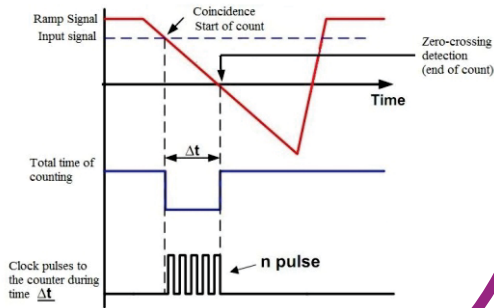




# INSTRUMENTATION ENGINEERING



## GATE I PSUs

MEASUREMENTS

***Classroom Practice solutions******To******Measurements******CONTENTS***

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# Chapter 1

# Error Analysis

## Class Room Practice Solutions

01. Ans: (a)

Sol: For 10V total input resistance

$$R_v = \frac{V_{fsd}}{I_{m fsd}} = 10/100\mu A \Rightarrow 10^5 \Omega$$

$$\text{Sensitivity} = R_v/V_{fsd} = 10^5/10 \\ \Rightarrow 10k\Omega/V$$

$$\text{For } 100V \quad R_v = 100/100\mu A \Rightarrow 10^6 \Omega$$

$$\text{Sensitivity} = R_v/V_{fsd} = 10^6/100 \\ \Rightarrow 10 k\Omega/V$$

02. Ans: (d)

Sol: Variables are measured with accuracy

$x = \pm 0.5\%$  of reading 80 (limiting error)

$Y = \pm 1\%$  of full scale value 100  
(Guaranteed error)

$Z = \pm 1.5\%$  reading 50 (limiting error)

The limiting error for Y is obtained as  
Guaranteed

$$\text{Error} = 100 \times (\pm 1/100) = \pm 1$$

Then % L.E in Y meter

$$20 \times \frac{x}{100} = \pm 1$$

$$x = 5\%$$

Given  $w = xy/z$ , Add all %L.E s

$$\text{Therefore } \pm (0.5\% + 5\% + 1.5\%) \\ = \pm 7\%$$

03.

Sol: Mean = 41.97

$$SD = \sqrt{\frac{\sum d_n^2}{n}} \quad d_1 = -0.27$$

$$\sqrt{\frac{(-0.27)^2 + (0.03)^2 + (0.17)^2 + (0.03)^2 + (0.13)^2 + \\ (-0.07)^2 + (0.53)^2 + (0.03)^2 + (0.13)^2 + (-0.17)^2}{10}} \\ = 0.2128$$

$$\text{Probable error} = 0.6745 \times SD \\ = 0.143$$

04.

Sol: Dial resistance of 1000  $\Omega$

$$\text{Error} = \pm 4000 \times \frac{0.02}{100} = 0.8$$

Dial resistance of 100  $\Omega$

$$\text{Error} = \pm 300 \times \frac{0.05}{100} = 0.15 \Omega$$

Dial resistance of 10  $\Omega$

$$\text{Error} = \pm 20 \times \frac{0.1}{100} = 0.02 \Omega$$

Dial resistance of 1  $\Omega$

$$\text{Error} = \pm 5 \times \frac{0.2}{100} = 0.01 \Omega$$

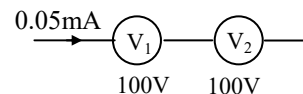
$$\text{Hence total error} = \pm (0.8 + 0.15 + 0.02 + 0.01) \\ = \pm 0.98 \Omega$$

Relative limiting error

$$= \pm \frac{0.98}{4325} = \pm 2.26 \times 10^{-4}$$

05.

Sol:



$V_1 :$

$V_2 :$

$$S.d.c_1 = 10k\Omega/V$$

$$S.d.c_2 = 20k\Omega/V$$

$$I_{fsd} = \frac{1}{S_{dc}}$$

$$I_{fsd} = \frac{1}{S_{dc2}}$$

$$= 0.1mA$$

$$= 0.05 mA$$

The maximum allowable current in this combination is 0.05mA, since both are connected in series.

$$\text{Maximum D.C voltage can be measured as} \\ 0.05 mA (10 k \Omega/V \times 100 + 20 \Omega/V \times 100) \\ = 3000 \times 0.05 = 150 V$$



06.

**Sol:** Internal impedance of 1<sup>st</sup> voltmeter

$$= \frac{100\text{V}}{5\text{mA}} = 20\text{ k}\Omega$$

Internal impedance of 2<sup>nd</sup> voltmeter

$$= 100 \times 250 \Omega/\text{V} = 25\text{ k}\Omega$$

Internal impedance of 3<sup>rd</sup> voltmeters,

$$= 5\text{ k}\Omega$$

Total impedance across 120 V

$$= 20 + 25 + 5 = 50\text{ k}\Omega$$

$$\text{Sensitivity} = \frac{50\text{ k}\Omega}{120\text{V}} \Rightarrow 416.6\Omega/\text{V}$$

$\Rightarrow$  Reading of 1<sup>st</sup> voltmeter

$$= \frac{20\text{ k}\Omega}{416.6\Omega/\text{V}} = 48\text{ V}$$

Reading of 2<sup>nd</sup> voltmeter

$$= \frac{25\text{ k}\Omega}{416.6\Omega/\text{V}} = 60\text{ V}$$

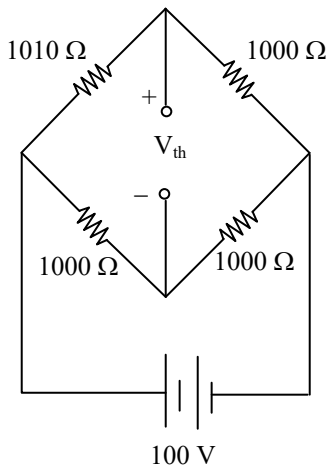
Reading of 3<sup>rd</sup> voltmeter

$$= \frac{5\text{ k}\Omega}{416.6\Omega/\text{V}} = 12\text{ V}$$

07. **Ans: (b)**

**Sol:** Bridge sensitivity =  $\frac{\text{Change in output}}{\text{Change in input}}$

$$= \frac{V_{th}}{10\Omega}$$



$$V_{th} = \frac{1010 \times 100}{2000} - \frac{1000 \times 100}{2000} = 0.25\text{V}$$

$$S_B = \frac{0.25\text{V}}{10\Omega} = 2.5\text{ mV}/\Omega$$

08. **Ans: (d)**

**Sol:**  $W_T = W_1 + W_2 = 100 - 50 = 50\text{ W}$

$$\frac{\partial W_T}{\partial W_1} = \frac{\partial W_T}{\partial W_2} = 1$$

$$\partial W_L = \frac{100}{100} \times 1\% = 1\%$$

$$\partial W_2 = \frac{100}{-50} \times 0.5\% = 1\%$$

$$W_T = W_1 + W_2 = 50 \pm 1.5\text{ W}$$

$$W_T = 50 \pm 3\%$$

09. **Ans: (b)**

**Sol:** Resolution =  $\frac{200}{100} \times \frac{1}{10} = 0.2\text{ V}$

10. **Ans: (b)**

**Sol:** % LE =  $\frac{\text{FSV}}{\text{true value}} \times \% \text{GAE}$

$$= \frac{200\text{V}}{100\text{V}} \times \pm 2\% \Rightarrow \pm 4\%$$

11. **Ans:  $\pm 2.45\%$**

**Sol:**  $R = 4 \frac{\rho L}{\pi D^2}$

Assuming errors are in standard deviations.

$$\% \sigma_\rho, \% \sigma_L \text{ and } \% \sigma_D = \pm 1\%,$$

$$\% \sigma_R =$$

$$\Delta R = \sqrt{\left(\frac{\partial R}{\partial \rho} \cdot \Delta \rho\right)^2 + \left(\frac{\partial R}{\partial L} \cdot \Delta L\right)^2 + \left(\frac{\partial R}{\partial D} \cdot \Delta D\right)^2}$$

And finally controlling zero into root sum square or standard deviation error analysis



$$\% \sigma_R = \sqrt{(\% \sigma_p)^2 + (\% \sigma_L)^2 + (2 \times \% \sigma_D)^2}$$

$$\Rightarrow \% \sigma_R = \sqrt{(1)^2 + (1)^2 + (2 \times 1)^2} = \sqrt{6}$$

$$\Rightarrow \% \sigma_R = \pm \sqrt{6} = \pm 2.45\%$$

**12. Ans: (a)**

**Sol:**  $R = \frac{V}{I}$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$$\frac{\Delta R}{R} = 2 + 1 = 3\%$$

**13. Ans: 2.5**

**Sol:** Given  $V_1 = 100 \text{ V}$

$$\sigma_1 = 1.5 \text{ V}$$

$$V_2 = 150 \text{ V}$$

$$\sigma_2 = 2 \text{ V}$$

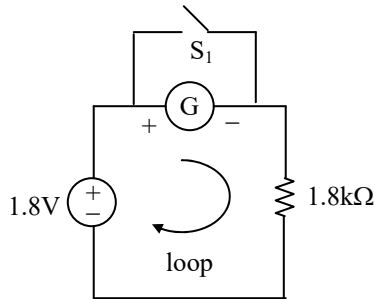
$$V_3 = V_1 + V_2$$

$$\sigma_3 = \sqrt{(2)^2 + (1.5)^2} = 2.5 \text{ Volt}$$

## Class Room Practice Solutions

01. Ans: (d)

Sol: The pointer swings to 1 mA and returns, settles at 0.9 mA i.e, pointer has oscillations. Hence, the meter is under-damped. Now the current in the meter is 0.9 mA.



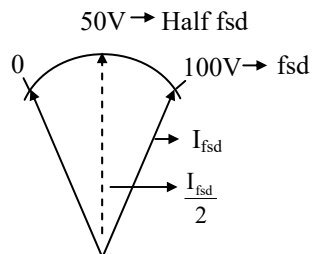
Applying KVL to circuit,  
 $1.8 \text{ V} - 0.9 \text{ mA} \times R_m - 0.9 \text{ mA} \times 1.8 \text{ k}\Omega = 0$   
 $1.8 \text{ V} - 0.9 \times 10^{-3} R_m - 1.62 = 0$   
 $R_m = \frac{0.18}{0.9 \times 10^{-3}} = 200 \Omega$

02. Ans: (c)

Sol:  $S = \frac{1}{1000} \Omega/\text{volt}$

$S = \frac{1}{I_{\text{fsd}}} \Omega/\text{V}$

$I_{\text{fsd}} = \frac{1}{S} = \frac{1}{1000} = 1 \text{ mA}$



$100 \text{ V} \rightarrow 1 \text{ mA}$

$50 \text{ V} \rightarrow ?$

$= 0.5 \text{ mA}$

03. Ans: (b)

Sol:  $\theta = \frac{5^2 \times \left(3 - \frac{2\theta}{4}\right) \times 10^{-6}}{25 \times 10^{-6}}$

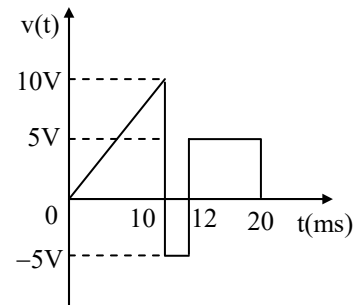
$\theta = 3 - \frac{\theta}{2}$

$\Rightarrow 2\theta = 6 - \theta$

$3\theta = 6 \Rightarrow \theta = 2$

04. Ans: (a)

Sol:



