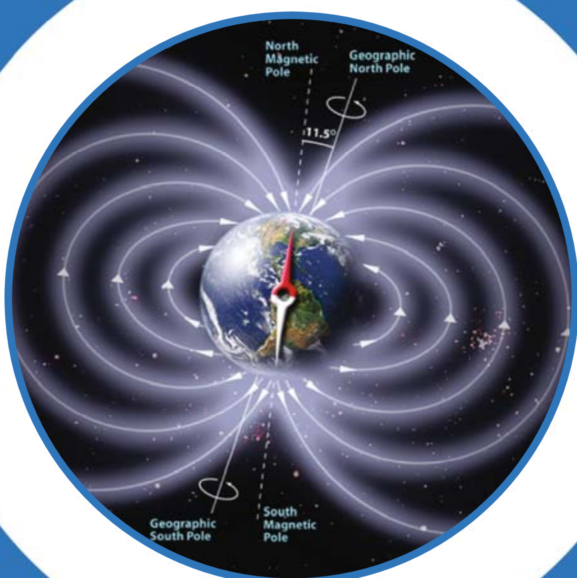
ELECTRONICS & COMMUNICATION ENGINEERING



GATE | PSUs

ELECTROMAGNETIC THEORY

Volume - I : Study Material with Classroom Practice Questions

Study Material with Classroom Practice solutions

To

Electromagnetic Theory

CONTENTS

Chapter **1**

Static Fields

Class Room Practice Solutions

01. Ans: 1

Sol: $\vec{V} = x\cos^2 y\hat{i} + x^2e^z\hat{j} + z\sin^2 y\hat{k}$

$= x\cos^2 y\hat{a}_x + x^2e^z\hat{a}_y + z\sin^2 y\hat{a}_z$

From divergence theorem

$$\oint\!\!\oint \overline{V}.\hat{n}\,ds = \int_v (\nabla.\overline{D})dv \text{ ....1}$$

$$\nabla.\overline{D} = \frac{\partial}{\partial x}(x\cos^2 y) + \frac{\partial}{\partial y}(x^2e^z) + \frac{\partial}{\partial z}(z\sin^2 y)$$

$= \cos^2 y + \sin^2 y = 1$

dv = dxdydz

Putting these value in equation 1 we have

$$\oint\!\!\oint \overline{V}.\hat{n}\,ds = \int_0^1\int_0^1\int_0^1 1\times dx\,dy\,dz$$

$$= \int_0^1 dx\int_0^1 dy\int_0^1 dz = 1$$

02. Ans: (c)

Sol: For the given $\vec{A} = xy\,\vec{a}_x + x^2\,\vec{a}_y$

Let $I = \oint \vec{A}.d\vec{\ell}$, I is evaluated over the path shown in the Fig., as follows

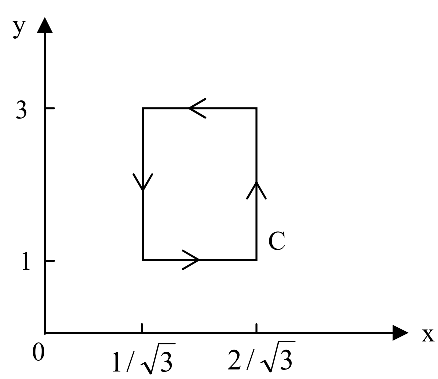


Fig.

$I = \oint \vec{A}.dx\,\vec{a}_x$, y = 1, x = from $\frac{1}{\sqrt{3}}$ to $\frac{2}{\sqrt{3}}$

$+\int \vec{A}.dy\,\vec{a}_y$, $x = \frac{2}{\sqrt{3}}$, y = from 1 to 3

$-\int \vec{A}.dx\,\vec{a}_x$, y = 3, x = from $\frac{1}{\sqrt{3}}$ to $\frac{2}{\sqrt{3}}$

$-\int \vec{A}.dy\,\vec{a}_y$, $x = 1/\sqrt{3}$, y = from 1 to 3

$$= \int xy\,dx + \int x^2\,dy - \int xy\,dx - \int x^2\,dy$$

$$= y\,\frac{x^2}{2}\Big|_{1/\sqrt{3}}^{2/\sqrt{3}} + x^2 y\Big|_1^3 - y\,\frac{x^2}{2}\Big|_{1/\sqrt{3}}^{2/\sqrt{3}} - x^2 y\Big|_1^3$$

at y = 1 $x = 2/\sqrt{3}$ y = 3 $x = 1/\sqrt{3}$

$$= \frac{1}{2}\left(\frac{4}{3} - \frac{1}{3}\right) + \frac{4}{3}(3-1) - \frac{3}{2}\left(\frac{4}{3} - \frac{1}{3}\right) - \frac{1}{3}(3-1)$$

$$= \frac{1}{2} + \frac{8}{3} - \frac{3}{2} - \frac{2}{3} = -1 + 2 = 1$$

03. Ans: (d)

Sol: $\overline{F} = \rho a_\rho + \rho\sin^2\phi\, a_\phi - z a_z$

$= F_\rho a_\rho + F_\phi a_\phi + F_z a_z$

$$\nabla.\overline{F} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho F_\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(F_\phi) + \frac{\partial}{\partial z}(F_z)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho^2) + \frac{1}{\rho}\frac{\partial}{\partial\phi}(\rho\sin^2\phi) + \frac{\partial}{\partial z}(-z)$$

$= 2 + 2\sin\phi\cos\phi - 1$

$= 1 + 2\sin\phi\cos\phi$

$$\nabla.F\big|_{\phi=\frac{\pi}{4}} = 2,\ \nabla.F\big|_{\phi=0} = 1$$

$$\nabla.F\big|_{\phi=\frac{\pi}{4}} = 2\nabla.F\big|_{\phi=0}$$

04. Ans: (c)

Sol: $\overline{D} = 2\hat{a}_x - 2\sqrt{3}\hat{a}_Z$ $\qquad \overline{D} = |\overline{D}|\overline{a}_n$

$|\overline{D}| = \sqrt{16} = 4$ $\qquad = \rho_s\hat{a}_n$

$$\therefore \overline{D} = 4\left\{\frac{2\hat{a}_x - 2\sqrt{3}\hat{a}_Z}{4}\right\}$$

$$= \rho_s \hat{a}_n \qquad \therefore \rho_s = 4\, C/m^2$$

05. Ans: (d)

Sol: $V = 10y^4 + 20x^3$

$$E = -\nabla V = -60x^2\hat{a}_x - 40y^3\hat{a}_y$$

$$D = \varepsilon_0 E = -60x^2\varepsilon_0\hat{a}_x - 40y^3\varepsilon_0\hat{a}_y$$

$$\nabla.D = \rho_v$$

$$\rho_v = \frac{\partial}{\partial x}(-60x^2\varepsilon_0) + \frac{\partial}{\partial y}(-40y^3\varepsilon_0)$$

$$= -120\, x\varepsilon_0 - 120\, y^2\varepsilon_0$$

$$\rho_v(\text{at } 2, 0) = -120\times 2\varepsilon_0 - 120\times 0^2\, \varepsilon_0$$

$$= -240\, \varepsilon_0$$

06. Ans: (d)

Sol: Given

$$V(x, y, z) = 50\, x^2 + 50\, y^2 + 50\, z^2$$

$\vec{E}\,(x, y, z)$ in free space $= -$grad (V)

$$= -\nabla V$$

$$= -\left[\frac{\partial}{\partial x}V\overrightarrow{a_x} + \frac{\partial}{\partial y}V\overrightarrow{a_y} + \frac{\partial}{\partial z}V\overrightarrow{a_z}\right]$$

$$= -\left[100x\,\overrightarrow{a_x} + 100y\,\overrightarrow{a_y} + 100z\,\overrightarrow{a_z}\right] V/m$$

$$\vec{E}\,(1, -1, 1) =$$

$$-\left[100\,\overrightarrow{a_x} - 100\,\overrightarrow{a_y} + 100\,\overrightarrow{a_z}\right] V/m$$

$$E(1,-1,1) = 100\sqrt{(-1)^2 + (1)^2 + (-1)^2}$$

$$= 100\sqrt{3}$$

Direction of the electric field is given by the unit vector in the direction of $\vec{E}$.

