MECHANICAL ENGINEERING

GATE | PSUs

HEAT TRANSFER

Volume - I : Study Material with Classroom Practice Questions

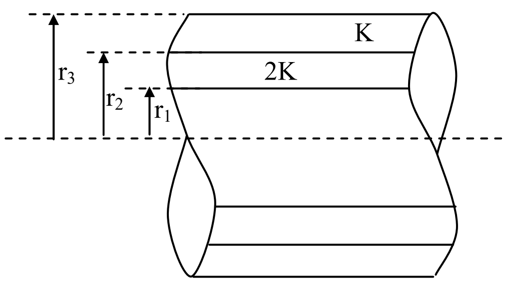
Heat Transfer

Solutions for Vol – I _ Classroom Practice Questions

Chapter- 1
Conduction

01. **Ans: (b)**

Sol: Case (1): Higher thermal conductive material is inside and low thermal conductive material is outside



$$Q_1 = \frac{2\pi L\Delta T}{\dfrac{\ln\frac{r_2}{r_1}}{K_1} + \dfrac{\ln\frac{r_3}{r_2}}{K_2}}$$

Let, $r_1 = 10$ mm,

$r_2 = 20$ mm,

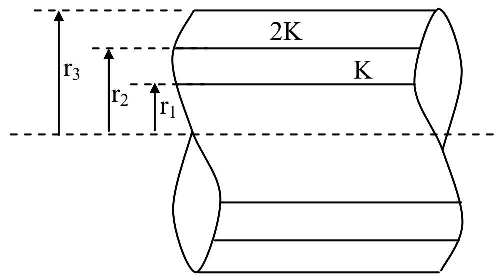
$r_3 = 30$ mm

$$Q_1 = \frac{2\pi L\Delta T}{\dfrac{\ln\left(\frac{20}{10}\right)}{2K} + \dfrac{\ln\frac{30}{20}}{K}}$$

$Q_1 = 1.32(2\pi KL\Delta T)$ …….. (1)

Now,

Case (2): Higher thermal conductivity material outside, lower thermal conductivity material is inside.



$$Q_2 = \frac{2\pi L\Delta T}{\dfrac{\ln\left(\frac{20}{10}\right)}{K} + \dfrac{\ln\frac{30}{20}}{2K}}$$

$Q_2 = 1.116(2\pi KL(\Delta T))$ ……… (2)

From (1) and (2)

$$Q_1 > Q_2$$

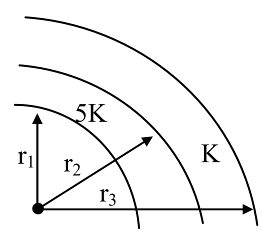
Means that, lower thermal conductivity material should be used for inner layer and higher thermal conductivity material should be used for outer layer so that heat transfer will be lower $\Rightarrow$ insulation is effective.

02. Ans: (a)

Sol: $r_1 = \frac{30}{2} = 15\text{mm}$

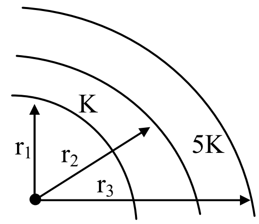
$r_2 = 15+25 = 40$ mm

$r_3 = 40 + 25 = 65$ mm



$$Q_1 = \frac{\Delta T}{\dfrac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi KL} + \dfrac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi KL}}$$

$$= \frac{\Delta T(2\pi K)}{\dfrac{\ln\left(\frac{40}{15}\right)}{5} + \dfrac{\ln\left(\frac{65}{40}\right)}{1}} = 1.466\Delta T(2\pi K)$$



$$Q_2 = \frac{2\pi K(\Delta T)}{\frac{\ln\left(\frac{40}{15}\right)}{1} + \frac{\ln\left(\frac{65}{40}\right)}{5}} = 0.927.\Delta T(2\pi K)$$

% decrease in heat transfer

$$= \frac{1.466 - 0.927}{1.466} \times 100 = 36\%$$

03. **Ans: (c)**

Sol: $H = 4m$, $L = 10m$, $\delta = 0.115$ m

$T_i = 30°C$, $T_0 = 10°C$, $K = 1.15$ W/mK

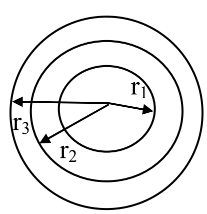
$h_i = 2.5 W/m^2K$, $h_0 = 4W/m^2K$

$$Q = \frac{(30-10)\times A}{\frac{1}{h_i} + \frac{\delta}{k} + \frac{1}{h_0}} \times 3600$$

$$= \frac{(20\times 40)\times 3600}{\frac{1}{2.5} + \frac{0.115}{1.15} + \frac{1}{4}} = 3840\,\text{kJ}$$

04. **Ans: (d)**

Sol: $\frac{K_1}{K_2} = \frac{1}{2}, \frac{r_1}{r_3} = 0.8$



$r_2 - r_1 = r_3 - r_2$

Due to steady state H.T.

