



ELECTRICAL ENGINEERING



GATE | PSU's

ELECTRIC
CIRCUITS

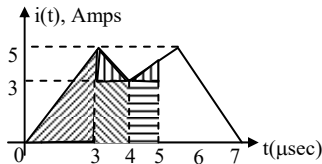
Volume - I : Study Material with Classroom Practice Questions

Electric Circuits

1. Basic Concepts

01. Ans: (c)

Sol:



We know that;

$$i(t) = \frac{dq(t)}{dt}$$

$$\Rightarrow dq(t) = i(t) \cdot dt$$

$$q = \int_0^{5 \mu\text{sec}} i(t) dt = \text{Area under } i(t) \text{ upto } 5 \mu\text{sec}$$

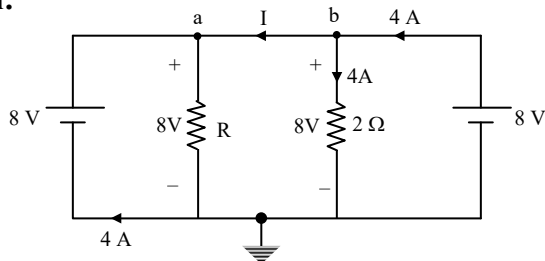
$$q = q_1 + q_2 + q_3$$

$$= \left(\frac{1}{2} \times 3 \times 5\right) + \left(\frac{1}{2} \times 1 \times 2 + (1 \times 3)\right) + \left(\frac{1}{2} \times 1 \times 1 + (1 \times 3)\right)$$

$$q = 15 \mu\text{C}$$

02. Ans: (a)

Sol:



Applying KCL at node 'b'

$$I + 4 = 4 \Rightarrow I = 0 \text{ A and } \frac{8}{R} = 4$$

$$\Rightarrow R = 2 \Omega$$

03. Ans: (a)

Sol: The energy stored by the inductor (1Ω , 2 H) upto first 6 sec:

$$E_{\text{stored upto 6 sec}} = \int P_L dt = \int \left(L \frac{di(t)}{dt} \cdot i(t) \right) dt$$

$$= \int_0^2 \left(2 \left[\frac{d}{dt} (3t) \right] \times 3t \right) dt + \int_2^4 \left(2 \left[\frac{d}{dt} (6) \right] \times 6 \right) dt$$

$$+ \int_4^6 \left(2 \left[\frac{d}{dt} (-3t + 18) \right] \times (-3t + 18) \right) dt$$

$$= \int_0^2 18t dt + \int_2^4 0 dt + \int_4^6 (-6[-3t + 18]) dt$$

$$= 36 + 0 - 36 = 0 \text{ J (or)}$$

$$E_{\text{stored upto 6 sec}} = E_L |_{t=6 \text{ sec}} = \frac{1}{2} L (i(t) |_{t=6})^2$$

$$= \frac{1}{2} \times 2 \times 0^2 = 0 \text{ J}$$

04. Ans: (d)

Sol: The energy absorbed by the inductor (1Ω , 2 H) upto first 6 sec:

$$E_{\text{absorbed}} = E_{\text{dissipated}} + E_{\text{stored}}$$

Energy is dissipated in the resistor

$$E_{\text{dissipated}} = \int P_R dt = \int (i(t))^2 R dt$$

$$= \int_0^2 (3t)^2 \times 1 dt + \int_2^4 (6)^2 \times 1 dt + \int_4^6 (-3t + 18)^2 \times 1 dt$$

$$= \int_0^2 9t^2 dt + \int_2^4 36 dt + \int_4^6 (9t^2 + 324 - 108t) dt$$

$$= 24 + 72 + 24 = 120 \text{ J}$$

$$\therefore E_{\text{dissipated}} = 120 \text{ J}$$

$$\text{And } E_{\text{stored upto 6 sec}} = 0 \text{ J}$$

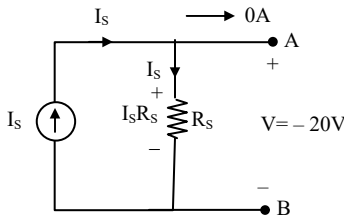


∴ E_{absorbed} = E_{dissipated} + E_{stored}

⇒ E_{absorbed} = 120J+0J=120J

05. Ans: (a)

Sol: Point (-20, 0) ⇒ V = -20V and I = 0A



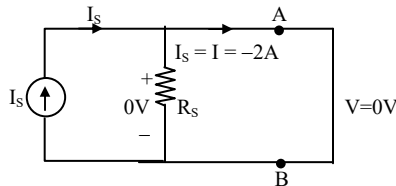
By KVL

⇒ I_s R_s - V = 0

⇒ I_s R_s + 20 = 0

⇒ I_s R_s = -20V (1)

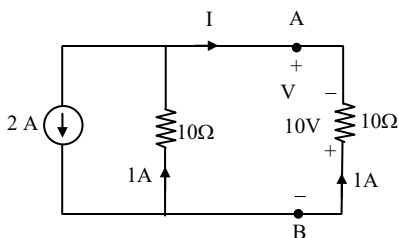
Point: (0, -2) ⇒ V = 0V and I = -2A



⇒ I_s = -2A

Substituting I_s in eq (1),

R_s = 10Ω

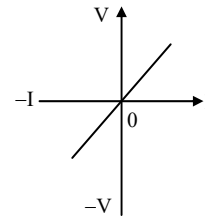


From the diagram;

I = -1A and V = -10V

06. Ans: (a)

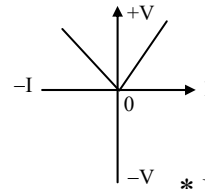
Sol:



- * linear
* Passive
* bilateral

07. Ans: (b)

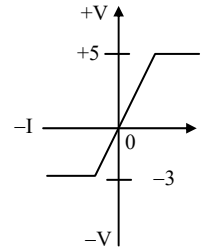
Sol:



- * Non linear
* Active
* Unilateral

08. Ans: (e)

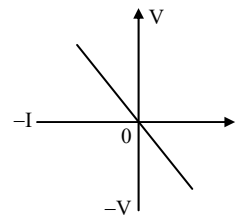
Sol:



- * Non linear
* Passive
* Unilateral

09. Ans: (c)

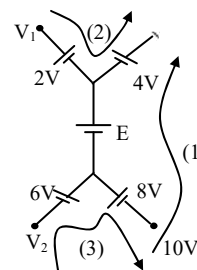
Sol:



- * Linear
* Active
* Bilateral

10.

Sol:





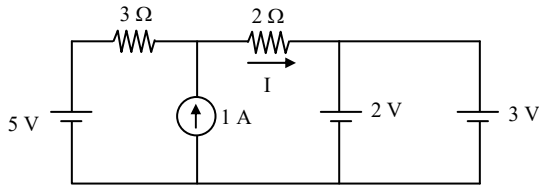
(1) By KVL $\Rightarrow +10 + 8 + E + 4 = 0$
 $E = -22V$

(2) By KVL $\Rightarrow +V_1 - 2 + 4 = 0$
 $V_1 = -2V$

(3) By KVL $\Rightarrow +V_2 + 6 - 8 - 10 = 0$
 $V_2 = 12V$

11. Ans: (d)

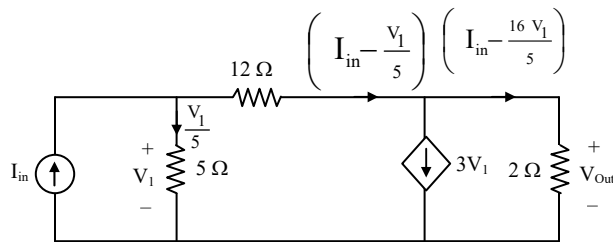
Sol:



Here the 2V voltage source and 3V voltage source are in parallel which violates the KVL. Hence such circuit does not exist. (But practical voltage sources will have some internal resistance so that when two unequal voltage sources are connected in parallel current can flow and such a circuit may exist).

12. Ans: (d)

Sol:



Applying KVL,

$$-V_1 + 12\left(I_{in} - \frac{V_1}{5}\right) + 2\left(I_{in} - \frac{16V_1}{5}\right) = 0$$

$$-V_1 + 12I_{in} - \frac{12V_1}{5} + 2I_{in} - \frac{32V_1}{5} = 0$$

$$14I_{in} = \frac{49}{5}V_1$$

$$\Rightarrow V_1 = \frac{70}{49}I_{in} \dots\dots\dots (1)$$

$$\therefore V_{out} = 2\left(I_{in} - \frac{16V_1}{5}\right) \dots\dots\dots (2)$$

Substitute equation (1) in equation (2)

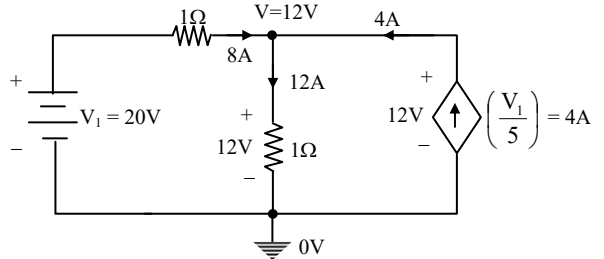
$$V_{out} = 2\left(I_{in} - \frac{16}{5} \times \frac{70}{49} I_{in}\right)$$

$$= 2\left(\frac{-25}{7}\right) I_{in} = \frac{-50}{7} I_{in}$$

$$\therefore V_{out} = -7.143 I_{in}$$

13. Ans: (c)

Sol:



By nodal \Rightarrow

$$V - 20 + V - 4 = 0$$

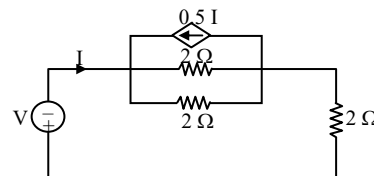
$$V = 12\text{volts}$$

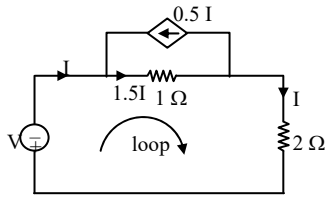
Power delivered by the dependent source is

$$P_{del} = (12 \times 4) = 48 \text{ watts}$$

14. Ans: (d)

Sol:

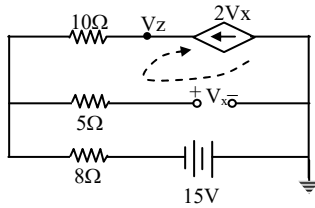




Applying KVL,
 $\Rightarrow V + 1.5I + 2I = 0$
 $\Rightarrow V = -3.5 I$

15. Ans: (c)

Sol:



By using KCL

$$\frac{V_x + 15}{8} - 2V_x = 0 \Rightarrow V_x = 4V$$

By using nodal Analysis at V_z node

$$\frac{V_z + 15}{18} - 2 = 0 \Rightarrow V_z = +21V$$

16. Ans: (a)

Sol: The given circuit is a bridge.

$I_R = 0$ is the bridge is balanced. i.e.,

$$Z_1 Z_4 = R_2 R_3$$

Where $Z_1 = R_1 + j\omega L_1$,

$$Z_4 = R_4 - \frac{j}{\omega C_4}$$

As $R_2 R_3$ is real, imaginary part of $Z_1 Z_4 = 0$

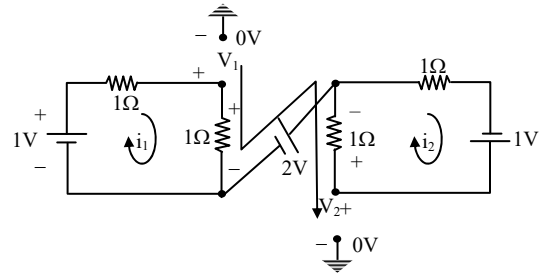
$$\omega L_1 R_4 - \frac{R_1}{\omega C_4} = 0 \quad \text{or} \quad \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$$

$$\text{or } Q_1 = Q_4$$

where Q is the Quality factor.

17.

Sol:



$$\text{By KVL } \Rightarrow 1 - i_1 - i_1 = 0$$

$$i_1 = 0.5A$$

$$\text{By KVL } \Rightarrow -i_2 - i_2 + 1 = 0$$

$$i_2 = 0.5A$$

$$\text{By KVL } \Rightarrow V_1 - 0.5 + 2 + 0.5 - V_2 = 0$$

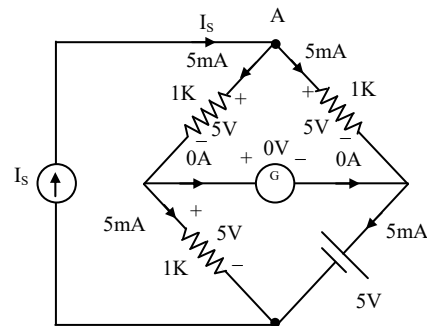
$$V_2 = V_1 + 2V$$

18.

Sol: As the bridge is balanced; voltage across (G) is "0V".

$$\text{By KCL at node "A"} \Rightarrow -I_s + 5m + 5m = 0$$

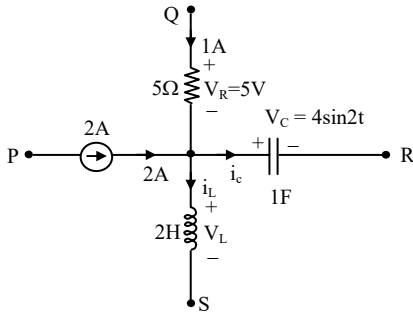
$$I_s = 10mA$$



19.

Sol: Given data:

$$V_R = 5V \text{ and } V_C = 4\sin 2t \text{ then } V_L = ?$$



$$i_c = \frac{CdV_c}{dt} = \frac{d}{dt}(4\sin 2t) = 8\cos 2t$$

By KCL; $-1 - 2 + i_L + i_c = 0$

$$i_L = 3 - 8\cos 2t$$

We know that;

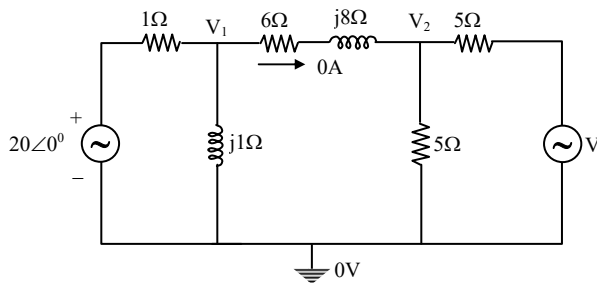
$$V_L = L \frac{di_L}{dt} = 2 \frac{d}{dt}(3 - 8\cos 2t)$$

$$= 2(-8)(-2)\sin 2t$$

$$V_L = 32\sin 2t \text{ volt}$$

20.

Sol: $V = ?$ If power dissipated in 6Ω resistor is zero.



$$P_{6\Omega} = 0 \text{ W (Given)}$$

$$\Rightarrow i_{6\Omega}^2 \cdot 6 = 0$$

$$\Rightarrow i_{6\Omega} = 0 \text{ (} V_{6\Omega} = 0 \text{)}$$

$$\frac{V_1 - V_2}{6 + j8} = 0; V_1 = V_2$$

$$\text{By Nodal} \Rightarrow \frac{V_1 - 20\angle 0^\circ}{1} + \frac{V_1}{j1} + 0 = 0$$

$$V_1 = 10\sqrt{2} \angle 45^\circ = V_2$$

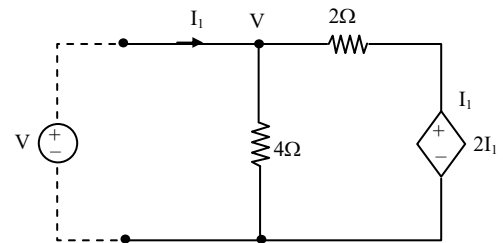
$$\text{By Nodal} \Rightarrow 0 + \frac{V_2}{5} + \frac{V_2 - V}{5} = 0$$

$$V = 2V_2 = 2(10\sqrt{2} \angle 45^\circ)$$

$$\therefore V = 20\sqrt{2} \angle 45^\circ$$

21. Ans: (d)

Sol:



Note: Since no independent source in the network, the network is said to be unenergised, so called a DEAD network".

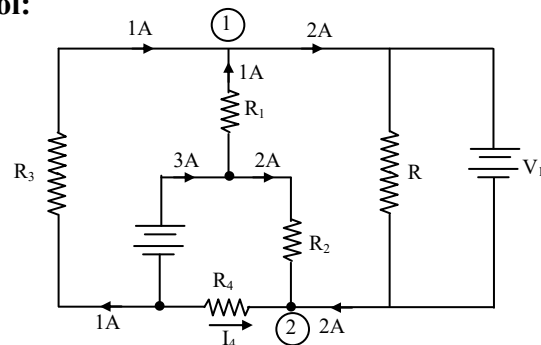
The behavior of this network is a load resistor behavior.

$$\text{By Nodal} \Rightarrow -I_1 + \frac{V}{4} + \frac{V - 2I_1}{2} = 0$$

$$3V = 8I_1 \Rightarrow R_{eq} = \frac{V}{I_1} = \frac{8}{3} \Omega$$

22. Ans: (a)

Sol:



Apply KCL at Node - 1,

$$I = I_{R_1} + I_{R_3} = 1 + 1 = 2A$$



Apply KCL at Node - 2,

$$I_4 = -I_2 - I = -2 - 2 = -4A$$

23.

Sol:

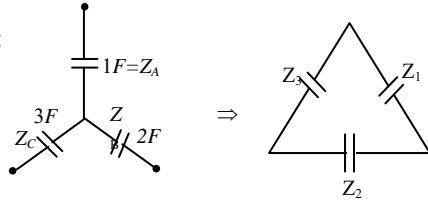


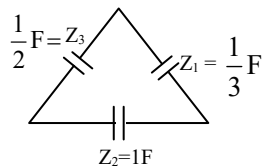
Fig.1

$$Z_1 = Z_A + Z_B + \left(\frac{Z_A Z_B}{Z_C} \right)$$

$$= \frac{1}{s} + \frac{1}{2s} + \left(\frac{1}{s} \right) \left(\frac{1}{2s} \right)$$

$$= \frac{1}{s} + \frac{1}{2s} + \left(\frac{1}{3s} \right)$$

$$Z_1 = \frac{1}{s \left(\frac{1}{3} \right)} ; \quad C = \frac{1}{3} F$$



$$Z_2 = Z_B + Z_C + \frac{Z_B Z_C}{Z_A} = \frac{1}{2s} + \frac{1}{3s} + \left(\frac{1}{2s} \right) \left(\frac{1}{3s} \right)$$

$$\left(\frac{1}{s} \right)$$

$$Z_2 = \frac{1}{s(1)} ; \quad C = 1F$$

$$Z_3 = Z_A + Z_C + \frac{Z_A Z_C}{Z_B}$$

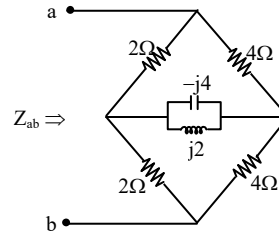
$$= \frac{1}{s} + \frac{1}{3s} + \left(\frac{1}{s} \right) \left(\frac{1}{3s} \right)$$

$$\left(\frac{1}{2s} \right)$$

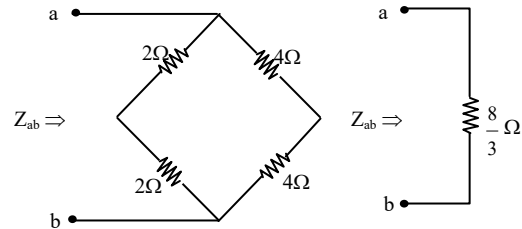
$$Z_3 = \frac{1}{s \left(\frac{1}{2} \right)} ; \quad C = \frac{1}{2} F$$

24.

Sol: $Z_{ab} = ?$



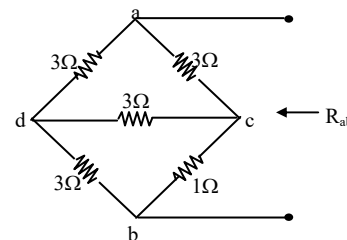
Since $2 * 4 = 4 * 2$; the given bridge is balanced one, therefore the current through the middle branch is zero. The bridge acts as below :



$$Z_{ab} = \frac{4 \times 8}{4 + 8} = \frac{8}{3} \Omega$$

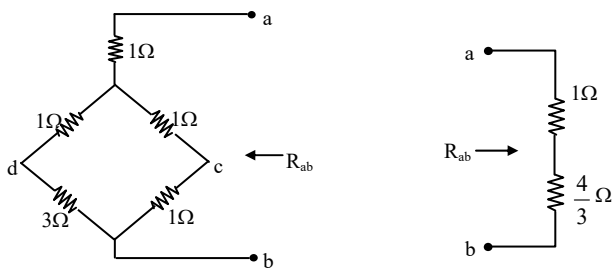
25.

Sol: Redraw the circuit diagram as shown below:





Using Δ to star transformation:



$$\therefore R_{ab} = 1 + \frac{4}{3} = \frac{7}{3} \Omega$$

26.

Sol: On redrawing the circuit diagram

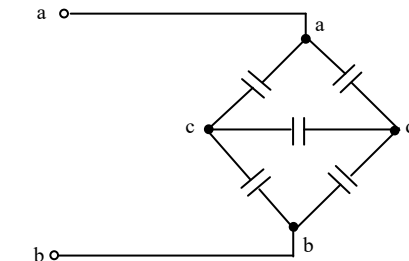
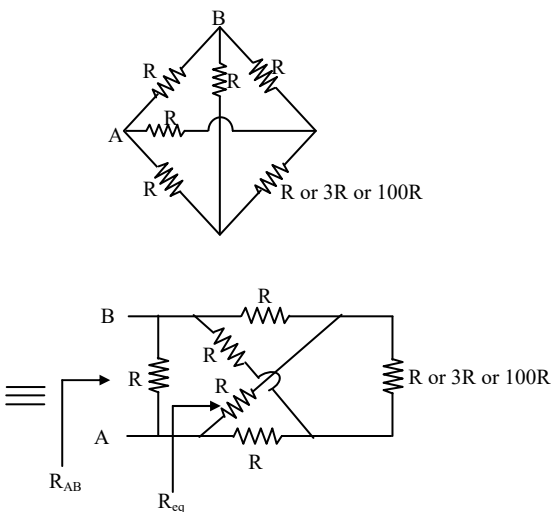
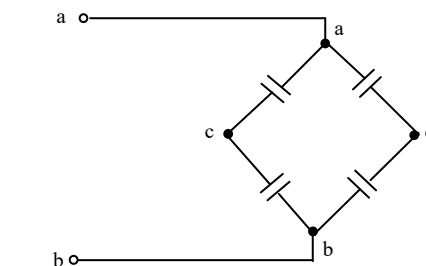
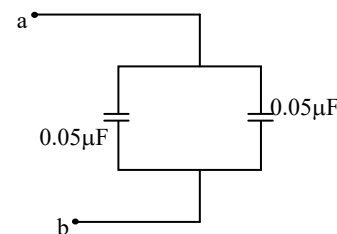


Fig. 1



$$= \frac{0.1 \times 0.1}{0.2} = 0.05 \mu\text{F}$$



$$C_{ab} = 0.1 \mu\text{F}$$

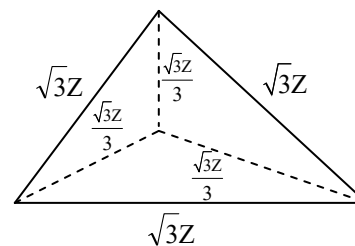
Note: The bridge is balanced and the answer is easy to get.

27. Ans: (b)

Sol: The equivalent capacitance across a, b is calculated by simplifying the bridge circuit as shown in Fig. 1 to Fig. 5. [$\because C = 0.1 \mu\text{F}$]

28. Ans:(a)

Sol: Consider a Δ connected network

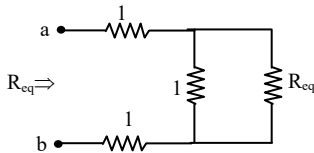




Then each branch of the equivalent λ connected impedance is $\frac{\sqrt{3}Z}{3} = \frac{Z}{\sqrt{3}}$

29. Ans: (a)

Sol: Network is redrawn as



$$R_{eq} = 1 + 1 + \frac{R_{eq}}{1 + R_{eq}}$$

$$= 2 + \frac{R_{eq}}{1 + R_{eq}} = \frac{2 + 2R_{eq} + R_{eq}}{1 + R_{eq}}$$

$$R_{eq} + R_{eq}^2 = 2 + 3R_{eq}$$

$$R_{eq}^2 - 2R_{eq} - 2 = 0$$

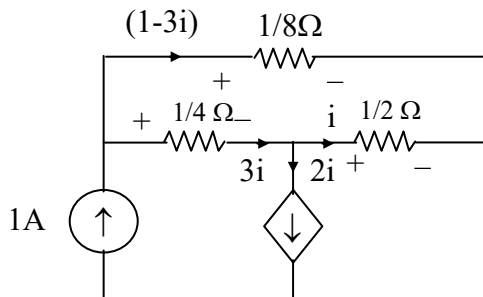
$$R_{eq} = (1 + \sqrt{3})\Omega$$

30. Ans: (c)

Sol: Applying KCL

$$I_{0.25\Omega} = 2i + i = 3i$$

$$I_{0.125\Omega} = (1 - 3i)A$$



Applying KVL in upper loop.

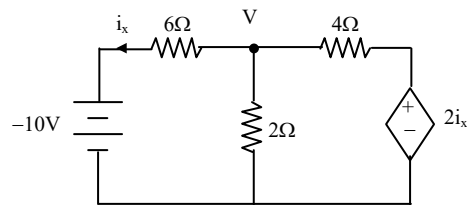
$$-\frac{(1-3i)}{8} + \frac{i}{2} + \frac{3i}{4} = 0$$

$$\frac{5i}{4} = \frac{1-3i}{8} \Rightarrow 10i = 1 - 3i \Rightarrow i = \frac{1}{13}A$$

$$V = \frac{3i}{4} = \frac{3}{4} \times \frac{1}{13} = \frac{3}{52}V$$

31. Ans: (a)

Sol:



Applying KCL at Node V

$$\frac{V}{2} + \frac{V - 2i_x}{4} + i_x = 0 \dots\dots\dots (1)$$

$$i_x = \frac{V+10}{6} \Rightarrow V = 6i_x - 10$$

Put in equation (1), we get

$$3i_x - 5 + i_x - 2.5i_x = 0$$

$$5i_x = 7.5$$

$$i_x = 1.5A$$

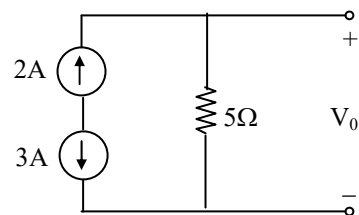
$$V = -1V$$

$$I_{\text{dependent source}} = \frac{V - 2i_x}{4} = \frac{-1 - 3}{4} = -1A$$

$$\therefore \text{Power absorbed} = (I_{\text{dependent source}})(2i_x) = (-1)(3) = -3W$$

32. Ans: (d)

Sol: $V_0 = ?$



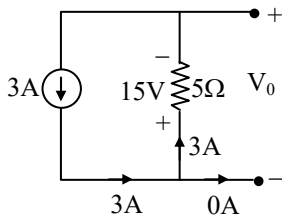


By KCL \Rightarrow $+2 + 3 = 0$
 $+5 \neq 0$

Since the violation of KCL in the circuit ; physical connection is not possible and the circuit does not exist.

33. Ans: (b)

Sol: Redraw the given circuit as shown below:



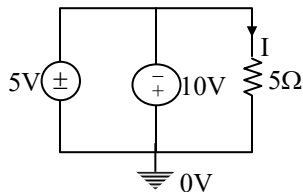
By KVL \Rightarrow
 $-15 - V_0 = 0$
 $V_0 = -15V$

34. Ans: (d)

Sol: Redraw the circuit diagram as shown below:

Across any element two different voltages at a time is impossible and hence the circuit does not exist.

Another method:

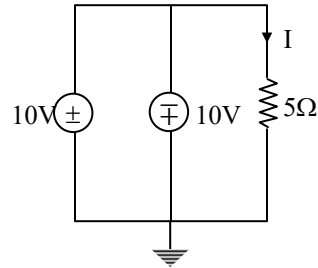


By KVL \Rightarrow
 $5 + 10 = 0$
 $15 \neq 0$

Since the violation of KVL in the circuit, the physical connection is not possible.

35. Ans: (d)

Sol: Redraw the given circuit as shown below:

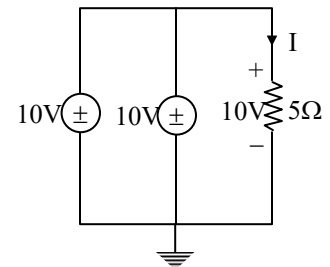


By KVL \Rightarrow
 $-10 - 10 = 0$
 $-20 \neq 0$

Since the violation of KVL in the circuit, the physical connection is not possible.

