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FLUID MECHANICS & HYDRAULIC MACHINES

Volume - I : Study Material with Classroom Practice Questions

Fluid Mechanics & Hydraulic Machines

Solutions for Vol – I Classroom Practice Questions

Chapter- 1
Properties of Fluids

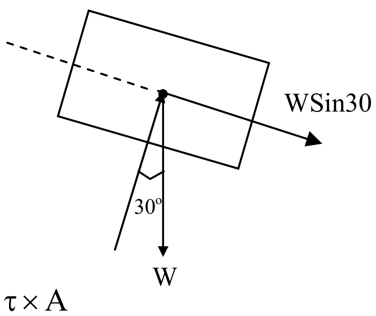
01. Ans: (d) **02. Ans: (c)**

03. Ans: 100

Sol: $\tau = \frac{\mu V}{h} = \frac{0.2\times1.5}{3\times10^{-3}} = 100\ \text{N/m}^2$

04. Ans: 1

Sol:



$F = \tau\times A$

$W\sin30 = \frac{\mu AV}{h}$

$\frac{100}{2} = \frac{1\times0.1\times V}{2\times10^{-3}}$

$V = 1\text{m/s}$

Common data Q. 05 & 06

05. Ans: (c)

Sol: $D_1 = 100\text{mm}$, $D_2 = 106\text{mm}$

$$\text{Radial clearance}, h = \frac{D_2 - D_1}{2}$$

$$= \frac{106-100}{2} = 3\text{mm}$$

$L = 2\text{m}$

$\mu = 0.2$ pa.s

$N = 240\text{rpm}$

$\omega = \frac{2\pi N}{60} = \frac{2\pi\times240}{60}$

$\omega = 8\pi$

$$\tau = \frac{\mu\omega r}{h} = \frac{0.2\times8\pi\times50\times10^{-3}}{3\times10^{-3}} = 83.77\text{N/m}^2$$

06. Ans: (b)

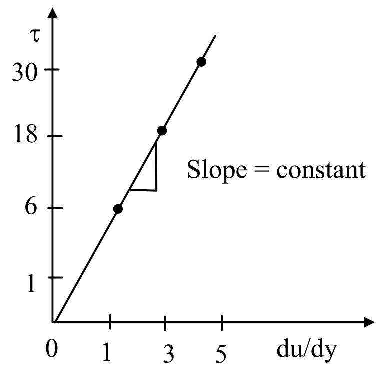
Sol: $\text{Power}, P = \frac{2\pi\omega^2\mu L r^3}{h}$

$$= \frac{2\pi\times(8\pi)^2\times0.2\times2\times(0.05)^3}{3\times10^{-3}}$$

$= 66$ Watt

07. Ans: (c)

Sol:



$\therefore$ Newtonian fluid

08. Ans: (d)

Sol: $$\tau = \mu \frac{du}{dy}$$

$$u = 3\sin(5\pi y)$$

$$\frac{du}{dy} = 3\cos(5\pi y)\times 5\pi = 15\pi\cos(5\pi y)$$

$$\tau\big|_{y=0.05} = \mu\frac{du}{dy}\bigg|_{y=0.05}$$

$$= 0.5\times 15\pi\cos(5\pi\times 0.05)$$

$$= 0.5\times 15\pi\times\cos\left(\frac{\pi}{4}\right) = 0.5\times 15\pi\times\frac{1}{\sqrt{2}}$$

$$= 7.5\times 3.14\times 0.707 \approx 16.6 N/m^2$$

$$\tau\big|_{y=0.12} = 0.5\times 15\pi\cos(5\pi\times 0.12)$$

$$= 7.5\times\pi\cos\left(5\pi\times\frac{3}{25}\right)$$

$$= 7.5\times\pi\cos\left(\frac{3\pi}{5}\right)$$

Which is negative so zero

09. Ans: (c) **10. Ans: (d)** **11. Ans: (a)**

12. Ans: (d)

Ans: Viscosity in liquids decreases and in gases decreases with rise in temperature.

13. Ans: (d)

Ans: Blood is a pseudoplastic fluid. So statement I is wrong.

14. Ans: (b)

Sol: Free surface is subjected to surface tension force in the plane of surface. It can resist small tensile loads.

15. Ans: (b)

Sol: $V = 0.01\ m^3$

$\beta = 0.75\times 10^{-9}\ m^2/N$

$dp = 2\times 10^7\ N/m^2$

$$k = \frac{1}{\beta} = \frac{1}{0.75\times 10^{-9}} = \frac{4}{3}\times 10^9$$

$$k = \frac{dp}{dv/v}$$

$$dv = \frac{2\times 10^7\times 10^{-2}\times 3}{4\times 10^9} = 1.5\times 10^{-4}$$

16. Ans: 320 Pa

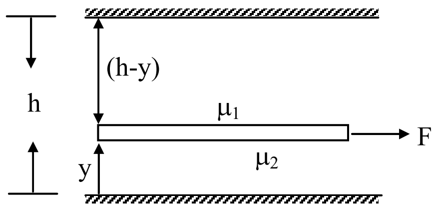
Sol: $$\Delta P = \frac{8\sigma}{D} = \frac{8\times 0.04}{1\times 10^{-3}} = \frac{32\times 10^{-2}}{10^{-3}}$$

$$\Delta P = 320\ N/m^2$$

Conventional Questions which can be asked as objective Questions

01.

Sol:



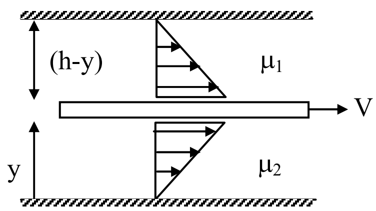
Assumption:

- Thin plate has negligible thickness.
- Velocity profile is linear. Because of narrow gap.
- Given fluid is a Newtonian fluid which obeys Newton's law of viscosity.

The force required to pull it is proportional to the total shear stress imposed by the two oil layers.

$F = F_1+F_2$, Where F_1 = Force on top sides of plate . F_2 = Force on bottom side of plate

The plate moves with velocity V



From Newton's law of viscosity,

$\tau = \dfrac{\mu du}{dy}$ Let A be area of plate

$\therefore$ $F_1 = \tau_1 \times$ Area of plate

$$F_1 = \mu_1 \times \frac{V}{h-y} \times A$$

$$F_2 = \mu_2 \times \frac{V}{y} \times A$$

(i) Shear force on two sides of the plate are equal:

$$F_1 = F_2$$

$$\frac{\mu_1 \times VA}{h-y} = \frac{\mu_2 VA}{y}$$

$$\frac{\mu_1}{\mu_2} = \frac{h-y}{y}$$

$$\frac{h}{y} = \frac{\mu_1}{\mu_2} + 1 \Rightarrow \frac{h}{y} = \frac{\mu_1 + \mu_2}{\mu_2}$$

$$y = \frac{\mu_2 h}{\mu_1 + \mu_2}$$

(ii) The position of plate so that pull required to drag the plate is minimum.

$F = \dfrac{\mu_1 VA}{h-y} + \dfrac{\mu_2 VA}{y}$, V, A, μ_1 & μ_2 , h are constant

For minimum force, $\dfrac{dF}{dy} = 0$

$$-\mu_1 VA(h-y)^{-2}(-1) - \mu_2 VAy^{-2} = 0$$

$$\frac{\mu_2 VA}{y^2} = \frac{\mu_1 VA}{(h-y)^2}$$

$$\frac{(h-y)^2}{y^2} = \frac{\mu_1}{\mu_2}$$

$$\frac{h-y}{y} = \sqrt{\frac{\mu_1}{\mu_2}}$$

$\frac{h}{y} = 1 + \sqrt{\frac{\mu_1}{\mu_2}}$ where y is the distance of the thin flat plate from the bottom flat surface.

$$y = \frac{h}{1+\sqrt{\frac{\mu_1}{\mu_2}}}$$

02. Ans: 0.372 Pa. sec

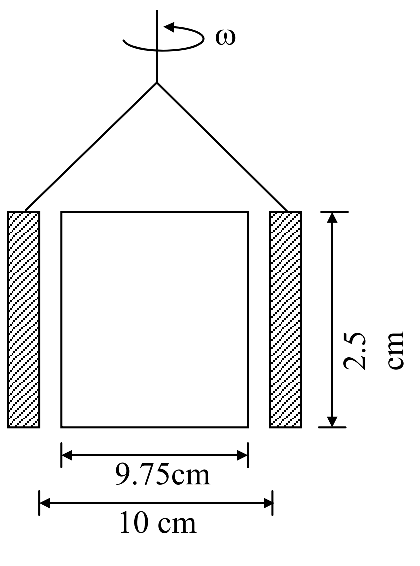
Sol: Torque = 1.2N-m

Speed, N = 90rpm

Diameter, D_1 = 10cm ,

D_2 = 9.75cm

H = 2.5cm



Assumption:

- The gap between two cylinders is narrow and hence velocity profile is assumed linear.
- No change in properties

Torque = Tangential force × radius

Force = shear stress×Area

$$= \frac{\mu \times VA}{h}$$

Where h is the clearance (radial)

$$h = \frac{15-14.75}{2}$$

$= 0.125\text{cm} = 1.25\times10^{-3}\text{m}$

Area = πDL

$= \pi\times0.15\times2.5\times10^{-2}$

$= 11.781\times10^{-3}\text{m}^2$

$$F_s = \frac{\mu\times\omega r\times A}{h}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\times600}{60} = 20\pi \text{ rad/s}$$

Torque = $F_s \times r$

$$= \frac{\mu\omega r A}{h}\times r$$

$$= \frac{\mu\omega r^2}{h}\times A$$

$$1.2 = \frac{\mu\times20\pi\times(0.15)^2\times11.781\times10^{-3}}{1.25\times10^{-3}\times4}$$

μ = 0.372 Pa.sec

Chapter- 2
Pressure Measurement & Fluid Statics

01. Ans: (a)

Sol: 1 millibar = $10^{-3}\times10^{5}$ = 100 N/m^2

One mm of Hg = $13.6\times10^{3}\times9.81\times1\times10^{-3}$

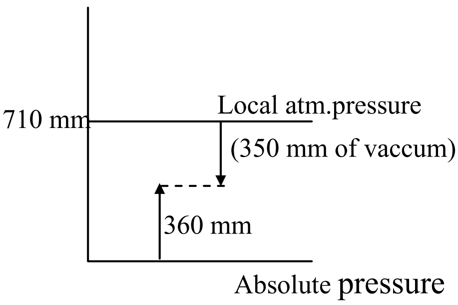
= 133.416 N/m^2

1 N/mm^2 = 1×10^{6} N/m^2

1 kgf/cm^2 = 9.81×10^{6} N/m^2

02. Ans: (b)

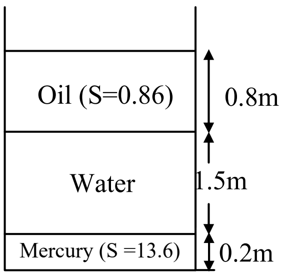
Sol:



03. Ans: (c)

04. Ans: 48.147

Sol:



$P_{bottom} = \rho_{oil}gh_{oil} + \rho_w gh_{water} + \rho_{Hg}gh_{Hg}$

$= (860\times9.81\times0.8)+(9810\times1.5)+(13600\times9.81\times0.2)$

= 48147.48 Pa

P_{bottom} = 48.147 kPa

05. Ans: (b)

06. Ans: 2.2

Sol: h_p in terms of oil

$s_o\,h_o = s_m h_m$

$0.85\times h_0 = 13.6\times0.1$

$h_0 = 1.6$m

$h_p = 0.6+1.6 \Rightarrow h_p = 2.2$m of oil

07. Ans: 750

Sol: $P_{atm}+\rho_w gh_w = P_{atm}+\rho_0 gh_0$

$1000\times6\times10^{-2} = \rho_0\times8\times10^{-2}$

$\rho_0 = 750$ kg/m^3

08. Ans: (b)

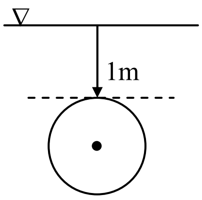
Sol: $h_M - \frac{s_w}{s_0}h_{w_1} = h_N - \frac{s_w h_{w_2}}{s_0} - h_0$

$$h_M - h_N = \frac{9}{0.83} - \frac{18}{0.83} - 3$$

$$h_M - h_N = -13.843\,\text{cm of oil}$$

09. Ans: 2.125

Sol:



$$h_P = h_G + \frac{I}{Ah_G}$$

$$= 2 + \frac{\pi D^4\times4}{64\times D^2\times2\times\pi}$$

$$= 2 + \frac{2^2\times4}{64\times2} = 2.125\text{m}$$

10. Ans: 61.6

Sol: F = P×A

$$F = \rho g\bar{h}A$$

$$= 9810\times 2\times \frac{\pi}{4}\times 2^2 = 61.6\text{ kN}$$

11. Ans: 10

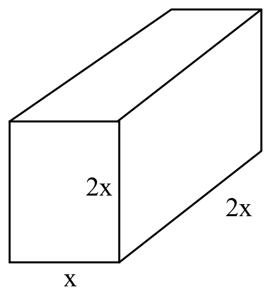
Sol: $F = \rho g\bar{h}A$

$$= 9810\times 1.625\times \frac{\pi}{4}\left(1.2^2 - 0.8^2\right)$$

F = 10kN

12. Ans: 1

Sol:



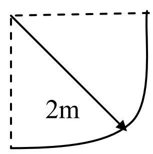
$$F_{bottom} = \rho g \times 2x \times 2x \times x$$

$$F_V = \rho g x \times 2x \times 2x$$

$$\frac{F_B}{F_V} = 1$$

13. Ans: 10

Sol:



$$F_V = x\times\pi$$

$$F_V = \rho g V = 1000\times 10\times \frac{\pi\times 2^2}{4}$$

$$F_V = 10\pi\text{kN}$$

$\therefore$ x = 10

14. Ans: (d)

Sol:

$$F_{net} = F_{H1} - F_{H2}$$

$$F_{H1} = \gamma\times\frac{D}{2}\times D\times 1 = \frac{\gamma D^2}{2}$$

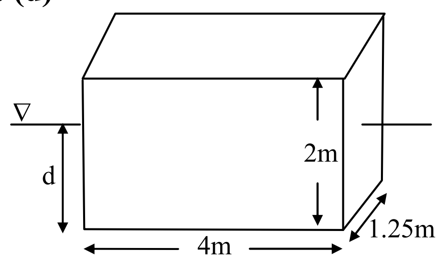
$$F_{H2} = \gamma\times\frac{D}{4}\times\frac{D}{2}\times 1 = \frac{\gamma D^2}{8}$$

$$= \gamma D^2\left(\frac{1}{2}-\frac{1}{8}\right) = \frac{3\gamma D^2}{8}$$

Chapter- 3
Buoyancy and Metacentric Height

01. Ans: (d)

Sol:



F_B = weight of body

$$\rho_b g V_b = \rho_f g V_f d$$

$$640\times 4\times 2\times 1.25 = 1025\times(4\times 1.25\times d)$$

$$d = 1.248\text{m}$$

$$V_{fd} = 1.248\times 4\times 1.25$$

$$V_{fd} = 6.24\text{m}^3$$

02. Ans: (c)

Sol: Surface area of cube = $6a^2$

Surface area of sphere = $4\pi r^2$

$4\pi r^2 = 6a^2$

$$\frac{2\pi}{3} = \left(\frac{a}{r}\right)^2$$

$F_{b,s} \propto V_s$

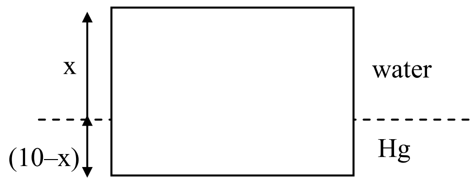
$$= \frac{\frac{4}{3}\pi r^3}{a^3} = \frac{4}{3}\frac{\pi r^3}{\left(r\sqrt{\frac{2\pi}{3}}\right)^3}$$

$$= \frac{4}{3}\frac{\pi r^3}{\left(\sqrt{\frac{2\pi}{3}} \times \sqrt{\frac{2\pi}{3}} r^3\right)} = \sqrt{\frac{6}{\pi}}$$

03. Ans: 4.76

Sol: $F_B = F_{B,M} + F_{B,W}$

$W_B = F_B$



$\rho_b g V_b = \rho_m g V_{fd,m} + \rho_\omega g V_{fd,w}$

$\rho_b V_b = \rho_m V_{fd,m} + \rho_w V_{fd,w}$

$S \times V_b = S_m V_{fd,m} + S_w V_{fdw}$

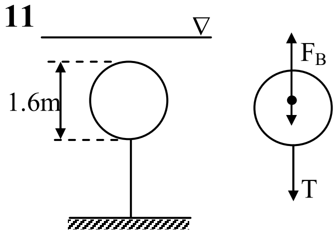
$7.6 \times 10^3 = 13.6 \times 10^2 (10 - x) + 10^2 \times x$

$-6000 = -1260x$

$x = 4.76$ cm

04. Ans: 11

Sol:



$F_B = W + T$

$W = F_B - T$

$= \rho_f g V_{fd} - T$

$= 10^3 \times 9.81 \times \frac{4}{3}\pi (0.8)^3 - (10 \times 10^3)$

$= 21 - 10$

$W = 11$ kN

05. Ans: 1.375

Sol: $W_{water} = 5N$

$W_{oil} = 7N$

$S = 0.85$

W – Weight in air

$F_{B1} = W - 5$

$F_{B2} = W - 7$

$W - 5 = \rho_1 g V_{fd}$.....(1)

$W - 7 = \rho_2 g V_{fd}$.....(2)

$V_{fd} = V_b$

$W - 5 = \rho_1 g V_b$

$W - 7 = \rho_2 g V_b$

$2 = (\rho_1 - \rho_2) g V_b$

$$V_b = \frac{2}{(1000 - 850)9.81}$$

$V_b = 1.3591 \times 10^{-3} m^3$

$W = 5 + (9810 \times 1.3591 \times 10^{-3})$

$W = 18.33N$

$W = \rho_b g V_b$

$$\frac{18.33}{9.81\times1.3591\times10^{-3}}=\rho_b$$

ρ_b = 1375.05 kg/m^3

S_b = 1.375

06. Ans: (d)

07. Ans: –14

Sol: GM = BM–BG

$$BM=\frac{I}{V}=\frac{3\times(1)^3}{12\times3\times1\times0.75}$$

$$BM=\frac{4}{12\times3},BG=\frac{1}{2}-\frac{3}{8}=\frac{1}{8}$$

$$BM=\frac{1}{9}$$

$$GM=\frac{1}{9}-\frac{1}{8}$$

GM = – 13.8mm ≈ – 14mm

08. Ans: (b)

Sol: W = F_B

$\rho_b g V_b = \rho_f g V_{fd}$

$\rho_b V_b = \rho_f V_{fd}$

$$0.6\times\frac{\pi}{4}d^2\times2d=1\times\frac{\pi}{4}d^2\times x$$

x = 1.2d

GM = BM – BG

$$BM=\frac{I}{V}=\frac{\pi d^4}{64\times\frac{\pi}{4}d^2\times1.2d}=\frac{d}{19.2}$$

BG = d–0.6d = 0.4d

GM < 0 unstable

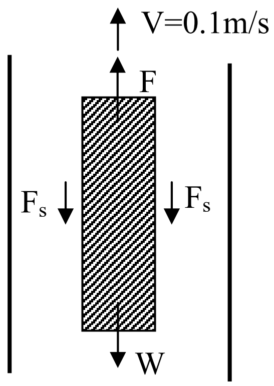
09. Ans: 20

Sol: $T=2\pi\sqrt{\dfrac{k^2}{g(GM)}}\Rightarrow2\pi\sqrt{\dfrac{(7.72)^2}{9.81\times0.6}}$

T = 20s

10. Ans: 122.475

Sol:



The thickness of the oil layer is same on either side of plate

y = thickness of oil layer

$$=\frac{23.5-1.5}{2}=11\text{mm}$$

Shear stress on one side of the plate

$$\tau=\frac{\mu dU}{dy}$$

F_s = total shear force (considering both sides of the plate)

$$=2A\times\tau=\frac{2A\mu V}{y}$$

$$=\frac{2\times1.5\times1.5\times2.5\times0.1}{11\times10^{-3}}$$

= 102.2727 N

Weight of plate, W = 50 N

Upward force on submerged plate,

$F_v = \rho g V = 900 \times 9.81 \times 1.5 \times 1.5 \times 10^{-3}$

$= 29.7978$ N

Total force required to lift the plate

$= F_s + W - F_V$

$= 102.2727 + 50 - 29.7978$

$= 122.4749$ N

Chapter- 4
Fluid Kinematics

01. Ans: (b)

02. Ans: (a)

Sol: Given, $u = -x$,

$v = 2y$

Stream line equation in 2 – D

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\frac{dx}{-x} = \frac{dy}{2y}$$

On integration

$$\int \frac{dx}{-x} = \int \frac{dy}{2y}$$

$$-\int \frac{1}{x} dx = \frac{1}{2}\int \frac{1}{y} dy$$

$$-\log x = \frac{1}{2}\log y + \log c$$

$$\log\left(\frac{1}{x}\right) = \log\sqrt{y} + \log c$$

$$\log\left(\frac{1}{x}\right) = \log\left(\sqrt{y}.c\right)$$

$$\frac{1}{x} = \sqrt{y}.c$$

At (1,1) point $= \frac{1}{1} = \sqrt{1}.c$

$$c = 1$$

$$x\sqrt{y} = 1$$

03. Ans: (a)

Sol: $\vec{V} = 2x\hat{i} + y\hat{j}$

Compare $\vec{V} = u\hat{i} + v\hat{j}$

Where, $u = 2x$, $v = y$

Velocity, $|\vec{v}| = \sqrt{u^2 + v^2}$

$$= \sqrt{(2x)^2 + (y)^2}$$

$$V = \sqrt{4x^2 + y^2}$$

$$V_{(1,1)} = \sqrt{4(1)^2 + (1)} = \sqrt{5}\,m/s$$

Acceleration, $\vec{a} = a_x\hat{i} + a_y\hat{j}$

$$|\vec{a}| = \sqrt{a_x^{\,2} + a_y^{\,2}}$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$= 0 + (2x)\frac{\partial}{\partial x}(2x) + (y)\frac{\partial(2x)}{\partial y}$$

$= 2x\ (2) + y(0)$

$= 4x$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= 0 + 2x\frac{\partial}{\partial x}(y) + y.\frac{\partial}{\partial y}(y)$$

$= 2x\ (0) + y\ (1)$

$= y$

$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{(4x)^2 + y^2}$

$= \sqrt{16x^2 + y^2}$

$= |\vec{a}|_{(1,1)} = \sqrt{16(1)^2 + (1)^2}$

$= \sqrt{17}\,\text{m/sec}^2$

Common Data for Questions 04 & 05

04. **Ans: 0.94**

Sol: $a_{Local} = \dfrac{\partial V}{\partial t}$

$$= \frac{\partial}{\partial t}\left(2t\left(1 - \frac{x}{2L}\right)^2\right)$$

$$= \left(1 - \frac{x}{2L}\right)^2 \times 2$$

$$(a_{Local})_{\text{at } x = 0.5,\ L = 0.8} = 2\left(1 - \frac{0.5}{2 \times 0.8}\right)^2$$

$= 2(1 - 0.3125)^2 = 0.945\ \text{m/sec}^2$

05. **Ans: –13.68**

Sol: $$a_{convective} = v.\frac{\partial v}{\partial x} = \left[2t\left[1 - \frac{x}{2L}\right]^2\right]\frac{\partial}{\partial x}\left[2t\left(1 - \frac{x}{2L}\right)^2\right]$$

$$= \left[2t\left[1 - \frac{x}{2L}\right]^2\right]2t\left[2\left(1 - \frac{x}{2L}\right)\left(-\frac{1}{2L}\right)\right]$$

At t = 3 sec; x = 0.5 m; L = 0.8 m

$$a_{convective} = 2\times3\left[1 - \frac{0.5}{2\times0.8}\right]^2 \times 2\times3\left[2\left(1 - \frac{0.5}{2\times0.8}\right)\right]\left[\frac{-1}{2\times0.8}\right]$$

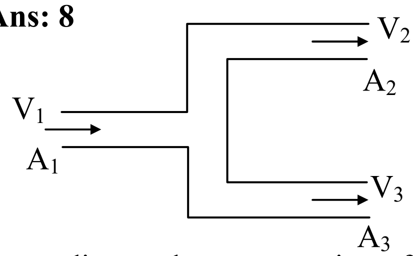
$a_{convective} = -\,14.62\ \text{m/sec}^2$

$a_{total} = a_{local} + a_{convective} = 0.94 - 14.62$

$= -13.68\ \text{m/sec}^2$

06. **Ans: 8**

Sol:



According to the conservation of mass

Total inward flow = Total outward flow

$Q_1 = Q_2 + Q_3$

$A_1V_1 = A_2V_2 + A_3V_3$

$A_2 = A_3$

$V_1 = 2$ m/s ; $V_2 = 3$ m/s ; $V_3 = 5$ m/s

$A_1 \times 2 = A_2 \times 3 + A_2 \times 5$

$A_1 = 4A_2$

At another instant $V_1 = 3$ m/s

$V_2 = 4$ m/s

$V_3 = ?$

$A_1V_1 = A_2V_2 + A_3V_3$

$4A_2 \times 3 = A_2 \times 4 + A_2 \times V_3$

$12\ A_2 = 4A_2 + A_2V_3$

$V_3 = 8$ m/s

07. **Ans: (d)**

Sol: $u = 6xy - 2x^2$

continuity equation for 2D flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = 6y - 4x$$

$$(6y - 4x) + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial V}{\partial y} = (4x - 6y) = 0$$

$$\partial V = (4x - 6y)\,dy$$

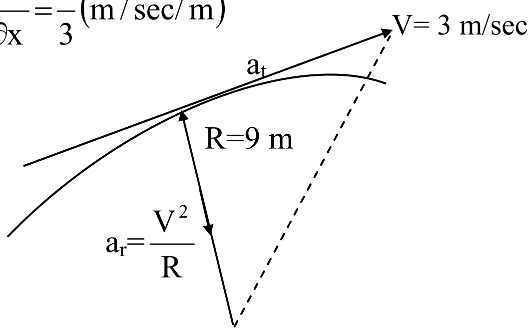
$$V = \int 4x\,dy - \int 6y\,dy$$

$$= 4xy - 3y^2 + c$$

$$= 4xy - 3y^2 + f(x)$$

08. Ans: $\sqrt{2}$

Sol: $\frac{\partial V}{\partial x} = \frac{1}{3}(m/sec/m)$



$$a_r = \frac{V^2}{R} = \frac{(3)^2}{9} = \frac{9}{9} = 1 \text{ m/s}^2$$

$$a_t = V\frac{\partial V}{\partial x} = 3 \times \frac{1}{3} = 1 \text{ m/s}^2$$

$$a = \sqrt{(a_r)^2 + (a_t)^2} = \sqrt{(1)^2 + (1)^2} = \sqrt{2} \text{ m/sec}^2$$

09. Ans: 13.75

Sol: $a_{t\,(conv)} = V_{avg} \times \frac{dV}{dx}$

$$a_{t\,(conv)} = \left(\frac{2.5 + 3}{2}\right)\left(\frac{3 - 2.5}{0.1}\right) = 2.75 \times 5$$

$$a_{t\,(conv)} = 13.75 \text{ m/s}^2$$

10. Ans: 1.5

Sol: $a_r = \frac{\partial V}{\partial \theta} \times \frac{\partial \theta}{\partial t}$

$$= \frac{\partial V}{\partial \theta} \times \omega \quad (\because V = r\omega)$$

$$= \frac{\partial V}{\partial \theta} \times \frac{V}{r}$$

$$= \frac{\partial}{\partial \theta}(3\sin\theta) \times \frac{3\sin\theta}{3} = 3\cos\theta \times \sin\theta$$

$$(a_r)_{(\theta=45)} = 3 \times \cos 45^0 \times \sin 45^0$$

$$= 3 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{3}{2} = 1.5 \text{ m/sec}^2$$

11. Ans: 0.3

Sol: $Q = Au$

$$a_{Local} = \frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\left(\frac{Q}{A}\right)$$

$$a_{local} = \frac{1}{A}\frac{\partial Q}{\partial t}$$

$$a_{Local} = \left(\frac{1}{0.4 - 0.1x}\right)\frac{\partial Q}{\partial t}$$

$$(a_{Local})_{at\ x=0} = \frac{1}{0.4} \times 0.12 \quad \left(\because \frac{\partial Q}{\partial t} = 0.12\right)$$

$$= 0.3 \text{ m/sec}^2$$

12. Ans: (b)

Sol: $\psi = x^2 - y^2$

$$a_{Total} = (a_x)\hat{i} + (a_y)\hat{j}$$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(x^2 - y^2) = -2y$$

$$v = \frac{-\partial \psi}{\partial x} = \frac{-\partial}{\partial x}(x^2 - y^2) = -2x$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$= (-2y)(0) + (-2x)(-2)$$

$$\therefore a_x = 4x$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}$$

$$= (-2y)\times(-2) + (-x)\times(0)$$

$$a_y = 4y$$

$$\therefore a = (4x)\hat{i} + (4y)\hat{j}$$

13. Ans: (b)

Sol: Given, The stream function for a potential flow field is $\psi = x^2 - y^2$

$\phi = ?$

$$u = \frac{-\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$$

$$u = \frac{\partial\psi}{\partial y} = \frac{\partial(x^2 - y^2)}{\partial y}$$

$$u = -2y$$

$$u = \frac{-\partial\phi}{\partial x} = 2y$$

$$\int\partial\phi = \int -2y\partial x$$

$$\phi = -2\,xy + c_1$$

Given, ϕ is zero at (0,0)

$\therefore\ c_1 = 0$

$\therefore\ \phi = -2xy$

14. Ans: 4

Sol: Given, 2D – flow field

Velocity, V = 3xi + 4xyj

$u = 3x$, $v = 4xy$

$$\omega_z = \frac{1}{2}\left(\frac{dv}{dx} - \frac{du}{dy}\right)$$

$$\omega_z = \frac{1}{2}(4y - 0)$$

$$(\omega_Z)_{at(2,2)} = \frac{1}{2}\times 4(2) = 4 \text{ rad/sec}$$

Chapter- 5
Energy Equation and its Applications

01. Ans: (c)

Sol: Applying Bernoulli's equation for ideal fluid

$$\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$$

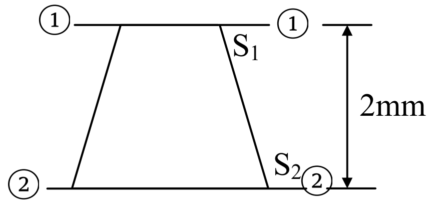
$$\frac{P_1}{\rho g} + \frac{(2)^2}{2g} = \frac{P_2}{\rho g} + \frac{(1)^2}{2g}$$

$$\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{4}{2g} - \frac{1}{2g}$$

$$\frac{P_2 - P_1}{\rho g} = \frac{3}{2g} = \frac{1.5}{g}$$

02. Ans: (c)

Sol:



$$\frac{V_1^2}{2g} = 1.27\text{m}\ ,\qquad \frac{P_1}{\rho g} = 2.5\text{m}$$

$$\frac{V_2^2}{2g} = 0.203\text{m} \ , \quad \frac{P_2}{\rho g} = 5.407\text{m}$$

$Z_1 = 2$ m , $Z_2 = 0$ m

Total head at (1) – (1)

$$= \frac{V_1^2}{2g} + \frac{P_1}{\rho g} + Z_1$$

$= 1.27 + 2.5 + 2 = 5.77$ m

Total head at (2) – (2)

$$= \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + Z_2$$

$= 0.203 + 5.407 + 0 = 5.61$ m

Loss of head = 5.77 – 5.61 = 0.16 m

∴ Energy at (1) – (1) > Energy at (2)– (2)

∴ Flow takes from higher energy to lower energy

i.e. from (1) to (2)

Top to bottom flow take place

03. Ans: 1.5

Sol: $A_1 = \frac{\pi}{4}d_1^2 = \frac{\pi}{4}(0.1)^2 = 7.85\times10^{-3}\ \text{mm}^2$

$$A_2 = \frac{\pi}{4}d_2^2 = \frac{\pi}{4}(0.05)^2 = 1.96\times10^{-3}\ \text{mm}^2$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$Z_1 = Z_2$, it is in Horizontal position

Since, at outlet atmospheric pressure,

$P_2 = 0$

$Q = 100$ lit/sec $= 0.1\ \text{m}^3$/sec

$$V_1 = \frac{Q}{A_1} = \frac{0.1}{7.85\times10^{-3}} = 12.73\,\text{m/sec}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.1}{1.96\times10^{-3}} = 51.02\,\text{m/sec}$$

$$\frac{P_{1\text{gauge}}}{\rho_{air}\times g} + \frac{(12.73)^2}{2\times10} = 0 + \frac{(51.02)^2}{2\times10}$$

$$\frac{P_1}{\rho_{air}.g} = 121.53$$

$$P_1 = 121.53\times\rho_{air}\times g$$

$= 1.51$ kPa

04. Ans: 395

Sol: $Q = 100$ litre/sec $= 0.1\ \text{m}^3$/sec

$V_1 = 100$ m/sec; $P_1 = 3\times10^5\ \text{N/m}^2$

$V_2 = 50$ m/sec; $P_2 = 1\times10^5\ \text{N/m}^2$

Power (P) = ?

Bernoulli's equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

$$\frac{3\times10^5}{1000\times10} + \frac{100^2}{2\times10} + 0 = \frac{1\times10^5}{1000\times10} + \frac{50^2}{2\times10} + 0 + h_L\ h_L$$

$= 395$ m

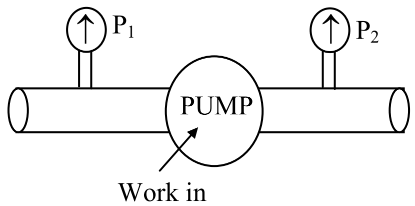
$P = \rho g Q.h_L$

$P = 1000\times10\times0.10\times395$

$P = 395$ kW

05. Ans: 51.5

Sol: Apply Bernoulli's equation to pump



$$\frac{P_1}{\rho g}+Z_1+\frac{V_1^2}{2g}+\text{Work in}$$

$$=\frac{P_2}{\rho g}+Z_2+\frac{V_2^2}{2g}+H_{Loss}$$

Where work in = Head raised = 10 m

Since pipes are same size

$V_1 = V_2$ and $Z_1 = Z_2$

$$\frac{P_1}{\rho g}+0+0+10=\frac{120\times10^3}{1000\times9.81}+0+0+3$$

$P_1 = (12.23 + 3 - 10)\times\rho g$

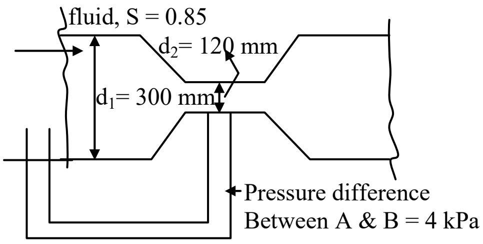
$P_1 = (5.234)(1000\times9.81)$

$= 51.33\times10^3\ N/m^2$

$= 51.33$ kPa

06. Ans: 35

Sol:



$$Q_{Th}=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh}$$

$$=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2g\left(\frac{\Delta P}{w}\right)}$$

$$A_1=\frac{\pi}{4}d_1^2=\frac{\pi}{4}(0.30)^2=0.07\,m^2$$

$$A_2=\frac{\pi}{4}d_2^2=\frac{\pi}{4}(0.12)^2=0.011\,m^2$$

$$Q_{Th}=\frac{0.07\times0.011}{\sqrt{(0.07)^2-(0.011)^2}}\sqrt{2\times9.81\times4\times10^3}$$

$\Delta P = 4$ kPa,

$$h=\frac{\Delta P}{w}=\frac{\Delta P}{P_f.g}$$

$$=\frac{\Delta P}{s_f\rho_w g}=\frac{4\times10^3}{0.85\times1000\times9.81}$$

$= 0.035\ m^3/sec$

$= 35.15$ ltr/sec.

07. Ans: 65

Sol: $h_{stag} = 0.30$ m

$h_{stat} = 0.24$ m

$$V=c\sqrt{2gh_{dyna}}$$

$$V=1\sqrt{2g(h_{stag}-h_{stat})}$$

$$=\sqrt{2(9.81)(0.30-0.24)}=65.09\ m/min$$

08. Ans: 81.5

Sol: x = 30 mm

$g = 10\ m/s^2$

$\rho_{air} = 1.23\ kg/m^3$; $\rho_{Hg} = 13600\ kg/m^3$

C = 1

$$V=\sqrt{2gh_D}$$

$$h_D=x\left(\frac{S_m}{S}-1\right)$$

$$h_D=30\times10^{-3}\left(\frac{13600}{1.23}-1\right)$$

$h_D = 331.67$ m

$$V=1\times\sqrt{2\times10\times331.67}$$

V = 81.5 m/sec

09. Ans: 140

Sol: $Q_a = C_d \dfrac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$

$C_d \propto \dfrac{1}{\sqrt{h}}$

$\dfrac{C_{d_{venturie}}}{C_{d_{orifice}}} = \sqrt{\dfrac{h_{orifice}}{h_{venture}}}$

$h_{venturi} = 140$ mm

Chapter- 6
Momentum equation and its Applications

01. Ans: 720

Sol: $P = \rho(g + a)h = \rho(g + 5g)h = 6\rho gh$

$= 6 \times 1200 \times 10 \times 10 = 720$ kPa

02. Ans: 1600

Sol: $S = 0.80$

$A = 0.02 \text{ m}^2$

$V = 10$ m/sec

$F = \rho.A.V^2$

$F = 0.80 \times 1000 \times 0.02 \times 10^2$

$F = 1600$ N

03. Ans: 6000

Sol: $A = 0.015 \text{ m}^2$

$V = 15$ m/sec

$U = 5$ m/sec

$F = \rho A\,(V + U)^2$

$F = 1000 \times 0.015\,(15+5)^2$

$F = 6000$ N

04. Ans: 19.6

Sol: $V = 100$ m/sec

$U = 50$ m/sec

$d = 0.1$ m

$F = \rho A\,(V - U)^2$

$F = 1000 \times \dfrac{\pi}{4} \times 0.1^2 \times (100 - 50)^2$

$F = 19.6$ kN

05. Ans: (b)

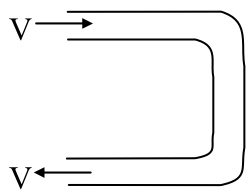
Sol: $F_x = \rho aV(\,V - 0)$

$= \rho a\, V^2$

$= 1000 \times 1 \times 10^{-4} \times 10^2 = 10$ N

06. Ans: (a)

Sol:



$F_x = \rho aV(\,V_{1x} - V_{2x})$

$= \rho aV(\,V - (-V))$

$= 2\,\rho a\, V^2$

$= 2\times 1000 \times 10^{-4} \times 5^2 = 5 \times 10^{-3}$ kN

07. Ans: (d)

Sol: $F_1 = \rho A(V-u)^2$

Power $(P_1) = F_1 \times u = \rho A(V-u)^2 \times u$

$F_2 = \rho.Q.V \times V_r$

$= \rho.A(V).(V-u)$

Power (P_2) = $F_2 \times u = \rho AV(V-u)u$

$$\frac{P_1}{P_2} = \frac{\rho A(V-u)^2 \times u}{\rho AV(V-u)\times u}$$

$$= 1 - \frac{5}{20} = 0.75$$

Chapter- 7
Laminar Flow

01. Ans: (d) **02. Ans: (d)**

03. Ans: (d)

Sol: $Q = A.V_{avg}$

$Q = A.\frac{V_{max}}{2}$ ($\because V_{max} = 2\ V_{avg}$)

$$Q = \frac{\pi}{4}\left(\frac{40}{1000}\right)^2 \times \frac{1.5}{2}$$

$$= \frac{\pi}{4}\times(0.04)^2 \times 0.75$$

$$= \frac{\pi}{4}\times\frac{4}{100}\times\frac{4}{100}\times\frac{3}{4} = \frac{3\pi}{10000}\ \text{m}^3/\text{sec}$$

04. Ans: 100000

Sol: $\tau = \frac{-dP}{dx}\times\frac{r}{2}$

$$250 = -\frac{dP}{10}\times\frac{0.1/2}{2}$$

$\therefore P_1 - P_2 = 1 \times 10^5\ \text{N/m}^2$

05. Ans: 1.92

Sol: $\rho = 1000\ \text{kg/m}^3$

$Q = 800\ \text{mm}^3/\text{sec} = 800 \times (10^{-3})^3\ \text{m}^3/\text{sec}$

$L = 2$ m

$D = 0.5$ mm

$P = 2$ MPa $= 2 \times 10^6$Pa

$\mu = ?$

$$P = \frac{128.\mu QL}{\pi D^4}$$

$$2\times10^6 = \frac{128\times\mu\times800\times(10^{-3})^3\times2}{\pi(0.5\times10^{-3})^4}$$

$\mu = 1.917$ millipa – sec

06. Ans: 0.75

Sol: $U_r = U_{max}\left(1-\left(\frac{r}{R}\right)^2\right)$

$$\left[\because \frac{U}{U_{max}} = 1-\left(\frac{r}{R}\right)^2\right]$$

$$= 1\left(1-\left(\frac{5}{10}\right)^2\right)$$

$$= 1\left(1-\frac{1}{4}\right) = \frac{3}{4} = 0.75\ \text{m/s}$$

07. Ans: 0.08

Sol: Given, $\rho = 0.8 \times 1000 = 800\ \text{kg/m}^3$

$\mu = 1$ Poise $= 10^{-1}$ N-s/m

$d = 50$ mm $= 0.05$ m

velocity = 2 m/s

Reynold's Number , $R_e = \frac{\rho VD}{\mu}$

$$= \frac{800\times2\times0.05}{10^{-1}} = 800$$

($\because R_e < 2000$) $\therefore$ Flow is laminar,

For laminar, Darcy friction factor

$$f = \frac{64}{R_e} = \frac{64}{800} = 0.08$$

08. Ans: (c)

09. Ans: 0.32

Sol: Given:

$\mu = 0.01\text{Poise} = 0.01\times10^{-1}$ N-s/m^2

$D = 10$ mm $= 10 \times 10^{-3}$ m

$V = 10$ mm/s $= 10\times10^{-3}$ m/sec

$L = 1$ km $= 1000$ m

$\rho = 1000$ kg/m^3

Reynolds Number, $R_e = \dfrac{\rho VD}{\mu}$

$$= \frac{1000\times10\times10^{-3}\times10\times10^{-3}}{0.01\times10^{-1}}$$

$R_e = 100 < 2000$

$\therefore R_e < 2000$, hence flow is laminar

For laminar flow, $h_f = \dfrac{32\mu VL}{\rho g D^2}$

$$= \frac{32\times0.01\times10^{-1}\times10\times10^{-3}\times10^3}{10^3\times10\times\left(10\times10^{-3}\right)^2}$$

$= 0.32$ m

10. Ans: 16

Sol: For fully developed laminar flow,

$$h_f = \frac{32\mu VL}{\rho g D^2} \quad (\therefore Q = AV)$$

$$h_f = \frac{32\mu\left(\frac{Q}{A}\right)L}{\rho g D^2} = \frac{32\mu QL}{AD^2\times\rho g}$$

$$h_f = \frac{32\mu QL}{\frac{\pi}{4}D^2\times D^2\times\rho g}$$

$$h_f \propto \frac{1}{D^4}$$

$$h_{f1}D_1^4 = h_{f_2}D_2^4$$

Given, $D_2 = \dfrac{D_1}{2}$

$$h_{f1}\times D_1^4 = h_{f2}\times\left(\frac{D_1}{2}\right)^4$$

$$h_{f_2} = 16 h_{f_1}$$

$\therefore$ Head loss, increase by 16 times if diameter halved.

11. Ans: 5.2

Sol: Oil viscosity, $\mu = 10$ poise $= 10\times0.1$
$= 1$ N-s/m^2

$y = 50\times10^{-3}$m

$L = 120$ cm $= 1.20$ m

$P = 3\times10^3$Pa

Width of plate = 0.2 m

$Q = ?$

$Q = A.V_{avg} = (\text{width of plate}\times y)v$

$$P = \gamma Q h_L = \frac{12\mu vL}{B^2}$$

$$3\times10^3 = \frac{12\times1\times v\times1.20}{\left(50\times10^{-3}\right)^2}$$

$V = 0.52$ m/sec

$Q = AV_{avg} = (0.2\times50\times10^{-3})\ (0.52)$

$= 5.2$ lit/sec

Chapter- 8
Flow Through Pipes

01. Ans: (d)

Sol: $\nu = 0.4\ cm^2/sec = 0.4\times10^{-4}\ m^2/sec$

$d = 8\ cm = 8\times10^{-2}$ m.

Lower critical Reynolds number for laminar flow is 2000

$$Re = \frac{V.D}{\nu}$$

$$2000 = \frac{V\times8\times10^{-2}}{0.4\times10^{-4}}$$

Average (or) Mean velocity (V) = 1 m/sec

For Laminar pipe flow; $V_{max} = 2V_{avg}$

$= 2\times1 = 2$ m/s

02. Ans: (a)

Sol: $\nu = 8\times10^{-4}\ m^2/sec$; d = 0.08 m

$Q = 3200\ \pi\times10^{-6}\ m^3/sec$

Type of flow = ?

$$Q = AV$$

$$3200\ \pi\times10^{-6} = \frac{\pi}{4}\left(0.08^2\right)\times V$$

Mean (or) Average velocity = 2 m/sec

$$Re = \frac{V.D}{\nu}$$

$$\therefore Re = \frac{2\times0.08}{8\times10^{-4}}$$

Re = 200 < 2000 (Critical Reynolds's number for laminar flow)

$\therefore$ Type of flow is "Laminar"

03. Ans: (a)

Sol: In pipes Net work, series arrangement

$$\therefore h_f = \frac{f.lV^2}{2gd} = \frac{f.lQ^2}{12.1\times d^5}$$

$$\frac{h_{f_A}}{h_{f_B}} = \frac{f_A.l_A.Q_a^2}{12.1\times d_A^2}\times\frac{12.1\times d_B^5}{f_B.l_B.Q_B^2}$$

Given $l_A = l_B$, $f_A = f_B$, $Q_A = Q_B$

$$\frac{h_{f_A}}{h_{f_B}} = \left(\frac{d_B}{d_A}\right)^5 = \left(\frac{d_B}{1.2d_B}\right)^5$$

$$= \left(\frac{1}{1.2}\right)^5 = 0.4018 \approx 0.402;$$

04. Ans: (a)

Sol: Given, $d_1 = 10$ cm; $d_2 = 20$ cm

$f_1 = f_2$; $l_1 = l_2 = l$

$l_e = l_1 + l_2 = 2l$

$$\frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} \Rightarrow \frac{2l}{d_e^5} = \frac{l}{10^5} + \frac{l}{20^5}$$

$\therefore d_e = 11.4$ cm

05. Ans: (b)

Sol: In parallel pipe arrangement;

$$h_{f_A} = h_{f_B}$$

$$\frac{f_A.l_A.Q_A^2}{12.1\times d_A^5} = \frac{f_B l_B.Q_B^2}{12.1\times d_B^5}$$

Given $d_A = d_B$; $l_A = l_B$, $f_A = 4f_B$

$$\left(\frac{Q_A}{Q_B}\right)^2 = \frac{f_B}{f_A}$$

$$\frac{Q_A}{Q_B} = \sqrt{\frac{f_B}{f_A}} = \sqrt{\frac{f_B}{4f_B}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 0.5$$

06. Ans: (d)

Sol: For parallel pipes

$h_{f_1} = h_{f_2}$

$$\frac{f_1 \times l_1 \times Q_1^2}{12.1 \times d_1^5} = \frac{f_2 \times l_2 \times Q_2^2}{12.1 \times d_2^5}$$

For given data

$$\frac{Q_1^2}{Q_2^2} = \left(\frac{d_1}{d_2}\right)^5$$

$$\left(\frac{Q_1}{Q_2}\right)^2 = \left(\frac{2d}{d}\right)^5 = (2)^5 = 32$$

$$\frac{Q_1}{Q_2} = \sqrt{32} = 4\sqrt{2}$$

07. Ans: (c)

Sol: $d_e = (n)^{2/5}.d$

$30 = (2)^{2/5} d$

$\therefore$ d = 22.73 cm

Select near higher size i.e. 25 cm

08. Ans: (b)

Sol: Power transmitted by the pipe,

$P = \rho g Q(H - h_f)$

For maximum power transmission, the condition is $h_f = \dfrac{H}{3}$

$$P = \rho g Q \left(H - \frac{H}{3}\right) = \rho g Q \frac{2H}{3}$$

$$= 1000 \times 10 \times 1 \times \left(2 \times \frac{99}{3}\right)$$

$= 660 \times 10^3$ Watt = 660 kW

09. Ans: (b)

Sol: Q = 100 m^3/sec

H = 75 m

$$1 \text{ HP} = 75 \frac{\text{kgf} - \text{m}}{\text{sec}} \approx 750 \frac{\text{Nm}}{\text{sec}}$$

1 HP = 750 Watt = 0.75 kW

Power (Theoretical) = $\rho g Q H$

$\approx 1000 \times 10 \times 100 \times 75$

= 75000000 W = 75000 kW

0.75 kW = 1 MHP

75000 kW = – ?

$$= \frac{75000}{0.75} = 100000 \text{ MHP}$$

10. Ans: (c)

Sol: $\eta_{pump} = \dfrac{\text{Fluid power}}{\text{Shaft power}}$

$$\eta_{pump} = \frac{\rho g Q(H + h_f)}{P_{shaft}}$$

Given H = 10 m

Q = 0.1 m^3/sec

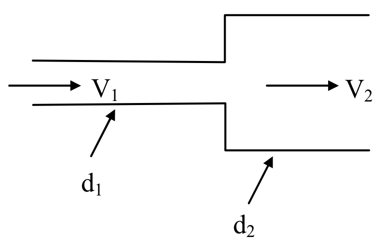
h_f = 5 m

$$1 = \frac{1000 \times 10 \times 0.1 \times (10 + 5)}{P_{Shaft}}$$

$\therefore P_{Shaft}$ = 15000 W = 15 kW

11. Ans: (c)

Sol:



Given $d_2 = 2d_1$

Losses due to sudden expansion,

$$h_L = \frac{(V_1 - V_2)^2}{2g}$$

$$= \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2$$

By continuity equation,

$Q = A_1V_1 = A_2V_2$

$$\therefore \frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$h_L = \frac{V_1^2}{2g}\left(1 - \frac{1}{4}\right)^2$$

$$h_L = \frac{9}{16} \times \frac{V_1^2}{2g}$$

$$\frac{h_L}{\frac{V_1^2}{2g}} = \frac{9}{16}$$

12. Ans: (b)

Sol: $K = 2 \times 10^9$ N/m^2

Given $\rho = 965$ kg/m^3

$$C = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{2 \times 10^9}{965}} \simeq 1440 \text{ m/sec}$$