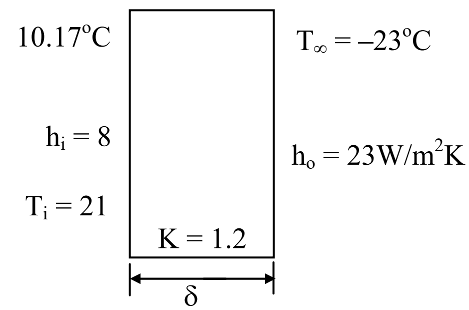
$Q_1 = Q_2$

$$\frac{4\pi K_1 r_1 r_2(\Delta T_1)}{(r_2 - r_1)} = \frac{4\pi K_2 r_2 r_3(\Delta T_2)}{(r_3 - r_2)}$$

$$\frac{\Delta T_1}{\Delta T_2} = \frac{K_2 r_3}{K_1 r_1} = \frac{2}{0.8} = 2.5$$

05. **Ans: (b)**

Sol: To avoid condensation in the building, the inside wall temp should be greater than or equal to DPT



H.T from inside air to inside wall = H.T from inside wall surface to outside air

$$\frac{21-10.17}{\frac{1}{8A}} = \frac{10.17-(-23)}{\frac{\delta}{1.2A} + \frac{1}{23A}}$$

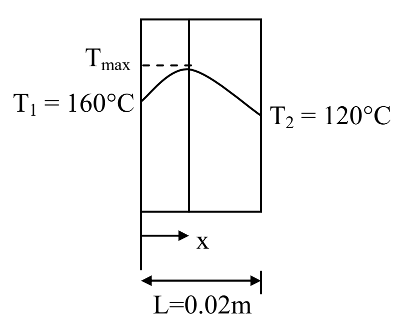
$$\frac{\delta}{1.2} + \frac{1}{23} = \frac{33.17}{86.64}$$

$$86.64 = \frac{33.17}{\frac{\delta}{1.2} + \frac{1}{23}} \Rightarrow \delta = 0.407\,\text{m}$$

Common Data for Q. 06 and Q. 07

06.

Sol:



$Q_G = 80\ MW/m^3 = 80 \times 10^6\ W/m^3$

$T_1 = 160°C$, $T_2 = 120°C$

$k = 200\ W/m^2K$

For 1-D steady state with heat generation equation is

$$\frac{d^2T}{dx^2} + \frac{Q_G}{k} = 0$$

$$\frac{d^2T}{dx^2} = -\frac{Q_G}{k} \Rightarrow \frac{dT}{dx} = -\frac{Q_G}{k}x + C_1$$

$$T = \frac{-Q_G}{2k}x^2 + C_1x + C_2 \text{ ------ (i)}$$

Where, C_1 & C_2 are constants that can be evaluated by boundary conditions.

At, $x = 0$, $T = 160°C$

$160 = C_2$

At $x = L = 0.02$ m,

$T = 120°C$,

$$120 = -\frac{80 \times 10^6}{2 \times 200}(0.02)^2 + C_1 \times 0.02 + 160$$

$C_1 = 2000$

$$T = \frac{-Q_G}{2k}x^2 + 2000x + 160 \text{ ------ (ii)}$$

To get maximum temperature

$$\frac{dT}{dx} = 0$$

$$\Rightarrow -\frac{Q_G}{2k} \times 2x + 2000 = 0$$

$$\Rightarrow x = \frac{2000 \times k}{Q_G} = \frac{2000 \times 200}{80 \times 10^6}$$

$$= 5 \times 10^{-3}\ m = 5\ mm$$

07.

Sol:

By putting the value of $x = 5\times10^{-3}$ m in eq.(ii)

$$T = -\frac{80 \times 10^6}{2 \times 200} \times \left(5 \times 10^{-3}\right)^2 + 2000 \times 5 \times 10^{-3} + 160$$

$= 165°C$

08. **Ans (c)**

Sol: $Q = \dfrac{(\Delta T)_{wall}}{\text{wall Resistance}}$

$$= \frac{130 - 30}{0.002}$$

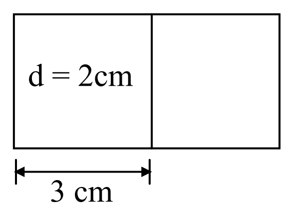
$= 50000W = 50\ kW$

09. **Ans: (a)**

Sol: $\Delta T = 100$,

$K = 20\ W/mK$,

$R_C = 1.016\ K/W$



$$Q = \frac{\Delta T}{R_1 + R_C + R_2}$$

$$= \frac{100}{\left(\dfrac{0.03}{20 \times \dfrac{\pi}{4} \times 0.02^2} \times 2\right) + 1.016} = 9.42W$$

10. **Ans: (b)**

Sol: $Q = \dfrac{T_i - T_0}{\dfrac{r_2 - r_1}{4\pi K r_1 r_2}}$

$$T_{i \text{ at radius } 12.5\text{ cm}} = Q \times \frac{r_0 - r_i}{4\pi K r_i r_0} + T_0$$

$$= 30 \times 10^3 \times \frac{0.15 - 0.125}{4\pi \times 70 \times 0.15 \times 0.125} + 40^0$$

$$= 45.47 + 40^0 = 85.47°$$

11. **Ans: (d)**

Sol: Critical radius of cylinder, $r_c = \dfrac{K}{h} = \dfrac{0.5}{20}$

$= 0.025$ m $= 25$ mm

Critical thickness = 25 – 10 = 15 mm

12. **Ans: (a)**

Sol: Critical radius, $r_c = \dfrac{k}{h} = \dfrac{0.1}{10}$

$= 10$ mm $= 1$ cm

For cable as r_1(=1.5 cm) > r_1(=1 cm), the heat transfer decreases by adding insulation.

13. **Ans: (c)**

Sol: $r_1 = 1$ mm ,

$$r_c\ (= r_2) = \frac{K}{h} = \frac{0.175}{125} = 1.4\times10^{-3} \text{ m}$$

$r_c = 1.4$ mm ,

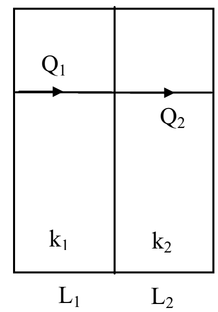
As r_1(= 1 mm) < r_c(= 1.4 mm)

The heat transfer increases by adding insulation till $r_2 = r_c$ and then it decreases.