36. Ans: (b)

Sol: Redraw the given circuit as shown below:



By KVL \Rightarrow
 $10 - 10 = 0$
 $0 = 0$

KVL is satisfied

$I_{5\Omega} = \frac{10}{5} = 2A$
 $I_{5\Omega} = 2A$

37. Ans: (d)

Sol:

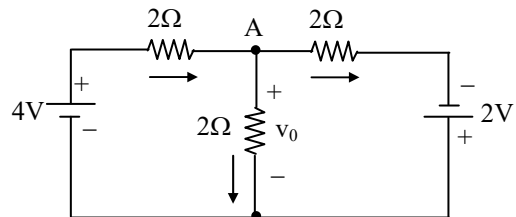


Fig. 1



The diode is forward biased. Assuming that the diode is ideal, the Network is redrawn with node A marked as in Fig. 1.

Apply KCL at node A

$$\frac{4 - v_0}{2} = \frac{v_0}{2} + \frac{v_0 + 2}{2}$$

$$\frac{3v_0}{2} = 1$$

$$v_0 = \frac{2}{3} \text{ V}$$

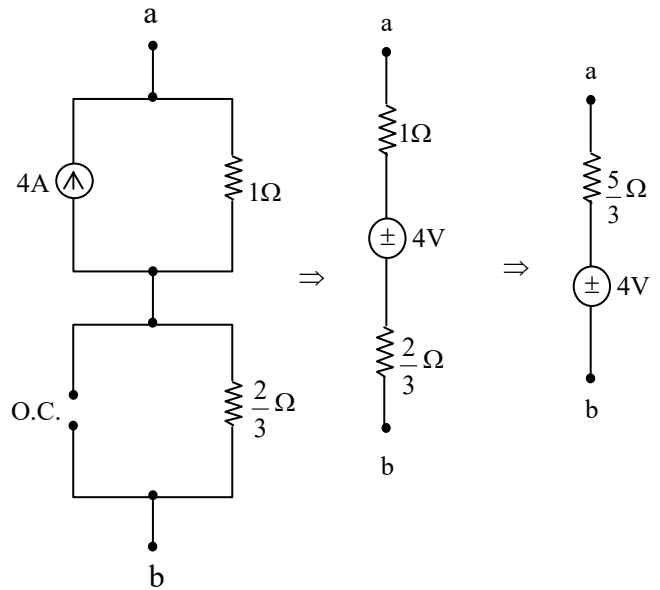
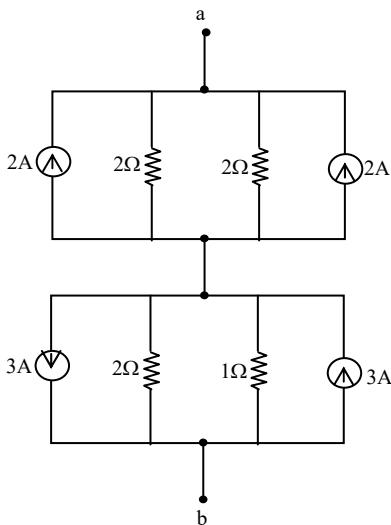
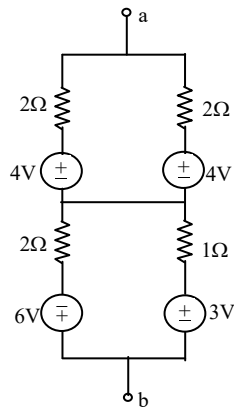
(Here polarity is different what we assume

$$\text{so } V_0 = \frac{-2}{3} \text{ V}$$

38.

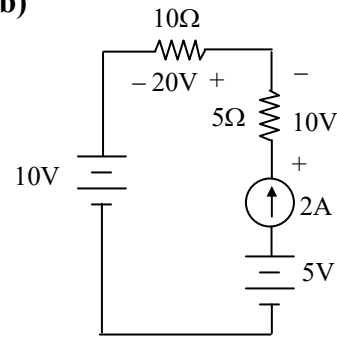
Sol: (in the question 4V voltage source polarity is reversed)

The actual circuit is



39. Ans: (b)

Sol:

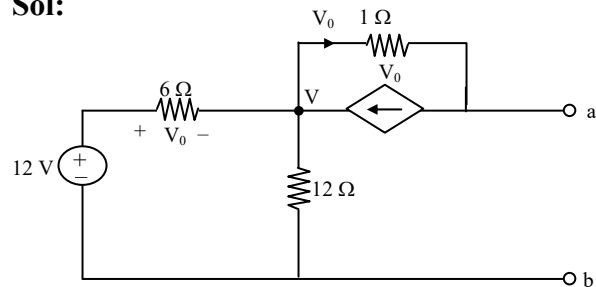


$$\text{Voltage across } 2\text{A} = 10 + 20 + 10 - 5 = 35 \text{ V}$$

$$\therefore \text{Power supplied} = VI = 35 \times 2 = 70 \text{ W}$$

40. Ans: (*)

Sol:





Applying KCL at node V

$$\frac{V-12}{6} + \frac{V}{12} - V_0 + V_0 = 0$$

$$\Rightarrow \frac{V}{6} + \frac{V}{12} = 2 \Rightarrow V = 8V$$

$$\therefore V_0 = 4V$$

Applying KVL in outer loop

$$\Rightarrow -V + 1(V_0) + V_{ab} = 0$$

$$\Rightarrow V_{ab} = V - V_0 = 8 - 4 = 4V$$

41.

Sol: By KVL

$$\Rightarrow V_i - 6 - 10 = 0$$

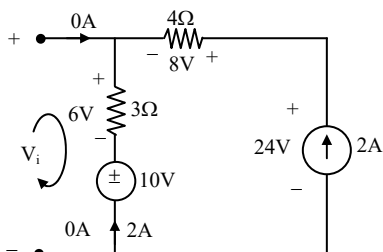
$$V_i = 16V$$

$$P_{4\Omega} = (8 * 2) = 16 \text{ watts} - \text{absorbed}$$

$$P_{2A} = (24 * 2) = 48 \text{ watts delivered}$$

$$P_{3\Omega} = (6 * 2) = 12 \text{ watts} - \text{absorbed}$$

$$P_{10V} = (10 * 2) = 20 \text{ watts} - \text{absorbed}$$



Since; $P_{del} = P_{abs} = 48$ watts. Tellegen's Theorem is satisfied.

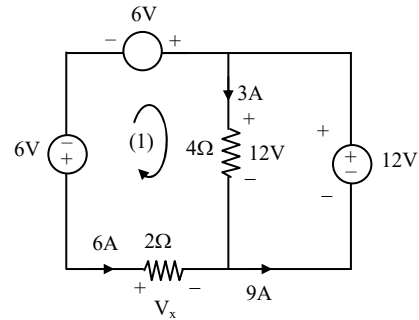
42.

Sol: By KVL in first mesh

$$\Rightarrow V_x - 6 + 6 - 12 = 0$$

$$V_x = 12V$$

$$P_{12V} = (12 \times 9) = 108 \text{ watts delivered}$$



$$P_{4\Omega} = (12 \times 3) = 36 \text{ watts} - \text{absorbed}$$

$$P_{6V} = (6 \times 6) = 36 \text{ watts} - \text{absorbed}$$

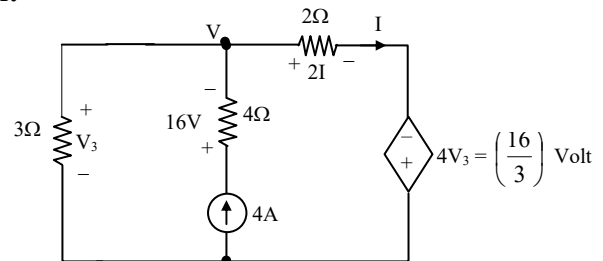
$$P_{6V} = (6 \times 6) = 36 \text{ watts} - \text{delivered}$$

$$P_{2\Omega} = (12 \times 6) = 72 \text{ watts} - \text{absorbed}$$

Since $P_{del} = P_{abs}$; Tellegen's theorem is satisfied.

43.

Sol:



By Nodal \Rightarrow

$$\frac{V}{3} - 4 + \frac{V}{2} + \frac{4V_3}{2} = 0$$

$$\frac{5V}{6} = 4 - 2V_3 \dots\dots\dots (1)$$

By KVL \Rightarrow

$$V_3 - 2I + 4V_3 = 0$$

$$5V_3 - 2I = 0 \dots\dots\dots (2)$$

By KVL \Rightarrow

$$V = V_3 \dots\dots\dots (3)$$

Substitute (3) in (1), we get



$$V_3 = \frac{24}{17}$$

$$V_3 = \frac{24}{17} \text{ Volt and } I = \frac{60}{17} \text{ A}$$

$P_{3\Omega} = 0.663 \text{ W}$ absorbed

$P_{4\Omega} = 64 \text{ W}$ absorbed

$P_{4A} = 69.64 \text{ W}$ delivered

$P_{2\Omega} = 24.91 \text{ W}$ absorbed

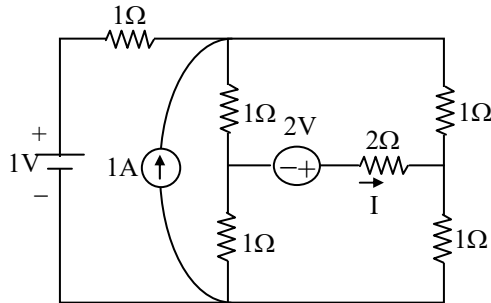
$P_{4V3} = 19.92 \text{ W}$ delivered

Since $P_{del} = P_{abs} = 89.57 \text{ W}$; Tellegen's Theorem is satisfied.

2. Circuit Theorems

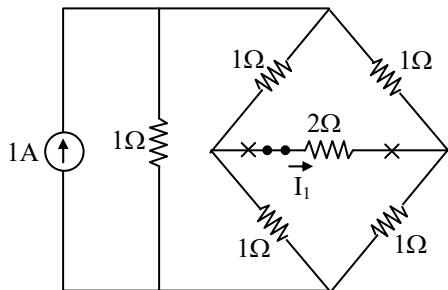
01.

Sol: The current "I" = ?

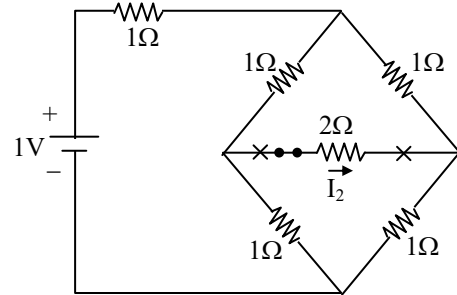


By superposition theorem, treating one independent source at a time.

(a) When 1A current source is acting alone.



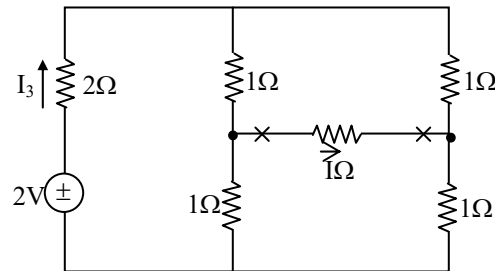
Since the bridge is balanced ; $I_1 = 0 \text{ A}$
(b) When 1V voltage source is acting alone



$I_2 = 0 \text{ A}$

Since the bridge is balanced.

(c) When 2V voltage source is acting alone



$$I_3 = \frac{2}{3} = 0.66 \text{ A}$$

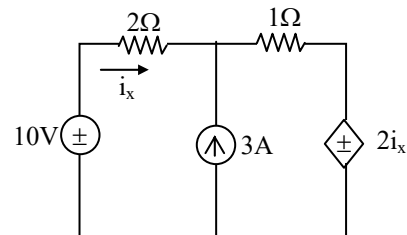
By superposition theorem ; $I = I_1 + I_2 + I_3$

$$I = 0 + 0 + 0.66 \text{ A}$$

$$I = 0.66 \text{ A}$$

02.

Sol:

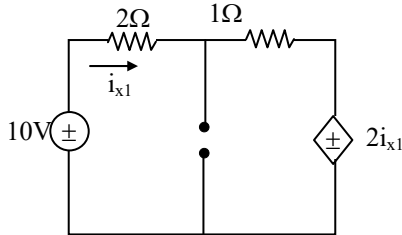




$i_x = ?$

By super position theorem; treating only one independent source at a time

(a) When 10V voltage source is acting alone

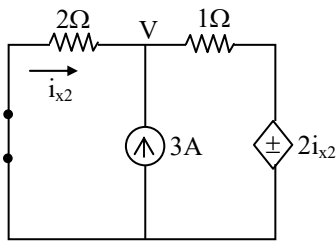


By KVL

$$10 - 2i_{x1} - i_{x1} - 2i_{x1} = 0$$

$$i_{x1} = 2A$$

(b) When 3A current source is acting alone



By Nodal

$$\frac{V}{2} - 3 + \frac{(V - 2i_{x2})}{1} = 0$$

$$3V - 4i_{x2} = 6 \dots\dots\dots (1)$$

And

$$i_{x2} = \frac{0 - V}{2} \Rightarrow V = -2i_{x2} \dots\dots(2)$$

Put (2) in (1), we get

$$i_{x2} = -\frac{3}{5}A$$

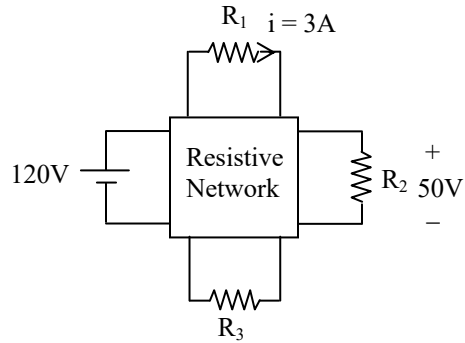
By SPT ;

$$i_x = i_{x1} + i_{x2} = 2 - \frac{3}{5} = \frac{7}{5}$$

$$\therefore i_x = 1.4A$$

03

Sol:



$$P_{R_3} = 60W$$

For 120 V $\rightarrow i_1 = 3A$

$$\text{For } 105V \rightarrow i_1 = \frac{105}{120} \times 3 = 2.625A$$

For 120 V $\rightarrow V_2 = 50V$

$$\text{For } 105V \rightarrow V_2 = \frac{105}{120} \times 50 = 43.75V$$

$$V_2 = 120V \Rightarrow I^2 R_3 = 60W \Rightarrow I = \sqrt{\frac{60}{R_3}}$$

For $V_s = 105V$

$$P_3 = \left(\frac{105}{120} \sqrt{\frac{60}{R_3}} \right)^2 \times R_3 = 45.9W$$

04. Ans: (b)

Sol: It is a liner network

$\therefore V_x$ can be assumed as function of i_{s1} and i_{s2}

$$V_x = Ai_{s1} + Bi_{s2}$$

$$80 = 8A + 12B \rightarrow (1)$$

$$0 = -8A + 4B \rightarrow (2)$$

From equation 1 & 2



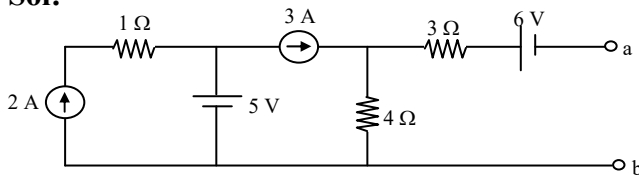
$A = 2.5; B = 5$

Now, $V_x = (2.5)(20) + (5)(20)$

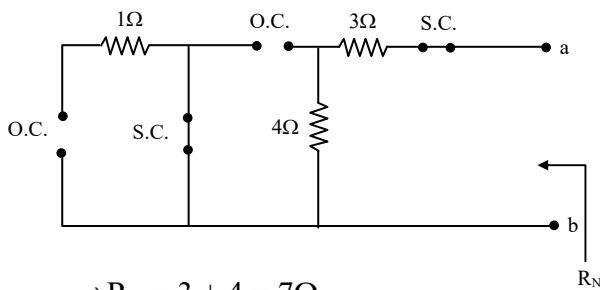
$V_x = 150V$

05. Ans: (c)

Sol:



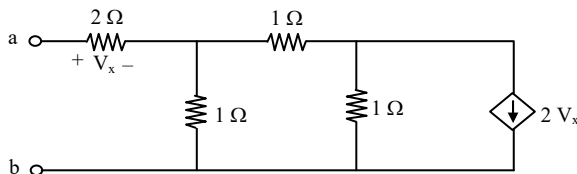
For finding Norton's equivalent resistance independent voltage sources to be short circuited and independent current sources to be open circuited, then the above circuit becomes



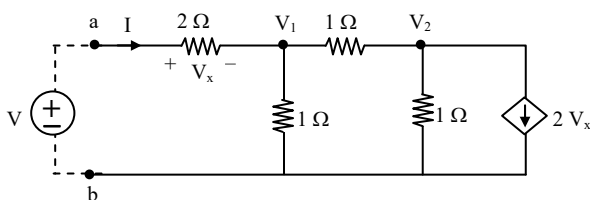
$\Rightarrow R_N = 3 + 4 = 7\Omega$

06. Ans: (b)

Sol:



Excite with a voltage source 'V'



Apply KCL at node V_1

$$-I + \frac{V_1}{1} + \frac{V_1 - V_2}{1}$$

$$\Rightarrow 2V_1 - V_2 - I = 0 \dots\dots(1)$$

Apply KCL at node V_2

$$\frac{V_2 - V_1}{1} + \frac{V_2}{1} + 2V_x = 0$$

$$2V_2 - V_1 + 2V_x = 0 \dots\dots (2)$$

But from the circuit,

$$V_x = 2I \dots\dots (3)$$

Substitute (3) in (2)

$$\Rightarrow 2V_2 - V_1 + 4I = 0$$

$$4V_2 - 2V_1 + 8I = 0$$

From (1),

$$2V_1 = V_2 + I$$

$$\therefore 4V_2 - (V_2 + I) + 8I = 0$$

$$\Rightarrow 3V_2 + 7I = 0 \Rightarrow V_2 = -\frac{7I}{3}$$

Substitute (2) in (1)

$$2V_1 - \left(-\frac{7I}{3}\right) - I = 0$$

$$2V_1 + \frac{7}{3}I - I = 0 \Rightarrow 2V_1 = \frac{-4I}{3}$$

$$\Rightarrow V_1 = \frac{-2I}{3}$$

$$\therefore V = V_x + V_1 = 2I + \left(-\frac{2I}{3}\right) = \frac{4I}{3}$$

$$\Rightarrow V = \frac{4I}{3}$$

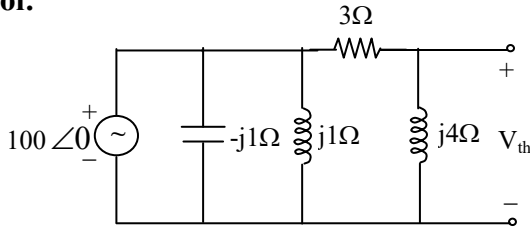
$$\Rightarrow \frac{V}{I} = \frac{4}{3}\Omega$$

$$\Rightarrow R_{eq} = \frac{4}{3}\Omega$$



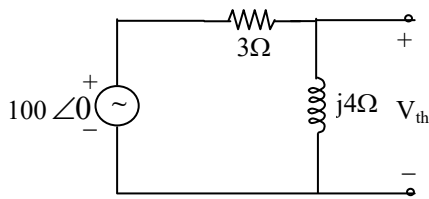
07.

Sol:



Here $j1\Omega$ and $-j1\Omega$ combination will act as open circuit.

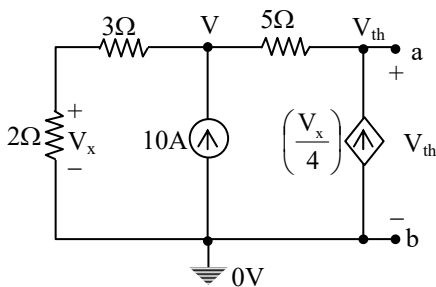
The circuit becomes



$$\Rightarrow V_{th} = \frac{100\angle 0^\circ \times j4}{3 + j4} = 80\angle 36.86^\circ \text{ V}$$

08.

Sol: Thevenin's and Norton's equivalents across a, b.



By Nodal

$$\frac{V}{5} - 10 + \frac{V}{5} - \frac{V_{th}}{5} = 0$$

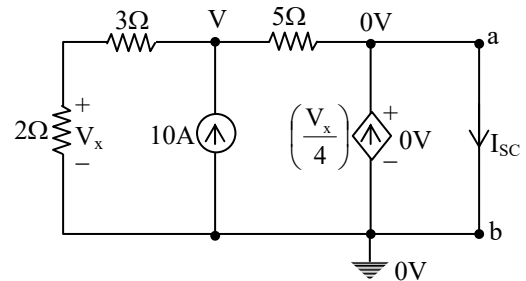
$$\frac{V_{th}}{5} - \frac{V}{5} - \frac{V_x}{4} = 0$$

$$\frac{2V}{5} = \left(10 + \frac{V_{th}}{5}\right)$$

$$\frac{V_{th}}{5} = \left(\frac{V}{10} + \frac{V}{5}\right)$$

$$V_x = \left(\frac{2V}{5}\right)$$

$$V_{th} = 150\text{V}, V = 100\text{V}$$



$$\frac{V}{5} - 10 + \frac{V}{5} = 0$$

$$\frac{2V}{5} = 10$$

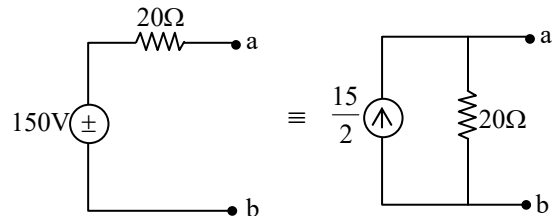
$$V = 25\text{V}$$

$$V_x = \frac{2V}{5} = \frac{2 \times 25}{5}$$

$$V_x = 10\text{V}$$

$$I_{sc} = \left(\frac{10}{4} + 5\right) = \frac{15}{2}\text{A}$$

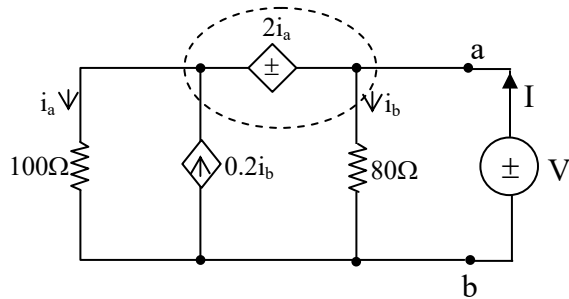
$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{150}{\frac{15}{2}} = 20\Omega$$





09.

Sol:



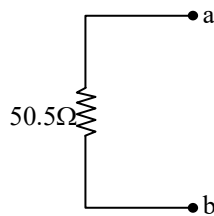
Super nodal equation

$$\Rightarrow i_a - 0.2i_b + i_b - I = 0$$

$$I = i_a + 0.8i_b$$

$$V = 80i_b ; i_b = \frac{V}{80}$$

- Inside the supernode, always the KVL is written.



By KVL \Rightarrow

$$100i_a + 2i_a - 80i_b = 0$$

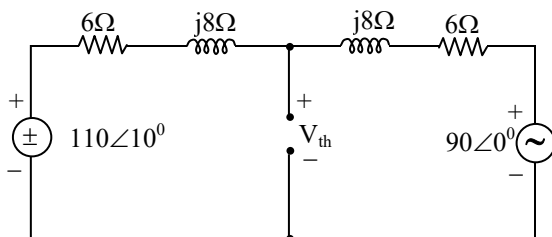
$$I = \frac{V}{102} + \frac{0.8 \times V}{80}$$

$$\frac{V}{I} = R_L = \frac{1}{\frac{1}{102} + \frac{1}{100}} = 50.5\Omega$$

$$R_L = 50.5\Omega$$

10.

Sol: V_{th} :

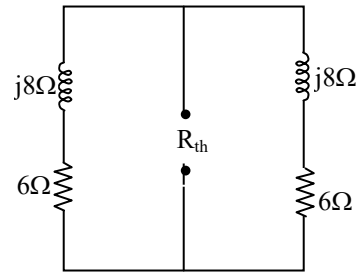


By Nodal \Rightarrow

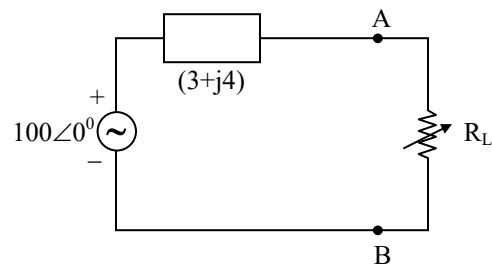
$$\frac{V_{th}}{(6 + j8)} - \frac{110\angle 0^\circ}{(6 + j8)} + \frac{V_{th}}{(6 + j8)} - \frac{90\angle 0^\circ}{(6 + j8)} = 0$$

$$2V_{th} = 200\angle 0^\circ \Rightarrow V_{th} = 100\angle 0^\circ$$

R_{th} :



$$R_{th} = (6 + j8) \parallel (6 + j8) = (3 + j4)\Omega$$



$$R_L = |3 + j4| = 5\Omega$$

$$I = \frac{100\angle 0^\circ}{(8 + j4)}$$

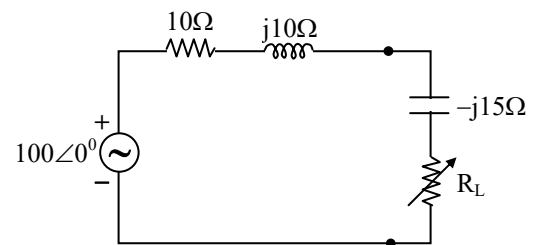
$$P = |I|^2 \times R_L$$

$$P_{max} = 125 \times 5 = 625 \text{ W}$$

$$\therefore P_{max} = 625 \text{ Watts}$$

11.

Sol:



The maximum power delivered to "R_L" is



$$R_L = \sqrt{R_S^2 + (X_S + X_L)^2}$$

Here $R_S = 10\Omega$; $X_S = 10\Omega$ & $X_L = -15$

$$R_L = \sqrt{10^2 + (10 - 15)^2}$$

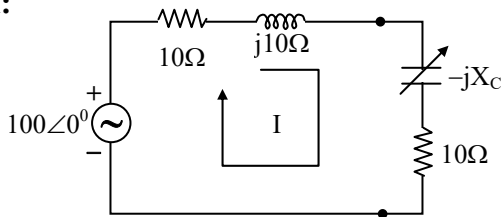
$$R_L = 5\sqrt{5}\Omega.$$

$$I = \frac{100\angle 0^\circ}{(10 + j10 - j15 + 5\sqrt{5})}$$

$$P_{\max} = |I|^2 \cdot 5\sqrt{5} = 236W$$

12.

Sol:



The maximum power delivered to 10Ω load resistor is:

$$Z_L = 10 - jX_C = 10 + j(-X_C)$$

$$X_L = -X_C$$

So for MPT; $(X_S + X_L) = 0$

$$10 - X_C = 0;$$

$$X_C = 10$$

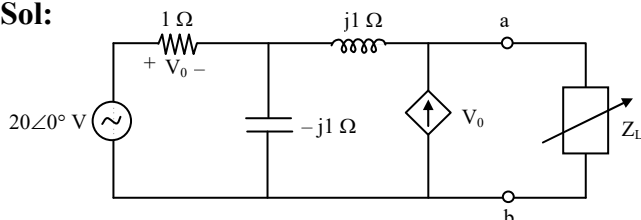
$$I = \frac{100\angle 0^\circ}{(10 + j10 - j10 + 10)} = 5\angle 0^\circ$$

$$P_{\max} = |I|^2 R_L = 5^2(10) = 250W$$

$$P_{\max} = 250 \text{ Watts}$$

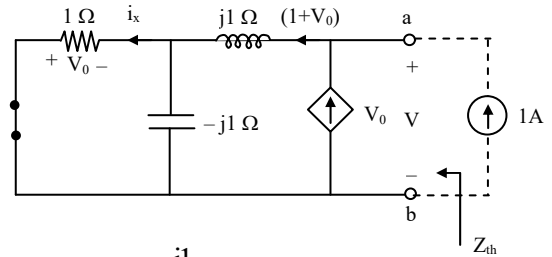
13. Ans: (b)

Sol:



For maximum power delivered to Z_L ,

$$Z_L = Z_{th}^*$$



$$i_x = (1 + V_0) \times \frac{-j1}{1 - j1} = (1 + V_0) (0.5 - j0.5)$$

But, $V_0 = -i_x$

$$= -(1 + V_0) (0.5 - j0.5)$$

$$(-1 - j) V_0 = 1 + V_0$$

$$\Rightarrow V_0 (-1 - j - 1) = 1$$

$$V_0 = \frac{1}{-2 - j} = -0.4 + j0.2$$

Applying KVL

$$+ V_0 - j1(1 + V_0) + V = 0$$

$$\Rightarrow V = -V_0 + j1(1 + V_0)$$

$$= 0.4 - j0.2 + j1(0.6 + j0.2)$$

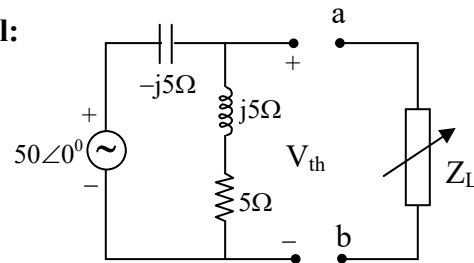
$$V = (0.2 + j0.4)V$$

$$\therefore Z_{th} = \frac{V}{1} = V = (0.2 + j0.4)\Omega$$

$$\therefore Z_L = Z_{th}^* = (0.2 - j0.4)\Omega$$

14.

