – non causal system &  $|z| < \frac{4}{3}$  - stable

31. **Ans: (d)**

$$\text{Sol: Poles} \Rightarrow 1 - 0.5 Z^{-1} = 0 \Rightarrow Z = 0.5$$

$$\text{Zeros} \Rightarrow 1 - 2Z^{-1} = 0 \Rightarrow Z = 2$$

It all zeros and poles are inside the unit circle  $[|Z| = 1]$  then it is a minimum phase system.

So given system is Non minimum phase system if all poles are inside unit circle then we can say system is causal and stable. So given system is stable.

32.

$$\text{Sol: (a). } H(Z) = \frac{Z - 2 - Z^{-1}}{1 + \frac{1}{2}Z^{-1}}$$

(b). IIR filter

33. **Ans: (c)**

**Sol:** Poles  $z = \pm 2j$

$$|\text{poles}| = 2$$

ROC =  $|z| < 2$  because system is stable .

In this case system is non-causal

34. **Ans: (c)**

$$\text{Sol: } H(z) = \frac{z}{z + \frac{1}{2}} \text{ is a stable system because}$$

pole  $z = -\frac{1}{2}$  is inside the unit circle.

The poles of  $H(z)$  should be inside the unit circle for a stable system.

$\therefore$  A is True but R is false.

35. **Ans: (c)**

$$\text{Sol: } H(z) = \frac{z^2 + 1}{(z + 0.5)(z - 0.5)}$$

(1) The system is stable because poles  $z = \pm 0.5$  are inside the unit circle.

$$(2) h(0) = \lim_{z \rightarrow \infty} H(z) = 1$$



$$(3) \omega = \frac{2\pi f}{f_s} = \frac{2\pi \times \frac{f_s}{4}}{f_s} = \frac{\pi}{2}$$

$$H(e^{j\omega}) = \frac{e^{2j\omega} + 1}{(e^{j\omega} + 0.5)(e^{j\omega} - 0.5)} \text{ at } \omega = \frac{\pi}{2} = 0$$

**36. Ans: (c)**

**Sol:** A causal LTI system is stable if and only if all of poles of  $H(z)$  lie inside the unit circle.

**37. Ans: (b)**

**Sol:** In (A) and (R) replace  $H(s)$  by  $H(z)$  for the question to be meaningful.

$$H(z) = \frac{z^3 - 2z^2 + z}{z^2 + \frac{1}{4}z + \frac{1}{8}} = \frac{N(z)}{D(z)}$$

As  $N(z)$  is of higher order than  $D(z)$ , the system is not causal, as  $\delta(n+1)$  is one of the terms in the output for the input  $\delta(n)$ .

If the  $N(z)$  is of lower order than the denominator, the system

(i) may be causal or

(ii) may not be causal as it depends upon the ROC of the given  $H(z)$ .

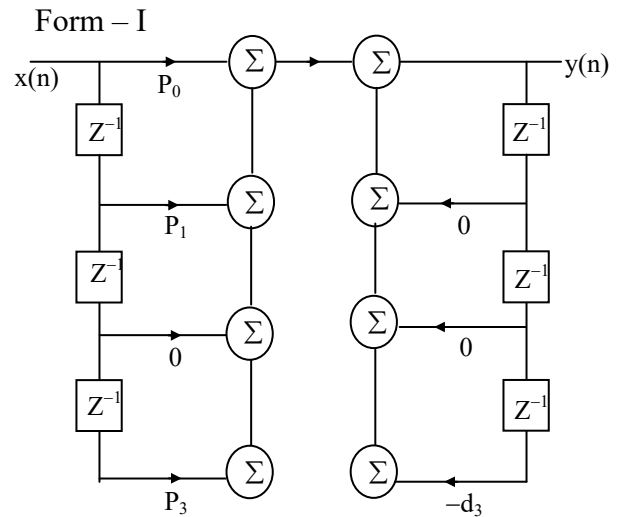
$\therefore$  Both A and R are individually true but R is not the correct explanation of A.

**38. Ans: (a)**

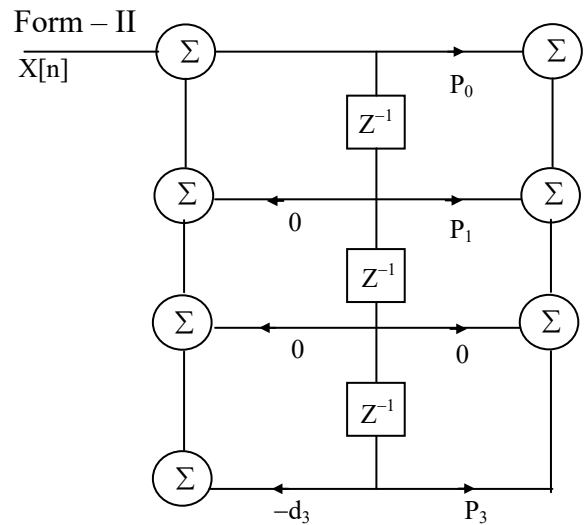
**Sol:** Both I & II are true and 'II' is the correct reason for 'I'

**39. Ans: (b)**

$$\text{Sol: } H(Z) = \frac{P_0 + P_1 Z^{-1} + P_3 Z^{-3}}{1 + d_3 Z^{-3}}$$



No. of delays = 6



No. of delay's = 3

**40.**

**Sol:**  $H(z) = z^{-1} = H_1(z) H_2(z)$

$$H_2(z) = z^{-1} \left[ \frac{1 - 0.6z^{-1}}{1 - 0.4z^{-1}} \right]$$



41. Ans: (a)

Sol:  $H(z) = \frac{1}{1 - 0.7z^{-1} + 0.13z^{-2}}$   
 $a_0 = 1, a_1 = 0.7, a_2 = -0.13$

42.