14. **Ans: (c)**

Sol: $Q_1 = Q_2 = Q$

$R_{eq} = R_1 + R_2$



$$\frac{2L}{kA} = \frac{L_1}{k_1A} + \frac{L_2}{k_2A}$$

$$\frac{2L}{k} = \frac{L}{k_1} + \frac{L}{k_2}$$

$$\frac{2}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$= \frac{k_1 + k_2}{k_1k_2} \Rightarrow k = \frac{2k_1k_2}{k_1 + k_2}$$

15. **Ans: (b)**

Sol: $r_2 = 5$ mm, $h_0 = 10$ W/m^{2}K, $k_i = 0.04$ W/mK

Max. heat flow occurs at critical radius or diameter

$$d_c = 2\ r_c = 2 \times \frac{2k_i}{h_0}$$

$$= \frac{4k_i}{h_0} = \frac{4 \times 0.04}{10} = \frac{0.16}{10} = 16 \text{ mm}$$

Chapter- 2 Convection

01. **Ans: (b)**

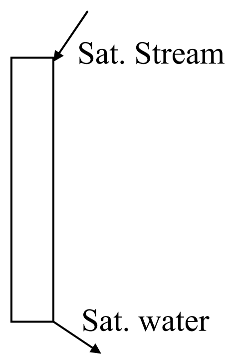
02. **Ans: (a)**

03. **Ans: (a)**

Sol: $C_f = 0.004$, $\rho = 0.88$

$\mu = 2.286 \times 10^{-5}$, $V = 50$ m/sec

$C_p = 1001$ J/kgK



$$P_r = \frac{\mu C_p}{k}$$

$$St \cdot Pr^{\frac{2}{3}} = \frac{C_f}{2}$$

$$\frac{h}{\rho V C_P} = \frac{C_f}{2 \times P^{\frac{2}{3}}}$$

$$h = \frac{C_f \times \rho V C_P}{2 \times P_r^{\frac{2}{3}}}$$

$$= \frac{0.004 \times 0.88 \times 50 \times 1001}{2 \times \left(\frac{2.286 \times 10^5 \times 1001}{0.035}\right)^{\frac{2}{3}}} = 117 \text{ W/m}^2\text{K}$$

04. Ans: (b)

Sol: $L_1 = 320$ cm, $T_p = 150°C$,

$T_\infty = 10^0C$, $Q_1 = 8$ kW

$Q = h.A\Delta T$

$$\text{Grashoff number (Gr)} = \frac{g\beta\Delta T L^3}{\nu^2}$$

$Gr \propto L^3$ (keeping others constant)

Nusselt number (Nu)

$= C\,(Gr.\,Pr)^{1/4} \rightarrow$ for laminar flow

$Nu \propto (Gr)^{1/4}$

$Nu \propto (L^3)^{1/4}$

$$\Rightarrow \frac{hL}{k} \propto L^{3/4} \Rightarrow h \propto L^{-1/4}$$

$Q = h\,A\,\Delta T$

$Q \propto h\,A$

$Q \propto h \times \pi dL$

$Q \propto h \times L$

$Q \propto L^{-1/4} \times L$

$Q \propto L^{3/4}$

$$\frac{Q_1}{Q_2} = \left(\frac{L_1}{L_2}\right)^{3/4} \Rightarrow \frac{8}{1} = \left(\frac{320}{L_2}\right)^{3/4}$$

$$\frac{320}{L_2} = (8)^{4/3} = 16 \Rightarrow L_2 = 20 \text{ cm}$$

05. **Ans: (c)**

06. **Ans: (b)**

Sol: δ = thickness of hydrodynamic boundary layer = 0.5

$$Pr = \frac{\mu C_p}{k} = 1 \Rightarrow \delta = \delta_t = 0.5\text{mm}$$

07. **Ans: (a)**

08. **Ans: (d)**

Sol: $Q_t = Q_{top} + Q_{bottom} + 4 \times Q_{side}$

$6 \times hA\,\Delta T = h_1A\,\Delta T + h_2A\,\Delta T + 4h_3A\,\Delta T$

$6h = h_1 + h_2 + 4h_3$

$$h = \frac{h_1 + h_2 + 4h_3}{6}$$

09. **Ans: (b)**

Sol: $\dfrac{T - T_w}{T_\infty - T_w} = 1 - e^{-3500y}$

At No slip region, $\dot{Q}_{cond} = \dot{Q}_{convection}$

$$-kA\left(\frac{dT}{dy}\right)_{y=0} = \bar{h} \times A(T_w - T_\infty) \ldots (1)$$

$$\frac{T - T_w}{T_\infty - T_w} = 1 - e^{-3500y}$$

$$T - T_w = (1 - e^{-3500y})(T_\infty - T_w)$$

$$\frac{dT}{dy} = 3500 \times e^{-3500y} \times (T_\infty - T_w) \ldots\ldots. (2)$$

Sub (2) in (1)

$$kA\,3500e^{-3500y}(T_\infty - T_w) = hA_s(T_\infty - T_w)$$

$$(\bar{h})_{y=0} = 0.03 \times 3500 \times e^{-3500 \times 0}$$

$$\bar{h} = 105 \text{ W/m}^2\text{K}$$

10.

Sol: $R_e = \dfrac{\rho V d}{\mu} = \dfrac{Vd}{\nu} = \dfrac{5 \times 0.01}{26.66 \times 10^{-6}}$

$= 1875.46 < 2300$

Hence flow is laminar

$$(\overline{Nu})_{T=constant} = 3.66$$

$$\frac{\bar{h}d}{k} = 3.66$$

$$\bar{h} = \frac{3.66 \times k}{d} = \frac{3.66 \times 0.1351}{0.01} = 49.44 \text{ W/m}^2\text{K}$$

11. (i) **Ans: (c)** , (ii) **Ans: (d)**

Sol: $k_w = 0.6$ W/m^{2}K

$k_g = 1.2$ W/m^{2}K , $T_w = 48^0$C

$$\left(\frac{\partial T}{\partial y}\right)_w = 1 \times 10^4 \text{ K/m}$$

As $Q_w = Q_g$

$$\Rightarrow -k_w A\left(\frac{dT}{dy}\right)_w = -k_g A\left(\frac{dT}{dx}\right)_g$$

$$\left(\frac{dT}{dx}\right)_g = \frac{0.6}{1.2} \times 1 \times 10^4 = 0.5 \times 10^4$$