13. Ans: (b)

Sol: Pipes are in parallel

$Q_e = Q_A + Q_B$ ------- (i)

$h_{Le} = h_{L_A} = h_{L_B}$

$L_e = 175$ m

$f_e = 0.015$

$$\frac{f_e L_e Q_e^2}{12.1 D_e^5} = \frac{f_A . L_A Q_A^2}{12.1 D_A^5} = \frac{f_B L_B Q_B^2}{12.1 D_B^5}$$

$$\frac{0.020 \times 150 \times Q_A^2}{12.1 \times (0.1)^5} = \frac{0.015 \times 200 \times Q_B^2}{12.1 \times (0.08)^5}$$

$Q_A = 1.747\ Q_B$ -------(ii)

From (i) $Q_e = 1.747\ Q_B + Q_B$

$Q_e = 2.747\ Q_B$ --------(iii)

$$\frac{0.015 \times 175 (2.747 Q_B)^2}{12.1 \times D_e^5} = \frac{0.015 \times 200 \times Q_B^2}{12.1 \times (0.08)^5}$$

$D_e = 116.6$ mm $\simeq 117$ mm

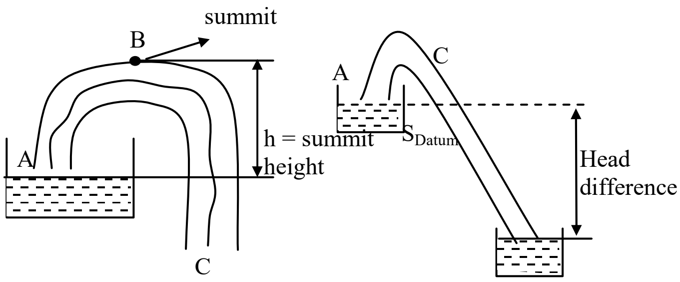


Fig. Siphoning Action

Chapter- 9
Elementary Turbulent Flow

01. Ans: (b)

Sol:

A constant stress layer exists in the near wall region. Due to the damping of the vertical velocity fluctuations near the wall, the Reynolds stress term will become negligible and we find that a linear velocity profile exists. This is only true for the very near wall region.

02. Ans: (d)

03. Ans: 2.4

Sol: Given: V = 2 m/s

$f = 0.02$

$V_{max} = ?$

$V_{max} = V(1 + 1.43\sqrt{f})$

$= 2\left(1+1.43\sqrt{0.02}\right)$

$= 2 \times 1.2 = 2.4$ m/s

04. Ans: (c)

Sol: Given data:

D = 30 cm = 0.3 m

$Re = 10^6$

$f = 0.025$

Thickness of laminar sub layer, $\delta' = ?$

$$\delta' = \frac{11.6\nu}{V^*}$$

Where V^* = shear velocity = $V\sqrt{\frac{f}{8}}$

ν = Kinematic viscosity

$$Re = \frac{V.D}{\nu}$$

$$\therefore \nu = \frac{V.D}{Re}$$

$$\delta' = \frac{11.6\times\frac{VD}{Re}}{V\sqrt{\frac{f}{8}}}$$

$$\delta' = \frac{11.6\times D}{Re\sqrt{\frac{f}{8}}}$$

$$= \frac{11.6\times 0.3}{10^6\times\sqrt{\frac{0.025}{8}}}$$

$= 6.22\times10^{-5}$ m $= 0.0622$ mm

05. Ans: 25

Sol: Given:

L = 100 m

D = 0.1 m

$h_L = 10$ m

$\tau = ?$

For any type of flow, the shear stress at wall/surface $\tau = \frac{-dP}{dx}\times\frac{R}{2}$

$$\tau = \frac{\rho g h_L}{L}\times\frac{R}{2}$$

$$\tau = \frac{\rho g h_L}{L}\times\frac{D}{4}$$

$$= \frac{1000\times9.81\times10}{100}\times\frac{0.1}{4}$$

$= 24.525$ N/m^2 = 25 Pa

06. Ans: 0.9

Sol: k = 0.15 mm; $\tau = 4.9$ N/m^2

ν = 1 centi-stoke

$$\frac{k}{\delta'} = \frac{0.15\times10^{-3}}{\left(\frac{11.6\times\nu}{V^*}\right)}$$

$$V^* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{4.9}{1000}} = 0.07 \text{ m/sec}$$

ν = 1 centi-stoke

$$= \frac{1}{100}\text{stoke} = \frac{10^{-4}}{100} = 10^{-6}\,m^2/sec$$

$$= \frac{\frac{0.15\times 10^{-3}}{11.6\times 10^{-6}}}{0.07} = 0.905$$

07. Ans: 480

Sol: Given:

d = 5 cm = 0.05 m

Flow rate $\dot{m} = \pi$ kg/sec

$\mu = 0.001$ N-s/m^2, $\rho = 1000$ kg/m^3

$f_D = \frac{64}{Re_d}$ ………… for laminar

$f_D = 0.316\ Re_d^{-0.25}$ …. for turbulent

g = 10 m/sec^2

$$\dot{m} = \rho AV = \rho\times\frac{\pi}{4}\times d^2\times V$$

$$\pi = 1000\times\frac{\pi}{4}(0.05)^2\times V$$

V = 1.6 m/sec

$$Re = \frac{\rho VD}{\mu} = \frac{1000\times 1.6\times 0.05}{0.001}$$

$= 80000 > 2000\ (R_{eD})$

$\therefore$ Flow is turbulent

$\therefore\ f_D = 0.316\ R_{eD}^{-0.25}$

$= 0.316\ (80000)^{-0.25} = 0.0187$

Pressure drop $(P_1 - P_2) = h_f \times \rho g$

$$= \frac{fLV^2}{2gD}\times\rho g = \frac{fLV^2\rho}{2D}$$

$$= \frac{0.0187\times 1\times(1.6)^2\times 1000}{2\times 0.05}$$

$= 478$ Pa / m $\approx$ 480 Pa/m

08. Ans: 20%

Sol: Since, Discharge decrease is associated with increase in friction.

$$\frac{df}{f} = -2\times\frac{dQ}{Q} = 2\left[-\frac{dQ}{Q}\right]$$

$$= 2\left[-\frac{Q-0.1Q}{Q}\right]$$

$$\frac{df}{f} = 2\times[-0.9] = -1.8$$

$df = -1.8\times f$

friction factor increased by 20%

09. Ans: 68.35

Sol: Power lost per one km length = $\gamma_w Q h_f$

$$h_f = \frac{f\ell Q^2}{12.1d^5}$$

$$Q = \frac{36}{60} = 0.6\,m^3/sec$$

$$\frac{1}{\sqrt{4f'}} = 2\log_{10}\left(\frac{R}{K}\right)+1.74$$

$$\frac{1}{\sqrt{4f'}} = 2\log_{10}\left(\frac{300}{3}\right)+1.74$$

$4f' = 0.03$

$f = 4f' = 0.03$

$$h_f = \frac{0.03\times 1000\times 0.6^2}{12.1\times 0.6^5} = 11.61\text{ m}$$

Power = $\gamma_w Q h_f$

$= 9.81\times 0.6\times 11.61$

$= 68.35$ kW

Chapter- 10
Boundary Layer Theory

01. Ans: (c)

Sol: $R_{e\ \text{Critical}} = \dfrac{U_\infty x_{\text{critical}}}{\nu_{\text{critical}}}$

Assume water properties

$$5\times10^5 = \frac{6\times x_{\text{critical}}}{1\times10^{-6}}$$

$x_{\text{critical}} = 0.08333$ m = 83.33 mm

02. Ans: 1.6

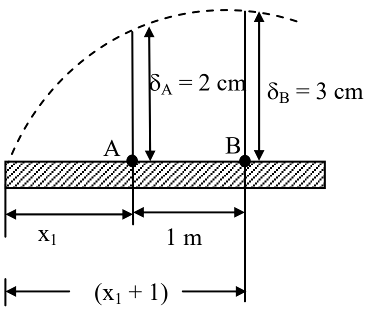
Sol: $\delta \propto \dfrac{1}{\sqrt{R_{e_1}}}$ (At given distance 'x')

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{R_{e_2}}{R_{e_1}}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{256}{100}} = \frac{16}{10} = 1.6$$

03. Ans: 80

Sol:



$$\delta \propto \sqrt{x}$$

$$\frac{\delta_A}{\delta_B} = \sqrt{\frac{x_1}{(x_1+1)}}$$

$$x = \frac{2}{3} = \sqrt{\frac{x_1}{x_1+1}}$$

$$\frac{4}{9} = \frac{x_1}{x_1+1}$$

$5x_1 = 4 = x_1 = 80$ cm

04. Ans: 1.5

Sol: $\tau = \mu\dfrac{du}{dy}$

(Newton's law of viscosity)

$$\tau = \mu\frac{d}{dy}\left(u_m\times1.5\frac{y}{\delta}\right)$$

$$\tau = \mu\times u_m\times1.5\times\frac{1}{\delta}$$

$$\tau = 1.5\frac{\mu u_m}{\delta}$$

$$\tau = K\frac{\mu u_m}{\delta}$$

By comparing, K = 1.5

05. Ans: 2

Sol: $\tau \propto \dfrac{1}{\delta}$

$$\tau \propto \frac{1}{\sqrt{x}} \quad \because \delta \propto \sqrt{x}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{4} = 2$$

06. Ans: 3

Sol: $\frac{U}{U_\infty} = \frac{y}{\delta}$

$\frac{\delta^*}{\theta} = ?$

$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$

$= \int_0^\delta \left(1 - \frac{y}{8}\right) dy$

$= y - \frac{y^2}{2\delta}\Big|_0^\delta = \delta - \frac{\delta}{2} = \frac{\delta}{2}$

$\theta = \int_0^\delta \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$

$= \int_0^\delta \frac{y}{8}\left(1 - \frac{y}{\delta}\right) dy$

$= \frac{y^2}{2\delta} - \frac{y^3}{3\delta}\Big|_0^\delta = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$

Shape factor $= \frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3$

07. Ans: 7.33

Sol: $R_{e(x=L)} = \frac{U_\infty L}{\nu}$

$R_{e(x=L)} = \frac{U_\infty L}{\nu} = \frac{6\times 1}{0.15\times 10^{-4}} = 4\times 10^5$

Since, $R_{e\,(x=L)} < 5\times 10^5$

Hence,

$\delta_{x=L} = \frac{Kx}{\sqrt{R_e}} = \frac{4.64L}{\sqrt{R_e}} = \frac{4.64\times 1}{\sqrt{4\times 10^5}} = 7.33\,\text{mm}$

08. Ans: 21

Sol: $\tau = \mu\left(\frac{du}{dy}\right)$

We know that $\frac{U_\infty}{\delta} = \frac{U}{y}$

On differentiating

$\tau = \frac{\mu . U_\infty}{\delta}$

$\tau_{x=L} = \frac{(\rho . v)U_\infty}{\delta_{\text{at } x=L}} (\because \mu = \rho v)$

$= \frac{1.226\times 0.15\times 10^{-4}\times 6}{7.33\times 10^{-3}} = 0.015\ \text{N/m}^2$

$\frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_1}} = \frac{\tau_{x=L}}{\tau_{X=L/2}} = \sqrt{\frac{(L/2)}{L}}$

$\tau_{x=L/2} = \sqrt{2}\tau_{x=L}$

$= \sqrt{2}\times 0.015\,\text{N/m}^2 = 21$ milli Pa

09. Ans: 22.6

Sol: Drag force, $F_D = \frac{1}{2} C_D.\rho.A_{Proj}.\,U_\infty^2$

B = 1.5 m, ρ = 1.2 kg/m^3

L = 3.0 m, ν = 0.15 stokes

U_∞ = 2 m/sec

$R_e = \frac{U_\infty L}{\nu} = \frac{2\times 3}{0.15\times 10^{-4}} = 4\times 10^5$

$C_D = \frac{1.328}{\sqrt{R_e}} = \frac{1.328}{\sqrt{4\times 10^5}} = 2.09\times 10^{-3}$

D.F, $F_D = \frac{1}{2}\times 2.09\times 10^{-3}\times 1.2\times(1.5\times 3)\times 2^2$

= 22.57 milli-Newton

10. Ans: 1.62

Sol: $\dot{m} = \rho A U_\infty = \rho(B\times\delta)U_\infty \quad (\because \delta = L)$

$$\dot{m}_{ab} = \dot{m}_{bc} + \dot{m}_{cd}$$

$$\dot{m}_{bc} = \frac{1}{2}\dot{m}_{ab} = \frac{1}{2}\rho(B\times\delta).U_\infty$$

$$= \frac{1}{2}\times 1.2\times 1\times 1.5\times 10^{-3}\times 30$$

$$= 1.62 \text{ kg/minute}$$

Chapter- 11
Force on Submerged Bodies

01. Ans: 8

Sol: Drag power = Drag Force × Velocity

$$P = F_D \times V$$

$$P = C_D \times \frac{\rho A V^2}{2}\times V$$

$$P \propto V^3$$

$$\frac{P_1}{P_2} = \left(\frac{V_1}{V_2}\right)^3$$

$$\frac{P_1}{P_2} = \left(\frac{V}{2V}\right)^3$$

$$P_2 = 8P$$

02. Ans: 4.56

Sol: $F_D = C_D.\dfrac{\rho A V^2}{2}$

$$W = 0.8\times 1.2\times \frac{\frac{\pi}{4}(D)^2\times V^2}{2}$$

(Note: A = Normal (or)

projected Area = $\frac{\pi}{4}D^2$)

$$80\times 9.81 = 0.8\times 1.2\times\frac{\pi}{4}(D)^2\times\frac{10^2}{2}$$

$$\therefore D = 4.56 \text{ m}$$

03. Ans: 0.054

Sol: Given data:

$V = 8$ m/s

$D = 0.06$ m

$\rho = 1.2$ kg/m^3

$\nu = 1.6\times 10^{-4}$ m^2/sec

$W = ?$

$$R_e = \frac{V.D}{\nu} = \frac{8\times 0.06}{1.6\times 10^{-4}} = 3000$$

For flow over sphere; $C_D = 0.5$

$1000 < R_e < 1\times 10^5$

$W = F_D$

$$W = C_D\times\frac{\rho A V^2}{2}$$

$$W = 0.5\times\frac{1.2\times\frac{\pi}{4}(0.06)^2\times(8)^2}{2}$$

$$W = 0.5\times 0.108 \quad = 0.054 \text{ N}$$

04. Ans: 4

Sol: Given data:

$l = 0.5$ km = 500 m

d = 1.25 cm

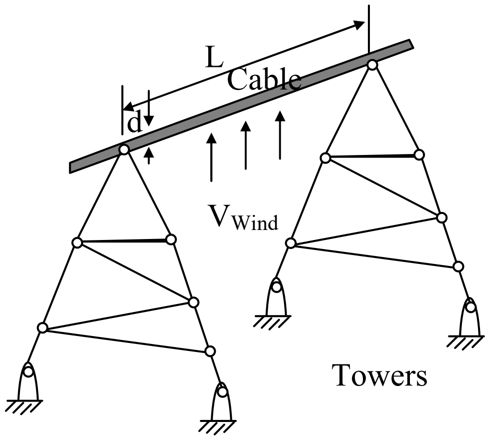
$V_{Wind} = 100$ km/hr

$\gamma_{Air} = 13.4$ N/m^3

$\nu = 1.4\times10^{-5}$ m^2/s

$C_D = 1.2$ for $R_e > 10000$

$C_D = 1.3$ for $R_e < 10000$



$$R_e = \frac{V.L}{\nu} = \frac{\left(\frac{100\times5}{18}\right)(500)}{1.4\times10^{-5}}$$

Note: The characteristic dimension for electric power transmission tower wire is "L"

$R_e = 992\times10^6 > 10{,}000$

$\therefore C_D = 1.2$

$$F_D = C_D\times\frac{\rho AV^2}{2}$$

$$= 1.2\times\frac{\left(\frac{13.4}{9.81}\right)(L\times d)V^2}{2}$$

$$= \frac{1.2\times\left(\frac{13.4}{9.81}\right)(500\times0.0125)\left(100\times\frac{5}{18}\right)^2}{2}$$

= 3952.4 N = 4 kN

05. Ans: $C_d = 0.1262$ & $C_L = 0.144$

Sol: Given data:

$W_{Kite} = 2.5$ N

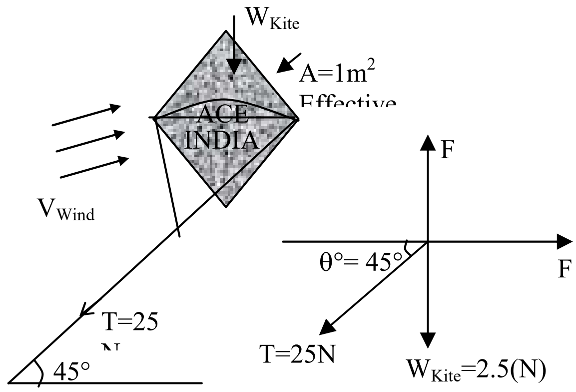
$A = 1$ m^2

$\theta = 45^o$

T = 25 N

$V_{Wind} = 54$ km/hr

$$= 54\times\frac{5}{18} = 15 \text{ m/s}$$



Resolving forces horizontally

$F_D = T\cos45^o$

$$C_D\times\frac{\rho AV^2}{2} = 25\times\cos45^o$$

$$\frac{C_D \times \left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 25 \times \frac{1}{\sqrt{2}}$$

$$\therefore\ C_D = 0.126$$

Resolving forces vertically

$F_L = W_{Kite} + T\sin 45^o$

$$\frac{C_L \rho A V^2}{2} = 2.5 + 25\sin 45^o$$

$$\frac{C_L\left(\frac{12.2}{9.81}\right)(1)(15)^2}{2} = 2.5 + \frac{25}{\sqrt{2}}$$

$$\therefore C_L = 0.144$$

06. Ans: (a)

Sol: Given data:

$C_{D_2} = 0.75\, C_{D_1}$ (25% reduced)

Drag power = Drag force × Velocity

$$P = F_D \times V = \frac{C_D \rho A V^2}{2} \times V$$

$$P = C_D \times \frac{\rho A V^3}{2}$$

Keeping ρ, A and power constant

$C_D V^3$ = constant = C

$$\frac{C_{D_1}}{C_{D_2}} = \left(\frac{V_2}{V_1}\right)^3$$

$$\left(\frac{C_{D_1}}{0.75 C_{D_1}}\right)^{1/3} = \frac{V_2}{V_1}$$

$$\therefore V_2 = 1.10064 V_1$$

% Increase in speed = 10.064%

07. Ans: 0.1875

Sol: Given:

$F_D = 300$N

$\rho = 0.8 \times 1000 = 800$ kg/m^3

L = 2 m

D = 80 mm = 0.08 m

V = 5 m/s

C_D = coefficient of drag

$$F_D = C_D . \frac{\rho V^2}{2} \times A$$

$$300 = C_D \times \frac{800 \times 5^2}{2} \times (0.08) \times (2)$$

$\Rightarrow C_D = 0.1875$

08. Ans: (c)

09. Ans: 60

Sol: Area = 45 m^2

Velocity = 5.6 kmph = 1.56 m/s

$\delta_{sea\ water} = 1.025$

$\nu = 1.67 \times 10^{-6}$ m^2/s

$C_D = 0.7$

Power required =?

Drag power = Drag force × Velocity

$$= \frac{C_D \rho A V^2}{2} \times V$$

$$= \frac{0.7 \times 1025 \times (1.56)^3}{2} \times 45$$

= 60.766 kW

10. Ans: 318

Sol: Width = 3 m

Height = 0.8 m

Velocity = 50 kmph = 13.89 m/s

= 1.25 kg/m^3

$C_D = 1.1$

Drag force $F = \frac{C_D \rho A V^2}{2}$

$= \frac{1.1 \times 1.25 \times 3 \times 0.8 \times 13.89^2}{2}$

= 318.33 N

Chapter- 12
Open channel Flow

01. Ans: (b)

02. Ans: (b)

Sol: $Q_1 = 15\text{m}^3/\text{sec}$, y = 1.5 m

$S_1 = \frac{1}{1690}, \text{if } S_2 = \frac{1}{1000}$

Then Q_2 = ?

$Q \propto \sqrt{S}$

$\frac{Q_2}{Q_1} = \sqrt{\frac{S_2}{S_1}}$

$\frac{Q_2}{Q_1} = \sqrt{\frac{\frac{1}{1000}}{\frac{1}{1690}}}$

$Q_2 = 1.3 \times 15 = 19.5$ m^3/s

03. Ans: (d)

Sol: $Q = AV$

$= B \times y \times \frac{1}{n} R^{2/3} S^{1/2}$

$= B \times y \times \frac{1}{n} y^{2/3} S^{1/2}$

$= R \approx y \rightarrow$ For wide rectangular channel $Q \propto y^{5/3}$

$\frac{Q_2}{Q_1} = \left(\frac{y_2}{y_1}\right)^{\frac{5}{3}}$

$\frac{Q_2}{Q_1} = \left(\frac{1.25 y_1}{y_1}\right)^{\frac{5}{3}}$

$\frac{Q_2}{Q_1} = 1.45$

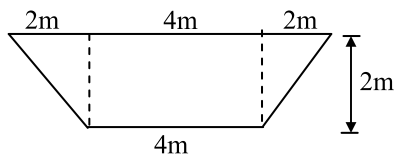
$Q_2 = 1.45\, Q_1$

It is increased by 45%

04. Ans: (c)

05. Ans: 24.33

Sol:



$\tau_{avg} = \gamma_w R S$

$R = \frac{A}{P}$

$A = 2 \times \left(\frac{1}{2} \times 2 \times 2\right) + 4 \times 2$

$= 2\times 2+4\times 2 = 12\ m^2$

$P = 4+2\sqrt{2^2+2^2}$

$= 9.66$ m

$R = \dfrac{12}{9.66} = 1.24\,m$

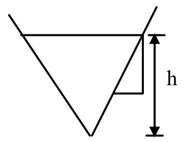
$\tau_{avg} = 9810 \times 1.24 \times 0.002$

$= 24.33\ N/m^2$

06. Ans: (d)

Sol:

Triangular:



Triangle

P = 2 (Inclined portion)

$P = 2I = 2h\sqrt{1+m^2} \quad \left(\because\ I = h\sqrt{1+m^2}\right)$

$= 2h\sqrt{1+1^2}$

$= 2h\sqrt{2}$

$\dfrac{P}{h} = 2\sqrt{2} = 2.83$

Trapezoidal: Efficient trapezoidal section is half of the Hexagon for which all sides are equal

Trapezoidal

$I = h\sqrt{1+m^2}$

$P = I = h\sqrt{(1)+\left(\dfrac{1}{\sqrt{3}}\right)^2} = h(1.15)$

$\dfrac{P}{h} = 1.15\times 3 = 3.46$ (3 sides are equal)

Rectangular:

$P = b+2h = 2h+2h = 4h\ (\ b = 2y\ )$

$\dfrac{P}{h} = 4$

07. Ans: 0.37

Sol:

A = y (b + my)

$A = \dfrac{Q}{y} = 4\,m^2$

$4 = \left(b+\dfrac{y}{\sqrt{3}}\right)y$ …….(I) $\left(\because\ m = \dfrac{1}{\sqrt{3}}\right)$

But b = I ($\because$ Efficient trapezoidal section)

$b = y\sqrt{1+m^2}$

$b = \dfrac{2y}{\sqrt{3}}$ ……………(II)

From (I) & (II)

y= 1.519 m

$\therefore\ D = \dfrac{(b+my)y}{b+2my} = 1.14\,m$

$\therefore F_r = \dfrac{V}{\sqrt{gD}}$

$F_r = 0.37$

08. Ans: (a)

Sol: Alternate depths

$y_1 = 0.4$ m

$y_2 = 1.6$ m

Specific energy at section =?

$$y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2g\,y_2^2}$$

$$0.4 + \frac{q^2}{2\times 9.81\times 0.4^2} = 1.6 + \frac{q^2}{2\times 9.81\times 1.6^2}$$

$$q^2\left(\frac{1}{3.1392} - \frac{1}{50.22}\right) = 1.6 - 0.4$$

$$q^2(0.298) = 1.2$$

$$q^2 = 4.02$$

$$q = 2\ m^3/s/m$$

$$E_1 = y_1 + \frac{q^2}{2gy_1^2}$$

$$E_1 = 0.4 + \frac{2^2}{2\times 9.81\times 0.4^2} = 1.68\ m$$

09. Ans: (b)

Sol: Depth = 1.6 m

Specific energy = 2.8 m

$$E_1 = \left[y_1 + \frac{v^2}{2g}\right] \Rightarrow 2.8 = 1.6 + \frac{V^2}{2\times 9.81}$$

$V = 4.85$ m/s

$$F_r = \frac{V}{\sqrt{gy}}$$

$$F_r = \frac{4.85}{\sqrt{9.81\times 1.6}} = 1.22 > 1\,(\text{Supercritical})$$

10. Ans: (c)

Sol: $F_r = 5.2$ (uniform flow)

The ratio of critical depth to normal depth $\frac{y_c}{y_n} = ?$

Note: The given two depths y_c & y_n are not alternate depths as they will have different specific energies.

