GATE | PSUs

COMPUTER SCIENCE & INFORMATION TECHNOLOGY

DISCRETE MATHEMATICS

Volume - I : Study Material with Classroom Practice Questions

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Discrete Mathematics

(Classroom Practice Booklet Solutions)

1. Logic

Propositional Logic

01. Ans: (a)

Sol: $(\sim (P \vee Q) \vee (\sim P \wedge Q) \vee P)$

$\Leftrightarrow (\sim P \wedge \sim Q) \vee (\sim P \wedge Q) \vee P$

(By Demorgan's law)

$\Leftrightarrow ((\sim P \wedge \sim Q) \vee (\sim P \wedge Q)) \vee P$

(By Associative law)

$\Leftrightarrow (\sim P \wedge (\sim Q \vee Q)) \vee P$

(By Distributive law)

$\Leftrightarrow (\sim P \wedge T) \vee P$

$(\because \sim Q \vee Q$ is a tautology)

$\Leftrightarrow \sim P \vee P\ (\because \sim P \wedge T \Leftrightarrow \sim P)$

$\Leftrightarrow T$

02. Ans: (c)

Sol: $((P \to Q) \leftrightarrow (\sim P \vee Q)) \wedge R$

$\Leftrightarrow (T) \wedge R \qquad (\because (P \to Q) \Leftrightarrow (\sim P \vee Q))$

$\Leftrightarrow R \qquad (\because T$ is a tautology)

03. Ans: (d)

Sol: In Boolean algebra notation, the given formula can be written as

$\left(\overline{P}.\left(\overline{Q}.R\right)\right) + (Q.R) + (P.R)$

$= \left(\overline{P}.\overline{Q}\right).R + (Q + P).R$

By associative law & distributive law

$= \left(\left(\overline{P}.\overline{Q}\right) + (Q + P)\right).R$

By distributive law

$= \left(\left(\overline{P + Q}\right) + (P + Q)\right).R$

By Demorgan's law & commutative law

$= 1\ .R$

$= R$

04. Ans: (d)

Sol: (a) L.H.S $\Leftrightarrow A \to (P \vee C)$

$\Leftrightarrow \sim A \vee (P \vee C) \qquad E_{16}$

$\Leftrightarrow (\sim A \vee P) \vee C \qquad$ Associative law

$\Leftrightarrow (A \wedge \sim P) \to C$

E_{16} & Demorgan's law

= R.H.S

(b) L.H.S $= (P \to C) \wedge (Q \to C)$

$\Leftrightarrow (\sim P \vee C) \wedge (\sim Q \vee C) \qquad$ By E_{16}

$\Leftrightarrow (\sim P \wedge \sim Q) \vee C \qquad$ Distributive law

$\Leftrightarrow (P \vee Q) \to C \qquad$ By E_{16}

= R.H.S

(c) $A \to (B \to C)$

$\Leftrightarrow \sim A \vee (\sim B \vee C) \qquad$ By E_{16}

$\Leftrightarrow (\sim A \vee \sim B) \vee C \qquad$ By associative law

$\Leftrightarrow (\sim B \vee \sim A) \vee C \qquad$ By commutative law

$\Leftrightarrow \sim B \vee (\sim A \vee C) \qquad$ By associative law

$\Leftrightarrow B \to (A \to C) \qquad$ By E_{16}

(d) When A is false and B is false, we have LHS is true and RHS is false.

$\therefore$ LHS $\neq$ RHS

05. Ans: (d)

Sol: The truth table of a propositional function in n variables contain 2^n rows. In each row the function can be true or false.

By product rule, number of non equivalent propositional functions (different truth tables) possible $= 2^{(2^n)}$

06. Ans: (c)

Sol: A set of connectives is said to be functionally complete, if equivalent form of every statement formula can be written with those connectives.

(a) $\{\vee, \sim\}$ is functionally complete because the other connectives can be expressed by these two connectives.

$(P \wedge Q) \Leftrightarrow \sim(\sim P \vee \sim Q)$

$(P \rightarrow Q) \Leftrightarrow (\sim P \vee Q)$

$(P \leftrightarrow Q) \Leftrightarrow \sim(\sim(\sim P \vee Q) \vee \sim(\sim Q \vee P))$

(b) $\{\wedge, \sim\}$ is functionally complete

$(P \vee Q) \Leftrightarrow \sim(\sim P \wedge \sim Q)$

$(P \rightarrow Q) \Leftrightarrow \sim(P \wedge \sim Q)$

$(P \leftrightarrow Q) \Leftrightarrow \sim(P \wedge \sim Q) \wedge \sim(Q \wedge \sim P)$

(c) The set $\{\wedge, \vee\}$ is not functionally complete, because, we cannot express 'not' operation using the connectives $\wedge$ and $\vee$

(d) We have, $(p \vee q) \Leftrightarrow (\sim p \rightarrow q)$

$\therefore$ $\{\rightarrow, \sim\}$ is a functionally complete set.

07. Ans: (b)

Sol: Argument I:

This argument is not valid, because it comes under fallacy of assuming the converse.

Argument II:

This argument is valid, by the rule of modus tollens.

Argument III:

Let P : It rains and Q : Erik is sick

In symbolic form the argument is

$P \rightarrow Q$

$\underline{\sim P}$

$\therefore \sim Q$

This argument is not valid, because when P is false and Q true, the premises are true but conclusion is false.

08. Ans: (d)

Sol: (a) The given formula is equivalent to the following argument

(1) $(a \wedge b) \rightarrow c$

$\underline{(2)\ (a \rightarrow b)}$

$\therefore (a \rightarrow c)$

Proof:

(3) a new premise to apply conditional proof

(4) b (2), (3), modus ponens

(5) $(a \wedge b)$ (3), (4), conjunction

(6) c (1), (5), modus ponens

$\therefore$ The argument is valid (c.p)

(b) The argument is

(1) $\sim(a \wedge b)$

(2) $(b \vee c)$

(3) $\underline{(c \to d)}$

$\therefore (a \to d)$

(4) a new premise to apply C.P

(5) ~ b (1), (4), conjunctive syllogism(c.s)

(6) c (2), (5), D.S

(7) d (3), (6), M.P

$\therefore$ The given argument is valid (conditional proof)

(c) The given formula is equivalent to the following argument

(1) a

$\underline{(2)\ (a \to b) \vee (c \wedge d)}$

$\therefore (\sim b \to c)$

Proof:

(3) ~ b now premise to apply c.p

(4) $(a \wedge \sim b)$ (1), (3), conjunction

(5) $\sim(a \to b)$ (4), E_{17}

(6) $(c \wedge d)$ (2), (5), D.S

(7) c (6), simplification

$\therefore$ The argument is valid (c.p)

(d) When a is false, b is true and c is false, the given formula has truth value false.

