



# CIVIL ENGINEERING



## GATE | PSUs

REINFORCED  
CEMENT  
CONCRETE

Volume - I : Study Material with Classroom Practice Questions

# Reinforced Cement Concrete

## Solutions for Volume : I Classroom Practice Questions

### Chapter- 3 Limit State – Singly Reinforced Beam

**01. Ans: (a)**

**Sol:** TOR 40 - Fe415,

$$\begin{aligned} M_{u \text{ limit}} &= \text{Equation (1) with } x_{u \text{ max}} \\ &= 0.138 f_{ck} b d^2 \\ &= 0.138 \times 15 \times 200 \times (500)^2 \\ &= 103.5 \text{ kN-m} \end{aligned}$$

**02. Ans: (c)**

**Sol:** Balanced (or) limiting % if steel (use  $x_{u \text{ max}}$ )

$$\begin{aligned} C &= T \\ 0.36 f_{ck} b x_{u \text{ max}} &= 0.87 f_y A_{st} \\ 0.36 f_{ck} b (0.48d) &= 0.87 \times 415 A_{st} \\ 0.36 \times 15 \times 200 \times 0.48 \times 300 &= 0.87 \times 415 A_{st} \\ A_{st} &= 430 \text{ mm}^2 \end{aligned}$$

**03. Ans: (b)**

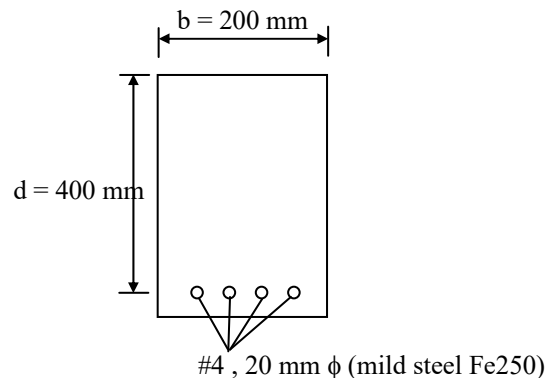
**Sol:**  $M_u = 138 \times 10^6 \text{ N-mm}$

$$\begin{aligned} M_u &= M_{u \text{ limit}} \\ &= 0.138 \times f_{ck} b d^2 - (\text{design as BS}) \\ 138 \times 10^6 &= 0.138 \times 20 \times 200 \times d^2 \\ d &= 500 \text{ mm} \end{aligned}$$

*Common data for Questions 04 & 05*

**04. Ans: (b)**

**Sol:**



$$\begin{aligned} \text{i) } x_{u \text{ max}} &= 0.53 \times d \\ &= 0.53 \times 400 \\ &= 212 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{ii) } x_u &= ? \quad C = T \\ 0.36 \times f_{ck} \times b \times x_u &= 0.87 \times f_y \times A_{st} \\ 0.36 \times 15 \times 200 \times x_u &= 0.87 \times 250 \times 4 \\ &\quad \times \left( \frac{\pi}{4} \times 20^2 \right) \end{aligned}$$

$$\begin{aligned} \Rightarrow 1080x_u &= 273318.5 \\ x_u &= 253.1 \text{ mm} \end{aligned}$$

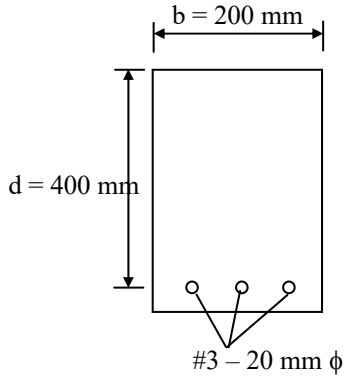
Over reinforcement section fails suddenly to avoid sudden fail  $\downarrow$  the MR to that of a balanced section

$$\begin{aligned} M_{u \text{ limit}} &= 0.148 \times f_{ck} b d^2 \\ &= 0.148 \times 15 \times 200 \times 400^2 \\ &= 71040000 \text{ N-mm} = 71.04 \text{ kN-m} \\ &\approx 72 \text{ kN-m} \end{aligned}$$



05. Ans: (d)

Sol:



i)  $x_{u \max} = 0.53 \times d$

$$= 0.53 \times 400 = 212 \text{ mm}$$

ii)  $C = T$

$$0.36 \times f_{ck} \times b \times x_u = 0.87 \times f_y \times A_{st}$$

$$0.36 \times 15 \times 200 \times x_u = 0.87 \times 250$$

$$\times \left( 3 \times \frac{\pi}{4} \times 20^2 \right)$$

$$1080 x_u = 204988.92$$

$$x_u = 190 \text{ mm}$$

$$x_u < x_{\max} \text{ under reinforced section}$$

$$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u)$$

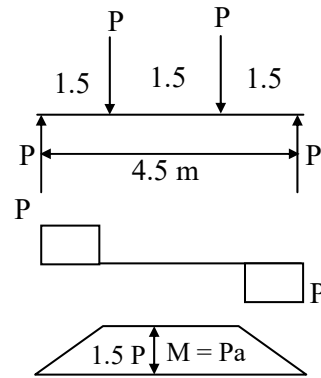
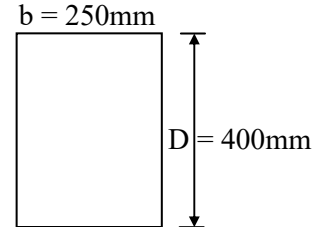
$$= 0.36 \times 15 \times 200 \times 190 (400 - 0.42 \times 190)$$

$$M_u = 65.7 \text{ kN.m} \approx 66 \text{ kN.m}$$

Common data for Questions 06 & 07

06. Ans: 8.86 kN

Sol:



Homogenous beam

$$f_{cr} = 2 \text{ MPa}$$

Modulus of rupture/tensile stress of concrete from bending equation

$$\frac{M}{I} = \frac{f}{y}$$

$$\Rightarrow M = f_{cr} \times z \quad \left[ \because z = \frac{bD^2}{6} \right]$$

$$= 2 \left[ \frac{250 \times 400^2}{6} \right] = 13.33 \times 10^6 \text{ N-mm}$$

$$M = P.a$$

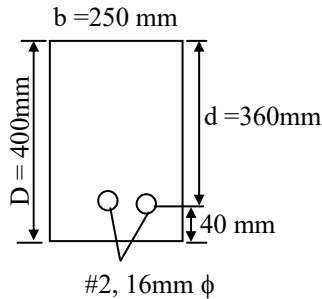
$$13.3 = P \times 1.5$$

$$P = \frac{13.3}{1.5} = 8.86 \text{ kN}$$



07. Ans: 31.6 kN

Sol:



Reinforced concrete beam

i)  $x_{u\max} = 0.48d$   
 $= 0.48 \times 360 = 172.8 \text{ mm}$

$C = T$

$0.36f_{ck}bx_u = 0.87 f_y A_{st}$

$0.36 \times 20 \times 250 \times x_u = 0.87 \times 415$   
 $\times \left( 2 \times \frac{\pi}{4} \times 16^2 \right)$

