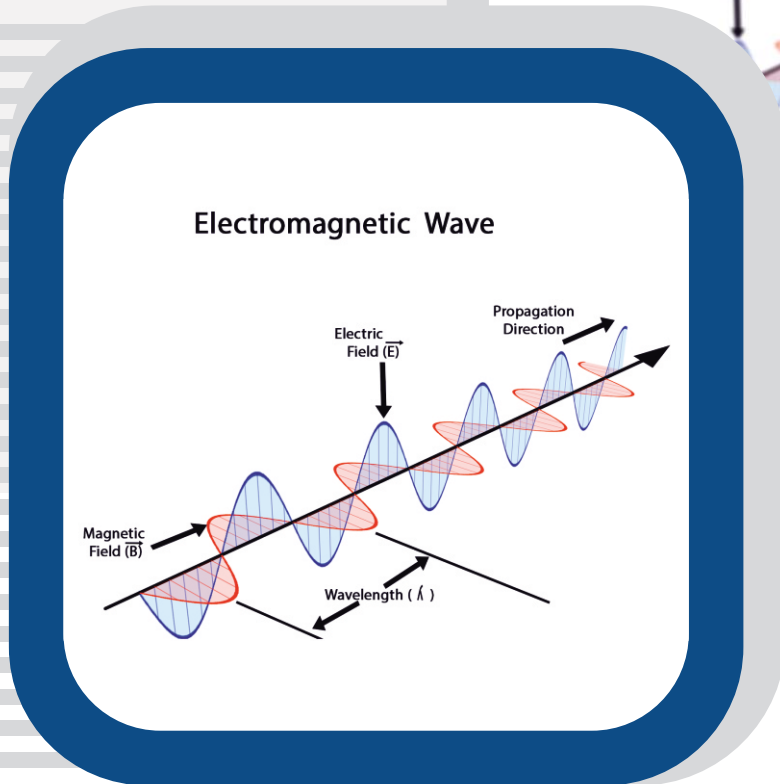




GATE | PSUs

ELECTRONICS & COMMUNICATION ENGINEERING



ELECTRONICS & COMMUNICATION ENGINEERING

ELECTROMAGNETIC THEORY

Volume-1 : Study Material with Classroom Practice Questions

01. Ans: 1

Sol: $\vec{V} = x \cos^2 y \hat{i} + x^2 e^z \hat{j} + z \sin^2 y \hat{k}$
 $= x \cos^2 y \hat{a}_x + x^2 e^z \hat{a}_y + z \sin^2 y \hat{a}_z$

From divergence theorem

$$\iiint_V \nabla \cdot \hat{n} \, ds = \int_V (\nabla \cdot \vec{D}) \, dv \dots\dots\dots 1$$

$$\nabla \cdot \vec{D} = \frac{\partial}{\partial x}(x \cos^2 y) + \frac{\partial}{\partial y}(x^2 e^z) + \frac{\partial}{\partial z}(z \sin^2 y)$$

$$= \cos^2 y + \sin^2 y = 1$$

$$dv = dx dy dz$$

Putting these value in equation 1 we have

$$\iiint_V \nabla \cdot \hat{n} \, ds = \int_0^1 \int_0^1 \int_0^1 1 \times dx \, dy \, dz$$

$$= \int_0^1 dx \int_0^1 dy \int_0^1 dz = 1$$

02. Ans: (c)

Sol: Given $\vec{A} = x y \vec{a}_x + x^2 \vec{a}_y$

Let $I = \oint_C \vec{A} \cdot d\vec{l}$, I is evaluated over the path shown in the Fig., as follows

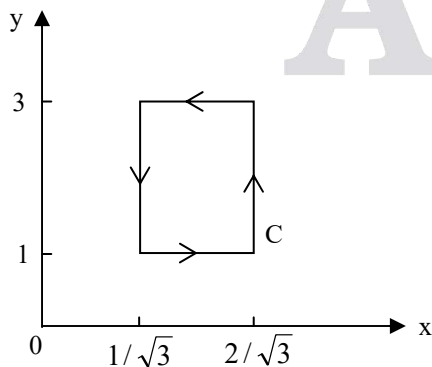


Fig.

$$I = \oint_C \vec{A} \cdot d\vec{x} \vec{a}_x, y = 1, x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$+ \int \vec{A} \cdot d\vec{y} \vec{a}_y, x = \frac{2}{\sqrt{3}}, y = \text{from } 1 \text{ to } 3$$

$$- \int \vec{A} \cdot d\vec{x} \vec{a}_x, y = 3, x = \text{from } \frac{1}{\sqrt{3}} \text{ to } \frac{2}{\sqrt{3}}$$

$$- \int \vec{A} \cdot d\vec{y} \vec{a}_y, x = 1/\sqrt{3}, y = \text{from } 1 \text{ to } 3$$

$$= \int x y \, dx + \int x^2 \, dy - \int x y \, dx - \int x^2 \, dy$$

$$= y \frac{x^2}{2} \Big|_{1/\sqrt{3}}^{2/\sqrt{3}} + x^2 y \Big|_1^3 - y \frac{x^2}{2} \Big|_{1/\sqrt{3}}^{2/\sqrt{3}} - x^2 y \Big|_1^3$$

at $y = 1 \quad x = 2/\sqrt{3} \quad y = 3 \quad x = 1/\sqrt{3}$

$$= \frac{1}{2} \left(\frac{4}{3} - \frac{1}{3} \right) + \frac{4}{3} (3-1) - \frac{3}{2} \left(\frac{4}{3} - \frac{1}{3} \right) - \frac{1}{3} (3-1)$$

$$= \frac{1}{2} + \frac{8}{3} - \frac{3}{2} - \frac{2}{3} = -1 + 2 = 1$$

03. Ans: (d)

Sol: $\vec{F} = \rho a_\rho + \rho \sin^2 \phi a_\phi - z a_z$
 $= F_\rho a_\rho + F_\phi a_\phi + F_z a_z$

$$\nabla \cdot \vec{F} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho F_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (F_\phi) + \frac{\partial}{\partial z} (F_z)$$

$$= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho^2) + \frac{1}{\rho} \frac{\partial}{\partial \phi} (\rho \sin^2 \phi) + \frac{\partial}{\partial z} (-z)$$

$$= 2 + 2 \sin \phi \cos \phi - 1$$

$$= 1 + 2 \sin \phi \cos \phi$$

$$\nabla \cdot \vec{F} \Big|_{\phi=\pi/4} = 2, \quad \nabla \cdot \vec{F} \Big|_{\phi=0} = 1$$

$$\nabla \cdot \vec{F} \Big|_{\phi=\pi/4} = 2 \nabla \cdot \vec{F} \Big|_{\phi=0}$$

04. Ans: (c)

Sol: $\vec{D} = 2\hat{a}_x - 2\sqrt{3}\hat{a}_z \quad \vec{D} = |\vec{D}|\hat{a}_n$

$$|\vec{D}| = \sqrt{16} = 4$$

$$= \rho_s \hat{a}_n$$



$$\therefore \vec{D} = 4 \left\{ \frac{2\hat{a}_x - 2\sqrt{3}\hat{a}_z}{4} \right\}$$

$$= \rho_s \hat{a}_n \quad \therefore \rho_s = 4 \text{ C/m}^2$$

05. Ans: (d)

Sol: $V = 10y^4 + 20x^3$

$$E = -\nabla V = -60x^2 \hat{a}_x - 40y^3 \hat{a}_y$$

$$D = \epsilon_0 E = -60x^2 \epsilon_0 \hat{a}_x - 40y^3 \epsilon_0 \hat{a}_y$$

$$\nabla \cdot D = \rho_v$$

$$\rho_v = \frac{\partial}{\partial x} (-60x^2 \epsilon_0) + \frac{\partial}{\partial y} (-40y^3 \epsilon_0)$$

$$= -120x \epsilon_0 - 120y^2 \epsilon_0$$

$$\rho_v(\text{at } 2, 0) = -120 \times 2 \epsilon_0 - 120 \times 0^2 \epsilon_0$$

$$= -240 \epsilon_0$$

06. Ans: (d)

Sol: Given

$$V(x, y, z) = 50x^2 + 50y^2 + 50z^2$$

$$\vec{E}(x, y, z) \text{ in free space} = -\text{grad}(V)$$

$$= -\nabla V$$

$$= - \left[\frac{\partial}{\partial x} V \vec{a}_x + \frac{\partial}{\partial y} V \vec{a}_y + \frac{\partial}{\partial z} V \vec{a}_z \right]$$

$$= - [100x \vec{a}_x + 100y \vec{a}_y + 100z \vec{a}_z] \text{ V/m}$$

$$\vec{E}(1, -1, 1) =$$

$$- [100 \vec{a}_x - 100 \vec{a}_y + 100 \vec{a}_z] \text{ V/m}$$

$$E(1, -1, 1) = 100 \sqrt{(-1)^2 + (1)^2 + (-1)^2}$$

$$= 100\sqrt{3}$$

Direction of the electric field is given by the unit vector in the direction of \vec{E} .