PMMC meter reads Average value

$V_{\text{avg}} = \frac{\left(\frac{1}{2} \times 10 \times 10\text{ms}\right) + (-5\text{V} \times 2\text{ms}) + (5\text{V} \times 8\text{ms})}{20\text{ms}}$

$= \frac{50 - 10 + 40}{20} = 4 \text{ V}$

(or)

Avg. value  $= \frac{1}{20} \left[ \int_0^{10} (1)t \, dt - \int_{10}^{12} 5 \, dt + \int_{12}^{20} 5 \, dt \right]$

$= \frac{1}{20} \left[ \left[ \frac{t^2}{2} \right]_0^{10} - 5[t]_{10}^{12} + 5[t]_{12}^{20} \right]$

$= 4 \text{ V}$



**05. Ans: (a)**

**Sol:**

	1°C↑	10°C	T <sub>c</sub>	θ
Spring stiffness(K <sub>c</sub> )	0.04%↓	0.4%↓	0.4%↓	0.4%↑
			T <sub>d</sub>	θ
Strength of magnet (B)	0.02%↓	0.2%↓	0.2%↓	0.2%↓

$$\text{Net deflection } (\theta_{\text{net}}) = 0.4\% \uparrow - 0.2\% \downarrow \\ = 0.2\% \uparrow$$

Increases by 0.2%

**06. Ans: 32.4° and 21.1°**

**Sol:** I<sub>1</sub> = 5 A, θ<sub>1</sub> = 90°; I<sub>2</sub> = 3 A, θ<sub>2</sub> = ?

θ ∝ I<sup>2</sup> (as given in Question)

(i) Spring controlled

$$\theta \propto I^2$$

$$\frac{\theta_2}{\theta_1} = \left(\frac{I_2}{I_1}\right)^2$$

$$\Rightarrow \frac{\theta_2}{90} = \left(\frac{3}{5}\right)^2$$

$$\theta_2 = 32.4^\circ$$

(ii) Gravity controlled

$$\sin \theta \propto I^2$$

$$\frac{\sin \theta_2}{\sin \theta_1} = \left(\frac{I_2}{I_1}\right)^2$$

$$\frac{\sin \theta_2}{\sin 90} = \left(\frac{3}{5}\right)^2$$

$$\Rightarrow \frac{\sin \theta_2}{1} = 0.36$$

$$\theta_2 = \sin^{-1}(0.36) = 21.1^\circ$$

**07. Ans: 3.6 MΩ**

**Sol:** V<sub>m</sub> = (0 – 200) V ; S = 2000 Ω/V

$$V = (0 - 2000) V$$

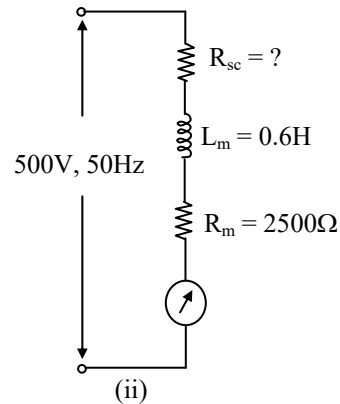
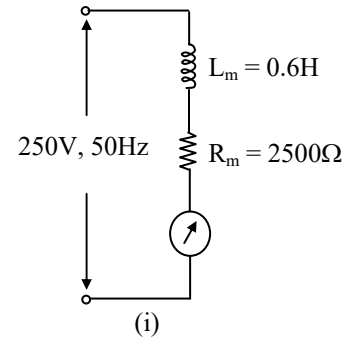
$$R_m = s \times V_m$$

$$= 2000 \Omega/V \times 200 V = 400000 \Omega$$

$$R_{sc} = R_m \left( \frac{V}{V_m} - 1 \right) \\ = 400000 \left( \frac{2000}{200} - 1 \right) = 3.6 \text{ M}\Omega$$

**08. Ans: 2511.5 Ω**

**Sol:**



Current is same in case (i) & (ii)

In case (i),

$$I_m = \frac{250 \text{ V}}{\sqrt{R_m^2 + (\omega L_m)^2}} \\ = \frac{250 \text{ V}}{\sqrt{(2500)^2 + (2\pi \times 50 \times 0.6)^2}} \\ = 0.0997 \text{ A}$$

In case (ii),

$$I_m = \frac{250 \text{ V}}{\sqrt{(R_m + R_{sc})^2 + (\omega L_m)^2}}$$



$$0.0997 \text{ A} = \frac{500 \text{ V}}{\sqrt{(2500 + R_{sc})^2 + (2\pi \times 50 \times 0.6)^2}}$$

$$\sqrt{(2500 + R_{sc})^2 + 35.53 \times 10^3} = \frac{500}{0.0997}$$

$$\sqrt{(2500 + R_{sc})^2 + 35.53 \times 10^3} = 5.015 \times 10^3$$

$$R_{sc} = 2511.5 \Omega$$

09. Ans: 0.1025  $\mu\text{F}$

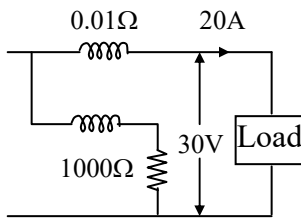
Sol:  $C = \frac{0.41 L_m}{R_{sc}^2}$

$$C = \frac{0.41 \times 1}{(2 \text{ k}\Omega)^2}$$

$$= 0.1025 \mu\text{F}$$

10. Ans: (c)

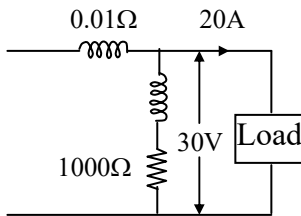
Sol: MC – connection



Error due to current coil

$$= \frac{20^2 \times 0.01}{(30 \times 20)} \times 100 = 0.667\%$$

LC – connection



Error due to potential coil

$$= \frac{(30^2 / 1000)}{(30 \times 20)} \times 100 = 0.15\%$$

As per given options, 0.15% high

11. Ans: (b)

Sol:  $\phi = \tan^{-1} \left[ \frac{\sqrt{3}(W_1 - W_2)}{(W_1 + W_2)} \right]$

Power factor =  $\cos \phi$   
= 0.917 lag (since load is inductive)

12. Ans: (c)

Sol:  $R_{load} = \frac{V}{I} = \frac{200}{20} = 10 \Omega$

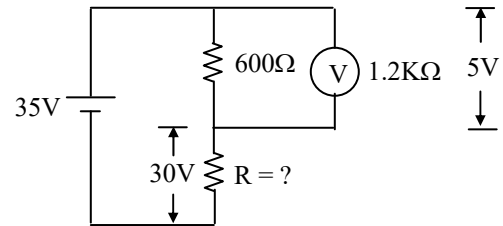
For same error  $R_L = \sqrt{R_C \times R_V}$

$\therefore 100 = 10 \times 10^3 \times R_C$

$\Rightarrow R_C = 0.01 \Omega$

13. Ans: (d)

Sol: By using voltage division, voltage across unknown resistance (R) is



$$30 \text{ V} = 35 \text{ V} \times \frac{R}{(600 \Omega \parallel 1.2 \text{ K}\Omega + R)}$$

$$30 \text{ V} = 35 \text{ V} \times \frac{R}{(400 + R)}$$

$$12000 + 30 R = 35 R$$

$$5 R = 12000 \Omega$$

$$R = 2.4 \text{ k}\Omega$$

14. Ans: (d)

Sol: Moving Iron Ammeter,  $\theta \propto I^2$

For 1 A dc  $\rightarrow 20^\circ$   $I_1 = 1 \text{ A}, \theta_1 = 20^\circ$

For  $3 \sin 314 t \rightarrow ?$

MI Ammeter measures the rms value of AC current

$$I_2 = \frac{I_m}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$I_2 = \frac{3}{\sqrt{2}}, \theta_2 = ?$$





$$\frac{\theta_2}{\theta_1} = \frac{I_2^2}{I_1^2}$$

$$\frac{\theta_2}{20} = \frac{(3/\sqrt{2})^2}{(1)^2} \Rightarrow \theta_2 = 90^\circ$$

**15. Ans: (a)**

**Sol:**  $V_{dc} = I_{dc} \times 10 \Omega$

$$= \left( \frac{12+5}{2} \right) \times 10 = 85 \text{ V}$$

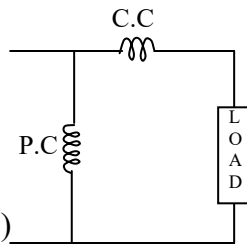
**16. Ans: (b)**

**Sol:** The connection diagram is as shown

$$\begin{aligned} P_T &= VI \cos \phi \\ &= 220 \times 20 \times 0.6 \\ &= 2640 \text{ W} \end{aligned}$$

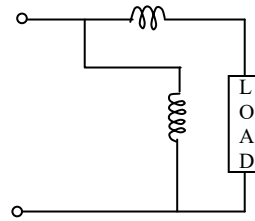
$$\begin{aligned} P_m &= P_T + I^2 R \\ &= 2640 + (20)^2 (0.03) \\ &= 2652 \text{ W} \end{aligned}$$

$$\begin{aligned} \% \text{ Error} &= \frac{P_m - P_T}{P_T} \times 100 \\ &= \frac{2652 - 2640}{2640} \times 100 \\ &= 0.45\% \end{aligned}$$



**17. Ans: (b)**

**Sol:** The connection diagram looks like



$$\begin{aligned} P_T &= VI \cos \phi \\ &= 2640 \text{ W} \end{aligned}$$

$$P_m = P_T + \frac{V^2}{R} = 2640 + \frac{(220)^2}{6000} = 2648 \text{ W}$$

$$\begin{aligned} \% \text{ Error} &= \frac{P_m - P_T}{P_T} \times 100 \\ &= \frac{2648 - 2640}{2640} \times 100 = 0.31\% \end{aligned}$$

**18. Ans: (d)**

**Sol:**  $W = 1250 \pm 1\%$

$$W = 1250 \pm \underbrace{1\%}_{\text{error w.r.t true value}}$$

$$\text{Error} = \pm \frac{1}{100} \times 1250 = \pm 12.5$$

$$\begin{aligned} W &= 1250 \pm 12.5 \\ &= 1237.5 \text{ W} - 1262.5 \text{ W} \end{aligned}$$

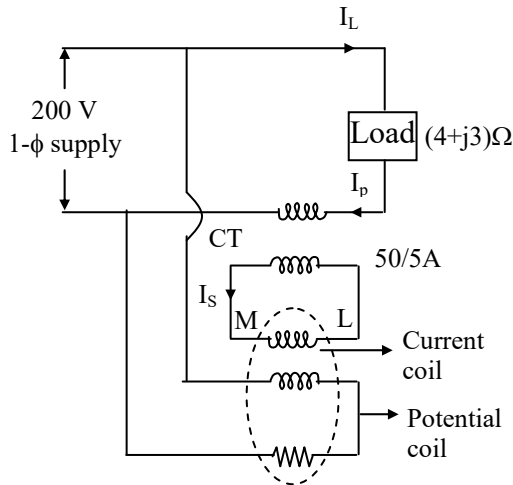
# 4 Measurement of Power & Energy

Chapter

## Class Room Practice Solutions

01. Ans: (b)

Sol:



Potential coil voltage = 200 V

C.T. primary current ( $I_p$ )

$$I_p = I_L = \frac{200 \text{ V}}{\sqrt{4^2 + 3^2} \tan^{-1}\left(\frac{3}{4}\right)}$$

$$I_p = I_L = \frac{200 \text{ V}}{5 \angle 36.86^\circ}$$

$$I_p = 40 \angle -36.86^\circ$$

$$\frac{I_p}{I_s} = \frac{50}{5}$$

$$\frac{40}{I_s} = \frac{50}{5}$$

$$I_s = \frac{5}{50} \times 40 = 4 \text{ A}$$

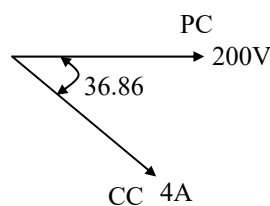
C.T secondary ( $I_s$ ) =  $4 \angle -36.86^\circ$

Wattmeter current coil =  $I_C = 4 \angle -36.86^\circ$

Wattmeter reading

$$= 200 \text{ V} \times 4 \times \cos(36.86^\circ)$$

$$= 640.08 \text{ W}$$



02. Ans: (a)

Sol: Energy consumed in 1 minute

$$= \frac{240 \times 10 \times 0.8}{1000} \times \frac{1}{60} = 0.032 \text{ kWh}$$

Speed of meter disc

= Meter constant in rev/kWhr  $\times$  Energy consumed in kWh/minute

$$= 400 \times 0.032$$

$$= 12.8 \text{ rpm (revolutions per minute)}$$

03. Ans: (a)

Sol: Energy consumed (True value)

$$= \frac{230 \times 5 \times 1}{1000} \times \frac{3}{60} = 0.0575 \text{ kWhr}$$

Energy recorded (Measured value)

$$= \frac{\text{No. of rev (N)}}{\text{meter constant (k)}}$$

$$= \frac{90 \text{ rev}}{1800 \text{ rev/kWh}} = 0.05 \text{ kWhr}$$

$$\% \text{Error} = \frac{0.05 - 0.0575}{0.0575} \times 100$$

$$= -13.04\% = 13.04\% \text{ (slow)}$$

04. Ans: (c)

$$\text{Sol: } W = \frac{1}{2} [E_1 I_1 \cos \phi_1 + E_3 I_3 \cos \phi_3]$$

05. Ans: (c)

Sol:  $V = 220 \text{ V}$ ,  $\Delta = 85^\circ$ ,  $I = 5 \text{ A}$

$$\text{Error} = VI [\sin(\Delta - \phi) - \cos \phi]$$

$$(1) \cos \phi = \text{UPF}, \phi = 0^\circ$$

$$\text{Error} = 220 \times 5 [\sin(85 - 0) - \cos 0] \\ = -4.185 \text{ W} \approx -4.12 \text{ W}$$

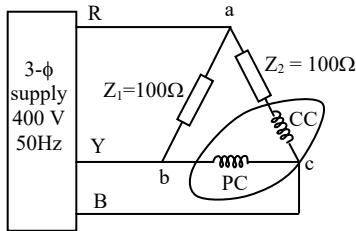
$$(2) \cos \phi = 0.5 \text{ lag}, \phi = 60^\circ$$

$$\text{Error} = 220 \times 5 [\sin(85 - 60) - \cos 60] \\ = -85.12 \text{ W}$$



06. Ans: (c)

Sol:



Based on R-Y-B

Assume abc phase sequence

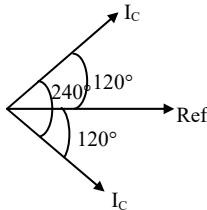
$$V_{ab} = 400 \angle 0^\circ ; V_{bc} = 400 \angle -120^\circ$$

$$V_{ca} = 400 \angle -240^\circ \text{ or } 400 \angle 120^\circ$$

$$\begin{aligned} \text{Current coil current (I}_c) &= \frac{V_{ca}}{Z_2} \\ &= \frac{400 \angle 120^\circ}{100 \Omega} = 4 \angle 120^\circ \end{aligned}$$

$$\text{Potential coil voltage (V}_{bc}) = 400 \angle -120^\circ$$

$$W = 400 \times 4 \times \cos(240) = -800 \text{ W}$$



07. Ans: (d)

Sol:  $V_L = 400 \text{ V}, I_L = 10 \text{ A}$

$$\cos \phi = 0.866 \text{ lag}, \phi = 30^\circ$$

$$W_1 = V_L I_L \cos(30 - \phi)$$

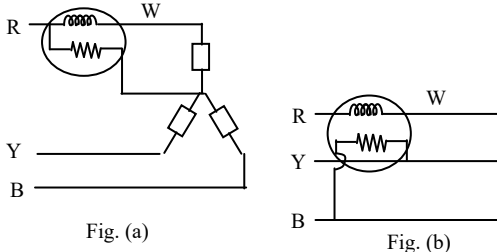
$$W_2 = V_L I_L \cos(30 + \phi)$$

$$W_1 = 400 \times 10 \times \cos(30 - 30) = 4000 \text{ W}$$

$$W_2 = 400 \times 10 \times \cos(30 + 30) = 2000 \text{ W}$$

08. Ans:  $W = 519.61 \text{ VAR}$

Sol:



$$W = 400 \text{ watt} ; W = V_{ph} I_{ph} \cos \phi$$

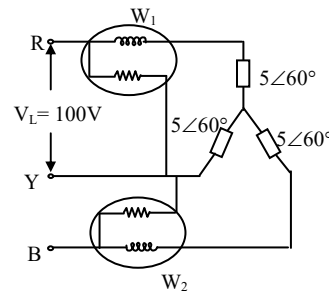
$$V_{ph} I_{ph} = 400/0.8$$

This type of connection gives reactive power

$$\begin{aligned} W &= \sqrt{3} V_p I_p \sin \phi = \sqrt{3} \times \frac{400}{0.8} \times 0.6 \\ &= 519.6 \text{ VAR} \end{aligned}$$

09. Ans: 0 & 1000 W

Sol:



Y-phase is made common.

Hence wattmeter readings are

$$W_1 = V_L I_L \cos(30 + \phi)$$

$$W_2 = V_L I_L \cos(30 - \phi)$$

In star-connection

$$I_L = I_{ph} ; V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_L = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_L / \sqrt{3}}{Z_{ph}}$$

$$I_L = I_{ph} = \frac{(100 / \sqrt{3})}{5} = \frac{20}{\sqrt{3}} = 11.54 \text{ A}$$

$$V_L = 100 \text{ V}, I_L = 11.54 \text{ A}, \phi = 60^\circ$$

$$W_1 = 100 \times 11.54 \times \cos(30 + 60) = 0 \text{ W}$$

$$W_2 = 100 \times 11.54 \times \cos(30 - 60)$$

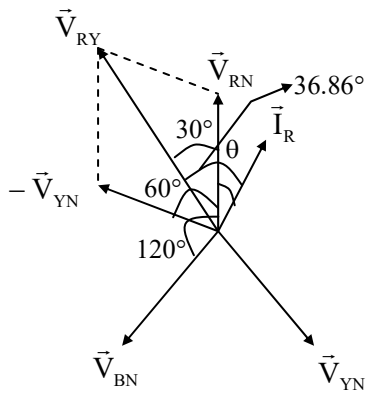
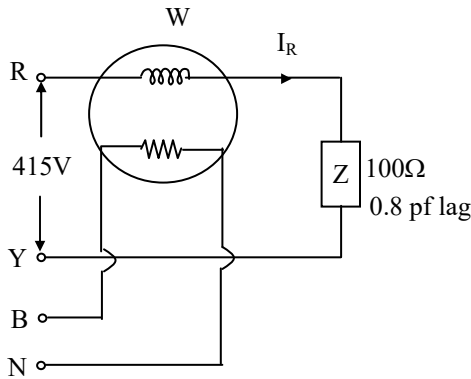
$$= 999.393 \text{ W} \approx 1000 \text{ W}$$

$$W_1 = 0 \text{ W}, W_2 = 1000 \text{ W}$$



10. Ans: 596.46 W

Sol:



Current coil is connected in 'R<sub>phase</sub>', it reads ' $\vec{I}_R$ ' current.

Potential coil reads phase voltage i.e.,  $\vec{V}_{BN}$

$$W = \vec{V}_{BN} \times \vec{I}_R \times \cos(\vec{V}_{BN} \cdot \vec{I}_R)$$

$$V_L = 415 \text{ V}, V_{BN} = \frac{415}{\sqrt{3}} \text{ V}$$

$$I_R = \frac{V_{RY}}{Z} = \frac{415}{100} = 4.15 \text{ A}$$

$$\cos \phi = 0.8$$

$$\Rightarrow \phi = 36.86 \text{ between } \vec{V}_{RY} \text{ \& } \vec{I}_R$$

$$\theta = 36.86^\circ - 30^\circ = 6.86^\circ$$

Now angle between  $\vec{V}_{BN}$  and  $\vec{I}_R$

$$= 120 + 6.86 = 126.86^\circ$$

$$W = \frac{415}{\sqrt{3}} \times 4.15 \times \cos(126.86)$$

$$= -596.467 \text{ W}$$

11. Ans: (c)

Sol: Meter constant = 14.4 A-sec/rev

$$= 14.4 \times 250 \text{ W-sec/rev}$$

$$= \frac{14.4 \times 250}{1000} \text{ kw - sec/rev}$$

$$= \frac{14.4 \times 250}{1000 \times 3600} \text{ kwhr/rev}$$

$$\text{Meter constant} = \frac{1}{1000} \text{ kwhr/rev}$$

Meter constant in terms of rev/kwhr = 1000

12. Ans: (d)

Sol:  $R_p = 1000 \Omega$ ,  $L_p = 0.5 \text{ H}$ ,  $f = 50 \text{ Hz}$ ,

$$\cos \phi = 0.7,$$

$$X_{Lp} = 2 \times \pi \times f \times L, \tan \phi = 1$$

$$= 2 \times \pi \times 50 \times 0.5$$

$$= 157 \Omega$$

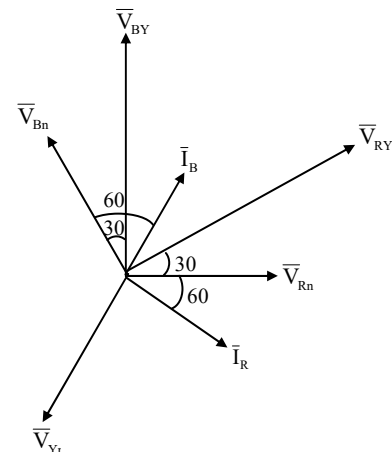
$$\% \text{ Error} = \pm (\tan \phi \tan \beta) \times 100$$

$$= \pm \left( 1 \times \frac{157}{1000} \right) \times 100$$

$$= 15.7\% \approx 16\%$$

13. Ans: (d)

Sol:



$$P = W_1 + W_2 + W_3 = 1732.05$$

$$\text{Power factor, } \cos \phi = \frac{1732.05}{3464} = 0.5 \text{ lag}$$



$$\sqrt{3} \times 400 \times I_L \times 0.5 = 1732.05$$

$$I_L = \frac{1732.05}{\sqrt{3} \times 400 \times 0.5} = 5 \text{ A}$$

When switch is in position N

$$W_1 = W_2 = W_3 = 577.35 \text{ W} \Rightarrow \text{balanced load}$$

$\therefore$  total power consumed by load is

$$W = W_1 + W_2 + W_3$$

$$W = 1732.05 \text{ W}$$

Given load is inductive

$$\text{And VA draw from source} = 3464 \text{ VA}$$

$$\begin{aligned} \therefore \text{power factor} &= \frac{W}{\text{VA}} \\ &= \frac{1732.05}{3464} = 0.5 \text{ lag} \end{aligned}$$

