MECHANICAL ENGINEERING

GATE | PSUs

IM & OR

Volume - I : Study Material with Classroom Practice Questions

IM & OR

Solutions for Vol – I _ Classroom Practice Questions

Chapter- 01
PERT & CPM

01. Ans: (a)

02. Ans: (a)

03. Ans: (a)

04. Ans: (a)

05. Ans: (b)

06. Ans: (a)

07. Ans: (c)

08. Ans: (c)

09. Ans: (b)

10. Ans: (c)

Sol: The earliest expected completion time,

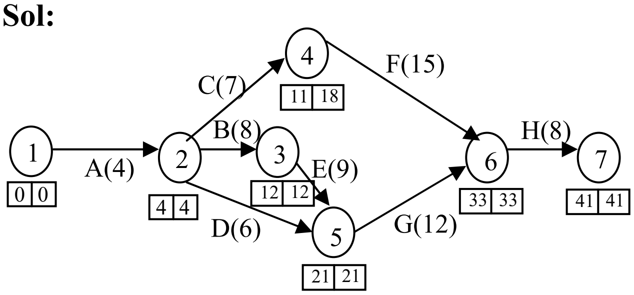
Critical path : A-B-C-D-F-E-H

$\Rightarrow 5 + 4 + 8 + 5 + 8 = 30$ days

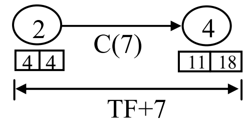
11. Ans: (a)

12. Ans:

Sol:



Path	**duration**
1-2-4-6-7	= 4 + 7 + 15 + 8 = 34
1-2-3-5-6-7	= 4 + 8 + 9 + 12 + 8 = **41 (days)**
1-2-5-6-7	= 4 + 6 + 12 + 8 = 30

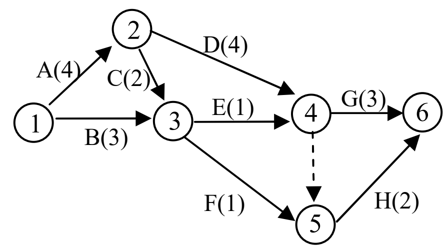


TF + 7 = 18 – 4

$\Rightarrow$ TF = 14 – 7 = 7

13. Ans:

Sol:



Critical path :

1-2-3-4-5-6	= 4 + 2 + 1 + 0 + 2 = 9
1-2-4-6	= 4 + 4 + 3 = **11 → CP**
1-2-3-4-6	= 4 + 2 + 1 + 3 = 10
1-3-5-6	= 3 + 1 + 2 = 6

14. Ans: (a)

15. Ans: (b)

16. Ans: (b)

Sol: $T_0 = 8$ min, $T_m = 10$, $T_p = 14$min,

$$T_e = \frac{T_o + 4T_m + T_p}{6}$$

$$= \frac{8 + 4\times 10 + 14}{6} = \frac{62}{6} = 10.33 \text{ min}$$

17. Ans: (c)

Sol: $\sigma_{CP} = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$

$T_e = 3 + 8 + 6 = 17$

$$Z = \frac{D - T_e}{\sigma_{CP}} = \frac{20 - 17}{3} = 1$$

From the normal distribution table,
P(Z=1) = 0.84

18. Ans: (a)

Sol: Take 4 – 3 , T_e = 6 days

Critical path = 1-2-4-3
= 5 + 14 + 4 = 23 days

$$\sigma_{\text{critical path}} = \sqrt{V_{1-2} + V_{2-4} + V_{4-3}}$$

$$= \sqrt{2^2 + 2.8^2 + 2^2} = 3.979$$

$$z = \frac{\text{Due date} - \text{critical path duration}}{\sigma_{\text{critical path}}}$$

$$z = \frac{27 - 23}{3.979} = 1.005$$

$\therefore$ P(z) = 0.841

19. Ans: (a)

20. Ans: (c)

Sol: D = 36 days, V = 4 days

$$Z = \frac{36 - 36}{\sqrt{4}} = 0$$

$\Rightarrow$ P(z) = 50%

21. Ans: (d)

Sol: Variance $= \left(\frac{t_p - t_o}{6}\right)^2 = \left(\frac{22 - 10}{6}\right)^2 = 4$

22. Ans: (a) **23. Ans: (b)**

24. Ans: (b)

25. Ans: (c)

Sol: $\sigma_{cp} = \sqrt{V_{a-b} + V_{b-c} + V_{c-d} + V_{d-e}}$

$$= \sqrt{4 + 16 + 4 + 1} = 5$$

26. Ans: (d)

Sol: *Critical path :*

1-3-4-6 = 20 days

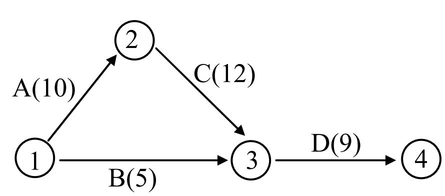
$$z = \frac{24 - 20}{\sqrt{4}} = \frac{4}{2} = 2$$

$\Rightarrow$P(z) = 97.7%

27. Ans:

Sol:

Activity	**Time estimated**	**Standard deviation**
	$T_e = \frac{T_o + 4T_m + T_p}{6}$	$\sigma = \frac{T_p - T_o}{6}$
A	$\frac{5 + 4\times10 + 15}{6} = 10$	$\frac{15 - 5}{6} = \frac{5}{3}$
B	$\frac{2 + 4\times5 + 8}{6} = 5$	$\frac{8 - 2}{6} = 1$
C	$\frac{10 + 4\times12 + 14}{6} = 12$	$\frac{14 - 10}{6} = \frac{2}{3}$
D	$\frac{6 + 4\times8 + 16}{6} = 9$	$\frac{16 - 6}{6} = \frac{5}{3}$



Critical path :

$$1\text{-}2\text{-}3\text{-}4 = 10 + 12 + 9 = 31 \text{ days}$$

$$\sigma_{cp} = \sqrt{V_{1-2} + V_{2-3} + V_{3-4}}$$

$$= \sqrt{\left(\frac{5}{3}\right)^2 + 1^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{5}{3}\right)^2} = \sqrt{7}$$

28. Ans: (d)

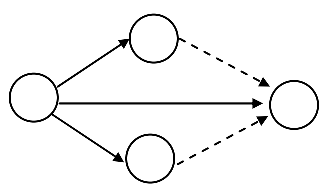
Sol: $T_e = \dfrac{T_o + 4T_m + T_p}{6}$

$$= \frac{5 + 4\times 5 + 5}{6} = \frac{30}{6} = 5$$

29. Ans: (b)

30. Ans: (c)

Sol:



31. Ans: (c)

32. Ans: (a)

33. Ans: (a)

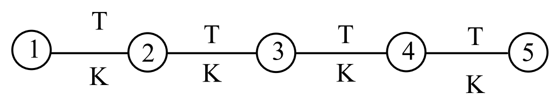
Sol:

9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
⟵ A (6) ⟶															
		⟵ B (4) ⟶													
						⟵ C (3) ⟶									
				⟵ D (7) ⟶											
6	6	10	10	17	17	**20**	**20**	14	14	14	14	10	10	7	7

Maximum resource load per week = 20

34. Ans: (b)

35. Ans: (d)



Given each activity having time mean duration 'T' and standard deviation 'K'.

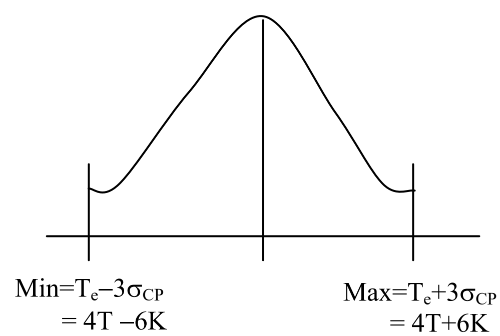
Total time estimate $T_e = 4T$

Variance of the path $(\Sigma \text{var})_{CP}$

$= R^2 + R^2 + R^2 + R^2$

$= 4R^2$

Standard deviation of CP= $\sqrt{\sum (\text{var})_{CP}}$



$\sigma_{CP} = \sqrt{4K^2}$

$\sigma_{CP} = \pm 2K$

Range of overall project duration likely to be in 4T + 6K and 4T – 6K

i.e., 4T ± 6K

Common solutions for Q.36 & Q.37

36. Ans: (b)

37. Ans: (b)

Sol:

Paths	Duration
1-2-4-5 = (AEF)	8+9+6=23
1-2-3-4-5=(ADF)	8+9+6=23
1-3-4-5 (BDF)	6+9+6 = 21
1-4-5 (CF)	16+6=22

$\therefore$ Highest time taken paths are AEF and ADF

$\therefore$ Critical path's are AEF and ADF

Critical paths are '2'.

Possible cases to crash

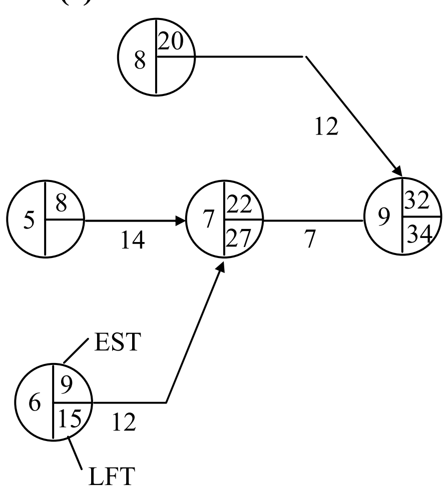
A by 1 day that cost = 80

F by 1 day that cost = 130

E and D by 1 day that cost = 20 + 40 = 60

38. Ans: (c)

Sol:



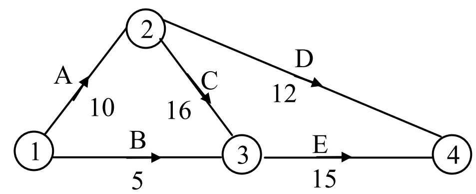
Total Float$)_{6\text{-}1}$ = TF$)_{6\text{-}7}$ = 27 –9 –12 = 6

Free float$)_{6=7}$ = 28 –9 –12 = 1

39.

Sol:

Paths	duration
AD	22
ACE	41←CP
BE	20



$$(\text{Var})_{cp} = (\text{Var})_A + (\text{Var})_C + (\text{Var})_E$$

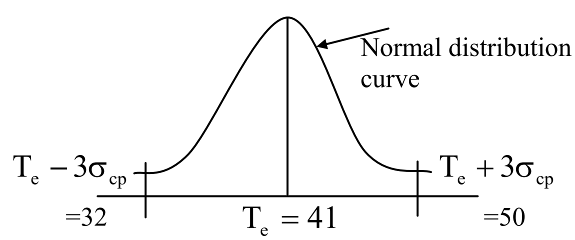
$$= \sigma_A^2 + \sigma_C^2 + \sigma_E^2$$

$$= 2^2 + 2^2 + 1^2 = 4 + 4 + 1 = 9$$

$$\sigma_{CP} = \sqrt{(\text{Var})_{CP}} = \sqrt{9} = 3$$

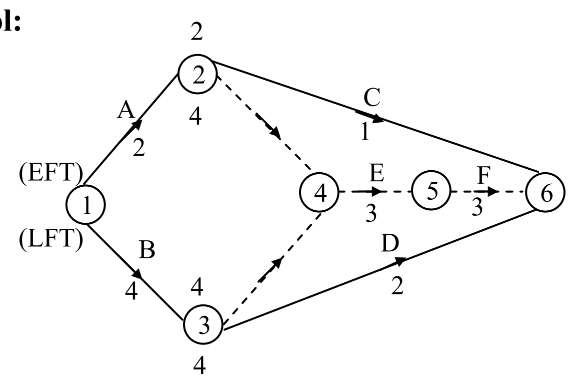
Minimum completion time = 32 days

Maximum completion time = 50 days



40.

Sol:



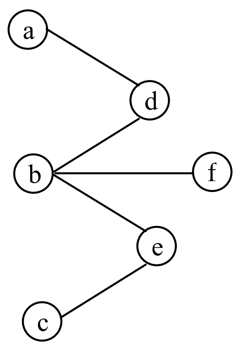
Paths	Duration
1-2-6 → AC	2 + 1 = 3
1-2-4-5-6→ AEF	2+3+2 = 7
1-3-6→ BD	4+2 = 6
1-3-4-5-6 → BEF	4+3+2 = 9

Highest Duration is '9'.

∴ CP is BEF

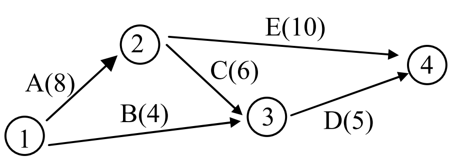
41.

Sol:



42. Ans:

Sol:



(a) critical path :

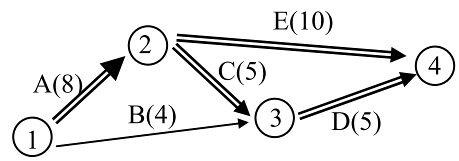
Path	Duration
A-E	8+10 = 18
A-C-D	8+6+5 = 19
B-D	4+5 = 9

(b) To reduce the project by 1 day the available option is crashing 'C' by 1 day

Option	Crashing possibilities ($N_T - C_T$)
A	8 – 8 = 0
C	6 – 5 = 1
D	5 – 5 = 0

By crashing activity C we can reduce the project duration by 1 day.

Network diagram



Path	Duration
A-E	8+10 = 18
A-C-D	8+6+5 = 19
B-D	4+5 = 9

Further crashing is not possible due to "A-C–D" critical path.