$$\vec{a}_E = \frac{\vec{E}(1, -1, 1)}{|E(1, -1, 1)|} = \frac{1}{\sqrt{3}} [-\vec{a}_x + \vec{a}_y - \vec{a}_z]$$

or in i, j, k notation, $\vec{a}_E = \frac{1}{\sqrt{3}} [-i + j - k]$

07. Ans: (b)

Sol: For valid B, $\nabla \cdot B = 0$

$$\left(\frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z \right) (x^2 a_x - xy a_y - Kxz a_z) = 0$$

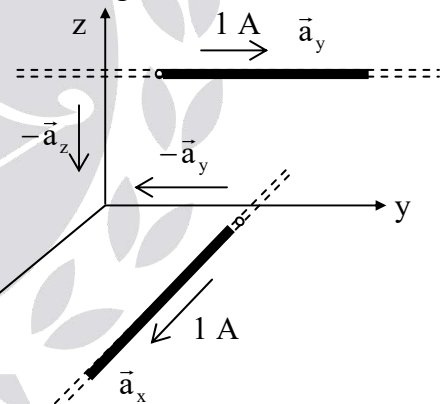
$$2x - x - Kx = 0$$

$$\Rightarrow 2 - 1 - K = 0$$

$$\therefore K = 1$$

08. Ans: (d)

Sol: The two infinitely long wires are oriented as shown in the Fig.



The infinitely long wire in the y-z plane carrying current along the \vec{a}_y direction produces the magnetic field at the origin in the direction of $\vec{a}_y \times -\vec{a}_z = -\vec{a}_x$.

The infinitely long wire in the x-y plane carrying current along the \vec{a}_x direction produces the magnetic field at the origin in the direction of $\vec{a}_x \times -\vec{a}_y = -\vec{a}_z$.

where \vec{a}_x , \vec{a}_y and \vec{a}_z are unit vectors along the 'x', 'y' and 'z' axes respectively.

\therefore x and z components of magnetic field are non-zero at the origin.



09. Ans: (a)

Sol: $\nabla \cdot \bar{B} = 0$

A divergence less vector may be a curl of some other vector

$$\bar{B} = \nabla \times \bar{A}$$

$$\nabla \times \bar{A} = \bar{B}$$

$$\oint_l \bar{A} \cdot d\bar{l} = \int_s \bar{B} \cdot d\bar{s}$$

$\int_s \bar{B} \cdot d\bar{s}$ is equal to magnetic flux ψ through a surface.

10. Ans: (c)

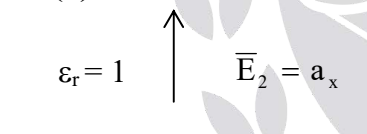
Sol: In general, for an infinite sheet of current density K A/m

$$H = \frac{1}{2} K \times \bar{a}_n$$

$$H = \frac{1}{2} (8\bar{a}_x \times \bar{a}_z) \\ = -4\bar{a}_y \quad (\because \bar{a}_x \times \bar{a}_z = -\bar{a}_y)$$

11. Ans: (b)

Sol:



$$D_{n_2} - D_{n_1} = \rho_s \rightarrow (a)$$

$$D_{n_2} = \epsilon E_{n_2} = \epsilon_0 a_x$$

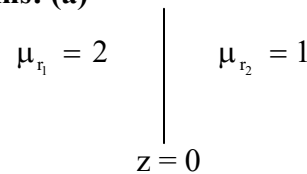
$$D_{n_1} = \epsilon_0 2 \times 2 a_x = 4\epsilon_0 a_x$$

From (a)

$$(\epsilon_0 - 4\epsilon_0) a_x = \rho_s \Rightarrow \rho_s = -3\epsilon_0$$

12. Ans: (a)

Sol:



$$B_1 = 1.2\bar{a}_x + 0.8\bar{a}_y + 0.4\bar{a}_z$$

$$B_{n_1} = 0.4\bar{a}_z$$

(Since $z = 0$ has normal component \bar{a}_x)

$$B_{t_1} = 1.2\bar{a}_x + 0.8\bar{a}_y$$

We know magnetic flux density is continuous

$$B_{n_1} = B_{n_2}$$

$$B_{n_2} = 0.4\bar{a}_z$$

Surface charge, $\bar{k} = 0$

$$H_{t_2} - H_{t_1} = 0$$

$$H_{t_2} = H_{t_1}$$

$$\mu_1 B_{t_2} = \mu_2 B_{t_1}$$

$$B_{t_2} = \frac{1}{2} (1.2\bar{a}_x + 0.8\bar{a}_y)$$

$$B_2 = B_{t_2} + B_{n_2} \\ = 0.6\bar{a}_x + 0.4\bar{a}_y + 0.4\bar{a}_z$$

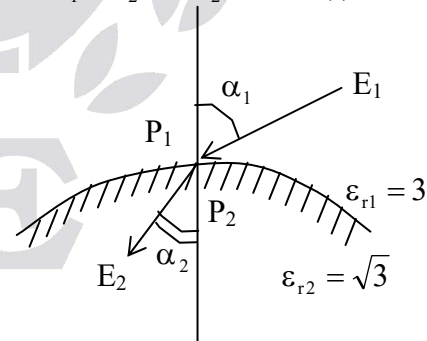
$$\mu_0 \mu_{r_2} H_2 = 0.6\bar{a}_x + 0.4\bar{a}_y + 0.4\bar{a}_z$$

$$H_2 = \frac{1}{\mu_0} [0.6\bar{a}_x + 0.4\bar{a}_y + 0.4\bar{a}_z] \text{ A/m}$$

13. Ans: (b)

Sol: Tangential components of electric fields are continuous ($E_{t_1} = E_{t_2}$)

$$E_1 \sin \alpha_1 = E_2 \sin \alpha_2 \text{ --- (1)}$$



Normal component of electric flux densities are continuous across a charge free interface

$$D_{n_1} = D_{n_2}$$

$$3E_1 \cos \alpha_1 = \sqrt{3}E_2 \cos \alpha_2 \text{ --- (2)}$$

$$\alpha_1 = 60^\circ$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\tan \alpha_1}{3} = \frac{\tan \alpha_2}{\sqrt{3}} \Rightarrow \tan \alpha_2 = 1$$

$$\alpha_2 = 45^\circ$$

Chapter 2 Maxwell Equations & EM Waves

**Identify polarization of following
(Page number 71 in Volume –I booklet)**

01. $\vec{E} = 20 \sin(\omega t - \beta x) \hat{a}_y$ V/m

Sol: At $x = 0$

$$\vec{E} = 20 \sin(\omega t) \hat{a}_y \text{ V/m}$$

Let $\theta = \omega t$

$$\theta = 0 \Rightarrow \vec{E} = 0$$

$$\theta = \frac{\pi}{2} \Rightarrow \vec{E} = 20 \hat{a}_y$$

$$\theta = \pi \Rightarrow \vec{E} = 0$$

$$\theta = \frac{3\pi}{2} \Rightarrow \vec{E} = -20 \hat{a}_y$$

$$\theta = 2\pi \Rightarrow \vec{E} = 0$$

i.e., linear polarization and also vertical polarization with respect to \hat{x} - axis

02. $\vec{H} = 45 \cos(\omega t - \beta z) \hat{a}_x$ A/m

Sol: This is linear polarization

03. $\vec{E} = 20 \sin(\omega t - \beta z) \hat{a}_x + 30 \sin(\omega t - \beta z) \hat{a}_y$

Sol: phase difference between \hat{a}_x component and \hat{a}_y component is 0°

So that it is linear polarization

Note: for phase difference 0° & 180° , irrespective of their amplitudes it must be in linear polarization.

04. $\vec{E} = 55 \cos(\omega t - \beta z) \hat{a}_x + 55 \sin(\omega t - \beta z) \hat{a}_y$

Sol: Phase difference between \hat{a}_x component and

$$\hat{a}_y \text{ component is } \frac{\pi}{2}$$

Amplitudes are same.