Sol:

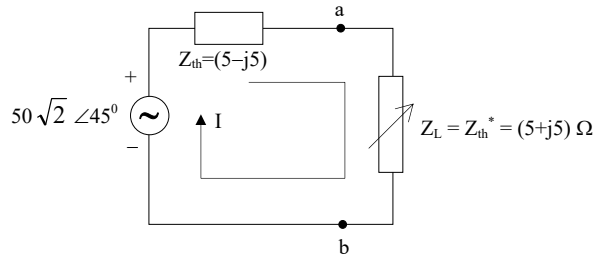


The maximum true power delivered to " Z_L " is:



$$V_{th} = \left(\frac{50 \angle 0^\circ}{-j5 + j5 + 5} \right) (j5 + 5) = 50\sqrt{2} \angle 45^\circ$$

$$Z_{th} = (-j5) \parallel (5 + j5) = (5 - j5) \Omega$$



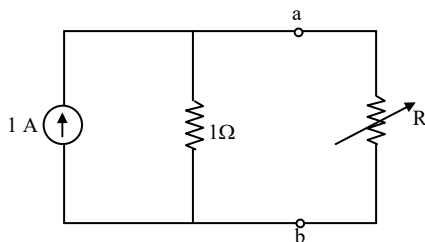
$$I = \frac{50\sqrt{2} \angle 45^\circ}{(5 - j5 + 5 + j5)} = 5\sqrt{2} \angle 45^\circ$$

$$P = |I|^2 5 = |5\sqrt{2}|^2 \cdot 5 = 250 \text{ Watts}$$

$$\therefore P_{max} = 250 \text{ Watts}$$

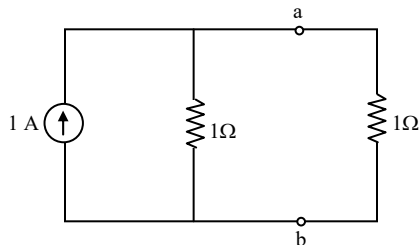
15. Ans: (c)

Sol:



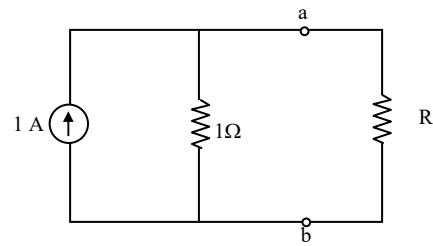
Maximum power will occur when $R = R_s$

$$\Rightarrow R = 1 \Omega$$



$$\therefore P_{max} = \left(\frac{1}{2} \right)^2 \times 1 = \frac{1}{4} \text{ W}$$

$$25\% \text{ of } P_{max} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16} \text{ W}$$



Current passing through 'R'

$$I = 1 \times \frac{1}{1 + R} = \frac{1}{1 + R}$$

$$\therefore P = I^2 R = \left(\frac{1}{1 + R} \right)^2 R = \frac{1}{16}$$

$$\Rightarrow (R + 1)^2 = 16R$$

$$\Rightarrow R^2 + 2R + 1 = 16R$$

$$\Rightarrow R^2 - 14R + 1 = 0$$

$$R = 13.9282 \Omega \text{ or } 0.072 \Omega$$

From the given options 72m\Omega is correct

16.

Sol: For, $E = 1V$, $I = 0A$ then $V = 3V$

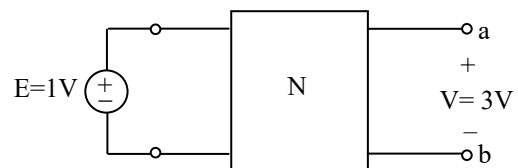


Fig.(b)

$V_{oc} = 3V$ (with respect to terminals a and b)

For, $E = 0V$, $I = 2A$ then $V = 2V$

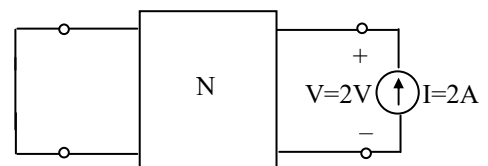
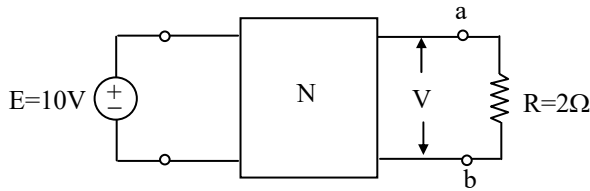


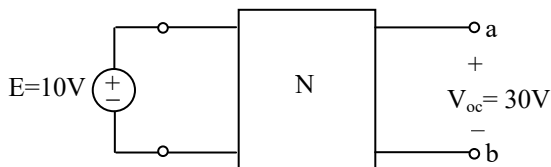
Fig.(c)



Now when $E = 10V$, and I is replaced by $R = 2\Omega$ then $V = ?$



When $E = 10V$,
From Fig.(b) using homogeneity principle



For finding Thevenin's resistance across ab independent voltage sources to be short circuited & independent current sources to be open circuited.

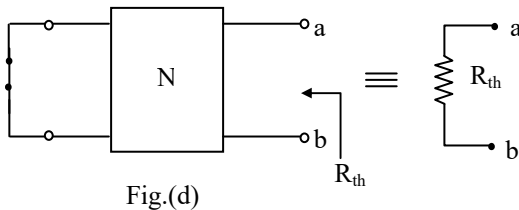
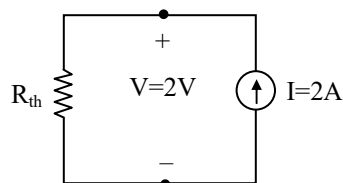


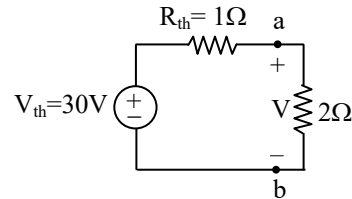
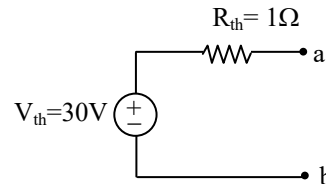
Fig.(d)

Fig.(c) is the energized version of Fig. (d)



$$\Rightarrow R_{th} = \frac{2}{2} = 1\Omega$$

\therefore with respect to terminals a and b the Thevenin's equivalent becomes.



$$V = 30 \times \frac{2}{2+1} = 20V$$

$$\therefore V = 20V$$

17.

Sol: Superposition theorem cannot be applied to fig (b)

Since there is only voltage source given:

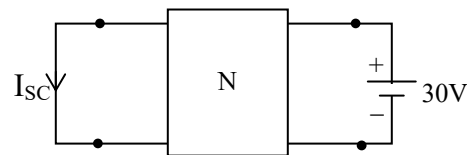
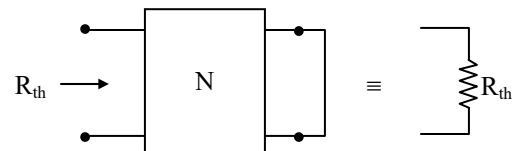


Fig (c)

By homogeneity and Reciprocity principles to fig (a);

$$I_{sc} = 6A$$

For R_{th} :



Statement: Fig (a) is the energized version of figure (d)

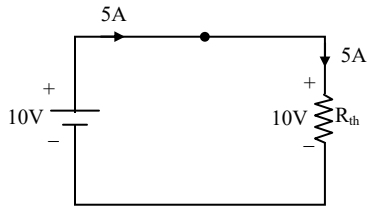


Fig (a)

$$10 = R_{th} \cdot 5 \quad \left| \begin{array}{l} \text{by ohm's law} \end{array} \right.$$

$$R_{th} = 2\Omega.$$

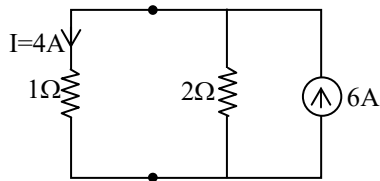


Fig (b)

$$I = \frac{6 \times 2}{(2+1)} = 4A$$

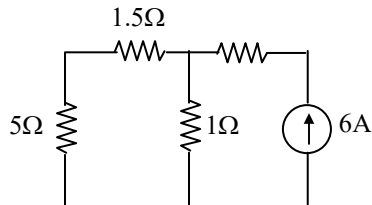
$$I = 4A$$

18. Ans: (b)

$$\text{Sol: } \begin{bmatrix} 10 \\ 4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$10 = Z_{11} (4) + Z_{12} (0)$$

$$4 = Z_{21} (4) + Z_{22} (0)$$



$$Z_{11} = \frac{10}{4} = 2.5$$

$$Z_{21} = \frac{4}{4} = 1$$

$$I_{5\Omega} = \frac{6 \times 1}{6.5+1} = \frac{6}{7.5} = 0.8A$$

19. Ans: (b)

Sol:

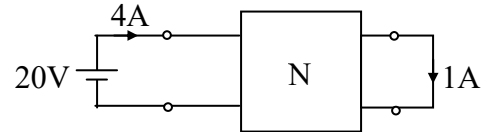


Fig.(a)

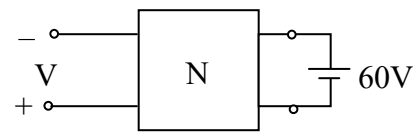


Fig.(b)

Using reciprocity theorem, for Fig.(a)

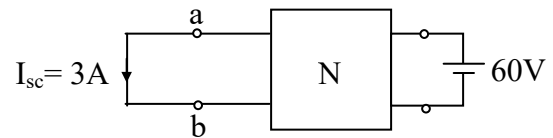
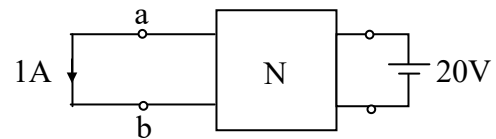


Fig.(c)

Norton's resistance between a and b is

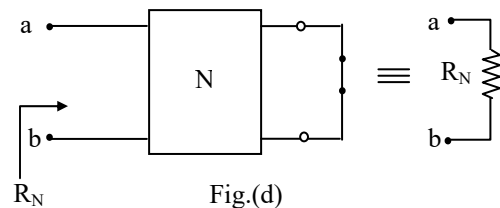
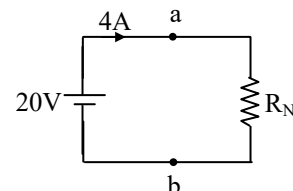


Fig.(d)

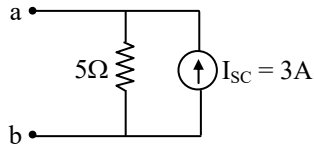
Fig.(a) is the energized version of Fig.(d)



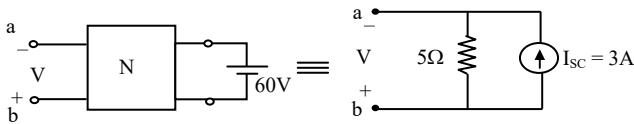


$$\Rightarrow R_N = \frac{20}{4} = 5\Omega$$

With respect to terminals a and b the Norton's equivalent of Fig.(b) is



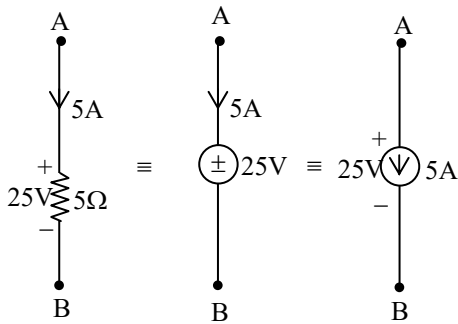
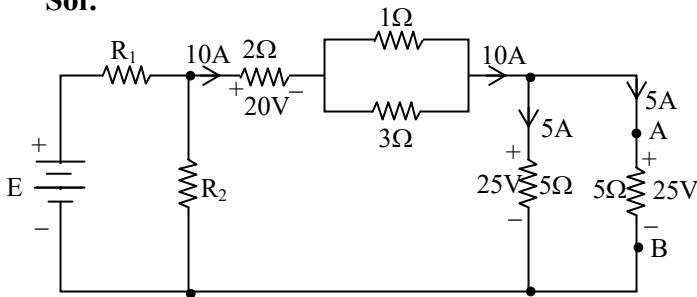
∴ From Fig.(b)



$$\Rightarrow V = -15V$$

20.

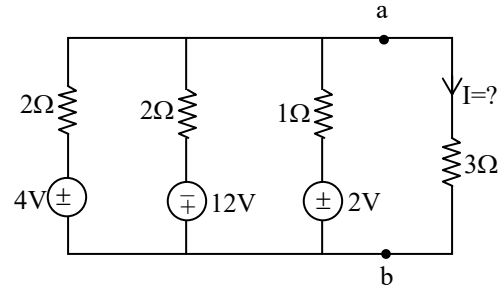
Sol:



$$P_{AB} = P_{5\Omega} = P_{25V} = P_{5A} = 5 \times 25 = 125 \text{ watts (ABSORBED)}$$

21.

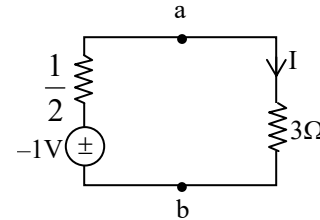
Sol:



By Mill Man's theorem ;

$$V' = \frac{V_1 G_1 + V_2 G_2 + V_3 G_3}{G_1 + G_2 + G_3}$$

$$\equiv \frac{\frac{4}{2} - \frac{12}{2} + \frac{2}{1}}{\left(\frac{1}{2} + \frac{1}{2} + 1\right)} = \frac{4 - 12 + 4}{2 * 2} \equiv -1V$$



$$\therefore V' = -1V$$

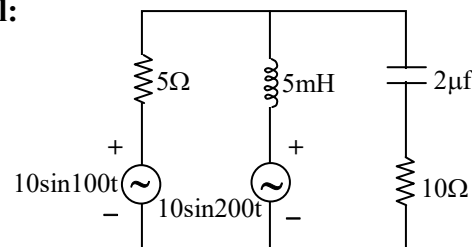
$$\frac{1}{R^1} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{2} + \frac{1}{2} + 1 = 2$$

$$\therefore R^1 = \frac{1}{2} \Omega$$

$$I = \frac{-1}{\left(\frac{1}{2} + 3\right)} \Rightarrow I = \frac{-2}{7} A$$

22. Ans: (d)

Sol:





Since the two different frequencies are operating on the network simultaneously; always the super position theorem is used to evaluate the responses since the reactive elements are frequency sensitive

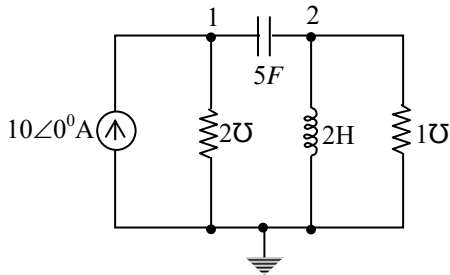
i.e., $Z_L = j\omega L$ and $Z_C = \frac{1}{j\omega C} \Omega$.

23.

Sol: In the above case if both the source are 100rad/sec , each then Millman's theorem is more conveniently used.

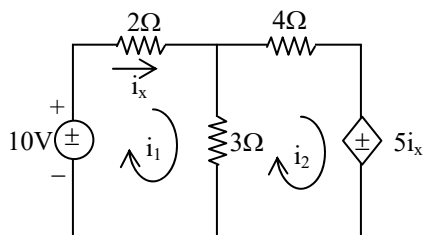
24.

Sol:



25.

Sol:



Nodal equations

$i = GV$

$i_x = i_1$

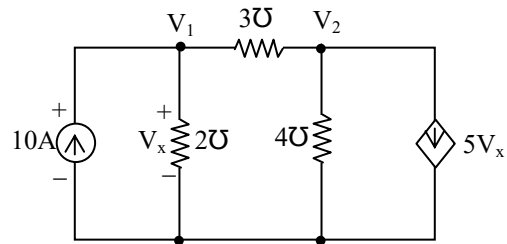
$10 = 2i_1 + 3(i_1 - i_2) \dots\dots\dots (1)$

$0 = 4i_2 + 2i_x + 3(i_2 - i_1) \dots\dots\dots (2)$

$V_x = V_1$

$10 = 2V_1 - 3(V_1 - V_2) \dots\dots\dots (3)$

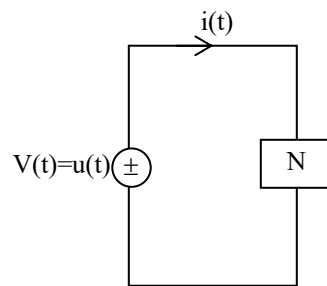
$0 = 4V_2 + 2V_x + 3(V_2 - V_1) \dots\dots\dots (4)$



3. Transient Circuit Analysis

01.

Sol:



$i(t) = e^{-3t} \text{ A for } t > 0 \text{ (given)}$

Determine the elements & their connection

$\frac{\text{Response Laplace transform}}{\text{Excitation Laplace transform}} = \text{System}$

transfer function

i.e., $\frac{I(s)}{V(s)} = H(s) = \frac{1}{\frac{1}{s+3}}$

$= \frac{s}{(s+3)} = y(s) = \frac{1}{Z(s)}$

$\therefore Z(s) = \left(\frac{s+3}{s} \right)$

$= 1 + \frac{1}{s\left(\frac{1}{3}\right)} = R + \frac{1}{SC}$

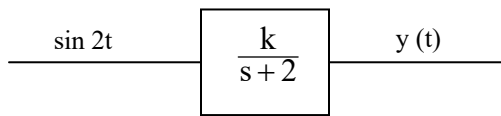


$\therefore R = 1\Omega$ and $C = \frac{1}{3}F$ are in series

02. Ans: (c)

Sol: The impulse response of first order system is Ke^{-2t} .

$$\text{So T/F} = L(I.R) = \frac{K}{s+2}$$



$$G(s) = \frac{K}{s+2}$$

$$|G(j\omega)| = \frac{K}{\sqrt{\omega^2 + 2^2}} = \frac{K}{2\sqrt{2}}$$

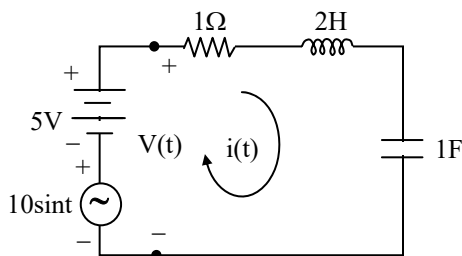
$$\angle G(j\omega) = -\tan^{-1} \frac{\omega}{2} = -\tan^{-1} 1 = -\frac{\pi}{4}$$

So steady state response will be

$$y(t) = \frac{K}{2\sqrt{2}} \sin\left(2t - \frac{\pi}{4}\right)$$

03.

Sol:



By KVL $\Rightarrow v(t) = (5 + 10\sin t)$ volt

Evaluating the system transfer function $H(s)$.

$$\frac{\text{Desired response L.T}}{\text{Excitation response L.T}} = \text{System transfer function}$$

$$\frac{I(s)}{V(s)} = H(s) = Y(s) = \frac{1}{Z(s)} = \frac{1}{\left(R + sL + \frac{1}{sC}\right)}$$

$$H(s) = \frac{s}{(2s^2 + s + 1)}$$

$$H(j\omega) = \frac{1}{\left(1 + \frac{1}{j\omega} + 2j\omega\right)}$$

II. Evaluating at corresponding ω_s of the input

$$H(j\omega)|_{\omega=0} = 0$$

$$H(j\omega)|_{\omega=1} = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$\text{III. } \frac{I(s)}{V(s)} = H(s)$$

$$I(s) = H(s)V(s)$$

$$i(t) = 0 \times 5 + \frac{1}{\sqrt{2}} \times 10 \sin(t - 45^\circ)$$

$$i(t) = 7.07 \sin(t - 45^\circ) A$$

OBS: DC is blocked by capacitor in steady state

04.

$$\text{Sol: } \frac{V(s)}{I(s)} = H(s) = Z(s) = \frac{1}{Y(s)} = \frac{1}{\left(\frac{1}{R} + \frac{1}{sL} + sC\right)}$$

$$H(s) = \frac{1}{\left(1 + \frac{1}{s} + s\right)}$$

$$H(j\omega)|_{\omega=1} = \frac{1}{\left(1 + \frac{1}{j} + j\right)} = 1$$

$$V(s) = I(s) H(s) = \sin t$$

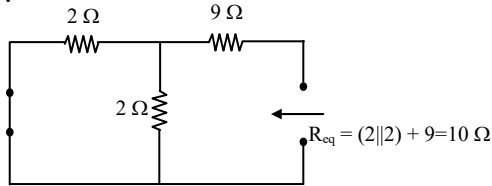
$$v(t) = \sin t \text{ Volts}$$



05.

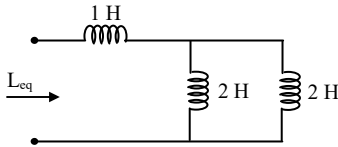
Sol: $\tau = \frac{L_{eq}}{R_{eq}}$

R_{eq} :



$R_{eq} = (2 \parallel 2) + 9 = 10 \Omega$

L_{eq} :



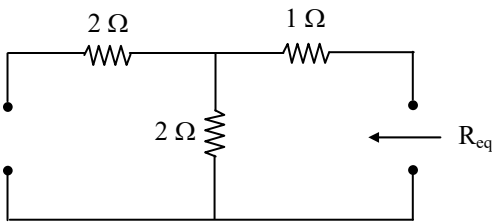
$L_{eq} = (2 \parallel 2) + 1 = 2 \text{ H}$

$\therefore \tau = \frac{L_{eq}}{R_{eq}} = \frac{2}{10} = 0.2 \text{ sec}$

06.

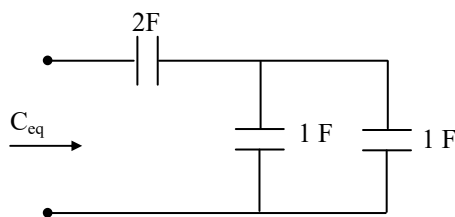
Sol: $\tau = R_{eq} C_{eq}$

R_{eq} :



$R_{eq} = 3 \Omega$

C_{eq} :

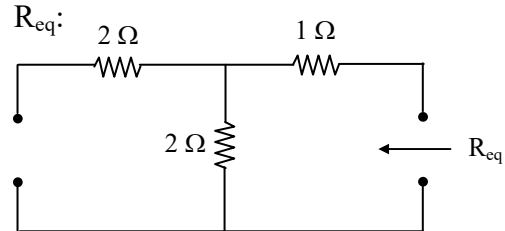


$C_{eq} = 1 \text{ F}$

$\therefore \tau = 3 \times 1 = 3 \text{ sec}$

07.

Sol: $\tau = R_{eq} C$



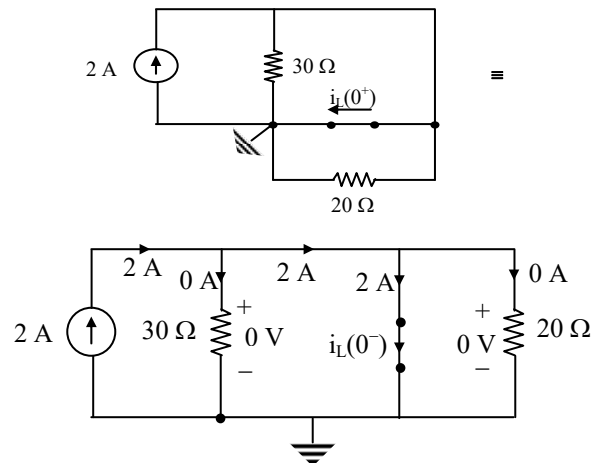
$R_{eq} = 3 \Omega$

$\therefore \tau = 3 \times 1 = 3 \text{ sec}$

08.

Sol: Let us assume that switch is closed at $t = -\infty$, now we are at $t = 0^-$ instant, still the switch is closed i.e., an infinite amount of time, the independent dc source is connected to the network and hence it is said to be in steady state.

In steady state, the inductor acts as short circuit and nature of the circuit is resistive.



At $t = 0^-$: Steady state: A resistive circuit



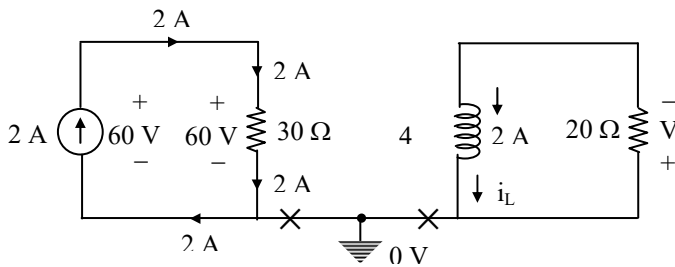
Note: The number of initial conditions to be evaluated at just before the switching action is equal to the number of memory elements present in the network.

(i) $t = 0^-$

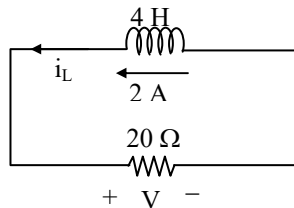
$$i_L(0^-) = 2 = i_L(0^+)$$

$$E_L(0^-) = \frac{1}{2} L i_L^2(0^-)$$

$$= \frac{1}{2} \times 4 \times 2^2 = 8 \text{ J} = E_L(0^+)$$



For $t \geq 0$

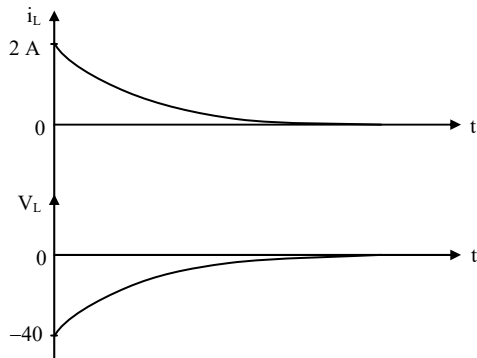


For $t \geq 0$: Source free circuit

$$I_0 = 2 \text{ A}; \tau = \frac{L}{R} = \frac{4}{20} = \frac{1}{5} \text{ sec}$$

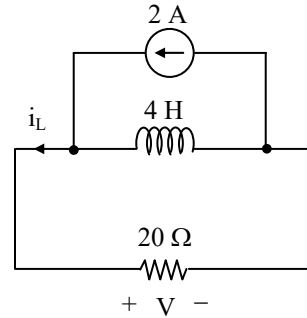
$$i_L = 2 e^{-5t} \text{ for } 0 \leq t \leq \infty$$

$$V_L = L \frac{di_L}{dt} = -40 e^{-5t} \text{ V for } 0 \leq t \leq \infty$$

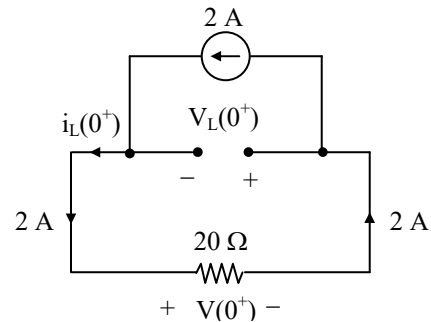


$t = 5\tau = 5 \times \frac{1}{5} = 1 \text{ sec}$ for steady state

practically i.e., with in 1 sec the total 8 J stored in the inductor will be delivered to the resistor.