∴ Minimum duration of project = 19

43. Ans: (c)

Sol:

Path	Duration
AB	7+5=12
CD	6+6=12
EF	8+4=12

Three critical paths, number of activities to be Crashed are 3

Chapter- 02
Linear Programming

01. Ans: (d) **02. Ans: (d)** **03. Ans: (c)**
04. Ans: (d) **05. Ans: (b)** **06. Ans: (a)**

07. Ans: (a)

Sol: $Z_{max} = x+2y$,

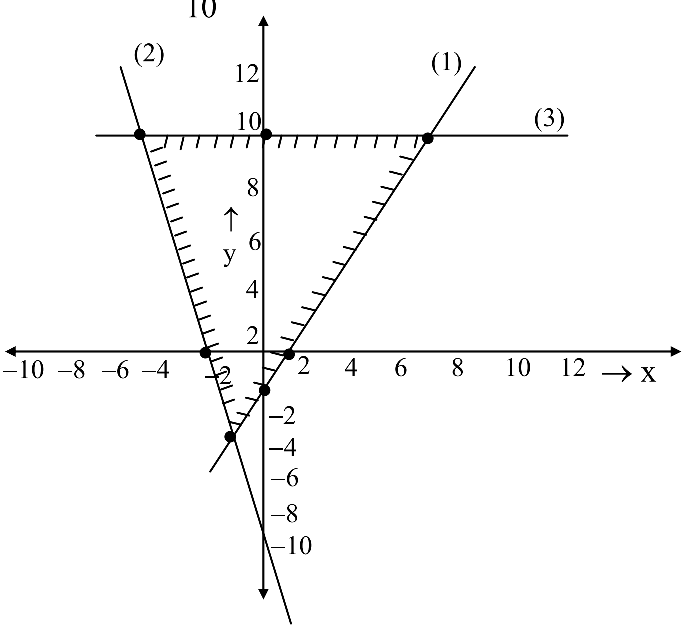
Subjected to

$4y - 4x \geq -1$……… (1)
$5x + y \geq -10$ ……….. (2)
$y \leq 10$ …………… (3)
x and y are unrestricted in sign

$$(1) \Rightarrow \frac{x}{\left(\frac{1}{4}\right)} + \frac{y}{\left(\frac{-1}{4}\right)} \leq 1$$

$$(2) \Rightarrow \frac{x}{(-2)} + \frac{y}{(-10)} \leq 1$$

$$(3) \Rightarrow \frac{y}{10} \leq 1$$



Only one value gives max value, then solution is unique.

08. Ans: (b)

Sol: $Z_{max} = 3x_1+2x_2$

Subjected to

$4x_1+x_2 \leq 60$……… (1)

$8x_1+x_2 \leq 90$ ……… (2)

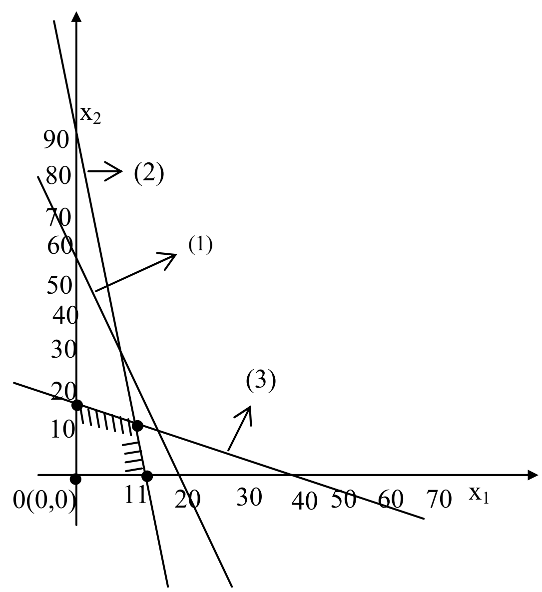
$2x_1+5x_2 \leq 80$ …….. (3)

$x_1, x_2 \geq 0$

$$(1) \Rightarrow \frac{x_1}{15} + \frac{x_2}{60} \leq 1$$

$$(2) \Rightarrow \frac{x_1}{11.25} + \frac{x_2}{90} \leq 1$$

$$(3) \Rightarrow \frac{x_1}{40} + \frac{x_2}{16} \leq 1$$



From the above graph the No. of corner points for feasible solutions are 4

09. Ans:

Sol: Let, x_1 be the number of ash trays

x_2 be the number of tea trays

Production to be maximized

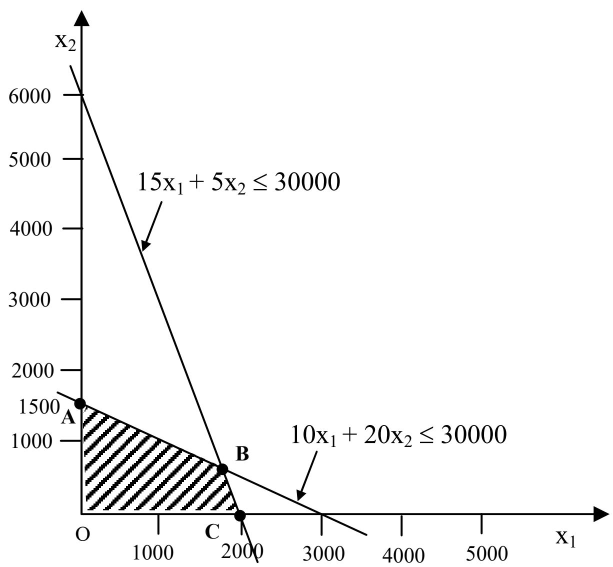
$Z = 20x_1 + 30\,x_2$

From the table given, constrained are

$10x_1 + 20x_2 \le 30000$

$15x_1 + 5x_2 \le 30000$

Fixed daily cost =Rs. 45000



From the graph, common feasible region is OABC O(0,0) , A(0,1500), C(2000,0)

B would be obtained by solving the constraints. B(1800 , 600)

Z	A(0,1500)	20×0+30×1500 = Rs.45000
	B(1800,600)	20×1800+30×600 = Rs.54000
	C(2000,0)	20×2000+30×0 = Rs.40000

Z_{max} = Rs. 54000 at B

Profit = Z_{max} – Fixed daily cost

= 54000 – 45000 = Rs.9000

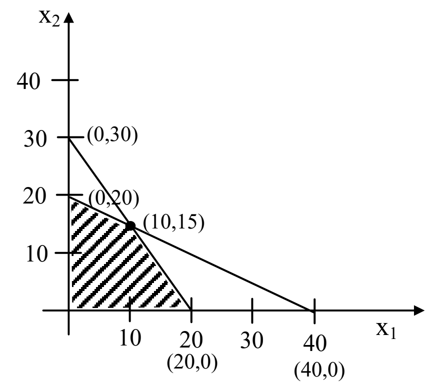
10.

Sol: $Z_{max} = 60x_1 + 50x_2$

s.t $\quad x_1 + 2x_2 \le 40$

$3x_1 + 2x_2 \le 60$

$$\frac{x_1}{40} + \frac{x_2}{20} \le 1\ , \quad \frac{x_1}{20} + \frac{x_2}{30} \le 1$$



$(Z_{max})_{(10,15)}$

$= 60\times10+50\times15 = 1350$ /-

11.

Sol:

Type of machine	Products		Total time available
	A	B	
P	10	7.5	75
Q	6	9	54
R	5	13	65

Profit for product, A = Rs. 60 per unit

Profit for product, B = Rs. 70 per unit

Let, x = number of A type products

y = number of B type products

∴ Maximization problem

$Z_{max} = 60x + 70y$

Constraints are, (in times)

$10x + 7.5y \le 75$

$6x + 9y \le 54$

$5x + 13y \le 65$

Common feasible region is OABCDO

O(0,0), A(0,5), D(7.5,0)

B is point of intersection of lines

$6x + 9y \le 54$,

$5x + 13y \le 65$

Solving this B = (3.55 , 3.64)

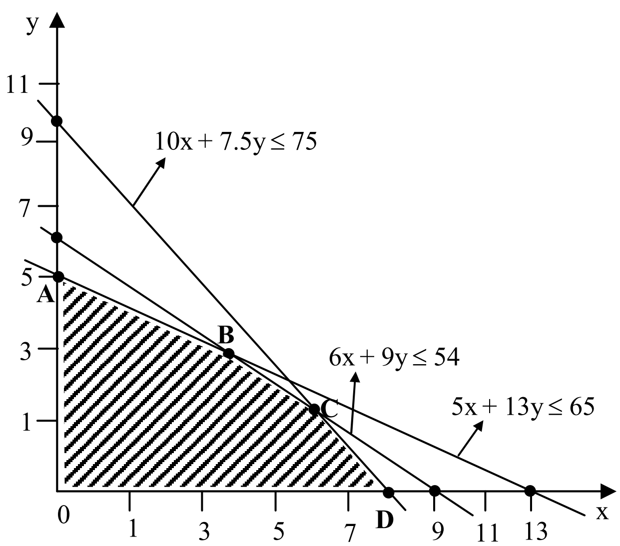
C is the point of intersection of the lines

$6x + 9y \le 54$,

$10x + 7.5y \le 75$

Solving these, C = (6,2)

Graphically solving :



Points	**Z=60x+70y**
A (0,5)	60×0+70×5 = 350
B (3.53,3.64)	3.55×60+70×3.64 = 464.8
C (6,2)	60×6+70×2 = 500
D (7.5,0)	7.5×60+0×70 = 450
O (0,0)	0×60+0×70 = 0

$\therefore Z_{max} = 500$ at C(6,2)

$\therefore$ A type products = 6 , B type products = 2

12.

Sol:

	Tables	Chairs	Availability
Wood	30	20	300
Labour	5	10	110
Profit/unit	8	6	
	x	y	

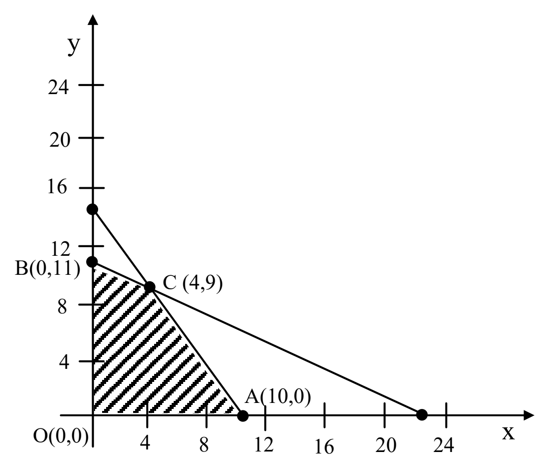
$Z_{max} = 8x + 6y$

Subject to

$30x + 20y \le 300$, $\frac{x}{10} + \frac{y}{15} \le 1$ ----- (1)

$5x + 10y \le 110$, $\frac{x}{22} + \frac{y}{11} \le 1$ ------ (2)

$x , y \ge 0$



"C' is the intersection of (1) and (2)

Solve equation (1) & (2) for x,y

We will get x = 4, y = 9

$Z = 8x + 6y$

$Z_0 = 0$

$Z_A = 8\times10 + 6\times0 = 80$

$Z_B = 8\times0 + 6\times11 = 66$

$Z_C = 8\times4 + 6\times9 = 86$

Solution is optimal at (c)

$Z_{max} = 86$ at x = 4 , y = 9

13.

Sol:

Demand	Products		Maximum available
	Chairs (x_1)	Tables (x_2)	
Wood	1	2	200
Chairs	1	-	150
Tables	-	1	80
Profit/loss	100	300	

$Z_{max} = 100x_1 + 300x_2$

Subject to

$x_1 + 2x_2 \le 200$

$x_1 \le 150$ and $x_2 \le 80$

14.

Sol:

Demand	Products		Maximum available
	A (x_1)	B (x_2)	
Raw material	1	1	850
Special type of buckle	1	-	200
Ordinary buckle	-	1	700
Time	1	½	500
Profits/unit	10/-	5/-	

Constraints :

x_1 = No. of belts of type 'A'

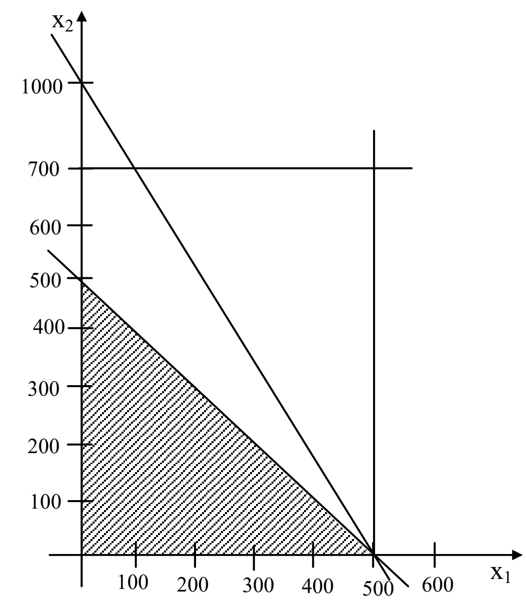
x_2 = No. of belts of type 'B'

$Z_{max} = 10x_1 + 5x_2$

s.t $x_1 + x_2 \le 500$

$x_1 \le 500$, $x_2 \le 700$

$x_1 + \frac{1}{2}x_2 \le 500$, $x_1 , x_2 \ge 0$



$Z_{max} = (10\times0) + (5\times500) = 2500$

15. Ans: (c)

Sol: Let, P type toys produced = x ,

Q type toys produced = y

	P	**Q**	
Time	1	2	2000
Raw material	1	1	1500
Electric switch	-	1	600
Profit	**3**	**5**	
	x	**y**	

$Z_{max} = 3x + 5y$

$x + 2y \le 2000$; $\frac{x}{2000} + \frac{y}{1000} \le 1$

$x + y \le 1500$; $\frac{x}{1500} + \frac{y}{1500} \le 1$

$y \le 600$; $\frac{y}{600} \le 1$

$x, y \ge 0$

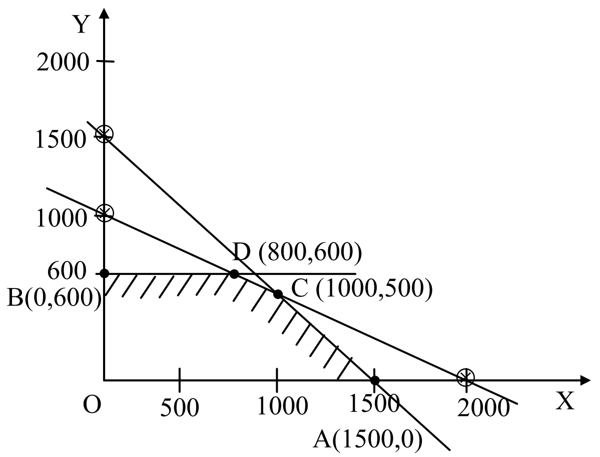
$Z_{max} = 3x + 5y$

$Z_A = 3 \times 1500 + 5 \times 0 = 4500$

$Z_B = 3 \times 0 + 5 \times 600 = 3000$

$Z_C = 3 \times 1000 + 5 \times 500 = 5500$

$Z_D = 3 \times 800 + 5 \times 600 = 5400$



C does not exist in answer.