$$\vec{a}_E = \frac{\vec{E}(1, -1, 1)}{|E(1, -1, 1)|} = \frac{1}{\sqrt{3}}\left[-\overrightarrow{a_x} + \overrightarrow{a_y} - \overrightarrow{a_z}\right]$$

or in i, j, k notation, $\vec{a}_E = \frac{1}{\sqrt{3}}\left[-i + j - k\right]$

07. Ans: (b)

Sol: For valid B, $\nabla.B = 0$

$$\left(\frac{\partial}{\partial x}a_x + \frac{\partial}{\partial y}a_y + \frac{\partial}{\partial z}a_z\right)\left(x^2a_x - xya_y - Kxza_z\right) = 0$$

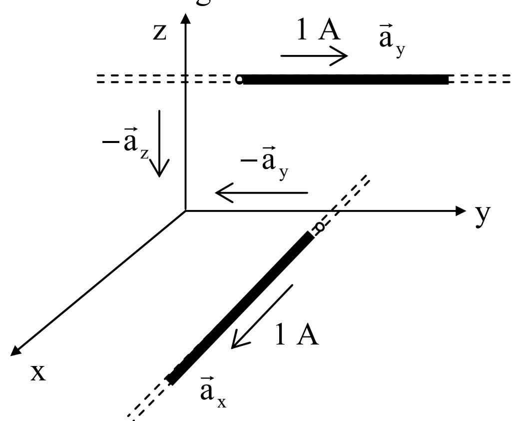
$$2x - x - Kx = 0$$

$$\Rightarrow 2 - 1 - K = 0$$

$$\therefore K = 1$$

08. Ans: (d)

Sol: The two infinitely long wires are oriented as shown in the Fig.



The infinitely long wire in the y-z plane carrying current along the $\vec{a}_y$ direction produces the magnetic field at the origin in the direction of $\vec{a}_y \times -\vec{a}_z = -\vec{a}_x$.

The infinitely long wire in the x-y plane carrying current along the $\vec{a}_x$ direction produces the magnetic field at the origin in the direction of $\vec{a}_x \times -\vec{a}_y = -\vec{a}_z$.

where $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$ are unit vectors along the 'x', 'y' and 'z' axes respectively.

$\therefore$ x and z components of magnetic field are non-zero at the origin.

09. Ans: (a)

Sol: $\nabla.\overline{B} = 0$

A divergence less vector may be a curl of some other vector

$\overline{B} = \nabla \times \overline{A}$

$\nabla \times \overline{A} = \overline{B}$

$\oint_l \overline{A}\,.\overline{dl} = \int_s \overline{B}\,.\,\overline{ds}$

$\int_s \overline{B}\,.\,\overline{ds}$ is equal to magnetic flux ψ through a surface.

10. Ans: (c)

Sol: In general, for an infinite sheet of current density k A/m

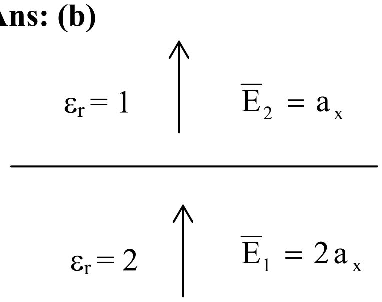
$H = \frac{1}{2} k \times a_n$

$H = \frac{1}{2}\left(8\overline{a}_x \times \overline{a}_z\right)$

$= -4\ \overline{a}_y\ \left(\because \overline{a}_x \times \overline{a}_z = -\overline{a}_y\right)$

11. Ans: (b)

Sol:



$D_{n_2} - D_{n_1} = \rho_S \rightarrow$ (a)

$D_{n_2} = \in E_{n_2} = \in_0 a_x$

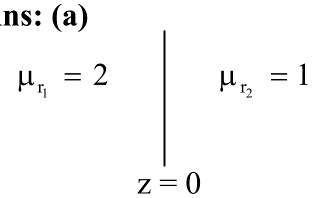
$D_{n_1} = \in_0 2 \times 2 a_x = 4 \in_0 a_x$

From (a)

$(\in_0 - 4\in_0)\, a_x = \rho_s \Rightarrow \rho_s = -3\in_0$

12. Ans: (a)

Sol:



$B_1 = 1.2\overline{a}_x + 0.8\overline{a}_y + 0.4\vec{a}_z$

$B_{n_1} = 0.4\ \overline{a}_z$

(Since z = 0 has normal component a_x)

$B_{t_1} = 1.2\ \overline{a}_x + 0.8\ \overline{a}_y$

We know magnetic flux density is continuous

$B_{n_1} = B_{n_2}$

$B_{n_2} = 0.4\ \overline{a}_z$

Surface charge, $\overline{k} = 0$

$H_{t_2} - H_{t_1} = 0$

$H_{t_2} = H_{t_1}$

$\mu_1 B_{t_2} = \mu_2 B_{t_1}$

$B_{t_2} = \frac{1}{2}(1.2 a_x + 0.8 a_y)$

$B_2 = B_{t_2} + B_{n_2}$

$= 0.6\ \overline{a}_x + 0.\overline{4} a_y + 0.4\overline{a}_z$

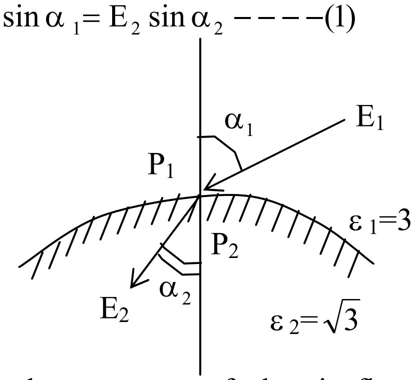
$\mu_0 \mu_{r_2} H_2 = 0.6\ \overline{a}_x + 0.\overline{4} a_y + 0.4\overline{a}_z$

$H_2 = \frac{1}{\mu_0}[0.6\ \overline{a}_x + 0.\overline{4} a_y + 0.4\overline{a}_z]\ A/m$

13. Ans: (b)

Sol: Tangential components of electric fields are continuous $\left(E_{t_1} = E_{t_2}\right)$

$E_1 \sin\alpha_1 = E_2 \sin\alpha_2$ ----(1)



Normal component of electric flux densities are continuous across a charge free interface

$D_{n_1} = D_{n_2}$

$3E_1 \cos\alpha_1 = \sqrt{3} E_2 \cos\alpha_2$ ----(2)

$\alpha_1 = 60^0$

$\frac{(1)}{(2)} \Rightarrow \frac{\tan\alpha_1}{3} = \frac{\tan\alpha_2}{\sqrt{3}} \Rightarrow \tan\ \alpha_2 = 1$

$\alpha_2 = 45^0$

Chapter **2**

Maxwell Equations & EM Waves

Example 2.12:

01. $\overline{E} = 20\sin(\omega t - \beta x)\hat{a}_y \text{ V/m}$

Sol: At x = 0

$\overline{E} = 20\sin(\omega t)\hat{a}_y \text{ V/m}$

Let $\theta = \omega t$

$\theta = 0 \Rightarrow \overline{E} = 0$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 20\hat{a}_y$

$\theta = \pi \Rightarrow \overline{E} = 0$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = -20\hat{a}_y$

$\theta = \pi \Rightarrow \overline{E} = 0$

i.e., linear polarization and also vertical polarization with respect to $\hat{x}$ – axis

02. $\overline{H} = 45\cos(\omega t - \beta z)\hat{a}_x \text{ A/m}$

Sol: This is linear polarization

03. $\overline{E} = 20\sin(\omega t - \beta z)\hat{a}_x + 30\sin(\omega t - \beta z)\hat{a}_y$

Sol: phase difference between $\hat{a}_x$ component and $\hat{a}_y$ component is 0°

So that it is linear polarization

Note: for phase difference 0° & 180°, irrespective of their amplitudes it must be in linear polarization.