$1800 x_u = 145186.8$

$x_u = 80.65 \text{ mm}$

$x_u < x_{\max}$

under reinforced section

$M.R = 0.36f_{ck} bx_u (d - 0.42x_u)$

$= 0.36 \times 20 \times 250 \times 80.65$

$(360 - 0.42 \times 80.65)$

$M_u = 47.5 \text{ kN-m}$

$M_u = P \times a$

$47.5 = P \times a$

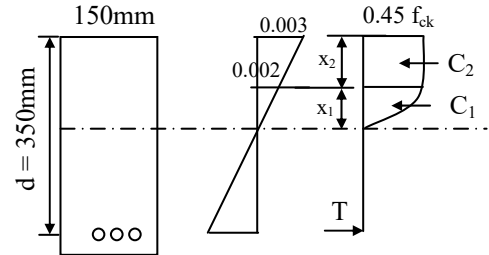
$P = \frac{47.5}{1.5}$

$P = 31.6 \text{ kN}$

Common data for Questions 08 & 09

08. Ans: 51 kN-m

Sol:



$x_{u\max} = 0.48 \times d$   
 $= 0.48 \times 350$   
 $= 168 \text{ mm}$

$M_{u\text{ limit}} = 0.36f_{ck} b x_{u\max} (d - 0.42 x_{u\max})$   
 $= 0.36 \times 20 \times 150 \times 168 (350 - 0.42 \times 168)$   
 $= 50.70 \times 10^6 \text{ N-m}$   
 $= 51 \text{ kN-m}$

09. Ans: 503 mm<sup>2</sup>

Sol:  $C = T$

$0.36 f_{ck} b x_{u\max} = 0.87 f_y A_{st}$

$A_{st} = \frac{0.36f_{ck} bx_{u\max}}{0.87 \times f_y}$

$= \frac{0.36 \times 20 \times 150 \times 168}{0.87 \times 415}$

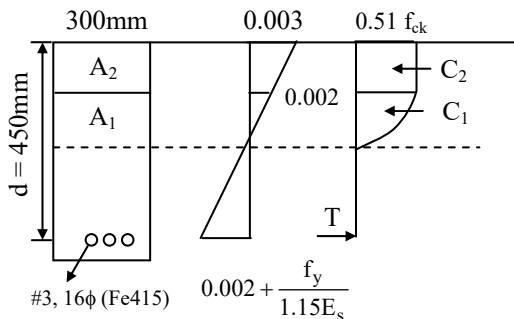
$= 502.53 \text{ mm}^2$

$A_{st} \approx 503 \text{ mm}^2$



10. Ans: 192 mm

Sol:



$$x_{u\max} = 0.003$$

$$d - x_{u\max} = \left( 0.002 + \frac{f_y}{1.15 E_s} \right)$$

$$(d - x_{u\max}) = \left( 0.002 + \frac{f_y}{1.1 E_s} \right)$$

$$450 - x_{u\max} = \left( 0.002 + \frac{415}{1.1 \times 2 \times 10^5} \right)$$

$$450 - x_{u\max} = 3.88 \times 10^{-6}$$

$$x_{u\max} = 196 \text{ mm}$$

### Chapter- 4 Limit State – Doubly Reinforced Beam

01. Ans: (c)

Sol: BM = 300 kN-m

Concrete  $M_{15} = f_{ck} = 15$

Steel  $f_y = 415$

$f_{sc} = 353.7 \text{ MPa}$

Eff. Cover  $d' = 50 \text{ mm}$

In LSM, we have to use

Factored moment

$M_u = M \times \gamma_f$

$$= 300 \times 1.5 = 450 \text{ kN-m}$$

Use  $\gamma_f = 1.5$

calculate  $M_{u \text{ limit}}$

$$M_{u \text{ limit}} = 0.138 f_{ck} b d^2$$

$$= 0.138 \times 15 \times 350 \times (700)^2$$

$M_{u \text{ limit}} = 355 \text{ kN-m}$

$M_u = 450 \text{ kN-m}$

$\therefore M_u > M_{u \text{ limit}}$

So we need to use 'DRB'

$$M_{u \text{ limit}} = 0.87 f_y A_{st} (d - 0.42 x_{u \text{ max}})$$

$$355 \times 10^6 = 0.87 \times 415 \times A_{st} (700 - 0.42 \times 0.48 \times 700)$$

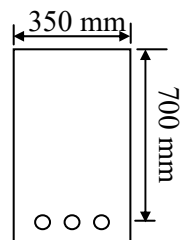
$$A_{st} = 1759.31 \text{ mm}^2$$

for extra moment we need to provide tensile steel & comp. steel

$$M_u - M_{u \text{ limit}} = 0.87 f_y (d - d') A_{st2}$$

$$(450 - 355) \times 10^6 = 0.87 \times 415 A_{st2} (700 - 50)$$

$$= 234682.5 A_{st2}$$





$$A_{st2} = 404.8 \approx 405 \text{ mm}^2$$

$$A_{st} = A_{st1} + A_{st2} = 2165 \text{ mm}^2$$

Now our purpose is to calculate 'A<sub>sc</sub>'

$$M_u - M_{u\text{limit}} = f_{sc} A_{sc} (d - d')$$

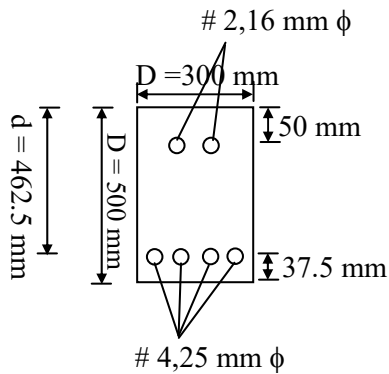
$$\text{(or)} f_{sc} A_{sc} = 0.87 f_y A_{st2}$$

$$A_{sc} = 413.2 \text{ mm}^2$$

**Common data for Questions 02 & 03**

**02. Ans: 270 kN-m**

**Sol:**



$$b = 300 \text{ mm}, D = 500 \text{ mm}, d = 462.5 \text{ mm}$$

$$f_{ck} = 25 \text{ N/mm}^2, f_y = 415 \text{ N/mm}^2,$$

$$f_{sc} = 0.8566 f_y$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1963.495 \text{ mm}^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$$

$$\Rightarrow C = T$$

$$\Rightarrow C_2 + C_2 = T$$

$$0.36 \times f_{ck} \times b \times x_u + f_{sc} A_{sc} = 0.87 f_y A_{st}$$

$$0.36 \times 25 \times 300 \times x_u + (0.8566 \times 415) \times$$

$$402.12 = 0.87 \times 415 \times 1963.495$$

$$x_u = 209.618 \text{ mm}$$

$$x_{u\text{max}} = 0.48 \times d$$

$$= 0.48 \times 462.5 = 222 \text{ mm}$$

$$x_u < x_{u\text{max}}$$

∴ under reinforcement section.