$$F_r = \frac{V}{\sqrt{gy}} \Rightarrow F_r^2 = \frac{V^2}{gy} = \frac{q^2}{gy^3}\left(\because v = \frac{q}{y}\right)$$

$$\frac{(F_{rn})^2}{(F_{rc})^2} = \frac{q^2}{gy_n^3}\times\frac{gy_c^3}{q^2} = \frac{y_c^3}{y_n^3}$$

$$\frac{y_c^3}{y_n^3} = \frac{(F_{rn})^2}{(F_{rc})^2} \Rightarrow \frac{y_c}{y_n} = \frac{(F_{rn})^{2/3}}{(F_{rc})^{2/3}}$$

$$\frac{y_c}{y_n} = (5.2)^{2/3} = 3$$

11. Ans: (c)

Sol: Rectangular channel

Alternate depths $y_1 = 0.2$, $y_2 = 4$m

$E_1 = E_2$ ($\because$ alternate depths), $F_r = \frac{V}{\sqrt{gD}}$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$y_1\left(1 + \frac{Fr_1^2}{2}\right) = y_2\left[1 + \frac{Fr_2^2}{2}\right]$$

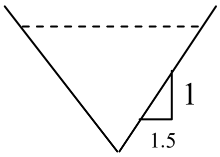
$$\frac{y_1}{y_2} = \left[\frac{1 + \frac{Fr_2^2}{2}}{1 + \frac{Fr_1^2}{2}}\right]$$

$$\frac{y_1}{y_2} = \left[\frac{1+\frac{4^2}{2}}{1+\frac{0.2^2}{2}}\right]$$

$$\frac{y_1}{y_2} = \left(\frac{2+16}{2+0.04}\right) = 8.8$$

12. Ans: (d)

Sol: Triangular channel



H:V = 1.5:1

Specific energy = 2.5 m

$$E_C = \frac{5}{4} y_c$$

$$\frac{4}{5} E_C = y_c$$

$y_c = 2\text{m}$

$$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5} \Rightarrow 2 = \left(\frac{2\times Q^2}{9.81\times 1.5^2}\right)^{1/5}$$

$Q = 18.79\ m^3/\text{sec}$

13. Ans: 0.47

Sol: $E_1 = E_2 + (\Delta z)$

$$V_1 = \frac{Q}{A_1} = \frac{12}{2.4\times 2} = 2.5\,\text{m/sec}$$

$A_2 = (b_2 + my_2)y_2 = (1.8 + 1\times 1.6)\ 1.6$

$= 5.44\ m^2$

$$V_2 = \frac{Q}{A_2} = \frac{12}{5.44} = 2.2\text{m/sec}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2 + \frac{(2.5)^2}{2\times 9.81} = 2.318\text{m}$$

$$E_2 = y_2 + \frac{V_2^2}{2g} = 1.6 + \frac{2.2^2}{2\times 9.81} = 1.846\ \text{m}$$

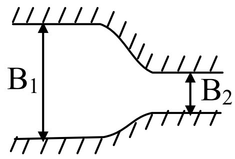
$2.318 = 1.846 + \Delta Z \Rightarrow \Delta Z = 0.47$ m

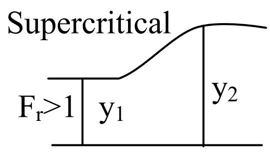
14. Ans: (c)

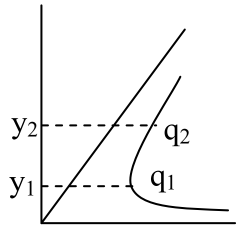
Sol: $F_r > 1$

$B_2 < B_1$

$q_2 > q_1$







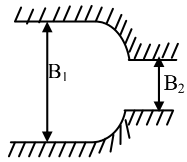
As Potential energy (y) increases then kinetic energy (ν) decreases

∴ 'y' increases and 'ν' decreases.

15. Ans: (a)

Sol: $Q = 3m^3/s$

$B_1 = 2m$, $D = 1.2$ m



Width reduce d to 1.5 m (B_2)

Assume channel bottom as horizontal

$\therefore E_1 = E_2$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g}$$

$$V_1 = \frac{Q}{B_1 y_1} = \frac{3}{2\times 1.2} = 1.25\,\text{m/sec}$$

$$V_2 = \frac{Q}{B_2 y_2} = \frac{3}{1.5 \times y_2} = \frac{2}{y_2}$$

$$1.2 + \frac{(1.25)^2}{2 \times 9.81} = y_2 + \frac{\left(\frac{2}{y_2}\right)^2}{2 \times 9.81}$$

$$1.27 = y_2 + \frac{4}{y_2^2 \times 19.62}$$

$$1.27 = y_2 + \frac{0.2}{y_2^2}$$

$$y_2^2(1.27) = y_2^3 + 0.2$$

$$y_2^3 - 1.27y_2^2 + 0.2 = 0$$

$$y_2 = 1.12 \text{ m}$$

$$F_{r_1} = \frac{1.25}{\sqrt{9.81 \times 1.2}}\left[\frac{V}{\sqrt{gD}} < 1\right] = 0.364 < 1$$

Approaching flow is sub critical. If approaching flow is sub critical the level at water falls in the throat portion.

16. Ans: (d)

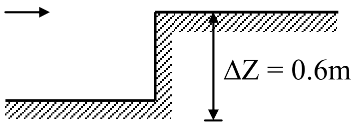
Sol: Rectangular Channel

$y_1 = 1.2$m

$V_1 = 2.4$m/s

$\Delta Z = 0.6$m

$$E_1 = y_1 + \frac{V_1^2}{2g} = 1.2 + \frac{(2.4)^2}{2 \times 9.81} = 1.49\text{m}$$



$Q = 2.4 \times 1.2 = 2.88$ m^3/s/m

Assuming channel width as constant, the critical depth

$$y_c = \left[\frac{Q^2}{gB^2}\right]^{\frac{1}{3}} = 0.94\text{m}$$

Critical specific energy for rectangular channel $E_C = \frac{3}{2} y_c$

$$E_c = \frac{3}{2}(0.94) = 1.41$$

We know for critical flow in the hump portion $E_1 = E_2 + (\Delta Z) = E_C + (\Delta Z)_C$

$\Rightarrow 1.49 = 1.41 + (\Delta Z)_C$

$\therefore (\Delta Z)_C = 0.08$m

If the hump provided is more than the critical hump height the u/s flow gets affected.

(or)

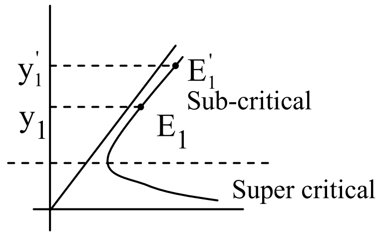
$$Fr_1 = \frac{v_1}{\sqrt{gy_1}} = \frac{2.4}{\sqrt{9.81 \times 1.2}} = 0.69 < 1$$

$\Rightarrow$ Hence sub-critical.

If the approaching flow is sub-critical the level of water will fall in the hump portion. Option (b) is correct if the hump height provided is less than critical hump height.

As the hump height provided is more than critical, the u/s flow gets affected with the increase of the specific energy from E_1 to E_1^1.

In the sub-critical region as the specific energy increases, the level of water rises from y_1 to y_1^1 in the form of a surge.



$$E_1^1 = y_1^1 + \frac{v^{1^1}}{2g}$$

$$E_1^1 = y_1^1 + \frac{q^2}{2gy_1^{1^2}} \quad \dots (1)$$

Also $E_1^1 = E_c + (\Delta Z)$ provided.

$= 1.41 + 0.6$

$= 2.01\text{m}$

$$\therefore 2.01 = y_1^1 + \frac{2.88^2}{2 \times 9.81 \times y_1^2}$$

Solve by trial & error

for $y_1^1 > 1.2\text{m}$

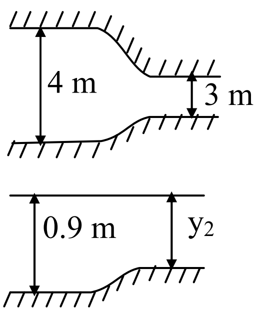
17. Ans: (c)

Sol: $B_1 = 4$ m

$B_2 = 3$ m

(U/S) $y_1 = 0.9$ m

$E_1 = E_2 + \Delta Z$



$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta Z$$

$V_1 = V_2$

According to continuity equation

$Q_1 = Q_2$

$A_1V_1 = A_2\,V_2$

$A_1 = A_2$

$B_2y_1 = B_2\,y_2$

$4 \times 0.9 = 3 \times y_2$

$y_2 = 1.2$ m

$y_1 = y_2 + \Delta Z$

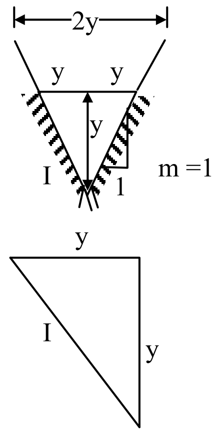
$0.9 = 1.2 + \Delta Z$

$\Delta Z = -0.3$ m

Negative indicates that the hump assumed is wrong infact it is a drop.

18. Ans: (a)

Sol: Given :



Top width = 2y

$$\text{Area} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 2y \times y$$

$A = y^2$

Wetted perimeter

$$I^2 = \sqrt{y^2 + y^2} = y\sqrt{2}$$

(Both sides) total wetted perimeter

$(P) = \sqrt{2}.y + \sqrt{2}.y = 2\sqrt{2}.y$

Hydraulic mean depth

$$(R) = \frac{A}{P} = \frac{y^2}{2\sqrt{2}y} = \frac{y}{2\sqrt{2}}$$

$y = y_n$(say)

Using Mannings formula

$$Q = A.\frac{1}{n}.(R)^{2/3}.(S)^{1/2}$$

$$0.2 = y_n^2 \frac{1}{0.015}\left[\frac{y_n}{2\sqrt{2}}\right]^{2/3}(0.001)^{1/2}$$

$$\frac{1}{y_n^{8/3}} = \frac{1}{0.015\times 0.2}\times\left[\frac{1}{2\sqrt{2}}\right]^{2/3}(0.001)^{1/2}$$

$$y_n^{8/3} = 0.2\times 0.015\times\left(2\sqrt{2}\right)^{2/3}\left[\frac{1}{0.001}\right]^{1/2}$$

$$(y_n)^{8/3} = 0.189$$

$$y_n = (0.189)^{3/8}$$

$y_n = 0.54$ m

$$\text{critical depth}(y_c) = \left[\frac{2Q^2}{g}\right]^{1/5}$$

(for triangle)

$$y_c = \left[\frac{2\times 0.2^2}{9.81}\right]^{1/5} = 0.382\,\text{m}$$

$y_n > y_c$ (0.54>0.38)

$\therefore$ mild slope

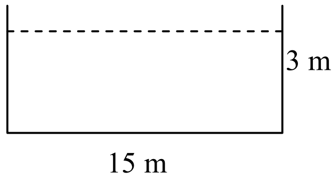
If (actual) depth at flow = 0.4m = y

$Y_n > y > y_c$ [0.54 > 0.4 > 0.38]

$\therefore$ Profile is M_2

19. Ans: 4.36×10^{-5}

Sol:



$\therefore$ Discharge, Q = 29 m^3/sec

Area of rectangular channel, A = 15 × 3 = 45 m^2

Perimeter, P = 15 + 2 × 3 = 21 m

Hydraulic radius, $R = \frac{A}{P} = \frac{45}{21} = 2.142$ m

$\therefore$ The basic differential equation governing the gradually varied flow is

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2T}{gA^3}}$$

$\frac{dy}{dx}$ = Slope of free water surface w.r.t to channel bottom

Velocity of flow $V = \frac{Q}{A} = \frac{29}{45}$

= 0.644 m/sec

$\therefore$ By Chezy's equation

Velocity, $V = C\sqrt{RS_f}$

$$0.644 = 65\sqrt{2.142\times S_f}$$

$S_f = 4.589 \times 10^{-5}$

$$S_o = \frac{1}{5000} = 2\times 10^{-4}$$

$$\frac{Q^2T}{gA^3} = \frac{29^2\times 15}{9.81\times 4^3} = 0.0141$$

$$\therefore \frac{dy}{dx} = \frac{2\times10^{-4} - 4.589\times10^{-5}}{1-0.0141}$$

$= 1.5631 \times 10^{-4}$

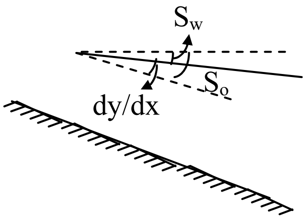
$$\therefore S_o = S_w + \frac{dy}{dx}$$

S_w water surface slope with respect to horizontal

$$S_w = S_o - \frac{dy}{dx}$$

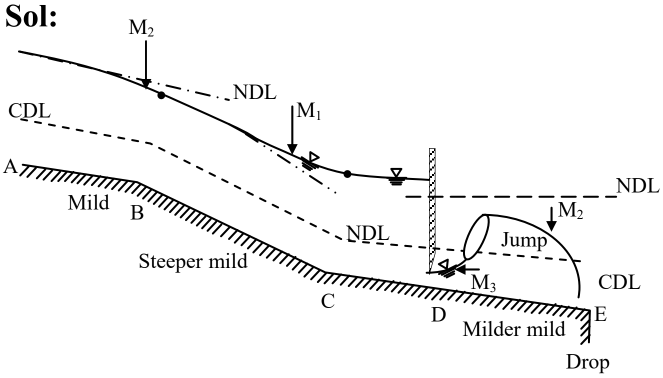
$= 2 \times 10^{-4} - 1.563 \times 10^{-4}$