$\Rightarrow$  power factor angle =  $-60^\circ$  ( $\because$  lag)

When switch is connected in Y position pressure coil of  $W_2$  is shorted

So  $W_2 = 0$  and phasor diagrams for other two are as follows

$$\begin{aligned} W_1 &= V_{RY} I_R \cos(\text{angle between } \bar{V}_{RY} \text{ and } \bar{I}_R) \\ &= 400 \times 5 \times \cos(90^\circ) = 0 \text{ W} \end{aligned}$$

$$\begin{aligned} W_3 &= V_{BY} I_B \cos(\text{angle between } \bar{V}_{BY} \text{ and } \bar{I}_B) \\ &= 400 \times 5 \times \cos(30^\circ) \\ &= 400 \times 5 \times \frac{\sqrt{3}}{2} = 1732 \text{ W} \end{aligned}$$

$$W_1 = 0, W_2 = 0, W_3 = 1732 \text{ W}$$

**14. Ans: (a)**

**Sol:**  $W_1 = 3 \text{ KW}$ ,

$W_2 = -1 \text{ KW}$  (after reversing connections of CC)

$$\phi = \tan^{-1} \left[ \sqrt{3} \left( \frac{W_1 - W_2}{W_1 + W_2} \right) \right]$$

$$\begin{aligned} &= \tan^{-1} \left[ \sqrt{3} \left( \frac{3 \text{ KW} - (-1 \text{ KW})}{3 \text{ KW} + (-1 \text{ KW})} \right) \right] \\ &= 73.89^\circ \end{aligned}$$

$$\begin{aligned} \text{Load power factor} &= \cos \phi \\ &= \cos (73.89) = 0.277 \end{aligned}$$

**15. Ans: (b)**

$$\text{Sol: Energy} = \frac{230 \text{ V} \times 20 \text{ A} \times 1 \times 2}{1000} = 9.2 \text{ kWhr}$$

$$\begin{aligned} \text{Meter constant (k)} &= \frac{\text{No. of revolutions (N)}}{\text{Energy in kWhr}} \\ &= \frac{1380 \text{ rev}}{9.2 \text{ kWhr}} = 150 \text{ rev/kWhr} \end{aligned}$$

**16. Ans: (b)**

$$\begin{aligned} \text{Sol: Energy} &= VI \cos \phi \times \text{time} \\ &= \frac{230 \times 10 \times 1 \times 3}{1000} = 6.9 \text{ kWhr} \end{aligned}$$

No. of revolutions (N)

$$\begin{aligned} &= \text{meter constant (k)} \times \text{Energy in kWhr} \\ &= 200 \times 6.9 = 1380 \end{aligned}$$

**17. Ans: (c)**

$$\begin{aligned} \text{Sol: Energy consumed in kWhr (True value)} \\ &= \frac{5 \text{ kw} \times 50}{3600} = 0.06944 \text{ kWhr} \end{aligned}$$

Energy recorded in kWhr (measured value)

$$= \frac{40 \text{ rev}}{500 \text{ rev/kWhr}} = 0.08 \text{ kWhr}$$

$$\begin{aligned} \% \text{Error} &= \frac{0.08 - 0.0694}{0.06944} \times 100 \\ &= 15.273\% \end{aligned}$$

**18. Ans: (b)**

**19. Ans: (b)**

$$\begin{aligned} \text{Sol: Energy consumed in kWhr (True value)} \\ &= \frac{230 \times 5 \times 1 \times 138}{1000 \times 3600} = 0.04408 \text{ kWhr} \end{aligned}$$



Energy recorded in kWhr

$$\begin{aligned} \text{(measured value)} &= \frac{\text{No. of rev}}{\text{meter constant(k)}} \\ &= \frac{80 \text{ rev}}{1800 \text{ rev/kWhr}} \\ &= 0.04444 \text{ kWhr} \end{aligned}$$

$$\begin{aligned} \% \text{Error} &= \frac{0.04444 - 0.04408}{0.04408} \times 100 \\ &= 0.817\% \end{aligned}$$

**20. Ans: (c)**

**Sol:** Energy recorded (kWhr)

$$= \frac{5 \text{ rev}}{1200 \text{ rev/kwhr}} = 4.1667 \times 10^{-3} \text{ kwhr}$$

Energy = 4.1667 Whr

$$\text{Load power} = \frac{4.1667 \text{ Whr}}{75 \text{ sec}} = \frac{4.1667 \text{ Whr}}{\frac{75}{3600} \text{ hr}}$$

Load power = 200 W

**21. Ans: (d)**

**Sol:** Energy recorded (measured value)

$$= \frac{51 \text{ rev}}{360 \text{ rev/kwhr}} = 0.141667 \text{ kwhr}$$

Energy consumed (True value)

$$= \frac{10 \text{ kw} \times 50}{3600} = 0.13889 \text{ kwhr}$$

$$\begin{aligned} \text{Error} &= \frac{0.141667 - 0.13889}{0.13889} \times 100 \\ &= 1.999\% \approx + 2\% \end{aligned}$$

**22. Ans: 3464.10**

$$\text{Sol: } \tan \theta = \sqrt{3} \left( \frac{W_2 - W_1}{W_2 + W_1} \right) = \sqrt{3} \times \frac{W_2}{W_2} = \sqrt{3}$$

$$\theta = \tan^{-1} \sqrt{3} = 60^\circ$$

$$\begin{aligned} P_T &= W_2 + W_1 = W_2 = \sqrt{3} V_L I_L \cos \theta \\ &= \sqrt{3} \times 400 \times 10 \times 0.5 = 3464.10 \text{ watt} \end{aligned}$$

# Chapter 5 Bridge Measurement of R, L & C

## Class Room Practice Solutions

01. Ans: (a)

Sol: It is Maxwell Inductance Capacitance bridge

$$R_x R_4 = R_2 R_3$$

$$R_x = \frac{R_2 R_3}{R_4}$$

$$R_x = \frac{750 \times 2000}{4000}$$

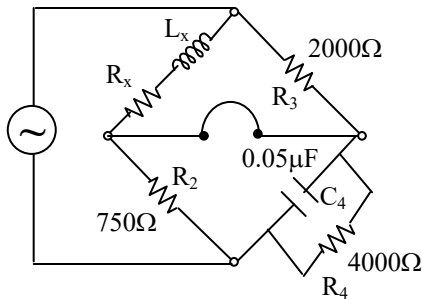
$$R_x = 375 \Omega$$

$$\frac{L_x}{C_4} = R_2 R_3$$

$$L_x = C_4 R_2 R_3$$

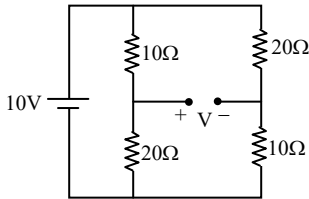
$$L_x = 0.05 \times 10^{-6} \times 750 \times 2000$$

$$L_x = 75 \text{ mH}$$



02. Ans: (d)

Sol:



$$V = 6.66 - 3.33$$

$$= 3.33 \text{ V}$$

03. Ans: (c)

Sol: The voltage across  $R_2$  is

$$= E \frac{R_2}{R_1 + R_2} = \frac{E}{2}$$

The voltage across  $R_1$  is

$$= E \frac{R_1}{R_1 + R_2} = \frac{E}{2}$$

Now,  $\frac{E}{2} = IR_3 + V$

$$I = \frac{E - 2V}{2R_3} \Rightarrow I = \frac{E - 2V}{2R}$$

and  $\frac{E}{2} = IR_4$

$$\frac{E}{2} = \left( \frac{E - 2V}{2R} \right) (R + \Delta R)$$

$$ER = (E - 2V)(R + \Delta R)$$

$$R + \Delta R = \frac{ER}{(E - 2V)}$$

$$\Delta R = \frac{ER}{(E - 2V)} - R$$

$$= \frac{ER - ER + 2VR}{(E - 2V)}$$

$$\Delta R = \frac{2VR}{(E - 2V)}$$

04. Ans: (a)

Sol: The deflection of galvanometer is directly proportional to current passing through circuit, hence inversely proportional to the total resistance of the circuit.

Let  $S$  = standard resistance

$R$  = Unknown resistance

$G$  = Galvanometer resistance

$\theta_1$  = Deflection with  $S$

$\theta_2$  = Deflection with  $R$

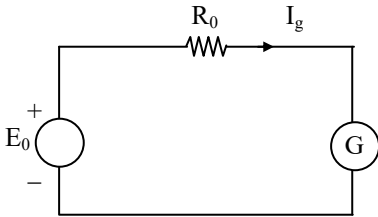
$$\therefore \frac{\theta_1}{\theta_2} = \frac{R + G}{S + G}$$



$$\begin{aligned} \Rightarrow R &= (S + G) \frac{\theta_1}{\theta_2} - G \\ &= (0.5 \times 10^6 + 10 \times 10^3) \left( \frac{41}{51} \right) - 10 \times 10^3 \\ &= 0.4 \times 10^6 \Omega \\ &= 0.4 \text{ M} \Omega \end{aligned}$$

05. Ans: (a)

Sol: Thevenin's equivalent of circuit is



$R_0$  = Resistance of circuit looking into terminals b & d with a & c short circuited.

$$\begin{aligned} &= \frac{RS}{R+S} + \frac{PQ}{P+Q} = \frac{1 \times 5}{1+5} + \frac{1 \times Q}{1+Q} \\ &= 0.833 + \frac{Q}{1+Q} \text{ K} \Omega \end{aligned}$$

$$\begin{aligned} \text{Now, } R_0 + G &= \frac{24 \times 10^{-3}}{13.6 \times 10^{-6}} \\ &= 1.765 \text{ k} \Omega \end{aligned}$$

$$\text{or } R_0 = 1765 - 100 = 1665 \Omega$$

$$0.833 + \frac{Q}{1+Q} = 1.665$$

$$\Rightarrow Q = 4.95 \text{ k} \Omega$$

06. Ans: (c)

$$\begin{aligned} \text{Sol: } R &= \frac{0.4343 \text{ T}}{C \log_{10} \left( \frac{E}{V} \right)} \\ &= \frac{0.4343 \times 60}{600 \times 10^{-2} \times \log_{10} \left( \frac{250}{92} \right)} \end{aligned}$$

$$= \frac{26.058}{260.49 \times 10^{-12}}$$

$$R = 100.03 \times 10^9 \Omega$$

07. Ans: 0.118  $\mu$ F, 4.26k $\Omega$

Sol: Given

$$R_3 = 1000 \Omega$$

$$C_1 = \frac{\epsilon_0 \epsilon_r A}{d} = \frac{2.3 \times 4\pi \times 10^{-7} \times 314 \times 10^{-4}}{0.3 \times 10^{-2}}$$

$$C_1 = 30.25 \mu\text{F}$$

$$\delta = 9^\circ \text{ for } 50 \text{ Hz}$$

$$\tan \delta = \omega C_1 r_1 = \omega L_4 R_4$$

$$\Rightarrow r_1 = 16.67 \Omega$$

$$\text{Variable resistor } (R_4) = R_3 \left( \frac{C_1}{C_2} \right)$$

$$R_4 = 4.26 \text{ k} \Omega$$

$$C_4 = 0.118 \mu\text{F}$$

08.