$$M_u = 0.36 f_{ck} \cdot b \cdot x_u (d - 0.42 x_u) + f_{sc} A_{sc} (d - d')$$

$$= 0.36 \times 25 \times 300 \times 209.6$$

$$(462.5 - 0.42 \times 209.6) + (0.8566 \times 415)$$

$$\times 402.12 (462.5 - 50)$$

$$= 270.9 \text{ kN-m}$$

**03. Ans: 18.82 kN/m**

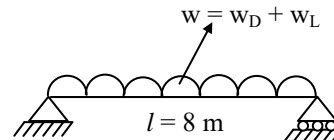
**Sol:** Working /line moment,

$$M = \frac{270.9}{1.5} = 180.6 \text{ kN-m}$$

Self weight of beam,  $w_D = (\gamma_c) b \times D$

$$= (25 \text{ kN/m}^3) \times (0.3 \times 0.5)$$

$$W = 3.75 \text{ kN/m}$$



$$M = \frac{(w_D + w_L) \times l^2}{8}$$

$$180.6 = \frac{(3.75 + w_L) \times 8^2}{8}$$

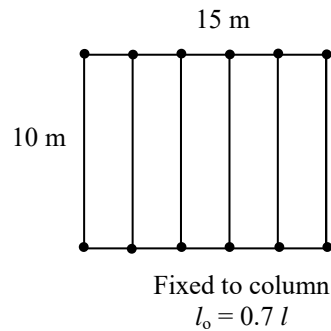
$$w_L = 18.825 \text{ kN/m}$$



**Chapter- 5**  
**Limit State Design**  
**- Flanged Beams**

**01. Ans: (c)**

**Sol:** For T-beams,



$$b_f = \frac{l_0}{6} + b_w + 6D_f$$

$$= \frac{0.7 \times 10}{6} + 0.25 + 6 \times 0.1$$

$$= 2.01 \text{ m} \not\geq c = 3 \text{ m}$$

$$\therefore b_f = 2.01 \text{ m}$$

**02. Ans: (d)**

**Sol:** L – beam

$$B_f = \frac{l_0}{12} + b_w + 3D_f$$

$$= \frac{10}{12} + 0.25 + 3 \times 0.1$$

$$= 1.38 \text{ m} \not\geq c = 3 \text{ m}$$

$$\therefore b_f = 1.38 \text{ m}$$

**03. Ans: (d)**

**Sol:**  $D_f = 100 \text{ mm}$ ,  $b_w = 300$ ,  $d = 500$ ,  $c = 3$

$$l = 6, l_0 = 3.6 \text{ m}, b_f = ?$$

$$b_f = \frac{l_0}{6} + b_w + 6D_f \not\geq c$$

$$= \frac{3.6}{6} + 0.3 + 6 \times 0.1$$

$$= 1.5 \text{ m} \not\geq c = 3 \text{ m}$$

$$= 1.5 \times 1000 \text{ m} = 1500 \text{ mm}$$

**Chapter- 6**  
**Limit State of Collapse - Shear**

*Common data Q 1 & 2*

**01. Ans: (b)**

**Sol:**

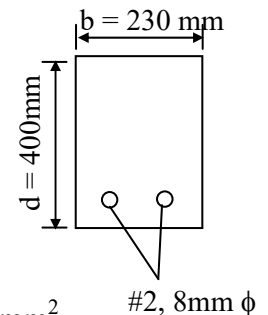
$$V_u = 120 \text{ kN}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$\text{Main steel, } f_y = 415 \text{ N/mm}^2$$

$$\text{Stirrups, } f_y = 250 \text{ N/mm}^2$$

$$\tau_c = 0.48 \text{ N/mm}^2$$



**i)** 8mm–2 legged

Stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2$$

$$= 100.53 \text{ mm}^2$$



$$\tau_v = \frac{V_u}{b \times d} = \frac{120 \times 10^3}{400 \times 230}$$

$$= 1.3 \text{ N/mm}^2$$

$\tau_v \leq \tau_{c \max}$  – safe in shear

**ii)**  $\tau_v > \tau_c$  – not safe in shear reinforcement

Minimum shear reinforcement is required

$$V_{us} = \frac{(0.87f_y)A_{sv} \times d}{S_v}$$

$$V_{us} = V_u - \tau_c b \cdot d$$

$$= 120 \times 10^3 - 0.48 \times 400 \times 230$$

$$= 75840 \text{ N} = 75.84 \text{ kN}$$

$$75.84 \times 10^3 = \frac{0.87 \times 250 \times 100.53 \times 400}{S_v}$$

$$S_v = 115 \text{ mm c/c}$$

**02. Ans: (c)**

**Sol:**  $T = 10.90 \text{ kN-m}$

$$V_e = V_u + \frac{1.6T_u}{b}$$

$$= 120 \times 10^3 + \frac{1.6 \times 10.90 \times 10^6}{230}$$

$$V_e = 196 \text{ kN}$$

Design shear force

$$V_{us} = V_e - \tau_c \cdot b \cdot d$$

$$= 196 \times 10^3 - 0.48 \times 230 \times 400$$

$$V_{us} = 151.84 \times 10^3 \text{ N}$$

**03. Ans: (d)**

**Sol:**  $b = 230 \text{ m}$ ,  $d = 450 \text{ mm}$

$$V_u = 50 \text{ kN}$$

$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 250 \text{ N/mm}^2$$

$$\tau_{c \max} = 2.8 \text{ MPa}, \tau_c = 0.75 \text{ MPa.}$$

$$\tau_v = \frac{V_u}{bd} = \frac{50 \times 10^3}{230 \times 450} = 0.483 \text{ MPa}$$

$\tau_v < \tau_{c, \max}$  safe in shear.

Provide minimum shear reinforcement.

$$\frac{A_{sv}}{bS_v} = \frac{0.4}{0.87f_y}$$

$$A_{sv} = 2 \times \frac{\pi \times 8^2}{4} = 100.53 \text{ mm}^2$$

$$S_v = \frac{100.53 \times 0.87 \times 250}{0.4 \times 230}$$

$$= 237.7 \text{ mm c/c}$$

$$S_v \not\geq 0.75 d = 0.75 \times 450 = 337.5 \text{ mm}$$

$$S_v \not\geq 300 \text{ mm}$$

∴ Provide min spacing of 230 mm c/c

**04. Ans: (c)**

$$V_u = 100 \text{ kN}$$

$$\tau_v = \frac{V_u}{b \times d} = \frac{100 \times 10^3}{230 \times 450} = 0.966$$

$\tau_v < \tau_{c \max}$  – shear reinforcement safe

$\tau_v > \tau_c$  not safe in shear reinforcement

Minimum shear reinforcement is required.

Design shear force for shear reinforcement

$$V_{us} = V_u - \tau_u bd$$

$$= 100 \times 10^3 - 0.75 \times 230 \times 450$$

$$= 22.375 \text{ kN}$$





For vertical stirrups,

$$V_{us} = \frac{0.87f_y A_{sv} d}{S_v}$$

$$S_v = \frac{0.87 \times 250 \times 100.53 \times 450}{22.375 \times 10^3}$$

$$= 439.75 \text{ mm}$$

**Min spacing:**

- i. 439.75 mm
- ii.  $0.75d = 0.75 \times 450 = 337.5 \text{ mm}$
- iii. 300 mm
- iv. min shear reinforcement

$$\frac{A_{sv}}{bS_v} = \frac{0.4}{0.87f_y} \Rightarrow S_v = 237.7 \text{ mm}$$

Provide min spacing of 230 mm c/c.

**05. Ans: (c)**

**Sol:**  $V_u = 150 \text{ kN}$

$$\tau_v = \frac{150 \times 10^3}{230 \times 450} = 1.449 \text{ MPa}$$

$\tau_v < \tau_{c,max}$  – safe in shear reinforcement

$\tau_v > \tau_c \rightarrow$  Shear reinforcement is required.