So it is circular polarization

at $z = 0$ and let $\theta = \omega t$

$$\theta = 0 \Rightarrow \vec{E} = 55 \hat{a}_x + 0 \hat{a}_y$$

$$\theta = \frac{\pi}{2} \Rightarrow \vec{E} = 0 \hat{a}_x + 55 \hat{a}_y$$

It is CCW direction i.e. RHCP

05. $\vec{E} = 40 \sin(\omega t - \beta y) \hat{a}_x + 50 \cos(\omega t - \beta y) \hat{a}_z$

Sol: Phase difference = $\frac{\pi}{2}$

Amplitudes = not same

So it is elliptical polarization. To decide direction of rotation follow below procedure.

At $y = 0$, and Let $\theta = \omega t$

$$\theta = 0 \Rightarrow \vec{E} = 0 \hat{a}_x + 50 \hat{a}_z$$

$$\theta = \frac{\pi}{2} \Rightarrow \vec{E} = 40 \hat{a}_x + 0 \hat{a}_z$$

$$\theta = \pi \Rightarrow \vec{E} = 0 \hat{a}_x - 50 \hat{a}_z$$

$$\theta = \frac{3\pi}{2} \Rightarrow \vec{E} = -40 \hat{a}_x + 0 \hat{a}_z$$

It is Anti clock wise direction i.e., Right Hand Elliptical Polarization.

06.

Sol: $\vec{E} = \text{Re} \{ [\hat{a}_x + j \hat{a}_y] e^{j(\omega t - \beta z)} \}$

$$\vec{E} = \text{Re} \left[(\cos(\omega t - \beta z) + j \sin(\omega t - \beta z)) \hat{a}_x + j (\cos(\omega t - \beta z) + j^2 \sin(\omega t - \beta z)) \hat{a}_y \right]$$

$$\vec{E} = (\cos(\omega t - \beta z) \hat{a}_x - \sin(\omega t - \beta z) \hat{a}_y)$$

Magnitudes of amplitudes are same, phase difference is $\frac{\pi}{2}$; So it is circular

polarization. Now we proceed to decide direction of rotation.

Here

$$\vec{E} = \cos(\omega t - \beta z) \hat{a}_x - \sin(\omega t - \beta z) \hat{a}_y$$

At $z = 0$ & let $\theta = \omega t$



$$\theta = 0 \Rightarrow \bar{E} = \hat{a}_x - 0\hat{a}_y$$

$$\theta = \frac{\pi}{2} \Rightarrow \bar{E} = 0\hat{a}_x - \hat{a}_y$$

$$\theta = \pi \Rightarrow \bar{E} = -\hat{a}_x + 0\hat{a}_y$$

$$\theta = \frac{3\pi}{2} \Rightarrow \bar{E} = 0\hat{a}_x - \hat{a}_y$$

i.e., we get clock wise rotation i.e.,
Left Hand Circular Polarization

07. not a valid EM wave representation

08.

Sol: $\bar{E} = 5 \cos(\omega t - \beta r) \hat{a}_0$

Let $r = 0$ & $\theta = \omega t$

at $\theta = 0 \Rightarrow \bar{E} = 5\hat{a}_0$

$$\theta = \frac{\pi}{2} \Rightarrow \bar{E} = 0\hat{a}_0$$

$$\theta = \pi \Rightarrow \bar{E} = -5\hat{a}_0$$

$$\theta = \frac{3\pi}{2} \Rightarrow \bar{E} = 0\hat{a}_0$$

i.e., linear polarization

09.

Sol: $\bar{E} = \text{Im}\{[\hat{a}_x + 2j\hat{a}_z]e^{j(\omega t - \beta y)}\}$
 $= \text{Im}\{[\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)]\hat{a}_x + [2j\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)]\hat{a}_z\}$
 $= \sin(\omega t - \beta y)\hat{a}_x + 2\cos(\omega t - \beta y)\hat{a}_z$

Let $y = 0$ & $\theta = \omega t$

$$\theta = 0 \Rightarrow \bar{E} = 0\hat{a}_x + 2\hat{a}_z$$

$$\theta = \frac{\pi}{2} \Rightarrow \bar{E} = \hat{a}_x + 0\hat{a}_z$$

$$\theta = \pi \Rightarrow \bar{E} = 0\hat{a}_x - 2\hat{a}_z$$

$$\theta = \frac{3\pi}{2} \Rightarrow \bar{E} = -\hat{a}_x + 0\hat{a}_z$$

So it is Right Hand Elliptical Polarization

10. $\bar{E} = 20 \sin(\omega t - \beta y) \hat{a}_x + 30 \sin(\omega t - \beta y + 45^\circ) \hat{a}_z$

Sol: let $y = 0$ & $\theta = \omega t$

At $\theta = 0$

$$\Rightarrow \bar{E} = 0\hat{a}_x + 30 \sin 45^\circ \hat{a}_z$$

$$= 0\hat{a}_x + \frac{30}{\sqrt{2}} \hat{a}_z$$

At $\theta = \frac{\pi}{2} \Rightarrow \bar{E} = 20\hat{a}_x + 30 \sin(135^\circ) \hat{a}_z$

$$= 20\hat{a}_x + \frac{30}{\sqrt{2}} \hat{a}_z$$

At $\theta = \pi \Rightarrow \bar{E} = 0\hat{a}_x + 30 \sin(225^\circ) \hat{a}_z$

$$= 0\hat{a}_x - \frac{30}{\sqrt{2}} \hat{a}_z$$

At $\theta = \frac{3\pi}{2} \Rightarrow \bar{E} = -20\hat{a}_x + 30 \sin(315^\circ) \hat{a}_z$

$$= -20\hat{a}_x - \frac{30}{\sqrt{2}} \hat{a}_z$$

Note: $\theta = 62.76^\circ$ is the maximum values direction obtained by

$$\frac{d\bar{E}}{d\theta} = 0 \text{ at } y = 0 \text{ \& } \omega t = \theta$$

at $\theta = -\frac{\pi}{4} \Rightarrow \bar{E} = \frac{-20}{\sqrt{2}} \hat{a}_x + 0\hat{a}_z$

at $\theta = \frac{\pi}{4} \Rightarrow \bar{E} = \frac{20}{\sqrt{2}} \hat{a}_x + 30\hat{a}_z$

So it is RHEP

11. $\bar{E} = 20 \sin(\omega t - \beta z) \hat{a}_x + 20 \sin(\omega t - \beta z + 45^\circ) \hat{a}_y$

Sol: Valid EM wave but polarization can not defined.

This is a valid EM wave representation but it is not satisfy anyone of the polarization principle



Class Room Practice Solutions

01. Ans: (c)

Sol: Given flux $\phi = (t^3 - 2t) \text{mWb}$

$$\text{Magnitude of induced emf } |e'| = \left| \frac{d\phi}{dt} \right|_{t=4\text{sec}}$$

$$|e'| = 3t^2 - 2 \Big|_{t=4\text{sec}}$$

$$= 3(4)^2 - 2$$

$$= 46 \text{mWb}$$

This 'e' for one turn; but for 100 turns

$$|e| = N|e'| = 100 \times 46 \text{mWb}$$

$$|e| = 4.6 \text{ volts}$$

02. Ans: (d)

Sol: Given,

$$E = 120 \pi \cos(10^6 \pi t - \beta x) \hat{a}_y \text{ V/m}$$

$$H = A \cos(10^6 \pi t - \beta x) \hat{a}_z \text{ A/m}$$

$$\epsilon_r = 8; \mu_r = 2$$

$$\text{We know that, } \frac{E_y}{H_z} = \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$H_z = \frac{E_y}{120\pi\sqrt{\frac{2}{8}}} = \frac{2E_y}{120\pi} = 2A/m$$

$$H_z = 2 \cos(10^6 \pi t - \beta x) \hat{a}_z \text{ A/m}$$

$$\therefore A = 2$$

$$\beta = \omega \sqrt{\mu\epsilon} = \frac{10^6 \pi \times \sqrt{2 \times 8}}{3 \times 10^8}$$

$$= 0.0418 \text{ rad/m}$$

03. Ans: (b)

Sol: This question relates to normal incidence of a UPW on the air (medium 1) to glass (medium 2) interface as shown in Fig.

Medium, 1	Medium, 2
Air	Glass slab
$n_1 = 1$	$n_2 = 1.5$
$\mu_1 = \mu_0$	$\mu_2 = \mu_0$
$\epsilon_1 = \epsilon_0$	$\epsilon_2 = \epsilon_0 \epsilon_r$

Fig.