04. $\overline{E} = 55\cos(\omega t - \beta z)\hat{a}_x + 55\sin(\omega t - \beta z)\hat{a}_y$

Sol: Phase difference between $\hat{a}_x$ component and $\hat{a}_y$ component is $\frac{\pi}{2}$

Amplitudes are same.

So it is circular polarization

at z = 0 and let $\theta = \omega t$

$\theta = 0 \Rightarrow \overline{E} = 55\hat{a}_x + 0\hat{a}_y$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 0\hat{a}_x + 55\hat{a}_y$

It is CCW direction i.e. RHCP

05. $\overline{E} = 40\sin(\omega t - \beta y)\hat{a}_x + 50\cos(\omega t - \beta y)\hat{a}_z$

Sol: Phase difference = $\frac{\pi}{2}$

Amplitudes = not same

So it is elliptical polarization. To decide direction of rotation follow below procedure.

At y = 0, and Let $\theta = \omega t$

$\theta = 0 \Rightarrow \overline{E} = 0\hat{a}_x + 50\hat{a}_z$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 40\hat{a}_x + 0\hat{a}_z$

$\theta = \pi \Rightarrow \overline{E} = 0\hat{a}_x - 50\hat{a}_z$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = -40\hat{a}_x + 0\hat{a}_z$

It is Anti clock wise direction i.e., Right Hand Elliptical Polarization.

06.

Sol: $\overline{E} = \text{Re}\left\{[\hat{a}_x + j\hat{a}_y]e^{j(\omega t - \beta Z)}\right\}$

$$\overline{E} = \text{Re}\begin{bmatrix}(\cos(\omega t - \beta z) + j\sin(\omega t - \beta z))\hat{a}_x + \\ j(\cos(\omega t - \beta z) + j\sin(\omega t - \beta z)\hat{a}_y)\end{bmatrix}$$

$\overline{E} = (\cos(\omega t - \beta z)\hat{a}_x - \sin(\omega t - \beta z)\hat{a}_y)$

Magnitudes of amplitudes are same, phase difference is $\frac{\pi}{2}$; So it is circular polarization. Now we proceed to decide direction of rotation.

Here

$\overline{E} = \cos(\omega t - \beta z)\hat{a}_x - \sin(\omega t - \beta z)\hat{a}_y$

At z = 0 & let $\theta = \omega t$

$\theta = 0 \Rightarrow \overline{E} = \hat{a}_x - 0\hat{a}_y$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 0\hat{a}_x - \hat{a}_y$

$\theta = \pi \Rightarrow \overline{E} = -\hat{a}_x + 0\hat{a}_y$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = 0\,\hat{a}_x - \hat{a}_y$

i.e., we get clock wise rotation i.e., Left Hand Circular Polarization

07. not a valid EM wave representation

08.

Sol: $\overline{E} = 5\cos(\omega t - \beta r)\hat{a}_\theta$

Let r = 0 & $\theta = \omega t$

at $\theta = 0 \Rightarrow \overline{E} = 5\hat{a}_\theta$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 0\,\hat{a}_\theta$

$\theta = \pi \Rightarrow \overline{E} = -5\hat{a}_\theta$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = 0\,\hat{a}_\theta$

i.e., linear polarization

09.

Sol: $\overline{E} = \text{Im}\{[\hat{a}_x + 2j\hat{a}_z]e^{j(\omega t - \beta y)}\}$

$= \text{Im}\left\{\begin{matrix}[\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)]\hat{a}_x + \\ 2j[\cos(\omega t - \beta y) + j\sin(\omega t - \beta y)\hat{a}_z]\end{matrix}\right\}$

$= \sin(\omega t - \beta y)\,\hat{a}_x + 2\cos(\omega t - \beta y)\,\hat{a}_z$

Let y = 0 & $\theta = \omega t$

$\theta = 0 \Rightarrow \overline{E} = 0\hat{a}_x + 2\hat{a}_z$

$\theta = \frac{\pi}{2} \Rightarrow \overline{E} = \hat{a}_x + 0\hat{a}_z$

$\theta = \pi \Rightarrow \overline{E} = 0\hat{a}_x - 2\hat{a}_z$

$\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = -\hat{a}_x + 0\hat{a}_z$

So it is Right Hand Elliptical Polarization

10. $\overline{E} = 20\sin(\omega t - \beta y)\hat{a}_x + 30\sin(\omega t - \beta y + 45^\circ)\hat{a}_z$

Sol: let y = 0 & $\theta = \omega t$

At $\theta = 0$

$\Rightarrow \overline{E} = 0\hat{a}_x + 30\sin 45^\circ\,\hat{a}_z$

$= 0\hat{a}_x + \frac{30}{\sqrt{2}}\hat{a}_z$

At $\theta = \frac{\pi}{2} \Rightarrow \overline{E} = 20\hat{a}_x + 30\sin(135^0)\hat{a}_z$

$= 20\hat{a}_x + \frac{30}{\sqrt{2}}\hat{a}_z$

At $\theta = \pi \Rightarrow \overline{E} = 0\hat{a}_x + 30\sin(225^\circ)\hat{a}_2$

$= 0\hat{a}_x - \frac{30}{\sqrt{2}}\hat{a}_z$

At $\theta = \frac{3\pi}{2} \Rightarrow \overline{E} = -20\hat{a}_x + 30\sin(315^\circ)\hat{a}_z$

$= -20\hat{a}_x - \frac{30}{\sqrt{2}}\hat{a}_z$

Note: $\theta = 62.76^\circ$ is the maximum values direction obtained by

$\frac{d\overline{E}}{d\theta} = 0$ at $y = 0$ & $\omega t = \theta$

at $\theta = -\frac{\pi}{4} \Rightarrow \overline{E} = \frac{-20}{\sqrt{2}}\hat{a}_x + 0\hat{a}_z$

at $\theta = \frac{\pi}{4} \Rightarrow \overline{E} = \frac{20}{\sqrt{2}}\hat{a}_x + 30\hat{a}_z$

So it is RHEP

11. $\overline{E} = 20\sin(\omega t - \beta z)\hat{a}_x + 20\sin(\omega t - \beta z + 45^\circ)\hat{a}_y$

Sol: Valid EM wave but polarization can not defined.

This is a valid EM wave representation but it is not satisfy anyone of the polarization principle

Class Room Practice Solutions

01. Ans: (c)

Sol: Given fulx $\phi = (t^3 - 2t)$mWb

Magnitude of inducted emf $|e'| = \left|\frac{d\phi}{dt}\right|_{t=4\,sec}$

$|e'| = 3t^2 - 2\Big|_{t=4\,sec}$

$= 3(4)^2 - 2$

= 46mWb

This ‘e’ for one turn; but for 100 turns

$|e| = N|e'| = 100 \times 46\text{mWb}$

$|e|$ = 4.6 volts

02. Ans: (d)

Sol: Given,

$$E = 120\,\pi \cos(10^6\,\pi\, t - \beta x)\,\hat{a}_y \text{ V/m}$$

$$H = A\cos(10^6\,\pi\, t - \beta x)\ \hat{a}_z \text{ A/m}$$

$\varepsilon_r = 8;\ \mu_r = 2$

We know that, $\dfrac{E_y}{H_z} = \eta = \sqrt{\dfrac{\mu}{\varepsilon}}$

$$H_z = \frac{E_y}{120\pi\sqrt{\frac{2}{8}}} = \frac{2E_y}{120\pi} = 2A/m$$

$$H_z = 2\cos(10^6\,\pi\, t - \beta x)\ \hat{a}_z \text{ A/m}$$

$\therefore$ A = 2

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{10^6\pi\times\sqrt{2\times 8}}{3\times 10^8} = 0.0418\text{rad/m}$$

03. Ans: (b)

Sol: This question relates to normal incidence of a UPW on the air (medium 1) to glass (medium 2) interface as shown in Fig.