Sol: (a)  $H(Z) = \frac{1 + 0.5Z^{-1}}{1 + 0.1Z^{-1} - 0.3Z^{-2}}$

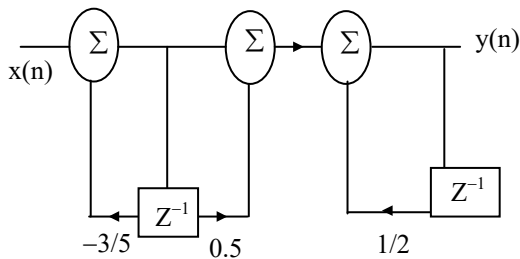
$$H(Z) = \frac{1 + 0.5Z^{-1}}{(-0.5 - 0.3Z^{-1})(-2 + Z^{-1})}$$

Cascaded form:

$$H(Z) = \frac{1 + 0.5Z^{-1}}{(-0.5 - 0.3Z^{-1})} \cdot \frac{1}{(-2 + Z^{-1})}$$

$$H(Z) = \frac{-\frac{1}{2}[1 + 0.5Z^{-1}]}{\left(1 + \frac{3}{5}Z^{-1}\right) \left(1 - \frac{1}{2}Z^{-1}\right)}$$

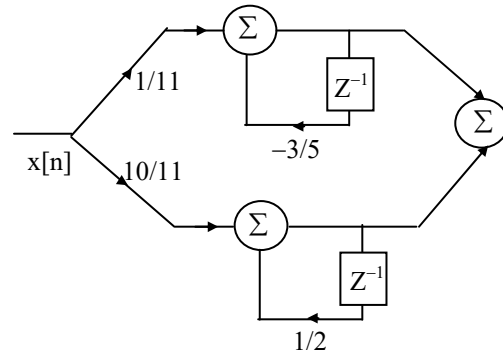
$$= \frac{1 + 0.5Z^{-1}}{1 + \frac{3}{5}Z^{-1}} \cdot \frac{1}{1 - \frac{1}{2}Z^{-1}}$$



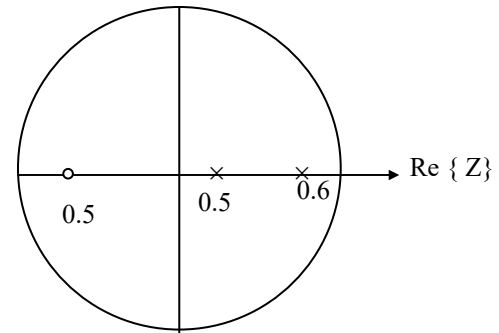
Parallel form:

$$H(Z) = \frac{(1 + 0.5Z^{-1})}{\left(1 + \frac{3}{5}Z^{-1}\right) \left(1 - \frac{1}{2}Z^{-1}\right)}$$

$$= \frac{1}{11} \cdot \frac{1}{1 + \frac{3}{5}Z^{-1}} + \frac{10}{11} \cdot \frac{1}{1 - \frac{1}{2}Z^{-1}}$$



(b)



$$|Z| = \frac{3}{5}, |Z| = 0.5 \text{ are poles}$$

$$|Z| = 0.5 \text{ is zero}$$

43.

Sol:  $H(z) = \frac{1 - \frac{k}{4}z^{-1}}{1 + \frac{k}{3}z^{-1}}$

$$\text{Pole} = \left| \frac{-k}{3} \right| < 1$$

$$|k| < 3$$

44. Ans: (c)

Sol: From signal below graph reduction

$$H(z) = \frac{2 + z^{-1}}{1 + 2z^{-1}} = \frac{2z + 1}{z + 2}$$

**45. Ans: (b)**

**Sol:**  $H(e^{j\omega}) = \frac{2e^{j\omega} + 1}{e^{j\omega} + 2}$

$|H(e^{j0})| = 1$

$|H(e^{j\pi/2})| = 1$

$|H(e^{j\pi})| = 1$

So, All pass filter

**46. Ans: (a)**

**Sol:**  $1 - k[z^{-1} + z^{-2}] = 0$

$z^2 - zk - k = 0$

$z_{1,2} = \frac{+k \pm \sqrt{k^2 + 4k}}{2}$

For causal & stable  $|\text{poles}| < 1$ 

$k = 1 \Rightarrow z_{1,2} = \frac{1 \pm \sqrt{5}}{2} = \frac{1 \pm 2.236}{2}$

(outside the unit circle)

$k = 2 \Rightarrow z_{1,2} = \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$

$= 1 \pm 1.732$

outside the unit circle

Here,  $k = [-1, 1/2]$