Sol: Resistance of unknown resistor required for balance

$$R = (P/Q)S = (1000/100) \times 200 = 2000 \Omega.$$

In the actual bridge the unknown resistor has a value of 2005  $\Omega$  or the deviation from the balance conditions is

$$\Delta R = 2005 - 2000 = 5 \Omega.$$

Thevenin source generator emf

$$\begin{aligned} E_0 &= E \left[ \frac{R}{R+S} - \frac{P}{P+Q} \right] \\ &= 5 \left[ \frac{2005}{2005+200} - \frac{1000}{1000+100} \right] \\ &= 1.0307 \times 10^{-3} \text{ V.} \end{aligned}$$

Internal resistance of bridge looking into terminals b and d.

$$\begin{aligned} R_0 &= \frac{RS}{R+S} + \frac{PQ}{P+Q} \\ &= \frac{2005 \times 200}{2005+200} + \frac{1000 \times 100}{1000+100} \\ &= 272.8 \Omega \end{aligned}$$





Hence the current through the galvanometer

$$I_g = \frac{E_0}{R_0 + G}$$

$$= \frac{1.0307 \times 10^{-3}}{272.8 + 100} \text{ A} = 2.77 \mu\text{A}$$

Deflection of the galvanometer

$$\theta = S_i I_g = 10 \times 2.77$$

$$= 27.7 \text{ mm}/\Omega$$

Sensitivity of bridge

$$S_B = \frac{\theta}{\Delta R}$$

$$= \frac{27.7}{5} = 5.54 \text{ mm}/\Omega$$

09. Ans: (c)

Sol: Sensitivity =  $\frac{\text{Change in output}}{\text{Change in input}}$

$$= \frac{3 \text{ mm}}{6 \Omega} = 0.5 \text{ mm}/\Omega$$

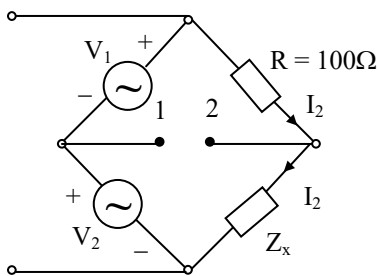
10. Ans: (d)

Sol: Changing the arms resistance by same value will not affect the Balance condition.

11. Ans: (a)

Sol:  $V_1 = \sqrt{2} \cos(1000t) \text{ V}$

$$V_2 = 2 \cos(1000t + 45^\circ) \text{ V}$$



Under balanced condition,

$$V_1 = I_2 R$$

$$I_2 = \frac{V_1}{R} = \frac{\sqrt{2} \cos 1000t}{100}$$

$$\left. \begin{aligned} I_2 &= 10^{-2} \times \sqrt{2} \cos(1000t) \\ V_2 &= 2 \cos(1000t + 45^\circ) \end{aligned} \right\} \text{ At } Z_x$$

' $I_2$ ' lags ' $V_2$ ' by  $45^\circ$ . So,  $Z_x$  has 'R' and 'L' in series.

$$R = Z \cos \theta$$

$$= \frac{2}{10^{-2} \times \sqrt{2}} \cos 45^\circ = 100 \Omega$$

$$X_L = Z \sin \theta$$

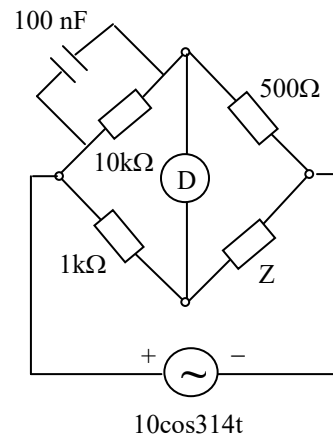
$$= \frac{2}{\sqrt{2} \times 10^{-2}} \sin 45^\circ = 100 \Omega$$

$$X_L = \omega L$$

$$L = \frac{X_L}{\omega} = \frac{100}{1000} = 0.1 \text{ H} = 100 \text{ mH}$$

12. Ans: (b)

Sol:



$$R \parallel \frac{1}{j\omega C} = \frac{R}{1 + j\omega RC}$$

$$\left( \frac{R}{1 + j\omega RC} \right) Z = 1k\Omega \times 500\Omega$$

$$\left( \frac{10k\Omega}{1 + j\omega \times 10k\Omega \times 100 \times 10^{-9}} \right) Z = 1k\Omega \times 500\Omega$$

$$\frac{10^4 \times Z}{1 + j\omega \times 10^{-3}} = 50 \times 10^4$$

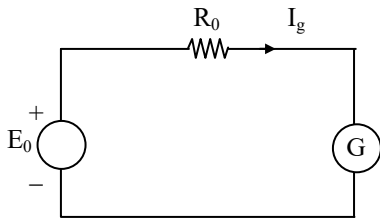
$$Z = 50 + j\omega \times 0.05$$

$$Z = R + j\omega L$$

$$R = 50 \Omega \text{ in series with } L = 0.05 \text{ H}$$

13. Ans: (a)

Sol: Thevenin's equivalent of circuit is



$R_0$  = Resistance of circuit looking into terminals b & d with a & c short circuited.

$$= \frac{RS}{R+S} + \frac{PQ}{P+Q} = \frac{1 \times 5}{1+5} + \frac{1 \times Q}{1+Q}$$

$$= 0.833 + \frac{Q}{1+Q} \text{ k}\Omega$$

Now,  $R_0 + G = \frac{24 \times 10^{-3}}{13.6 \times 10^{-6}}$

$$= 1.765 \text{ k}\Omega$$

or  $R_0 = 1765 - 100 = 1665 \Omega$

$$0.833 + \frac{Q}{1+Q} = 1.665$$

$$\Rightarrow Q = 4.95 \text{ k}\Omega$$

**14. Ans: (a)**

**Sol:**  $S = 100.03 \mu\Omega$ ,  $p = 100.31 \Omega$ ,  $q = 200 \Omega$

$P = 100.24 \Omega$ ,  $Q = 200 \Omega$ ,  $r = 700 \mu\Omega$

Unknown resistance (R)

$$= \frac{P}{Q} S + \frac{qr}{p+q+r} \left[ \frac{P}{Q} - \frac{p}{q} \right]$$

$$R = \frac{100.24}{200} \times 100.03 \times 10^{-6}$$

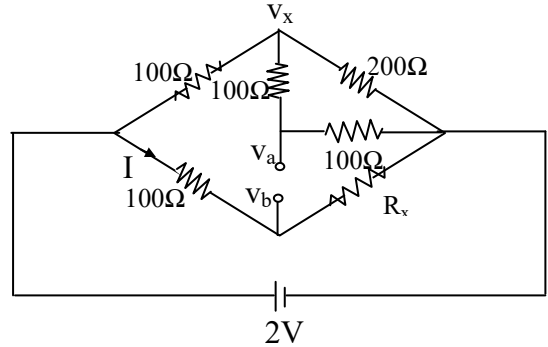
$$+ \frac{200 \times 700 \times 10^{-6}}{100.31 + 200 + 700 \times 10^{-6}} \left[ \frac{100.24}{200} - \frac{100.31}{200} \right]$$

$$= 50.135 \times 10^{-6} + 466.184 \times 10^{-6} [0.5012 - 0.50155]$$

$$R = 49.98 \mu\Omega$$

**15. Ans: 33 to 34**

**Sol:**



$V_0 = 0$  at balance

$$V_x = \frac{2 \times 100}{200} = 1V$$

$V_a = 0.5V$

at balance

$V_b = 0.5V$

Now

$$I = \frac{2 - 0.5}{100}$$

$$I = 15 \text{ mA}$$

$$R_x = \frac{0.5}{15 \times 10^{-3}}$$

$$R_x = 33.33 \Omega$$

## Class Room Practice Solutions

01. Ans: (d)

02. Ans: (d)

03. Ans: (a)

**Sol:** Since the instrument is a standardized with an emf of 1.018 V with sliding contact at 101.8 cm, it is obvious that a length 101.8 cm represents a voltage of 1.018.

Resistance of 101.8 cm length of wire  
 $= (101.8/200) \times 400 = 203.6 \Omega$

$\therefore$  Working current

$$I_m = 1.018/203.6 = 0.005 \text{ A} = 5 \text{ mA}$$

Total resistance of the battery circuit  
 $=$  resistance of rheostat

+ resistance of slide wire

$\therefore$  Resistance of rheostat

$R_h =$  total resistance

– resistance of slide wire

$$= \frac{3}{5 \times 10^{-3}} - 400 = 600 - 400 = 200 \Omega$$

04. Ans: (b)

**Sol:** Voltage drop per unit length

$$= \frac{1.45 \text{ V}}{50 \text{ cm}} = 0.029 \text{ V/cm}$$

Voltage drop across 75 cm length

$$= 0.029 \times 75 = 2.175 \text{ V}$$

Current through resistor (I)

$$= \frac{2.175 \text{ V}}{0.1 \Omega} = 21.75 \text{ A}$$

(or)

75 cm  $\rightarrow$  0.1  $\Omega$

50 cm  $\rightarrow$  ?

Slide wire resistance with standard cell

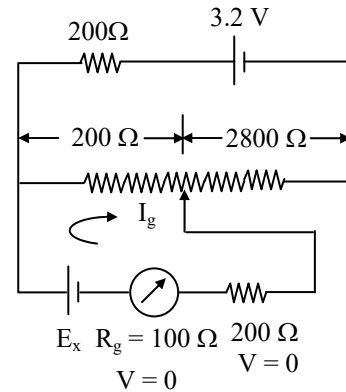
$$= \frac{50}{70} \times 0.1 = 0.067 \Omega$$