Medium, 1 Ai	Medium, 2 Glass slab
$n_1 = 1$	$n_2 = 1.5$
$\mu_1 = \mu_0$	$\mu_2 = \mu_0$
$\epsilon_1 = \epsilon_0$	$\epsilon_2 = \epsilon_0\ \epsilon_r$

Fig.

If n_1 and n_2 are the refractive indices and v_1 and v_2 are the velocities

$$\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\sqrt{\mu_1\ \epsilon_1}}{\sqrt{\mu_2\ \epsilon_2}}$$

$$= \sqrt{\frac{\epsilon_1}{\epsilon_2}} \quad \text{for } \mu_1 = \mu_2 = \mu_0$$

For $n_1 = 1$, $n_2 = 1.5$

$$\sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{1}{1.5} = \frac{2}{3}$$

Reflection coefficient,

$$\frac{E_r}{E_i} = \frac{\sqrt{\frac{\epsilon_1}{\epsilon_2}} - 1}{\sqrt{\frac{\epsilon_1}{\epsilon_2}} + 1} = \frac{\frac{2}{3} - 1}{\frac{2}{3} + 1} = -\frac{1}{5}$$

$$\therefore\ \frac{P_r}{P_i} = \frac{|E_r|^2}{|E_i|^2} = \frac{1}{25} = 4\%$$

04. Ans: (d)

Sol: Normal incidence is shown in Fig.

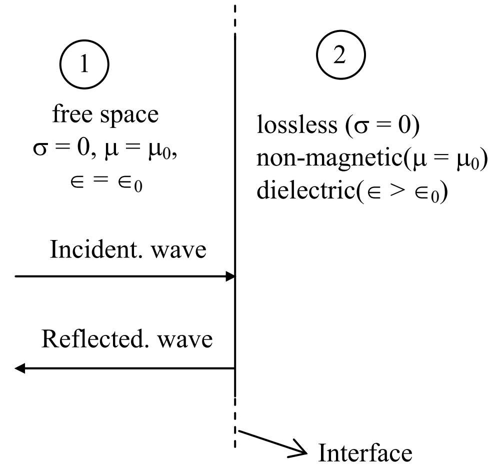


Fig.

Given: $E_{max} = 5\ E_{min}$ in medium 1.

$$\therefore\ \text{VSWR, S} = \frac{E_{max}}{E_{min}} = 5$$

$$|K| = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}$$

Reflection coefficient,

$$K = \frac{E_r}{E_i} = \frac{\frac{\eta_2}{\eta_1} - 1}{\frac{\eta_2}{\eta_1} + 1} = \frac{-2}{3}$$

$$-3\frac{\eta_2}{\eta_1} + 3 = 2\frac{\eta_2}{\eta_1} + 2$$

$$\therefore\ \frac{\eta_2}{\eta_1} = \frac{1}{5},\quad \eta_2 = \frac{1}{5}\eta_1$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^{9}}$$

$$= (120\pi)\ \Omega$$

$\therefore$ Intrinsic impedance of the dielectric medium, $\eta_2 = \frac{1}{5} \times 120\pi = 24\pi$

05. Ans: (a)

Sol: Given:

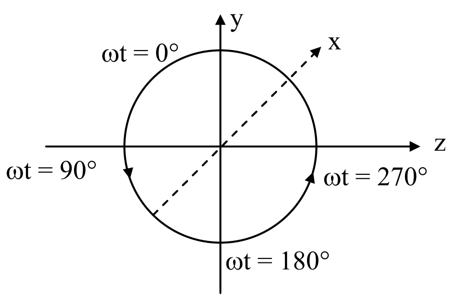
$\vec{E} = 10(\hat{a}_y + j\hat{a}_z)\, e^{-j25x}$ in free space.

$\vec{E} = (E_y\, \vec{a}_y + E_z\, \vec{a}_z)\, e^{-j\beta x}$

$\beta = 25 = \frac{\omega}{c} \Rightarrow$

$\omega = 25\, c = 25\times3\times10^8$ rad/s

f = 1.19 GHz ≈ 1.2 GHz



$E_y = 10$, $E_z = j\,10$

E_z leads E_y by 90°

At x = 0

Let $E_y = 10\cos(\omega t)$

then $E_z = 10\cos(\omega t + 90°)$

A Left Hand screw is to be turned in the direction along the circle as time increases so that the screw moves in the direction of propagation, ‘x’.

$\therefore$ The wave is left circularly polarized.

06. Ans: (b)

Sol: $\overline{H} = 0.2\cos(\omega t - \beta x)\, \hat{a}_z$

Wave is progressing along + X direction

$\rightarrow$ (+X)

$$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$$

$\therefore \overline{E} = 0.2\eta\cos(\omega t - \beta x)\, \hat{a}_y$

$\overline{E}_s = 0.2\eta e^{-j\beta x}\, \hat{a}_y \quad \overline{H}_s = 0.2e^{-j\beta x}\, \hat{a}_z$

$$\overline{P}_{avg} = \frac{1}{2}\overline{E}_s \times \overline{H}_s^*$$

$$= \frac{1}{2}(0.2)^2\, \eta\, \hat{a}_x$$

$$= \frac{1}{2}(0.2)^2\,(120\pi)\, \hat{a}_x \ \text{w/m}^2$$

x = 1 plane $\Rightarrow$ $\overline{ds} = dydz\hat{a}_x$

$$W_{avg} = \int_S \overline{P}_{avg}.\overline{ds} \ \text{watts}$$

$$= \frac{1}{2}(0.2)^2(120\pi)\iint dydz$$

$$= \left[\frac{1}{2}\left((0.2)^2(120\pi)\right)\right]\left[\pi(5)^2\right]\times10^{-4}$$

= 0.0592 Watts

= 59.2 mW ≃ 60 mW

07. Ans: (a)

Sol: $P \propto \frac{1}{r^2}$

$$\frac{P_Q}{P_p} = \frac{r_p^2}{r_Q^2} = \frac{(R)^2}{\left(\frac{R}{2}\right)^2}$$

$$\frac{P_Q}{P_P} = \frac{4}{1} = 4:1$$

08. Ans: (b)

Sol: $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$

$$\delta\ \alpha\ \sqrt{\frac{1}{f}} \Rightarrow \frac{\delta_1}{\delta_2} = \sqrt{\frac{f_2}{f_1}}$$

$$\frac{1.5}{\delta} = \sqrt{\frac{8\times10^9}{2\times10^9}}$$

$$\delta = \frac{1.5}{2} = 0.75\ \mu m$$

Similarly

$$\frac{1.5}{\delta} = \sqrt{\frac{18\times10^9}{2\times10^9}} = 3$$

$$\delta = \frac{1.5}{3} = 0.5\mu m$$

09. Ans: (b)

Sol: $\frac{\sigma}{\omega\varepsilon} = \frac{5}{2\times\pi\times25\times10^3\times80\times8.854\times10^{-12}}$

$= 44938.7$

Since $\frac{\sigma}{\omega\varepsilon} >> 1$ hence sea water is a good conductor

Where attenuation is 90%, transmission is 10%, then $e^{-\alpha x} = 0.1$

Where α is attenuation constant

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{2\times\pi\times25\times10^3\times4\pi\times10^{-7}\times5}{2}}$$

$\alpha = 0.7025$

$-\alpha x = ln(0.1)$

$-0.7025x = -2.3$

$x = 3.27m$

10. Ans: (b)

Sol: $\delta = \frac{1}{\alpha} = \frac{1}{2\pi} = 0.159$

11. Ans: (c)

Sol: E is minimum

H is maximum

i.e., 'c' is the option

$E_{Tan_1} = E_{Tan_2} = 0$

[perfect conductor $E_{Tan_2} = 0$]

$H_{Tan_1} = J_S \times a_n + H_{Tan_2}$

$H_{Tan_1} = J_S \times a_n$

[perfect conductor $H_{Tan_2} = 0$]

12. Ans: (d)

Sol: $\vec{H} = 0.5e^{-0.1x}\cos(10^6t-2x)\hat{a}_z$ A/m $\rightarrow$ (+X)

$$\frac{E_y}{H_z} = \eta = -\frac{E_z}{H_y}$$

Wave frequency = 10^6 radians/s

Phase constant β = 2 rad/m

$$\beta = \frac{2\pi}{\lambda} = 2\text{ rad/m}$$

$\lambda = \pi = 3.14$m.