$\therefore$ It is not valid

09. Ans: (b)

Sol: From the truth table

$(x * y) \Leftrightarrow (x \wedge \sim y)$

$(p \vee q) \Leftrightarrow \sim(\sim p \wedge \sim q)$

$\Leftrightarrow \sim(\sim p * q)$

10. Ans: (d)

Sol: If $\{(a \to b) \to (a \to c)\}$ has truth value false, then 'a' has truth value true, b has truth value true, and c has truth value false.

For these truth values, only the compound proposition given in option (d) has truth value true.

11. Ans: (c)

Sol: Let us denote the formula by $P \to Q$,

Where $P = (a \wedge b) \to c$ and $Q = a \to (b \vee c)$.

(a) Here Q is false only when a is *true*, b is *false*, and c is *false*. For these truth values P has truth value *true*.

$\therefore$ $(P \to Q)$ has truth value *false*.

$\therefore$ The given formula is not a tautology.

(b) When a is *true*, b is *true* and c is *false*; $(P \to Q)$ has truth value *true* .

$\therefore$ The given formula is not a contradiction

(c) The given formula true in one case and false in other cases

$\therefore$ The given formula is a contingency.

12. Ans: (b)

Sol: We have

$(p \leftrightarrow q) \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$

$\therefore$ Option (b) is true .

13. Ans: (a)

Sol: Argument I

1. $\sim p \rightarrow (q \rightarrow \sim w)$	premise
2. $\sim s \rightarrow q$	premise
3. $\sim t$	premise
4. $(\sim p \vee t)$	premise
5. $\sim p$	(3),(4), disjunctive syllogism
6. $q \rightarrow \sim w$	(1), (5), modus ponens
7. $(\sim s \rightarrow \sim w)$	(2), (6), transitivity
8. $(w \rightarrow s)$	(7), contrapositive property

$\therefore$ Argument I is valid.

Argument II

We can not derive the conclusion from the premises, by applying the rules of inference. Further, when p, q, r, t and w has truth values true, we have all the premises are true but conclusion is false.

$\therefore$ Argument II is not valid

14. Ans: (d)

Sol: (a) L. H. S $\Leftrightarrow \sim a \vee (\sim b \vee c)$ Equivalence

$\Leftrightarrow (\sim a \vee \sim b) \vee c$ Associativity

$\Leftrightarrow (a \wedge b) \rightarrow c$ Equivalence

= R. H. S.

(b) L. H. S $\Leftrightarrow (a \wedge b) \rightarrow (\sim c \rightarrow d)$

Equivalence

$\Leftrightarrow (a \wedge b \wedge \sim c) \rightarrow d$ Equivalence

= R. H. S

(c) L. H. S. $= (a \rightarrow (a \vee b)) = T$

R. H. S. $= \sim a \rightarrow (a \rightarrow b)$

$= a \vee (\sim a \vee b)$

$= (a \vee \sim a) \vee b$

$= T \vee b$

$= T$

$\therefore$ L. H. S = R. H. S

(d) When a is false and b is true

We have L. H. S. = F and R. H. S. = T

$\therefore$ Option (d) is not true because

L. H. S. $\neq$ R. H. S.

15. Ans: (a)

Sol: The given formula is equivalent to the following argument

1. $\sim p \rightarrow (q \rightarrow \sim w)$	Premise
2. $\sim s \rightarrow q$	Premise
3. $\sim (w \rightarrow t)$	Premise
4. $\underline{(\sim p \vee t)}$	Premise

$\therefore s$

5. $(w \wedge \sim t)$	3), Equivalence
6. w	5), Simplification
7. $\sim t$	5), Simplification
8. $\sim p$	4), 7) Disjunctive syllogism
9. $(q \rightarrow \sim w)$	1), 8) Modus ponens
10. $\sim q$	9), 6) Modus tollens
11. s	2), 10) Modus tollens

$\therefore$ The argument is valid

Hence, the given statement is a tautology

16. Ans: (d)

Sol: (a) When p is true, q is false, and r is true; the premises are true and conclusion is false. Therefore, the argument is not valid.

(b) When p is false and r is true, the premises are true and conclusion is false.

$\therefore$ The argument is not valid.

(c) When p is true, q is true, r is true, and s is false; then the given argument is not valid

(d) The premises are true only when p is true, q is true, r is true, s is false and t is true.

$\therefore$ Whenever the premises are true, the conclusion is also true.

$\therefore$ The argument is valid

17. Ans: (c)

Sol: If $(p \leftrightarrow q)$ is a contradiction, then p and q have different truth values.

S_1 is valid, because $(p \wedge q)$ has truth value false.

S_2 is valid, because $(p \vee q)$ is true

S_3 is not valid, because when p is true and q is false, S_3 has truth value false.

S_4 has truth value 'true', in both the cases, i.e., {p true and q false} or {p false and q true}

$\therefore$ S_4 is valid.

18. Ans: (c)

Sol: Argument I is equivalent to the following argument.

$$\{p \rightarrow r, q \rightarrow r, p \vee q\} \Rightarrow r$$

This argument is valid by the rule of dilemma.

$\therefore$ The given argument is also valid by Conditional proof (C.P).

Argument II is equivalent to the following argument

$$\{p \rightarrow q, p \rightarrow r, p\} \Rightarrow (q \wedge r)$$

1) $p \rightarrow q$	premise
2) $p \rightarrow r$	premise
3) p	new premise to apply (C.P)
$\therefore (q \wedge r)$	
4) q	(1), (3), modus ponens

5) r (2), (3), modus ponens

6) $q \wedge r$ (4), (5), conjunction

$\therefore$ Argument II is valid (Conditional Proof).

19. Ans: (a)

Sol: The given formula can be written as

$((a\to b) \wedge (c\to d) \wedge (\sim b \vee \sim d)) \to (\sim a \vee \sim c)$

This formula is valid, by the rule of destructive dilemma.

First order Logic

20. Ans: (a)

Sol: To negate a statement formula we have to replace $\forall_x$ with $\exists_x$, $\exists_x$ with $\forall_x$ and negate the scope of the quantifiers.