If n_1 and n_2 are the refractive indices and v_1 and v_2 are the velocities

$$\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}}$$

$$= \sqrt{\frac{\epsilon_1}{\epsilon_2}} \text{ for } \mu_1 = \mu_2 = \mu_0$$

For $n_1 = 1, n_2 = 1.5$

$$\sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{1}{1.5} = \frac{2}{3}$$

Reflection coefficient,

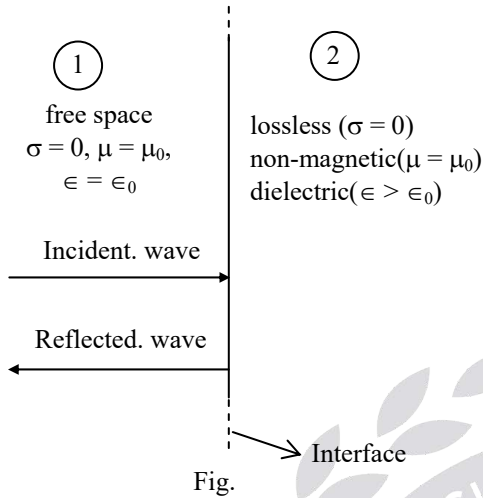
$$\frac{E_r}{E_i} = \frac{\sqrt{\frac{\epsilon_1}{\epsilon_2}} - 1}{\sqrt{\frac{\epsilon_1}{\epsilon_2}} + 1} = \frac{\frac{2}{3} - 1}{\frac{2}{3} + 1} = -\frac{1}{5}$$

$$\therefore \frac{P_r}{P_i} = \frac{|E_r|^2}{|E_i|^2} = \frac{1}{25} = 4\%$$



04. Ans: (d)

Sol: Normal incidence is shown in Fig.



Given: $E_{\max} = 5 E_{\min}$ in medium 1.

$$\therefore \text{VSWR, } S = \frac{E_{\max}}{E_{\min}} = 5$$

$$|K| = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

Reflection coefficient,

$$K = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{-2}{3}$$

$$-3 \frac{\eta_2}{\eta_1} + 3 = 2 \frac{\eta_2}{\eta_1} + 2$$

$$\therefore \frac{\eta_2}{\eta_1} = \frac{1}{5}, \quad \eta_2 = \frac{1}{5} \eta_1$$

$$\begin{aligned} \eta_1 &= \sqrt{\frac{\mu_0}{\epsilon_0}} \\ &= \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^9} \\ &= (120\pi) \Omega \end{aligned}$$

\therefore Intrinsic impedance of the dielectric medium, $\eta_2 = \frac{1}{5} \times 120\pi = 24\pi$

05. Ans: (a)

Sol: Given:

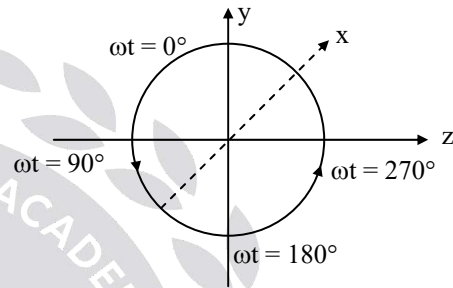
$$\vec{E} = 10(\hat{a}_y + j\hat{a}_z) e^{-j25x} \text{ in free space.}$$

$$\vec{E} = (E_y \hat{a}_y + E_z \hat{a}_z) e^{-j\beta x}$$

$$\beta = 25 = \frac{\omega}{c} \Rightarrow$$

$$\omega = 25 c = 25 \times 3 \times 10^8 \text{ rad/s}$$

$$f = 1.19 \text{ GHz} \approx 1.2 \text{ GHz}$$



$$E_y = 10, E_z = j 10$$

$$E_z \text{ leads } E_y \text{ by } 90^\circ$$

At $x = 0$

$$\text{Let } E_y = 10 \cos(\omega t)$$

$$\text{then } E_z = 10 \cos(\omega t + 90^\circ)$$

A Left Hand screw is to be turned in the direction along the circle as time increases so that the screw moves in the direction of propagation, 'x'.

\therefore The wave is left circularly polarized.

06. Ans: (b)

$$\text{Sol: } \vec{H} = 0.2 \cos(\omega t - \beta x) \hat{a}_z$$

Wave is progressing along + X direction

$\rightarrow (+X)$

$$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$$

$$\therefore \vec{E} = 0.2\eta \cos(\omega t - \beta x) \hat{a}_y$$

$$\vec{E}_s = 0.2\eta e^{-j\beta x} \hat{a}_y \quad \vec{H}_s = 0.2 e^{-j\beta x} \hat{a}_z$$

$$\begin{aligned} \vec{P}_{\text{avg}} &= \frac{1}{2} \vec{E}_s \times \vec{H}_s^* \\ &= \frac{1}{2} (0.2)^2 \eta \hat{a}_x \end{aligned}$$



$$= \frac{1}{2}(0.2)^2 (120\pi)\hat{a}_x \text{ w/m}^2$$

$$x = 1 \text{ plane} \Rightarrow \bar{ds} = dydz\hat{a}_x$$

$$W_{\text{avg}} = \int_s \bar{P}_{\text{avg}} \cdot \bar{ds} \text{ watts}$$

$$= \frac{1}{2}(0.2)^2 (120\pi) \iint dydz$$

$$= \left[\frac{1}{2}((0.2)^2 (120\pi)) \right] [\pi(5)^2] \times 10^{-4}$$

$$= 0.0592 \text{ Watts}$$

$$= 59.2 \text{ mW} \simeq 60 \text{ mW}$$

07. Ans: (a)

Sol: $P \propto \frac{1}{r^2}$

$$\frac{P_Q}{P_p} = \frac{r_p^2}{r_Q^2} = \left(\frac{R}{\frac{R}{2}} \right)^2$$

$$\frac{P_Q}{P_p} = \frac{4}{1} = 4:1$$

08. Ans: (b)

Sol: $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$

$$\delta \propto \sqrt{\frac{1}{f}} \Rightarrow \frac{\delta_1}{\delta_2} = \sqrt{\frac{f_2}{f_1}}$$

$$\frac{1.5}{\delta} = \sqrt{\frac{8 \times 10^9}{2 \times 10^9}}$$

$$\delta = \frac{1.5}{2} = 0.75 \mu\text{m}$$

Similarly

$$\frac{1.5}{\delta} = \sqrt{\frac{18 \times 10^9}{2 \times 10^9}} = 3$$

$$\delta = \frac{1.5}{3} = 0.5 \mu\text{m}$$

09. Ans: (b)

Sol: $\frac{\sigma}{\omega\epsilon} = \frac{5}{2 \times \pi \times 25 \times 10^3 \times 80 \times 8.854 \times 10^{-12}}$
 $= 44938.7$

Since $\frac{\sigma}{\omega\epsilon} \gg 1$ hence sea water is a good conductor

Where attenuation is 90%, transmission is 10%, then $e^{-\alpha x} = 0.1$

Where α is attenuation constant

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{2 \times \pi \times 25 \times 10^3 \times 4\pi \times 10^{-7} \times 5}{2}}$$

$$\alpha = 0.7025$$

$$-\alpha x = \ln(0.1)$$

$$-0.7025x = -2.3$$

$$x = 3.27\text{m}$$

10. Ans: (b)

Sol: $\delta = \frac{1}{\alpha} = \frac{1}{2\pi} = 0.159$

11. Ans: (c)

Sol: E is minimum

H is maximum

i.e., 'c' is the option

$$E_{\text{Tan}_1} = E_{\text{Tan}_2} = 0$$

[perfect conductor $E_{\text{Tan}_2} = 0$]

$$H_{\text{Tan}_1} = J_s \times a_n + H_{\text{Tan}_2}$$

$$H_{\text{Tan}_1} = J_s \times a_n$$

[perfect conductor $H_{\text{Tan}_2} = 0$]

12. Ans: (d)

Sol: $\vec{H} = 0.5 e^{-0.1x} \cos(10^6 t - 2x)\hat{a}_z \text{ A/m} \rightarrow (+X)$

$$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$$

Wave frequency = 10^6 radians/s

Phase constant $\beta = 2 \text{ rad/m}$



$$\beta = \frac{2\pi}{\lambda} = 2 \text{ rad/m}$$

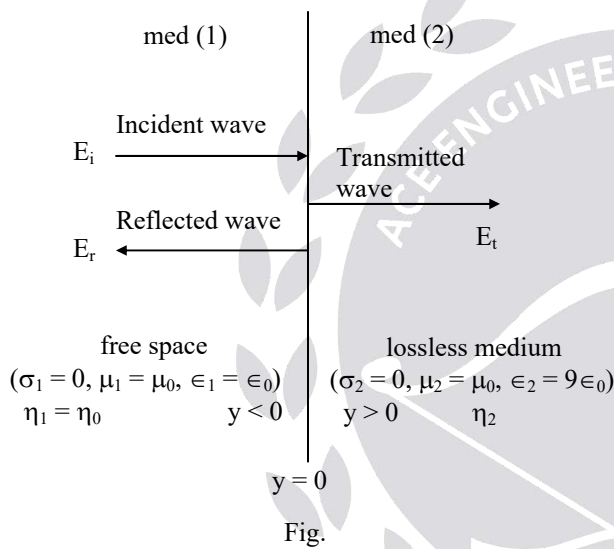
$$\lambda = \pi = 3.14 \text{ m.}$$

The wave is traveling along +X direction,
Given wave is polarized along Y.