For $t \geq 0$



At $t = 0^+$: Resistive circuit :
Network is in transient state

By KCL:

$$-2 + i_L(0^+) = 0$$

$$i_L(0^+) = 2 \text{ A}$$

$$V(0^+) = R i_L(0^+) \text{ |By Ohm's law}$$

$$V(0^+) = 20 (2) = 40 \text{ V}$$

By KVL:

$$V_L(0^+) + V(0^+) = 0$$

$$V_L(0^+) = -V(0^+) = -40 \text{ V} = V_L(t) \Big|_{t=0^+}$$

Observations:

$$t = 0^-$$

$$i_L(0^-) = 2 \text{ A}$$

$$t = 0^+$$

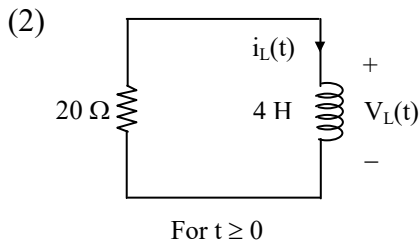
$$i_L(0^+) = 2 \text{ A}$$



$$\begin{aligned} i_{20\Omega}(0^-) &= 0 \text{ A} & i_{20\Omega}(0^+) &= 2 \text{ A} \\ V_{20\Omega}(0^-) &= 0 \text{ V} & V_{20\Omega}(0^+) &= 40 \text{ V} \\ V_L(0^-) &= 0 \text{ V} & V_L(0^+) &= -40 \text{ V} \end{aligned}$$

Conclusion:

To keep the same energy as $t = 0^-$ and to protect the KCL and KVL in the circuit (i.e., to ensure the stability of the network), the inductor voltage, the resistor current and its voltage can change instantaneously i.e., within zero time at $t = 0^+$.



$$\begin{aligned} i_L(t) &= 2 e^{-5t} \text{ A for } 0 \leq t \leq \infty \\ V_L(t) &= -40 e^{-5t} \text{ V for } 0 \leq t \leq \infty \end{aligned}$$

Conclusion:

For all the source free circuits, $V_L(t) = -ve$ for $t \geq 0$, since the inductor while acting as a temporary source (upto 5τ), it discharges from positive terminal i.e., the current will flow from negative to positive terminals. (This is the must condition required for delivery, by Tellegan's theorem)

$$(3) V_L(0^+) = -40 \text{ V}$$

$$V_L(t) \Big|_{t=0^+} = -40 \text{ V}$$

$$L \frac{d i_L(t)}{d t} \Big|_{t=0^+} = -40$$

$$\frac{d i_L(t)}{d t} \Big|_{t=0^+} = -\frac{40}{L} = -\frac{40}{4} = -10 \text{ A/sec}$$

Check :

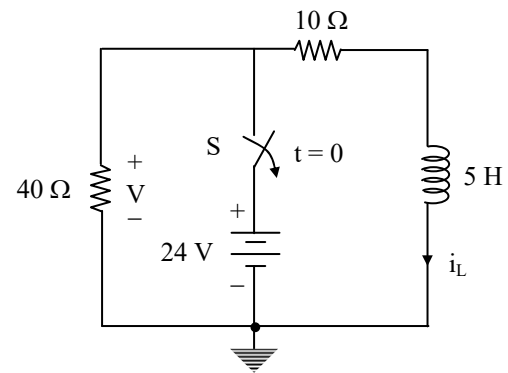
$$i_L(t) = 2 e^{-5t} \text{ A for } 0 \leq t \leq \infty$$

$$\frac{d i_L(t)}{d t} = -10 e^{-5t} \text{ A/sec for } 0 \leq t \leq \infty$$

$$\frac{d i_L(t)}{d t} \Big|_{t=0^+} = -10 \text{ A/sec}$$

09.

Sol:



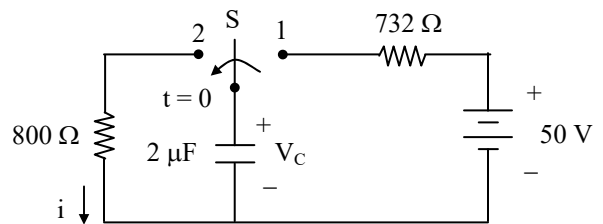
$$i_L(0^+) = 2.4 \text{ A}$$

$$V(0^+) = -96 \text{ V}$$

$$i_L(t) = 2.4 e^{-10t} \text{ A for } 0 \leq t \leq \infty$$

10.

Sol:



$$V_C(0^+) = 50 \text{ V ; } i(0^+) = 62.5 \text{ mA}$$

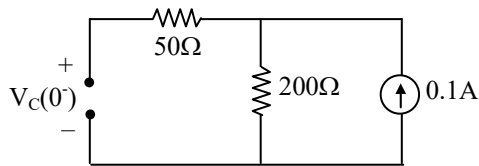
$$V_C(t) = 50 e^{-\frac{t}{1.6 \times 10^{-3}}} \text{ V for } t \geq 0$$

$$i_C = C \frac{d V_C}{d t} \Big|_{\text{By Ohm's law}}$$



11.

Sol: Case(i): $t < 0$

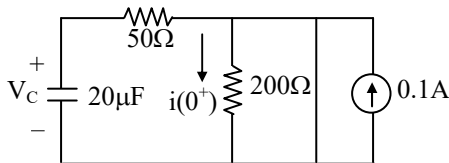


$$V_C(0^-) = 20V \text{ \& } i(0^-) = 0.1A$$

∴ capacitor never allows sudden changes in voltages

$$V_C(0^-) = V_C(0) = V_C(0^+) = 20V$$

Case (ii): $t > 0$



To find the time constant $\tau = R_{eq}C$

After switch closed

$$R_{eq} = 50\Omega \quad C = 20\mu F$$

$$i(0^+) = 0A$$

$$\tau = 50 \times 20\mu$$

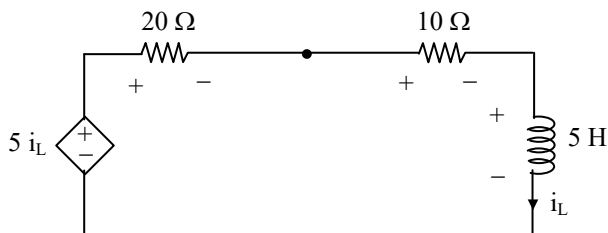
$$\tau = 1\text{msec}$$

$$V_C(t) = V_0 e^{-t/\tau} = 20e^{-t/1\text{m}}$$

$$V_C(t) = 20e^{-t/1\text{m}}V; \quad 0 \leq t \leq \infty$$

12.

Sol: After performing source transformation;



By KVL;

$$5 i_L - 30 i_L - 5 \frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} + 5 i_L = 0$$

$$(D + 5) i_L = 0$$

$$i_L(t) = K e^{-5t} \text{ A for } 0 \leq t \leq \infty$$

$$\tau = \frac{1}{5} \text{ sec}$$

13.

Sol: $i_{L_1}(0) = 10 \text{ A} ; i_{L_2}(0) = 2 \text{ A}$

$$i_{L_1}(t) = I_0 e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R} = \frac{1}{1} = 1 \text{ sec}$$

$$i_{L_1}(t) = 10 e^{-t} \text{ A}$$

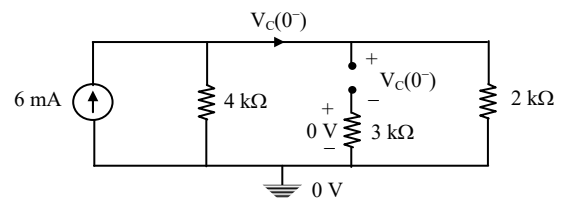
$$\text{Similarly, } i_{L_2}(t) = I_0 e^{-\frac{t}{\tau}}$$

$$\tau = \frac{L}{R} = 2 \text{ sec}$$

$$i_{L_2}(t) = 20 e^{-\frac{t}{2}} \text{ A}$$

14.

Sol:



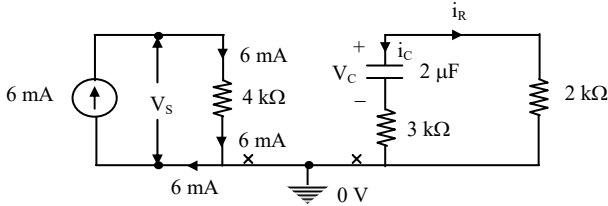
At $t = 0^-$: Steady state: A resistive circuit

By Nodal:

$$-6 \text{ mA} + \frac{V_C(0^-)}{4\text{K}} + \frac{V_C(0^-)}{2\text{K}} = 0$$



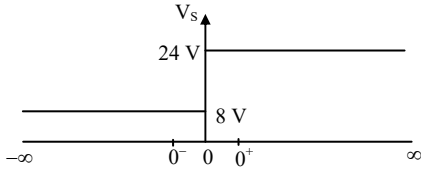
$$V_C(0^-) = 8 \text{ V} = V_C(0^+)$$



For $t \geq 0$: A source free circuit

$$V_s = 6 \text{ m} \times 4 \text{ K} = 24 \text{ V}$$

$$\tau = R_{\text{eq}} C = (5 \text{ K}) 2 \mu = 10 \text{ m sec}$$



$$V_C = 8 e^{-\frac{t}{10\text{m}}} = 8 e^{-100t} \text{ V for } 0 \leq t \leq \infty$$

$$i_C = C \frac{dV_C}{dt} \Big|_{\text{By Ohm's law}} = -1.6 e^{-100t} \text{ mA for } 0 \leq t \leq \infty$$

By KCL:

$$i_C + i_R = 0$$

$$i_R = -i_C = 1.6 e^{-100t} \text{ mA for } 0 \leq t \leq \infty$$

Observation:

In all the source free circuit, $i_C(t) = -ve$ for $t \geq 0$ because the capacitor while acting as a temporary source it discharges from the +ve terminal i.e., current will flow from -ve to +ve terminals.

15.

Sol: By KCL:

$$i(t) = i_R(t) + i_L(t)$$

$$= \frac{V_R(t)}{R} + \frac{1}{L} \int_{-\infty}^t V_L(t) dt$$

$$= \frac{V_s(t)}{10} + i_L(0) + \frac{1}{L} \int_0^t V_s(t) dt$$

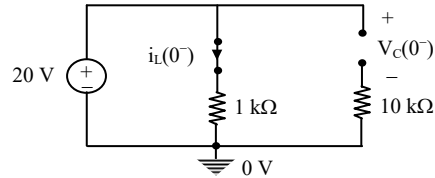
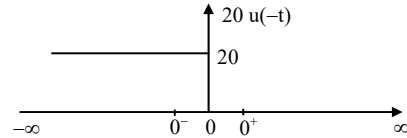
$$i(t) = 4t + 5 + 4t^2$$

$$i(t) |_{t=2 \text{ sec}} = 8 + 16 + 5 = 29 \text{ A} = 29000 \text{ mA}$$

16. Ans: (c)

17.

Sol:

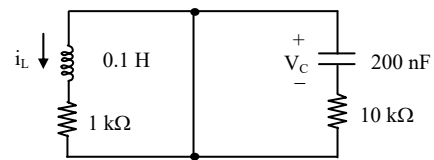


At $t = 0^-$: steady state: A resistive circuit.

(i) $t = 0^-$

$$V_C(0^-) = 20 \text{ V} = V_C(0^+)$$

$$i_L(0^-) = \frac{20}{1\text{K}} = 20 \text{ mA} = i_L(0^+)$$



For $t \geq 0$: A source free RL & RC circuit

$$\tau = \frac{0.1}{1\text{K}} = 100 \mu \text{ sec}$$

$$\tau_C = 200 \times 10^{-9} \times 10 \times 10^3 = 2 \text{ m sec}$$

$$\frac{\tau_C}{\tau_L} = 20 ; \tau_C = 20 \tau_L$$



Observation:

$\tau_L < \tau_C$; therefore the inductive part of the circuit will achieve steady state quickly i.e., 20 times faster.

$$V_C = 20 e^{-\frac{t}{\tau_C}} \text{ V for } 0 \leq t \leq \infty$$

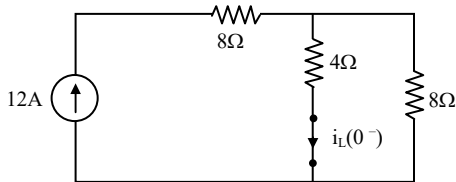
$$i_L = 20 e^{-\frac{t}{\tau_L}} \text{ mA for } 0 \leq t \leq \infty$$

$$V_L = L \frac{di_L}{dt} \quad \left| \begin{array}{l} \text{By Ohm's law} \end{array} \right.$$

$$i_C = C \frac{dV_C}{dt} \quad \left| \begin{array}{l} \text{By Ohm's law} \end{array} \right.$$

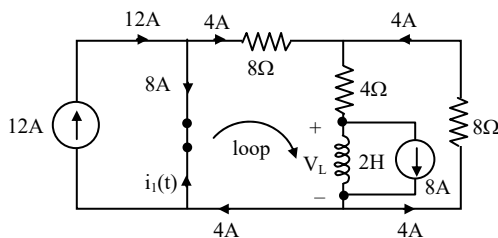
18.

Sol: At $t=0^-$



$$\Rightarrow i_L(0^-) = \frac{12 \times 8}{8+4} = 8\text{A}$$

At $t=0^+$



$$\therefore i_1(0^+) = -8\text{A}$$

Applying KVL in the loop,

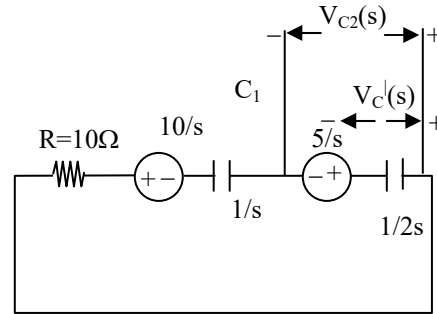
$$\Rightarrow 8(4) + 4(8) + V_L = 0$$

$$\Rightarrow V_L = -64$$

$$\Rightarrow L \frac{di_L}{dt} = -64 \Rightarrow \frac{di_L}{dt} = -32 \text{ A/sec}$$

19. Ans: (c)

Sol:



$$V_c^1(s) = \frac{\frac{5}{s} \left(\frac{1}{2s} \right)}{R + \frac{1}{s} + \frac{1}{2s}}$$

$$= \frac{5}{\frac{2s^2}{2Rs + 2 + 1}} = \frac{5}{s(2Rs + 3)}$$

$$V_{c_2}(\infty) - V_c^1(s) - \frac{5}{s} = 0$$

$$V_c(\infty) = V_c^1(s) + \frac{5}{s}$$

$$V_c(\infty) = \lim_{s \rightarrow 0} s \left[\frac{5}{s(2Rs + 3)} + \frac{5}{s} \right]$$

$$= \frac{5}{3} + 5 = \frac{20}{3}$$

20. Ans: (d)

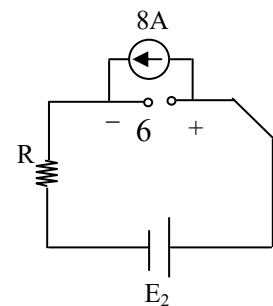
Sol: At $t=0$

$$L \frac{di(0)}{dt} = V_L(0)$$

$$V_L = 2 \times 3 = 6$$

$$V_L = 6\text{V}$$

$$E_2 + 6 - 8R = 0$$





$$E_2 = 8R - 6$$

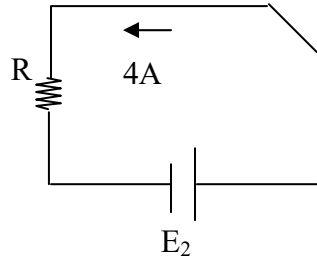
$$E_2 - 4R = 0$$

$$E_2 = 4R$$

$$8R - 6 = 4R$$

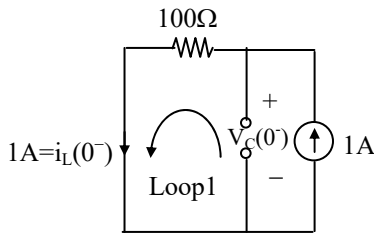
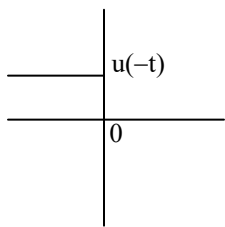
$$4R = 6$$

$$R = 1.5\Omega$$



21. Ans: (d)

Sol: at $t < 0$



Apply KVL in loop1 $\Rightarrow V_C(0^-) - 100 = 0$

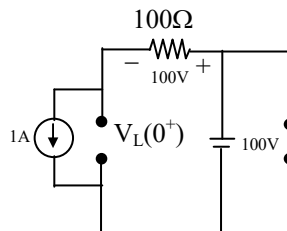
$$\Rightarrow V_C(0^-) = 100V$$

At $t = 0^+$

$$V_L(0^+) = 0$$

$$L \frac{di(0^+)}{dt} = 0$$

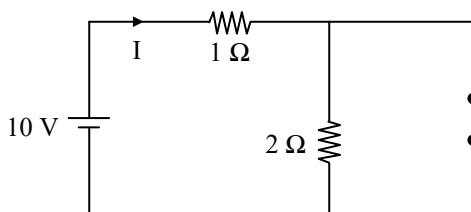
$$\frac{di(0^+)}{dt} = 0$$



22.

Sol: Case -1 at $t = 0^+$

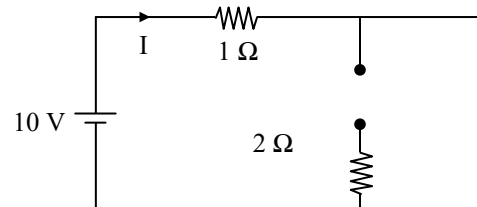
By redrawing the circuit



Current through the battery at $t = 0^+$ is

$$\frac{10}{3} \text{ Amp}$$

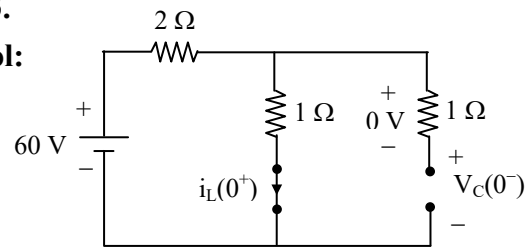
Case -2 at $t = \infty$



Current through the battery at $t = \infty$ is 10 A

23.

Sol:

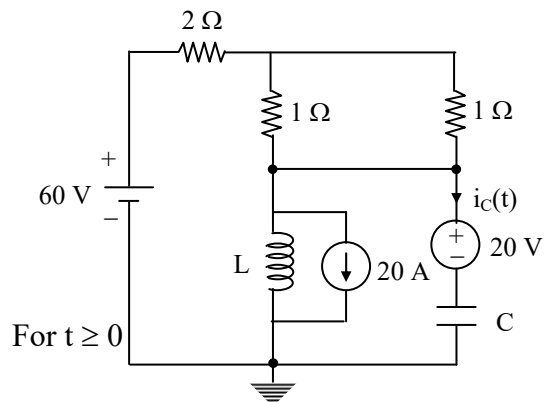


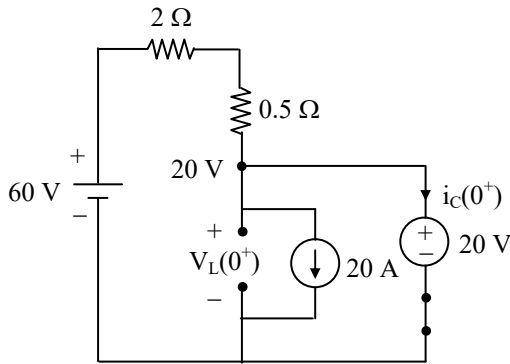
At $t = 0^-$: Steady state: A resistive circuit

(i) $t = 0^-$:

$$i_L(0^-) = \frac{60}{3} = 20 \text{ A} = i_L(0^+)$$

$$V_{1\Omega} = 20 \text{ V} = V_C(0^-) = V_C(0^+)$$





At $t = 0^+$: A resistive circuit :
Network is in transient state

$$V_L(0^+) = 20 \text{ V}$$

Nodal :

$$\frac{20 - 60}{2.5} + 20 + i_c(0^+) = 0$$

$$i_c(0^+) = -4 \text{ A}$$

24.

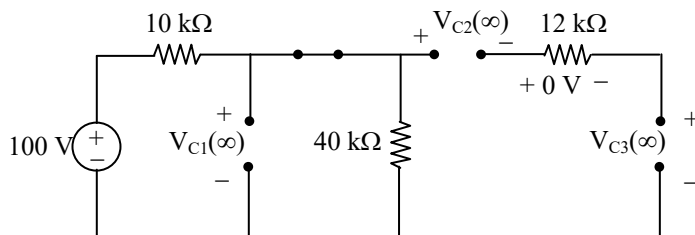
Sol: Repeat the above problem procedure :

$$\left. \frac{di_L(t)}{dt} \right|_{t=0^+} = \frac{V_L(0^+)}{L} = 0 \text{ A/sec}$$

$$\left. \frac{dV_C(t)}{dt} \right|_{t=0^+} = \frac{i_C(0^+)}{C} = -10^6 \text{ V/sec}$$

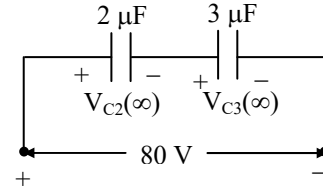
25.

Sol: **Observation:** So, the steady state will occur either at $t = 0^-$ or at $t = \infty$, that depends where we started i.e., connected the source to the network.



At $t = \infty$: Steady state: A Resistive circuit

$$V_{C_1}(\infty) = \frac{100}{50K} \times 40K = 80 \text{ V}$$

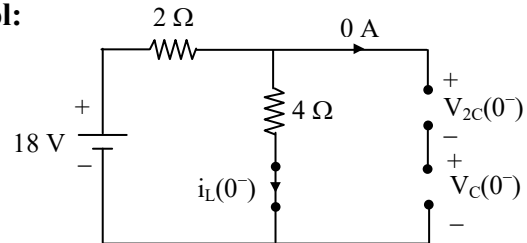


$$V_{C_2}(\infty) = \frac{80 \times 3\mu\text{F}}{(2+3)\mu\text{F}} = 48 \text{ V}$$

$$V_{C_3}(\infty) = \frac{80 \times 2\mu\text{F}}{5\mu\text{F}} = 32 \text{ V}$$

26.

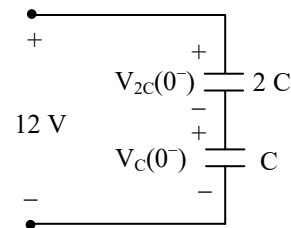
Sol:



At $t = 0^-$: Circuit is in Steady state: Resistive circuit

$$i_L(0^-) = 3 \text{ A} = i_L(0^+)$$

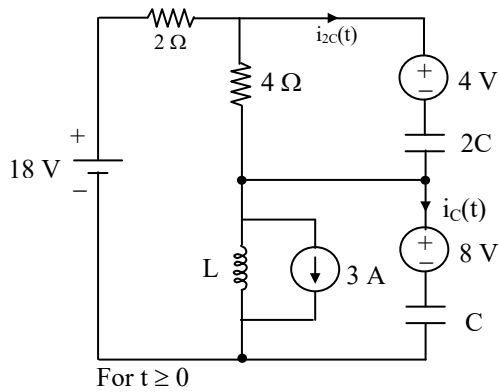
$$V_{4\Omega} = 4 \times 3 = 12 \text{ V}$$



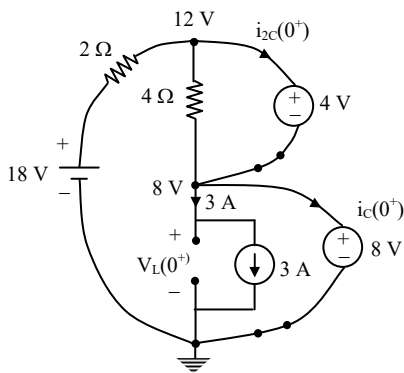
$$V_{2C}(0^-) = \frac{12 \times C}{2C + C}$$

$$= 4 \text{ V} = V_{2C}(0^+)$$

$$V_C(0^-) = 8 \text{ V} = V_C(0^+)$$



and redrawing the circuit



By Nodal;

$$\frac{12 - 18}{2} + \frac{12 - 8}{4} + i_{2c}(0^+) = 0$$

$$\frac{-6}{2} + \frac{4}{4} + i_{2c}(0^+) = 0$$

$$i_{2c}(0^+) = 2 \text{ A} = i_{2c}(0^-)$$

$$\frac{8 - 12}{4} - i_{2c}(0^+) + 3 + i_C(0^+) = 0$$

$$i_C(0^+) = 0 \text{ A} = i_C(0^-)$$

27.

Sol: $t = 0^-$ $t = 0^+$ $t = 0^+$

$$i_L(0^-) = 5 \text{ A} \quad i_L(0^+) = 5 \text{ A}$$

$$\frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L} = 40$$

$$i_R(0^-) = -5 \text{ A} \quad i_R(0^+) = -1 \text{ A}$$

$$\frac{di_R(0^+)}{dt} = -40 \text{ A/sec}$$

$$i_C(0^-) = 0 \text{ A} \quad i_C(0^+) = 4 \text{ A}$$

$$\frac{di_C(0^+)}{dt} = -40 \text{ A/sec}$$

$$V_L(0^-) = 0 \text{ V}$$

$$V_L(0^+) = 120 \text{ V}$$

$$\frac{dV_L(0^+)}{dt} = 1098 \text{ V/sec}$$

$$V_R(0^-) = -150 \text{ V}$$

$$V_R(0^+) = -30 \text{ V}$$

$$\frac{dV_R(0^+)}{dt} = -1200 \text{ V/sec}$$

$$V_C(0^-) = 150 \text{ V}$$

$$V_L(0^+) = 150 \text{ V}$$

$$\frac{dV_C(0^+)}{dt} = 108 \text{ V/sec}$$

(i). $t = 0^-$

$$\text{By KCL} \Rightarrow i_L(t) + i_R(t) = 0$$

$$t = 0^- \Rightarrow i_L(0^-) + i_R(0^-) = 0$$

$$i_R(0^-) = -5 \text{ A}$$

$$V_R(t) = R i_R(t) \text{ |By Ohm's law}$$

$$V_R(0^-) = R i_R(0^-) = 30(-5) = -150 \text{ V}$$

$$\text{By KVL} \Rightarrow V_L(t) - V_R(t) - V_C(t) = 0$$

$$V_C(0^-) = V_L(0^-) - V_R(0^-) = 150 \text{ V}$$

(ii). At $t = 0^+$

$$\text{By KCL at 1}^{\text{st}} \text{ node} \Rightarrow$$

$$-4 + i_L(t) + i_R(t) = 0$$

$$-4 + i_L(0^+) + i_R(0^+) = 0$$

$$i_R(0^+) = -i_L(0^+) + 4$$

$$i_R(0^+) = -5 + 4 = -1 \text{ A}$$

$$V_R(t) = R i_R(t) \text{ |By Ohm's law}$$



$$V_R(0^+) = R i_R(0^+)$$

$$V_R(0^+) = -30 \text{ V}$$

$$\text{By KVL} \Rightarrow V_L(t) - V_R(t) - V_C(t) = 0$$

$$V_L(0^+) = V_R(0^+) + V_C(0^+)$$

$$= 150 - 30 = 120 \text{ V}$$

By KCL at 2nd node;

$$-5 + i_C(t) - i_R(t) = 0$$

$$i_C(0^+) = 4 \text{ A}$$

(iii). $t = 0^+$

By KCL at 1st node \Rightarrow

$$-4 + i_L(t) + i_R(t) = 0$$

$$0 + \frac{di_L(t)}{dt} + \frac{d}{dt} i_R(t) = 0$$

$$V_R(t) = R i_R(t) \text{ |By Ohm's law}$$

$$\frac{d}{dt} V_R(t) = R \frac{d}{dt} i_R(t)$$

By KVL \Rightarrow

$$V_L(t) - V_R(t) - V_C(t) = 0$$

$$\frac{dV_L(t)}{dt} - \frac{dV_R(t)}{dt} - \frac{dV_C(t)}{dt} = 0$$

By KCL at node 2:

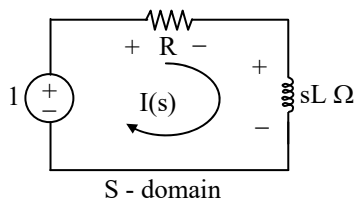
$$-5 + i_C(t) - i_R(t) = 0$$

$$0 + \frac{d}{dt} i_C(t) - \frac{d}{dt} i_R(t) = 0$$

$$\frac{d}{dt} i_C(0^+) = -(-40) = 40 \text{ A/sec}$$

28.

Sol: Transform the network into Laplace domain



$$V(s) = Z(s) I(s)$$

By KVL in S-domain \Rightarrow

$$1 - R I(s) - s L I(s) = 0$$

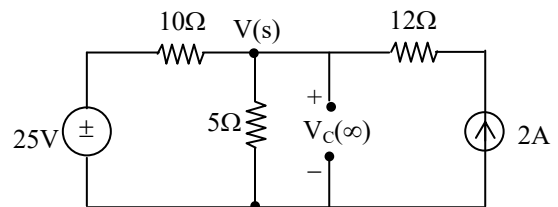
$$I(s) = \frac{1}{L} \frac{1}{\left(s + \frac{R}{L}\right)}$$

$$i(t) = \frac{1}{L} e^{-\frac{R}{L}t} \text{ A for } t \geq 0$$

29.

Sol: By Time domain approach;

$$V_C(0^-) = 5 \times 2 = 10 \text{ V} = V_C(0^+)$$



At $t = \infty$: Steady state: A resistive circuit

$$\text{Nodal} \Rightarrow \frac{V_C(\infty) - 25}{10} + \frac{V_C(\infty)}{5} - 2 = 0$$

$$V_C(\infty) = 15 \text{ V}$$

$$\tau = R_{eq} C = (5 \parallel 10) \cdot 1 = (10/3) \text{ sec}$$

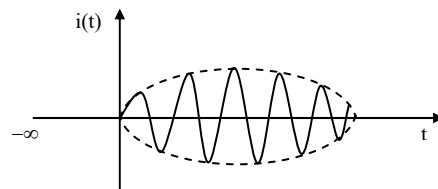
$$V_C = 15 + (10 - 15) e^{-\frac{t}{(10/3)}}$$

$$V_C = 15 - 5 e^{-3t/10} \text{ V for } t \geq 0$$

$$i_C = C \frac{dV_C}{dt} = 1.5 e^{-3t/10} \text{ A for } t \geq 0$$

30.

Sol:



That is the response is oscillatory in nature



31.