$\sim\{\exists_x \{P(x) \wedge \sim Q(x)\}\} = \forall x \{P(x) \to Q(x)\}$

(Use the equivalence

$\sim (P \to Q) \Leftrightarrow (P \wedge \sim Q))$

21. Ans: (c)

Sol: $\forall_x (B (x) \to I (x))$

$\Leftrightarrow \sim \sim [\forall_x (B(x) \to I (x))]$

$\Leftrightarrow \sim (\exists_x (B(x) \wedge \sim I (x)))$

22. Ans: (a)

Sol: To negate a statement formula we have to replace $\forall_x$ with $\exists_x$, $\exists_x$ with $\forall_x$ and negate the scope of the quantifiers. Use the equivalence

$\sim (P \to Q) \Leftrightarrow (P \wedge \sim Q)$

23. Ans: (d)

Sol: (a) valid (see Relationship diagram in material book)

(b) valid (see Relationship diagram in material book)

(c) valid (see Relationship diagram in material book)

(d) not valid (see Relationship diagram in material book)

24. Ans: (d)

Sol: **S_1 is true**

Once we select any integer n, the integer $m = 5 - n$ does exist and

$n + m = n + (5 - n) = 5$

S_2 is true, because if we choose $n=1$ the statement $nm = m$ is true for any integer m.

S_3 is false, for example, when $m = 0$ the statement is false for all n

S_4 is false, here we can not choose $n = - m$, because m is fixed.

25. Ans: (b) & (d)

Sol: (a) L.H.S $\Leftrightarrow \exists_x (A (x) \to B(x))$

$\Leftrightarrow \exists_x (\sim A(x) \vee B(x))$, E_{16}

$\Leftrightarrow \exists_x \sim A(x) \vee \exists_x B(x)$, E_{23}

$\Leftrightarrow \forall_x A(x) \to \exists_x B(x)$, E_{16}

= R.H.S

(b) L.H.S $\Leftrightarrow \{\forall_x \sim A(x) \vee \forall_x B(x))$

$\Rightarrow \forall_x (\sim A(x) \vee B(x))$

$\Rightarrow \forall_x (A(x) \rightarrow B(x) = $ R.H.S

But converse is not true

$\therefore$ (b) is false

(c) valid equivalence

(d) not valid (converse is not true)

26. Ans: (b)

Sol: (a) The given formula is valid by conditional proof, if the following argument is valid.

(1) $\forall x \{ P(x) \rightarrow Q(x) \}$

$\underline{(2)\ \forall\ x\ P(x)}$ new premise to apply C.P

$\therefore\ \forall x\ Q(x)$

Proof:

(3) $P(a) \rightarrow Q(a)$	(1) , U.S
(4) $P(a)$	(2), U.S
(5) $Q(a)$	(3),(4), M.P
(6) $\forall x\ Q(x)$	(5) , U.S

$\therefore$ The given formula is valid (C.P)

(b) The statement need not be true.

Let c and d are two elements in the universe of discourse, such that P(c) is true and P(d) is false and Q(c) is false and Q(d) is false.

Now, the L.H.S of the given statement is true but R.H.S is false.

$\therefore$ The given statement is not valid.

(c) $\forall_x (P(x) \vee Q(x)) \Rightarrow (\forall_x P(x) \vee \exists_x Q(x))$

Indirect proof:

1) $\forall_x (P(x) \vee Q(x))$ Premise

2) $\sim (\forall_x P(x) \vee \exists_x Q(x))$

New premise to apply Indirect proof

3) $\exists_x \sim P(x) \wedge \forall_x \sim Q(x)$

(2), Demorgan's law

4) $\exists_x \sim P(x)$	(3), Simplification
5) $\forall_x \sim Q(x)$	(3), Simplification
6) $\sim P(a)$	(4), E.S
7) $\sim Q(a)$	(5), U.S
8) $(\sim P(a) \wedge \sim Q(a))$	(6), (7),Conjunction
9) $\sim (P(a) \vee Q(a))$	(8), Demorgan's law
10) $(P(a) \vee Q(a))$	(1), U.S
11) F	(9), (10), Conjunction

$\therefore$ valid (Indirect proof)

S_2: The argument is

1) $\forall_x \forall_y (P(x, y) \rightarrow W(x, y))$

$\underline{2) \sim W(a, b)}$

$\therefore \sim P(a, b)$

(d) $\forall_x \{ P(x) \vee Q(x) \}$ follows from $(\forall_x P(x) \vee \forall_x Q(x))$

$\therefore$ The given statement is valid.

27. Ans: (c)

Sol: Consider

Argument I

1. $\forall x\{p(x) \vee q(x)\}$	premise
2. $\forall x[\{\sim p(x) \wedge q(x)\} \rightarrow r(x)]$	premise
3. $\{p(a) \vee q(a)\}$ (1),	universal specification
4. $\{\sim p(a) \wedge q(a)\} \rightarrow r(a)$	(2), U. S.
5. $\sim r(a)$	new premise to apply C.P
6. $\sim\{\sim p(a) \wedge q(a)\}$	(5), (4), disjunctive syllogism
7. $\{ p(a) \vee \sim q(a)\}$	(6), demorgan's law
8. $\{ p(a) \vee q(a)\} \wedge \{p(a) \vee \sim q(a)\}$	(3), (7) conjunction
9. $p(a) \vee \{q(a) \wedge \sim q(a)\}$	(8), distributive law
10. $p(a) \vee F$	from (9)
11. $p(a)$	from (10)
12. $\{\sim r(a) \rightarrow p(a)\}$	from (11),
13. $\forall x\{\sim r(x) \rightarrow p(x)\}$	from (12), U.G

$\therefore$ The argument is valid (C.P)

Argument II

1. $\forall x[p(x) \rightarrow \{q(x) \wedge r(x)\}]$	premise
2. $\exists x\{p(x) \wedge s(x)\}$	premise
3. $p(a) \wedge s(a)$	(2), E. S
4. $p(a)$	(3), simplification
5. $s(a)$	(3), simplification
6. $p(a) \rightarrow \{q(a) \wedge r(a)\}$	(1), U.S
7. $q(a) \wedge r(a)$	(4), (6), modus ponens
8. $r(a)$	(7), simplification
9. $r(a) \wedge s(a)$	(5), (8), conjunction
10. $\exists x\{r (x) \wedge s(x)\}$	(9), E. G

$\therefore$ The argument is valid (C.P)

28. Ans: (d)

Sol: The given statement can be represented by S_2.