But, $Q_w = Q_{conv}$

$$k_w A\left(\frac{dT}{dy}\right)_y = hA(48 - 40)$$

$$h = \frac{0.6 \times 1 \times 10^4}{8} = 750 \text{ W/m}^2\text{K}$$

12. **Ans: (b)**

13. **Ans: (b)**

Sol: k = 1.0 W/mK

$R_e = 1500 \rightarrow$ means that the flow is laminar

D = 10 cm = 0.1m

For a fully developed laminar flow

Through pipe

(i) With constant heat flux

$Nu = 4.364 \quad \rightarrow$ constant

$$= \frac{hD}{k}$$

$$h = \frac{4.364 \times 1.0}{0.1} = 43.64 \text{ W/mK}$$

(ii) With constant wall temp

$$Nu = 3.66 = \frac{hD}{k}$$

$$h = \frac{3.66 \times 1.0}{0.1} = 36.6 \text{ W/m}^2\text{K}$$

Chapter- 3
Extended Surfaces - FINS

01. **Ans: (c)**

Sol: $T_\infty = 40^\circ C,\ \ T_0 = 100^\circ C,$

$\theta_o = 100 - 40 = 60$

$\theta = 55 - 40 = 15$

In case of rod of length "L" with insulated tip, the temp at the end of tip = 55C

$$\frac{\theta}{\theta_o} = \frac{\text{Cosh}(m(L-x))}{\text{Cosh}(mL)} = \frac{1}{\text{Cosh}\,mL}$$

$$\frac{15}{60} = \frac{1}{\text{Cosh}\,mL} \Rightarrow \text{Cosh}(mL) = \frac{60}{15} = 4$$

mL = Cosh^{-1} (4) = 2.063

$$m = \frac{2.063}{L}$$

In case of rod of length "2L" with both ends maintained at same temp of 100^0C, at the center the temp gradient is zero, means that from one side upto the center of long rod, it can be treated as short fin of length

$\frac{2L}{2} = L$, for which temp distribution is

$$\theta = \theta_o \frac{\text{Cosh}(m(L-x))}{\text{Cosh}(mL)} = \frac{60\,\text{Cosh}(0)}{\text{Cosh}(m \times L)}$$

$$\theta = \frac{60 \times 1}{\text{Cosh}\left(\frac{2.063}{L} \times L\right)}$$

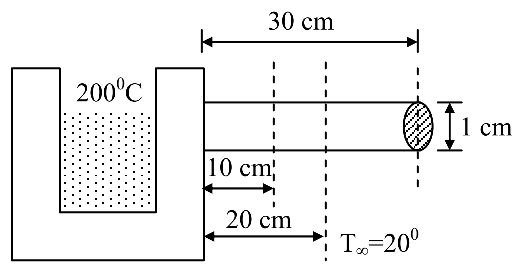
$$= \frac{60}{\text{Cosh}(2.063)} = \frac{60}{4} = 15$$

$$\theta = T - 40 = 15 \Rightarrow T = 55^\circ C$$

02. **Ans: (d)**

03. **Ans: (b)**

Sol:



$$L_C = L + \frac{D}{4}$$

$$= 30 + \frac{1}{4} = 30.25 \text{ cm} = 0.3025 \text{ m}$$

$$m = \sqrt{\frac{4h}{k\,d}} = \sqrt{\frac{4 \times 15}{65 \times 0.01}} = 9.607$$

$$\frac{T_1 - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L_C - x_1)}{\cosh(mL_C)}$$

$$\frac{T_1 - 20}{200 - 20} = \frac{\cosh(9.607(0.3025 - 0.1))}{\cosh(9.607 \times 0.3025)}$$

$$T_1 - 20 = 0.389 \times 180 \Rightarrow T_1 = 90$$

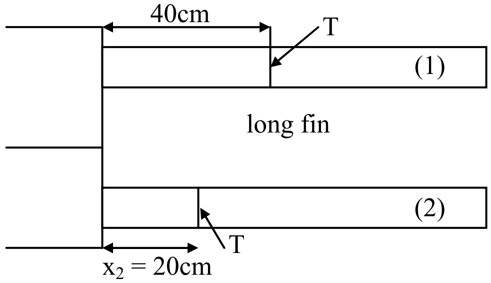
$$\frac{T_2 - 20}{200 - 20} = \frac{\cosh m(L_C - x_2)}{\cosh mL_C}$$

$$\frac{T_2 - 20}{200 - 20} = \frac{\cosh(9.607(0.3025 - 0.2))}{\cosh(9.607 \times 0.3025)}$$

$T_2 = 49.93 \approx 50$

04. **Ans: (d)**

Sol:



Given, $k_1 = 200$ W/mK, $k_2 = ?$

$x_1 = 40$ cm, $x_2 = 20$ cm

$\theta_1 = \theta_0 e^{-m_1 x_1}$

$\theta_2 = \theta_0 e^{-m_2 x_2}$

$\theta_1 = \theta_2$

$\theta_0 \quad e^{-m_1 x_1} = \theta_0 e^{-m_2 x_2}$

$m_1 x_1 = m_2 x_2$

$$\frac{x_2}{x_1} = \frac{m_1}{m_2} \qquad \left(\because m = \sqrt{\frac{4h}{kd}}\right)$$

$$\frac{x_2}{x_1} = \sqrt{\frac{k_2}{k_1}} \Rightarrow k_2 = k_1\left(\frac{x_2}{x_1}\right)^2$$