Design shear force,

$$V_{us} = V_u - \tau_c b d$$

$$= 150 \times 10^3 - 0.75 \times 230 \times 450$$

$$= 72.375 \text{ KN}$$

Shear force taken by bent-up bars.

$$V_{us1} = 0.87f_y A_{sv} \sin \alpha$$

$$= 0.87 \times 415 \times 2 \times \frac{\pi}{4} \times 16^2 \times \sin 45^\circ$$

$$= 102.66 \text{ kN}$$

$$\geq 0.5 V_{us} = 36.18 \text{ kN}$$

$$\therefore V_{us1} > 0.5 V_{us}$$

As per IS: 456 ;  $V_{us1} \geq 0.5 V_{us}$ . In this case  $V_{us1}$  is exceeding  $0.5 V_{us}$ . Therefore limit  $V_{us1}$  as 36.18 kN, the remaining S.F i.e 36.18 kN should be resisted by vertical stirrups.

**Vertical stirrups:**

For  $V_{us2} = 36.18 \text{ kN}$

$$36.18 \times 10^3 = \frac{0.87f_y A_{sv} d}{S_v}$$

$$S_v = \frac{0.87 \times 250 \times \left(2 \times \frac{\pi}{4} \times 8^2\right) \times 450}{36.18 \times 10^3}$$

$$= 271.958 \text{ mm}$$

Provide minimum center to center spacing of 230 mm c/c

**06. Ans: (a)**

**Sol: Beam -P**

$$\tau_{c,max} = 2.1 \text{ MPa}$$

$$f_{ck} = 30 \text{ N/mm}^2$$

$$\tau_c = 0.75 \text{ MPa}$$

$$V_u = 400 \text{ kN}$$

$$\tau_v = \frac{V_u}{b \times d} = \frac{400 \times 10^3}{750 \times 400}$$

$$\tau_v = 1.33 \text{ N/mm}^2$$

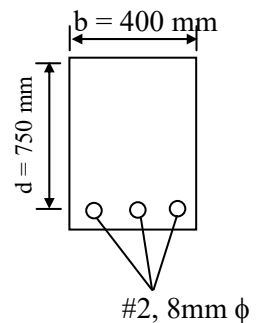
**i)**  $\tau_v < \tau_{c,max}$  –shear reinforcement safe

**ii)**  $\tau_v > \tau_c$  minimum shear reinforcement is required

$$V_{us} = V_u - \tau_c b d$$

$$= 400 \times 10^3 - 0.75 \times 400 \times 750$$

$$V_{us} = 175 \text{ kN}$$





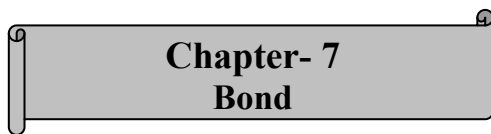
**Beam -Q**

$$V_u = 750 \text{ kN}$$

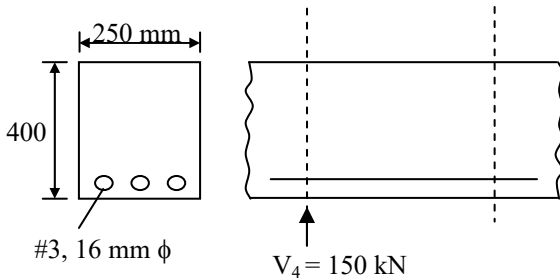
$$\tau_v = \frac{V_u}{b \times d} = \frac{750 \times 10^3}{750 \times 400} = 2.5 \text{ N/mm}^2$$

$$\tau_v > \tau_{c \text{ max}}$$

The beam is not safe in shear . It should be revised.



01. Ans: (c)



**Flexural bond:**

Steel in tension ( sagging moment)

$$L_d \geq \frac{M_1}{V_u} + l_0 \rightarrow \text{continuous beam}$$

$$l_0 = 12 \phi = 12 \times 16$$

$$\left. \begin{array}{l} = 192 \text{ mm} \\ d = 400 \text{ mm} \end{array} \right\} \text{Which is greater}$$

Take  $l_0 = 400 \text{ mm}$

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 250 \times 16}{4 \times 1} = 870 \text{ mm}$$

$$x_{u, \text{ max}} = 0.53 \times 400 = 212 \text{ mm}$$

$$x_u = \frac{0.87 \times 250 \times 3 \times \frac{\pi}{4} \times 16^2}{0.36 \times 15 \times 250}$$

$$= 97.18 \text{ mm}$$

$$x_u < x_{u, \text{ max}} \rightarrow \text{Under reinforcement section.}$$

$$M_1 = 0.36 \times 15 \times 250 \times 97.18 (400 - 0.42 \times 97.18)$$

$$= 47.12 \times 10^6 \text{ N-mm}$$

$$L_d \geq \frac{47.12 \times 10^6}{150 \times 10^3} + 400 = 714.15 \text{ mm}$$

$$L_d > 714.15$$

not safe in bond.

02. Ans: (d)

Sol:  $d = 12 \text{ mm}$

$$f_y = 415 \text{ N/mm}^2$$

$$f_{ck} = 30 \text{ N/mm}^2, \tau_{bd} = 2.4 \text{ MPa}$$

$$L_d = \frac{\phi \sigma_s}{\tau_{bd} \times 4}$$

$$= \frac{12 \times 0.87 \times 415}{(1.6 \times \tau_{bd}) \times 4} = 282.0703$$

$$L_d = 282.0703 \text{ mm}$$

$$L_d \text{ with } 90^\circ \text{ bend} = 282.0703 - 8\phi$$

$$= 282.0703 - 8 \times 12$$

$$= 186.1 \text{ mm}$$

03. Ans: (d)

Sol: Axially loaded short column

$$d = 20 \text{ mm}, \text{ spliced} = 16 \text{ mm}$$

$$f_y = 415 \text{ N/mm}^2$$

$$\tau_{bd} = 1.2 \text{ MPa}$$

$$\left. \begin{array}{l} \text{lap} < l_d \\ < 24\phi \end{array} \right\} \text{max}$$



Use smaller  $\phi = 16$  mm

$$L_d = \frac{\phi \sigma_s}{4 \times \tau_{bd}} = \frac{16 \times 0.87 \times 415}{1.25 \times 4 \times 1.2 \times 1.6}$$

$$= 601.75 \text{ mm}$$

Lap length  $\leq L_d = 601.75$  mm

$$\leq 24 \phi = 384 \text{ mm}$$

Use maximum, i.e., 601.75 mm

04. Ans: (d)

Sol: 1) pull out (bond fail)

$$P_1 = \tau_{bd}[\pi D l]$$

2) breaking of steel bar

$$P_2 = \sigma_{st} \left[ \frac{\pi}{4} \times D^2 \right]$$

} minimum

05. Ans: 46.8

Sol:  $f_{ck} = 20 \text{ N/mm}^2$ ,  $\tau_{bd} = 1.2 \text{ MPa}$

↑60% - HYSD bars

Steel bar is in tension

$$L_d = \frac{\phi \sigma_s}{4 \times \tau_{bd}} = \frac{\phi \times 360}{4 \times 1.6 \times 1.2} = 46.8 \phi$$