Further, $S_1 \equiv S_2 \equiv S_3$

$\therefore$ Option (d) is correct

29. Ans: (b)

Sol: $\mathbf{S_1}$ is equivalent to the following argument

1) $\exists_x P(x)$ premise

2) $\exists_x \{P(x) \to Q(x)$ premise

$\therefore \exists_x Q(x)$

Here, we cannot combine (1) and (2), to get the conclusion, because in both the formulae existential quantifiers are used.

$\mathbf{S_2}$ is equivalent to the following argument

1) $\forall_x P(x)$ premise

2) $\forall_x \{P(x) \to Q(x)$ premise

$\therefore \exists_x Q(x)$

3) $\forall_x P(x)$ from (1) and (2), by modus ponens,

4) $\exists_x P(x)$ from (3)

$\therefore S_2$ is valid

$\mathbf{S_3}$**:**

1) $\forall_x P(x)$ premise

2) $\exists_x Q(x)$ premise

$\therefore \exists_x \{(P(x) \wedge Q(x)$

(3) Q(a) (2), E.S.

(4) P(a) (1) U.S.

(5) $P(a) \wedge Q(a)$ (3), (4), conjunction

(6) $\exists_x \{P(x) \wedge Q(x)\}$ (5), U.G.

$\therefore S_3$ is valid

30. Ans: (d)

Sol: I) Let D (x) : x is a doctor

C (x) : x is a college graduate

G (x) : x is a golfer

The given argument can be written as

1) $\forall_x \{D(x) \to C(x)\}$

2) $\exists_x \{D(x) \wedge \sim G(x)\}$

$\therefore \exists_x \{G(x) \wedge \sim C(x)\}$

3) $\{D(a) \wedge \sim G(a)\}$ 2), Existential Specification

4) $\{D(a) \to C(a)\}$ 1), Universal Specification

5) D (a) 3), Simplification

6) $\sim G(a)$	3), Simplification
7) $C(a)$	4), 5), Modus ponens Conjunction
8) $C(a) \wedge \sim G(a)\}$	7) ,6), Conjunction
9) $\exists_x \{G(x) \wedge \sim C(x)\}$	8), Existential Generalization

The argument is not valid

II) Let M (x) = x is a mother

N (x) = x is a male

P (x) : x is a politician

The given argument is

1) $\forall_x \{M(x) \rightarrow \sim N(x)\}$

2) $\exists_x \{N(x) \wedge P(x)\}$

$\therefore \exists_x \{P(x) \wedge \sim M(x)\}$

3) $N(a) \wedge P(a)$	2), Existential Specification
4) $M(a) \rightarrow \sim N(a)$	1), Universal Specification
5) $N(a)$	3), Simplification
6) $P(a)$	3), Simplification
7) $\sim M(a)$	4), 5), Modus tollens
8) $\{P(a) \wedge \sim M(a)\}$	6), 7), Conjunction
9) $\exists_x\{P(x) \wedge \sim M(x)\}$	8), Existential Generalization

$\therefore$ The argument is valid.

31. Ans: (b)

Sol: S_1 is false. For x = 0. There is no integer y such that '0 is a divisor of y'

S_2 is true. If we choose, x = 1, then the statement is true for any integer y

S_3 is true. If we choose, x = 1, then the statement is true for any integer y

S_4 is false, because there is no integer y which is divisible by all integers.

32. Ans: (b)

Sol: $P(x): x^2 - 7x + 10 = 0$

$\Rightarrow x = 2, 5$

$Q(x): x^2 - 2x - 3 = 0$

$\Rightarrow x = -1, 3$

S_1) For x = 3, Q (x) true and R(x) is false.

$\therefore S_1$ is not true.

S_2) For x = –1, Q(x) is true and R(x) is true.

$\therefore S_2$ is true

S_3) For the integer x = 1, P(x) is false

$\therefore P(x) \rightarrow R(x)$ is true for integer

$\therefore S_3$ is true.

33. Ans: (c)

Sol: (a) When y = 2, the given statement is false

(b) When x = 2 the given statement is false

(c) Solving the equations

$2x + y = 5$ and $x - 3y = -8$

we get x = 1 and y = 3

$\therefore$ The given statement is true

(d) Solving the equations

$3x - y = 7$ and $2x + 4y = 3$

we get $x = \frac{31}{14}$ and $y = -\frac{5}{14}$

$\therefore$ The given statement is false

34. Ans: (b)

Sol: Argument I:

1. $\exists_x A(x)$,

2. $\underline{\forall_x \sim\{A(x) \wedge Q(x)}$

$\therefore \exists_x Q(x)$

3. A(a)	from (1), by existential specification
4. $\sim \{A(a) \wedge Q(a)\}$	from (2), by universal specification
5. $(\sim A(a) \vee \sim Q(a))$	(4), demorgan's law
6. $\sim Q(a)$	from (2) and (5) by disjunctive syllogism.

$\therefore$ given argument is not valid.

Argument II:

1. $\{\exists_x \{(P(x) \vee Q(x)) \rightarrow R(x)\}$,

2. $\underline{\forall_x Q(x)\}}$

$\therefore \exists_x R(x)$

3. $\{(P(a) \vee Q(a)) \rightarrow R(a)\}$,	from (1) by existential specification.
4. Q(a)	from (2), by universal specification
5. $P(a) \vee Q(a)$	from (4) by addition
6. R(a)	from (3) and (5) by modus ponens
7. $\exists_x R(x)$	from (6) by existential genralization

$\therefore$ Argument II is valid.

35. Ans: (b)

Sol: (a) 1. $\forall x\ \{P(x) \vee Q(x)\}$ Premise

2. $\exists x \sim Q(x)$ Premise
3. $\sim Q(a)$ (2) Existential Specification (E.S)
4. $\{P(a) \vee Q(a)\}$ (1), Universal Specification (U.S)
5. $P(a)$ (3), (4), Disjunctive Syllogism (D.S)
6. $\exists x\ P(x)$ (5), Existential Generalization (E.G)

$\therefore$ The argument is not valid

(b) 1. $\exists x \sim P(x)$ Premise

2. $\forall x\ \{P(x) \to Q(x)\}$ Premise
3. $\sim P(a)$ (1), E.S
4. $P(a) \to Q(a)$ (2), U.S

From (3) and (4) we cannot derive the conclusion $\sim Q(a)$

$\therefore$ The argument is not valid

(c) 1. $\forall x\ \{P(x) \vee Q(x)\}$ Premise

2. $\forall x\ Q(x)$ Premise
3. $P(a) \vee Q(a)$ (1), U.S
4. $Q(a)$ (2), U.S

From (3) and (4), we cannot derive the conclusion.