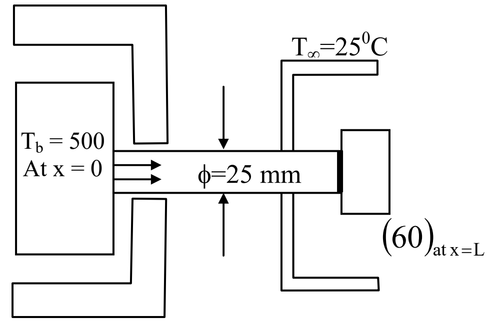
$$k_2 = k_1\left(\frac{20}{40}\right)^2 = 200\times\left(\frac{1}{2}\right)^2 = 50\,\text{W/mK}$$

05. **Ans: (b)**

Sol: $$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

If, $x = L$

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh mL} \qquad (\because \cosh 0 = 0)$$



$$\frac{60-25}{500-25} = \frac{1}{\cosh(mL)}$$

Cosh(mL) = 13.57

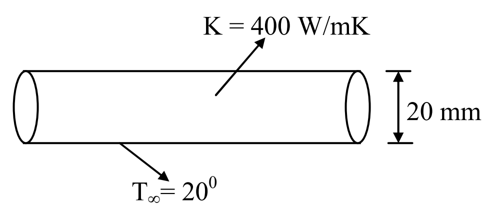
$$mL = 3.299 \Rightarrow L = \frac{3.299}{m}$$

$$L = \frac{3.299}{\sqrt{\frac{4h}{kd}}} = \frac{3.299}{\sqrt{\frac{4\times 15.7}{35\times 0.025}}} = 0.3894\,\text{m}$$

$= 38.94$ cm ≈ 39 cm

06. **Ans: (c)**

Sol:



$h = 8.5$ W/m^{2}K

$Q_{loss} = \sqrt{hPAK}\times\theta_b$

$P = \pi\times d = \pi \times 0.02 = 0.0628$ m

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}\times 0.02^2$$

$= 3.14 \times 10^{-4} = 0.000314$

$\theta_b = 100 - 20 = 80^0$C

$$\dot{Q}_{loss} = \sqrt{8.54 \times 0.0628 \times 0.000314 \times 400} \times 80$$

$$= 20.76 \text{ W}$$

07. **Ans: (c)**

Sol: $e^{-mx} = \dfrac{T - T_\infty}{T_0 - T_\infty}$

$$\Rightarrow m = \frac{1}{x} \ln\left(\frac{T_0 - T_\infty}{T - T_\infty}\right) = \sqrt{\frac{4h}{kd}}$$

$$\frac{1}{100 \times 10^{-3}} \ln\left(\frac{125 - 28}{91 - 28}\right) = \sqrt{\frac{4 \times 17.45}{k \times 0.02}}$$

$\Rightarrow k = 187.37$ W/m.K

08. Ans: (b)

Sol: Effective length, $L_c = 100 + \dfrac{5}{4} = 101.25\,\text{mm}$

$L_c = 0.10125$ m

$$\dot{Q} = \sqrt{hpkA}\,\theta_b \tanh(mL_c)$$

$$m = \sqrt{\frac{4h}{kd}} = \sqrt{\frac{4 \times 40}{400 \times 5 \times 10^{-3}}} = 8.94$$

$$\dot{Q} = \sqrt{40 \times \pi \times 5 \times 10^{-3} \times 400 \times \frac{\pi}{4}\left(5 \times 10^{-3}\right)^2} \times (130 - 30) \times \tanh(8.94 \times 0.10125)$$

$$\dot{Q} = 5\,\text{W}$$

09. Ans: 31

Sol: $\dot{Q}_{gen} = 480 \times 10^{-3}\,\text{W}$

$T_b = 70°C$, $T_\infty = 30°C$

$k_{Al} = 170$ W/mK ,

$h = 12$ W/m^{2}K

$$\dot{Q}_{single\,fin} = \sqrt{hpkA_s}\,\theta_b \tanh(mL)$$

$$m = \sqrt{\frac{12 \times 2 \times 1.4 \times 10^{-3}}{170 \times 0.7 \times 0.7 \times 10^{-6}}} = 20.08$$

$L = 12 \times 10^{-3}$ m

$P = 2 \times 1.4 \times 10^{-3}$ m

$A = 0.49 \times 10^{-6}$ m^2

$$\dot{Q}_{single} = 0.0158\,\text{W} = 15.81 \times 10^{-3}\,\text{W}$$

Total heat generated = n × $\dot{Q}_{dissipated\,in\,single\,fin}$

$$n = \frac{480 \times 10^{-3}}{15.81 \times 10^{-3}} = 30.36 \approx 31$$

10. Ans: (c)

Chapter- 4
Transient Heat Conduction

01. Ans: (c)

Sol: $D = 1\text{ mm} = 10^{-3}\text{m}$

$K = 25\text{ W/mK}$, $\rho = 8400\text{ Kg/m}^3$

$G = 400\text{ J/kgK}$, $h = 560\text{ W/m}^2\text{K}$

$$\frac{t-t_0}{t_b-t_0} = e^{\frac{-hA}{\rho VC}\tau}$$

Given, $t - t_0 = 0.01\,(t_b - t_0)$

$$\therefore 0.01 = e^{\frac{-560\times 6}{8400\times 400\times 10^{-3}}\tau}$$

$$e^{-\tau} = 0.01$$

$$\tau = 4.6\text{ sec}$$

02. Ans: (b)

Sol: $$Q_{entering} = -kA\left(\frac{dT}{dx}\right)_{x=0}$$

$$T = 50 - 50x + 12x^2 + 15x^3 - 15x^4$$

$$\frac{dT}{dx} = -50 + 24x + 45x^2 - 60x^3$$

$$\frac{d^2T}{dx^2} = 24 + 90x - 180x^2$$

$$\frac{d^3T}{dx^3} = 90 - 360x$$

Cooling rate is $\frac{dT}{d\tau}$

Cooling rate or heating rate to be maximum or minimum $\frac{d}{dx}\left(\frac{dT}{d\tau}\right) = 0$

$$\Rightarrow \frac{d}{dx}\left(\frac{d^2T}{dx^2}\right) = 0 \Rightarrow \frac{d^3T}{dx^3} = 0$$

$$\Rightarrow 90 - 360x = 0$$

$$\Rightarrow x = 0.25$$

03. **Ans: (c)**

Sol: $T_0 = 530\,^0\text{C}$, $T_\infty = 30^0\text{C}$

$m = 500\text{ gm}$, Time = 10 sec

$T = 430°\text{C}$

$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho} = \frac{0.5}{9000}$$

According to the Newton's law of cooling the rate of cooling is directly proportional to difference in temperature.