The wave is traveling along +X direction, Given wave is polarized along Y.

$\because$ It has Y-component of electric field

13. Ans: (a)

Sol: The normal incidence of a plane wave traveling in positive y – direction is shown at the interface y = 0 in Fig.

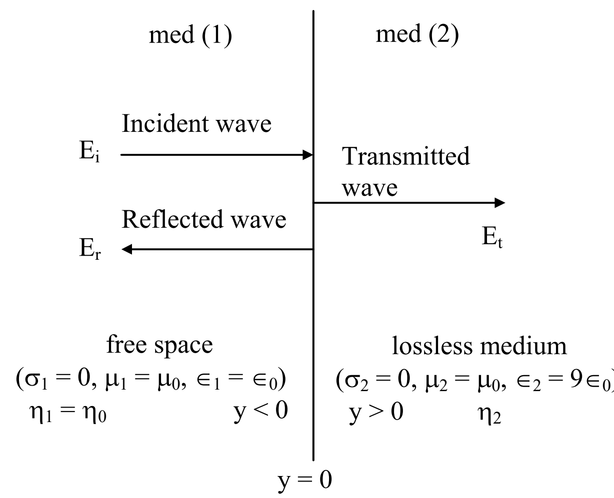


Fig.

Given: $\vec{E}_i = E_{iz}\,\vec{a}_z$

where $E_{iz} = 24\cos(3\times10^8 t - \beta y)$ V/m

$\omega = 3\times10^8$ rad/s, $\beta = \frac{\omega}{v}$,

For free space, $v = v_0 = 3\times10^8$ m/s

$\therefore$ $\beta = 1$ rad/m

$$\eta_1 = \eta_0 = \frac{E_{iz}}{H_{ix}}$$

$\therefore\ H_{ix} = \dfrac{E_{iz}}{\eta_0} = \dfrac{24\cos(3\times10^8 t - \beta y)}{120\pi}$

$\vec{H}_i = H_{ix}\,\vec{a}_x$

$$\frac{H_r}{H_i} = \frac{\eta_1-\eta_2}{\eta_1+\eta_2} = \frac{\frac{\eta_1}{\eta_2}-1}{\frac{\eta_1}{\eta_2}+1},$$

Where $\dfrac{\eta_1}{\eta_2} = \dfrac{\sqrt{\mu_1\in_2}}{\sqrt{\in_1\mu_2}} = \sqrt{\dfrac{\in_2}{\in_1}} = \sqrt{\dfrac{9\in_0}{\in_0}} = 3$

$\therefore\ \dfrac{H_r}{H_i} = \dfrac{3-1}{3+1} = \dfrac{1}{2}$

$$\therefore\ \vec{H}_r = \frac{1}{2}\,\frac{24}{120\,\pi}\cos(3\times10^8 t + 1y)\,\vec{a}_x$$

$$= \frac{1}{10\pi}\cos(3\times10^8 t + 1y)\,\vec{a}_x \ \text{A/m}$$

Note that $\vec{H}_r$ is reflected wave which travels in negative y direction, which corresponds to + βy term with β = 1 in the expression for $\vec{H}_r$.

14. Ans: (b)

Sol: Brewster's angle $\theta_B = \tan^{-1}\sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}}$

$$\theta_B = \tan^{-1}\sqrt{\frac{1}{3}} = 30^o$$

At this angle there is no reflected wave when wave is parallel polarized.

$n_1\sin\theta_i = n_2\sin\theta_t$

$\sqrt{\in_1}\sin\theta_i = \sqrt{\in_2}\sin\theta_t$

$\sin\theta_t = \sqrt{\dfrac{\in_1}{\in_2}}\sin\theta_i$

$\sin\theta_t = \sqrt{3}\,\dfrac{1}{2}\left(\theta_i = 30^o\right)$

$\theta_t = 60^\circ$

15. Ans: (d)

Sol: Given that

$E_t = -2E_r$

Where

E_t is electric field of transmitted wave

E_r is electric field of reflected wave

$$\frac{E_t}{E_r} = -2$$

If E_i is electric field of incident wave.

But $-\dfrac{2E_r}{E_i} = \dfrac{2\eta_2}{\eta_2+\eta_1}$

and $\dfrac{E_r}{E_i} = \dfrac{-\eta_2}{\eta_1+\eta_2}$

and also $\dfrac{E_r}{E_i} = \dfrac{\eta_2-\eta_1}{\eta_2+\eta_1}$

so $\dfrac{\eta_2-\eta_1}{\eta_2+\eta_1} = \dfrac{-\eta_2}{\eta_2+\eta_1}$

$\eta_1 = 2\eta_2$

$\dfrac{\eta_1}{\eta_2} = 2 \qquad \sqrt{\dfrac{\varepsilon_2}{\varepsilon_1}} = 2 \quad \Rightarrow \quad \dfrac{\varepsilon_2}{\varepsilon_1} = 4$

Chapter 3 Transmission Lines

Example 3.7:

(i) $15\frac{\lambda}{4}$ line, the impedance at the junction is inverse impedance of short circuit i.e ∞ for 50 meter line the impedance at the junction is same as load impedance 100 Ω because for this line characteristic impedance is equal to load impedance so the net impedance at the junction is parallel combination of infinity and 100 i.e 100 only

Now the Z_{in} is 100 Ω only because for this line characteristic impedance is equal to load impedance

(ii) For the $\frac{\lambda}{4}$ line the impedance at the junction is inverse of load impedance i.e 0 Ω

For the $\frac{\lambda}{2}$ line the impedance at the junction is same as load impedance i.e 0 Ω so the net impedance at the junction is parallel combination of 0 and 0 i.e 0 only

So now the Z_{in} is 0 Ω because for the $15\frac{\lambda}{2}$ the Z_{in} is same as junction impedance because the impedance is repeated for every $n\frac{\lambda}{2}$ where n is an integer

(iii) As based on the above analysis the Z_{in} is 100 Ω

Example 3.14:

$$Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S = [U - Y]\,[U + Y]^{-1}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} \frac{1}{3} = \frac{1}{3}\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

Example 3.15:

$$Z = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$S = [Z - U]\,[Z + U]^{-1}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} \frac{1}{3} = \frac{1}{3}\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$$

Class Room Practice Solutions

01. Ans: (b)

Sol: $Z_{in} = Z_0 \dfrac{Z_R + jZ_0 \tan \beta\ell}{Z_0 + jZ_R \tan \beta\ell}$

Phase velocity

$$\upsilon_p = \frac{\omega}{\beta}$$

$$\upsilon_p = \frac{2\pi f}{\beta}$$

$$\beta = \frac{2\pi f}{\upsilon_p} = \frac{2\times\pi\times10^8}{2\times10^8} = \pi$$

$\beta\ell = \pi.\ell \Rightarrow \pi$ (Given l=1m)