$\therefore$ The argument is not valid

(d) 1. $\forall x\ P(x)$ Premise

2. $\exists x \sim \{P(x) \wedge Q(x)\}$ Premise
3. $\sim \{P(a) \wedge Q(a)\}$ (2), E.S
4. $P(a)$ (1), U.S
5. $\sim Q(a)$ (3), (4), Conjunctive Syllogism
6. $\exists x \sim Q(x)$ (5), E.G

$\therefore$ The argument is valid

36. Ans: (c)

Sol: (a) If x is not free then

$\forall x\ \{W \vee A(x)\} \Leftrightarrow W \vee \forall x A(x)$ is valid

Refer page 14 of material book.

(b) L.H.S $\Leftrightarrow \exists x\ \{\sim A(x) \vee W\}$

$\Leftrightarrow \{\exists x \sim A(x)\} \vee \exists x\ W$

$\Leftrightarrow \forall x\ A(x) \rightarrow W$

(c) $\forall x\ \{A(x) \rightarrow W\}$

$\Leftrightarrow \forall x\ \{\sim A(x) \vee W\}$ E_{16}

$\Leftrightarrow \{\forall x \sim A(x)\} \vee W$ using (a)

$\Leftrightarrow \{\exists x\ A(x)\} \rightarrow W$ E_{16}

$\neq$ R.H.S

(d) valid refer derivation of option (c).

2. Combinatorics

01. Ans: 2

Sol:

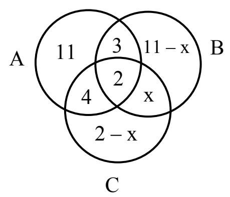
$\{n(A \vee B \vee C)\}$ = sum of the elements in all the regions of the diagram.

$= 33 - x$

$\Rightarrow n(A \vee B \vee C) = 31 = 33 - x$

$\Rightarrow x = 2$

$\therefore n(\overline{A} \wedge B \wedge C) = 2$



02. Ans: (a)

Sol: $n(B \cup P \cup C) = n(B) + n(P) + n(C) - n(B \cap P)$

$-n(P \cap C) - n(B \cap C) + n(B \cap P \cap C)$

$\Rightarrow 34 = 14 + 21 + 13 - n(B \cap P) - n(P \cap C)$

$- n(B \cap C) + 3$

$\Rightarrow n(B \cap P) + n(P \cap C) + n(B \cap C) = 17$

Number of students received awards in exactly two subjects = 17 – 3 – 3 – 3

= 8

03. Ans: (a)

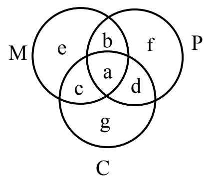
Sol: Number of students received awards in only one subject = 34 – 8–3

= 23

Option (a) is correct.

04. Ans: (d)

Sol:



Given:

$$a + b + c + d = \frac{50}{100}$$

$$b + c + d = \frac{40}{100}$$

$$\text{p. m. c} = a = \frac{50}{100} - \frac{40}{100}$$

$$= \frac{1}{10}$$

$$p + m + c = (a+b+c+d+e+f+g)+(b+c+d)+2a$$

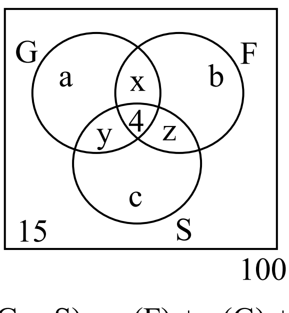
$$= \frac{75}{100} + \frac{40}{100} + \frac{20}{100} = \frac{27}{20}$$

06. Ans: (d)

Sol: I. Number of students who can speak atleast one language = 100 – (number of students who can speak none of the 3 languages)

$= 100 - 15 = 85$

II.



$$n(F \vee G \vee S) = n(F) + n(G) + n(S) - n(F \wedge G) - n(G \wedge S) - n(F \wedge S) + n(F \wedge G \wedge S)$$

$$\Rightarrow 85 = 42 + 42 + 36 - n(F \wedge G) - n(G \wedge S) - n(F \wedge S) + 4$$

$$\Rightarrow n(F \wedge G) + n(G \wedge S) + n(F \wedge S) = 39$$

$$\Rightarrow (x + 4) + (y + 4) + (z + 4) = 39$$

$$\Rightarrow x + y + z = 27$$

Number of students who can speak atleast two languages = $x + y + z + 4$

$= 31$

III. Number of students who can speak exactly two languages = $x + y + z = 27$

IV. Number of students who can speak only one of the 3 languages

$= a + b + c$

$= 85 - (x + y + z + 4)$

$= 85 - 31 = 54$

07. Ans: 18

Sol: $n(3 \wedge 5)$ = Number of integers divisible by 3 and 5

= Number of integers divisible by L.C.M of 3 and 5

$$= \frac{300}{15} = 20$$

$n(3 \wedge 5 \wedge 7)$ = Number of integers in the set divisible by L.C.M of {7, 3, 5}

= Number of integers in the set divisible by 105 = 2

Required number of integers = 20 – 2

= 18

08. Ans: 4096

Sol: In a binary matrix of order 3×4 we have 12 elements. Each element we can choose in 2 ways.

By product rule,

Required number of matrices = $2^{12} = 4096$

09. Ans: 188

Sol: An English movie and a telugu movie can be selected in (6)(8) = 48 ways

A telugu movie and a hindi movie can be selected in (8).(10) = 80 ways

A hindi movie and an English movie can be selected in (10)(6) = 60 movies

Required number of ways = 48 + 80 + 60

= 188

10. Ans: 738

Sol: Number of 1-digit integers = 9

Number of 2-digit integers with distinct digits = (9) (9)

= 81

Number of 3-digit integers with distinct digits = (9).(9).(8)

= 648

Required number of integers = 9+81+648

= 738

11. Ans: 2673

Sol: **Case(i):** If the first digit is 6, then each of the remaining digits we can choose in 9 ways.

Case(ii): If the first digit is not 6, then first digit we can choose in 8 ways, digit 6 can appear in 3 ways and each of the remaining digits we can choose in 9 ways.

Required number of integers

= (9)(9)(9) + (8)(3)(9)(9)

= 2673

12. Ans: 2940

Sol: Consider an integer with 5 digits.

Digit 3 can appear in 5 ways

Digit 4 can appear in 4 ways

Digit 5 can appear in 3 ways

Each of the remaining digits we can choose in 7 ways.

By product rule,

Required number of integers

= (5)(4)(3)(7)(7) = 2940

13. Ans: 89

Sol: Each Tennis match eliminates one player and we have to eliminate 89 players.