06. Ans: 288.7 mm

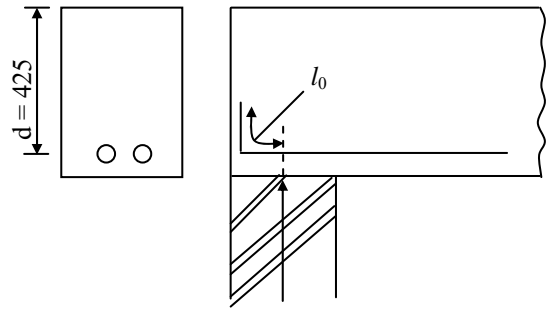
Sol: Given,  $V_u = 220 \text{ kN}$

$$A_{st} = 2 \times \frac{\pi}{4} \times 16^2 = 402.12 \text{ mm}^2$$

$b = 250 \text{ mm}$ ,  $d = 425 \text{ mm}$

Fe 415,  $M_{20}$ ,  $\tau_{bd} = 1.2 \text{ MPa}$

$l_0 = ?$  for  $90^\circ$  bond



$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times 16}{4 \times 1.6 \times 1.2}$$

$$= 752.1875 \text{ mm}$$

$$L_d (\text{req}) = 752.1875 - 8 \times 16$$

$$= 624.1875 \text{ mm}$$

$$M_1 = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$x_{u \text{ max}} = 0.48 \times 425 = 204$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$= \frac{0.87 \times 415 \times 402.12}{0.36 \times 20 \times 250}$$

$$= 80.65 \text{ mm}$$

$x_u < x_{u \text{ max}} \rightarrow$  Under reinforcement section

$$M_1 = 0.87 f_y A_{st} (d - 0.42 x_u)$$

$$= 0.87 \times 415 \times 402.12 (425 - 0.42 \times 80.65)$$

$$= 56.78 \times 10^6 \text{ N-mm}$$

$$L_d = \frac{1.3 M_1}{V} + l_0$$

$$624.1875 = 1.3 \times \frac{56.78 \times 10^6}{220 \times 10^3} + l_0$$

$$l_0 = 288.66 \text{ mm}$$

Minimum extension beyond centre of support.



**Chapter- 8**  
**Limit State of Collapse - Torsion**

**01. Ans: (d)**

**Sol: i)** size – 300 × 1000 mm

$$V_u = 150 \text{ kN}; \quad M_u = 150 \text{ kN-m}$$

$$T_u = 30 \text{ kN-m}$$

$$V_e = V_u + \frac{1.6T_u}{b}$$

$$= 150 \times 10^3 + \frac{1.6 \times 30 \times 10^6}{300} = 310 \text{ kN}$$

$$M_{e1} = M_u + M_T$$

$$= M_u + \frac{T_u \left[ 1 + \frac{D}{b} \right]}{1.7}$$

$$= 150 + \frac{30 \left[ 1 + \frac{1000}{300} \right]}{1.7} = 226.47 \text{ kN-m}$$

**02. Ans: (d)**

$$b = 300 \text{ mm}, \quad D = 600 \text{ mm}$$

$$V = 100 \text{ kN}, \quad M = 100 \text{ kN-m}$$

$$T = 34 \text{ kN-m}$$

$$M_{e1} = M_u + M_T$$

$$= M_u + \frac{T_u \left[ 1 + \frac{D}{b} \right]}{1.7}$$

$$= 100 + \frac{34 \left[ 1 + \frac{600}{300} \right]}{1.7}$$

$$= 160 \text{ kN-m}$$

**03. Ans: (a)**

**Sol:**  $T = 68 \text{ kN-m}$

$$M_{e2} = M_T < M_u$$

Here  $M_T < M_u$

No need of  $A_{sc}$

$$M_T = \frac{T_u \left( 1 + \frac{D}{b} \right)}{1.7} = \frac{68 \left( 1 + \frac{600}{300} \right)}{1.7}$$

$$= 120 \text{ kN-m}$$

$M_T > M_u$  – additional compression steel is required for  $M_{e2}$  i.e  $M_{e2} = M_T - M_u$   
 $= 120 - 100$   
 $= 20 \text{ kN-m}$

**04. Ans: (a)**

**Sol:**  $b = 500$ ,  $D = 700 \text{ mm}$

$$d = 35 \text{ mm}, \quad V = 15 \text{ kN}$$

$$M = 100 \text{ kN-m}, \quad T = 10 \text{ kN-m}$$

$$\tau_c = 1.5 \text{ MPa}$$

If  $\tau_{ve} > \tau_c$  ignore torsion

If  $\tau_{ve} < \tau_c$  consider torsion for  $A_{st}$

$$V_e = V_u + V_T$$

$$= v_u + 1.6 \frac{T_u}{b}$$

$$= 15 + 1.6 \left( \frac{10}{0.5} \right)$$

$$= 47 \text{ kN}$$

$$\tau_{ve} = \frac{v_e}{b.d} = \frac{47}{0.5 \times 0.7} = \frac{47 \times 10^3}{500 \times (700 - 35)}$$

$$= 0.14 \text{ MPa}$$

$$\tau_{ve} < \tau_c$$

∴ Design BM for  $A_{st}$  is  $M_u$  only

$$M_u = 100 \text{ kN-m}$$



*Common data Solutions for Q05 & Q06*

**05. Ans: (d)**

**Sol:**  $V = 20 \text{ kN}$ ,  $T = 9 \text{ kN-m}$   
 $b = 300 \text{ mm}$ ,  $M = 200 \text{ kN-m}$   
 gross depth = 125 mm

$$d = 25 \text{ mm}$$

$$V_e = V_u + V_T$$

$$= V_u + 1.6 \frac{T_u}{b} = 20 + 1.6 \left( \frac{9}{0.3} \right)$$

$$= 68 \text{ kN}$$

**06. Ans: (b)**

**Sol:** As  $\tau_{ve} < \tau_c$

$$T_u = 0$$

$$M_{e1} = M_u = 200 \text{ kN-m}$$

$A_{st}$  based on  $M_u$  only

**Chapter- 10**  
**Limit State of Collapse**  
**- Compression**

**01. Ans: (c)**

**Sol:**  $b = 300 \text{ mm}$

$$d = 600 \text{ mm}$$

$$f_y = 415 \text{ MPa}$$

$$f_{ck} = 20 \text{ MPa}$$

$$P_u = 0.40 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$A_{sc} = 0.8\% A_g$$

$$= \frac{0.8}{100} (300 \times 600)$$

$$= 1440 \text{ mm}^2$$

$$A_c = A_g - A_{sc}$$

$$= 300 \times 600 - 1440$$

$$= 178560 \text{ mm}^2$$

$$P_u = 0.4 \times 20 \times 178560 + 0.67 \times 415 \times 1440$$

$$P_u = 1829 \text{ kN}$$

**02. Ans: (d)**

**Sol:**  $d = 300 \text{ mm}$ ;  $f_{ck} = 20 \text{ N/mm}^2$

$$f_y = 415 \text{ N/mm}^2;$$

$$P_u = 1.05 [0.4 f_{ck} A_c + 0.67 f_y A_{sc}]$$

$$A_{sc} = \left( \frac{\pi}{4} \times 300^2 \right) \times \frac{1}{100} = 706.85 \text{ mm}^2$$

$$A_c = A_g - A_{sc}$$

$$= \left( \frac{\pi}{4} \times 300^2 \right) - 706.85$$

$$= 69978.98 \text{ mm}^2$$

$$P_u = 1.05 (0.4 \times 20 \times 69978.98 + 0.67 \times 415 \times 706.85)$$

$$= 794.19 \text{ kN}$$

**03. Ans: (d)**

**Sol:**  $A_c = A_g = 300 \times 300 \text{ mm}$

$$f_{ck} = 20 \text{ N/mm}^2,$$

$$A_c = A_g = A_{sc}$$

$$f_y = 415 \text{ N/mm}^2$$

$$A_{sc} = 4 \times \frac{\pi}{4} \times 20^2 = 1256.63$$

$$P_u = 0.4 \times 20 \times 300 \times 300 + 0.67 \times 415 \times 1256.63$$

$$= 1069 \text{ kN}$$



04. Ans: (d)

Sol: 
$$m = \frac{E_{\text{strong}}}{E_{\text{weak}}} = \frac{E_{\text{steel}}}{E_{\text{conc}}}$$