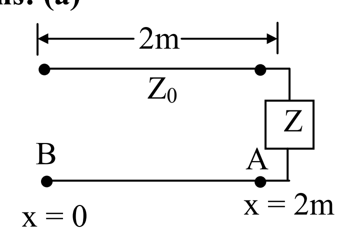
$$\tan\beta\ell = 0$$

$$Z_{in} = Z_R$$

$$= (30 - j40)\Omega$$

02. Ans: (a)

Sol:



$$K_x = \frac{C_2}{C_1} e^{2j\beta X}$$

$$K_A = \frac{C_2}{C_1} e^{j4\beta} \text{ at } (x = 2)$$

$$K_B = \frac{C_2}{C_1} e^{2j\beta(0)} \text{ at } (x = 0)$$

$$\frac{K_B}{K_A} = \frac{\frac{C_2}{C_1} e^{2j\beta(0)}}{\frac{C_2}{C_1} e^{j4\beta}} = e^{-j4\beta}$$

$$\upsilon_P = \frac{\omega}{\beta} \Rightarrow \beta = \frac{\pi}{2}$$

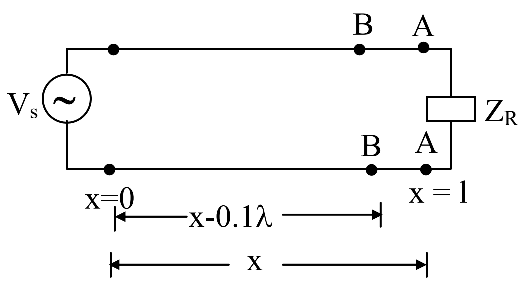
Given f = 50 MHz

$$\upsilon_p = 2 \times 10^8 \text{ m/s}$$

$$\frac{K_B}{K_A} = e^{-j4\left(\frac{\pi}{2}\right)} = e^{-j2\pi} = 1 \text{(or)} \frac{\Gamma_i}{\Gamma_R} = 1$$

03. Ans: (b)

Sol:



$$V = C_1 e^{-j\beta x} + C_2 e^{+j\beta x}$$

$$K_x = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_A = 0.3 e^{-j30^0} = \frac{C_2}{C_1} e^{2j\beta x}$$

$$K_B = \frac{C_2}{C_1} e^{2j\beta(x-0.1\lambda)}$$

$$\frac{K_B}{K_A} = \frac{\frac{C_2}{C_1} e^{2j\beta x} e^{-j4\frac{\pi}{\lambda}0.1\lambda}}{\frac{C_2}{C_1} e^{2j\beta x}}$$

$$K_B = K_A . e^{-j.4\pi}$$

$$= 0.3 e^{-j30^0} e^{-72^0} = 0.3\, e^{-j102^0}$$

Note: In the options $0.3\,e^{j102^0}$ is given. But correct answer is $0.3\,e^{-j102^0}$

04. Ans: (c)

Sol: From the voltage SW pattern,

$V_{min} = 1$, $V_{max} = 4$, VSWR = S = 4

$Z_0 = R_0 = 50\ \Omega$

Let the resistive load be R_L

For Resistive loads

$$S = \frac{R_L}{R_0} \quad \text{for } R_L > R_0$$

$$= \frac{R_0}{R_L} \quad \text{for } R_0 > R_L$$

$\therefore$ $R_L = S\,R_0 = 4 \times 50 = 200\ \Omega$ for $R_L > R_0$

$R_L = R_0/S = 50/4 = 12.5\ \Omega$ for $R_0 > R_L$

As voltage minimum is occurring at the load point, $R_L = 12.5\ \Omega$

05. Ans: (a)

Sol: Reflection coefficient:

$$\Gamma = \frac{R_L - R_0}{R_L + R_0} = \frac{12.5 - 50}{12.5 + 50} = -0.6$$

06. Ans: (d)

Sol: The interconnection of TL's is shown in Fig.

$$Z_{i1} = \frac{(50)^2}{100} = 25\Omega$$

$$Z_{i2} = \frac{(50)^2}{200} = 12.5\Omega$$

$$Z_L = 25 \parallel 12.5 = \frac{25}{3}\ \Omega$$

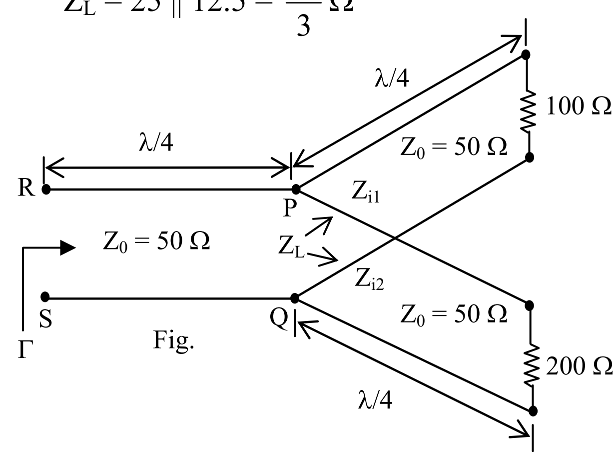


Fig.

Reflection coefficient at PQ $= \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{\frac{25}{3} - 50}{\frac{25}{3} + 50} = -\frac{125}{175} = -\frac{5}{7}$$

$\therefore$ At the input RS,

Reflection coefficient, $\Gamma = -\dfrac{5}{7} e^{-j2\beta\ell}$

$$\text{As } \beta\ell = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\Gamma = -\frac{5}{7} e^{-j\pi} = \frac{5}{7}$$

07. Ans: (d)

Sol: $Z_{in} = Z_0\left[\dfrac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}\right]$

i) For a shorted line,

$Z_L = 0$

$\ell = \lambda/8$

$$\beta\ell = \frac{2\pi}{\lambda}\times\frac{\lambda}{8} = \frac{\pi}{4}$$

$$Z_{in} = Z_0\left[\frac{0 + jZ_0}{Z_0 + 0}\right]$$

$Z_{in} = j\,Z_0$

ii) For a shorted line means $Z_L = 0$

Given that $\ell = \dfrac{\lambda}{4}$

$$\beta\ell = \frac{2\pi}{\lambda}\times\frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = \frac{Z_0^{\ 2}}{Z_L} = \frac{Z_0^{\ 2}}{0}$$

$Z_{in} = \infty$

iii) Open line means $Z_L = \infty$,

Given that $\ell = \dfrac{\lambda}{2}$

$$\therefore \beta\ell = \frac{2\pi}{\lambda}.\frac{\lambda}{2} = \pi \Rightarrow \tan\pi = 0$$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\pi}{Z_0 + jZ_L \tan\pi}\right]$$

$Z_{in} = Z_L$

iv) For a matched line of any length

$Z_L = Z_0$

$$Z_{in} = Z_0\left[\frac{Z_0 + jZ_0 \tan\beta\ell}{Z_0 + jZ_0 \tan\beta\ell}\right] = Z_0$$

08. Ans: (c)

Sol: The line is matched as $Z_L = Z_0 = 50\ \Omega$ and hence reflected wave is absent.

For the traveling wave, given:

Phase difference for a length of 2 mm = $\pi/4$ rad

Frequency of excitation = 10 GHz

Phase velocity, $v_p = \dfrac{\omega}{\beta}$

$\omega = 2\pi \times 10 \times 10^9$ rad/sec

β = Phase-shift per unit length

$$= \frac{\pi}{4\times2\times10^{-3}}\ \text{rad/m}$$

$$v_p = \frac{2\pi\times10^{10}\times8}{\pi\times10^3} = 1.6\times10^8\ \text{m/s}$$

09. Ans: (b)

Sol: $[S] = \begin{bmatrix} 0.3\angle0^0 & 0.9\angle90^0 \\ 0.9\angle90^0 & 0.2\angle0^0 \end{bmatrix}$

For reciprocal; $S_{11} = S_{22}$

It is satisfied.