Hence, Z_{max} is at D, i.e., Z_{max} @ D = 5400

16. Ans: (c)

Sol: $Z_{max} = x_1 + 1.5\, x_2$

Subject to

$2x_1+3x_2 \le 6$ ------- (1)

$x_1 + 4x_2 \le 4$ -------- (2)

$x_1 , x_2 \ge 0$

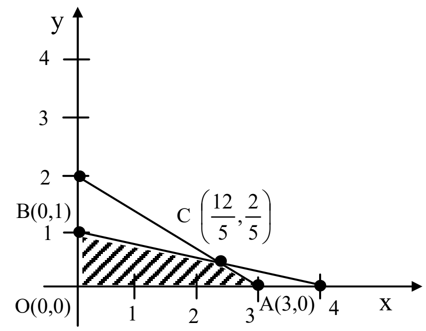
$$\frac{x_1}{3}+\frac{x_2}{2}=1$$

$$\frac{x_1}{4}+\frac{x_2}{1}=1$$

Let, "c" in the intersection of (1) and (2)

Solve (1) & (2) for 'c'.

It follows, $x_1 = \frac{12}{5};\ x_2 = \frac{2}{5}$



$Z_{max} = x_1 + 1.5x_2$

$Z_0 = 0$

$Z_A = 3 + 1.5 \times 0 = 3$

$Z_B = 3\times0+1.5\times1 = 1.5$

$$Z_C = \frac{12}{5}+\frac{3}{2}\times\frac{2}{5}=3$$

Problem is having multiple solutions and it is Optimal at (A) and (C)

17. Ans: (a)

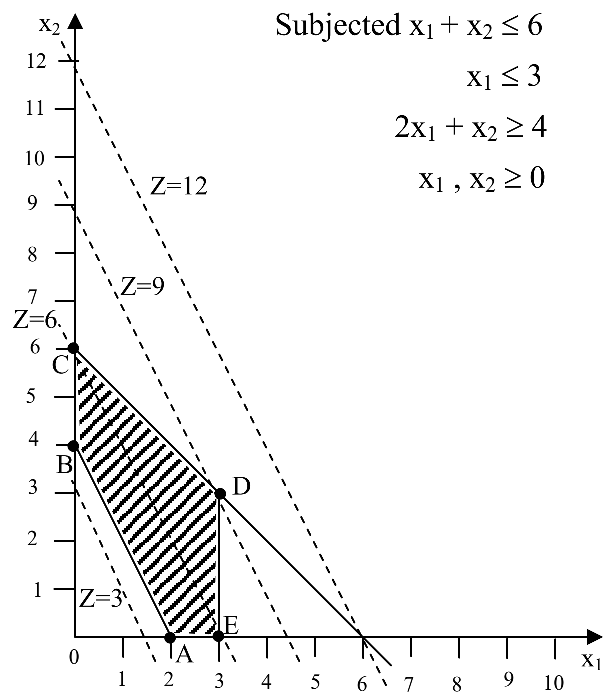
Sol: $Z_{max} = 2x_1 + x_2$

Subjected $x_1 + x_2 \le 6$

$x_1 \le 3$

$2x_1 + x_2 \ge 4$

$x_1 , x_2 \ge 0$



But feasible region is ABCDEA ($\because x_1, x_2 > 0$)

A(2,0) B(0,4) C(0,6) E(3,0)

D can be obtained by solving

$x_1 \le 3$ & $x_1 + x_2 \le 6$

$\Rightarrow x_1 = 3$ and $x_2 = 3$ and D (3,3)

Z_{max}	A(2,0)	$2\times2+1\times0 = 4$
	B (0,4)	$0\times2+1\times4 = 4$
	G(0,6)	$0\times2+1\times6 = 6$
	E(3,0)	$3\times2+0\times1 = 6$
	D(3,3)	$3\times2+1\times3 = 9$

$Z_{max} = 9$ at D (3,3)

18. Ans: (a) **19. Ans: (b)** **20. Ans: (d)**

21. Ans: (a)

Sol: $Z_{max} = 4x_1 + 6x_2 + x_3$

s.t

$2x_1 - x_2 + 3x_3 \le 5$

$x_1, x_2, x_3 \ge 0$

$2x_1 - x_2 + 3x_3 + s_1 = 5$

$Z_{max} = 4x_1 + 6x_2 + x_3 + 0\,s_1$

$c_j \rightarrow$ s v$\downarrow$	4 x_1	6 x_2	1 x_3	0 s_1	B_0	min Ratio
0 s_1	2	-1	3	1	5	– 5
z_j c_j - z_j	0 4	0	0 1	0 0	0	

6 ↑EV

Entering vector exists but leaving vector doesn't exist as minimum ratio column is having negative values. It is a case of unbounded solution space and unbounded optimal solution to problem.

22. Ans: (a)

Sol:

$C_j \rightarrow$		107	1	2	0	0	0	B_0	Min Ratio
C_B	SV	x_1	x_2	x_3	x_4	s_1	s_2		
0	x_4	0	$\frac{17}{9}$	$\frac{-4}{9}$	1	0	$\frac{-14}{9}$	$\frac{7}{3}$	$\frac{-21}{4}$
0	S_1	0	$\frac{35}{6}$	$\frac{-2}{3}$	0	1	$\frac{-16}{3}$	5	$\frac{-15}{2}$
107	x_1	1	$\frac{-1}{3}$	$\frac{-1}{3}$	0	0	$\frac{1}{3}$	10	$\frac{-10}{3}$
Z_j		107	$\frac{-107}{3}$	$\frac{-107}{3}$	0	0	$\frac{107}{3}$		
$C_j - Z_j$		107	$\frac{110}{3}$	$\frac{113}{3}$ (EV)↑	0	0	$\frac{107}{3}$		

Minimum ratio column has all negative values, so can not decide outgoing variables. Problem has unbounded solutions.

23. Ans: (b)

Sol: Solution is optimal; but Number of zeros are greater than the number of basic Variables in $C_j - Z_j$(net evaluation row) hence multiple optimal solutions.

24. Ans: (b)

Sol:

$C_j \rightarrow$		3	2	5	0	0	0	B_0
C_B	SV	x_1	x_2	x_3	s_1	s_2	s_3	
2	x_2	$\frac{-1}{4}$	1	0	$\frac{1}{2}$	$\frac{-1}{4}$	0	100
5	x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
0	S_3	–2	0	0	–2	1	1	20
Z_j		7	2	5	1	2	0	1350
$C_j - Z_j$		–4	0	0	–1	–2	0	

As it contain 0's and '–ve' values in

$C_j - Z_j$ row, hence the problem is optimal.

$Z_{max} = (2 \times 100) + (5\times230) + (0\times20)$

$= 1350/-$

25. Ans: (a)

Common Data

26. Ans: (d) **27. Ans: (a)** **28. Ans: (a)**

Sol:

$C_j\rightarrow$		6	4	0	0	0	–M	B_0	Min Ratio
C_B	SV	x_1	x_2	s_1	s_2	s_3	A_1		
0	S_1	0	$\frac{5}{3}$	1	$\frac{-2}{3}$	0	x	14	
0	S_3	0	$\frac{-1}{3}$	0	$\frac{1}{3}$	1	x	5	
6	x_1	1	$\frac{2}{3}$	0	$\frac{1}{3}$	0	x	8	
Z_j		6	4	0	2	0	x	48	
$C_j - Z_j$		0	0	0	–2	0	x		

As the No. of zeros greater than No. of basic variables in $C_j–Z_j$ row, hence it is a case of multiple solutions or alternate optimal solution exists.

If Non basic variable x_2 is having a zero evaluation at optimality, with that variable if we enter and performs simplex procedure in alternate solution to the problem is obtained as follows.

$C_j\rightarrow$		6	4	0	0	0	–M	B_0	Min Ratio
C_B	SV	x_1	x_2	s_1	s_2	s_3	A_1		
0	S_1	0	$\frac{5}{3}$	1	$\frac{-2}{3}$	0	x	14	$\frac{14}{5/3}$
0	S_2	0	$\frac{-1}{3}$	0	$\frac{1}{3}$	1	x	5	–15
6	x_1	1	$\frac{2}{3}$	0	$\frac{1}{3}$	0	x	8	12
Z_j		6	4	0	2	0	x	48	
$C_j - Z_j$		0	0	0	–2	0	x		

Entering vector column

Minimum ratio having row will become entering vector row.

Hence the alternate solution is

$x_2 = \frac{42}{5}, \quad x_1 = 12$

Entering vector x_2 and leaving vector s_1 because of s_1 row had minimum ratio.

29. Ans: (c) **30. Ans: (c)**

31. Ans: (a)

32. Ans: (c)

Sol: $Z_{min} = 10x_1+x_2+5x_3+0S_1$

Dual , $W_{min}= 50y_1$

subjected to

$5y_1 \le 10, \quad y_1 \le 2 , \quad W_{max} = 100$

$3y_1 \le 5 \;, \quad y_1 \le 5/3 , W_{max} = 250/3$

$y_1 , \; y_2 \ge 0$

$\Rightarrow Z_{max} = 250 / 3$

Common Data for Questions

33. Ans: (c)

Sol: Given, $Z_{max} = 5x_1 + 10x_2 + 8x_3$

Subjected to

$3x_1 + 5x_2 + 2x_3 \le 60 \rightarrow$ material

$4x_1 + 4x_2 + 4x_3 \le 72 \rightarrow$ Machine hours

$2x_1 + 4x_2 + 5x_3 \le 100 \rightarrow$ labour hours

$x_1, x_2, x_3 \ge 0$

$3x_1 + 5x_2 + 2x_3 + s_1 = 60$

$4x_1 + 4x_2 + 4x_3 + s_2 = 73$

$2x_1 + 4x_2 + 5x_3 + s_3 = 100$

$Z_{max} = 5x_1 + 10x_2 + 8x_3 + 0s_1 + 0s_2 + 0s_3$

$C_j \rightarrow$		5	10	8	0	0	0	B_0	Min Ratio
C_B	SV	x_1	x_2	x_3	s_1	s_2	s_3		
10	x_2	$\frac{1}{3}$	1	0	$\frac{1}{3}$	$\frac{-1}{6}$	0	8	
8	x_3	$\frac{2}{3}$	0	1	$\frac{-1}{3}$	$\frac{5}{12}$	0	10	
0	S_3	$\frac{-8}{3}$	0	0	$\frac{1}{3}$	$\frac{-17}{12}$	1	18	
Z_j		$\frac{26}{3}$	10	8	$\frac{2}{3}$	$\frac{5}{3}$	0	160	
$C_j - Z_j$		$\frac{-11}{3}$	0	0	$\frac{-2}{3}$	$\frac{-5}{3}$	0		
$\frac{C_j - Z_j}{x_2}$		−11	0	0	−2	10	0	LL=2 UL=10	10−2=8 10+10 =20
$\frac{C_j - Z_j}{x_3}$		$\frac{-11}{2}$	0	0	2	−4	0	LL=4 UL=2	8−4=4 8+2=10

In $C_j - Z_j$ row all elements are negatives or zeros, hence the solution is optimal and unique..

Basic variables are:

$x_2 = 8$, $x_3 = 10$, $s_3 = 18$

i.e., production of B = 8 units, C = 10 units

18 labours hours remained unutilized

Non Basic variable

$x_1 = 0$, $s_1 = 0$, $s_2 = 0$

Resource materials and resource machine hours are fully utilized. In $(C_j - Z_j)$ row at optimality, the values under s_1, s_2 and s_3 columns represents the shadow prices.

So, If 1 kg material increases, contribution increases by $\frac{2}{3}$.

If 1 kg material decreases, contribution decreases by $\frac{2}{3}$.

If 1 kg material increases, then production B increases by $\frac{1}{3}$ and production C decreases by $\frac{1}{3}$

If m/c hr increases by 1 units, contribution increases by 5/3.