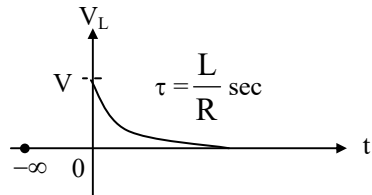
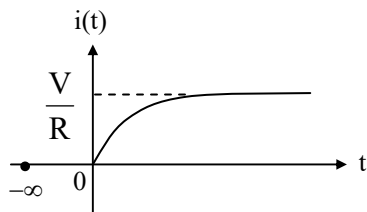
Sol: $i(0^-) = 0 \text{ A} = i(0^+)$

$$i(\infty) = \frac{V}{R} \text{ A}$$

$$\tau = \frac{L}{R} \text{ sec}$$

$$i(t) = \frac{V}{R} + \left(0 - \frac{V}{R}\right) e^{-t/\tau} = \frac{V}{R} (1 - e^{-t/\tau})$$

$$V_L = \frac{L di(t)}{dt} = V e^{-Rt/L} \text{ for } t \geq 0$$



Exponentially Increasing Response

32.

Sol: $V_C(0^-) = 0 = V_C(0^+)$

$$V_C(\infty) = V$$

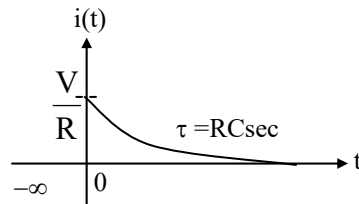
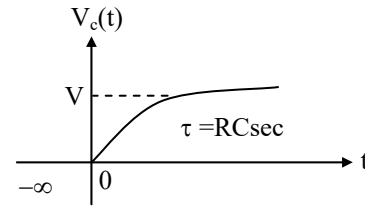
$$\tau = RC$$

$$V_C = V + (0-V)e^{-t/\tau}$$

$$= V(1 - e^{-t/RC}) \text{ for } t \geq 0$$

$$i_C = C \frac{dv_C}{dt} = \frac{V}{R} e^{-t/RC} \text{ for } t \geq 0$$

$$= i(t)$$

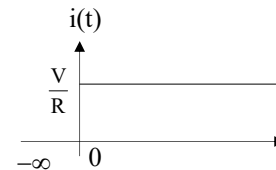


Exponentially Decreasing Response

33.

Sol: It's an RL circuit with $L = 0 \Rightarrow \tau = 0 \text{ sec}$

$$i(t) = \frac{V}{R}, \forall t \geq 0 \text{ So, } \tau = 0 \text{ sec}$$



i.e. the response is constant

34.

Sol: $i_1 = \frac{100u(t) - V_L}{10}$

$$i_1 = \left(10u(t) - \frac{1}{100} \frac{di_L}{dt}\right) \text{ A}$$

Nodal \Rightarrow

$$-i_1 + i_L + \frac{V_L - 20i_1}{20} = 0$$

$$-2i_1 + i_L + \frac{1}{200} \frac{di_L}{dt} = 0$$

Substitute i_1 ;



$$\frac{di_L}{dt} + 40i_L = 800u(t)$$

$$sI_L(s) - i_L(0^+) + 40I_L(s) = \frac{800}{s}$$

$$i_L(0^-) = 0A = i_L(0^+)$$

$$I_L(s) = \frac{800}{s(s+40)} = \frac{20}{s} - \frac{20}{s+40}$$

$$i_L(t) = 20u(t) - 20e^{-40t} u(t)$$

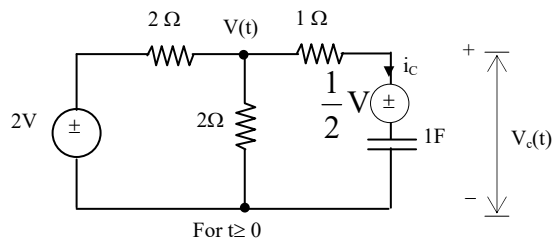
$$i_L(t) = 20(1 - e^{-40t}) u(t)$$

$$i_1 = 10u(t) - \frac{1}{100} \frac{di_L}{dt}$$

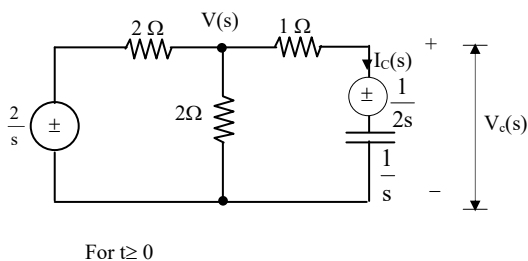
$$i_1 = (10 - 8e^{-40t}) u(t)$$

35.

Sol: By Laplace transform approach:



Transform the above network into the Laplace domain



Nodal \Rightarrow

$$\frac{V(s) - \frac{2}{s}}{2} + \frac{V(s)}{2} + \frac{V(s) - \frac{1}{2s}}{1 + \frac{1}{s}} = 0$$

$$I_c(s) = \left(\frac{V(s) - \frac{1}{2s}}{1 + \frac{1}{s}} \right)$$

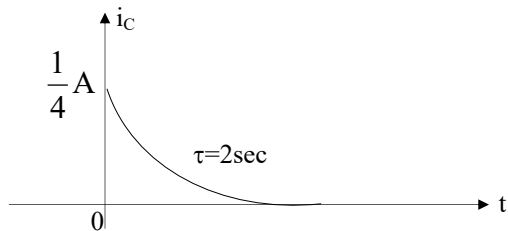
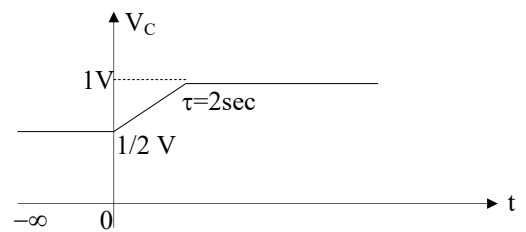
$$\Rightarrow i_c(t) = \frac{1}{4} e^{-\frac{t}{2}} \text{ A for } t \geq 0$$

By KVL \Rightarrow

$$V_c(s) - \frac{1}{2s} - \frac{1}{s} I_c(s) = 0$$

$$V_c(s) = \frac{1}{2s} + \frac{1}{s} I_c(s)$$

$$v_c(t) = 1 - \frac{1}{2} e^{-\frac{t}{2}} \text{ V for } t \geq 0$$

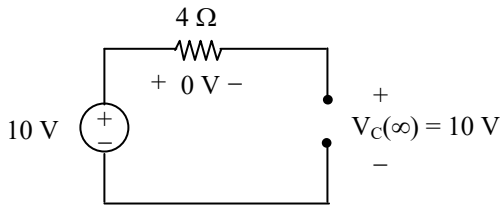


36.

Sol: By Time domain approach ;

$$V_c(0) = 6 \text{ V (given)}$$

$$V_c(\infty) = 10 \text{ V}$$



At $t = \infty$: Steady state : Resistive circuit

$$\tau = RC = 8 \text{ sec}$$

$$V_C = 10 + (6 - 10) e^{-t/8}$$

$$V_C = 10 - 4 e^{-t/8}$$

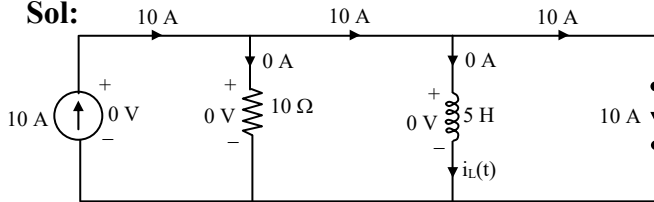
$$V_C(0) = 6 \text{ V}$$

$$i_C = C \frac{dV_C}{dt} = e^{-t/8} = i(t)$$

$$E_{4\Omega} = \int_0^{\infty} (e^{-t/8})^2 4 dt = 16 \text{ J}$$

37.

Sol:

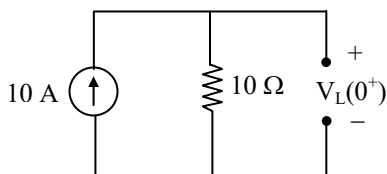


At $t = 0^-$: Network is not in steady state i.e., unenergised

$t = 0^-$:

$$i_L(0^-) = 0 \text{ A} = i_L(0^+)$$

$$V_L(0^+) = 10 \times 10 = 100 \text{ V}$$



At $t = 0^+$: Network is in transient state : A resistive circuit

$$i_L(\infty) = 10 \text{ A (since inductor becomes short)}$$

$$\tau = \frac{L}{R} = \frac{5}{10} = 0.5 \text{ sec}$$

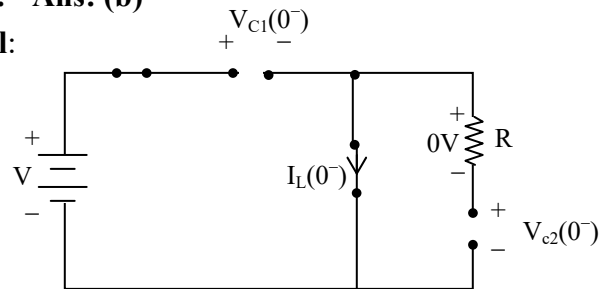
$$i_L(t) = 10 + (0 - 10) e^{-t/\tau} = 10 (1 - e^{-t/0.5}) \text{ A for } 0 \leq t \leq \infty$$

$$V_L(t) = L \frac{d}{dt} i_L(t) = 100 e^{-2t} \text{ V for } 0 \leq t \leq \infty$$

$$E_L \Big|_{t=5\tau \text{ or } t=\infty} = \frac{1}{2} Li^2 = \frac{1}{2} \times 5 \times 10^2 = 250 \text{ J}$$

38. Ans: (b)

Sol:



At $t = 0^-$: Steady state: A resistive circuit

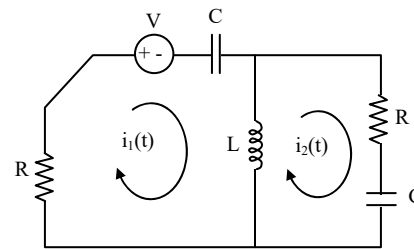
By KVL \Rightarrow

$$V - V_{C1}(0^-) = 0$$

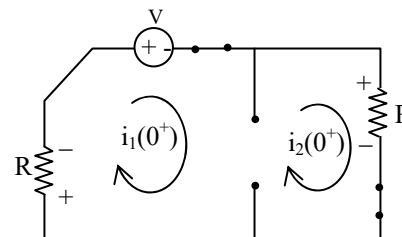
$$V_{C1}(0^-) = V = V_{C1}(0^+)$$

$$V_{C2}(0^-) = 0 \text{ V} = V_{C2}(0^+)$$

$$i_L(0^-) = 0 \text{ A} = i_L(0^+)$$



For $t \geq 0$ Fig (a)





At $t = 0^+$: A resistive circuit: Network is in transient state.

$$i_1(0^+) = i_2(0^+)$$

By KVL \Rightarrow

$$-Ri_1(0^+) - V - Ri_1(0^+) = 0$$

$$i_1(0^+) = \frac{-V}{2R} = i_2(0^+)$$

OBS: $i_L(t) = i_1(t) \sim i_2(t)$

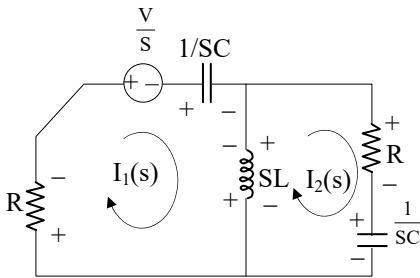
At $t = 0^+ \Rightarrow$

$$i_L(0^+) = i_1(0^+) \sim i_2(0^+)$$

$$= 0A \Rightarrow \text{Inductor: open circuit}$$

39.

Sol: (b) Transform the network given in fig. (a) into the S-domain.



$$V(s) = Z(s) \cdot I(s)$$

By KVL in S-domain \Rightarrow

$$-RI_1(s) - \frac{V}{s} - \frac{I_1(s)}{sC} - SL(I_1(s) - I_2(s)) = 0$$

Similarly:

$$-RI_2(s) - \frac{I_2(s)}{sC} - SL(I_2(s) - I_1(s)) = 0$$

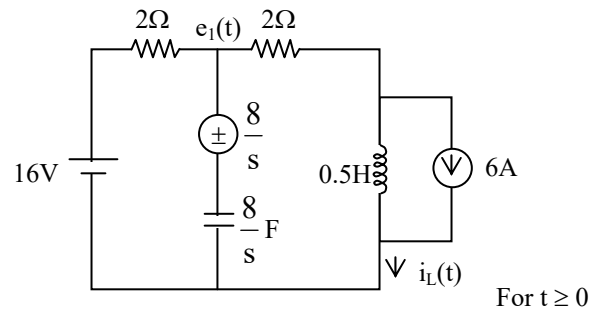
$$\begin{bmatrix} R + SL + \frac{1}{sC} & -SL \\ -SL & R + SL + \frac{1}{sC} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -V/s \\ 0 \end{bmatrix}$$

40.

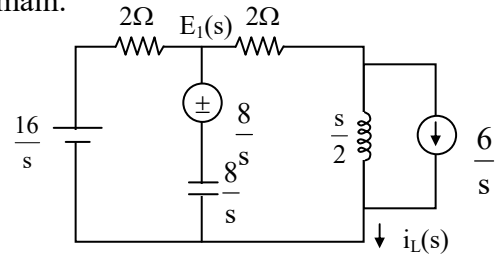
Sol: Evaluation of $i_L(t)$ and $e_1(t)$ for $t \geq 0$ by Laplace transform approach.

$$i_L(0^+) = 6A; i_L(\infty) = 4A$$

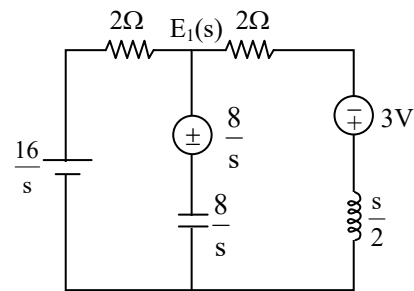
$$e_1(0^+) = 8V; e_1(\infty) = 8V$$



Transform the above network into Laplace domain.



S-domain:



Nodal in S-domain

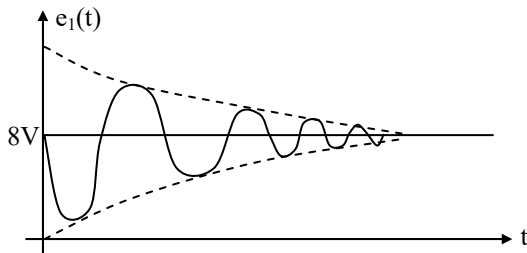
$$\frac{E_1(s) - 16/s}{2} + \frac{E_1(s) - \frac{8}{s}}{\frac{8}{s}} + \frac{E_1(s) + 3}{2 + \frac{s}{2}} = 0$$



$$E_1(s) = \frac{8}{s} \left(\frac{s^2 + 6s + 32}{s^2 + 8s + 32} \right)$$

$$E_1(s) = \frac{8}{s} \left(1 - \frac{2s}{(s+4)^2 + 4^2} \right)$$

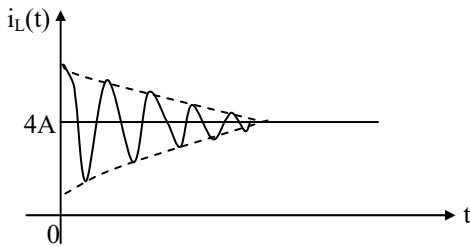
$$e_1(t) = 8 - 4e^{-4t} \sin 4t \text{ V for } t \geq 0$$



$$I_L(s) = \frac{E_1(s) + 3}{2 + \frac{s}{2}}$$

$$i_L(t) = 4 + 2e^{-4t} \cos 4t \text{ A}$$

$$\text{for } t \geq 0 \quad \omega_n = 4 \text{ rad/sec}$$



OBS: $\tau = \frac{1}{4} \text{ sec} = \frac{1}{\xi \omega_n} \quad \left| \quad \omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2} \times \frac{1}{8}}} = 4 \right.$

$$\frac{1}{4} \times \omega_n = \frac{1}{\xi}$$

$$\xi = \frac{4}{\omega_n} = \frac{4}{4} = 1$$

$\xi = 1$ (A critically damped system)

41.

Sol: $\omega t|_{t=t_0} = \tan^{-1} \left(\frac{\omega L}{R} \right)$

$$\omega t_0 = \tan^{-1} \left(\frac{\omega L}{R} \right)$$

$$2\pi(50)t_0 = \tan^{-1} \left(\frac{2\pi(50)(0.01)}{5} \right)$$

$$t_0 = 32.14 \times \frac{\pi}{180^\circ}$$

$$t_0 = 1.78 \text{ msec.}$$

So, by switching exactly at 1.78msec from the instant voltage becomes zero, the current is free from Transient.

42.

Sol: $\omega t_0 + \phi = \tan^{-1}(\omega CR) + \frac{\pi}{2}$

$$2t_0 + \frac{\pi}{4} = \tan^{-1}(\omega CR) + \frac{\pi}{2}$$

$$2t_0 + \frac{\pi}{4} = \tan^{-1} \left(2 \left(\frac{1}{2} \right) (1) \right) + \frac{\pi}{2} = \frac{\pi}{4} + \frac{\pi}{2}$$

$$2t_0 = \frac{\pi}{2} \Rightarrow t_0 = 0.785 \text{ sec}$$

4. AC Circuit Analysis

01.

Sol: $I_{\text{avg}} = I_{\text{dc}} = \frac{1}{T} \int_0^T i(t) dt = 3 + 0 + 0 = 3\text{A}$

$$I_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$= \sqrt{3^2 + \left(\frac{4\sqrt{2}}{\sqrt{2}} \right)^2 + \left(\frac{5\sqrt{2}}{\sqrt{2}} \right)^2 + 0 + 0 + 0}$$

$$= 5\sqrt{2}\text{A}$$



02.

Sol: $V_{dc} = V_{avg} = \frac{1}{T} \int_0^T V(t) dt = 2V$

Here the frequencies are same, by doing simplification

$$v(t) = 2 - 3\sqrt{2} \left(\cos 10t \times \frac{1}{\sqrt{2}} - \sin 10t \times \frac{1}{\sqrt{2}} \right)$$

$$+ 3\cos 10t = 2 + 3\sin 10t \text{ V}$$

So $V_{rms} = \sqrt{(2)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = \sqrt{8.5} \text{ V}$

03.

Sol: $X_{avg} = X_{dc} = \frac{1}{T} \int_0^T x(t) dt = 0$

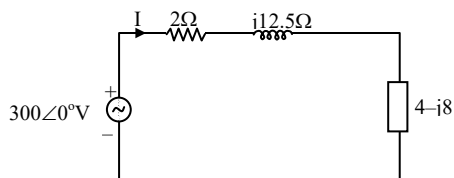
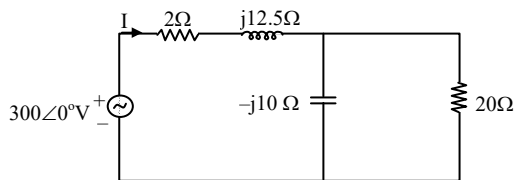
$$X_{rms} = \sqrt{\frac{1}{T} \int_0^T x^2(t) dt} = \frac{A}{\sqrt{3}}$$

04. **Ans: (a)**

Sol: For a symmetrical wave (i.e., area of positive half cycle = area of negative half cycle.) The RMS value of full cycle is same as the RMS value of half cycle.

05.

Sol: Complex power, $S = VI^*$



$$\Rightarrow I = \frac{300\angle 0^\circ}{2 + j12.5 + 4 - j8}$$

$$\Rightarrow I = 40\angle -36.86^\circ$$

∴ Complex power, $S = VI^*$

$$= 300 \angle 0^\circ \times 40 \angle 36.86^\circ$$

$$= 9600 + j7200$$

∴ Reactive power delivered by the source

$$Q = 72000 \text{ VAR}$$

$$= 7.2 \text{ KVAR}$$

06.

Sol: $Z = j1 + (1-j1) \parallel (1+j2) = 1.4 + j0.8$

$$I = \frac{E_1}{Z} \Big|_{\text{By ohm's law}} = \frac{10\angle 20}{1.4 + j0.8}$$

$$= 6.2017\angle -9.744^\circ \text{ A}$$

$$I_1 = \frac{I(1+j2)}{1-j1+1+j2} = 6.2017\angle 27.125^\circ \text{ A}$$

$$I_2 = \frac{I(1-j1)}{1-j1+1+j2} = 3.922\angle -81.31^\circ \text{ A}$$

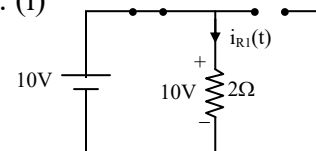
$$E_2 = (1-j1)I_1 = 8.7705\angle -17.875^\circ \text{ V}$$

$$E_0 = 0.5I_2 = 1.961\angle -81.31^\circ \text{ V}$$

07.

Sol: Since two different frequencies are operating on the network simultaneously always the super position theorem is used to evaluate the response.

By SPT: (i)

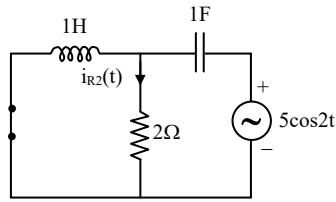


Network is in steady state, therefore the

network is resistive. $I_{R1}(t) = \frac{10}{2} = 5A$



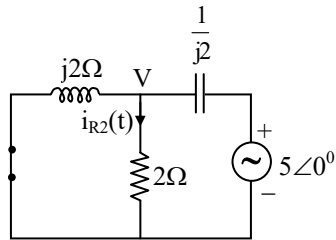
(ii)



Network is in steady state

As impedances of L and C are present because of $\omega = 2$. They are physically present.

$$Z_L = j\omega L; Z_C = \frac{1}{j\omega C} \Big|_{\omega=2}$$



Network is in phasor domain

$$\text{Nodal} \Rightarrow \frac{V}{j2} + \frac{V}{2} + \frac{V - 5\angle 0^\circ}{-j0.5} = 0$$

$$V = 6.32\angle 18.44^\circ$$

$$I_{R2} = \frac{V}{2} = 3.16\angle 18.44^\circ = 3.16 e^{j18.14^\circ}$$

$$i_{R2}(t) = R.P[I_{R2}e^{j2t}]A \\ = 3.16\cos(2t + 18.44^\circ)$$

By super position theorem,

$$i_R(t) = i_{R1}(t) + i_{R2}(t) \\ = 5 + 3.16\cos(2t + 18.44^\circ)A$$

08. Ans: (c)

$$\text{Sol: } \frac{1}{s^2 + 1} - I(s) \left(2 + 2s + \frac{1}{s} \right) = 0$$

$$I(s) \left(\frac{2s + 2s^2 + 1}{s} \right) = \frac{1}{s^2 + 1}$$

$$I(s) + 2s^2 I(s) + 2s I(s) = \frac{s}{s^2 + 1}$$

$$i(t) + \frac{2d^2 i}{dt^2} + 2 \frac{di}{dt} = \cos t$$

$$2 \frac{d^2 i}{dt^2} + 2 \frac{di}{dt} + i(t) = \cos t$$

09.

$$\text{Sol: } V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = V_R = I.R$$

$$100 = I.20; I = 5A$$

$$\text{Power factor} = \cos\phi = \frac{V_R}{V} = \frac{V_R}{V_R} = 1$$

So, unity power factor.

10.

Sol: By KCL in phasor – domain \Rightarrow

$$-I_1 - I_2 - I_3 = 0$$

$$I_3 = -(I_1 + I_2)$$

$$i_1(t) = \cos(\omega t + 90^\circ)$$

$$I_1 = 1\angle 90^\circ = j1$$

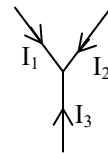
$$I_2 = 1\angle 0^\circ = (1 + j0)$$

$$I_3 = \sqrt{2} \angle \pi + 45^\circ = \sqrt{2} e^{j(\pi + 45^\circ)}$$

$$i_3(t) = \text{Real part}[I_3 \cdot e^{j\omega t}]mA$$

$$= -\sqrt{2} \cos(\omega t + 45^\circ + \pi)mA$$

$$i_3(t) = -\sqrt{2} \cos(\omega t + 45^\circ)mA$$



11.

$$\text{Sol: } I = \frac{V}{R} + \frac{V}{Z_L} + \frac{V}{Z_C} = 8 - j12 + j18$$

$$I = 8 + 6j$$



$$|I| = \sqrt{100} = 10A$$

12.

Sol: By KCL \Rightarrow

$$-I + I_L + I_C = 0$$

$$I = I_L + I_C$$

$$I_L = \frac{V}{Z_L} = \frac{V}{j\omega L} = \frac{3\angle 0^\circ}{j(3)\left(\frac{1}{3}\right)}$$

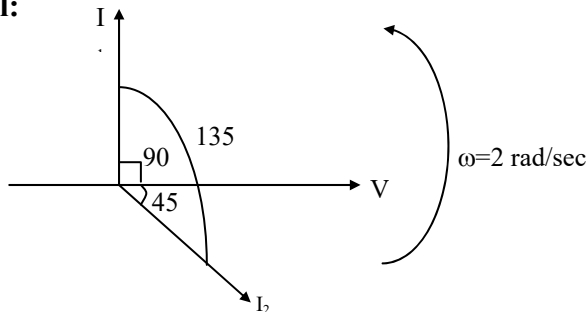
$$I_L = \frac{3\angle 0^\circ}{j} = \frac{3\angle 0^\circ}{\angle 90^\circ} = 3\angle -90^\circ$$

$$I = 3\angle -90^\circ + 4\angle 90^\circ$$

$$= -j3 + j4 = j1 = 1\angle 90^\circ$$

13. Ans: (d)

Sol:



$$I_1 = I_C = \frac{V}{Z_C} = \frac{V}{X_C} \angle 90^\circ$$

$$I_2 = \frac{V}{2 + j\omega L} = \frac{V}{2 + j2} = \frac{V}{2\sqrt{2}} \angle 45^\circ$$

Therefore, the phasor I_1 leads I_2 by an angle of 135°

14.

Sol: $I_2 = \sqrt{I_R^2 + I_C^2} \Rightarrow 10 = \sqrt{I_R^2 + 8^2}$

$$I_R = 6A$$

$$I_1 = I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$10 = \sqrt{6^2 + (I_L - I_C)^2}$$

$$I_L - I_C = \pm 8A$$

$$I_L - 8 = \pm 8$$

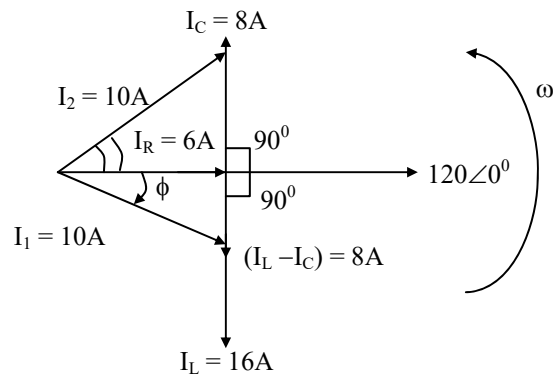
$$I_L - 8 = -8 \text{ (Not acceptable)}$$

Since $I_L = \frac{V}{Z_L} \neq 0$.

$$I_L - 8 = 8$$

$$I_L = 16A$$

$$I_L > I_C$$



$$I_2 \text{ leads } 120\angle 0^\circ \text{ by } \tan^{-1}\left(\frac{8}{6}\right)$$

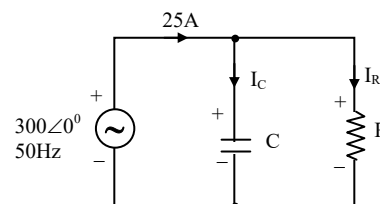
$$I_1 \text{ lags } 120\angle 0^\circ \text{ by } \tan^{-1}\left(\frac{8}{6}\right)$$

$$\text{Power factor } \cos\phi = \frac{I_R}{I} = \frac{I_R}{I}$$

$$= \frac{6}{10} = 0.6 \text{ (lag)}$$

15.

Sol:





Network is in steady state.

$$|I_C| = \left| \frac{V}{Z_C} \right| = \left| \frac{300 \angle 0^\circ}{(1/j\omega C)} \right| = v\omega C$$

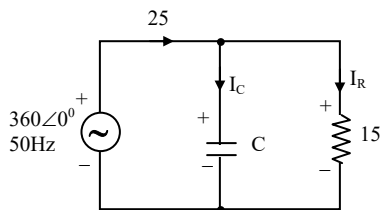
$$= 300 \times 2\pi \times 50 \times 159.23 \times 10^{-6}$$

$$I_C = 15A$$

$$I = \sqrt{I_R^2 + I_C^2}$$

$$25 = \sqrt{I_R^2 + 15^2}$$

$$I_R = 20A$$



$V_R = RI_R$ | By ohm's law

$$300 = R \cdot 20$$

$$R = 15\Omega$$

Network is in steady state

$$I_R = \frac{360}{15} = 24A$$

$$\text{So the required } I_C = \sqrt{25^2 - 24^2}$$

$$v\omega C = 7$$

$$360 \times 2\pi \times f \times 159.23 \times 10^{-6} = 7$$

$$f = 19.4\text{Hz}$$

$$\text{OBS: } I_C = \frac{V}{Z_C}$$

$$Z_C = \frac{1}{j\omega C} \Omega$$

$$\text{As } f \downarrow \Rightarrow Z_C \uparrow \Rightarrow I_C \downarrow$$

16.