$\therefore$ We have to conduct 89 matches.

14. Ans: 243

Sol: Each element of A can appear in the subsets in 3 ways.

Case 1: The element appears in P but does not appear in Q.

Case 2: The element appears in Q and does not appear in P.

Case 3: The element does not appear in P and does not appear in Q.

By product rule,

Required number of ways = $3^n = 3^5 = 243$

15. Ans: 150

Sol: Required number of ways = Number of onto functions possible from persons to rooms

$= 3^5 - C(3, 1)\, 2^5 + C(3, 2) \,.\, 1^5$

$= 243 - 3\,(32) + 3$

$= 150$

16. Ans: P(10,6) = 151200

Sol: Required number of ways

= Number of ways we can map the 6 persons to 6 of the 10 books

= P(10,6)

= 151200

17. Ans: (a)

Sol: The 3 women can speak as a group in $\angle 3$ ways.

The women group can speak with other 4 men in $\angle 5$ ways.

$\therefore$ Required number of ways

$= \angle 5.\ \angle 3$

$= 720$

18. Ans: 2880

Sol: First girls can sit around a circle in $\angle 4$ ways.

Now there are 5 distinct places among the girls, for the 4 boys to sit.

Therefore, the boys can sit in P(5, 4) ways.

By product rule,

Required number of ways = $\angle 4.P(5, 4)$

$= 2880$

19. Ans: 1152

Sol: Consider 8 positions in a row marked 1, 2, 3,….., 8.

Case 1: Boys can sit in odd numbered positions in $\angle 4$ ways and girls can sit in even numbered positions in $\angle 4$ ways.

Case 2: Boys can sit in even numbered positions in $\angle 4$ ways and girls can sit in odd numbered positions in $\angle 4$ ways.

Required number of ways

$= \angle 4.\angle 4 + \angle 4\ \angle 4 = 1152$

20. Ans: 325

Sol: Number of signals we can generate using 1 flag = 5

Number of signals we can generate using two flags = P(5,2) = 5.4 = 20 and so on.

Required number of signals

= 5 + P(5,2) + P(5,3) + P(5,4) + P(5,5)

= 325

21. Ans: 10^6

Sol: Each book we can give in 10 ways.

By product rule, Required number of ways $= 10^6$

22. Ans: 243

Sol: Each digit of the integer we can choose in 3 ways.

By product rule,

Required number of integers $= 3^5$

$= 243$

23. Ans: 12600

Sol: Required number of permutations

$$= \frac{\angle 10}{\angle 2.\angle 3.\angle 4} = 12{,}600$$

24. Ans: 210

Sol: Required number of binary sequences

$$= \frac{\angle 10}{\angle 6.\angle 4} = 210$$

25. Ans: 252

Sol: Required number of outcomes

$$= \frac{\angle 10}{\angle 5.\angle 5} = 252$$

26. Ans: 2520

Sol: Required number of ways

$$= \frac{\angle 10}{\angle 3.\angle 2.\angle 5}$$

$= 2520$

27. Ans: 2520

Sol: Required number of ways

= number of ordered partitions

$$= \frac{\angle 10}{\angle 3.\angle 2.\angle 5} = 2520$$

28. Ans: 945

Sol: Required number of ways = Number of unordered partitions of a set in to 5 subjects of same size $= \dfrac{\angle 10}{(\angle 2.\angle 2.\angle 2.\angle 2.\angle 2).\angle 5}$

$= 945$

29. Ans: 600

Sol: We can select 4 men in C(5, 4) ways. Those 4 men can be paired with 4 women in P(5, 4) ways.

$\therefore$ Number of possible selections

= C(5, 4). P(5, 4)

= 5. (120) = 600

30. Ans: 120

Sol: The 3 zeros can appear in the sequence in C(10,3) ways. The remaining 7 positions of the sequence can be filled with ones in only one way.

Required number of binary sequences

= C(10,3).1 = 120

31. Ans: 35

Sol: Consider a string of 6 ones in a row. There are 7 positions among the 6 ones for placing the 4 zeros. The 4 zeros can be placed in C(7,4) ways.

Required number of binary sequences

= C(7,4) = C(7,3)

= 35

32. Ans: 252

Sol: To meet the given condition, we have to choose 5 distinct decimal digits and then arrange them in descending order. We can choose 5 distinct decimal digits in C(10,5) ways and we can arrange them in descending order in only one way.

Required number of ways = C(10,5).1 = 252

33. Ans: 2n(n–1)

Sol: We have 2n persons.

Number of handshakes possible with 2n persons = C(2n,2)

If each person shakes hands with only his/her spouse, then number of handshakes possible

= n

Required number of handshakes

= C(2n, 2) – n = 2n(n –1)

34. Ans: 1092

Sol: In a chess board, we have 9 horizontal lines and 9 vertical lines. A rectangle can be formed with any two horizontal lines and any two vertical lines.

Number of rectangles possibles

= C(9,2). C(9,2) = (36)(36) = 1296

Number of squares in a chess board

$= 1^2 + 2^2 + 3^2 + \ldots + 8^2 = 204$

Every square is also a rectangle.

Required number of rectangles which are not squares = 1296 – 204 = 1092

35. Ans: 84

Sol: Between H and R, we have 9 letters. We can choose 3 letters in C(9,3) ways and then arrange them between H and R in alphabetical order in only one way.

Required number of letter strings = C(9,3).1

= 84

36. Ans: 210

Sol: We can choose 6 persons in C(10, 6) ways

We can distinct 6 similar books among the 6 persons in only one ways

∴ Required number of ways

= C(10, 6). 1

= C(10, 4) = 210

37. Ans: 1001

Sol: Required number of ways = V(5,10)

V(n,k) = C(n–1+k, k)

⇒ V(5,10) = C(14,10)

= C(14,4)

= 1001

38. Ans: 455

Sol: To meet the given condition, let us put 1 ball in each box, The remaining 12 balls we can distribute in V(4,12) ways.