$$\frac{-dT}{d\tau} \propto (T - T_\infty)$$

$$\frac{dT}{d\tau} = -K(T - T_\infty)$$

$$\int_{T=T_0}^{T} \frac{dT}{T-T_\infty} = -K\int_{\tau=0}^{\tau} d\tau$$

$$ln\left(\frac{T-T_\infty}{T_0-T_\infty}\right) = -K\tau$$

$$\Rightarrow \frac{T-T_\infty}{T_0-T_\infty} = e^{-Kt}$$

$$\frac{430-30}{530-30} = e^{-10K}$$

$$\Rightarrow K = 0.0223/\text{sec}$$

Now next 10 sec means that at t = 20

Temp – ?

$$\frac{T-30}{530-30} = e^{-0.0223(20)}$$

$$T = 350.09^0\text{ C}$$

04. **Ans: (a)**

Sol: For lumped analysis to be used

$Bi \le 0.1$

$\frac{hL_C}{k} = 0.1$ ($\because L_C$ for solid cube is $\frac{a}{6}$)

Where, 'a' is a side of cube)

$\frac{ha_{max}}{6k} = 0.1$

$a_{max} = \frac{0.1 \times 6k}{h}$

$= \frac{0.1 \times 6 \times 206}{25} = 4.944$ m

05. **Ans: (d)**

Sol: d = 5 cm = 0.05 m

$T_i = 450°C$, $T_\infty = 90°C$, $T = 150°C$

Lumped capacity analysis can be applied be cause internal temperature gradients are neglected.

$$\frac{T_i - T_\infty}{T - T_\infty} = e^{\left(\frac{hA}{\rho V C_p}\tau\right)}$$

$$\frac{450-90}{150-90} = e^{\left(\frac{115}{8000\times420}\left(\frac{A}{V}\right)\times\tau\right)}$$

$[\frac{V}{A} = \frac{d}{6}$ (for sphere)]

$$\frac{450-90}{150-90} = e^{\left(\frac{115}{8000\times420}\left(\frac{6}{0.08}\right)\times\tau\right)}$$

$\Rightarrow \tau = 436.254$ sec = 7.27 min

06. Ans: (a)

Sol: $T = 65 + 80\,r - 425\,r^2$

$r_i = 0.25$ m $\quad r_o = 0.4$ m

L – 1.5 m $\quad$ k = 5.5 W/m.K

$\alpha = 0.04\ m^2/hr$

$$\frac{d\tau}{dt} = \alpha\frac{1}{r}.\frac{d}{dr}\left(r.\frac{dT}{dr}\right)$$

$$\left(\frac{d\tau}{dt}\right)_{inside} = 0.004\times\frac{1}{r_i}\frac{d}{dr}\left(r_i\frac{dT}{dr}\right)$$

$$= \frac{0.004}{0.25}\frac{d}{dr}\left(r_i\left(80 - 850\,r_i\right)\right)$$

$= 0.016\ (80 - 1700\ r_i)$

$= 0.016\ [80 - 1700\ (0.25)]$

$= -5.52°C/\ hr$

$$\left(\frac{d\tau}{dt}\right)_{outside} = \alpha\frac{1}{r_o}.\frac{d}{dr}\left(r_o.\frac{dT}{dr}\right)$$

$$= \frac{\alpha}{r_o}\frac{d}{dr}\left(80\,r_o - 850\,r_o^2\right)$$

$$= \frac{0.004}{0.4}\left(80 - \left(1700\times0.4\right)\right)$$

$= -6°C/hr$

07. Ans: (d)

08. Ans: (c)

09. Ans: 2.5 (range 2.4 to 2.6)

Sol: It is given in the question that at all times the surface is remaining at temperature of ambient.

$(F_O)_A = (F_O)_B$

$$\left(\frac{\alpha\tau}{L_c^{\,2}}\right)_A = \left(\frac{\alpha\tau}{L_c^{\,2}}\right)_B$$

$$\alpha_A = \left(\frac{k}{\rho c}\right)_A$$

$$= \frac{40}{2\times10^6} = 2\times10^{-5}$$

$$\alpha_B = \left(\frac{k}{\rho c}\right)_B$$

$$= \frac{20}{2\times10^7} = 1\times10^{-6}$$

$$\frac{2\times10^{-5}\times2}{(0.4)^2} = \frac{1\times10^{-6}\,\tau_B}{(0.1)^2}$$

$\Rightarrow \tau_B = 2.5$ hours

Chapter- 5
Radiation

01. **Ans: (b)**

Sol: $T_1 = 800$ K, $T_2 = 500$ K

$\in_1 = 0.8,\ \in_2 = 0.6$

$$Q_{12} = \frac{\sigma_b(T_1^4 - T_2^4)}{\frac{1}{\in_1}+\frac{1}{\in_2}-1}$$

$$= \frac{5.67\times10^{-8}(800^4 - 500^4)}{\frac{1}{0.8}+\frac{1}{0.6}-1}$$

$= 10.26$ kW/m^2

02.