Sol: $P_{5\Omega} = 10\text{Watts}$ (Given)

$$= P_{\text{avg}} = I_{\text{rms}}^2 R$$

$$10 = I_{\text{rms}}^2 \cdot 5$$

$$I_{\text{rms}} = \sqrt{2} A$$

Power delivered = Power observed

(By Tellegen's Theorem)

$$P_T = I_{\text{rms}}^2 (5 + 10)$$

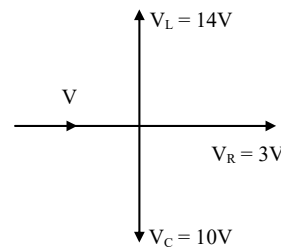
$$V_{\text{rms}} I_{\text{rms}} \cos\phi = (\sqrt{2})^2 (15)$$

$$\frac{50}{\sqrt{2}} \times \sqrt{2} \cos\phi = 2 \times 15$$

$$\cos\phi = 0.6 \text{ (lag)}$$

17. Ans: (d)

Sol:



$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(3)^2 + (14 - 10)^2}$$

$$V = 5 V$$

18.

$$\text{Sol: } Y = Y_1 + Y_c = \frac{1}{Z_L} + \frac{1}{Z_C}$$

$$= \frac{1}{30 \angle 40^\circ} + \frac{1}{\left(\frac{1}{j\omega C} \right)}$$

$$= j\omega C + \frac{1}{30} \angle -40^\circ$$

$$= j\omega C + \frac{1}{30} (\cos 40^\circ - j \sin 40^\circ)$$

Unit power factor $\Rightarrow j$ -term = 0

$$\omega C = \frac{\sin 40^\circ}{30}$$



$$C = \frac{\sin 40^\circ}{2\pi \times 50 \times 30} = 68.1 \mu\text{F}$$

$$C = 68.1 \mu\text{F}$$

19. Ans: (b)

Sol: To increase power factor shunt capacitor is to be placed.

VAR supplied by capacitor

$$\begin{aligned} &= P (\tan\phi_1 - \tan\phi_2) \\ &= 2 \times 10^3 [\tan(\cos^{-1} 0.65) - \tan(\cos^{-1} 0.95)] \\ &= 1680 \text{ VAR} \end{aligned}$$

$$\text{VAR supplied} = \frac{V^2}{X_C} = V^2 \omega C = 1680$$

$$\therefore C = \frac{1680}{(115)^2 \times 2\pi \times 60} = 337 \mu\text{F}$$

20.

$$\text{Sol: } Z = \frac{V}{I} = \frac{160 \angle 10^\circ - 90^\circ}{5 \angle -20^\circ - 90^\circ} = 32 \angle 30^\circ$$

$\phi = 30^\circ$ (Inductive)

$$V_{\text{rms}} = \frac{160}{\sqrt{2}} \text{ V, } I_{\text{rms}} = \frac{5}{\sqrt{2}}$$

$$\begin{aligned} \text{Real power (P)} &= \frac{160}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \cos 30^\circ \\ &= 200 \sqrt{3} \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Reactive power (Q)} &= \frac{160}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times \frac{1}{2} \\ &= 200 \text{ VAR} \end{aligned}$$

$$\text{Complex power} = P + jQ = 200(\sqrt{3} + j1) \text{ VA}$$

21.

$$\text{Sol: } V = 4 \angle 10^\circ \text{ and } I = 2 \angle -20^\circ$$

Note: When directly phasors are given the magnitudes are taken as rms values since they are measured using rms meters.

$$V_{\text{rms}} = 4 \text{ V and } I_{\text{rms}} = 2 \text{ A}$$

$$Z = \frac{V}{I} = 2 \angle 30^\circ; \phi = 30^\circ \text{ (Inductive)}$$

$$P = 10 \sqrt{3} \text{ W, } Q = 10 \text{ VAR}$$

$$S = 10(\sqrt{3} + j1) \text{ VA}$$

22. Ans: (a)

$$\text{Sol: } S = VI^*$$

$$= (10 \angle 15^\circ) (2 \angle 45^\circ)$$

$$= 10 + j17.32$$

$$S = P + jQ$$

$$P = 10 \text{ W } \quad Q = 17.32 \text{ VAR}$$

23. Ans: (c)

$$\text{Sol: } P_R = (I_{\text{rms}})^2 \times R$$

$$I_{\text{rms}} = \frac{10}{\sqrt{2}}$$

$$P_R = \left(\frac{10}{\sqrt{2}} \right)^2 \times 100$$

24.

$$\text{Sol: } P_{\text{avg}} = \frac{V_{\text{rms}}^2}{R} = \frac{\left(\frac{240}{\sqrt{2}} \right)^2}{60} = 480 \text{ Watts}$$

$$V = 240 \angle 0^\circ$$

$$I_R = \frac{V}{R} = \frac{240}{60} = 4 \text{ A}$$

$$I_L = \frac{V}{Z_L} = \frac{V}{X_L} = \frac{240}{40} = 6 \text{ A}$$



$$I_C = \frac{V}{Z_C} = \frac{V}{X_C} = \frac{240}{80} = 3A$$

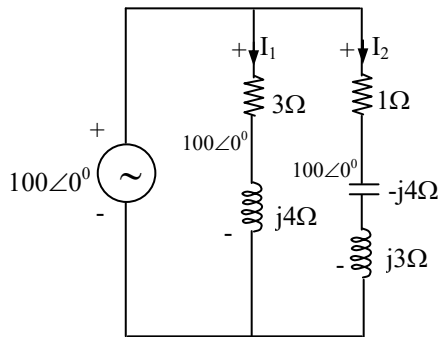
$I_L > I_C$: Inductive nature of the circuit.

$$I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{4^2 + 3^2} = 5A$$

$$\text{Power factor} = \frac{I_R}{I} = \frac{4}{5} = 0.8 \text{ (lagging)}$$

25. Ans: (a)

Sol:



NW is in Steady state.

$$V = 100\angle 0^\circ \Rightarrow V_{\text{rms}} = 100V$$

$$I_1 = \frac{100\angle 0^\circ}{(3 + j4)\Omega} \Rightarrow |I_1| = 20 = I_{1\text{rms}}$$

$$I_2 = \frac{100\angle 0^\circ}{(1 - j1)\Omega} \Rightarrow |I_2| = \frac{100}{\sqrt{2}} A = I_{2\text{rms}}$$

$$\begin{aligned} P &= P_1 + P_2 \\ &= (I_{1\text{rms}})^2 \cdot 3 + (I_{2\text{rms}})^2 \cdot 1 \\ &= 20^2 \cdot 3 + \left(\frac{100}{\sqrt{2}}\right)^2 \cdot 1 \end{aligned}$$

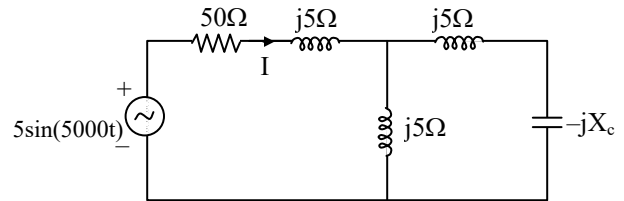
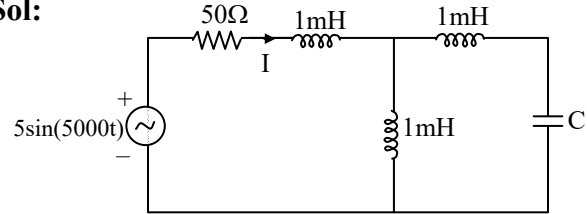
$$P = 6200 \text{ W}$$

$$\begin{aligned} Q &= Q_1 + Q_2 \\ &= (I_{1\text{rms}})^2 \cdot 4 + (I_{2\text{rms}})^2 \cdot (1) \\ &= 3400 \text{ VAR} \end{aligned}$$

$$\text{So, } S = P + jQ = (6200 + j3400) \text{ VA}$$

26.

Sol:



when $I = 0$,

\Rightarrow impedance seen by the source should be infinite

$$\Rightarrow Z = \infty$$

$$\begin{aligned} \therefore Z &= (50 + j5) + (j5) \parallel j(5 - X_c) \\ &= 50 + j5 + \frac{j5 \times j(5 - X_c)}{j5 + j(5 - X_c)} = \infty \end{aligned}$$

$$\Rightarrow j(10 - X_c) = 0$$

$$\Rightarrow X_c = 10 \Rightarrow \frac{1}{\omega C} = 10$$

$$\Rightarrow C = \frac{1}{5000 \times 10} = 20 \mu\text{F}$$

27. Ans: (c)

$$\text{Sol: } I_{\text{rms}} = \sqrt{3^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \sqrt{25} = 5A$$

$$\text{Power dissipation} = I_{\text{rms}}^2 R = 5^2 \times 10 = 250 \text{ W}$$

28.

Sol: $X_C = X_L \Rightarrow \omega = \omega_0$, the circuit is at resonance

$$V_C = QV_S \angle -90^\circ$$



$$Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = 2 \Rightarrow \frac{1}{\omega_0 C R} = \frac{X_C}{R} = 2$$

$$\Rightarrow V_C = 200 \angle -90^\circ = -j200V$$

29.

Sol: Series RLC circuit

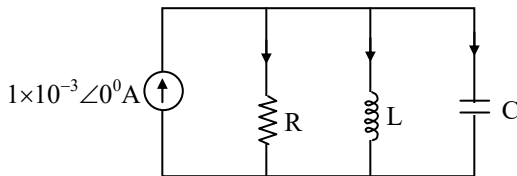
$$f = f_L, \text{ PF} = \cos \phi = 0.707(\text{lead})$$

$$f = f_H, \text{ PF} = \cos \phi = 0.707(\text{lag})$$

$$f = f_0, \text{ PF} = \cos \phi = 1$$

30. **Ans: (b)**

Sol: Network is in steady state (since no switch is given)



$$\text{Let } I = 1\text{mA}$$

$$\omega = \omega_0(\text{Given})$$

$$\Rightarrow I_R = I$$

$$I_L = QI \angle -90^\circ = -jQI$$

$$I_C = QI \angle 90^\circ = jQI$$

$$I_L + I_C = 0$$

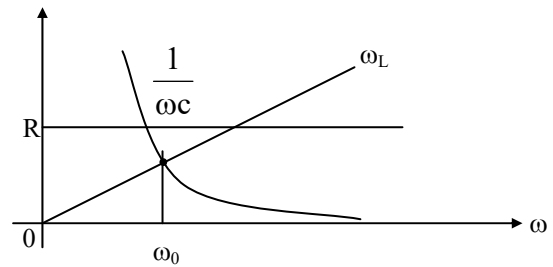
$$|I_R + I_L| = |I - jQI|$$

$$= I\sqrt{1+Q^2} > I$$

$$|I_R + I_C| = |I + jQI| = I\sqrt{1+Q^2} > I$$

31. **Ans: (c)**

Sol: Since, "I" leads voltage, therefore capacitive effect and hence the operating frequency ($f < f_0$)



32.

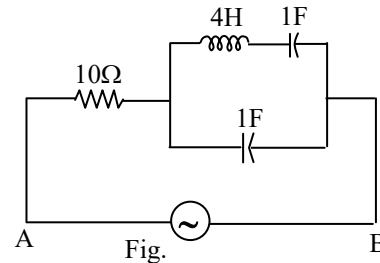
$$\begin{aligned} \text{Sol: } Y &= \frac{1}{R_L + j\omega L} + \frac{1}{R_C - \frac{j}{\omega C}} \\ &= \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + \frac{R_C + j/\omega C}{R_C^2 + (1/\omega C)^2} \end{aligned}$$

j - term $\Rightarrow 0$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}} \text{ rad/sec}$$

33.

Sol:



The given circuit is shown in Fig.

$$Z_{AB} = 10 + Z_1$$

$$\text{where, } Z_1 = \left(\frac{-j}{\omega} \right) \parallel \left(j4\omega - \frac{j}{\omega} \right)$$

$$\begin{aligned} &= \frac{\left(\frac{-j}{\omega} \right) \left(j4\omega - \frac{j}{\omega} \right)}{\frac{-j}{\omega} + j4\omega - \frac{j}{\omega}} \end{aligned}$$



$$= \frac{4 - \frac{1}{\omega^2}}{j4\omega - \frac{j2}{\omega}}$$

For circuit to be resonant i.e., $\omega^2 = \frac{1}{4}$

$$\omega = \frac{1}{2} = 0.5 \text{ rad/sec}$$

$$\therefore \omega_{\text{resonance}} = 0.5 \text{ rad/sec}$$

34.

Sol: (i) $\frac{L}{C} = R^2 \Rightarrow$ circuit will resonate for all the frequencies, out of infinite number of frequencies we are selecting one frequency.

$$\text{i.e., } \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{2} \text{ rad/sec}$$

$$\text{then } Z = R = 2\Omega.$$

$$I = \frac{V}{Z} = \frac{10\angle 0^\circ}{2} = 5\angle 0^\circ$$

$$i(t) = 5\cos\frac{t}{2} \text{ A}$$

$$Z_L = j\omega_0 L = j2\Omega ; Z_C = \frac{1}{j\omega_0 C} = -j2\Omega.$$

$$I_L = \frac{I(2 - j2)}{2 + j2 + 2 - j2} = \frac{I}{\sqrt{2}} \angle -45^\circ$$

$$i_L = \frac{5}{\sqrt{2}} \cos\left(\frac{t}{2} - 45^\circ\right) \text{ A}$$

$$i_C = \frac{I(2 + j2)}{2 + j2 + 2 - j2} = \frac{I}{\sqrt{2}} \angle 45^\circ$$

$$i_C = \frac{5}{\sqrt{2}} \cos\left(\frac{t}{2} + 45^\circ\right) \text{ A}$$

$$P_{\text{avg}} = I_{L(\text{rms})}^2 \cdot R + I_{C(\text{rms})}^2 \cdot R$$

$$= \left(\frac{5/\sqrt{2}}{\sqrt{2}}\right)^2 \cdot 2 + \left(\frac{5/\sqrt{2}}{\sqrt{2}}\right)^2 \cdot 2$$

$$= 25 \text{ watts}$$

(ii) $\frac{L}{C} \neq R^2$ circuit will resonate at only one frequency.

$$\text{i.e., at } \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{4} \text{ rad/sec}$$

$$\text{Then } Y = \frac{2R}{R^2 + \frac{L}{C}} \text{ mho}$$

$$Y = \frac{2(2)}{2^2 + \frac{4}{4}} = \frac{4}{5} \text{ mho}$$

$$Z = \frac{5}{4} \Omega$$

$$I = \frac{V}{Z} = \frac{10\angle 0^\circ}{5/4} = 8\angle 0^\circ$$

$$i(t) = 8\cos\frac{t}{4} \text{ A}$$

$$Z_L = j\omega_0 L = j1\Omega$$

$$Z_C = \frac{1}{j\omega_0 C} = -j1\Omega$$

$$I_L = \frac{I(2 - j1)}{2 + j1 + 2 - j1} = \frac{\sqrt{5}}{4} I \angle \tan^{-1}\left(\frac{1}{2}\right)$$

$$i_L = \frac{8\sqrt{5}}{4} \cos\left(\frac{t}{4} - \tan^{-1}\left(\frac{1}{2}\right)\right)$$

$$I_C = \frac{I(2 + j1)}{2 + j1 + 2 - j1} = \frac{\sqrt{5}}{4} I \angle \tan^{-1}\left(\frac{1}{2}\right)$$

$$i_C = \frac{8\sqrt{5}}{4} \cos\left(\frac{t}{4} + \tan^{-1}\left(\frac{1}{2}\right)\right)$$



$$P_{avg} = I_{Lrms}^2 R + I_{Crms}^2 R$$

$$= \left(\frac{2\sqrt{5}}{\sqrt{2}}\right)^2 \cdot 2 + \left(\frac{2\sqrt{5}}{\sqrt{2}}\right)^2 \cdot 2 = 40 \text{ Watts}$$

35.

Sol: (i) $Z_{ab} = 2 + (Z_L \parallel Z_C \parallel 2)$

$$= 2 + jX_L \parallel -jX_C \parallel 2$$

$$= \frac{2 + 2X_L X_C (X_L X_C - j2(X_L - X_C))}{(X_L X_C)^2 + 4(X_L - X_C)^2}$$

$$j\text{-term} = 0$$

$$\Rightarrow -2(X_L - X_C) = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.4}} = \frac{1}{4} \text{ rad/sec}$$

At resonance entire current flows through 2Ω only.

(ii) $Z_{ab}|_{\omega=\omega_0} = 2 + 2 = 4\Omega$

$$X_L = X_C$$

(iii) $V_i(t) = V_m \sin\left(\frac{t}{4}\right)V$

$$Z = 4\Omega$$

$$i(t) = \frac{V_i(t)}{Z} = \frac{V_m}{4} \sin\left(\frac{t}{4}\right) = i_R$$

$$V = 2i_R = \frac{V_m}{2} \sin\left(\frac{t}{4}\right)V = V_C = V_L$$

$$i_C = C \frac{dV_C}{dt} = \frac{V_m}{2} \cos\left(\frac{t}{4}\right)$$

$$i_c = \frac{V_m}{2} \sin\left(\frac{t}{4} + 90^\circ\right)A$$

$$i_L = \frac{1}{L} \int V_L dt = \frac{-V_m}{2} \cos\left(\frac{t}{4}\right)$$

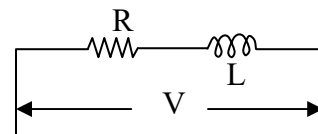
$$i_L = \frac{V_m}{2} \sin\left(\frac{t}{4} - 90^\circ\right)A$$

OBS: Here $i_L + i_C = 0$

\Rightarrow LC Combination is like an open circuit.

36. Ans: (d)

Sol:



$$Q = \frac{\omega L}{R}$$

$$Q = \frac{2\omega L}{R} = 2 \times \text{original} \rightarrow Q - \text{doubled}$$

$$S = V.I$$

$$= V \cdot \frac{V}{R + j\omega L} \times \frac{R - j\omega L}{R - j\omega L}$$

$$S = \frac{V^2}{R^2 + (\omega L)^2} - \frac{V^2 \cdot j\omega L}{R^2 + (\omega L)^2}$$

$$S = P + jQ$$

$$\text{Active power (P)} = \frac{V^2}{R^2 + (\omega L)^2}$$

$$P = \frac{V^2}{R^2(1 + Q^2)}$$

$$P \approx \frac{V^2}{R^2 Q^2}$$

as Q is doubled, P decreases by four times.

37.

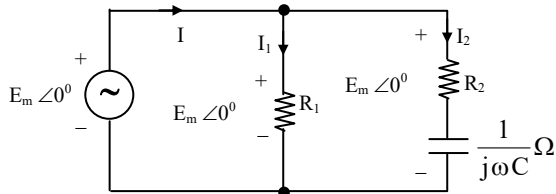
Sol: $Z_C = \frac{1}{j\omega C}$



$$\omega = 0; Z_C = \infty \Rightarrow C : \text{open circuit} \Rightarrow i_2 = 0$$

$$\omega = \infty; Z_C = 0 \Rightarrow C : \text{Short Circuit} \Rightarrow i_2 = \frac{E_m}{R_2} \angle 0^\circ$$

Transform the given network into phasor domain.



Network is in phasor domain.

By KCL in P-d $\Rightarrow I = I_1 + I_2$

$$I_1 = \frac{E_m \angle 0^\circ}{R_1}$$

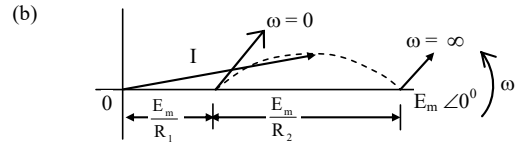
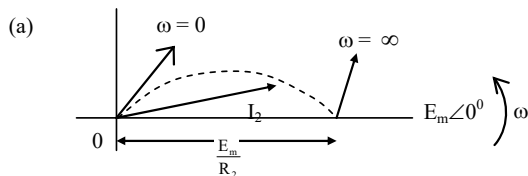
$$I_2 = \frac{E_m \angle 0^\circ}{R_2 + \frac{1}{j\omega C}} = \frac{E_m \angle 0^\circ}{R_2 - \frac{j}{\omega C}}$$

$$I_2 = \frac{E_m \angle \tan^{-1}\left(\frac{1}{\omega C R_2}\right)}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

$$\omega = \infty \Rightarrow I_2 = \frac{E_m \angle 0^\circ}{R_2}$$

$$\omega = 0 \Rightarrow I_2 = 0A$$

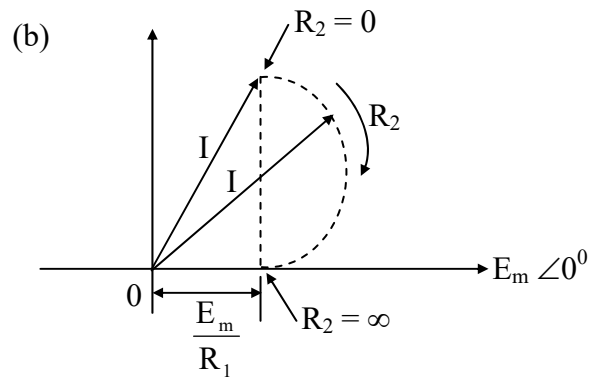
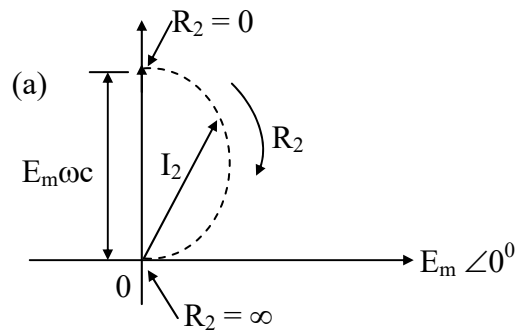
$\omega : (0 \text{ and } \infty)$ j the current phasor I_2 will always lead the voltage $E_m \angle 0^\circ$.



38.

$$\text{Sol: } R_2 = 0 \Rightarrow I_2 = \frac{E_m \angle 0^\circ}{0 + \frac{1}{j\omega C}} = E_m \omega C \angle 90^\circ$$

$$R_2 = \infty \Rightarrow I_2 = 0A$$

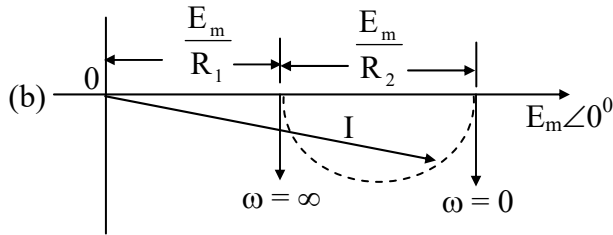
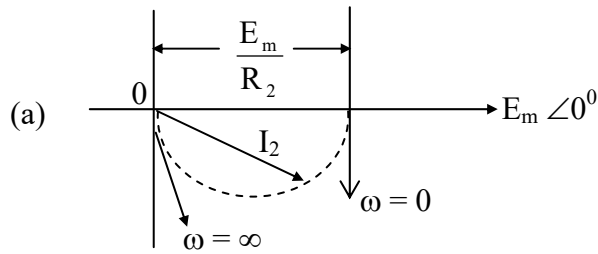


39.

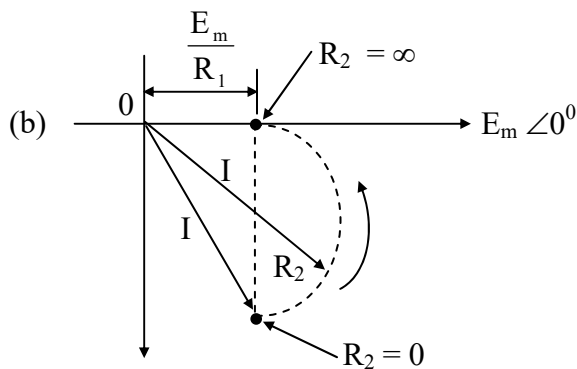
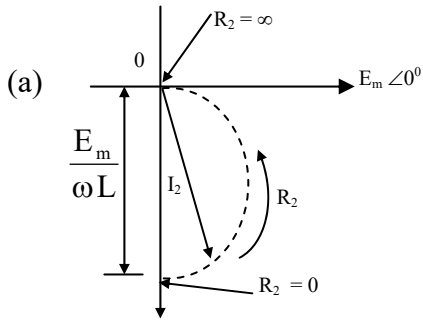
$$\text{Sol: } I = I_1 + I_2; I_1 = \frac{E_m \angle 0^\circ}{R_1}$$

$$I_2 = \frac{E_m \angle 0^\circ}{R_2 + j\omega L} = \frac{E_m}{\sqrt{R_2^2 + (\omega L)^2}} \angle -\tan^{-1}\left(\frac{\omega L}{R_2}\right)$$

(i) If " ω " Varied



ii. If “ R_2 ” is varied



5. Magnetic Circuits

01.

Sol: $X_C = 12$ (Given)

$X_{eq} = 12$ (must for series resonance)

So the dot in the second coil at point “Q”

$$L_{eq} = L_1 + L_2 - 2M$$

$$L_{eq} = L_1 + L_2 - 2K\sqrt{L_1 L_2}$$

$$\omega L_{eq} = \omega L_1 + \omega L_2 - 2K\sqrt{L_1 L_2} \omega$$

$$12 = 8 + 8 - 2K\sqrt{8 \cdot 8} \Rightarrow K = 0.25$$

02.

Sol: $X_C = 14$ (Given)

$X_{Leq} = 14$ (must for series resonance)

So the dot in the 2nd coil at “P”

$$L_{eq} = L_1 + L_2 + 2M$$

$$L_{eq} = L_1 + L_2 + K\sqrt{L_1 L_2}$$

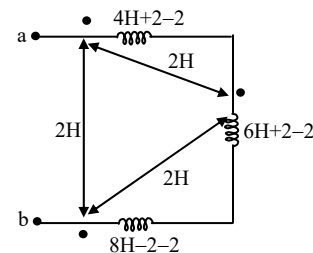
$$\omega L_{eq} = \omega L_1 + \omega L_2 + 2K\sqrt{\omega L_1 L_2}$$

$$14 = 2 + 8 + 2K\sqrt{2(8)}$$

$$\Rightarrow K = 0.5$$

03.

Sol: $L_{ab} = 4H+2-2+6H+2-2+8H-2-2$
 $= 14H$





04. Ans: (c)

Sol: Impedance seen by the source

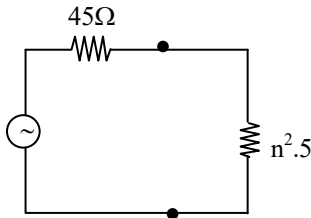
$$Z_s = \frac{Z_L}{16} + (4 - j2)$$

$$= \frac{10 \angle 30^\circ}{16} + (4 - j2) = 4.54 - j1.69$$

05.

Sol: $Z_{in} = \left(\frac{N_1}{N_2}\right)^2 \cdot Z_L$

$$R'_{in} = n^2 \cdot 5$$

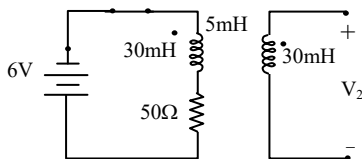


For maximum power transfer; $R_L = R_s$

$$n^2 \cdot 5 = 45 \Rightarrow n = 3$$

06. Ans: (b)

Sol:



Apply KVL at input loop

$$-6 - 30 \times 10^{-3} \frac{di_1}{dt} + 5 \times 10^{-3} \frac{di_2}{dt} - 50i_1 = 0 \dots (1)$$

Take Laplace transform

$$-\frac{6}{s} + [-30 \times 10^{-3}(s) - 50]I_1(s) + 5 \times 10^{-3} s I_2(s) = 0 \dots (2)$$

Apply KVL at output loop

$$V_2(s) - 30 \times 10^{-3} \frac{di_2}{dt} + 5 \times 10^{-3} \frac{di_1}{dt} = 0$$

Take Laplace transform

$$V_2(s) - 30 \times 10^{-3} s I_2(s) + 5 \times 10^{-3} s I_1(s) = 0$$

Substitute $I_2(s) = 0$ in above equation

$$V_2 + 5 \times 10^{-3} s I_1(s) = 0 \dots \dots \dots (3)$$

From equation (2)

$$-\frac{6}{s} + (-30 \times 10^{-3}(s) + 50)I_1(s) = 0$$

$$I_1(s) = \frac{-6}{s(30 \times 10^{-3}(s) + 50)} \dots \dots \dots (4)$$

Substitute eqn (4) in eqn (3)

$$V_2(s) = \frac{-5 \times 10^{-3}(s)(-6)}{s(30 \times 10^{-3}(s) + 50)}$$

Apply Initial value theorem

$$\lim_{s \rightarrow \infty} s \frac{-5 \times 10^{-3}(s)(-6)}{s(30 \times 10^{-3}(s) + 50)}$$

$$v_2(t) = \frac{-5 \times 10^{-3} \times (-6)}{30 \times 10^{-3}} = +1$$

07.