Required number of ways = V(4,12).1

$= C(15,12) = C(15,3) = 455$

39. Ans: 3003

Sol: The number of solutions to the inequality is same as the number or solutions to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$$

Where $x_6 \geq 0$

The required number of solutions = V(6,10)

$= C(15,10) = C(15,5) = 3003$

40. Ans: 10

Sol: Let $x_1 = y_1 + 3$, $x_2 = y_2 - 2$, $x_3 = y_3 + 4$

The given equation becomes

$y_1 + y_2 + y_3 = 3$

Number of solutions to this equation

$= V(3, 3)$

$= C(5, 3) = 10$

$\therefore$ Required number of solutions = 10

41. Ans: 63

Sol: Let X_1 = units digit, X_2 = tens digit and X_3= hundreds digit

Number of non negative integer solutions to the equation

$$X_1 + X_2 + X_3 = 10 \text{ is}$$

$V(3, 10) = C(12, 10) = C(12, 2) = 66$

We have to exclude the 3 cases where $X_i = 10$ $(i = 1, 2, 3)$

Required number of integers = 66 – 3 = 63

42. Ans: 1001

Sol: Let $x_1 = y_1 + 2$, $x_2 = y_2 + 3$, $x_3 = y_3 + 4$, $x_4 = y_4 + 5$ and $x_5 = y_5 + 6$

Required number of solutions = Number of non negative integer solutions to the equation

$$y_1 + y_2 + y_3 + y_4 + y_5 = 10.$$

$= V(5, 10) = C(5 - 1 + 10, 10)$

$= C(14, 10)$

$= C(14, 4) = 1001$

43. Ans: 10

Sol: To meet the given conditions, let us put 2 books on each of the 4 shelves. Now we are left with 2 books to distribute among the 4 shelves. Which ever way we distribute the remaining books, the number of books on any shelf cannot exceed 4.

$\therefore$ Required number of ways = V (4, 2)

$= C(4 - 1 + 2, 2)$

$= C(5, 2) = 10$

44. Ans: (a)

Sol: This is similar to distributing n similar balls in k numbered boxes, so that each box contains atleast one ball.

If we put 1 ball in each of the k boxes, then we are left with (n–k) balls to distribute in k boxes.

Required number of ways = V(k, n – k)

= C(k – 1 + n – k, n – k)

= C(n – 1, k – 1)

45. Ans: 84

Sol: Required number of ways = Number of non negative integer solutions to the equation,

$$x_1 + x_2 + x_3 + x_4 = 10$$

where $x_i \geq 1$ (i = 1, 2, 3, 4)

This is equivalent to number of non negative integer solutions to the equation.

$$x_1 + x_2 + x_3 + x_4 = 6$$

$\therefore$ Required number of ways = V(4, 6)

Where V(n, k) = C(n – 1 + k, k)

$\Rightarrow$ V(4, 6) = C(9, 6) = C(9, 3) = 84

46. Ans: S_1, S_2, S_5, S_6

Sol: Average number of letters received by an apartment = $A = \frac{410}{50}$

$= 8.2$

Here, $\lceil A \rceil = 9$ and $\lfloor A \rfloor = 8$

By pigeonhole principle, S_1 and S_2 are necessarily true.

S_5 follows from S_1 and S_6 follows from S_2.

S_3 and S_4 need not be true.

47. Ans: 97

Sol: If we have n pigeon holes, then minimum number of pigeons required to ensure that atleast (k+1) pigeons belong to same pigeonhole = kn + 1

For the present example, n=12 and k+1= 9

Required number of persons = kn + 1

= 8(12) +1 = 97

48. Ans: 26

Sol: By Pigeonhole principle,

Required number of balls = kn + 1

= 5(5) + 1 = 26

49. Ans: 39

Sol: The favourable colors to draw 9 balls of same color are green, white and yellow.

We have to include all red balls and all green balls in the selection of minimum number of balls. For the favourable colors we can apply pigeonhole principle.

Required number of balls = 6 + 8 + (kn + 1)

Where k +1 = 9

and n = 3

$6 + 8 + (8 \times 3 + 1) = 39$

50. Ans: 4

Sol: Suppose x ≥ 6,

Minimum number of balls required = kn +1=16

where k + 1 = 6 and n = 3.

$\Rightarrow 5(3) + 1 = 16$

Which is impossible

$\therefore\ x < 6$

Now, minimum number of balls required

$$= x + (kn + 1) = 15$$

where $k + 1 = 6$ and $n = 2$

$$\Rightarrow x + 5(2) + 1 = 15$$

$$\Rightarrow x = 4$$

51. Ans: 7

Sol: For sum to be 9, the possible 2-element subsets are $\{0,9\}$, $\{1, 8\}$, $\{2, 7\}$, $\{3, 6\}$, $\{4, 5\}$

If we treat these subsets as pigeon holes, then any subset of S with 6 elements can have at least one of these subsets.

Since we need two such subsets, the required value of $k = 7$.

52. Ans: 7

Sol: If we divide a number by 10 the possible remainders are 0, 1, 2, …, 9.

Here, we can apply pigeonhole principle. The 6 pigeonholes are

$\{0\}$, $\{5\}$, $\{1,9\}$, $\{2,8\}$, $\{3,7\}$, $\{4,6\}$

In the first two sets both $x + y$ and $x - y$ are divisible by 10. In the remaining sets either $x + y$ or $x - y$ us divisible by 10.

$\therefore$ The minimum number of integers we have to choose randomly is 7.

53. Ans: 14

Sol: Every positive integer 'n' can be written as, $n = 2^k m$ where 'm' is odd and $k \geq 0$. Let us call m the odd part.

If we treat the odd numbers 1, 3, 5, …, 25 as pigeonholes then we have 13 pigeonholes.

Every element in S has on odd part and associated with one of the 13 pigeonholes.

The minimum value of $k = 14$

54. Ans: 6

Sol: For the difference to be 5, the possible combinations are $\{1, 6\}$, $\{2, 7\}$, $\{3, 8\}$, $\{4, 9\}$, $\{5, 0\}$.

If we treat them as pigeonholes, then we have 5 pigeonholes.

By pigeonhole principle, if we choose any 6 integers in S, then the difference of the two integers is 5.

55. Ans: 40

Sol: The distinct prime factors of 110 are 2, 5 and 11.

Required number of +ve integers

$$= \phi(110)$$

$$= 110\left[\frac{(2-1).(5-1).(11-1)}{2.5.11}\right]$$

$$= 40$$

56. Ans: 48

Sol: The distinct prime factors of 180 are 2, 3 and 5.

Required number of +ve integers

$$= \phi(180)$$

$$= 180\left[\frac{(2-1).(3-1).(5-1)}{2.3.5}\right] = 48$$

57. Ans: 288

Sol: The distinct prime factors of 323 are 17 and 19.