For lossless line $|S_{11}|^2 + |S_{12}|^2 = 1$

$$(0.3)^2 + (0.9)^2 = 0.9 \neq 1$$

$\therefore$ It is a lossy line

Chapter 4

Waveguides

Class Room Practice Solutions

01. Ans: (b)

Sol: Evanescent modes means no wave propagation.
Dominant mode means, the guide has lowest cut-off frequency.
TM_{01} and TM_{10} not possible, the minimum values of m, n for TM are at least 1, 1 respectively

02. Ans: (a)

Sol: The mode which has lowest cutoff frequency is called dominant mode TE_{10}.

At 4GHz all modes are evanescent.

At 7GHz degenerate modes are possible

TE_{11} and TM_{11} are degenerate.

$$f_{c\,TE_{10}} = \frac{c}{2a} = \frac{3\times10^{8}}{2\times3\times10^{-2}} = 5\text{ GHz.}$$

At 6 GHz dominant mode will propagate.
At 11 GHz higher order modes are possible

03. Ans: (a)

Sol: Given: In a rectangular WG of cross-section : (a × b)

$$\vec{E} = \frac{\omega\mu}{h^2}\left(\frac{\pi}{a}\right) H_0 \sin\left(\frac{2\pi}{a}x\right)\sin(\omega t - \beta z)\,\hat{y}$$

The wave is traveling in the z-direction having E_y component only as function of 'x'. As there is no component of $\vec{E}$ in the direction of propagation, $\vec{a}_z$ the wave is Transverse Electric (TE). Comparing the 'sin' term in $\vec{E}$ with the general expression: $\sin\left(\frac{m\pi}{a}x\right)$

$$m = 2$$

As there is no function of 'y' in $\vec{E}$, n = 0
∴ The mode of propagation in the WG is TE_{20}

04. Ans: (d)

Sol: Given

a = 4.755, b = 2.215,

f = 12 GHz, c = 3 × 10^8 m/s

Cut off frequency

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

For TE_{10}, mode

$$f_c = \frac{c}{2a} = 3.15\text{ GHz}$$

$f > f_c$ (TE_{10} mode) so it propagates

For TE_{20} mode

$$f_C(TE_{20}) = \frac{c}{2}\sqrt{\left(\frac{2}{a}\right)^2}$$

$$= 2\,[f_c(TE_{10})] = 6.30\text{ GHz}$$

$f > f_c$ [TE_{20}] so it propagates

For TE_{01} mode

$$f_{C\,(TE01)} = \frac{c}{2}\sqrt{\frac{1}{b^2}}$$

$$= \frac{c}{2b} = 6.77\text{GHz}$$

∴ $f > f_c$ (TE_{01}] so it propagate

For TE_{11} mode

$$f_{c[TE11]} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 7.47\text{ GHz}$$

$f > f_c$ (TE_{11}) so it propagate

So, all modes are possible to propagate.

05. Ans: (a)

Sol: Given a = 6cm, b = 4 cm f = 3 GHz

Cut off frequency

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

TE_{10}: $f_c = \frac{c}{2a} = 2.5$ GHz

TE_{01}: $f_c = \frac{c}{2b} = 3.75$ GHz

TE_{11}: $f_c = \frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = 4.50$ GHz

TM_{11}: $f_c = \frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = 4.50$ GHz

06. Ans: (a)

Sol: $\frac{m\pi}{a} = \frac{2\pi}{a} \Rightarrow m = 2$

$\frac{n\pi}{b} = \frac{3\pi}{b} \Rightarrow n = 3$

For TM wave propagating along z-direction

$E_z \neq 0$ and $H_z = 0$

TM_{23}

$$TM_{23} \Rightarrow f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Substitute $c = 3 \times 10^{10}$ cm/sec

$m = 2, \quad a = 6$ cm

$n = 3, \quad b = 3$ cm

we get $f_c = 15.811$ GHz

$$\eta_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$\omega = 10^{12} \Rightarrow f = \frac{10^{12}}{2\pi} = \frac{10^3}{2\pi}\text{GHz}$$

and $\eta = 120\ \pi$. & $f_c = 15.811$ GHz

Substitute all the above values and we get $\eta_{TM} = 375\ \Omega$

07. Ans: (c)

Sol: $W_{avg} = \frac{1}{4}\frac{E_{yo}^2}{\eta_{TE_{10}}}a.b$; $\eta_{TE_{10}} = \frac{\eta}{\sqrt{1-(\lambda/\lambda_c)^2}}$

$\eta = 120\pi$, $\lambda = \frac{c}{f} = \frac{3\times10^{10}}{11\times10^9} = 2.72$cm

$\lambda_c = 2a = 2\times2.29 = 4.58$cm

So we get $\eta_{TE_{10}} = 469.52\Omega$

Putting all the values

$\therefore W_{avg} = 31.32$kW

08. Ans: (a)

09. Ans: (a)

Sol: $\frac{1}{\lambda^2} = \frac{1}{\lambda_g^2} + \frac{1}{\lambda_c^2}$

$f_c = 0.908\,\text{GHz}$

$$\Rightarrow \lambda_c = \frac{3\times10^{10}}{0.908\times10^9} = 33.03\,\text{cm}$$

Substitute $\lambda_g = 40$ cm, $\lambda_c = 33.03$ cm

We get, $\lambda = 25.47$ cm

$$\Rightarrow f = \frac{3\times10^{10}}{25.47}$$

$= 1.18$ GHz

10. Ans: (a)

Sol: $\frac{c}{2a} = 0.908\,\text{GHz}$

$$\Rightarrow a = \frac{3\times10^{10}}{2\times(0.908)\times10^9}$$

$= 16.51\,\text{cm}$

$$\Rightarrow b = \frac{a}{2} = 8.26\,\text{cm}$$

11. Ans: (a)

Sol: $\overline{\beta} = \beta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$$= \frac{2\pi}{25.47}\sqrt{1-\left(\frac{0.908}{1.18}\right)^2}$$

$= 0.157$ rad/cm

$= 15.7$ rad/m

Chapter **5**

Elements of Antennas

Class Room Practice Solutions

01. Ans: (c)

Sol: Antenna receives 2 μW of power: $P_r = 2\ \mu W$

RMS value of incident E field = 20 mV/m

Power density, P_d

$$= \frac{E^2}{\eta} = \frac{(20\times10^{-3})^2}{377} W/m^2$$

Effective aperture area, $A_e = \frac{P_r}{P_d}$

$$= \frac{2\times10^{-6}}{\frac{(20\times10^{-3})^2}{377}} = \frac{377\times 2}{400} = 1.885\ m^2$$

02. Ans: (b)

Sol: Lossless antenna directive gain = 6 dB = 4

Input power to the antenna = 1 mW

for lossless we get 100% efficiency

$$\frac{W_{rad}}{W_{in}} = \frac{G_o}{D_o} = 1$$

$W_{rad} = W_{in}$

$W_{rad} = 1mW$

03. Ans: (c)

Sol: $P_{rad} = \frac{A_0 \sin^2\theta}{r^2}\hat{a}_r \quad W/m^2$

$$W_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{A_0\sin^2\theta}{r^2} r^2 \sin\theta d\theta d\phi$$

$$= A_0\, 2\pi \int_{\theta=0}^{\pi} \sin^3\theta d\theta$$

$$= A_0\, 2\pi \frac{4}{3}$$

$$W_{rad} = A_0 \frac{8\pi}{3}$$

$$U = r^2 P_{rad} = r^2 \frac{A_0 \sin^2\theta}{r^2} = A_0 \sin^2\theta$$

$$D_{max} = \frac{U_{max}}{W_{rad}} 4\pi = \frac{|A_0\sin^2\theta|_{max}}{\frac{8\pi}{3}A_0} \times 4\pi$$

$$= \frac{4\pi A_0}{8\pi A_0}\times 3$$

$$= \frac{3}{2} = D_{max} = 1.5$$

04. Ans: (d)

Sol: Where $W_{rad} = \oiint \bar{P}_{rad}.\bar{ds}$

$$\bar{P}_{rad} = \frac{W_{rad}}{2\pi r^2}.\hat{a}_r = \frac{40}{\pi}\hat{a}_r\ \ \mu W/m^2$$