If m/c hr decreases by 1 units, contribution decreases by $\frac{5}{3}$

If m/c hr increases by 1 units, production B decreases by $\frac{1}{6}$ and production increases by $\frac{5}{12}$.

If m/c hr decreases by 1 units, production B increases by $\frac{1}{6}$ and production C decreases by $\frac{5}{12}$

If 1 unit of A produces, contribution decreases by $\frac{11}{3}$, production B decreases by $\frac{1}{3}$, production C decreases by $\frac{2}{3}$.

34. Ans: (a)

Sol: If 3 kg material increases, contribution increases by $3\times\frac{2}{3}$ = Rs. 2

35. Ans: (a)

Sol: Present profit = 160 $\Rightarrow 160 - \frac{5}{3}\times 12 = 140$/-

36. Ans: (b)

Sol: New production of B

$$= 8 - \left(12\times\frac{-1}{6}\right) = 8 + \left(12\times\frac{1}{6}\right)$$

8 + 2 = 10 units

37. Ans: (c)

Sol:
$$= 10 + \left(3\times\frac{-1}{3}\right)$$
$$= 10 - \left(3\times\frac{1}{3}\right) = 10 - 1 = 9$$

38. Ans: (a)

Sol: If 1 unit of A produces, contribution decreases by $\frac{11}{3}$

39. Ans: (a)

Sol: $160 - \left(6\times\frac{11}{3}\right) = 138$

40. Ans: (a)

Sol: Production of B , $3\times\frac{1}{3} = 1$

Production of C, $3\times\frac{2}{3} = 2$

Common data 41 & 42

41. Ans: (b) , 42.Ans: (b)

Sol: Basic variables

$x_1 = 20$, $x_2 = 10$

Non-basic variables

$s_1 = 0 \Rightarrow$ first constraint is fully consumed.

$s_2 = 0 \Rightarrow$ second constraint is fully consumed.

$x_3 = 0$ (unwanted variable)

	x_1	x_2	x_3	s_1	s_2	RHS
z-row	0	0	2	1	2	110
x_1	1	0	1	1	-1	20
x_2	0	0	0	–1	2	10

	s_1
z-row	1
x_1	1
x_2	–1

If RHS value of 1st constraint increases by 1 unit then

From the table

z increases by 1 unit, x_1 increases by 1 unit, x_2 decreases by 1 unit,

If RHS value of 2nd constraint increases by 1 unit then

	s_2
z-row	2
x_1	-1
x_2	2

From the table

z increases by 2 units, x_1 decreases by 1 unit

x_2 decreases by 2 units,

If RHS value of 1st constraint decreases by 10 units then z decreases by 10 units,

The new objective value ,

$$Z_{max} = 110 - 10 = 100$$

Chapter- 03
Inventory Control

01. Ans: (b)

Sol: $EOQ = \sqrt{\dfrac{2AS}{CI}}$

$$EOQ_1 = \sqrt{2} \times \sqrt{\dfrac{2AS}{CI}}$$

$$EOQ_1 = \sqrt{2} \times EOQ$$

02. Ans: (c)

03. Ans: (b)

Sol: A = 900 unit

S = 100 per order

CI = 2 per unit per year

$$EOQ = ELS = \sqrt{\dfrac{2AS}{CI}}$$

$$= \sqrt{\dfrac{2 \times 900 \times 100}{2}} = 300$$

04. Ans: (c) **05. Ans: (b)** **06. Ans: (d)**

07. Ans: (a)

Sol: A = 800 , S = 50/- , C_s = 2 per unit = CI

$$(TIC)_{EOQ} = \sqrt{2ASCI}$$

$$= \sqrt{2 \times 800 \times 50 \times 2} = 400$$

08. Ans:

Sol: $EOQ = \sqrt{\frac{2AS}{CI}}$

$= \sqrt{\frac{2\times5000\times16}{(0.02+0.12+0.06)\times20}}$

= 200 units

$TVC = \sqrt{2ASCI}$

$= \sqrt{2\times5000\times16\times0.2\times20}$ **= Rs. 800/-**

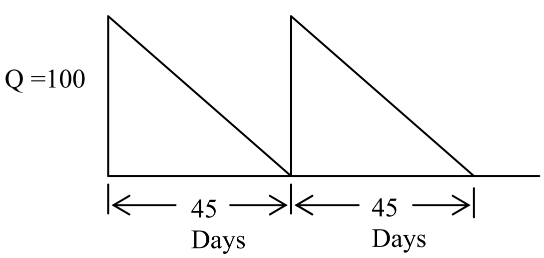
09. Ans: (c)

Sol: $\frac{EOQ_1}{EOQ_2} = \sqrt{\left(\frac{2AS}{CI}\right)_A} \times \sqrt{\left(\frac{CI}{2AS}\right)_B}$

$(EOQ)_A : (EOQ)_B = 1:4$

10. Ans: (d)

Sol: $\left(\text{No of orders} = \frac{A}{Q} = \frac{12\,\text{months}}{45\,\text{days}} = \frac{12}{1.5} = 8\right)$



$TVC = \frac{A}{Q}S + \frac{Q}{2}CI.$

$= 8\times100 + \frac{100}{2}\times120 = \text{Rs. } 6800$

11. Ans: (b)

Sol: Average inventory

$= \frac{Q}{2} = \frac{6000}{2} = 3000$ per year

$= 250$ per month

12.

Sol: Given, A = 1000 units/year, S = 40/-

I = 0.1, C = 500/-

a) $EOQ = \sqrt{\frac{2AS}{CI}} = \sqrt{\frac{2\times1000\times40}{500\times0.1}} = 40\,\text{units}$

b) No. of annual orders $= \frac{A}{Q} = \frac{1000}{40} = 25$

Frequency of ordering (or) time between orders

$= \frac{Q}{A}\times\text{time period}$

$= \frac{40}{1000}\times360 = \frac{360}{25} = 14.6$ days

c) $(TAC)_{EOQ} = AC + \sqrt{2ACSI}$

$= 1000\times500 + \sqrt{2\times1000\times500\times40\times0.1}$

$= 5{,}02{,}000/-$

Order per month $= \frac{1000}{12} = 83.33$units.

$(TAC)_Q = AC + \frac{A}{Q}.S + \frac{Q}{2}.CI$

$(TAC)_{83.38} = 1000\times500 + \frac{1000}{83.33}\times40 + \frac{83.33}{2}\times500\times0.1$

$= 5{,}02{,}563/-$

In percentage $= \frac{(TAC)_Q}{(TAC)_{EOQ}}\times100$

$= \frac{5{,}02{,}563}{5{,}02{,}000}\times100 = 100.11\%$

13. Ans: (b)

Sol: $P = 1000$, $r = 500$, $Q = 1000$

$I_{max} = \frac{1000}{1000}(1000-500) = 500$

14.

Sol: Simultaneous consumption producing Model

A = 15,000 units, C.I = 5/ units/year

S = 25/–, P = 100 units/day

No. of working days = 250/year

$$\text{Consumption rate} = r = \frac{15,000}{250}$$

$$= 60\,\text{units/day}$$

EBQ = EPQ = ELS

$$EPQ = \sqrt{\frac{2AS}{CI}\left(\frac{P}{P-r}\right)}$$

$$Q = \sqrt{\frac{2\times 15000\times 25}{5}\left(\frac{100}{100-60}\right)}$$

Q = 612.37 units

$$(TVC)_{EPQ} = \sqrt{2CSI\frac{(P-r)}{P}}$$

$$= \sqrt{15000\times 2\times 5\times 25\left(\frac{100-60}{100}\right)}$$

= 1225/-

$Q = t_p . P$

$$t_p = \frac{Q}{P} = \frac{612.37}{100} = 6.1237 \text{ days}$$

$$I_{max} = t_p(P-r) = \frac{Q}{P}.(P-r)$$

$$= 612.77\times\frac{(100-60)}{100} = 245$$

$$\text{No of production runs} = \frac{A}{Q}$$

$$= \frac{15000}{612.37} = 25$$

15.

Sol: A = 10,000 units S = 200/order

CI = 4/unit/year C = 20/-

a) $EOQ = \sqrt{\dfrac{2AS}{CI}}$

$$= \sqrt{\frac{2\times 10000\times 200}{4}} = 1000 \text{ units.}$$

Total annual cost at EOQ,

$$(TAC)_{EOQ} = AC + \sqrt{2ACSI}$$

$$= 10000(20) + \sqrt{2(10000)4(200)}$$

= 2,04,000/-

b) $(TAC)_Q = AC + \dfrac{A}{Q}S + \dfrac{Q}{2}.CI$

$$(TAC)_{Q=2000} = (10000)(20) + \frac{10000}{2000}(200) + \frac{2000}{2}(4)$$

= 2,05,000 /-

For 2000 orders to be economical the total annual cost for 2000 order with r% discount must be less than TAC at EOQ

$${}_{r\%}(TAC)2000 \le (TAC)_{EOQ}$$

$${}_{r\%}(TAC)_{2000} = (TAC)_{EOQ}$$

$$AC\left[1-\frac{r}{100}\right] + \frac{A}{Q}S + \frac{Q}{2}CI\left(1-\frac{r}{100}\right)$$

$$= (TAC)_{EOQ}$$

$$AC\left[1-\frac{r}{100}\right] + \frac{10000}{2000}\times(200) + \frac{2000}{2}4^2\left(1-\frac{r}{100}\right)$$

$$= 2,04,000$$

$$\left(1-\frac{r}{100}\right)[2,04,000] = 2,03,000$$

$$\left(1-\frac{r}{100}\right) = \frac{2,03,000}{2,04,000} - \frac{203}{204}$$

r = 0.49 %

Preferred discount rate is $r \ge 0.49\%$

16. Ans: (c)

Sol: A = 1000 units, S = Rs.100/order,

C_c = 100/unit/year, C_s = 400/unit/year

$$Q_{max} = EOQ_s \times \frac{C_S}{C_C + C_S}$$

$$= \sqrt{\frac{2\times 1000\times 100}{100}} \times \frac{400}{500} = 40\,\text{units}$$

17.

Sol: Given :

C = Rs. 5/unit , A = 4000 units

S = Rs. 30/order , CI = Rs. 1.5

$$EOQ = \sqrt{\frac{2\times 4000\times 30}{1.5}} = 400 \text{ units}$$

$$\text{No. of order per year} = \frac{4000}{400} = 10 \text{ runs}$$

$$(\text{Total yearly cost})_{EOQ} = AC + \sqrt{2ASCI}$$

$$= (4000\times 5) + \sqrt{2\times 4000\times 30\times 1.5}$$

= Rs. 20600/-

$$(TC)_{Q1@R,\%} = AC\left(1-\frac{R_1}{100}\right) + \frac{D}{Q_1}S + \frac{Q_1}{2}CI\left(1-\frac{R_1}{100}\right)$$

$$= (4000\times 5)\left(1-\frac{2}{100}\right) + \frac{4000}{1000}\times 30 + \frac{1000}{2}\times 1.5\left(1-\frac{2}{100}\right)$$

= Rs. 20455/-

$$(TC)_{Q_2@\%} = 4000\times 5\left(1-\frac{3}{100}\right) + \frac{4000}{2000}\times 30 + \frac{2000}{2}\times 1.5\times\left(1-\frac{3}{100}\right)$$

= Rs. 20915/-

Among all 2% discount for ordering quantities of 1000 or more

18.

Sol: Given:

A = 2000 units/year , S = Rs. 20/-,

I = 25%

C_u = Rs. 8/- (Lowest with unit price)

$$EOQ\,|_{C_u=8\%} = \sqrt{\frac{2\times 2000\times 20}{8\times 0.25}} = 200 \text{ units}$$

The $EOQ_{\text{at } Cu = Rs. 8/-}$ is satisfying the Quantity range hence it is declared as an optimal order quantity.

19.

Sol:

Daily sales	No. of days	Probability P_i	SL	SOR
10	15	0.15	0.15	1
11	20	0.20	0.35	0.85
12	40	0.40	0.75	0.65
13	25	0.25	1	0.25

Cus = SP – CP = 5 – 2 = 3

Cos = CP = 2

$$SL = \frac{Cus}{Cus + Cos}$$

$$= \frac{3}{3+2} = 0.6$$

SOR = 1 – SL = 1 – 0.6 = 0.4

As SL = 0.6 falling in the range 11 to 12 sales, hence order 12 for 40 days.

(Cus) = Cost of under stock

(Cos) = Cost of over stock

(SL) = Service levels

(SOR) = Stock out risk

20.

Sol: Cus = SP –CP = 2 – 0.8 = 1.2

Cos = CP – Rebate = 0.8 – 0.2 = 0.6

$$SL = \frac{Cus}{Cus + +Cos} = \frac{1.2}{1.2+0.6} = 0.6$$

For 60% – Service levels

$Q_{Optimum} = I_{min} + SL\,(I_{max} - I_{min})$

$= 20000 + 0.6(24000 - 2000)$

$= 22400$

21.