∴ It has Y-component of electric field

13. Ans: (a)

Sol: The normal incidence of a plane wave traveling in positive y – direction is shown at the interface $y = 0$ in Fig.



Given: $\vec{E}_i = E_{iz} \vec{a}_z$

where $E_{iz} = 24 \cos(3 \times 10^8 t - \beta y) \text{ V/m}$

$\omega = 3 \times 10^8 \text{ rad/s}, \beta = \frac{\omega}{v}$,

For free space, $v = v_0 = 3 \times 10^8 \text{ m/s}$

∴ $\beta = 1 \text{ rad/m}$

$$\eta_1 = \eta_0 = \frac{E_{iz}}{H_{ix}}$$

$$\therefore H_{ix} = \frac{E_{iz}}{\eta_0} = \frac{24 \cos(3 \times 10^8 t - \beta y)}{120 \pi}$$

$$\vec{H}_i = H_{ix} \vec{a}_x$$

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\eta_1 - 1}{\eta_1 + 1}$$

Where $\frac{\eta_1}{\eta_2} = \frac{\sqrt{\mu_1 \epsilon_2}}{\sqrt{\epsilon_1 \mu_2}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sqrt{\frac{9\epsilon_0}{\epsilon_0}} = 3$

$$\therefore \frac{H_r}{H_i} = \frac{3 - 1}{3 + 1} = \frac{1}{2}$$

$$\begin{aligned} \therefore \vec{H}_r &= \frac{1}{2} \frac{24}{120 \pi} \cos(3 \times 10^8 t + 1y) \vec{a}_x \\ &= \frac{1}{10 \pi} \cos(3 \times 10^8 t + 1y) \vec{a}_x \text{ A/m} \end{aligned}$$

Note that \vec{H}_r is reflected wave which travels in negative y direction, which corresponds to + βy term with $\beta = 1$ in the expression for \vec{H}_r .

14. Ans: (b)

Sol: Brewster's angle $\theta_B = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$$\theta_B = \tan^{-1} \sqrt{\frac{1}{3}} = 30^\circ$$

At this angle there is no reflected wave when wave is parallel polarized.

$$n_1 \sin \theta_i = n_2 \sin \theta_t$$

$$\sqrt{\epsilon_1} \sin \theta_i = \sqrt{\epsilon_2} \sin \theta_t$$

$$\sin \theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i$$

$$\sin \theta_t = \sqrt{3} \frac{1}{2} (\theta_i = 30^\circ)$$

$$\theta_t = 60^\circ$$



15. Ans: (d)

Sol: Given that

$$E_t = -2E_r$$

Where

E_t is electric field of transmitted wave

E_r is electric field of reflected wave

$$\frac{E_t}{E_r} = -2$$

If E_i is electric field of incident wave.

$$\text{But } -\frac{2E_r}{E_i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$

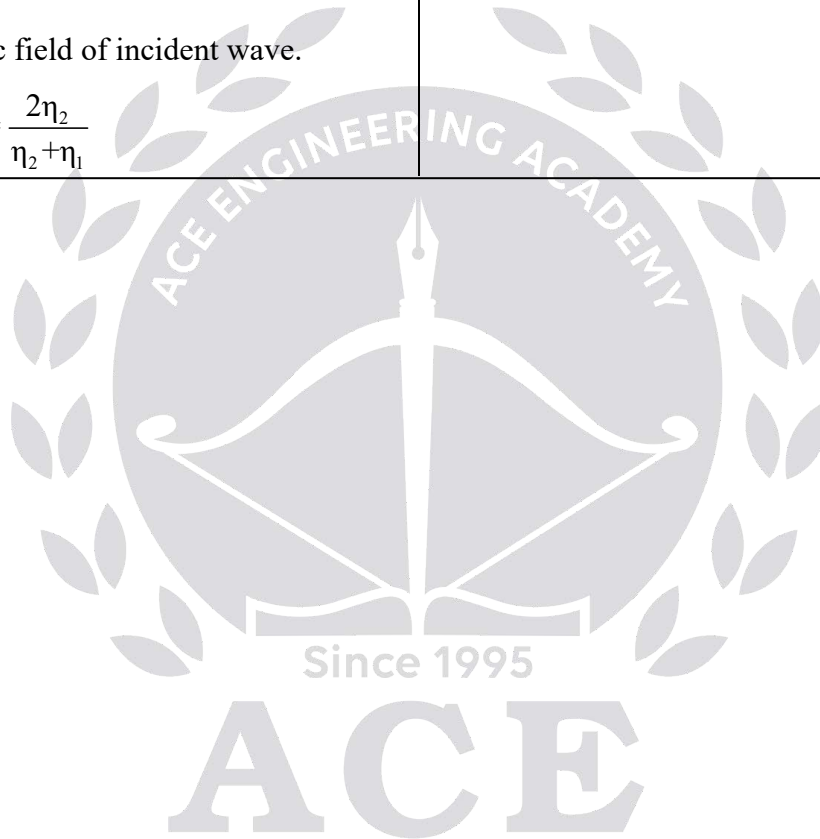
$$\text{and } \frac{E_r}{E_i} = \frac{-\eta_2}{\eta_1 + \eta_2}$$

$$\text{and also } \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\text{so } \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{-\eta_2}{\eta_2 + \eta_1}$$

$$\eta_1 = 2\eta_2$$

$$\frac{\eta_1}{\eta_2} = 2 \Rightarrow \sqrt{\frac{\epsilon_2}{\epsilon_1}} = 2 \Rightarrow \frac{\epsilon_2}{\epsilon_1} = 4$$



01. Ans: (b)

$$\text{Sol: } Z_{\text{in}} = Z_0 \frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l}$$

Phase velocity

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{2\pi f}{\beta}$$

$$\beta = \frac{2\pi f}{v_p} = \frac{2 \times \pi \times 10^8}{2 \times 10^8} = \pi$$

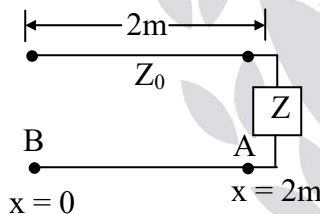
$$\beta l = \pi \cdot l \Rightarrow \pi \text{ (Given } l=1\text{m)}$$

$$\tan \beta l = 0$$

$$Z_{\text{in}} = Z_R \\ = (30 - j40)\Omega$$

02. Ans: (a)

Sol:



$$K_x = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_A = \frac{C_2}{C_1} e^{j4\beta} \text{ at } (x = 2)$$

$$K_B = \frac{C_2}{C_1} e^{2j\beta(0)} \text{ at } (x = 0)$$

$$\frac{K_B}{K_A} = \frac{\frac{C_2}{C_1} e^{2j\beta(0)}}{\frac{C_2}{C_1} e^{j4\beta}} = e^{-j4\beta}$$

$$v_p = \frac{\omega}{\beta} \Rightarrow \beta = \frac{\pi}{2}$$

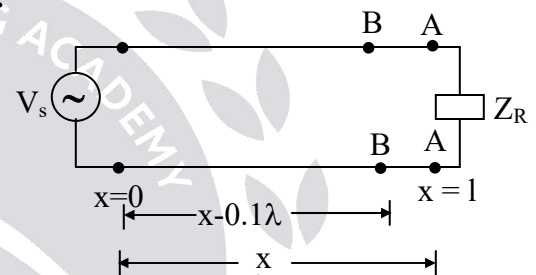
Given $f = 50 \text{ MHz}$

$$v_p = 2 \times 10^8 \text{ m/s}$$

$$\frac{K_B}{K_A} = e^{-j4\left(\frac{\pi}{2}\right)} = e^{-j2\pi} = 1 \text{ (or) } \frac{\Gamma_i}{\Gamma_R} = 1$$