Then  $0.067 \times I_w = 1.45 \text{ V}$

$$I_w = \frac{1.45}{0.067} = 21.75 \text{ A}$$

05. Ans: (a)

**Sol:**



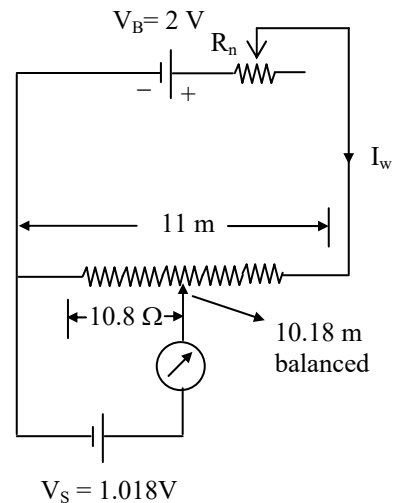
Under balanced,  $I_g = 0$

$$E_x = 3.2 \text{ V} \times \frac{200}{(200 + 200 + 2800)} = 0.2 \text{ V}$$

$E_x = 200 \text{ mV}$

06. Ans: (a)

**Sol:**



Resistance 1  $\Omega$ /cm

For 11 m  $\rightarrow$  11  $\Omega$



For 10m + 18cm → 10.8Ω

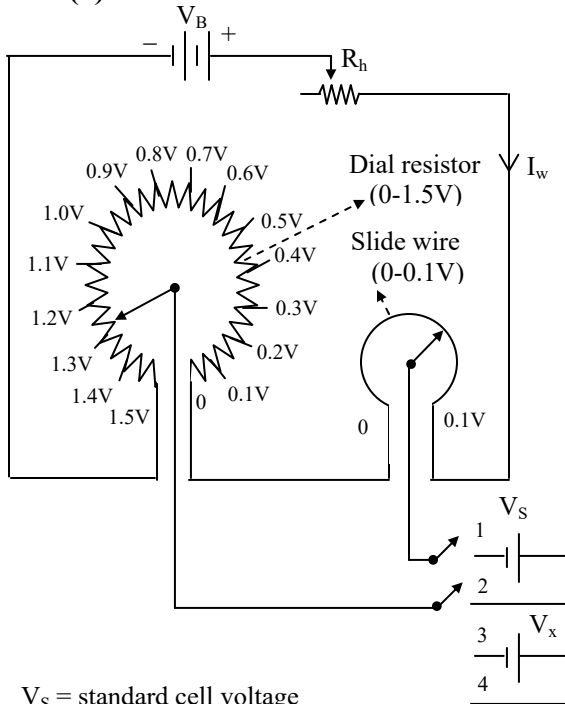
$$I_w \times 10.8\Omega = 1.018 \text{ V}$$

$$I_w = \frac{V_B}{R_n + I_r} \Rightarrow 0.1 = \frac{2}{R_n + 11\Omega}$$

$$R_n = \frac{2}{0.1} - 11 = 9 \Omega$$

**07. Ans: (a)**

**Sol:**



$V_s$  = standard cell voltage

Dial resistor has 15 steps and each step is  $10 \Omega = 15 \times 10 \Omega = 150 \Omega$

Slide wire resistance =  $10 \Omega$

Total resistance =  $150 + 10 = 160 \Omega$

Working current ( $I_w$ ) =  $10 \text{ mA}$

Range of potentiometer

$$= 10\text{mA} \times 160 \Omega = 1.6 \text{ V}$$

Resolution of potentiometer

$$= \frac{\text{working current} \times \text{slide wire resistance}}{\text{slide wire length}}$$

$$= \frac{10 \text{ mA} \times 10 \Omega}{100 \text{ cm}} = 0.001 \text{ V/cm}$$

(1 div = 1 cm)

One fifth of a division can be read certainly.

$$\text{Resolution} = \frac{1}{5} \times 0.001 = 0.2 \text{ mV/cm}$$

**08. Ans: (a)**

**Sol:** Since the instrument is a standardized with an emf of 1.018 V with sliding contact at 101.8 cm, it is obvious that a length 101.8 cm represents a voltage of 1.018.

Resistance of 101.8 cm length of wire

$$= (101.8/200) \times 400 = 203.6 \Omega$$

∴ Working current

$$I_m = 1.018/203.6 = 0.005 \text{ A} = 5 \text{ mA}$$

Total resistance of the battery circuit

= resistance of rheostat + resistance of slide wire

∴ Resistance of rheostat

$R_h$  = total resistance – resistance of slide wire

$$= \frac{3}{5 \times 10^{-3}} - 400 = 600 - 400 = 200 \Omega$$

**09. Ans: (a)**

**10. Ans: (a)**

**Sol:** Bar primary ( $N_p$ ) = 1 turn

$N_s$  = 500 turns

$I_s$  = 5 A

$Z_s$  =  $1 \Omega$

$N_p I_m = 200$

$$I_m = 200 \text{ A}, n = \frac{N_s}{N_p} = \frac{500}{1} = 500$$

Phase angle error ( $\theta$ )

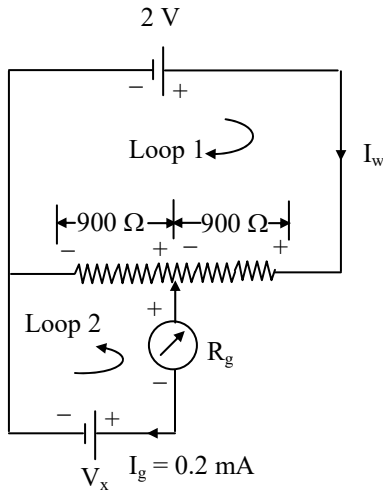
$$= \frac{I_m}{n I_s} \times \frac{180}{\pi} \text{ degrees}$$

$$= \frac{200}{500 \times 5} \times \frac{180}{\pi} = 4.58^\circ$$



11. Ans: (d)

Sol:



Write KVL for loop 1

$$2V - 900 I_w - 900(I_w - 0.2\text{mA}) = 0$$

$$I_w = 1.211 \text{ mA}$$

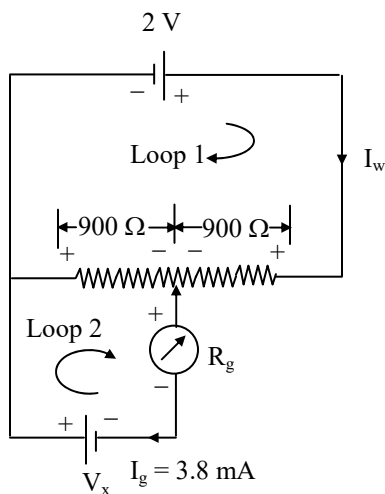
Write KVL for loop 2

$$V_x + 0.2 \text{ mA} \times R_g - 900(I_w - 0.2\text{mA}) = 0$$

$$V_x + 0.2 \times 10^{-3} R_g - 900(1.211 \times 10^{-3} - 0.2 \times 10^{-3}) = 0$$

$$V_x = 0.909 - 0.2 \times 10^{-3} R_g \dots\dots (1)$$

When  $V_x$  is reversed, the circuit is



Write KVL for loop (1)

$$2 - 900I_w - 900(I_w - 3.8\text{mA}) = 0$$

$$I_w = 3.011 \text{ mA}$$

Write KVL for loop (2)

$$V_x - 900(3.8\text{mA} - I_w) - 3.8\text{mA} \times R_g = 0$$

$$V_x - 900(3.8 \times 10^{-3} - 3.011 \times 10^{-3}) - 3.8 \times 10^{-3} R_g = 0$$

$$V_x = 0.710 + 3.8 \times 10^{-3} R_g \dots\dots\dots(2)$$

Substitute (2) in eqn (1)

$$0.710 + 3.8 \times 10^{-3} R_g = 0.909 - 0.2 \times 10^{-3} R_g$$

$$R_g = 49.72 \Omega$$

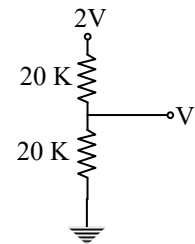
$$R_g \approx 50 \Omega$$

Substitute ' $R_g$ ' value in eqn (2)

$$V_x = 0.710 + 3.8 \times 10^{-3} \times 50 = 0.9001\text{V}$$

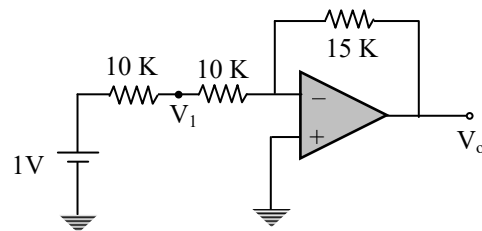
12. Ans: (c)

Sol: Apply thevenin's at  $V_1$



$$V_{th} = 1\text{V} \quad R_{th} = 10\text{K}$$

Then equivalent circuit



$$V_o = -1\text{V} \left( \frac{15}{20} \right) = -0.75$$

13. Ans: (b)

14. Ans: (a)

16. Ans: (a)

Sol: Given data

$$\frac{R_L}{R_T} = 1, \quad R_{PQ} = R_C, \quad \frac{V_o}{V_s} = 0.5$$



From circuit  $R_{PQ} // R_L$

$$V_0 = V_S \left[ \frac{\frac{R_L R_C}{R_L + R_C}}{\frac{R_L R_C}{R_L + R_C} + R_T - R_C} \right]$$

$$= V_S \left[ \frac{R_T R_C}{R_T R_C + (R_T - R_C)(R_T + R_C)} \right]$$

$$R_T^2 - R_C^2 + R_T R_C = 2R_T R_C$$

Divide above equation on both sides with  $R_T^2$

$$1 - \left( \frac{R_C}{R_T} \right)^2 = \frac{R_C}{R_T}$$

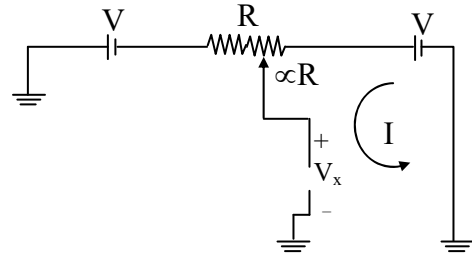
$$\therefore \frac{R_C}{R_T} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

Here resistances can not has negative value.

$$\text{Therefore } \frac{R_C}{R_T} = \frac{-1 + \sqrt{5}}{2}$$

17. Ans: (a)

Sol:



$$I = \frac{2V}{R}$$

Apply KVL in loop

$$-V + I\alpha R + V_x = 0$$

$$-V + \frac{2V}{R} \times \alpha R + V_x = 0$$

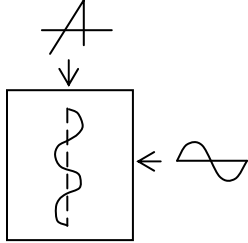
$$V - 2V\alpha = V_x$$

$$V_x = [1 - 2\alpha] V$$

## Class Room Practice Solutions

01. Ans: (a)

Sol: Frequency ratio is 2



∴ Two cycles of sine wave displayed on vertical time base

02. Ans: (b)

Sol: Time period of one cycle =  $\frac{8.8}{2} \times 0.5$   
 $= 2.2 \text{ msec}$

Therefore frequency =  $\frac{1}{T} = \frac{1}{2.2 \times 10^{-3}}$   
 $= 454.5 \text{ Hz}$

The peak to peak Voltage =  $4.6 \times 100$   
 $= 460 \text{ mV}$

Therefore the peak voltage  $V_m = 230 \text{ mV}$

R.M.S voltage =  $\frac{230}{\sqrt{2}} = 162.6 \text{ mV}$

03. Ans: (c)

Sol: In channel 1

The peak to peak voltage is 5V and peak to peak divisions of upper trace voltage = 2

Therefore for one division voltage is 2.5V

In channel 2, the no. of divisions for unknown voltage = 3

Divisions = 3, voltage/division = 2.5

∴ voltage =  $2.5 \times 3 = 7.5 \text{ V}$

Similarly frequency of upper trace is 1kHz

So the time period T

(for four divisions) =  $\frac{1}{f}$

$$T = \frac{1}{10^3} \Rightarrow 1 \text{ msec}$$

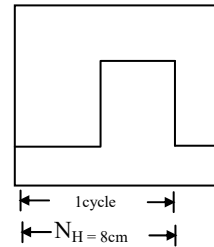
i.e for four divisions time

period = 1m sec

In channel 2, for eight divisions of unknown waveform time period = 2m sec.