Sol: Given,

Daily demand – D. D ,

Lead Time – L.T

Re-order Level - ROL

For Item A

$$EOQ = \sqrt{\frac{2AS}{CI}}$$

$$= \sqrt{\frac{2\times 8000\times 15}{0.06}} = 2000 \text{ units}$$

R.O.L = daily demand × Lead Time

$$= \frac{8000}{250}\times 10 = 320 \text{ units}$$

For Item B

ROL = D.D × L.T

A = 9000 units

$$EOQ = \sqrt{\frac{2AS}{CI}}$$

$$= \sqrt{\frac{2\times 9000\times 40}{0.18}} = 2000 \text{ units}$$

For Item C

$$EOQ = \sqrt{\frac{2AS}{CI}}$$

$$300 = \sqrt{\frac{2\times 7500\times S}{30}}$$

S = Rs. 180/order

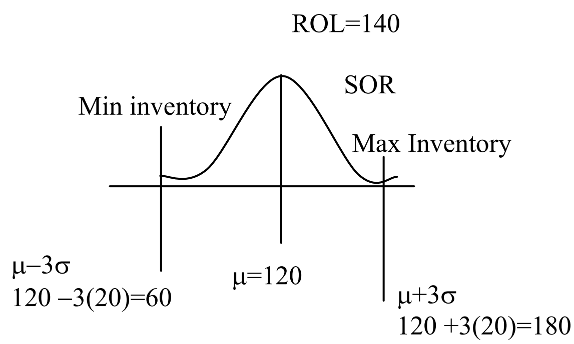
ROL = D.D × L.T

$$210 = \frac{7500}{250}\times LT$$

Lead Time = 7 days

22.

Sol:



a) SOR = 2%,

For service level (SL) = 98% to be safety factor on σ basis, $SF_\sigma = 2.05$

Safety stock (SS) = $SF_\sigma \times \sigma$

$= 2.05 \times 20 = 41$

Re-order point (ROP)

= Avg lead time demand + SS

= 120 + 41 = 161

b) Given, ROP = 140 units, SF_σ = ?

$140 = 120 + SF_\sigma\, 20$

$SF_\sigma = 1$

i.e., as SF_σ basis is 1 will achieve service levels (SL) 84.13%.

Stock out risk (SOR) = 100 – SL

($\because$ SOR + SL = 100%)

= 100 – 84.13

SOR = 15.87%

Stock out = 140 – 100 = 40 units

23.

Sol: Expected demand during lead time

$$= \frac{(80\times 0.2)+(100\times 0.25)+(120\times 0.3)+(140\times 0.25)}{0.2+0.25+0.3+0.25}$$

= 112

Reorder level = 1.25 × 112 = 140

For Re order level > Expected lead time demand

24.

Sol: $\sigma = 60$ units , $SL = \frac{51}{52} = 98\%$

(Consider 52 weeks/year)

$SS = SF_\sigma \times \sigma = 2.05 \times 60 = 123$

ROL = ALTd + SS

$= ALT \times CR + SF_\sigma \sigma$

= 500 ×1 + 123 = 623 units

Where, CR = consumption rate

ALT = Average lead time

25.

Sol: Lead Time > order cycle

$\sigma_{OC} = \sqrt{n\sigma^2} = \sqrt{6\times 5^2} = 12.21$

Safety stock (SS) = $SF_\sigma \times \sigma$

= 1.28 × 12.21 = 15.67 m ≈ 16.

($\because$ For 90% SL $\rightarrow SF_\sigma = 1.28$)

ROL = ALTd + SS = 40 + 16 = 56

26. Ans: (b) **27. Ans: (d)**

28. Ans: (d)

Sol: C – Class means these class items will have very less consumption values. – least consumption values

$B \rightarrow 300 \times 0.15 = 45$

$F \rightarrow 300 \times 0.1 = 30$

$C \rightarrow 2 \times 200 = 400$

$E \rightarrow 5 \times 0.3 = 1.5$

$J \rightarrow 5 \times 0.2 = 1.0$

$G \rightarrow 10 \times 0.05 = 0.5$

$H \rightarrow 7 \times 0.1 = 0.7$

$\therefore$ G, H items are classified as C class items because they are having least consumption values.

29.

Sol: Raking of items according to their usage values

Part code	Price per unit Rs	Units/year	Total cost (Rs)	% of total cost	Ranking
P01	100	100	10000	0.2	X
P02	200	300	60000	1.2	VI
P03	50	700	35000	0.7	IV
P04	300	400	120000	2.4	IV
P05	500	1000	500000	10	III
P06	3000	30	60000	1.2	VII
P07	1000	100	100000	2	V
P08	7000	500	3500000	70.5	I
P09	5000	105	525000	10.6	II
P10	60	1000	60000	1.2	VIII
Total			4970000	100	

ABC PLAN

RANK	Part code	% of total cost%	Cumulative percentage
I	P08	70.5	70.5
II	P09	10.6	81.1
III	P05	10	91.1
IV	P04	2.4	93.5
V	P07	2	95.5
VI	P02	1.2	96.7
VII	P06	1.2	97.9
VIII	P10	1.2	99.1
IX	P03	0.7	99.8
X	P01	0.2	100

Class A items → Nil
Class B items → I, II
Class C items → III,IV,V,VI,VII,VIII,IX,X

30. Ans: (b)

Chapter- 04
Forecasting

01. Ans: (d) **02. Ans: (d)**

03. Ans:

Sol:

(i) $F_7 = \dfrac{60+50+40}{3} = 50$

(ii) $F_7 = \dfrac{60\times0.5+50\times0.25+40\times0.25}{0.5+0.25+0.25} = 52.5$

(iii) 2 period moving average $= \dfrac{60+50}{2} = 55$

4 period moving average

$$= \frac{60+50+40+20}{4} = 42.5$$

5 period moving average

$$= \frac{60+50+40+20+30}{5} = 40$$

04. Ans: (a)

Sol: 3 period moving avg $= \dfrac{100+99+101}{3}$

$= 100$

4 period moving average

$$= \frac{102+100+99+101}{4} = 100.5$$

5 period moving average

$$= \frac{99+102+100+99+101}{5} = 100.2$$

Arithmetic mean

$$= \frac{101+99+102+100+99+101}{6} = 100.33$$

05. Ans: (a)

Sol: D_t = 100 units , F_t = 105 units

$\alpha = 0.2$

$F_{t+1} = 105 + 0.2\,(100 - 105) = 104$

06. Ans: (c)

Sol: $D_t = 105$, $F_t = 97$, $\alpha = 0.4$

$F_{t+1} = 97 + 0.4\,(105 - 97) = 100.2$

07. Ans: (c)

Sol: $F_{t+1} = F_t + a\,(X_t - F_t)$

08. Ans: (c)

09.

Sol: At , $\alpha = 0.2$

$F_{may} = 100 + 0.2\,(200 - 100) = 120$

$F_{june} = 120 + 0.2\,(50 - 120) = 106$

$F_{july} = 106 + 0.2\,(150 - 106) = 114.8$

Time	Demand	Forecast
April	200	100
May	50	120
June	150	106
July	-	114.8

$$\alpha = \frac{2}{n+1}$$

$$n + 1 = \frac{2}{\alpha} \Rightarrow n = \frac{2}{0.2} - 1 = 9\,\text{period}$$

10.

Sol: In, Jun, July, Aug, Sep demand is Stable

In Oct, Nov, Dec – demand is Fluctuating

$$F_{Jan} = \frac{327 + 339 + 355}{3} = 340.33 \text{ units.}$$

Last '3' months average is forecast for next month

The inflation start only from October hence considering last 3 months data was highly significant

Simple exponential $\alpha = 0.1$

$F_{Jan} = F_{Dec} + \alpha(D_{Dec} - F_{Dec})$

$= 307 + 0.1(355 - 307)$

$= 311.8$

11.

Sol: Simple exponential method

$\alpha = 0.2$, $D_{Jan} = 200$

$F_{Jan} = 175$, $D_{Feb} = 170$

$F_{feb} = F_{Jan} + \alpha(D_{Jan} - F_{Jan})$

$= 175 + 0.2\,(200 - 175) = 180$

$F_{march} = F_{Feb} + \alpha(D_{Feb} - F_{Feb})$

$= 180 + 0.2(170 - 180) = 178$

12.

Sol: Linear Regression model:

(x)	y (Rs)	xy	x^2
1	450	450	1
2	550	1110	4
3	625	1875	9
4	650	2600	16
5.	750	3750	25
6.	775	4650	36
Σx=21	Σy=3800	Σxy=14450	|Σx^2=91|

$y = na + bx \Rightarrow \Sigma y = na + b\Sigma x$

$xy = ax + bx^2 \Rightarrow \Sigma xy = a\Sigma x + b\Sigma x^2$

$3800 = 6a + 21b$ …….. (1)

$14425 = 21a + 91b$…….. (2)

Now, solve (1) and (2) for a, b

$a = 408.3, \quad b = 64.28$

Forecast equ. $\quad y_c = a + bx$

$y_c = 408.3+64.28x$

Forecast for month – 7,

$y_7 = 408.3 + 64.28(7) = 858.26$

Forecast For month-8

$y_8 = 408.3+64.28(8) = 952.5$

13. Ans: (a)

14. Ans: (b)

Sol:

Period	D_i	F_i	$(D_i - F_i)^2$
14	100	75	625
15	100	87.5	156.25
16.	100	93.75	39.0625
			$\Sigma(D_i-F_i)^2=820.31$

$F_{15} = F_{14} + \alpha(D_{14} - F_{14})$

$= 75 + 0.5(100 - 75) = 87.5$

$F_{16} = F_{15} + \alpha(D_{15} - F_{15})$

$= 87.5 + 0.5(100 - 87.5) = 93.75$

Mean square error (MSE) $= \dfrac{\sum(D_i - F_i)^2}{n}$

$= \dfrac{820.31}{3} = 273.13$

15. Ans: (a)

Sol:

Period	D_i	Fi	|(Di–Fi)|
1	10	9.8	0.2
2	13	12.9	0.3
3	15	15.6	0.6
4	18	18.5	0.5
5	22	21.4	0.6

$\Sigma|Di-Fi| = 2.2$

16.

Sol: Deviation $= D_i - F_i$

$$\text{MAD} = \frac{7.5+18+0+28.+12}{6}$$

$$= \frac{70}{6} = 11.66$$

$$\text{Tracking signal} = \left|\frac{\text{Cumulative deviation}}{\text{MAD}}\right|$$

$$= \left|\frac{-24}{11.66}\right| = 2.05<4$$

If tracking signal < 4 – No significant deviation in data

If tracking signal > 4 – significant deviation in data

17. Ans: (c)

18. Ans: (d)

19. Ans: (d)

Chapter- 05
Queuing Theory

01. Ans: (b)

Sol: $\lambda = 3$ per day

$\mu = 6$ per day

$$W_s = \frac{1}{\mu-\lambda} = \frac{1}{6-3} = \frac{1}{3}\text{day}$$

02. Ans: (c)

Sol: $\lambda = 0.35\ \text{min}^{-1}$, $\mu = 0.5\ \text{min}^{-1}$

$$P_n = \left[1-\frac{\lambda}{\mu}\right]\left[\frac{\lambda}{\mu}\right]^n$$

$$= \left[1-\frac{0.35}{0.5}\right]\left[\frac{0.35}{0.5}\right]^8 = 0.0173$$

03. Ans: (a)

Sol: $\lambda = 10\text{hr}^{-1}$,

$\mu = 15\ \text{hr}^{-1}$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{10^2}{15(15-10)} = 1.33$$

04. Ans: (b)

Sol: $\lambda = 4\ \text{hr}^{-1}$, $\mu = \frac{60}{12} = 5\ \text{hr}^{-1}$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{4^2}{5(5-4)} = \frac{16}{5} = 3.2$$

05. Ans: (b)

Sol: $$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{\lambda^2}{\mu^2\left(1-\frac{\lambda}{\mu}\right)} = \frac{\rho^2}{(1-\rho)}$$

06. Ans: (d)

Sol: $\lambda = \frac{1}{4} = 0.25\text{min}^{-1}$

$\mu = \frac{1}{3} = 0.33\ \text{min}^{1}$

$\rho = \frac{\lambda}{\mu} = \frac{0.25}{0.33} = 0.75$

07. Ans: (b)

Sol: $\lambda = \frac{1}{10} = 0.1\,\text{min}^{-1}$

$\mu = \frac{1}{4} = 0.25\,\text{min}^{-1}$

System busy $\Rightarrow (\rho) = \frac{\lambda}{\mu} = \frac{0.1}{0.25} = 0.4$

08. Ans: (c)

Sol: $\lambda = 4\text{hr}^{-1}, \mu = 6\ \text{hr}^{-1}$

$$P(Q_S \geq 2) = \left(\frac{\lambda}{\mu}\right)^2$$

$$= \left(\frac{4}{6}\right)^2 = \frac{4}{9}$$

09.

Sol: $\lambda = 2\ \text{hr}^{-1}$, $\mu = 5\ \text{hr}^{-1}$

a) Traffic intensity $(\rho) = \frac{\lambda}{\mu} = \frac{2}{5} = 0.4$

b) No customer $\Rightarrow$ service facility idle

$P_0 = 1 - \rho = 1 - 0.4 = 0.6$

(c) & (d)

Customer being served $\Rightarrow$ No one waiting

$$P_0 + P_1 = \left(1-\frac{\lambda}{\mu}\right) + \left(1-\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)$$

$$= \left(1-\frac{\lambda}{\mu}\right)\left(1+\frac{\lambda}{\mu}\right) = 1 - \left(\frac{\lambda}{\mu}\right)^2$$

$$= 1 - 0.16 = 0.84$$

e) $L_q = \dfrac{\lambda^2}{\mu(\mu-\lambda)}$

$$= \frac{2^2}{5(5-2)} = 0.266$$

As $\mu > \lambda \Rightarrow L_q$ is finite

If $\mu = \lambda \Rightarrow L_q$ is infinite

10.