Sol: $R'_{in} = \frac{8}{2^2} = 2\Omega$

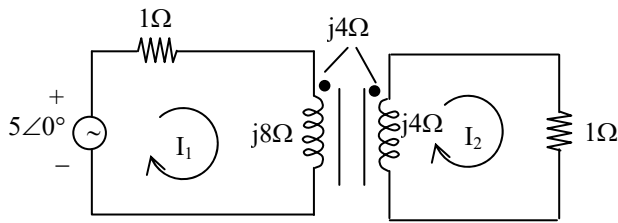
$$R_{in} = 3 + R'_{in} = 3 + 2 = 5\Omega$$

$$I_1 = \frac{10 \angle 20^\circ}{5} = 2 \angle 20^\circ$$

$$\frac{I_1}{I_2} = n = 2 \Rightarrow I_2 = 1 \angle 20^\circ A$$

08.

Sol: Transform the above network into phasor domain



Network is in Phasor -domain

$$V = Z.I$$

By KVL in p-d \Rightarrow

$$5\angle 0^\circ = I_1 + j8.I_1 - j4.I_2$$

$$0 = I_2 + j4I_2 - j4I_1$$

$$I_1 = \frac{\Delta_1}{\Delta}; i_1(t) = \text{Re al part} [I_1 e^{j2t}] \text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta}; i_2(t) = \text{Re al part} [I_2 e^{j2t}] \text{A}$$

$$I_1(t) = 1.072 \cos (2t + 114.61^\circ) \text{A}$$

$$I_2(t) = 1.416 \cos (2t + 128.65^\circ) \text{A}$$

09.

Sol: Evaluation of Initial conditions:

$$i_1(0^-) = 0 \text{A} = i_1(0^+)$$

$$i_2(0^-) = 0 \text{A} = i_2(0^+)$$

Evaluation of final conditions:

$$i_1(\infty) = 5 \text{A} \quad ; \quad i_2(\infty) = 0 \text{A}$$

By KVL \Rightarrow

$$5 = i_1(t) + \frac{4di_1(t)}{dt} - 2 \frac{di_2(t)}{dt}$$

By Laplace transform to the above equations.

$$\frac{5}{s} = I_1(s) + 4[sI_1(s) - i_1(0^+)] - 2[sI_2(s) - i_2(0^+)]$$

By KVL \Rightarrow

$$0 = 1.i_2(t) + 2 \frac{di_2(t)}{dt} - 2 \frac{di_1(t)}{dt}$$

$$0 = 1.I_2(s) + 2[sI_2(s) - i_2(0^+)] - [sI_1(s) - i_1(0^+)]$$

On solving, we can obtain $i_1(t)$ and $i_2(t)$

$$i_1(t) = 5 - e^{-\frac{3t}{4}} \left[5 \cosh\left(\frac{\sqrt{5}}{4}t\right) - \sqrt{5} \sinh\left(\frac{\sqrt{5}}{4}t\right) \right] \text{A}$$

10.

Sol: By the definition of KVL in phasor domain

$$V_S - V_0 - V_2 = 0$$

$$V_0 = V_S - V_2 = V_S \left(1 - \frac{V_2}{V_S} \right)$$

$$V = ZI$$

By KVL

$$V_S = j\omega L_1.I_1 + j\omega M(0)$$

$$V_2 = j\omega L_2(0) + j\omega M I_1$$

$$V_0 = V_S \left(1 - \frac{M}{L_1} \right)$$

6. Two Port Networks

01.

Sol: The defining equations for open circuit impedance parameters are:

$$V_1 = Z_{11}I_1 + Z_{12}I_2; \quad V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$[Z] = \begin{bmatrix} \frac{10}{s} & \frac{4s+10}{s} \\ \frac{s}{10} & \frac{s}{3s+10} \end{bmatrix} \Omega$$

02. **Ans: (b)**

Sol: The matrix given is $\begin{bmatrix} 0 & -1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

since $y_{11} \neq y_{22}$

\Rightarrow Asymmetrical, and

$$Y_{12} \neq Y_{21} \text{ '}$$



⇒ Non reciprocal network

03.

Sol: Convert Y to Δ :

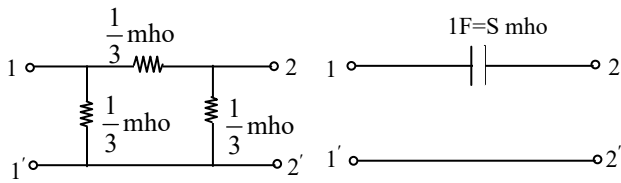
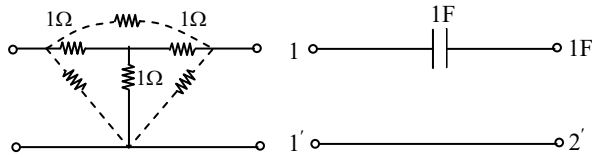


Fig:A

$$Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

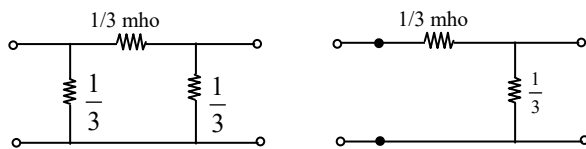
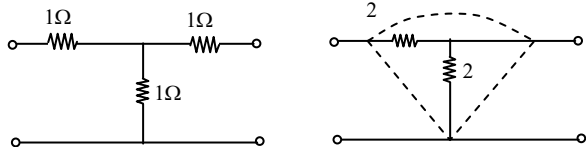
Fig:B

$$Y_B = \begin{bmatrix} S & -S \\ -S & S \end{bmatrix}$$

$$Y = \begin{bmatrix} S + \frac{2}{3} & -S - \frac{1}{3} \\ -S - \frac{1}{3} & S + \frac{2}{3} \end{bmatrix} \text{ mho}$$

04.

Sol:



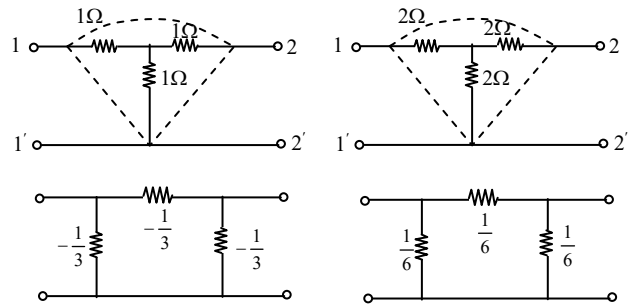
$$Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \quad Y_B = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & -\frac{2}{3} \\ -\frac{2}{3} & \frac{4}{3} \end{bmatrix}$$

05.

Sol: Convert Y to Δ :

Convert Y to Δ :



$$Y_A = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix} \text{ mho} \quad Y_B = \begin{bmatrix} \frac{2}{6} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{2}{6} \end{bmatrix} \text{ mho}$$

$$Y = \begin{bmatrix} \frac{6}{6} & -\frac{3}{6} \\ -\frac{3}{6} & \frac{6}{6} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 1 \end{bmatrix}$$

06.

Sol:

$$T_1 = T_2 = \begin{bmatrix} 1 + \frac{1}{-j1} & 1 \\ \frac{1}{-j1} & 1 \end{bmatrix} = \begin{bmatrix} 1 + j & 1 \\ j & 1 \end{bmatrix}$$

$$T_3 \Rightarrow Z_1 = 1\Omega; Z_2 = \infty$$

$$T_3 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$



$$T = (T_1)(T_2)(T_3)$$

$$T = \begin{bmatrix} j3 & 2+j4 \\ -1+j2 & j3 \end{bmatrix}$$

07.

Sol: $T_1 : Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$T_1 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$T_2 : Z_1 = 0 ; Z_2 = 2 \Omega$$

$$T_2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$T = [T_1] [T_2] \Rightarrow T = \begin{bmatrix} 3.5 & 3 \\ 2 & 2 \end{bmatrix}$$

08. **Ans: (a)**

Sol: For $I_2 = 0$ (O/P open), the Network is shown in Fig.1

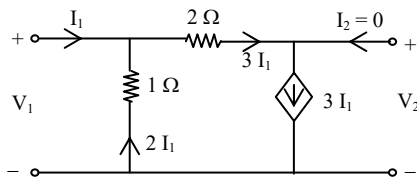


Fig. 1

$$V_1 = -2 I_1 \dots\dots\dots (1)$$

$$Z_{11} = \frac{V_1}{I_1} = -2$$

$$V_2 = -6 I_1 + V_1 \dots\dots\dots (2)$$

From (1) and (2)

$$V_2 = -6 I_1 - 2 I_1$$

$$\text{or } V_2 = -8 I_1$$

$$Z_{21} = \frac{V_2}{I_1} = -8$$

For $I_1 = 0$ (I/P open), the network is shown in Fig.2

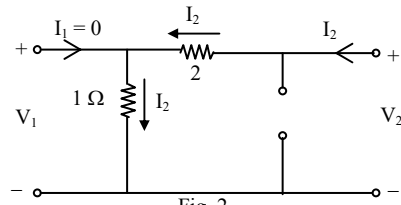


Fig. 2

Note: that the dependent current source with current $3 I_1$ is open circuited.

$$V_1 = 1 I_2, \quad Z_{12} = \frac{V_1}{I_2} = 1$$

$$V_2 = 3 I_2, \quad Z_{22} = \frac{V_2}{I_2} = 3$$

$$[Z] = \begin{bmatrix} -2 & 1 \\ -8 & 3 \end{bmatrix}$$

09.

Sol: By Nodal

$$-I_1 + V_1 - 3V_2 + V_1 + 2V_1 - V_2 = 0$$

$$-I_2 + V_2 + V_2 - 2V_1 = 0$$

$$Y = \begin{bmatrix} 4 & -4 \\ -3 & 2 \end{bmatrix} \text{U}$$

$$[Z] = Y^{-1}$$

We can also obtain [g], [h], [T] and $[T]^{-1}$ by re-writing the equations.

10.

Sol: The defining equations for open-circuit impedance parameters are:

$$V_1 = Z_{11}I_1 + Z_{12}I_2; \quad V_2 = Z_{21}I_1 + Z_{22}I_2$$

In this case, the individual Z-parameter matrices get added.



$$(Z) = (Z_a) + (Z_b) \Rightarrow [Z] = \begin{bmatrix} 10 & 2 \\ 2 & 7 \end{bmatrix} \Omega$$

11.

Sol: For this case the individual y-parameter matrices get added to give the y-parameter matrix of the overall network.

$$Y = Y_a + Y_b$$

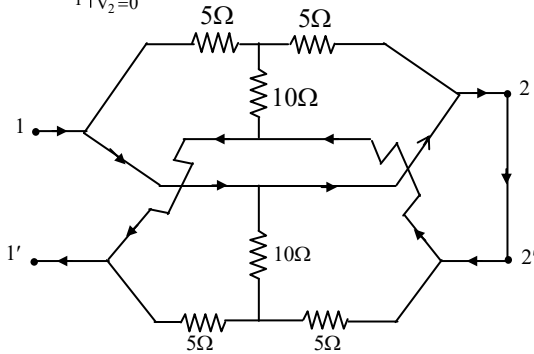
The individual y-parameters also get added

$$Y_{11} = Y_{11a} + Y_{11b} \text{ etc}$$

$$[Y] = \begin{bmatrix} 1.4 & -0.4 \\ -0.4 & 1.4 \end{bmatrix} \text{ mho}$$

12. **Ans: (c)**

$$\text{Sol: } Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}$$



$$Y_{11} = \frac{I_1}{0} = \infty$$

13.

$$\text{Sol. (i). } [T_a] = \begin{bmatrix} 1 + \frac{Z_1}{Z_2} & Z_1 \\ \frac{1}{Z_2} & 1 \end{bmatrix}$$

$$\text{(ii). } [T_a] = \begin{bmatrix} 1 & Z_1 \\ \frac{1}{Z_2} & 1 + \frac{Z_1}{Z_2} \end{bmatrix}$$

$[T_a]$ and $[T_b]$ are obtained by defining equations for transmission parameters.

14.

Sol: In this case, the individual T-matrices get multiplied

$$(T) = (T_1) \times (T_{N1})$$

$$(T) = (T_1)(T_{N1}) = \begin{pmatrix} 1+s/4 & s/2 \\ 1/2 & 1 \end{pmatrix} \begin{pmatrix} 8 & 4 \\ 2 & 5 \end{pmatrix} \\ = \begin{pmatrix} 3s+8 & 3.5s+4 \\ 6 & 7 \end{pmatrix}$$

15.

$$\text{Sol: } Z_{in} = R_{in} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{V_2 - 2I_2}{V_2 - 3I_2},$$

$$V_2 = 10(-I_2)$$

$$Z_{in} = R_{in} = \frac{12}{13} \Omega$$

16.

$$\text{Sol: } \left. \frac{V_1}{I_1} \right|_{I_2=0} = Z_{11}$$

$$\Rightarrow V_1 = (4 \parallel 4) I_1 |_{I_2=0}$$

$$\Rightarrow Z_{11} = 2\Omega$$

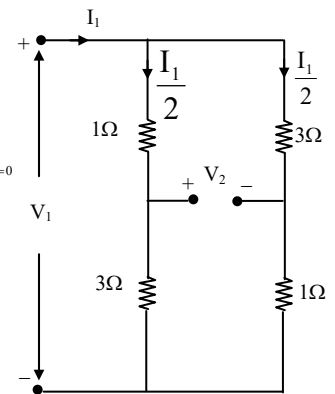
$$V_2 = (4 \parallel 4) I_2 |_{I_1=0}$$

$$\Rightarrow Z_{22} = 2\Omega$$

By KVL \Rightarrow

$$\frac{3I_1}{2} - V_2 - \frac{I_1}{2} = 0$$

$$V_2 = I_1$$





$$\Rightarrow Z_{21} = 1\Omega = Z_{12}$$

$$Z = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Omega$$

$$Y = Z^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{-1}{3} \\ \frac{-1}{3} & \frac{2}{3} \end{bmatrix} \text{U}$$

Now [T] parameters;

$$V_1 = 2I_1 + I_2 \dots\dots\dots (1)$$

$$V_2 = I_1 + 2I_2 \dots\dots\dots (2)$$

$$\Rightarrow I_1 = V_2 - 2I_2 \dots\dots\dots (3)$$

Substituting (3) in (1):

$$V_1 = 2(V_2 - 2I_2) + I_2 = 2V_2 - 3I_2 \dots\dots\dots (4)$$

$$T = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$T^{-1} = T^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Now h parameters

$$2I_2 = -I_1 + V_2$$

$$I_2 = \frac{-I_1 + V_2}{2} \dots\dots\dots (5)$$

Substitute (5) in (1)

$$V_1 = 2I_1 \frac{-I_1 + V_2}{2} + \frac{V_2}{2}$$

$$V_1 = \frac{3}{2}I_1 + \frac{1}{2}V_2 \dots\dots\dots (6)$$

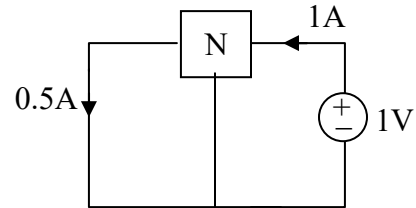
$$h = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{1}{2} \end{bmatrix}$$

$$g = [h]^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

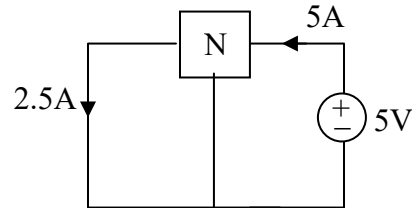
17. Ans: (a)

$$\text{Sol: } Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

just use reciprocity of fig (a)



Now use Homogeneity



$$\text{So, } Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{5}{5} = 1 \text{ mho}$$

This has nothing to do with fig (b) since fig (b) also valid for some specific resistance of 2Ω at port-1, but $Y_{22}, V_1=0$. So S.C port-1

18.

$$\text{Sol: } \frac{V_2}{V_1} = \frac{N_2}{N_1} = n = \frac{-I_1}{I_2}$$

$$\frac{V_2}{V_1} = n$$

$$\Rightarrow V_1 = \frac{1}{n}V_2 - (0)I_2$$

$$\Rightarrow T = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & n \end{bmatrix}$$

$$T^{-1} = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$



$$T^1 = T^{-1} = \begin{bmatrix} n & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

Now h-parameters

$$V_1 = (0)I_1 + \frac{1}{n}V_2$$

$$I_2 = \frac{-I_1}{n} + (0)V_2$$

$$h = \begin{bmatrix} 0 & \frac{1}{n} \\ \frac{-1}{n} & 0 \end{bmatrix}$$

$$g = \begin{bmatrix} 0 & -n \\ n & 0 \end{bmatrix}$$

Note: In an ideal transformer, it is impossible to express V_1 and V_2 in terms of I_1 and I_2 , hence the 'Z' parameters do not exist. Similarly, the y-parameters.

19. Ans: (c)

Sol: $Z_{22} = \left. \frac{V_2}{I_2} \right|_{V_1=0}$

$$\frac{V_1}{V_2} = \frac{1}{n} = \frac{I_2}{I_1}$$

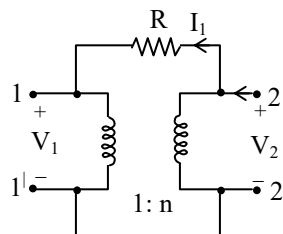
$$V_1 = \frac{1}{n}V_2$$

$$\frac{V_2 - V_1}{R} = I_1$$

$$I_2^1 = I_2 + I_1$$

$$\frac{1}{n} = \frac{I_2}{I_1} = \frac{I_2^1 - I_1}{I_1} = \frac{I_2^1}{I_1} - 1$$

$$\frac{I_2^1}{I_1} = \frac{1}{n} + 1 = \frac{1+n}{n}$$



$$I_2^1 = \left(\frac{1+n}{n} \right) I_1$$

$$I_2^1 = \left(\frac{1+n}{n} \right) \left(\frac{V_2 - V_1}{R} \right)$$

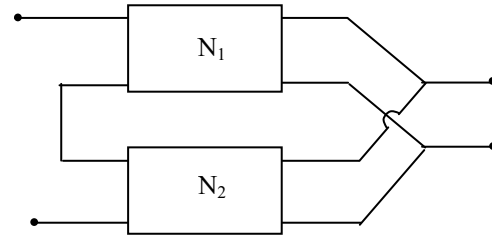
$$I_2^1 = \left(\frac{1+n}{n} \right) \left(\frac{V_2 - \frac{1}{n}V_2}{R} \right)$$

$$\frac{I_2^1}{V_2} = \left(\frac{1+n}{n} \right) \left(\frac{n-1}{nR} \right)$$

$$\frac{V_2}{I_2^1} = \frac{n^2 R}{n^2 - 1}$$

20.

Sol:



For series parallel connection individual h-parameters can be added.

∴ For network 1, $h_1 = g_1^{-1}$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

For network 2, $h_2 = g_2^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\therefore h = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

∴ overall g-parameters,

$$g = h^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$



$$g = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

7. Graph Theory

01. Ans: (c)

Sol: $n > \frac{b}{2} + 1$

Note: Mesh analysis simple when the nodes are more than the meshes.

02. Ans: (c)

Sol: Loops = $b - (n-1) \Rightarrow \text{loops} = 5$
 $n = 7$
 $\therefore b = 11$

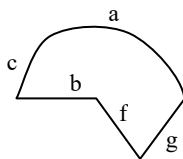
03. Ans: (a)

04.

Sol: Nodal equations required = f-cut sets
 $= (n-1) = (10-1) = 9$
 Mesh equations required = f-loops
 $= b - n + 1 = 17 - 10 + 1 = 8$
 So, the number of equations required
 $= \text{Minimum (Nodal, mesh)} = \text{Min}(9, 8) = 8$

05. Ans: (c)

Sol: not a tree (Because trees are not in closed path)



06. Ans: (a)

07.

Sol: For a complete graph ;

$$b = n_{C_2} \Rightarrow \frac{n(n-1)}{2} = 66$$

$$n = 12$$

$$\text{f-cut sets} = (n-1) = 11$$

$$\text{f-loops} = (b-n+1) = 55$$

$$\text{f-loop} = \text{f-cutset matrices} = n^{(n-2)}$$

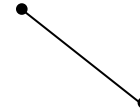
$$= 12^{12-2} = 12^{10}$$

08. Ans: (a)

Sol: Let $N=1$

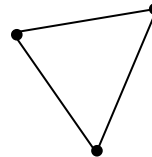
Nodes=1, Branches = 0 ; f-loops = 0

Let $N=2$



Nodes = 2; Branches = 1; f-loop= 0

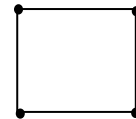
Let $N=3$



Nodes = 3; Branches = 3; f-loop = 1

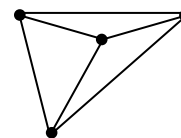
$$\Rightarrow \text{Links} = 1$$

Let $N = 4$



Nodes=4; Branches = 4; f-loops=Links=1

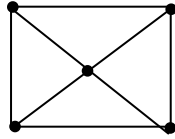
Still $N = 4$





Branches = 6; f-loops = Links = 3

Let $N = 5$

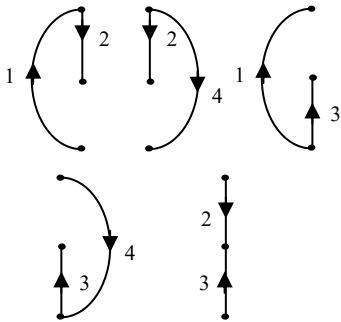


Nodes = 5; Branches = 8; f-loops = Links = 4 etc

Therefore, the graph of this network can have at least "N" branches with one or more closed paths to exist.

09. Ans: (b)

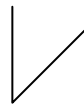
Sol:



10. Ans: (d)

Sol:

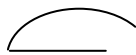
(a) 1,2,3,4 →



(b) 2,3,4,6 →



(c) 1,4,5,6 →



(d) 1,3,4,5 →



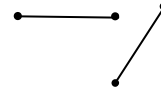
11. Ans: (b)

Sol: $m = b - n + 1 = 8 - 5 + 1 = 4$

12. Ans: (d)

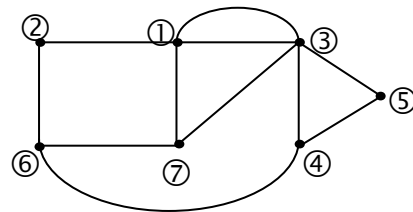
13. Ans: (d)

Sol: The valid cut-set is (1,3,4,6)



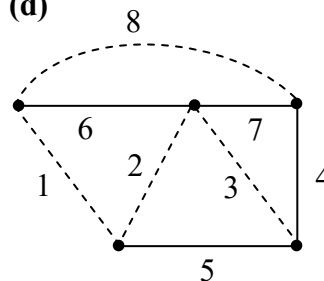
14. Ans: (b)

Sol:



15. Ans: (d)

Sol:



Fundamental loop should consist only one link, therefore option (d) is correct.

8. Passive Filters

01.

Sol:

$$\left. \begin{aligned} \omega = 0 &\Rightarrow V_0 = V_i \\ \omega = \infty &\Rightarrow V_0 = 0 \end{aligned} \right\} \Rightarrow \text{Low pass filter}$$



02.

Sol: $\omega=0 \Rightarrow V_0 = \frac{V_i R_2}{R_1 + R_2}$

“ V_0 ” is attenuated $\Rightarrow V_0 = 0$

$\omega = \infty \Rightarrow V_0 = V_i$

It represents a high pass filter characteristics.

03.

Sol: $H(s) = \frac{V_i(s)}{I(s)} = \frac{S^2 LC + SRC + 1}{SC}$

Put $s = j\omega$ $= -\frac{\omega^2 LC + j\omega RC + 1}{j\omega C}$

$\omega=0 \Rightarrow H(s)=0$

$\omega=\infty \Rightarrow H(s)=0$

It represents band pass filter characteristics

04.

Sol: $\omega = 0 \Rightarrow V_0 = 0$

$\omega = \infty \Rightarrow V_0 = 0$

It represents Band pass filter characteristics

05.

Sol: $\omega = 0 \Rightarrow V_0 = 0$

$\omega = \infty \Rightarrow V_0 = V_i$

It represents High Pass filter characteristics.

06.

Sol: $H(s) = \frac{1}{s^2 + s + 1}$

$\omega = 0 : S = 0 \Rightarrow H(s) = 1$

$\omega = \infty : S = \infty \Rightarrow H(s) = 0$

It represents a Low pass filter characteristics

07.

Sol: $H(s) = \frac{s^2}{s^2 + s + 1}$

$\omega = 0 : S = 0 \Rightarrow H(s) = 0$

$\omega = \infty : S = \infty \Rightarrow H(s) = 1$

It represents a High pass filter characteristics

08.

Sol: $\omega=0; V_0 = V_i$

$\omega=\infty; V_0 = 0$

It represents a low pass filter characteristics.

09.

Sol: $\omega = 0 \Rightarrow V_0 = V_{in}$

$\omega = \infty \Rightarrow V_0 = V_{in}$

It represents a Band stop filter or notch filter.

10.

Sol: $H(s) = \frac{S}{s^2 + s + 1}$

$\omega = 0 : S = 0 \Rightarrow H(s) = 0$

$\omega = \infty : S = \infty \Rightarrow H(s) = 0$

It represents a Band pass filter characteristics

11.

Sol: $H(s) = \frac{S^2 + 1}{s^2 + s + 1}$

$\omega = 0 \Rightarrow S = 0 \Rightarrow H(s) = 1$

$\omega = \infty \Rightarrow S = \infty \Rightarrow H(s) = 1$

It represents a Band stop filter



12.

Sol: $H(s) = \frac{1-s}{1+s}$

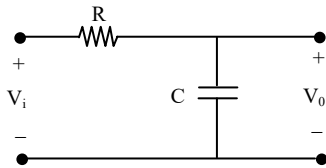
$\omega = 0 \Rightarrow S = 0 \Rightarrow H(s) = 1$

$\omega = \infty \Rightarrow S = \infty \Rightarrow H(s) = -1 = 1 \angle 180^\circ$

It represents an All pass filter

13. Ans: (c)

Sol.



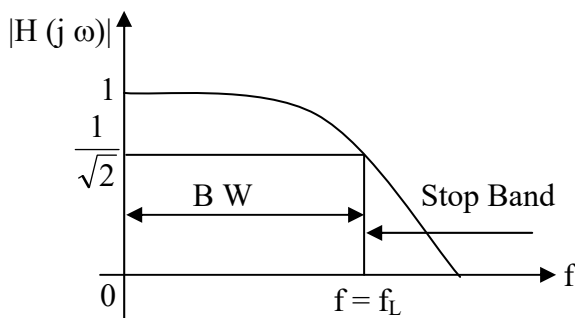
$\omega = 0 \Rightarrow V_0 = V_i$

$\omega = \infty \Rightarrow V_0 = 0$

$$V_0(s) = \left(\frac{V_i(s)}{R + \frac{1}{sC}} \right) \left(\frac{1}{sC} \right)$$

$$\frac{V_0(s)}{V_i(s)} = H(s) = \frac{1}{sCcR + 1}$$

$$H(j\omega) = \frac{1}{1 + j\omega c R} = \frac{1}{1 + j \frac{f}{f_L}}$$



Where $f_L = \frac{1}{2\pi RC}$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_L}\right)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{f}{f_L}\right)$$

$f = 0 \Rightarrow \phi = 0^\circ = \phi_{\min}$

$f = f_L \Rightarrow \phi = -45^\circ = \phi_{\max}$

14. Ans: (b)

15. Ans: (d)

16. Ans: (a)

17. Ans: (c)

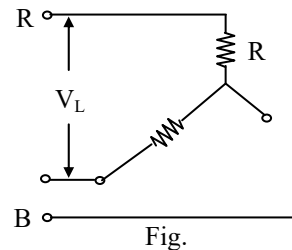
9. Three Phase Circuits

01. Ans: (c)

Sol: $Z_p(\text{star}) = \frac{9 \angle 30^\circ \ 9 \angle 30^\circ}{27 \angle 30^\circ} = 3 \angle 30^\circ \Omega$

02. Ans: (c)

Sol:



Let V_L be the line to line voltage $V_p = \frac{V_L}{\sqrt{3}}$

Let the total power in star connected load with phase resistance as R be P_1

$$P_1 = 3 \frac{V_p^2}{R} = 3 \frac{V_L^2}{3R} = \frac{V_L^2}{R}$$

When one of the phase resistance is removed, the relevant star load is shown in Fig.

Power in this star load



$$= P_2 = 2 \left(\frac{V_L}{2} \right)^2 \frac{1}{R} = \frac{V_L^2}{2R}$$

$$\therefore \frac{P_2}{P_1} = 50\%$$

03. Ans: (d)

Sol: $I_n = 15\angle 0^\circ + 15\angle -120^\circ + 15\angle -240^\circ = 0$

04. Ans: (a)

05.