03. Ans: (b)

Sol:



$$V = C_1 e^{-j\beta x} + C_2 e^{+j\beta x}$$

$$K_x = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_A = 0.3e^{-j30^\circ} = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_B = \frac{C_2}{C_1} e^{2j\beta(x-0.1\lambda)}$$

$$\frac{K_B}{K_A} = \frac{\frac{C_2}{C_1} e^{2j\beta x} e^{-j4\frac{\pi}{\lambda} \cdot 0.1\lambda}}{\frac{C_2}{C_1} e^{2j\beta x}}$$

$$K_B = K_A \cdot e^{-j.4\pi} \\ = 0.3e^{-j30^\circ} e^{-72^\circ} \\ = 0.3 e^{-j102^\circ}$$

Note: In the options $0.3 e^{j102^\circ}$ is given. But correct answer is $0.3 e^{-j102^\circ}$



04. Ans: (c)

Sol: From the voltage SW pattern,

$$V_{\min} = 1, V_{\max} = 4, \text{VSWR} = S = 4$$

$$Z_0 = R_0 = 50 \Omega$$

Let the resistive load be R_L

For Resistive loads

$$S = \frac{R_L}{R_0} \quad \text{for } R_L > R_0$$

$$= \frac{R_0}{R_L} \quad \text{for } R_0 > R_L$$

$$\therefore R_L = S R_0 = 4 \times 50 = 200 \Omega \quad \text{for } R_L > R_0$$

$$R_L = R_0/S = 50/4 = 12.5 \Omega \quad \text{for } R_0 > R_L$$

As voltage minimum is occurring at the load point, $R_L = 12.5 \Omega$

05. Ans: (a)

Sol: Reflection coefficient:

$$\Gamma = \frac{R_L - R_0}{R_L + R_0} = \frac{12.5 - 50}{12.5 + 50} = -0.6$$

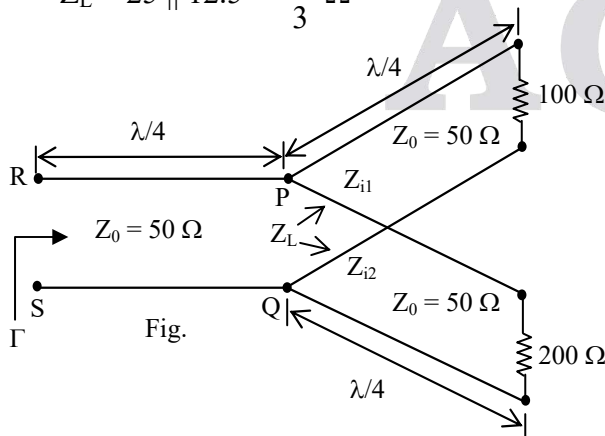
06. Ans: (d)

Sol: The interconnection of TL's is shown in Fig.

$$Z_{i1} = \frac{(50)^2}{100} = 25 \Omega$$

$$Z_{i2} = \frac{(50)^2}{200} = 12.5 \Omega$$

$$Z_L = 25 \parallel 12.5 = \frac{25}{3} \Omega$$



$$\text{Reflection coefficient at PQ} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\begin{aligned} &= \frac{\frac{25}{3} - 50}{\frac{25}{3} + 50} = -\frac{125}{175} = -\frac{5}{7} \end{aligned}$$

\therefore At the input RS,

$$\text{Reflection coefficient, } \Gamma = -\frac{5}{7} e^{-j2\beta\ell}$$

$$\text{As } \beta\ell = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\Gamma = -\frac{5}{7} e^{-j\pi} = \frac{5}{7}$$

07. Ans: (d)

$$\text{Sol: } Z_{\text{in}} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell} \right]$$

i) For a shorted line,

$$Z_L = 0$$

$$\ell = \lambda/8$$

$$\beta\ell = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$$

$$Z_{\text{in}} = Z_0 \left[\frac{0 + jZ_0}{Z_0 + 0} \right]$$

$$Z_{\text{in}} = jZ_0$$

ii) For a shorted line means $Z_L = 0$

$$\text{Given that } \ell = \frac{\lambda}{4}$$

$$\beta\ell = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{\text{in}} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{0}$$

$$Z_{\text{in}} = \infty$$

iii) Open line means $Z_L = \infty$,

$$\text{Given that } \ell = \frac{\lambda}{2}$$



$$\therefore \beta \ell = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi \Rightarrow \tan \pi = 0$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \pi}{Z_0 + jZ_L \tan \pi} \right]$$

$$Z_{in} = Z_L$$

iv) For a matched line of any length

$$Z_L = Z_0$$

$$Z_{in} = Z_0 \left[\frac{Z_0 + jZ_0 \tan \beta \ell}{Z_0 + jZ_0 \tan \beta \ell} \right] = Z_0$$

08. Ans: (c)

Sol: The line is matched as $Z_L = Z_0 = 50 \Omega$ and hence reflected wave is absent.

For the traveling wave, given:

Phase difference for a length of
2 mm = $\pi/4$ rad

Frequency of excitation = 10 GHz

Phase velocity, $v_p = \frac{\omega}{\beta}$

$$\omega = 2\pi \times 10 \times 10^9 \text{ rad/sec}$$

β = Phase-shift per unit length

$$= \frac{\pi}{4 \times 2 \times 10^{-3}} \text{ rad/m}$$

$$v_p = \frac{2\pi \times 10^{10} \times 8}{\pi \times 10^3} = 1.6 \times 10^8 \text{ m/s}$$

09. Ans: (b)

$$\text{Sol: } [S] = \begin{bmatrix} 0.3 \angle 0^\circ & 0.9 \angle 90^\circ \\ 0.9 \angle 90^\circ & 0.2 \angle 0^\circ \end{bmatrix}$$

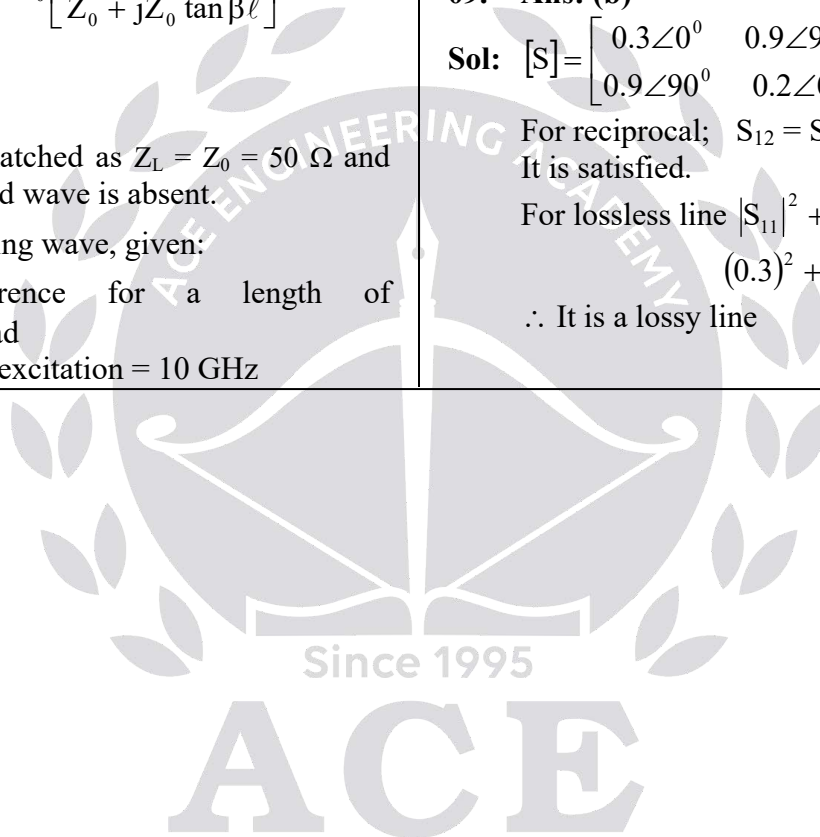
For reciprocal; $S_{12} = S_{21}$

It is satisfied.

For lossless line $|S_{11}|^2 + |S_{12}|^2 = 1$

$$(0.3)^2 + (0.9)^2 = 0.9 \neq 1$$

\therefore It is a lossy line



4

Waveguides

Chapter

01. Ans: (b)

Sol: Evanescent modes means no wave propagation.