Sol:

A	**B**
$\lambda = 3\ hr^{-1}$	$\lambda = 3\ hr^{-1}$
$\mu = 6\ hr^{-1}$	$\mu = 4\ hr^{1}$
NPC/hr = 15 Rs	NPC/hr = 15
LC/hr = 20	LC/hr = 12

L_S represents non productive machining

$L_S = \dfrac{\lambda}{\mu-\lambda}$	$L_S = \dfrac{\lambda}{\mu-\lambda}$
$= \dfrac{3}{6-3} = 1$	$= \dfrac{3}{4-3} = 3$ m/c

NPC/hr $=1\times15$Rs NPc/hr $=3\times15 = 45$ Rs

LC/hr = 20/-

"A" should be hired

Chapter- 06
Sequencing & Scheduling

01. Ans: (a)

Sol: SPT rule

Job	Process time (days)	Completion time
1	4	4
3	5	9
5	6	15
6	8	23
2	9	32
4	10	42
	ΣC_i	125

$$\text{Average Flow Time} = \frac{\sum C_i}{n}$$

$$= \frac{125}{6} = 20.83$$

02. Ans: (a)

Sol: According to SPT rule total inventory cost is minimum.

03.

Sol: SPT rule is used for minimizing mean flow time

Job	t_i	C_i	d_i	$C_i - d_i$	
4	2	2	9	-7	→ E J
2	3	5	12	-7	→ E J
1	5	10	10	--	→ OS
5	6	16	8	8	→ T J
3	8	24	20	4	→ T J

$\boxed{\sum C_i = 57}$

EJ - EARLY JOB ,
OS - ON SCHEDULE
TJ - TARDY JOB
Minimum total cost = 57 × 60 = 3,420
Number of jobs which fail to meet due date are 2.

04.

Sol: **SPT** – rule minimizes average flow time
EDD – rule minimizes mean tardiness

Job	T_i	C_i	D_i	$C_i - D_i$
5	2	2	15	-13
2	2	4	21	-17
1	3	7	17	-10
4	4	11	12	-1
6	4	15	24	-9
3	9	24	5	19
		$\sum C_i = 63$		$\sum C_i - D_i = 49$

Job	T_i	C_i	D_i	$C_i - D_i$
3	9	9	5	4
4	4	13	12	1
5	2	15	15	--
1	3	18	17	1
2	2	20	21	-1
6	4	24	24	0
		$\sum C_i = 99$		$\sum C_i - D_i = 6$

$$MFT = \frac{63}{6} = 10.5;$$

$$MT = \frac{19}{6} = 3.17$$

$$MFT = \frac{\Sigma C_i}{n} = \frac{99}{6} = 16.5$$

$$MT = \frac{\Sigma C_i - D_i}{n} = \frac{6}{6} = 1$$

T_i = Process Time
C_i = Completion Time
D_i = Due Date
MFT = Mean Flow Time
MT = Mean Tardiness

05.

Sol:

FCFS	**EDD**		**SPT**	**LPT**	**STACK**	**STACK**	
	(or)					(or)	
A	A	F	C	A	1–10=–9	A	A
B	F	A	F	B	9 – 7=2	E	F
C	E	E	E	D	7 – 2 =5	F	E
D	C	C	D	E	7 – 6 =1	D	D
E	D	D	B	F	2 – 5=–3	B	B
F	B	B	A	C	1 – 4=–3	C	C

Note:
Stack=Due Date (DD) – Processing time (P.T)

06. Ans: F-C-G-B-E-D-A

Sol: Calendar date required (CDR)
Processing time (PT)
Process time remained (PTR)

Job	CDR	PT	Critical ratio $= \frac{\text{CDR} - \text{Todays date}}{\text{PTR}}$
A	190	5	(190–175)/5=3→Ahead of schedule
B	178	2	(178–175)/2 = 1.5→Ahead of schedule

C	184	10	(184–175)10 = 0.9→ Behind schedule
D	181	3	(181–175)/3 =2 → Ahead of schedule
E	205	17	(205–175)/17 = 1.76→Ahead of schedule
F	187	15	(187–175)/15 = 0.8→ Behind schedule
G	184	9	(184–175)/9 = 1→on schedule

If critical ratio is one job will be on schedule.

If critical ratio is less than one job will be behind schedule.

If critical ratio is greater than one job will be ahead of schedule.

07.

Sol:

Job	T_j	F_j	D_j	L_j	T_j = max of (0, L_j)
a	8	8	9	-1	0
b	7	15	18	-3	0
c	9	24	21	3	3
d	12	36	38	-2	0
e	14	50	41	9	9
f	10	60	60	0	0

(i) Make-span time = 60 days

(ii) Mean flow time = $\frac{\Sigma F_y}{n} = \frac{193}{6} = 32.16$

(iii) No. of tardy jobs = 2 (c & e)

(iv) Mean tardiness, $\overline{T} = \frac{\Sigma T_j}{n} = \frac{12}{6} = 2$

08. Ans: (d)

Sol: EDD rule can minimize maximum lateness.

The job sequence is **R – P – Q – S**

09. Ans: (d)

Sol: Johnson's rule :

Optimum job sequence

III	I	IV	II

Do the job 1st if the minimum time happens to be on the machine (M) and do it on the end if .it is on second machine (N). Select either in case of a tie.

10. Ans: (b)

Sol:

Job	**M**			**N**			**Idle**
	In	**PT**	**Out**	**In**	**PT**	**Out**	
III	0	1	1	1	2	3	-
I	1	3	4	4	6	10	1
IV	4	7	11	11	5	16	1
II	11	5	16	16	2	18	-

Total idle time on machine (N) = 3

11. Ans: (a)

Sol: Optimum sequence of jobs

2	3	1	4

12. Ans: (b)

Sol: Optimum sequence is

R	T	S	Q	U	P

Job	**M_1**			**M_2**		
	In	**PT**	**Out**	**In**	**PT**	**Out**
R	0	8	8	8	13	21
T	8	11	19	21	14	35
S	19	27	46	46	20	66

Q	46	32	78	78	9	87
U	78	16	94	94	7	101
P	94	15	109	109	6	115

The optimal make-span time = 115 days

13.

Sol: Sequence by Johnson's Rule is:

6, 3, 4, 1, 2, 5

Job	**DENTER** T_{in}	T_{out}	**PAINTER** T_{in}	T_{out}
6	0	1	1	7
3	1	3	7	12
4	3	8	12	16
1	8	12	16	19
2	12	22	22	24
5	22	28	28	30

Minimum Make Span = 30

14.

Sol: Optimum Sequence :

A	C	D	B	E

PT = processing time

Job	**Machine – 1**			**Machine – 2**			**Idle Time**
	In	**PT**	**Out**	**In**	**PT**	**Out**	
A	0	2	2	2	4	6	-
C	2	5	7	7	6	13	1
D	7	6	13	13	7	20	-
B	13	7	20	20	8	28	-
E	20	5	25	28	3	31	-

Minimum time for completion of all jobs = 31

15.

Sol: *Condition :* Max (t_{2j}) ≤ Min (t_{ij} or t_{3j})

4 ≤ 4 or 4

Comp	X	M	W
N	8	3	5
A	4	4	6
O	7	3	7
L	5	4	8
E	6	4	4

Since the condition is satisfied, we can create two virtual Machines 'G' & 'H'.

X = t_{1j} , M = t_{2j} , W = t_{3j}

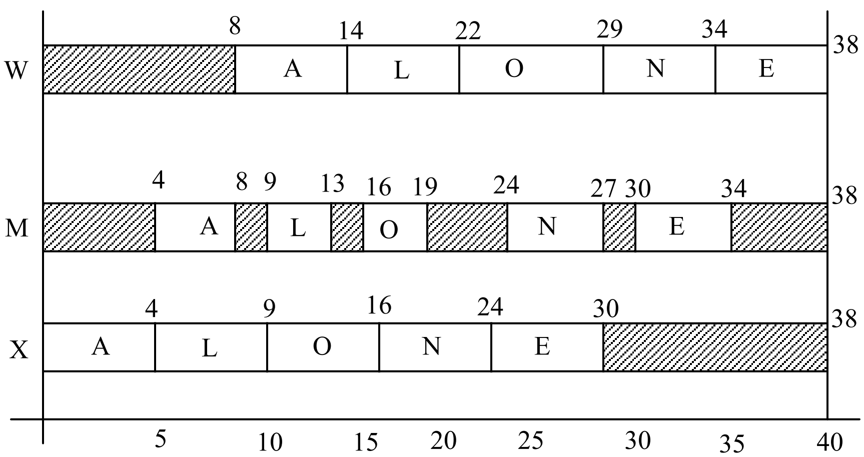
Comp	Machine G (X+M)	Machine H (M+W)
N	11	8
A	8	10
O	10	10
L	9	12
E	10	8

Optimum sequence

A	L	O	N	E

Comp	Machine X			Machine M			Idle	Machine W			Idle
	In	PT	Out	In	PT	Out		In	PT	Out	
A	0	4	4	4	4	8	4	8	6	14	8
L	4	5	9	9	4	13	1	14	8	22	-
O	9	7	16	16	3	19	3	22	7	29	-
N	16	8	24	24	3	27	5	29	5	34	-
E	24	6	30	30	4	34	3	34	4	38	-

Gantt Chart :



(iii) % utilization :

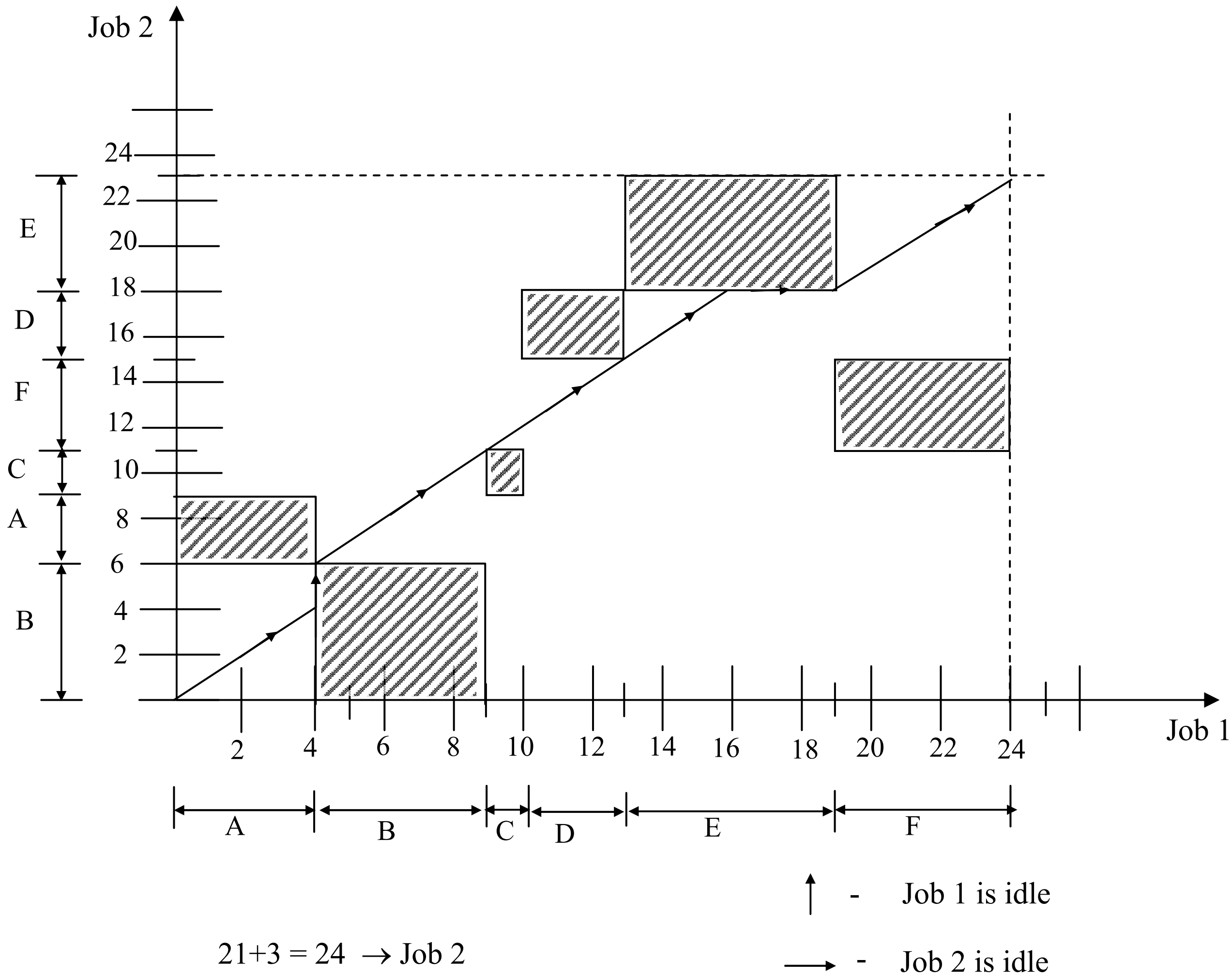
$$\text{Machine X } = \frac{30}{38} \times 100 = 78.94\%$$

$$\text{Machine m } = \frac{38-20}{38} \times 100 = 47.73\%$$

$$\text{Machine W } = \frac{38-8}{38} \times 100 = 78.94\%$$

16.

Sol:



$21+3 = 24 \rightarrow$ Job 2

$21+2 = 23 \rightarrow$ Job 1

Chapter- 7
Transportation Model

01. Ans: (c)

Sol: A no. of allocations : m + n − 1 $\Rightarrow$ 5 + 3 −1 = 7

02. Ans: (a) **03. Ans: (b)**

04.