04. Ans: (b)

Sol:



→ Given data: Y input signal is a symmetrical square wave

$$f_{\text{signal}} = 25\text{KHz}, V_{pp} = 10\text{V}$$

→ Screen has 10 Horizontal divisions & 8 vertical divisions which displays 1.25 cycles of Y-input signal.

$$\rightarrow V_{pp} = N_v \times \frac{\text{VOLT}}{\text{div}}$$

$$\Rightarrow \frac{\text{VOLT}}{\text{div}} = \frac{V_{pp}}{N_v} = \frac{10\text{V}}{5\text{cm}} = 2 \text{ Volt/ c.m}$$

$$\rightarrow T_{\text{signal}} = N_H \text{ per cycle} \times \frac{\text{TIME}}{\text{div}}$$

$$\Rightarrow \frac{\text{TIME}}{\text{div}} = \frac{T_{\text{signal}}}{N_H \text{ per cycle}} = \frac{1}{25\text{kHz} \times 8\text{cm}} = 5 \frac{\mu\text{s}}{\text{cm}}$$



05. Ans: (a)

Sol:  $t_r = \frac{0.35}{10\text{MHz}} \Rightarrow t_r = 35 \text{ nsec}$

06. Ans: (c)

Sol: No. of cycles of signal displayed  
 $= f_{\text{signal}} \times T_{\text{sweep}}$   
 $= 200\text{Hz} \times \left(10 \text{ cm} \times \frac{0.5\text{ms}}{\text{cm}}\right) = 1$

i.e, one cycle of sine wave will be displayed.

We know  $V_{\text{rms}} = \frac{V_{\text{p-p}}}{2\sqrt{2}}$

$V_{\text{rms}} = \frac{N_v \times \text{Volt / div}}{2\sqrt{2}}$

$\Rightarrow N_v = \frac{2\sqrt{2} \times V_{\text{rms}}}{\text{Volt / div}}$

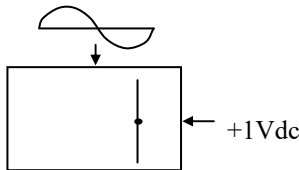
$\Rightarrow N_v = \frac{2\sqrt{2} \times 300\text{mV}}{100\text{mv / cm}}$

$\Rightarrow N_v = 8.485\text{cm}$

i.e 8.485cm required to display peak to peak of signal. But screen has only 8cm (vertical)  
 As such, peak points will be clipped.

07. Ans: (a)

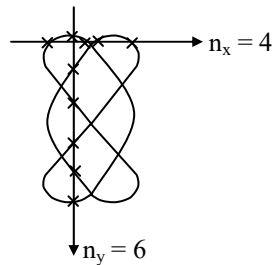
Sol:



Vertical straight line

08. Ans: (b)

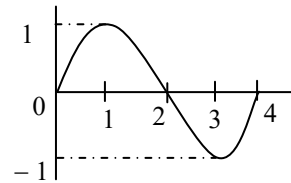
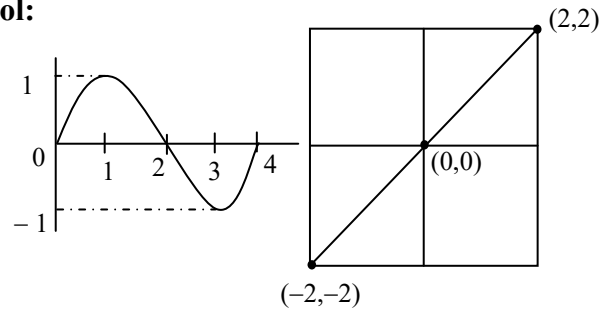
Sol:  $f_y = \frac{n_x}{n_y} f_x$   
 $= \frac{4}{6} \times 600\text{Hz}$   
 $= 400 \text{ Hz}$



09. Ans: (b)

10. Ans: (d)

Sol:



Let  $K_y = K_x = 2 \text{ Volt/div}$

t	V <sub>y</sub>	V <sub>x</sub>	d <sub>y</sub> = k <sub>y</sub> V <sub>y</sub>	d <sub>x</sub> = k <sub>x</sub> V <sub>x</sub>	points
0	0	0	0	0	(0,0)
1	1	1	2	2	(2,2)
2	0	0	0	0	(0,0)
3	-1	-1	-2	-2	(-2,-2)
4	0	0	0	0	(0,0)

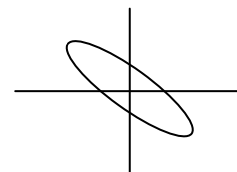
By using these points draw the line which is a diagonal line inclined at  $45^\circ$  w.r.t the x-axis.

11. Ans: (a)

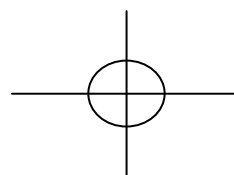
12. Ans: (d)

Sol:

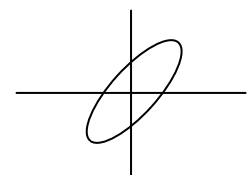
$90^\circ < \phi < 180^\circ$



$\phi = 90^\circ$



$0^\circ < \phi < 90^\circ$







Because of phase difference only figures changes from ellipse to circle and back to ellipse.

**13. Ans: (c)**

**Sol:** For either  $30^\circ$  or  $330^\circ$  phase difference, an ellipse having major axis in 1<sup>st</sup> and 3<sup>rd</sup> quadrants will be displayed on the screen.

**14. Ans: (a)**

**Sol:** Given data :  $R_i = 1M\Omega$ ,  $C_i = 45$  PF  
Probe used is 10 : 1 attenuation probe

The probe offers 10 times attenuation i.e.,

$$V_i = \frac{1}{10} \times V_s$$

The input resistance increases by 10 times and the input capacitance decreases by 10 times

$$C_{\text{eff}} = \frac{C_i}{10} = \frac{45\text{PF}}{10} = 4.5\text{PF}$$

**15. Ans: (a)**

**Sol: Given data:**

$$V_y(t) = 2\cos(100\pi t + 30^\circ)\text{V}$$

$$\frac{\text{Volt}}{\text{div}} \text{ is set as } 1 \frac{\text{V}}{\text{cm}} \text{ \& } \frac{\text{TIME}}{\text{div}}$$

$$\text{is set as } 2 \frac{\text{ms}}{\text{cm}}$$

Trigger voltage level is 0V & slope is negative. Screen dimensions are 10cm×8cm

Number of cycles of signal displayed:

$$N = f_{\text{signal}} \times T_{\text{sweep}}$$

$$= 50\text{Hz} \times 10\text{cm} \times 2 \frac{\text{ms}}{\text{cm}} = 1 \text{ cycle}$$

As trigger voltage level is set as 0V, the signal will be displayed from 0V onward and from falling side since slope is negative.

Therefore option is (a)

**16. Ans: (a)**

**Sol: Given data:**

$$V_y(t) = 4\cos(200\pi t - 45^\circ)\text{V}$$

$$\frac{\text{Volt}}{\text{div}} \text{ is set as } \frac{1\text{V}}{\text{div}} \text{ \& } \frac{\text{TIME}}{\text{div}} \text{ is set as } 1 \frac{\text{ms}}{\text{div}}$$

Internal triggering is chosen trigger voltage level is 0V & Slope is positive screen dimensions are 10div × 8div

Number of cycles of signal displayed

$$N = f_{\text{signal}} \times T_{\text{sweep}}$$

$$= 100\text{Hz} \times 10\text{div} \times 1\text{ms/div} = 1 \text{ cycle}$$

The test signal is used as triggering signal since internal triggering is chosen. As the trigger voltage level is 0V the signal will be displayed from 0V onwards and from rising side since trigger slope, is positive.

Therefore option is 'a'

# 8

# Digital Voltmeters

## Chapter

### Class Room Practice Solutions

**01. Ans: (b)**

**Sol:** Resolution =  $\frac{1}{\text{scale}} \times \text{voltage range}$  --- for N digit DVM

$$\text{Resolution} = \frac{1}{\text{extended scale}} \times \text{EVR}$$

EVR (Extended Voltage Range)

---- for  $N \frac{1}{2}$  (or)  $N \frac{3}{4}$  (or)  $N \frac{5}{6}$  DVM

$$\therefore \text{Resolution} = \frac{1}{2 \times 10^3} \times \text{range}$$

$$100\text{mV} = \frac{1}{2 \times 10^3} \times \text{range}$$

$$\Rightarrow \text{range} = 2 \times 10^3 \times 100\text{mV} = 200\text{V}$$

**02. Ans: (b)**

**Sol:** Sensitivity = resolution  $\times$  lowest voltage range

$$= \frac{1}{10^4} \times 100 \text{ mV} = 0.01 \text{ mV}$$

**03. Ans: (c)**

**Sol:** The DVM has  $3 \frac{1}{2}$  digit display

Total count 2000

$$\text{Resolution} = \frac{\text{Max. vtg}}{\text{Total count}}$$

$$\Rightarrow \frac{200\text{mV}}{2000}$$

$$\Rightarrow 0.1 \text{ mV}$$

**04. Ans: (a)**

$$\text{Sol: Resolution} = \frac{\text{max. voltage}}{\text{max. count}} = \frac{3.999}{3999} = 1 \text{ mV}$$

**05. Ans: (a)**

**Sol:** Triangular wave of  $V_m = 2 \times 10\text{V} = 20\text{V}$   
rms meter reading for the same triangular wave input =  $20/\sqrt{3} \text{ V}$

**06. Ans: (b)**

**Sol:** To eliminate the 50 Hz sinusoidal noise component riding on input signal (i.e. power line noise) the first integration time period must be selected as integer multiples of the period of that sine component

$\therefore$  Due to this it will be arranged out to zero. This is known as line frequency signal rejection or signal mode rejection.

$$\Rightarrow T = \text{period of line frequency}$$

$$\Rightarrow T = \frac{1}{50} \Rightarrow T = 0.02 \text{ Sec} = 20 \text{ msec}$$

**07. Ans: (d)**

**Sol:** 0.2% of reading + 10 counts  $\rightarrow$  (1)

$$= 0.2 \times \frac{100}{100} + 10(\text{sensitivity} \times \text{range})$$

$$= 0.2 \times \frac{100}{100} + 10 \left( \frac{1}{10^4} \times 200 \right)$$

$$= 0.2 + 0.2 = \pm 0.4\%$$

**08. Ans: (a)**

**09. Ans: (a)**

**10. Ans: (d)**

**Sol:** Dual slope DVM always measures average value.

**11. Ans: (b)**

**12. Ans: (b)**

**Sol:** In a dual slope type DVM, the analog dc voltage (to be measured) is integrated for a fixed period  $T_1$ . To eliminate or reject power-line noise:  $T_1 = n T_s$



Where

$T_1 = 1^{\text{st}}$  integration period

$T_s =$  Time period of sinusoidal component riding on dc voltage to be measured, having a frequency of  $f_s$ . And  $n \geq 1$

Given data :  $f_s = 50\text{Hz}$  &

$$T_1 = 100t_{\text{clk}}$$

$$T_1 = n \times T_s$$

$$\Rightarrow 100t_{\text{clk}} = n \times T_s$$

$$\Rightarrow 100 \times \frac{1}{f_{\text{clk}}} = n \times \frac{1}{f_s}$$

$$\Rightarrow f_{\text{clk}} = \frac{100}{n} \times f_s$$

$n$  should minimum (i.e.,1) for maximum clock frequency

$$\begin{aligned} \therefore f_{\text{clk}}(\text{max}) &= \frac{100}{1} \times 50\text{Hz} = 5000\text{Hz} \\ &= 5 \text{ kHz} \end{aligned}$$