Dominant mode means, the guide has lowest cut-off frequency.

TM₀₁ and TM₁₀ not possible, the minimum values of m, n for TM are at least 1, 1 respectively.

02. Ans: (a)

Sol: The mode which has lowest cutoff frequency is called dominant mode TE₁₀.

At 4GHz all modes are evanescent.

At 7GHz degenerate modes are possible

TE₁₁ and TM₁₁ are degenerate.

$$f_{c \text{ TE}_{10}} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 3 \times 10^{-2}} = 5 \text{ GHz.}$$

At 6 GHz dominant mode will propagate.

At 11 GHz higher order modes are possible

03. Ans: (a)

Sol: Given: In a rectangular WG of cross-section : (a × b)

$$\vec{E} = \frac{\omega\mu}{h^2} \left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{2\pi}{a} x\right) \sin(\omega t - \beta z) \hat{y}$$

The wave is traveling in the z-direction having E_y component only as function of 'x'. As there is no component of \vec{E} in the direction of propagation, \vec{a}_z the wave is Transverse Electric (TE).

Comparing the 'sin' term in \vec{E} with the general expression: $\sin\left(\frac{m\pi}{a} x\right)$

$$m = 2$$

As there is no function of 'y' in \vec{E} , n = 0

∴ The mode of propagation in the WG is TE₂₀

04. Ans: (d)

Sol: Given

$$a = 4.755, b = 2.215,$$

$$f = 12 \text{ GHz, } c = 3 \times 10^8 \text{ m/s}$$

Cut off frequency

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For TE₁₀, mode

$$f_c = \frac{c}{2a} = 3.15 \text{ GHz}$$

f > f_c (TE₁₀ mode) so it propagates

For TE₂₀ mode

$$f_c (\text{TE}_{20}) = \frac{c}{2} \sqrt{\left(\frac{2}{a}\right)^2} = 2 [f_c (\text{TE}_{10})] = 6.30 \text{ GHz}$$

f > f_c [TE₂₀] so it propagates

For TE₀₁ mode

$$f_c (\text{TE}_{01}) = \frac{c}{2} \sqrt{\frac{1}{b^2}} = \frac{c}{2b} = 6.77 \text{ GHz}$$

∴ f > f_c (TE₀₁) so it propagate

For TE₁₁ mode

$$f_{c[\text{TE}_{11}]} = \frac{c}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 7.47 \text{ GHz}$$

f > f_c (TE₁₁) so it propagate

So, all modes are possible to propagate.

05. Ans: (a)

Sol: Given a = 6cm, b = 4 cm f = 3 GHz

Cut off frequency

$$f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$



$$TE_{10}: f_c = \frac{c}{2a} = 2.5 \text{ GHz}$$

$$TE_{01}: f_c = \frac{c}{2b} = 3.75 \text{ GHz}$$

$$TE_{11}: f_c = \frac{c}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 4.50 \text{ GHz}$$

$$TM_{11}: f_c = \frac{c}{2} \sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 4.50 \text{ GHz}$$

06. Ans: (a)

Sol: $\frac{m\pi}{a} = \frac{2\pi}{a} \Rightarrow m = 2$

$$\frac{n\pi}{b} = \frac{3\pi}{b} \Rightarrow n = 3$$

For TM wave propagating along z-direction

$$E_z \neq 0 \text{ and } H_z = 0$$

TM₂₃

$$TM_{23} \Rightarrow f_c = \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Substitute $c = 3 \times 10^{10}$ cm/sec

$$m = 2, a = 6 \text{ cm}$$

$$n = 3, b = 3 \text{ cm}$$

we get $f_c = 15.811 \text{ GHz}$

$$\eta_{TM} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\omega = 10^{12} \Rightarrow f = \frac{10^{12}}{2\pi} = \frac{10^3}{2\pi} \text{ GHz}$$

and $\eta = 120 \pi$. & $f_c = 15.811 \text{ GHz}$

Substitute all the above values and we get

$$\eta_{TM} = 375 \Omega$$

07. Ans: (c)

Sol: $W_{avg} = \frac{1}{4} \frac{E_{y0}^2}{\eta_{TE_{10}}} a.b$; $\eta_{TE_{10}} = \frac{\eta}{\sqrt{1 - (\lambda/\lambda_c)^2}}$

$$\eta = 120\pi, \lambda = \frac{c}{f} = \frac{3 \times 10^{10}}{11 \times 10^9} = 2.72 \text{ cm}$$

$$\lambda_c = 2a = 2 \times 2.29 = 4.58 \text{ cm}$$

So we get $\eta_{TE_{10}} = 469.52 \Omega$

Putting all the values

$$\therefore W_{avg} = 31.32 \text{ kW}$$

08. Ans: (a)

Sol: $f_{c_{10}} = \frac{c}{2a} = \frac{3 \times 10^{10}}{2 \times 2} = 7.5 \text{ GHz}$

For $b = a/2$, the next high order mode is TE₀₁ or TE₂₀.

$$\therefore f_{c_{01}} = f_{c_{20}} = \frac{3 \times 10^{10}}{2} = 15 \text{ GHz}$$

So the range of single mode (dominant mode propagation) is

$$7.5 < f < 15 \text{ GHz}$$

09. Ans: (a)

Sol: $\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$

$$f_c = 0.908 \text{ GHz}$$

$$\Rightarrow \lambda_c = \frac{3 \times 10^{10}}{0.908 \times 10^9} = 33.03 \text{ cm}$$

Substitute $\lambda_g = 40 \text{ cm}$, $\lambda_c = 33.03 \text{ cm}$

We get, $\lambda = 25.47 \text{ cm}$

$$\Rightarrow f = \frac{3 \times 10^{10}}{25.47} = 1.18 \text{ GHz}$$

10. Ans: (a)

Sol: $\frac{c}{2a} = 0.908 \text{ GHz}$

$$\Rightarrow a = \frac{3 \times 10^{10}}{2 \times (0.908) \times 10^9}$$

$$= 16.51 \text{ cm}$$

$$\Rightarrow b = \frac{a}{2} = 8.26 \text{ cm}$$



11. Ans: (a)

Sol: $\bar{\beta} = \beta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$

$$= \frac{2\pi}{25.47} \sqrt{1 - \left(\frac{0.908}{1.18}\right)^2}$$

$$= 0.157 \text{ rad/cm}$$

$$= 15.7 \text{ rad/m}$$



01. Ans: (c)

Sol: Antenna receives $2 \mu\text{W}$ of power: $P_r = 2 \mu\text{W}$
RMS value of incident E field
 $= 20 \text{ mV/m}$

Power density, P_d

$$= \frac{E^2}{\eta} = \frac{(20 \times 10^{-3})^2}{377} \text{ W/m}^2$$

Effective aperture area, $A_e = \frac{P_r}{P_d}$

$$= \frac{2 \times 10^{-6}}{(20 \times 10^{-3})^2} = \frac{377 \times 2}{400} = 1.885 \text{ m}^2$$

02. Ans: (b)

Sol: Lossless antenna directive gain = 6 dB = 4

Input power to the antenna = 1 mW
for lossless we get 100% efficiency

$$\frac{W_{\text{rad}}}{W_{\text{in}}} = \frac{G_o}{D_o} = 1$$

$$W_{\text{rad}} = W_{\text{in}}$$

$$W_{\text{rad}} = 1 \text{ mW}$$

03. Ans: (c)

Sol: $P_{\text{rad}} = \frac{A_0 \sin^2 \theta}{r^2} \hat{a}_r \text{ W/m}^2$

$$W_{\text{rad}} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{A_0 \sin^2 \theta}{r^2} r^2 \sin \theta d\theta d\phi$$

$$= A_0 2\pi \int_{\theta=0}^{\pi} \sin^3 \theta d\theta$$

$$= A_0 2\pi \frac{4}{3}$$

$$W_{\text{rad}} = A_0 \frac{8\pi}{3}$$

$$U = r^2 P_{\text{rad}} = r^2 \frac{A_0 \sin^2 \theta}{r^2} = A_0 \sin^2 \theta$$

$$D_{\text{max}} = \frac{U_{\text{max}}}{W_{\text{rad}}} 4\pi = \frac{|A_0 \sin^2 \theta|_{\text{max}}}{\frac{8\pi}{3} A_0} \times 4\pi$$

$$= \frac{4\pi A_0}{8\pi A_0} \times 3$$

$$= \frac{3}{2} = D_{\text{max}} = 1.5$$

04. Ans: (d)

Sol: Where $W_{\text{rad}} = \iint \bar{P}_{\text{rad}} \cdot \bar{d}\bar{s}$

$$\bar{P}_{\text{rad}} = \frac{W_{\text{rad}}}{2\pi r^2} \hat{a}_r = \frac{40}{\pi} \hat{a}_r \mu\text{W/m}^2$$