$d/s = d/c$

$\delta_s = \delta_c$

comparability condition for composite (RCC) members

$$\frac{P_s l}{A_s t_s} = \frac{P_c l}{A_c t_c}$$

$$\frac{P_s}{P_c} = \frac{A_s}{A_c} \left( \frac{\epsilon_s}{\epsilon_c} \right) = \frac{1\% A_c}{A_c} \times 10 = 10\%$$

**Chapter- 11**  
**Footings**

Common data for Questions 1,2 & 3

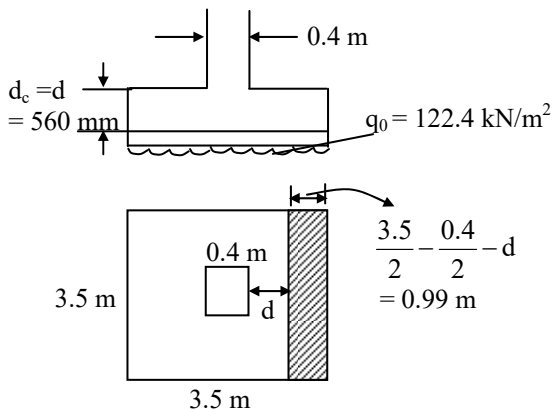
01. Ans: (b)

Sol: B = 3.5m

column size = 400 mm

d = 560 mm

$q_0 = 122.4 \text{ kN/m}^2$



For one way shear

$$V_u = q_0 [\text{hatched area}]$$

$$= 122.4 [0.99 \times 3.5]$$

$$= 425 \text{ kN}$$

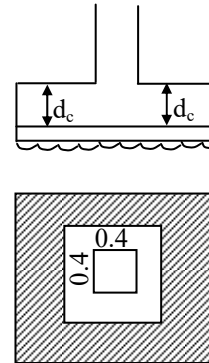
$$\tau_v = \frac{v_{us}}{b.d_c} = \frac{425 \times 10^3}{3500 \times 560}$$

$$= 0.22 \text{ N/mm}^2$$

$$= 0.22 \text{ Mpa}$$

02. Ans: (c)

Sol:



$$B = 0.4 + \frac{0.56}{2} + \frac{0.56}{2}$$

$$= 0.96$$

$$v_u = q_0 [\text{hatched area}]$$

$$= 122.4 \times [3.5^2 - 0.96^2]$$

$$= 1386 \text{ kN}$$

$$\tau_v = \frac{v_u}{pd} = \frac{1386 \times 10^3}{(4 \times 960)(560)}$$

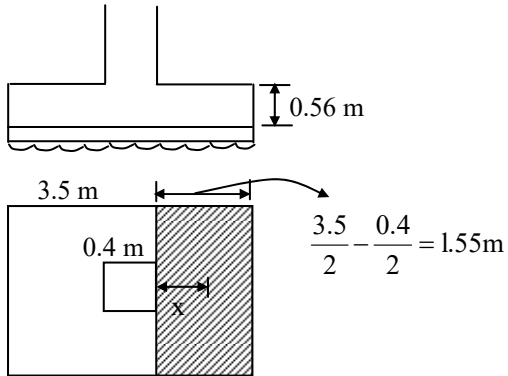
$$= 0.64 \text{ MPa}$$

$V_u$  is more for 2-way  
2-way shear is critical



03. Ans: (a)

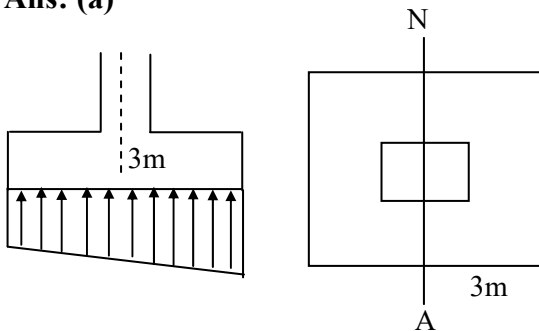
Sol:



$$M_u = q_0 [\text{hatched area} \times \bar{x}]$$

$$= 122.4 \left[ 3.5 \times 1.55 \times \frac{1.55}{2} \right] = 515 \text{ kN}$$

04. Ans: (a)



$$\left. \begin{array}{l} \sigma_{\max} \\ \sigma_{\min} \end{array} \right\} = \frac{P}{A} \pm \frac{M}{Z}$$

$$= \frac{450}{3 \times 2} \pm \frac{60}{\left( \frac{2 \times 3^2}{6} \right)}$$

$$\sigma_{\max} = 95 \text{ kN/m}^2 \text{ compression}$$

$$\sigma_{\min} = 55 \text{ kN/m}^2 \text{ compression}$$

As per IS 456 -2000 the assumed pressure distain below the footing is uniform

05. Ans: (a)

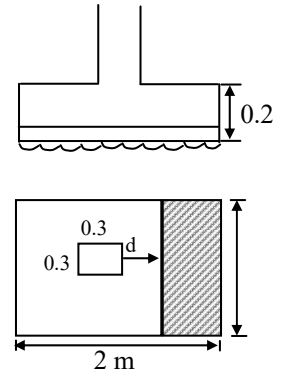
Sol:  $l = 2\text{m}$  ;  $d = 200 \text{ mm}$

column size =  $300 \times 300 \text{ mm}$

$$q_0 = 320 \text{ kN}$$

$$\tau_v = ?$$

$$q_0 = \frac{320}{2 \times 2} = 80 \text{ kN/m}^2$$



One way shear  $V_u = q_0 [\text{hatched area}]$

$$= 80 [0.65 \times 2] = 104$$

$$\tau_v = \frac{V_u}{bd_c} = \frac{104 \times 10^3}{2000 \times 200} = 0.26$$

### Chapter- 14 Analysis of Prestressed Concrete Members

01. Ans: (b)

Sol: Prestressing force,  $P = 2500 \text{ kN}$

Effective span,  $l = 10 \text{ m}$

Udl on the beam,  $w = 40 \text{ kN/m}$

For load balancing

$$P.e = \frac{wl^2}{8}$$

$$(2500)(e) = \frac{(40)(10)^2}{8}$$

$$e = 0.2 \text{ m} = 200 \text{ mm}$$

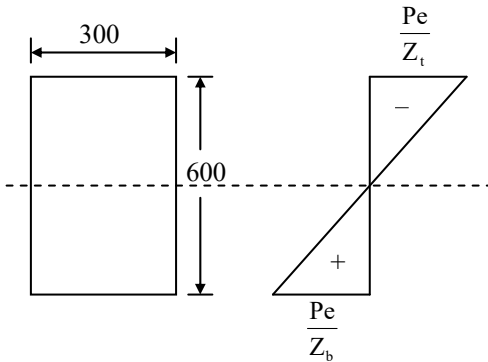


02. Ans: (b)

Sol:  $\gamma_c = 24 \text{ kN/m}^3$

$$\sigma_t = 2 \text{ MPa}$$

$$\sigma_b = 20 \text{ MPa}$$



$$\sigma_b = \frac{P}{A} + \frac{Pe}{z} \text{----- (1)}$$

$$\sigma_t = \frac{P}{A} - \frac{Pe}{z} \text{----- (2)}$$

Adding (1) & (2)