Sol: The circuit is redrawn with switch open as shown in Fig.1

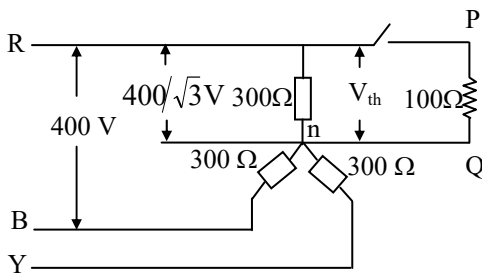


Fig.1

Open circuit voltage, when the switch is open = Thevenin voltage

Phase voltage, $V_{Rn} = \frac{400}{\sqrt{3}}$ V

To find Thevenin's equivalent impedance short circuit the voltage sources (Fig. 2 & 3)

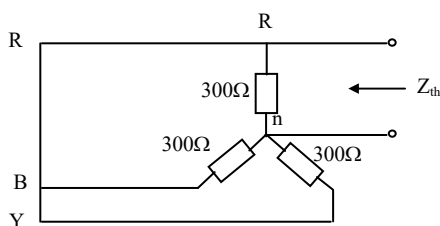


Fig.2

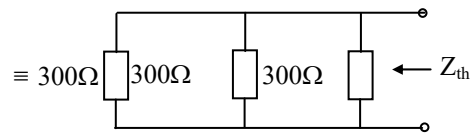


Fig.3

$$\therefore Z_{th} = \frac{300}{3} = 100 \Omega$$

\therefore Thevenin's equivalent circuit across R, n is shown in Fig. 4 with the switch closed and 100 Ω load across P, Q

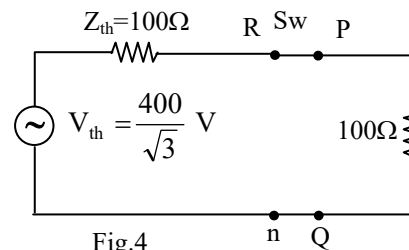


Fig.4

\therefore RMS value of voltage across 100 Ω resistor = $\frac{400}{2\sqrt{3}}$ V = 115.5 V

06.

Sol:

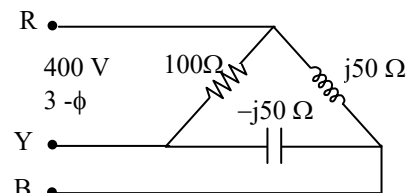


Fig.1

The unbalanced load is shown in Fig. 1. Power is consumed only in 100 Ω resistor.

Power consumed in the delta connected unbalanced load shown in Fig.1 is given by

$$P_1 = \frac{V_{ph}^2}{R} = \frac{(400)^2}{100} = 1600 \text{ W}$$



The star connected load with 'R_x' in each phase is shown in Fig.2.

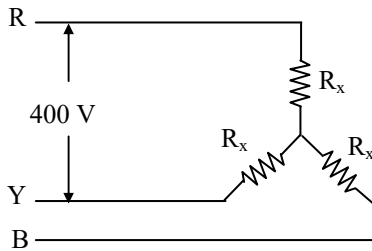


Fig.2

Power consumed in balanced star connected load as in Fig.2 is

$$P_2 = 3 \times \left[\frac{\left(\frac{400}{\sqrt{3}} \right)^2}{R_x} \right] = \frac{400^2}{R_x}$$

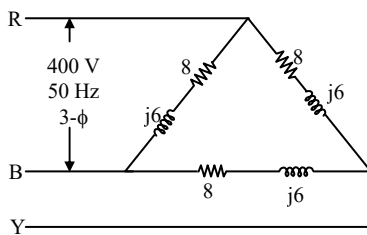
But given $P_1 = P_2$

$$\therefore 1600 = \frac{400^2}{R_x}$$

$$\therefore R_x = \frac{400 \times 400}{1600} = 100 \Omega$$

07. Ans: (b)

Sol:



Power factor angle of load (ϕ)

$$= \tan^{-1} \left(\frac{6}{8} \right) = 36.86^\circ$$

Active power consumed by the delta connected balanced load as in Fig. is

$$P = 3 \times V_{ph} \times I_{ph} \times \cos \phi$$

$$= 3 \times 400 \times \frac{400}{\sqrt{8^2 + 6^2}} \times \cos 36.86 = 38400 \text{ W}$$

Reactive power consumed by the delta connected load is

$$\begin{aligned} Q_1 &= 3 \times V_{ph} \times I_{ph} \times \sin \phi \\ &= 3 \times 400 \times \frac{400}{\sqrt{8^2 + 6^2}} \times \sin 36.86 \\ &= 28800 \text{ VAR} \end{aligned}$$

Active power consumption remains same even after capacitor bank is connected

Reactive power consumed by the delta connected load at a powerfactor of 0.9

$$\begin{aligned} Q_2 &= \frac{P}{0.9} \times \sin(\cos^{-1} 0.9) \\ &= \frac{38400}{0.9} \times \sin 25.84 = 18597.96 \text{ VAR} \end{aligned}$$

$$\therefore Q_2 = 18597.96 \text{ VAR}$$

$$\begin{aligned} \therefore \text{Reactive power supplied by star} \\ \text{connected capacitor bank} &= Q_1 - Q_2 \\ &= 28800 - 18597.96 \\ &= 10202.04 \approx 10.2 \text{ kVAR} \end{aligned}$$

08. Ans: (d)

Sol: The rating of star connected load is given as

$$12\sqrt{3} \text{ kVA, } 0.8 \text{ p.f (lag)}$$

Active power consumed by the load,

$$P = 12\sqrt{3} \times 0.8 \times 10^3 = 16.627 \text{ kW}$$

Reactive power consumed by the load

$$= 12\sqrt{3} \times \sin(\cos^{-1} 0.8) \times 10^3$$

$$Q_1 = 12.47 \text{ kVAR}$$

Reactive power consumed by the load at unity power factor is

$$Q_2 = \frac{P}{(1)} \times \sin(\cos^{-1} 1) = 0$$

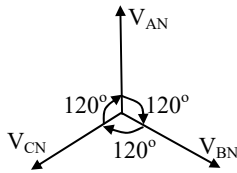
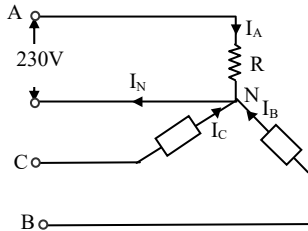


\therefore kVAR to be supplied by the delta connected capacitor bank = $Q_1 - Q_2$

$$Q_C = 12.47 \text{ kVAR}$$

09. Ans: (b)

Sol:



Assume V_{AN} as reference

$$V_{AN} = 230 \angle 0^\circ$$

$$V_{BN} = 230 \angle -120^\circ$$

$$V_{CN} = 230 \angle +120^\circ$$

$$\frac{V_{AN}^2}{R} = 4000 \Rightarrow R = \frac{230^2}{4000} = 13.225 \Omega$$

$$I_A = \frac{V_{AN}}{R} = \frac{230}{13.225} = 17.3913 \text{ A}$$

$$\therefore I_A = 17.3913 \angle 0^\circ \text{ A}$$

Given neutral current $I_N = 0$

$$\Rightarrow I_A + I_B + I_C = 0$$

$$\Rightarrow I_B + I_C = -(I_A)$$

$$I_B + I_C = -17.3913$$

$$\Rightarrow \frac{V_{BN}}{Z_B} + \frac{V_{CN}}{Z_C} = -17.3913$$

$$\Rightarrow \frac{230 \angle -120^\circ}{Z_B} + \frac{230 \angle +120^\circ}{Z_C} = -17.3913$$

$$\Rightarrow \frac{230 \angle -120^\circ}{Z_B} + \frac{230 \angle +120^\circ}{Z_C} = 17.3913 \angle 180^\circ$$

Assume that pure capacitor in phase B and pure inductor in phase C we will get

$$\begin{aligned} I_B + I_C &= \frac{230 \angle -120^\circ}{X_C \angle -90^\circ} + \frac{230 \angle +120^\circ}{X_L \angle 90^\circ} \\ &= \frac{230 \angle -30^\circ}{X_C} + \frac{230 \angle +30^\circ}{X_L} \end{aligned}$$

When we add the two phasors I_B and I_C with angles -30° and $+30^\circ$ we will get the resultant vector with the angle between -30° and $+30^\circ$

But,

$$I_B + I_C \text{ should be equal to } 17.3913 \angle 180^\circ$$

Which has angle of 180°

\therefore We have taken wrong assumption

\therefore Now take pure inductor in phase B and pure capacitor in phase C we will get

$$\begin{aligned} I_B + I_C &= \frac{230 \angle -120^\circ}{X_L \angle 90^\circ} + \frac{230 \angle +120^\circ}{X_C \angle -90^\circ} \\ &= \frac{230 \angle -210^\circ}{X_L} + \frac{230 \angle +210^\circ}{X_C} \\ &= \frac{230}{(2\pi \times 50 \times L)} \angle -210^\circ + \frac{230}{\left(\frac{1}{(2\pi \times 50 \times C)}\right)} \angle +210^\circ \end{aligned}$$

$$= \frac{0.7321}{L} \angle -210^\circ + 72256.63 \times C \angle +210^\circ$$

\therefore From the given options by substituting $L = 72.95 \text{ mH}$ and $C = 139.02 \mu\text{F}$ we will get $I_B + I_C \approx 17.3913 \angle 180^\circ$

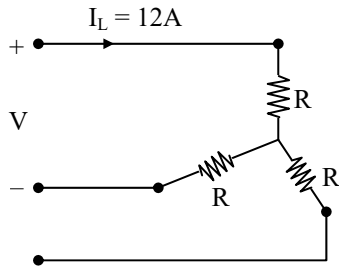
$L = 72.95 \text{ mH}$ in phase B and $C = 139.02 \mu\text{F}$ in phase C should be placed.



10. Ans: (c)

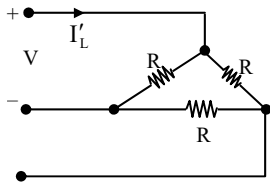
11. Ans: (d)

Sol:

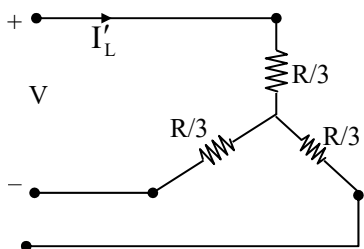


$$\Rightarrow I_L = \frac{\left(\frac{V}{\sqrt{3}}\right)}{R} \Rightarrow \frac{V}{\sqrt{3}R} = 12A$$

Now if the same resistances are connected in delta across the same supply



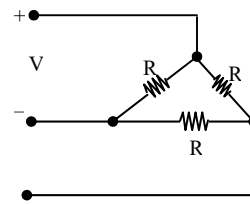
Transforming Δ into equivalent Y



$$\Rightarrow I'_L = \frac{\left(\frac{V}{\sqrt{3}}\right)}{\left(\frac{R}{3}\right)} = \frac{3V}{\sqrt{3}R} = 3(12) = 36 A$$

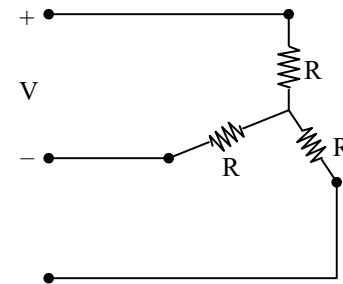
12. Ans: (b)

Sol: Assume the resistances are equal



$$\Rightarrow P_{\text{absorbed } \Delta} = 3 \frac{V^2}{R} = 60 \text{ kW} \dots\dots(1)$$

Now, if the resistors are connected in star,



$$\Rightarrow P_{\text{absorbed Y}} = 3 \frac{\left(\frac{V}{\sqrt{3}}\right)^2}{R} = 3 \times \frac{V^2}{3R} = \frac{V^2}{R}$$

From equation(1), $\Rightarrow \frac{V^2}{R} = 20 \text{ kW}$

$$\therefore P_{\text{absorbed Y}} = \frac{V^2}{R} = 20 \text{ kW}$$

10. Synthesis of Passive Networks

01. Ans: (c)

Sol: $F(s) = \frac{(s+2)}{(s+1)(s+3)}$

The given $F(s)$ has pole-zero structure as P-Z-P-Z alternating on the negative real axis of the s -plane, with a pole nearest the origin at $s = -1$ and a zero at $s = -2$. This $F(s)$



corresponds to RC impedance or RL admittance.

02. Ans: (b)

Sol: For RC and RL driving point functions, the poles and zeros should alternate on the negative real axis, where as for LC driving point functions the poles and zeros should alternate the imaginary axis.

03. Ans: (c)

F(s)		Type of F(s)
A. $\frac{(s^2 - s + 4)}{s^2 + s + 4}$	zeros in the right half plane	Non-minimum phase (2)
B. $\frac{(s + 4)}{s^2 + 3s - 4}$	poles in the right half plane	Unstable (4)
C. $\frac{s + 4}{s^2 + 6s + 5}$	Poles and zeros alternate on the negative real axis with first critical frequency near the origin as a pole.	RC impedance (3)
D. $\frac{s^3 + 3s}{s^4 + 2s^2 + 1}$	multiple poles on the imaginary axis	Non-positive real (1)

04. Ans: (b)

Sol: Remember that parallel LC networks in cascade is Foster – I form and series LC networks in shunt is Foster – II form. Ladder NW with series elements as inductors and shunt elements as capacitors is Cauer-I form and the ladder NW with capacitors as series elements and inductors as shunt elements is

Cauer – II form. The given circuit in this question is Foster-I form.

05. Ans: (c)

Sol: Given : $Z(s) = \frac{s(s^2 + 1)}{s^2 + 4}$

Location of Poles : $s = \pm j2$

Location of Zeros : $s = 0, \pm j1$

Poles and Zeros are simple and lie on the imaginary axis, but they do not alternate.

Hence the given Z(s) is not realizable.

06. Ans: (b)

Sol: Poles and zeros of driving point function [Z(s) or Y(s)] of LC network are simple and alternate on the $j\omega$ axis.

07. Ans: (c)

Sol: $V = I Z(s)$

$$V = I \sqrt{\frac{\omega^2 + \alpha^2}{\omega^2 + \beta^2}} \angle \tan^{-1}\left(\frac{\omega}{\alpha}\right) - \tan^{-1}\left(\frac{\omega}{\beta}\right)$$

voltage load the current

$$\tan^{-1}\left(\frac{\omega}{\beta}\right) < \tan^{-1}\left(\frac{\omega}{\alpha}\right) < \frac{\omega}{\alpha} (\alpha < \beta) (\beta > \alpha)$$

08. Ans: (d)

09. Ans: (b)

Sol: $s = -1 \pm j$

$$(s + 1) ((s+1)+j) ((s+1) -j)$$

$$(s + 1)^2 + (1)^2 = s^2 + 2s + 2$$

$$Z(s) = \frac{K(s + 3)}{s^2 + 2s + 2}$$



$$Z(0) = \frac{2(3)}{2} = 3 = \frac{3(s+3)}{s^2 + 2s + 2}$$

10. Ans: (d)

Sol:

$$s^2 + 2s)s^2 + 4s + 3(1 = \frac{1}{R}$$

$$\frac{s^2 + 2s}{2s + 3}$$

$$2s + 3)s^2 + 2s(\frac{s}{2} = sL$$

$$\frac{s^2 + \frac{3s}{2}}{2s + 3}$$

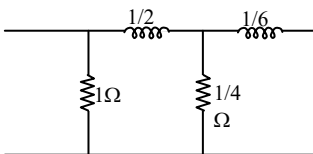
$$\frac{\frac{s}{2}2s + 3(4 = \frac{1}{R}$$

$$2s + 3$$

$$3) \frac{s}{2} (\frac{s}{6} = sL$$

$$\frac{s}{2}$$

$$\frac{2}{0}$$



$$y(s) = \frac{s^2 + 4s + 3}{s^2 + 2s}$$

No. of elements = 4

11. Ans: (b)

Sol:

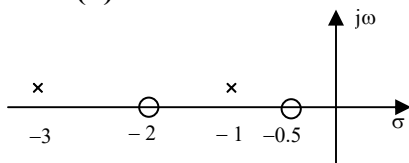


Fig.

$$\text{Given } Y(s) = \frac{s^2 + 2.5s + 1}{s^2 + 4s + 3}$$

$$Y(s) = \frac{(s+0.5)(s+2)}{(s+1)(s+3)}$$

Its pole-zero pattern is shown in Fig.

From the pattern it can be observed that

→ Poles and zeros alternate on the negative real axis of s-plane.

→ The lowest critical frequency is a zero.

→ From the given $Y(s)$, $Y(0) = 1/3$ and $Y(\infty) = 1$, $Y(0) < Y(\infty)$, $Y(\sigma)$ has +ve slope.

It is an admittance of the RC network, as the above properties are true for RC admittance.

12. Ans: (b)

13. Ans: (a)

Sol:

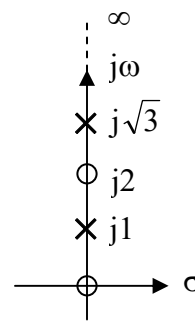


Fig.

$$F(s) = \frac{s(s^2 + 4)}{(s^2 + 1)(s^2 + 6)}$$
 represents an

LC immittance function with pole-zero pattern as shown in Fig. Hence it is p.r.

$$F(s) = \frac{s(s^2 - 4)}{(s^2 + 1)(s^2 + 6)}$$
 is not p.r as it has a

zero in the RH at $s = 2$

$$F(s) = \frac{s^3 + 3s^2 + 2s + 1}{4s}$$
 is not p.r

as the difference in degrees of highest degree terms in $N(s)$ and $D(s)$ is more than 1. For this $F(s)$, difference is 2.



$$F(s) = \frac{s(s^4 + 3s^2 + 1)}{(s + 1)(s + 2)(s + 3)(s + 4)}$$

14. Ans: (a)

Sol: Given $Z(s) = \frac{(s^2 + 4)(s^2 + 16)}{s(s^2 + 9)}$

Out of the given figs., Foster – I form should be either (1) or (4) and Foster –II form should be either (2) or (3). Foster–I form can be confirmed as Fig. 1 by seeing the behavior of Z(s) at $s = \infty$ and $s = 0$.

$$Z(s) = 1 \text{ at } s = \infty, L = 1 \text{ H}$$

$$Z(s) = \frac{64}{9s} \text{ at } s = \infty, C = \frac{9}{64} \text{ F}$$

Foster – II form can be confirmed as fig. (3)

$$\text{as } L = \frac{12}{7} \parallel \frac{12}{5} = 1 \text{ H, at } s = \infty$$

$$\text{and } C = \frac{7}{192} + \frac{5}{48} = \frac{9}{64} \text{ F at } s = 0.$$

The exact realization can be done as shown below. Foster–I form is obtained by expanding the given Z(s) in partial fractions.

$$Z(s) = k_1 s + \frac{k_2}{s} + \frac{k_3 s}{s^2 + 9} = 1s + \frac{64}{9s} + \frac{35}{9} \frac{s}{s^2 + 9}$$

.....(1)

$$\text{As } k_1 = \lim_{s \rightarrow \infty} \frac{Z(s)}{s} = 1$$

$$k_2 = s Z(s) \Big|_{s=0} = \frac{64}{9}$$

$$k_3 = \frac{(s^2 + 9)}{s} Z(s) \Big|_{s^2 = -9} = \frac{(-9 + 4)(-9 + 16)}{-9} = \frac{35}{9}$$

It can be seen from equation (1), the first Foster form corresponds to Fig. I (not Fig. IV) Foster – II form is obtained by taking partial fractions of

$$Y(s) = \frac{s(s^2 + 9)}{(s^2 + 4)(s^2 + 16)}$$

$$= \frac{k_1 s}{(s^2 + 4)} + \frac{k_2 s}{(s^2 + 16)} = Y_1(s) + Y_2(s)$$

$$k_1 = \frac{(s^2 + 4)}{s} Y(s) \Big|_{s^2 = -4} = \frac{-4 + 9}{-4 + 16} = \frac{5}{12}$$

$$k_2 = \frac{(s^2 + 16)}{s} Y(s) \Big|_{s^2 = -16} = \frac{-16 + 9}{-16 + 4} = \frac{7}{12}$$

$$Y_1(s) = \frac{\frac{5}{12} s}{s^2 + 4} = \frac{1}{\frac{12}{5}s + \frac{48}{5s}} = \frac{1}{Ls + \frac{1}{Cs}}$$

$$L = \frac{12}{5} \text{ H, } C = \frac{5}{48} \text{ F}$$

∴ It can be seen that Foster – II form corresponds to Fig. III (not Fig. II) It is instructive to find out the remaining elements in Fig. I and III.

15. Ans: (a)

16. Ans: (d)

Sol: Given:

$$Z_D(s) = \frac{2(s^2 + 1)(s^2 + 3)}{s(s^2 + 2)} = \frac{2s^4 + 8s^2 + 6}{s^3 + 2s}$$

Out of the figs. given (d) is in the form of Cauer-I network and (a) is in the form of Cauer-II. The Cauer network can be confirmed as (d) by seeing the behaviour of

Z(s) at $s = \infty$ and at $s = 0$



$Z(s) = 2$, at $s = \infty$, giving $L = 2 \text{ H}$

$Z(s) = \frac{3}{s}$, at $s \rightarrow \infty$, giving

$$C = \frac{1}{3} \text{ F} = \left(\frac{1}{4} + \frac{1}{12} \right) \text{ F}$$

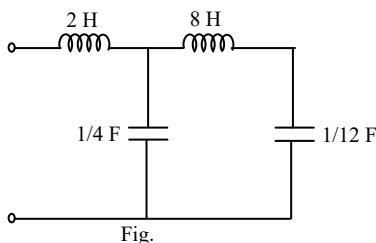
Exact realizations of Cauer – I and Cauer – II forms can be obtained as shown below:

Cauer-I Network is obtained by successive removal of poles at $s = \infty$. As the given $Z_D(s)$ has a pole at $s = \infty$, removal of it gives the first element as $L=2\text{H}$. Follow the Continued Fraction (CF) expansion given below, which confirms to the Network in (d).

$$\begin{array}{r} s^3 + 2s \Big] 2s^4 + 8s^2 + 6 \Big[2s, \\ \underline{2s^4 + 4s^2} \\ 4s^2 + 6 \Big] s^3 + 2s \Big[\frac{1}{4} s, \\ \underline{s^3 + \frac{3}{2}s} \\ \frac{s}{2} \Big] 4s^2 + 6 \Big[8s, \\ \underline{4s^2} \\ 6 \Big] \frac{s}{2} \Big[\frac{s}{12}, \\ \underline{\frac{s}{2}} \\ 0 \end{array}$$

Quotient values

$$L = 2\text{H}, C = \frac{1}{4} \text{ F}, L = 8\text{H}, C = \frac{1}{12} \text{ F}$$

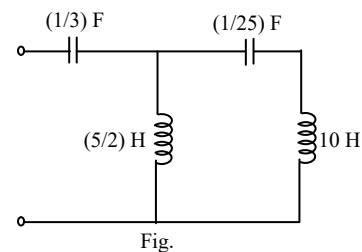


Cauer – II NW is obtained by successive removal of poles at $s = 0$.

$Z_D(s)$ also has a pole at $s=0$, removal of it gives the first element as $C = \frac{1}{3} \text{ F}$.

Follow the CF expansion below.

$$\begin{array}{r} 2s + s^3 \Big] 6 + 8s^2 + 2s^4 \Big[\frac{3}{s}, C = \frac{1}{3} \text{ F} \\ \underline{6 + 3s^2} \\ 5s^2 + 2s^4 \Big] 2s + s^3 \Big[\frac{2}{5s}, L = \frac{5}{2} \text{ H} \\ \underline{2s + \frac{4}{5}s^3} \\ \frac{1}{5}s^3 \Big] 5s^2 + 2s^4 \Big[\frac{25}{s}, C = \frac{1}{25} \text{ F} \\ \underline{5s^2} \\ 2s^4 \Big] \frac{1}{5}s^3 \Big[\frac{1}{10s}, L = 10 \text{ H} \\ \underline{\frac{1}{5}s^3} \\ 0 \end{array}$$



So the answer must be the Cauer – I NW in (d).

It is instructive to find the Cauer – I and Cauer–II structures by completing the CF expansions above



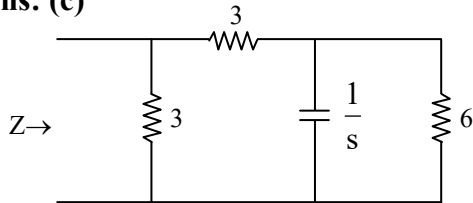
17. Ans: (c)

Sol:

A. $\frac{s^2 - s + 1}{s^2 + s + 1}$	zeros in the right half plane	Non minimum phase (4)
B. $\frac{s^2 + s + 1}{s^2 - s + 1}$	poles in the right half plane	Unstable (3)
C. $\frac{s^2 - 4s + 3}{s^2 + 6s + 8}$	poles and zeros alternate on the negative real axis with first critical frequency as zero.	RL impedance (2)

18. Ans: (c)

Sol:



$$\begin{aligned} &= \frac{6}{6 + \frac{1}{s}} + 3 = 3 + \frac{\frac{6}{s}}{\frac{6s+1}{s}} = \frac{6}{6s+1} + 3 \\ &= \frac{18s+3+6}{6s+1} = \frac{9+18s}{6s+1} = \frac{3\left(\frac{18s+9}{6s+1}\right)}{3 + \frac{18s+9}{6s+1}} \\ &= \frac{3(18s+9)}{18s+3+18s+9} = \frac{3(18s+9)}{36s+12} \\ &= \frac{3 \times 18 \left(s + \frac{1}{2}\right)}{36 \left(s + \frac{1}{3}\right)} = \frac{\left(s + \frac{1}{2}\right)}{s + \frac{1}{3}} \end{aligned}$$

19. Ans: (b)

Sol: $p(s) = s^4 + 2s^2 + 4s + 3$

$$y(s) = \frac{\text{even part}}{\text{odd part}} = \frac{s^4 + 2s^2 + 3}{s^3 + 4s}$$

$$\frac{s^3 + 4s}{s^4 + 4s^2}$$

$$-2s^2 + 3) s^3 + 4s \left(-\frac{s}{2}\right) \Rightarrow -ve \text{ quotients}$$

$$\frac{s^3 + \frac{3s}{2}}{2}$$

$p(s)$ is not Hurwitz

$$Q(s) = s^5 + 3s^2 + s \text{ missing terms}$$

$Q(s)$ is not Hurwitz

20. Ans: (a)

21. Ans: (d)

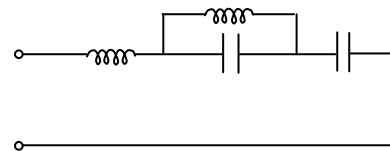
22. Ans: (d)

Sol: Foster – I form consists of LC tank circuits in series to realize $Z_{LC}(s)$.

This form is obtained by taking partial fractions of $Z(s)$.

$$Z(s) = 4 \left[1s + \frac{A}{s} + \frac{Bs}{s^2 + 4} \right]$$

$$n = 3$$



23. Ans: (a)

Sol: Assertion given is the necessary condition for $Y(s)$ to be positive real because the definition of positive real function includes the statement that $Y(s)$ is real for real s .



24. Ans: (d)

Sol: The function $10 \frac{(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)}$ is a valid

reactance function as poles and zeros alternate on the $j\omega$ -axis.

A is false, R is true.

25. Ans: (a)

Sol: For a reactance function, either a pole or zero should occur at the origin.

As $\frac{dX}{d\omega} > 0$, poles and zeros alternate on the $j\omega$ -axis.

26. Ans: (a)

Sol: The poles and zeros of driving point function should be in the left half of the s -plane. A is True.

Only PR function can be realized as the driving point function of a network and PR function has its poles and zeros in the left half of the s -plane. R is True and is the correct explanation of A

27. Ans: (c)

Sol: For a system to be stable, all coefficients of the characteristic polynomial must be positive. This is a necessary condition for stability, but not a sufficient condition.

A is true, R is false.

28. Ans: (a)

Sol: $Z(s) = \frac{k(s^2 + 1)(s^2 + 5)}{(s^2 + 2)(s^2 + 10)}$

For $Z(s)$ to be an LC function, the highest powers of numerator and denominator should differ by 1. For the given $Z(s)$, the highest powers of numerator and denominator are not differing by one. They are same equal to 2.

29. Ans: (a)

Sol: $Q \propto \frac{1}{\xi}$

For circuits with high Q , ξ is less. If damping is less, the real part of the poles are close to the $j\omega$ -axis in the left-half plane.

30. Ans: (a)

Sol:

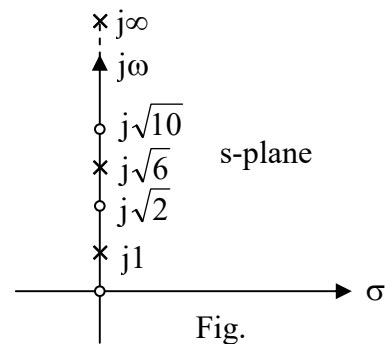


Fig.

$$\text{Given: } Z(s) = \frac{Ks(s^2 + 2)(s^2 + 10)}{(s^2 + 1)(s^2 + 6)}$$

It represents an LC driving point impedance function because it satisfies the property: Poles and zeros interlace on the imaginary axis of the complex s – plane as shown in Fig.

31. Ans: (b)