05. Ans: (b)

Sol: $R_{\text{rad}} = 30 \Omega$, $R_l = 10 \Omega$

$$G_D = 4, G_p = ?$$

$$\eta = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_l} = \frac{30}{40} = 0.75$$

$$G_p = \eta G_D$$

$$= 0.75 \times 4 = 3$$

06. Ans: (c)

Sol: $D_g = 30 \text{ dB} = 1000$

$$P_T = 7.5 \text{ kW}$$

$$D_g = \frac{4\pi \times \text{Radiation intensity}}{\text{Radiated Power}}$$



$$D_g = 4\pi \frac{U}{W_{\text{rad}}}$$

$$\therefore U = \frac{7.5 \times 10^3 \times 1000}{4\pi}$$

$$\Rightarrow U = r^2 P_{\text{rad}}$$

P_{rad} : Power density we have to find

P_{rad} at $r = 40 \times 10^3$ m

$$P_{\text{rad}} = \frac{U}{r^2} = \frac{7.5 \times 10^3 \times 1000}{4\pi \times (40 \times 10^3)^2} \text{ W/m}^2$$

07. Ans: (d)

Sol: $W_{\text{rad}} = 10 \text{ kW}$

$E_{\text{max}} = 120 \text{ mV/m}$

$R = 20 \text{ km}$

$\eta = 98\%$

$$P_{\text{rad}} = \frac{E_0^2}{2\eta_0}$$

$$= \frac{(120 \times 10^{-3})^2}{2 \times 120\pi}$$

$$= 1.909 \times 10^{-5}$$

$$U_{\text{max}} = (20 \times 10^3)^2 \times 1.909 \times 10^{-5} = 7636$$

$$D_0 = 4\pi \frac{U_{\text{max}}}{W_{\text{rad}}}$$

$$D_0 = 4\pi \frac{7639.43}{10 \times 10^3} = 9.59$$

$$\eta = \frac{G_0}{D_0} = 0.98$$

$$G_0 = 0.98 \times 9.59 = 9.407$$

08. Ans: 0.21

Sol: Given:

Antenna length, $l = 1 \text{ cm}$

Frequency, $f = 1 \text{ GHz}$

Distance, $r = 100\lambda$

Wave length, $\lambda = \frac{C}{f}$

$$= \frac{3 \times 10^8}{10^9}$$

$$= 30 \text{ cm}$$

$\frac{d\ell}{\lambda} = \frac{1}{30}$, hence the given antenna is Hertzian dipole.

In the far field, the tangential electric field

is given by, $E_\theta = \frac{j\eta I d\ell \sin \theta \beta}{4\pi r}$

$$= \frac{j377 \times 100 \times 10^{-3} \times 2\pi \times 10^{-2} \times 1}{30 \times 10^{-2} \times 4\pi \times 100 \times 30 \times 10^{-2}}$$

$$\therefore |E_\theta| = 0.21 \text{ V/cm}$$

09. Ans: (c)

Sol: Given:

Length of dipole, $\ell = 0.01\lambda$

As it is very small, compared with wavelength, hence it can be approximated to Hertzian dipole

$$R_{\text{rad}} = 80\pi^2 \left(\frac{d\ell}{\lambda} \right)^2$$

$$= 80 \pi^2 (0.01)^2$$

$$R_{\text{rad}} = 0.08 \Omega$$



10. Ans: (d)

$$\text{Sol: AF} = \frac{\sin \frac{n\phi}{2}}{\sin \frac{\phi}{2}}$$

take limit

$$\lim_{\frac{n\phi}{2} \rightarrow 0} \frac{\sin \frac{n\phi}{2} \cdot \frac{n\phi}{2}}{\frac{n\phi}{2} \cdot \frac{\phi}{2}} = n$$

$$\lim_{\frac{\phi}{2} \rightarrow 0} \frac{\sin \frac{\phi}{2} \cdot \frac{\phi}{2}}{\frac{\phi}{2} \cdot \frac{\phi}{2}}$$

11. Ans: (b)

Sol: In broad side array the BWFN is given by

$$\text{BWFN} = \frac{2\lambda}{L} \text{ (rad)}$$

Where, L = length of the array

$$L = (n-1)d$$

Given: n = 9

$$\text{Spacing, } d = \frac{\lambda}{4}$$

$$\text{BWFN} = \frac{2\lambda}{(9-1) \frac{\lambda}{4}}$$

$$= \frac{2\lambda}{2\lambda} \times \frac{180}{\pi}$$

$$\therefore \text{BWFN} = 57.29^\circ$$

12. Ans: (d)

Sol: The directivity of n-element end fire array is given by

$$D = \frac{4L}{\lambda}$$

Where, L = (n-1)d

$$L \cong nd \quad (\because n = 1000, \text{ very large})$$

$$D = \frac{4 \times nd}{\lambda} \\ = \frac{4 \times 1000\lambda}{\lambda \times 4}$$

$$\therefore D \cong 1000$$

$$\text{Directivity, (in dB)} = 30$$

13. Ans: 7.78

$$\text{Sol: Directivity, } D = 4\pi \frac{U_{\max}}{P_{\text{rad}}}$$

$$\text{Given: } U(\theta, \phi) = 2\sin\theta \sin^3\phi; \quad 0 \leq \theta \leq \pi, \\ 0 \leq \phi \leq \pi$$

$$U_{\max} = 2$$

$$P_{\text{rad}} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{\pi} 2\sin\theta \sin^3\phi \sin\theta d\theta d\phi$$

$$= 2 \int_{\theta=0}^{\pi} \int_{\phi=0}^{\pi} \sin^2\theta \sin^3\phi d\theta d\phi$$

$$= 2 \left(\frac{\pi}{2} \right) \left(\frac{4}{3} \right)$$

$$= \frac{4\pi}{3}$$

$$D = 4\pi \times \frac{2}{\left(\frac{4\pi}{3} \right)}$$

$$D = 6$$

$$\text{Directivity, (in dB)} = 10\log 6 = 7.7815$$

14. Ans: 2793

Sol: For Hertzian dipole the directivity, D is given by D = 1.5

$$D = \left(\frac{4\pi}{\lambda^2} \right) A_e$$



$$A_e = 1.5 \times \frac{\lambda^2}{4\pi}$$

$$A_e = 0.119 \lambda^2$$

$$\text{Wavelength, } \lambda = \frac{3 \times 10^8}{10^8} = 3\text{m}$$

$$\therefore A_e = 0.119 \times 9$$

$$A_e = 1.074 \text{ m}^2$$

Aperture area of antenna is given by

$$A_e = \frac{P_r}{P}$$

Where, P_r = power received at the antenna load terminals.

P = power density of incident wave

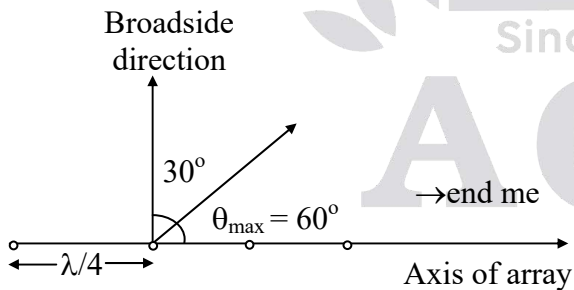
$$P = \frac{P_r}{A_e}$$

$$= \frac{3 \times 10^{-6}}{1.074}$$

$$\therefore P = 2.793 \mu\text{W/m}^2 \text{ (or) } 2793 \text{ nW/m}^2$$

15. Ans: (c)

Sol:



Given: No. of elements, $n = 4$

$$\text{Spacing, } d = \frac{\lambda}{4}$$

Direction of main beam (or) principal lobe,

$$\theta_{\max} = 60^\circ$$

Array phase function, ψ is given by

$$\psi = \beta d \cos \theta + \alpha$$

To form a major lobe. $\psi = 0$

$$\alpha = -\beta d \cos \theta_{\max}$$

$$\alpha = -\frac{2\pi}{\lambda} \times \frac{\lambda}{4} \cos 60$$

$$\alpha = -\frac{\pi}{4}$$

The phase shaft between the elements

$$\text{required is } \alpha = -\frac{\pi}{4}$$