Sol: Total supply = 80+ 60 + 40 + 20 =200

Total demand = 60 + 60 + 30 + 40 +10 = 200

$\therefore$ Total supply = Total demand

The problem is balanced

Destination / source	1	2	3	4	5	Available
A	4	3	1	2	6	80
B	5	2	3	4	5	60
C	3	5	6	3	2	40
D	2	4	4	5	3	20
Required	60	60	30	40	10	200

(i) By North West Corner rule

	1	2	3	4	5	Supply
A	$\boxed{60}$ 4	$\boxed{20}$ 3	1	2	6	80 / 20 /0
B	5	$\boxed{40}$ 2	$\boxed{20}$ 3	4	5	60 / 20 / 0
C	3	5	$\boxed{10}$ 6	$\boxed{30}$ 3	2	40 / 30 / 0
D	2	4	4	$\boxed{10}$ 5	$\boxed{10}$ 3	20 / 0
Demand	60 / 0	60 / 40 / 0	30 / 10 / 0	40 / 10 / 0	10 / 0	

Total transportation cost = 4×60 + 3×20 + 2×40 + 3×20 + 6×10 + 3×30 + 5×10 + 3×10 = 670 /-

05.

Sol: Total supply = 14 + 16 + 5 = 35

Total demand = 6 + 10 + 15 + 4 = 35

$\therefore$ Total supply = Total demand

It is a balanced transportation model

(i) By North West corner rule

	S_1	S_2	S_3	S_4	Supply
W_1	$\boxed{6}$ 15	$\boxed{8}$ 22	26	25	14 / 8 /0
W_2	36	$\boxed{2}$ 38	$\boxed{14}$ 18	40	16 / 14 / 0
W_3	45	35	$\boxed{1}$ 60	$\boxed{4}$ 52	5/ 0
Demand	6 / 0	10 / 2 / 0	15 / 1 / 0	4 / 0	35 / 35

Transportation cost = 15 × 6 + 22 × 8 + 38 ×2 + 18 × 14 + 60 × 1 + 52 × 4 = 862 /-

(ii) Least Cost Method :

	S_1	S_2	S_3	S_4	Supply
W_1	$\boxed{6}$ 15	$\boxed{8}$ 22	26	25	14 / 8 /0
W_2	36	38	$\boxed{15}$ 18	$\boxed{1}$ 40	16 / 1 / 0
W_3	45	$\boxed{2}$ 35	60	$\boxed{3}$ 52	5/ 3 / 0
Demand	6 / 0	10 / 2 / 0	15 / 0	4 / 3 / 0	35 / 35

Transportation cost = 15×6 + 22×8 + 18×15 + 40×1 + 35×2 + 52×3 = Rs. 803 /-

(iii) VAM

Step 1: Find out the difference between least and next highest numbers for rows and columns. Which is called as the penalty.

Step 2: Select the maximum penalty row and column and allocate the maximum possible amount to the box with least cost.

	S_1	S_2	S_3	S_4	Supply				
W_1	$\boxed{6}$ 15	$\boxed{5}$ 22	26	$\boxed{3}$ 25	14 / 8 / 3 / 0	7	3	3	1
W_2	36	38	$\boxed{15}$ 18	$\boxed{1}$ 40	16 / 1 / 0	18	10	10	22
W_3	45	$\boxed{5}$ 35	60	52	5 / 0	10	17	-	-
Demand	6 / 0	10 / 5 / 0	15 / 0	4 / 0	35 / 35				
	21	13	8	15					
	-	13	8	15					
	-	16	6	15					
	-	-	8	15					

Transportation cost = 15×6 + 22×5+ 25 × 3 + 18×15 + 40×1 + 35 ×5 = 760 /-

06. Ans: (a)

Sol: No. of allocations : 5

$\therefore$ no. of allocations : m + n – 1

m + n – 1 = 4 + 3 –1

$\therefore$ It is a degenerate solution

07. Ans: (a)

Sol:

	1	2	3	4	Supply
A	$\boxed{10}$	$\boxed{2}$ 5	$\boxed{20}$	$\boxed{11}$ 10	15
B	$\boxed{12}$	$\boxed{7}$ 10	$\boxed{9}$ 15	$\boxed{20}$	25
C	$\boxed{5}$ 5	$\boxed{14}$	$\boxed{16}$	$\boxed{18}$ 5	10
Demand	5	15	15	15	50 / 50

Evaluation of empty cells:

Cell (A1) Evaluation = $C_{A1}–C_{A4} + C_{C4}–C_{C1}$

=10 – 11 + 18 – 5 = 12

Cell (A3) Evaluation = $C_{A3} – C_{A2}+C_{B2} – C_{B3}$

= 20 – 9 + 7 – 2 = 16

Cell (B1) Evaluation =12–7+2–11+18–4

= 10

Cell (B4) Evaluation = 20 – 7 + 2 – 11= 4

Cell (C2) Evaluation = 14 – 2 + 11–18= 5

Cell (C3) Evaluation = 16–9 +7–2–18 = 5

If cell cost evaluation value is '–ve', indicates further unit transportation cost is decreasing and if cost evaluation value is '+ve' indicates further unit transportation cost is increases. If cost evaluation value is zero, unit transportation cost doesn't change.

∴ As for A3 cell cost evaluation is +16, means that, if we transport goods to A3 the unit transportation cost is increased by 16/-.

Common Data for Questions Q8, Q09 & Q10 :

08. Ans: (b)

09. Ans: (a)

10. Ans: (b)

Sol:

	1	2	3	4
A	[6]	[1]	[9] 25	[3] 45
B	[11] 30	[5]	[2] 25	[8]
C	[10] 55	[12] 35	[4]	[7]

No. of allocations = 6

R + C – 1 = 6

As No. of allocations = R + C – 1

Hence the problem is not degeneracy case.

Opportunity cost of cell (i, j) is

$C_{ij} - (U_i + V_j)$

If $C_{ij} - (U_i + V_j) \geq 0 \Rightarrow$ problem is optimal,

Empty cell evaluation (or) Opportunity cost of cells:

$A_1 = -12$, $A_2 = -19$, $B_2 = -8$

$B_4 = 12$, $C_3 = 3$, $C_4 = 12$

From the above as A2 has opportunity cost '–19' indicates unit transportation cost is decreased by 19/-

By forming loop A2, A3, B2, B3 it is observed that to transport minimum quantity is 25 among 25, 30, 35.

∴ The reduction in the transportation cost is $25 \times 19 = 475$

11. Ans: (c)

12. Ans: (c)

Chapter- 8
Assignment Model

01. Ans: (a)

02. Ans: (a)

03.

Sol: Step-1:

Take the row minimum of substract it from all elements of corresponding row

1	0	2	3
0	2	2	1
8	5	0	1
0	6	2	4

Step – 2 :

Take the column minimum & substract it from all elements of corresponding column.

1	0	2	2
0	2	2	0
8	5	0	0
0	6	2	3

Step – 3 :

Select single zero row or column and assign at the all where zero exists. If there is no single zero row or column. Then use straight line method.

	A	B	C	D
1	1	$\boxed{0}$	2	2
2	0	2	2	$\boxed{0}$
3	8	5	$\boxed{0}$	0
4	$\boxed{0}$	6	2	3

1 – B : 7

2 – D : 8

3 – C : 2

4 – A : 5

Total cost = 22

04.

Sol: Assignment problem is the special case of the transportation problem

	A	B	C	D
1	10	8	10	8
2	10	7	9	10
3	11	9	8	7
4	12	14	13	10

Step (1): Select the small element in a row and subtract it from all other numbers in that row.

	A	B	C	D
1	2	0	2	0
2	3	0	2	3
3	4	2	1	0
4	2	4	3	0

Step (2): Now in columns, subtract the small number from all other elements in that column.

	A	B	C	D
1	0	0	1	0
2	1	0	1	3
3	2	2	0	0
4	0	4	2	0

Step (3): Now select the single zero cell in a row if possible and assign that cell and cross off other zero corresponding to that cell's row and column. Here (2,B) with single zero.

	A	B	C	D
1	0	$\cancel{0}$	1	0
2	1	$\boxed{0}$	1	3
3	2	2	0	0
4	0	4	2	0

Step (4): Now in the third row, if we select (3,B) then it would not be possible to get assignment in the 3rd column . So assign (3,C) cell.

	A	B	C	D
1	0	$\cancel{0}$	1	0
2	1	$\boxed{0}$	1	3
3	2	2	$\boxed{0}$	$\cancel{0}$
4	0	4	2	0

Step (5): In the remaining assignment, if we assign (1, A) then other assignment would be (4, D). If the assignment is (1, D) then other assignment would be (4, A) and in the both cases, total cost is same.

So assign (1,A) and (4,D)

	A	B	C	D
1	$\boxed{0}$	$\cancel{0}$	1	$\cancel{0}$
2	1	$\boxed{0}$	1	3
3	2	2	$\boxed{0}$	$\cancel{0}$
4	$\cancel{0}$	4	2	$\boxed{0}$

Assignment is 1-A, 2-B, 3-C, and 4-D
Optimal cost = 10 + 7 + 8 + 10 = 35 Euros.

05. Ans: (c)

Sol:

	S_1	S_2	S_3		S_1	S_2	S_3	
P	110	120	130		$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	Column Transaction
Q	115	140	140		0	15	5	
R	125	145	165		0	10	20	
P	0	10	20		$\cancel{5}$	$\cancel{0}$	$\cancel{0}$	
Q	0	25	25	Row Transaction	$\cancel{0}$	$\cancel{10}$	$\cancel{0}$	
R	0	20	40		$\cancel{0}$	$\cancel{5}$	$\cancel{15}$	

$P\text{-}S_2 - 120$

$Q\text{-}S_3 - 140$

$R\text{-}S_1 - 125$

Total = 385

06.

Sol:

	A	B	C	D
1	10	$\boxed{5}$	15	13
2	3	9	8	$\boxed{18}$
3	10	7	$\boxed{2}$	3
4	$\boxed{5}$	11	7	9

Step – 1 :

5	0	10	8
0	6	5	15
8	5	0	1
0	6	2	4

Step – 2 :

5	0	10	7
0	6	5	14
8	5	0	0
0	6	2	3

Step – 3

$$\begin{array}{|cccc|} \hline 5 & \boxed{0} & 10 & 7 \\ \boxed{0} & 6 & 5 & 14 \\ 8 & 5 & \boxed{0} & 0 \\ 0 & 6 & 2 & 3 \\ \hline \end{array}$$

It may be noted there are no remaining zeroes and row – 4 and column – 4 each has no assignment. Thus optimal solution is not reached at this stage. Therefore, proceed to following important steps.

Step – 4 :

Draw the minimum number of horizontal and vertical lines necessary to cover all zeroes at least once.

Take the above Table

$$\begin{array}{c|cccc|c} & J_1 & J_2 & J_3 & J_4 & \\ \hline C_1 & 5 & 0 & 10 & 7 & L_2 \\ C_2 & 0 & 6 & 5 & 14 & \\ C_3 & 8 & 5 & 0 & 0 & L_3 \\ C_4 & 0 & 6 & 2 & 3 & \\ \hline & L_1 & & & & \end{array}$$

(i) Mark row – 4 in which there is no assignment

(ii) Mark column 1 which have zeroes in marked column.

(iii) Next mark row 2 because this row contains assignment in marked column 1.

No further rows or columns will be required to mark during this procedure.

(iv) Draw the required lines as follows.

(a) Draw L_1 through marked column 1

(b) Draw L_2 and L_3 through unmarked row (1 and 3)

Step – 5 :

Select the smallest element (2).

Among all the uncovered elements of the above table and substract this value from all the elements of the matrix not covered by lines and add to every element that lie at the intersection of the lines L_1, L_2,and L_3 and leaving the remaining element unchange.

$$\begin{array}{c|cccc|} & J_1 & J_2 & J_3 & J_4 \\ \hline C_1 & 7 & \boxed{0} & 10 & 7 \\ C_2 & \boxed{0} & 4 & 3 & 12 \\ C_3 & 10 & 5 & 0 & \boxed{0} \\ C_4 & 0 & 4 & \boxed{0} & 5 \\ \hline \end{array}$$

It may be added that there are no remaining zeroes and every row and column has an assignment.

Since, the no. of assignment = no. of row or column

$\therefore$ The solution is optimal

The pattern of assignment at which job has been assigned to each contractor.

Contractor	**Job**	**Amount (Rs)×1000**
C_1	J_2	5
C_2	J_1	3
C_3	J_4	3
C_4	J_3	7
		18×1000=18000

Minimum amount = Rs. 18,000/-

07.
Sol:

	Job 1	Job 2	Job 3	Job 4	
A	20	36	31	27	
B	24	34	45	22	
C	22	45	38	18	
D	37	40	35	28	
A	0	16	11	7	Row Transaction
B	2	12	23	0	
C	4	27	20	0	
D	9	12	7	0	
A	$\boxed{0}$	4	4	7	Column Transaction
B	2	$\boxed{0}$	16	0	
C	4	15	13	$\boxed{0}$	
D	9	0	$\boxed{0}$	0	
	$A - J_1 \rightarrow 20$				
	$B - J_2 \rightarrow 34$				
	$C - J_4 \rightarrow 18$				
	$D - J_3 \rightarrow 35$				
	107				

08.
Sol: Here no. of rows ≠ no. of column

∴ The algorithm is not balanced so add one dummy column.