Sol: $\varepsilon_1 = \varepsilon_2 = 0.8$, $T_1 = 400$K, $T_2 = 600$K

$\varepsilon_s = 0.005$

$Q_{1s} = Q_{s2}$

$$\frac{\sigma_b\left(T_1^4 - T_s^4\right)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_s}-1} = \frac{\sigma_b\left(T_s^4 - T_2^4\right)}{\frac{1}{\varepsilon_2}+\frac{1}{\varepsilon_s}-1}$$

$T_s = 527$K

$$q = Q_{1s} = \frac{\sigma_b\left(T_1^4 - T_s^4\right)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_s}-1} = 14.7\,W/m^2$$

03. **Ans: (b)**

Sol: $$1000 = \frac{\sigma_b\left(T_1^4 - T_2^4\right)}{\frac{1}{0.5}+\frac{1}{0.5}-1}$$

$$\Rightarrow \sigma_b\left(T_1^4 - T_2^4\right) = 3000$$

$\varepsilon_2' = 0.25$

$$Q_{12}' = \frac{\sigma_b\left(T_1^4 - T_2^4\right)}{\frac{1}{0.5} + \frac{1}{0.25} - 1}$$

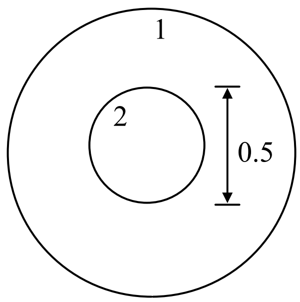
$$= \frac{3000}{2+4-1} = \frac{3000}{5} = 600\ \text{W/m}^2$$

04. **Ans: (b)**

Sol: $F_{21}+F_{22} = 1$

$F_{22} = 0$

$F_{21} = 1$



$A_1F_{12} = A_2F_{21}$

$$F_{12} = \frac{A_2}{A_1} = \frac{\pi DL + 2\times\frac{\pi}{4}D^2}{4\pi R^2}$$

$$= \frac{\pi\times0.5\times0.5 + \frac{\pi}{2}0.5^2}{4\pi\times0.5^2}$$

$$= \frac{1+\frac{1}{2}}{4} = \frac{1.5}{4} = 0.375$$

$F_{11}+F_{12} = 1 \Rightarrow F_{11} = 1- 0.375 = 0.625$

05. **Ans: (d)**

Sol: $T_1 = 1000^0$ K, $T_2 = 500^0$K

$\varepsilon_1 = 1$, $\varepsilon_2 = 0.7$

Irradiation of body 1

$= 0.7 \times 5.67 \times 10^{-8} \times 500^4 + 0.3 \times 5.67 \times 10^{-8} \times 1000^4$

$= 19.5\ \text{kW/m}^2$

06. **Ans: (d)**

Sol: $T_1 = 1000$ K, $T_2 = 500$ K

$\varepsilon_1 = 0.7$, $\varepsilon_2 = 0.8$

$$Q_{12} = \frac{\sigma_b\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = 31.7\ \text{kW/m}^2$$

07. **Ans: (c)**

Sol: $\dfrac{\dot{Q}_S}{\dot{Q}_{W.S}} = \dfrac{1}{79}$

$Q_{WS} = 79Q_S$

$$\frac{E_{b1} - E_{b2}}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} = \frac{79\left(E_{b1} - E_{b2}\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 + N\left(\frac{2}{\varepsilon_S} - 1\right)}$$

Given $\varepsilon_S = 0.05$

$\varepsilon_2 = \varepsilon_1 = 0.8$

$$\left(\frac{2}{0.8} - 1\right) + N\left(\frac{2}{0.05} - 1\right) = 79\left(\frac{2}{0.8} - 1\right)$$

$\Rightarrow N = 3$

08. **Ans: (c)**

Sol: % Reduction = 75%

$$1 - \frac{\dot{Q}_S}{\dot{Q}_{W.S}} = 0.75$$

$$\frac{\dot{Q}_S}{\dot{Q}_{WS}} = 0.25 = \frac{1}{4}$$

$$\frac{1}{n+1} = \frac{1}{4}$$

$\Rightarrow n = 3$

09. Ans: (a)

Sol: $\varepsilon = 0.9$

T_c = recorded temperature

= 20 + 273 + = 293 K

T_w = wall temperature

= 5 + 273 += 278 K

T_∞ = fluid temperature = ?

$h = 8.3$ W/m^2 K

$Q_{conv} = Q_{radation}$

$$h\left(T_\infty - T_c\right) = \varepsilon\sigma_b\left(T_c^4 - T_w^4\right)$$

$$\left(T_\infty - T_c\right) = 0.9 \times 5.67 \times 10^{-8} \times \frac{\left[293^4 - 278^4\right]}{8.3}$$

T_∞ = 301.59 K = 28.59°C

10. Ans: (b)

Sol: $F_{11} + F_{12} + F_{13} + F_{14} = 1$

$F_{14} = 1 - (0.1 + 0.4 + 0.25)$

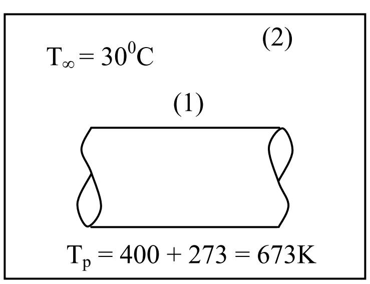
$= 1 - 0.75 = 0.25$

$A_1 F_{14} = A_4 F_{41}$

$$F_{41} = \frac{A_1}{A_4} \times F_{14} = \frac{4}{2} \times 0.25 = 0.5$$

11. Ans: (b)

Sol:



d_{pipe} = 20 cm , $\varepsilon_{pipe} = 0.8$, L_p = 1 m

$$\frac{Q}{A_{pipe}} = \frac{\sigma\left(T_P^4 - T_\infty^4\right)}{\dfrac{1}{\dfrac{1}{\varepsilon_{pipe}} - 1 + 1 + \dfrac{A_p}{A_{wall}} \times \left(\dfrac{1}{\varepsilon_{wall}} - 1\right)}}$$