**13. Ans: (d)**

**14. Ans: (b)**

**Sol:** For N-decade counter

$$\text{Pulse width}_{(\text{max})} = \frac{10^N}{f_{\text{clk}}}$$

Resolution  $\Rightarrow 1 \text{ count} \Rightarrow 1.T_{\text{clk}}$

$$\text{Resolution} = \frac{1}{f}$$

Range of pulse width

$$\Rightarrow 0 \text{ to } \left( \frac{10^N - 1}{F} \right)$$

**15. Ans:  $\pm 2$**

**Sol:** Given data:  $3\frac{1}{2}$  digit DMM,

Accuracy specification =  $\pm 1\%$  of full scale plus accuracy class 1

Reading = 100 mA on its 200 mA Full scale  
100mV reading on the 200mV full scale. 1 0 0 .0mV

$$\% \text{ error in reading} = \frac{\pm 1}{100} \times 200 \text{ mA} = \pm 2 \%$$

Therefore, error can be calculated as

$$\text{Error} = \pm \left[ \frac{2}{100} \times 100.0\text{mA} \right] = 2\text{mA}$$

**16. Ans: 200**

**Sol:** Given data:

$3\frac{1}{2}$  Digit DMM, 200 mV full scale range

$V_{\text{ref}} = 100 \text{ mV}$ ,  $T_1 = 100 \text{ ms}$

$V_{\text{in}} = (100 + 10 \cos(100\pi t)) \text{ mV}$

$T_{\text{conv}} = ?$

We know

$$V_{\text{in}} T_1 = V_{\text{ref}} T_2$$

$$100 \text{ mV} \times 100 \text{ ms} = 100 \text{ mV} \times T_2$$

$$T_2 = 100 \text{ ms}$$

$$\therefore t_{\text{conv}} = T_1 + T_2$$

$$= 100 \text{ ms} + 100 \text{ ms}$$

$$= 200 \text{ ms}$$

## Class Room Practice Solutions

**01. Ans: (a)**

**Sol:**  $c_1 = 300\text{pF}$        $c_2 = 200\text{ pF}$   
 $Q = 1/(\omega c_1 R) = 120 = 1/(c_2 + c_x)R$   
 $C_1 = c_2 + c_x$   
 $\therefore C_x = 100\text{ pF}$

**02. Ans: (b)**

**03. Ans: (c)**

**04. Ans: (b)**

**Sol:** Given data:  $C_d = 820\text{PF}$ ,  
 $\omega = 10^6\text{rad/sec}$  &  $C = 9.18\text{nF}$   
 We know,  $L = \frac{1}{\omega^2 [C + C_d]}$   
 $= \frac{1}{(10^6)^2 [9.18\text{nF} + 820\text{PF}]} = 100\mu\text{H}$

The inductance of coil tested with a Q-meter is  $100\mu\text{H}$ .

**05. Ans: (b)**

**Sol:** A series RLC circuit exhibits voltage magnification property at resonance. i.e., the voltage across the capacitor will be equal to Q-times of applied voltage.

Given that  $V =$  applied voltage and  
 $V_0 =$  Voltage across capacitor

There fore,  $Q = \frac{V_{c\text{max}}}{V_{in}} \Rightarrow Q = \frac{V_0}{V}$

**06. Ans: (b)**

**Sol:**  $f_1 = 500\text{ kHz}$  ;     $f_2 = 250\text{kHz}$   
 $C_1 = 36\text{ pF}$  ;     $C_2 = 160\text{ pF}$

$$n = \frac{250\text{ kHz}}{500\text{ kHz}} \Rightarrow n = 0.5$$

$$C_d = \frac{36\text{pF} - (0.5)^2 160\text{pF}}{(0.5)^2 - 1} = 5.33\text{pF}$$

**07. Ans: (c)**

**Sol:**  $Q = \frac{\text{capactor voltmeter reading}}{\text{Input voltage}}$   
 $= \frac{10}{500 \times 10^{-3}} = 20$

**08. Ans: i  $\rightarrow$  (c), ii  $\rightarrow$  (a)**

**Sol:** (i)  $C_d = \frac{C_1 - n^2 C_2}{n^2 - 1} = \frac{360 - 288}{3} = 24\text{ pF}$

(ii)  $L = \frac{1}{\omega_1^2 [C_1 + C_d]}$   
 $= \frac{1}{[2\pi \times 500 \times 10^3]^2 [24 + 360] \times 10^{-6}} = 264\mu\text{H}$

**09. Ans: (d)**

**10. Ans: 10 pF**

**Sol:** Given data:  
 $C_1 = 110\text{ pF}$   
 $C_2 = 20\text{ pF}$   
 $n = \frac{2f}{f} = 2$   
 $C_d = \frac{C_1 - n^2 C_2}{n^2 - 1} = \frac{110\text{pF} - 4 \times 20\text{pF}}{4 - 1}$   
 $= \frac{110\text{pF} - 80\text{pF}}{3} = \frac{30\text{pF}}{3} = 10\text{pF}$

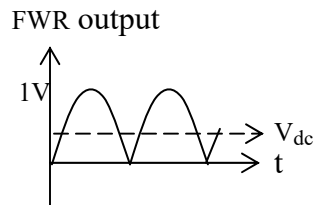
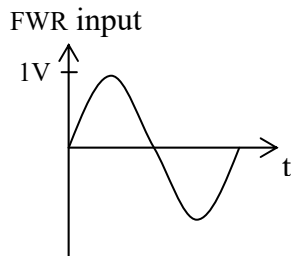
## Class Room Practice Solutions

**01. Ans: (a)**

**Sol:** The full wave Rectifier type electronic AC voltmeter has a scale calibrated to read r.m.s value for square wave inputs. As such, the scale calibration factor used for deriving rms volt scale from DC volt scale is 1.

Reading =  $1 \times V_{dc}$  Where  $V_{dc}$  is Average voltage of output of full wave Rectifier for given input.

This voltmeter is used to measure a sinusoidal voltage



DC. voltmeter measures  $V_{dc}$  of output of FWR

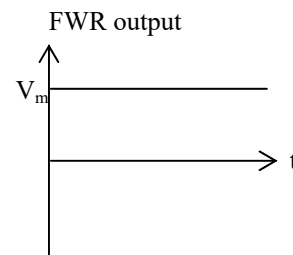
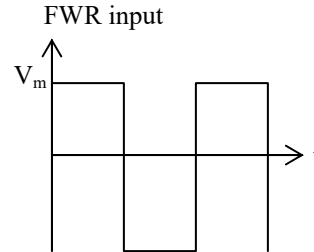
$$V_{dc} = \frac{2V}{\pi}$$

$$\text{Therefore, reading} = 1 \times V_{dc} = \frac{2}{\pi} V$$

**02. Ans: (b)**

**Sol:** The scale of a full wave rectifier type voltmeter is calibrated to read r.m.s for ideal sine wave i.e, reading =  $1.11V_{dc}$  where  $V_{dc}$  is average voltage of out put of FWR for given input.

This voltmeter is used for square wave input



DC voltmeter measures  $V_{dc}$  of output of FWR

$$V_{dc} = V_m$$

$$\begin{aligned} \text{reading} &= 1.11V_{dc} \\ &= 1.11V_m \end{aligned}$$

i.e., indicated rms is  $1.11V_m$  where true rms of square wave is  $V_m$ .

Therefore, the multiplying factor of the meter for correction is

$$\frac{1}{1.11} \left( \because 1.11V_m \times \frac{1}{1.11} \Rightarrow V_m \right)$$

**03. Ans: (b)**

**Sol:** Given data : Voltmeter sensitivity is  $20\text{k}\Omega/\text{V}$  Reading of  $4.5\text{V}$  on its  $5\text{V}$  full scale Reading of  $6\text{V}$  on its  $10\text{V}$  full scale \*Say, voltage source is  $V_s$  and its internal resistance is  $R_s$ .

**5V range:**

$$\begin{aligned} R_v &= 20 \frac{\text{K}\Omega}{\text{V}} \times 5 \\ &= 100\text{K}\Omega \end{aligned}$$



$$\text{Reading} = V_s \times \frac{100\text{k}\Omega}{R_s + 100\text{k}\Omega}$$

$$4.5\text{V} = V_s \times \frac{100\text{k}\Omega}{R_s + 100\text{k}\Omega}$$

$$\therefore V_s = \frac{4.5\text{V}}{100\text{k}\Omega} (R_s + 100\text{k}\Omega) \rightarrow (1)$$

**10V Range:**

$$R_v = 20 \frac{\text{K}\Omega}{\text{V}} \times 10\text{V}$$

$$= 200\text{k}\Omega$$

$$\text{Reading} = V_s \times \frac{200\text{k}\Omega}{R_s + 200\text{k}\Omega}$$

$$6\text{V} = V_s \times \frac{200\text{k}\Omega}{R_s + 200\text{k}\Omega}$$

$$\therefore V_s = \frac{6\text{V}}{200\text{k}\Omega} (R_s + 200\text{k}\Omega) \rightarrow (2)$$

Solving equation (1) & (2)

$$\frac{6\text{V}}{200\text{k}\Omega} (R_s + 200\text{k}\Omega)$$

$$= \frac{4.5\text{V}}{100\text{k}\Omega} (R_s + 100\text{k}\Omega)$$

$$R_s + 200\text{k}\Omega = 1.5(R_s + 100\text{k}\Omega)$$

$$0.5R_s = 50\text{k}\Omega$$

$$R_s = 100\text{k}\Omega$$

Putting the value of  $R_s$  in equation (1)

$$V_s = \frac{4.5\text{V}}{100\text{k}\Omega} (100\text{k}\Omega + 100\text{k}\Omega)$$

$$= 4.5\text{V} \times 2$$

$$= 9\text{V}$$

Therefore, the voltage source is 9V and its internal resistance is 100k $\Omega$

**04. Ans: (c)**

**Sol:** Given data: Full wave Bridge Rectifier AC voltmeter's AC volt range is 0-100V. The PMMC ammeter used in the design has full scale current rating of 1mA and internal resistance of 100 $\Omega$  & diodes are ideal

$$R_s = 0.9 \times \frac{V_{\text{rmsFSD}}}{I_{\text{dcFSD}}} - 2R_d - R_m$$

$$= 0.9 \times \frac{100\text{V}}{1\text{mA}} - 100\Omega$$

$$= 90\text{k}\Omega - 100\Omega$$

$$= 89.9\text{k}\Omega$$

**05. Ans: (b)**

**Sol: Given data:** PMMC ammeter full scale current range is 100 $\mu\text{A}$ , and internal resistance is 100 $\Omega$ .

Required current range is 1A

$$R_{\text{sh}} = \frac{100\Omega}{\frac{1\text{A}}{100\mu\text{A}} - 1}$$

$$\Rightarrow R_{\text{sh}} = 10\text{m}\Omega$$

$\therefore$  10m $\Omega$  in parallel with the meter

**06. Ans: (c)**

**Sol:** PMMC ammeter will read average value of current.

$$I_{\text{dc}} = 0.636 I_m$$

( $\because$  full wave rectified sinusoidal)

$$= 0.636 \times \frac{1\text{V}}{10\text{k}\Omega}$$

$$= 0.0636\text{mA}$$

$$= 63.6\mu\text{A}$$