$S_w = 4.36 \times 10^{-5}$



20. Ans: (a)

Sol:



21. Ans: (d)

22. Ans: 0.74

Sol: Free fall →2nd profile

Critical depth, $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$

$$y_c = \left(\frac{2^2}{9.81}\right)^{\frac{1}{3}} = 0.74 \text{ m}$$

$$V = \frac{q}{y_n}$$

$$\frac{2}{y_n} = \frac{1}{n} y_n^{2/3} S^{1/2}$$

$$\frac{2}{y_n} = \frac{1}{0.012} \times y_n^{2/3} (0.0004)^{1/2}$$

$y_n = 1.11$ m

$y_n > y_c$

Hence the water surface will have a depth equal to y_c

$y_c = 0.74$ m

23. Ans: (d)

Sol: $q = 2$ m^2/sec

$y_A = 1.5$ m; $y_B = 1.6$ m

$\Delta E = 0.09$

$$S_o = \frac{1}{2000}$$

$\overline{S_f} = 0.003$

$$\Delta x = \frac{\Delta E}{S_o - \overline{S_f}} = \frac{0.09}{\frac{1}{2000} - 0.003} = -36 \text{ m}$$

24. Ans: (d)

Sol: Given $q_1 = Q/B = 10$ m^3/s

$v_1 = 20$ m/s

$$\therefore y_1 = \frac{q_1}{v_1} = \frac{10}{20} = 0.5 \text{ m}$$

We know that relation between y_1 and y_2 for hydraulic jump is

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8Fr_1^2}\right]$$

$$Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{20}{\sqrt{9.81\times 0.5}} = 9.03$$

$$\therefore \ \frac{y_2}{0.5} = \frac{1}{2}\left[-1+\sqrt{1+8\times(9.03)^2}\right]$$

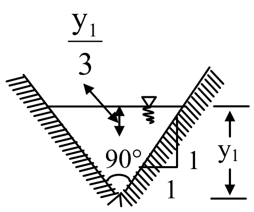
$y_2 = 6.14$ m

25. Ans: (c)

Sol: $Q = 1 m^3/s$

$y_1 = 0.5m$

$y_2 = ?$



As it is not a rectangular channel, let us work out from fundamentals by equating specific force at the two sections.

$$\left[\frac{Q^2}{gA}+A\bar{z}\right]_1 = \left[\frac{Q^2}{gA}+A\bar{z}\right]_2$$

$$\frac{1^2}{9.81\times y_1^2} + y_1^2\times\frac{y_1}{3} = \frac{1^2}{9.81y_2^2} + y_2^2\times\frac{y_2}{3}$$

$$0.449 = \frac{1}{9.81y_2^2} + \frac{y_2^3}{3}$$

$y_2 = 1.02$ m

26. Ans: (b)

Sol: Given:

Head = 5m = (ΔE)

Froud number = 8.5

Approximate sequent depths =?

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1+\sqrt{1+8F_{r1}^2}\right]$$

$$= \frac{1}{2}\left[-1+\sqrt{1+8(8.5)^2}\right]$$

$= 11.5$ m

$y_2 = 11.5\ y_1$

$$\left.\begin{array}{l}\text{(a) } y_2 = 11.5(0.3) = 3.45\\ \text{(b) } y_2 = 11.5(0.2) = 2.3\text{ m}\end{array}\right\}\text{from options}$$

$y_1 = 0.2$, $y_2 = 2.3$ m

(or)

$\Delta E = 5m$

$$\Delta E = \frac{(y_2-y_1)^3}{4y_1y_2}$$

$$\frac{(11.5y_1-y_1)^3}{4(11.5y_1)y_1} = 5$$

$$(10.5y_1)^3 = 230y_1^2$$

$$1157.625\ y_1 = 230$$

$y_1 = 0.2$ m

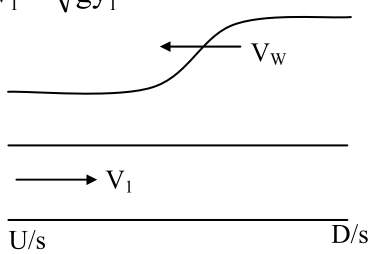
$y_2 = 11.5(0.2)$

$y_2 = 2.3$ m

27. Ans: 1.43

Sol: $y_1 = 1.2$ m

$$V_w + V_1 = \sqrt{gy_1}$$



$$V_1 = \sqrt{9.81\times 1.2} - 2$$

$V_1 = 1.43$ m/s

In this problem if the wave moves downstream the velocity of wave is

$$V_w - V_1 = \sqrt{gy_1}$$

$$V_w = \sqrt{gy_1} + V_1$$

$$= \sqrt{9.81 \times 1.2} + 2 = 5.43 \text{ m/s}$$

28. Ans: (b) **29. Ans: (c)**

Chapter- 13
Dimensional Analysis

01. Ans: (c) **02. Ans: (b)** **03. Ans: (b)**

04. Ans: (c) **05. Ans: (c)**

06. Ans: (a)

Sol: $L_r = \dfrac{1}{16}$

$Q_P = 1024$; $Q_m = ?$

$$Q_r = L_r^{5/2} = \frac{Q_m}{Q_P} = \left(\frac{1}{16}\right)^{5/2}$$

$$\frac{Q_m}{Q_p} = \left(\frac{1}{16}\right)^{5/2}$$

$Q_m = 1 \text{ m}^3/\text{sec}$

07. Ans: (b)

Sol: According to Froude's law

$$T_r = \sqrt{L_r}$$

$$\frac{t_m}{t_p} = \sqrt{L_r}$$

$$t_p = \frac{t_m}{\sqrt{L_r}} = \frac{10}{\sqrt{1/25}}$$

$t_p = 50$ min

08. Ans: (a)

Sol: $V_P = 10$ m/s dia = 3m

$V_m = 5$ m/s, $F_m = 50$ N, $F_p = ?$

Acc to Froude's law:- $F_r = L_r^3$

(But L_r is not given)

$$P \propto \rho V^2 = \frac{F}{A}$$

$\boxed{\rho A V^2 = F}$ Reynolds law

Now scale ratio:

$$\frac{F_m}{F_P} = \frac{V_m^2}{V_P^2} \times \frac{A_m}{A_p} \times \frac{\rho_m}{\rho_p}$$

$$\frac{50}{F_P} = \left(\frac{1}{10}\right)^2 \times \left(\frac{5}{10}\right)^2 \left(A = L_r^2\right) \quad (\because \text{same fluid})$$

$F_P = 20000$ N

09. Ans: (a)

Sol: L = 100 m

$V_P = 10\,\text{m/s}$,

$L_r = \dfrac{1}{25}$

As viscous parameters are not discussed follow Froude's law

Acc to Froude

$$V_r = \sqrt{L_r}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{1}{25}}$$

$$V_m = \frac{1}{5}\times 10 = 2 \text{ m/s}$$

10. Ans: (c)

Sol: $L_r = 1 : 25$, $F_m = 5$ N, $F_p = ?$

$$\frac{F_m}{F_p} = L_r^3$$

$F_p = 78.125$ kN

11. Ans: (a)

Sol: $L_r = \dfrac{1}{100}$

$a_m = 0.013$

$$\frac{a_m}{a_p} = (L_r)^{\frac{1}{6}}$$

$$a_p = \frac{a_m}{(L_r)^{\frac{1}{6}}} = \frac{0.013}{\left(\frac{1}{100}\right)^{1/6}}$$

$a_p = 0.028$

12. Ans: (a)

Sol: $L_r = \dfrac{1}{9}$

$y_{p1} = 0.5$ m , $y_{p2} = 1.5$ m

$q_m = ?$, $q_p = ?$

$$\frac{2q_P^2}{g} = y_{1p}.y_{2p}(y_{1p} + y_{2p})$$

$$\frac{2q_P^2}{9.81} = 0.5\times 1.5\times(0.5 + 1.5)$$

$$\frac{2q_P^2}{9.81} = (0.5)(1.5)(2)$$

$q_p = 2.71$

$$q_r = \frac{q_m}{q_p} = L_r^{3/2}$$

$$q_m = \left(\frac{1}{9}\right)^{3/2}\times q_p = 0.1 \text{ m}^3\text{/s/m}$$

13. Ans: (c)

Sol: For distorted model according to Froude's law

$Q_r = L_H L_V^{3/2}$

$L_H = 1:1000$,

$L_V = 1:100$

$Q_m = 0.1$ m^3/s

$$Qr = \frac{1}{1000}\times\left(\frac{1}{100}\right)^{3/2} = \frac{0.1}{Q_p}$$

$Q_P = 10^5$ m^3/s

14. Ans: (a)

Sol: $L_H = 1:1000$, $L_V = 1:100$

$q_m = 0.1$ m^3/sec

$q_P = ?$

$q_r = (L_V)^{3/2}$

$$\frac{q_m}{q_P} = \left(\frac{1}{100}\right)^{3/2}$$

$Q_P = q_m\times 100$

$= 0.1\times 1000 = 10^2$

$$\left[\begin{aligned} q_r &= \frac{Q_r}{L_H} \\ &= \frac{L_H.L_V^{3/2}}{L_H} \\ &= L_V^{3/2} \end{aligned}\right]$$

15. Ans: (b)

Sol: $L_H = \frac{1}{150}$, $T_r = \frac{L_H}{\sqrt{L_V}}$

$L_V = \frac{1}{60}$

$\frac{T_m}{T_P} = \frac{1}{150} \times \frac{\sqrt{60}}{1}$

$T_m = 0.0516 \times T_P$

The actual time interval between two successful high tides in a sea → 12 hour 24 min

$T_P = (12\times60) + 24$

$= 744$ min

$T_m = 0.0516 \times 744 \simeq 40$ min

16. Ans: (d)

Sol: Froude number = Reynolds number.

$\nu_r = 0.0894$

If both gravity & viscous forces are important then

$\nu_r = (L_r)^{3/2}$

$\sqrt[3]{\frac{\nu_m}{\nu_p}} = L_r$

$L_r = 1:5$

Chapter- 14
Turbomachinery

01. Ans: 1000

Sol: T = Moment of momentum of water in a turbine = Torque developed = 15915 N-m

Speed (N) = 600 rpm

Power developed $= \frac{2\pi NT}{60}$

$= \frac{2\times\pi\times600\times15915}{60}$

$= 1000 \times 10^3$ W = 1000 kW

02. Ans: 4000

Sol: $Q = 50$ m^3/sec

$H = 7.5$ m

$\eta_{Turbine} = 0.8$

$\eta_{Turbine} = \frac{P_{shaft}}{P_{water}} = \frac{P_{shaft}}{\rho g Q(H - h_f)}$

$0.8 = \frac{P_{shaft}}{1000\times9.81\times50(7.5-0)}$

$P_{shaft} = 2943\times10^3$ W = 2943 kW

$= \frac{2943}{0.736}$ HP = 4000 HP

03. Ans: 1

Sol: We know that

$$U = \frac{\pi DN}{60} = k_u.\sqrt{2gH}$$

Where D = diameter of wheel

N = speed of turbine = 600 rpm

H = Head available of pelton wheel turbine = 300 m

$$\therefore \frac{\pi \times D \times 600}{60} = 0.41\sqrt{2 \times 9.81 \times 300}$$

$$D = 1.0 \text{ m}$$

04. Ans: (b)

05. Ans: (b)

Sol: P = 8.1 MW = 8100 kW

H = 81 m

N = 540 rpm

Specific speed $N_S = \dfrac{N\sqrt{P}}{(H)^{5/4}}$

$$= \frac{540 \times \sqrt{8100}}{(81)^{5/4}}$$

$$= \frac{540 \times 90}{243} = 200$$

$60 < N_S < 300$ (Francis Turbine)

06. Ans: (a) **07. Ans: (b)** **08. Ans: (a)**

09. Ans: (d) **10. Ans: (d)**

11. Ans: 1000

Sol: Given $N_p = 500$ rpm

$$\frac{D_m}{D_p} = \frac{1}{2}$$

We know that

$$\left(\frac{ND}{\sqrt{H}}\right)_m = \left(\frac{ND}{\sqrt{H}}\right)_P$$

Given H is constant

$$\therefore \frac{N_m}{N_p} = \frac{D_p}{D_m}$$

$$\therefore \frac{N_m}{500} = 2$$

$$\Rightarrow N_m = 1000 \text{ rpm}$$

12. Ans: 72

Sol: Given $P_1 = 100$ kW

$H_1 = 100$ m and $H_2 = 81$ m

We know that

$$\left(\frac{P}{(H)^{3/2}}\right) = \left(\frac{P}{(H)^{3/2}}\right)_2$$

$$\therefore \frac{100}{(100)^{3/2}} = \frac{P_2}{(80)^{3/2}}$$

$P_2 = 71.55$ kW $\simeq$ 72 kW

$\therefore$ New power developed by same turbine = 72 kW