Operates	Machine			
	A	B	C	Dummy
1	9	26	15	0
2	13	27	6	0
3	35	20	15	0
4	18	30	20	0

Step – 1:

9	26	15	0
13	27	6	0
35	20	15	0
18	30	20	0

Step – 2:

$\boxed{0}$	6	9	0
4	7	$\boxed{0}$	0
26	$\boxed{0}$	9	0
9	10	14	$\boxed{0}$

Here the operator – 4 is assigned to dummy column.

∴ He is the idle worker.

09.
Sol :

Step 1: Take the row minimum and subtract it from its corresponding row's elements

	I	**II**	**III**	**IV**	**V**	**Row Min.**
A	11	17	8	16	20	8
B	9	7	12	6	15	6
C	13	16	15	12	16	12
D	21	24	17	28	26	17
E	14	10	12	11	15	10

	I	II	III	IV	V
A	11-8	17-8	8-8	16-8	20-8
B	9-6	7-6	12-6	6-6	15-6
C	13-12	16-12	15-12	12-12	16-12
D	21-17	24-17	17-17	28-28	26-26
E	14-10	10-10	12-10	11-10	15-10

Step 2: Take the column minimum and subtract it from its corresponding row's elements

	I	II	III	IV	V
A	3	9	0	8	12
B	3	1	6	0	9
C	1	4	3	0	4
D	4	7	0	0	0
E	4	0	2	1	5
Column min	1	0	0	0	0

	I	II	III	IV	V
A	3-1	9-0	0-0	8-0	12-0
B	3-1	1-0	6-0	0-0	9-0
C	1-1	4-0	3-0	0-0	4-0
D	4-1	7-0	0-0	0-0	0-0
E	4-1	0-0	2-0	1-0	5-0

Step 3 : After row and Column reduction

	I	II	III	IV	V
A	2	9	0	8	8
B	2	1	6	0	5
C	0	4	3	0	0
D	3	7	0	11	5
E	3	0	2	1	1

Step 4: Allocation marking and optimally check.

	I	II	III	IV	V	
A	2	9	$\boxed{0}$	8	8	✓
B	2	1	6	$\boxed{0}$	5	
C	$\boxed{0}$	4	3	$\cancel{0}$	$\cancel{0}$	
D	3	7	$\cancel{0}$	11	5	✓
E	3	$\boxed{0}$	2	1	1	
	✓		✓			

Step 5: Revising and improving

	I	II	III	IV	V
A	$\boxed{0}$	7	$\cancel{0}$	6	6
B	2	1	8	$\boxed{0}$	5
C	$\cancel{0}$	4	5	$\cancel{0}$	$\boxed{0}$
D	1	5	$\boxed{0}$	9	3
E	3	$\boxed{0}$	4	1	1

$A \to 1$, $B \to 4$, $C \to 5$, $D \to 3$, $E \to 2$;

Total min cost = 60

Chapter- 9
PPC & Aggregate Planning

01. Ans: (d) **02. Ans: (b)**

03. Ans: (b)

Sol:

Months		Month 1	Month 2	Month 3	Unused capacity	Capacity Available
1	RT	$\boxed{20}$ 90 $\boxed{24}$	$\boxed{20}$ 10 $\boxed{26}$	$\boxed{24}$ $\boxed{28}$	10	100
	OT	(1)	$\boxed{20}$	$\boxed{22}$		20
2	RT		100 $\boxed{24}$	$\boxed{36}$		100
	OT		20	$\boxed{20}$		20
3	RT			$\boxed{24}$ 80		80
	OT			30	10	40
	RT	90	130	110		
	OT					

Level of planned production in overtimes in 3rd period is ‘30’.

RT = Regular time

OT = Over time

04. Ans: (b)

Sol:

Month	Cumulative Production	Cumulative Demand	Inventory		Cost	
			End	Stock out	End inventory	Stock out cost
1	100	80	20	-	40	-
2	180	180	-	-	-	-
3	250	260	-	10	-	100
4	320	300	20	-	40	-
					80	100
				Total	180	

05. Ans: (b) **06. Ans: (d)**

07.

Sol:

Supply from		Demand for: Period 1	Period 2	Period 3	Period 4	Un used capacity	Total Capacity Available (supply)
Beginning inventory		^{200} 0	5	10	15	-	200
1	Regular	^{700} 60	65	70	75	0	700
	Overtime	70	75	80	85	300	300
2	Regular		^{500} 60	65	^{200} 70	0	700
	Overtime		70	75	80	300	300
3	Regular			^{200} 60	^{500} 65	0	700
	Overtime			70	^{200} 75	100	300
4	Regular				^{700} 60	0	700
	Overtime				^{300} 70	0	300
		900	500	200	1900	700	4200 / 4200

Total cost = (700 ×60) + (500 ×60) + (200 ×70) + (200 × 60) + (500 ×65) + (200 ×75) + (700 ×60)+ (300 ×70) = Rs 2,08,500/-

08.
Sol:

	Demand for Total Supply from	Period1	Period2	Period3	Period4	Unused capacity	Capacity Available (supply)
	Beginning Inventory	**150** 0	2	4	6	-	150
1	Regular	**900** 25	27	29	31	-	900
	Overtime	**150** 30	32	34	36	-	150
	Subcontract	**200** 35	-	-	-	**100** -	300
2	Regular		**600** 25	27	29	-	600
	Overtime		**125** 30	32	34	-	125
	Subcontract		**175** 35	-	-	**125** -	300
3	Regular			**700** 25	27	-	700
	Overtime			**100** 30	**50** 32	-	150
	Subcontract			35	-	**300** -	300
4	Regular				**800** 25	-	800
	Overtime				**200** 30	-	200
	Subcontract				**250** 35	**50** -	300
		1400	900	800	1200+100	575	4975 / 4975

Total cost = $(900 \times 25) + (150 \times 30) + (200 \times 35) + (600 \times 25) + (125 \times 30) + (175 \times 35) + (700 \times 25) + (100 \times 30) + (50 \times 32) + (800 \times 25) + (200 \times 30) + (250 \times 35)$

= Rs 1,15,725/-

Chapter- 10
Material Requirement & Planning

01. Ans: (b) **02. Ans: (c)** **03. Ans: (d)** **04. Ans: (c)**

05. Ans: (c) **06. Ans: (c)** **07. Ans: (b)** **08. Ans: (b)**

09.

Sol: $A \rightarrow 1 \times 10 = 10$

$B \rightarrow 2 \times 10 = 20$

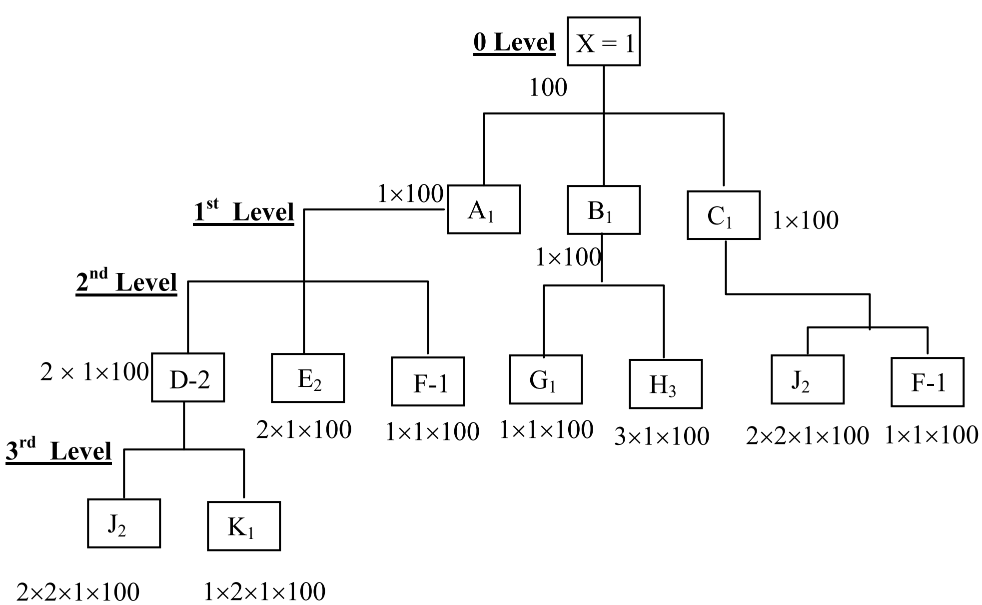
$C \rightarrow (1 \times 2 \times 10) + (3 \times 4 \times 2 \times 10) = 260$

$D \rightarrow (4 \times 2 \times 10) = 80$

$E \rightarrow (3 \times 4 \times 2 \times 10) + (2 \times 2 \times 10) + (4 \times 10) = 320$

10.

Sol:



11.

Sol:

Order Quantity = 200 LT = 3 Weeks	Week							
	1	2	3	4	5	6	7	8
Project required	40	85	10	60	130	110	50	170
Receipts				200		200		200
On hand inventory	100	15	5	145	15	105	55	85
Planned order release	200		200		200			

(On hand inventory)$_t$

1st week = 140+0-40 = 100

2nd week = 100+0-85 = 15

3rd week = 15 + 0 – 10 = 5

4th week = 5 + 200 – 60 = 145

5th week = 145 + 0 – 130 = 15

6th week = 15 + 200 – 110 = 105

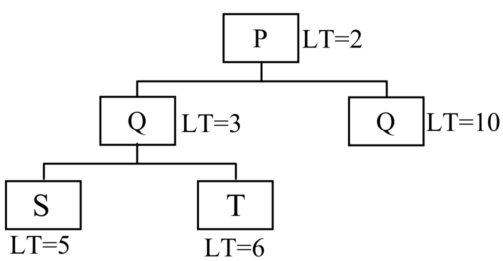
7th week = 105 + 0 – 50 = 55

8th week = 55 + 200 – 170 = 85

$\therefore$ Order before 3-weeks

12. Ans: (c)

Sol:



Maximum Lead time = 12 weeks

13.

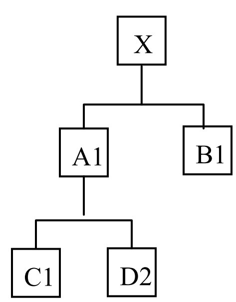
Sol:

Net required A = (1 × 1 × 20 –10) = 10

C = (1 × 1 × 20–1×10 –10) = 0

B = 1 × 20 –1×5 = 15

D = 2 × 1 × 20 – 2 × 10 –10 = 10



Chapter- 11
Break Even Analysis

01. Ans: (c)

Sol: Total fixed cost, TFC = Rs 5000/-

Sales price, SP = Rs 30/-

Variable cost, VC = Rs 20/-

Break even production per month,

$$Q^* = \frac{TFC}{SP - VC} = \frac{5000}{30 - 20} = 500 \text{ units}$$

02. Ans: (a)

Sol: Total cost = 20 + 3

2X = 30

$\therefore$ X = 15 units

When X = 10 units

$TC_1 = 20 + (3 \times 10) = \text{Rs } 50/-$

$TC_2 = 50 + (1 \times 10) = \text{Rs } 60/-$

Among both, total cost for process is less

So process-1 is choose.

03. Ans: (c)

Sol: In automated assembly there are less labour, so variable cost is less, but fixed is more because machine usage is more. In job shop production, labour is more but machine is less. So variable cost is more and fixed cost is less.

04. Ans: (c)

Sol: TC = Total cost

TC_A = Total cost for jig-A

TC_B = Total for jig-B

$TC_A = TC_B$

$800 + 0.1X = 1200 + 0.08X$

$0.02X = 400$

$$\therefore X = \frac{400}{0.02} = \frac{400}{2} \times 100 = 20{,}000 \text{ units}$$

05. Ans: (d)

Sol: Sales price – Total cost = Profit

$(C_P \times 14000) - (47000 + 14000 \times 15) = 23000$

$\therefore C_P = 20$

06. Ans: (b)

07. Ans: (a)

08. Ans: (c)

09.

Sol:

X	**Y**
$S_1 = 100$	$S_2 = 120$
$F_1 = 20{,}000$	$F_2 = 8000$
$V_1 = 12$	$V_2 = 40$

$P = q(S - V) - F$

$P_1 = q(100 - 12) - 20{,}000$

$P_2 = q(120 - 40) - 80,000$

$P_1 = P_2$

$88q - 20,000 = 80q - 80,000$

$12000 = 8q$

$\Rightarrow q = 1500$

10. Ans:

Sol: Preparation cost for

Conventional lathe = 30,

CNC lathe = 150

Production time of

Conventional lathe = 30 min,

Variable cost per hour

Conventional lathe = 75 per hour

$$= \frac{75}{60} \times 30 \text{ per product}$$

CNC lathe = 120 per hour

$$= \frac{120}{60} \times 15 \text{ per product}$$

Total cost for Q products

Conventional lathe = 30 + 37.5 Q

CNC lathe = 150 + 30 Q

At break even quantities

$(TC)_1 = (TC)_2$

$\Rightarrow 30 + 37.5\ Q = 150 + 30\ Q$

$\Rightarrow 7.5\ Q = 120$

$\Rightarrow \quad Q = 16$

$\therefore$ CNC lathe is economical when production per day is above 16.

11. Ans: (b)

12. Ans: (c)

13. Ans: (d)

Sol:

	Standard machine tool	Automatic machine tool
F_1 = F.C.	$\frac{30}{60} \times 200 = \text{Rs.}100$	$2 \times 800 = \text{Rs.}1600 = F_2$
V.C	$= \frac{20}{60} \times 200$ = Rs. 73.33	$= \frac{5}{60} \times 800$ = Rs. 66.67

$$q = \frac{1600 - 100}{73.33 - 66.67} = 225 \text{ volts}$$

If greater than 225 units then automatic machine tool is economic.