$$20 = \frac{P}{A} + \frac{Pe}{z}$$

$$2 = \frac{P}{A} - \frac{Pe}{z}$$

$$18 = \frac{2P}{A}$$

$$P = 1620 \text{ kN}$$

$$\sigma_b = \frac{P}{A} + \frac{Pe}{z}$$

$$20 = \frac{1620 \times 10^3}{300 \times 600} + \frac{1620 \times 10^3 \times 6 \times e}{300 \times 600^2}$$

$$e = 122 \text{ mm}$$

$$e \approx 135 \text{ mm}$$

Common data for Questions 03 & 04

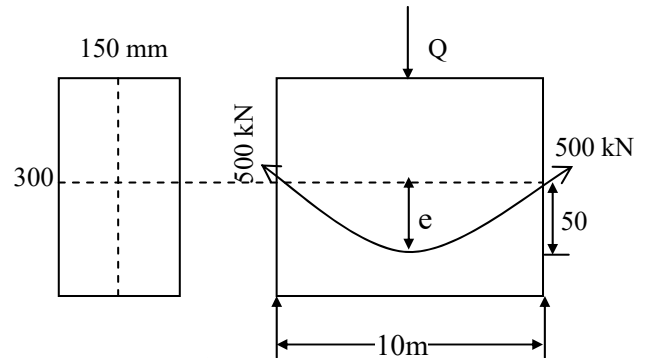
03. Ans: (a)

Sol:  $150 \times 300 \text{ mm}$

$$l = 10 \text{ m, } e = \text{at support} = 50 \text{ mm}$$

$$e = 50 \text{ mm center, } P = 500 \text{ kN}$$

$$Q = ? \text{ (at center of span)}$$



$$Pe = \frac{Q \times l}{4}$$

$$500 \times \frac{50}{1000} = \frac{Q \times 10}{4}$$

$$100 = Q \times 10$$

$$Q = 10 \text{ kN}$$

04. Ans: (b)

Sol: Self weight of external load

$$w_D = \gamma_c \times b \times D$$

$$= (24 \text{ kN/m}^3) \times 0.15 \times 0.3$$

$$= 1.08 \text{ kN/m}$$

P – line at upper kern point ( $\sigma_b = 0$ )

$$M_D = \frac{w_D l^2}{8} = \frac{1.08 \times 10^2}{8} = 13.5$$

$$\sigma_b = 0 = \frac{P}{A} + \frac{Pe}{z} - \frac{M_D}{z} - \frac{M_L}{z}$$





$$= \frac{500 \times 10^3}{300 \times 150} + \frac{500 \times 10^3 \times 50}{\left(\frac{150 \times 300^2}{6}\right)} - \frac{13.5 \times 10^6}{\left(\frac{150 \times 300^2}{6}\right)}$$

$$- \frac{M_L}{\left(\frac{150 \times 300^2}{6}\right)}$$

$$0 = 11.11 + 11.11 - 6 = \frac{M_L}{225 \times 10^4}$$

$$M_L = 16.22 \times 225 \times 10^4$$

$$M_L = 36.5 \text{ kN-m,}$$

$$M_L = \frac{Ql}{4}$$

$$36.5 = \frac{Q \times 10}{4}$$

$$146 = Q \times 10$$

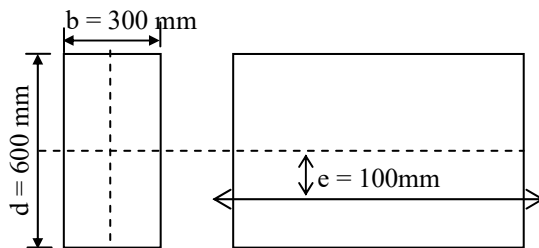
$$Q = 14.6 \text{ kN}$$

**05. Ans: (c)**

**Sol:**  $l = 6 \text{ m, } b = 300 \text{ mm, } d = 600 \text{ mm}$

$e = 100 \text{ mm, } P = 1000 \text{ kN,}$

neglect self weight



$$\sigma_b = \frac{P}{A} + \frac{Pe}{z}$$

$$= \frac{1000 \times 10^3}{300 \times 600} + \frac{1000 \times 10^3 \times 100}{\left(\frac{300 \times (600)^2}{6}\right)}$$

$$= 5.55 + 5.55 = 11.11 \text{ MPa}$$

**06. Ans: (b)**

**Sol:**  $b = 200 \text{ mm, } D = 250 \text{ mm}$

$A = 500 \text{ mm}^2, P = 1000 \text{ MPa}$

$m = 10$

$\epsilon_s = (\text{Strain in concrete}) \text{ same}$

$$\epsilon_s = \epsilon_c$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_c = \sigma_s \left( \frac{\epsilon_c}{\epsilon_s} \right)$$

$$= \frac{\sigma_s}{m} = \frac{1000}{10}$$

$$\sigma_c = 100 \text{ MPa}$$

Prestressing force on steel  $= \sigma_s \cdot A_s$

$$= 1000 \times 500$$

$$= 500 \times 10^3 \text{ N}$$

Compression force in concrete  $= 5000 \text{ kN}$

$$= \sigma_c \cdot A_c$$

Compression stress in concrete  $\sigma_c = \frac{P_c}{A_c}$

$$= \frac{500 \times 10^3}{200 \times 250} = 10 \text{ MPa}$$

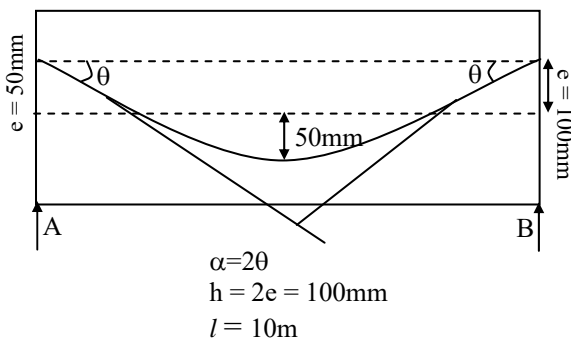


## Chapter- 15 Losses of Prestress

Common data for Questions 1,2 &3

01. Ans: (b)

Sol:  $l = 10 \text{ m}$ ,  $b = 100 \text{ mm}$ ,  
 $D = 300 \text{ mm}$   $A = 200 \text{ sq-mm}$ ,  
 $e = 50 \text{ mm}$ ,  $\mu = 0.35$ ;  
 $k = 0.0015 \text{ per m}$