05. Ans: (b)

Sol: $R_{rad} = 30\ \Omega$, $R_l = 10\Omega$

$G_D = 4$, $G_p = ?$

$$\eta = \frac{R_{rad}}{R_{rad}+R_\ell} = \frac{30}{40} = 0.75$$

$G_p = \eta\ G_D$

$= 0.75 \times 4 = 3$

06. Ans: (c)

Sol: $D_g = 30$ dB=1000

$P_T = 7.5$ kW

$$D_g = \frac{4\pi\times \text{Radiation intensity}}{\text{Radiated Power}}$$

$$D_g = 4\pi \frac{U}{W_{rad}}$$

$$\therefore U = \frac{7.5\times10^3 \times 1000}{4\pi}$$

$$\Rightarrow U = r^2 P_{rad}$$

P_{rad} : Power density we have to find

P_{rad} at $r = 40\times10^3$ m

$$P_{rad} = \frac{U}{r^2}$$

$$= \frac{7.5\times10^3\times1000}{4\pi\times(40\times10^3)^2}\ \text{W/m}^2$$

07. Ans: (d)

Sol: $W_{rad} = 10\text{kW}$

$E_{max} = 120$ mV/m

R = 20km

$\eta = 98\%$

$$P_{rad} = \frac{E_0^2}{2\eta_0}$$

$$= \frac{(120\times10^{-3})^2}{2\times120\pi}$$

$$= 1.909\times10^{-5}$$

$$U_{max} = (20\times10^3)^2\times1.909\times10^{-5}$$

$$= 7639.43$$

$$D_0 = 4\pi\frac{U_{max}}{W_{rad}}$$

$$D_0 = 4\pi\frac{7639.43}{10\times10^3} = 9.59$$

$$\eta = \frac{G_0}{D_0} = 0.98$$

$$G_0 = 0.98\times9.59$$

$$= 9.407$$

08. Ans: 0.21

Sol: Given:

Antenna length, l = 1cm

Frequency, f = 1 GHz

Distance, r = 100λ

Wave length, $\lambda = \frac{C}{f}$

$$= \frac{3\times10^8}{10^9}$$

$$= 30 \text{ cm}$$

$\frac{d\ell}{\lambda} = \frac{1}{30}$, hence the given antenna is Hertzian dipole.

In the far field, the tangential electric field is given by, $E_\theta = \frac{j\eta I d\ell \sin\theta}{4\pi}\frac{\beta}{\gamma}$

$$= \frac{j377\times0.1}{200\times30\times30}$$

$$\therefore |E_\theta| = 0.21\text{V/cm}$$

09. Ans: (c)

Sol: Given:

Length of dipole, $\ell = 0.01\lambda$

As it is very small, compared with wavelength, hence it can be approximated to Hertzian dipole

$$R_{rad} = 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2$$

$$= 80\,\pi^2\,(0.01)^2$$

$$R_{rad} = 0.08\ \Omega$$

10. Ans: (d)

Sol: $$AF = \frac{\sin\frac{n\varphi}{2}}{\sin\frac{\varphi}{2}}$$

take limit

$$\frac{\underset{\frac{n\varphi}{2}\to 0}{Lt}\ \frac{\sin\frac{n\varphi}{2}}{\frac{n\varphi}{2}}.\frac{n\varphi}{2}}{\underset{\frac{\varphi}{2}\to 0}{Lt}\ \frac{\sin\frac{\varphi}{2}}{\frac{\varphi}{2}}.\frac{\varphi}{2}} = n$$

11. Ans: (b)

Sol: In broad side array the BWFN is given by

$$BWFN = \frac{2\lambda}{L}(rad)$$

Where, L = length of the array

$$L = (n-1)\,d$$

Given: n = 9

Spacing, $d = \frac{\lambda}{4}$

$$BWFN = \frac{2\lambda}{(9-1)\frac{\lambda}{4}}$$

$$= \frac{2\lambda}{2\lambda}\times\frac{180}{\pi}$$

$\therefore$ BWFN = 57.29°

12. Ans: (d)

Sol: The directivity of n-element end fire array is given by

$$D = \frac{4L}{\lambda}$$

Where, L = (n–1)d

$L \cong nd\ \ (\because n = 1000, \text{very large})$

$$D = \frac{4\times nd}{\lambda}$$

$$= \frac{4\times 1000\lambda}{\lambda\times 4}$$

$\therefore D \simeq 1000$

Directivity, (in dB) = 30

13. Ans: 7.78

Sol: Directivity, $D = 4\pi\frac{U_{max}}{P_{rad}}$

Given: $U(\theta, \phi) = 2\sin\theta\,\sin^3\phi;\ 0 \le \theta \le \pi,$
$0 \le \phi \le \pi$

$U_{max} = 2$

$$P_{rad} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{\pi} 2\sin\theta\sin^3\phi\sin\theta\,d\theta\,d\phi$$

$$= 2\int_{\theta=0}^{\pi}\int_{\phi=0}^{\pi}\sin^2\theta\sin^3\phi\,d\theta\,d\phi$$

$$= 2\left(\frac{\pi}{2}\right)\left(\frac{4}{3}\right)$$

$$= \frac{4\pi}{3}$$

$$D = 4\pi\times\frac{2}{\left(\frac{4\pi}{3}\right)}$$

D = 6

Directivity, (in dB) = 10log6 = 7.7815

14. Ans: 2793

Sol: For Hertzian dipole the directivity, D is given by D = 1.5

$$D = \left(\frac{4\pi}{\lambda^2}\right)A_e$$

$$A_e = 1.5\times\frac{\lambda^2}{4\pi}$$

$A_e = 0.119\ \lambda^2$

$$\text{Wavelength, } \lambda = \frac{3\times10^8}{10^8} = 3\text{m}$$

$\therefore A_e = 0.119 \times 9$

$A_e = 1.074\ m^2$

Aperture area of antenna is given by

$$A_e = \frac{P_r}{P}$$

Where, P_r = power received at the antenna load terminals.

P = power density of incident wave

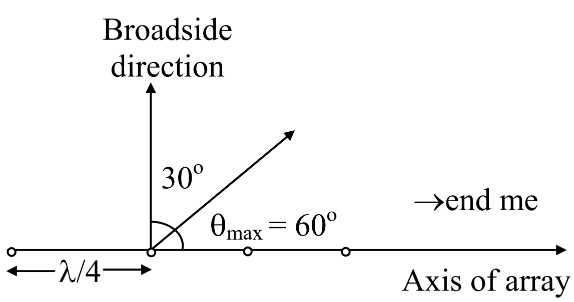
$$P = \frac{P_r}{A_e}$$

$$= \frac{3\times10^{-6}}{1.074}$$

$\therefore$ P = 2.793 $\mu W/m^2$ (or) 2793 nW/m^2

15. Ans: (c)

Sol:



Given: No. of elements, n = 4

$$\text{Spacing, } d = \frac{\lambda}{4}$$

Direction of main beam (or) principal lobe, $\theta_{max} = 60^o$

Array phase function, ψ is given by

$\psi = \beta d\cos\theta + \alpha$

To form a major lobe. $\psi = 0$

$\alpha = -\beta d\cos\theta_{max}$

$$\alpha = -\frac{2\pi}{\lambda}\times\frac{\lambda}{4}\cos 60$$

$$\alpha = -\frac{\pi^C}{4}$$

The phase shaft between the elements required is $\alpha = -\frac{\pi^C}{4}$