As room area is very large compared to pipe, hence $\dfrac{A_p}{A_{wall}} \to 0$

$$\frac{Q}{\pi d_p L_p} = \frac{\sigma\left(T_p^4 - T_\infty^4\right)}{\dfrac{1}{\varepsilon_{pipe}}}$$

$$\frac{Q}{L_p} = \pi d_p \times \sigma\left(T_p^4 - T_\infty^4\right)\varepsilon_{pipe}$$

$= \pi \times 0.2 \times 5.67 \times 10^{-8}(673^4 - 303^4) \times 0.8$

= 5.6 kW

12. Ans: (d)

13. Ans: (c)

Chapter- 6
Heat Exchangers

01. **Ans: (a)**

Sol: $C_c = 4180 \times 2 = C_{max}$

$C_h = 1030 \times 5.25 = C_{min}$

$A = 32.5 m^2$,

$U = 200\ W/m^2{}^\circ C$

$$NTU = \frac{UA}{C_{min}} = \frac{200 \times 32.5}{1030 \times 5.25} = 1.2$$

02. **Ans: (c)**

Sol: $NTU = 0.5$, $C_h = C_c = C = 1$

For counter flow

$$\varepsilon = \frac{NTU}{1+NTU} = \frac{0.5}{1+0.5} = \frac{0.5}{1.5} = \frac{1}{3} = 0.33$$

03. **Ans: (d)**

Sol: Given, $C = 1$, $NTU = 2$

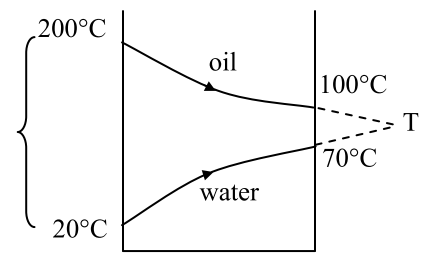
$$\varepsilon_c = \frac{NTU}{1+NTU} = \frac{2}{3} = 0.667$$

$$\varepsilon_P = \frac{1-e^{-2NTU}}{2} = \frac{1-e^{-2(2)}}{2} = 0.49$$

$$\frac{\varepsilon_c}{\varepsilon_p} = \frac{0.667}{0.49} = 1.36$$

04. **Ans: (b)**

Sol:



Parallel flow Heat exchanger

$$\dot{m}_0 C_0 (200-100) = \dot{m}_w C_w (70-20)$$

$$\Rightarrow 2\dot{m}_0 C_0 = \dot{m}_W C_W$$

Now

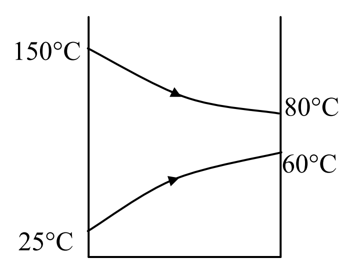
$$\dot{m}_0 C_0 (200-T) = \dot{m}_w C_w (T-20)$$

$$\Rightarrow 200-T = 2T-40$$

$$T = 80^\circ C$$

05. **Ans: (c)**

Sol:



Parallel flow Heat exchanger

$\theta_1 = 150-25 = 125^\circ C$

$\theta_2 = 80-60 = 20^\circ C$

$(\Delta T)_{max} = 150-80 = 70^\circ C$

$(\Delta T)_{min} = 60-25 = 35^\circ C$

$$LMTD = \frac{\theta_1 - \theta_2}{\ell n\left(\frac{\theta_1}{\theta_2}\right)} = \frac{125-20}{\ell n\left(\frac{125}{20}\right)} = 57.29$$

$$NTU = \frac{(\Delta T)_{max}}{LMTD} = \frac{150-80}{57.29} = 1.22$$

06. **Ans: (b)**

Sol: $\varepsilon = \frac{NTU}{1+NTU}$

$$0.8 = \frac{NTU}{1+NTU} \Rightarrow NTU = 4$$

07. **Ans: (b)**

Sol: $C_h = C_C = 1 \times 4000 = 4000$,

$T_{h_1} = 102^0C$

$T_{C_1} = 15^0C$, $U = 1000\ W/m^2K$,

$A = 5m^2$, parallel flow H.E

$2\varepsilon = 1 - \exp(-2NTU)$

$$NTU = \frac{UA}{C_{min}} = \frac{1000\times5}{4000} = 1.25$$

$$\varepsilon = \frac{1-\exp(-2\times1.25)}{2} = \frac{C_C(T_{c2}-T_{c1})}{C_{min}(T_{h1}-T_{c1})}$$

$$0.4589 = \frac{4000(Tc_2-15)}{4000(102-15)}$$

$Tc_2 = 40+15 = 55^0C$

08. **Ans: (a)**

Sol: $C_1 = 30°C$, $m_c = \frac{1500}{3600} = 0.417\ kg/sec$

$T_{h_1} = 120°C$, $C_{P_C} = 4187\ J/kg\ K$

$T_{C_2} = 80°C$, $U = 2000\ W/m^2K$

Because the hot fluid is steam and whose temp remains constant, the flow is immaterial

$120 \rightarrow 120$

$30 \rightarrow 80$

$\theta_1 = 120 - 30 = 90$, $\theta_2 = 120 - 80 = 40$

$$\Delta\theta_m = \frac{90-40}{\ell n\left(\frac{90}{40}\right)} = 61.66^0C$$

$$m_C C_{P_C}\left(T_{C_2} - T_{C_1}\right) = UA\Delta\theta_m$$

$$A = \frac{\frac{1500}{3600}\times4187(80-30)}{2000\times61.66} = 0.707\ m^2$$