Initial stress in wires = 1200 MPa

$$\begin{aligned} \text{Loss of stress in wires} &= \sigma(\mu\alpha + kx) \\ &= 1200[0.35 \times \alpha + 0.0015 \times 10] \end{aligned}$$

From equation of parabola

$$\theta = \frac{4 \times 0.1}{10} = 0.04 \text{ radians}$$

$$\alpha = 2 \times \theta = 0.08$$

$$\text{Loss} = 1200[0.35 \times 0.08 + 0.0015 \times 10]$$

Loss of stress = 51.6 MPa

$$\% \text{ of loss} = \frac{51.6}{1200} \times 100 = 4.28 \approx 4.3\%$$

02. Ans: (b)

Sol: both the ends

Tensioning from both the ends

$$\frac{\% \text{ of loss}}{2} = \frac{4.28}{2} = 2.15$$

03. Ans: (b)

Sol: Straight tendon tensioned from one end

$$\text{Loss of stress in wires} = \sigma[\mu\alpha + kx]$$

$$(\because \alpha = 0)$$

$$1200(0.35 \times 0 + 0.0015 \times 10) = 18 \text{ MPa}$$

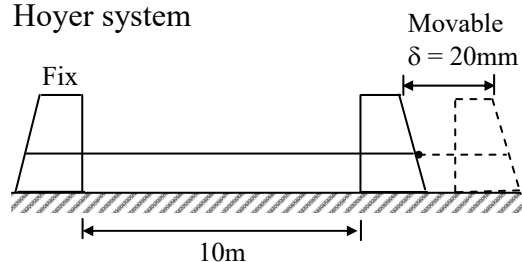
$$\begin{aligned} \% \text{ of loss} &= \frac{18}{1200} \times 100 \\ &= 1.5\% \end{aligned}$$

If tensioned from two ends

$$\frac{\% \text{ of loss}}{2} = \frac{1.5}{2} = 0.75\%$$

04. Ans: (c)

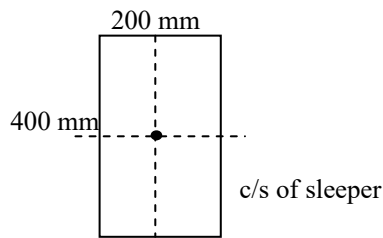
Sol: Hoyer system



$$\delta = \frac{PL}{AE} \quad \left( \text{as } \sigma = \frac{P}{A} \right)$$

Prestress induced in steel wire,  $\sigma = \frac{\delta E}{L}$

$$\sigma = \frac{20 \times 2 \times 10^5}{10,000} = 400 \text{ MPa}$$



Eccentricity of Prestress,  $e = 0$

$$\begin{aligned} \text{Prestressing force in steel wire} &= P = \sigma_s \cdot A_s \\ &= 400 \times 500 \text{ mm}^2 \\ &= 200 \text{ kN} \end{aligned}$$

$$f_c = \frac{P}{A} + \frac{Pe}{I} (e) = \frac{200}{200 \times 400} = 2.5 \text{ MPa}$$

$$\text{Loss due to shorting} = m \times f_c = \left( \frac{E_s}{E_c} \right) f_c$$

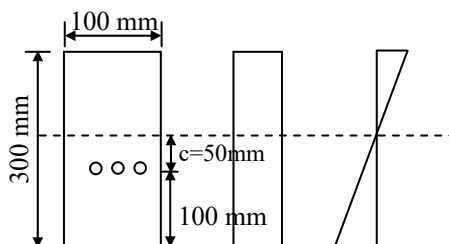
$$\sigma = \left( \frac{200,000}{20,000} \right) \times 2.5 = 25 \text{ MPa}$$

$$\% \text{ of loss of Prestress} = \frac{25}{400} \times 100 = 6.25\%$$

**Common data for Questions 05 & 06**

**05. Ans: (d)**

**Sol:**



$$f_c = \frac{P}{A} + \frac{P}{I} (e)^2$$

Initial stresses in steel wire = 1200 MPa

Prestressing force in each steel wire

$$P = \sigma_s \cdot A_s$$

$$P = 1200 \times 50 = 60 \text{ kN}$$

$$f_c = \frac{60 \times 10^3}{100 \times 300} + \frac{60 \times 10^3}{\left( \frac{100 \times 300^3}{12} \right)} \times (50)^2$$

$$f_c = 2.66 \text{ MPa}$$

Simultaneous tension = loss of Prestress is zero

**06. Ans: (a)**

**Sol:** Successive tensioning of the 3 cables

$$= \frac{n(n-1)}{2} (m \cdot f_c)$$

$$= \frac{3(3-1)}{2} (6 \times 2.66)$$

$$= 48.0 \text{ MPa}$$

$$\% \text{ of loss} = \frac{48.0}{1200} \times 100 = 4\%$$

For pretensioning system

$$\text{Loss} = n(m \cdot f_c)$$

$$= 3(6 \times 2.66) = 48.0 \text{ MPa}$$

**07. Ans: (c)**

**Sol:** Anchorage slip = 3 mm

$$l = 30 \text{ m}, \sigma = 1200 \text{ MPa}$$

$$E = 2.1 \times 10^5 \text{ MPa}$$

$$E = \frac{\delta E}{l} = \frac{3 \times 2.1 \times 10^5}{30}$$

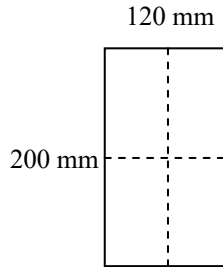
$$\sigma = 21 \text{ MPa}$$

$$\% \text{ of loss} = \frac{21}{1200} \times 100 = 1.73\%$$



08. Ans: (b)

Sol:



$$P = 150 \text{ kN}, e = 20 \text{ mm}$$

$$E_s = 187.5 \text{ mm}^2$$

$$E_s = 2.1 \times 10^5 \text{ MPa}$$

$$E_c = 3.0 \times 10^4 \text{ MPa}$$

% of steel

$$f_c = \frac{P}{A} + \frac{Pe}{I} \cdot e$$

$$= \frac{150 \times 10^3}{1878} + \frac{150 \times 10^3 \times 20^2}{\left(\frac{120 \times 200^3}{12}\right)}$$

$$= 800 + 0.75 \text{ MPa}$$

$$f_c = 800.75 \text{ MPa}$$

losses due to elastic starting =  $m \cdot f_c$

$$= \left(\frac{E_s}{E_c}\right) \cdot f_c$$

$$= \frac{2.1 \times 10^5}{3.0 \times 10^4} \times 7 = 4.9$$

Percentage loss in the prestressing steel due to elastic deformation

$$= \frac{4.9}{800.75} \times 100$$

$$= 6.12\%$$

09. Ans: (c)

$$\text{Sol: } \varepsilon = \varepsilon_{\text{shrink}} + \varepsilon_{\text{creep}}$$

$$= 0.0008$$

Loss of prestress on steel =  $\varepsilon \times E_s$

$$= 0.0008 \times 200 \times 10^3$$

$$= 160 \text{ MPa}$$

Stress remaining after loss = Initial stress - Loss

$$= 200 - 160$$

$$= 40 \text{ MPa}$$