Required number of +ve integers

$$= \phi(323)$$

$$= 323\left[\frac{(17-1).(19-1)}{17.19}\right] = 288$$

58. Ans: 265

Sol: Required number of 1 – 1 functions

= number of derangements possible with 6 elements

$$= D_6 = \angle 6\left(\frac{1}{\angle 2} - \frac{1}{\angle 3} + \frac{1}{\angle 4} - \frac{1}{\angle 5} + \frac{1}{\angle 6}\right)$$

$$= 265$$

59. Ans: (i) 44 (ii) 76 (iii) 20 (iv) 89 (v) 119 (vi) 0

Sol: (i) Number of ways we can put 5 letters, so that no letter is correctly placed

$$= D_5 = \angle 5\left(\frac{1}{\angle 2} - \frac{1}{\angle 3} + \frac{1}{\angle 4} - \frac{1}{\angle 5}\right)$$

$$= 44$$

(ii) Number of ways in which we can put 5 letters in 5 envelopes = $\angle 5$

Number of ways we can put the letters so that no letter is correctly placed = D_5

Required number of ways = $\angle 5 - D_5$

$= 120 - 44$

$= 76$

(iii) Number of ways we can put the 2 letters correctly = C(5,2) = 10

The remaining 3 letters can be wrongly placed in D_3 ways.

Required number of ways = C(5,2) D_3

$= (10)\ 2$

$= 20$

(iv) Number of ways in which no letter is correctly placed = D_5

Number of ways in which exactly one letter is correctly placed = C(5,1) D_4

Required number of ways

$= D_5 + C(5,1)D_4$

$= 44 + 5\ (9)$

$= 89$

(v) There is only one way in which we can put all 5 letters in correct envelopes.

Required number of ways = $\angle 5 - 1$

$= 119$

(vi) It is not possible to put only one letter in wrong envelope.

Required number of ways = 0

60. Ans: (i) 1936 (ii) 14400

Sol: (i) The derangements of first 5 letters in first 5 places = D_5

Similarly, the last 5 letters can be deranged in last 5 places in D_5 ways.

The required number of derangements

$= D_5D_5 = (44)\ (44)$

$= 1936$

(ii) Any permutation of the sequence in which the first 5 letters are not in first 5 places is a derangement. The first 5 letters can be arranged in last 5 places in $\angle 5$ ways. Similarly, the last 5 letters of the given sequence can be arranged in first 5 places in $\angle 5$ ways.

Required number of derangements

$= \angle 5.\ \angle 5$

$= 14400$

61. Ans: 4!.D_4 = 216

Sol: First time, The books can be distributed in $\angle 4$ ways.

Second time, we can distribute the books in D_4 ways.

Required number of ways = $\angle 4\ D_4 = 216$

62. Ans: 31250

Sol: Let $b_n = a_n^2$

The given recurrence relation becomes

$b_{n+1} - 5\,b_n = 0$

The solution is $b_n = 4\ (5^n)$

$\Rightarrow a_n = 2\left(\sqrt{5}\right)^n$

$\Rightarrow a_{12} = 31{,}250$

63. Ans: (a)

Sol: The recurrence relation is

$a_n - a_{n-1} = 2n - 2$ ………… (1)

The characteristic equation is $t - 1 = 0$

Complementary function = $C_1\ .\ 1^n$

Here, 1 is a characteristic root with multiplicity 1.

Let particular solution = $(c\,n^2 + d\,n)$

Substituting in (1),

$(cn^2 + d\,n) - \{c\,(n-1)^2 + d(n-1)\} = 2n - 2$

$n = 1 \Rightarrow c + d = 0$

$n = 0 \Rightarrow -c + d = -2$

$\Rightarrow c = 1$ and $d = -1$

$\therefore$ P. S = $n^2 - n$

The solution is

$a_n = C_1 + n^2 - n$ …………… (1)

Using the initial condition, we get $C_1 = 1$

Substituting C_1 value in equation (1), we get

$\therefore\ a_n = n^2 - n + 2$

64. Ans: (b)

Sol: **Case 1:** If the first digit is 1, then the remaining digits we can choose in a_{n-1} ways

Case 2: If the first digit is 0 and second digit is 1, then the remaining digits we can choose in a_{n-2} ways.

Case 3: If the first two digits are zeros, then each of the remaining digits should be 1.

$\therefore$ By sum rule,

The recurrence relation for a_n is

$$a_n = a_{n-1} + a_{n-2} + 1$$

65. Ans: (c)

Sol: Let a_n = number of n-digit quaternary sequences with even number of zeros

Case 1: If the first digit is not 0, then we can choose first digit in 3 ways and the remaining digits we can choose in a_{n-1} ways. By product rule, number of quaternary sequences in this case is $3a_{n-1}$.

Case 2: If the first digit is 0, then the remaining digits should contain odd number of zeros.

Number quaternary sequences in this case is $(a_{n-1} - 4^{n-1})$

$\therefore$ By sum rule, the recurrence relation is

$$\Rightarrow a_n = 3a_{n-1} + (4^{n-1} - a_{n-1})$$

$$\Rightarrow a_n = 2a_{n-1} + 4^{n-1}$$

66. Ans: (a)

Sol: The given recurrence relation is

$$(E^2 - 5E + 6)\, a_n = 2.$$

The characteristic equation is $t^2 - 5t + 6 = 0$

$$\Rightarrow t = 3, 2$$

Complementary function = $C_1 . 3^n + C_2 . 2^n$

Particular solution = $\dfrac{2}{E^2 - 5E + 6}$

$$= 2\left(\frac{1^n}{E^2 - 5E + 6}\right) = 2\left(\frac{1^n}{1^2 - 5(1) + 6}\right)$$

$$= 1$$

The solution is

$$a_n = C_1\, 3^n + C_2\, 2^n + 1$$

Using initial conditions, we get

$$C_1 = 2 \text{ and } C_2 = 0$$

$$\therefore a_n = 2\,(3^n) + 1$$

Now, $a_{100} = 2\,(3^{100}) + 1$

67. Ans: (d)

Sol: **Case: (i)** If the first square is not red then it can be colored in 2 ways and the remaining squares can be colored in a_{n-1} ways.

Case (ii) If the first square is colored in red, then second square can be colored in two ways and remaining squares can be colored in a_{n-2} ways.

By sum rule, the recurrence relation is

$$a_n = 2\, a_{n-1} + 2\, a_{n-2}$$

$$a_n = 2\,(a_{n-1} + a_{n-2})$$

68. Ans: 8617

Sol: $a_n = a_{n-1} + 3(n^2)$

$n = 1 \Rightarrow a_1 = a_0 + 3(1^2)$

$n = 2 \Rightarrow a_2 = a_1 + 3(2^2)$

$\quad = a_0 + 3(1^2 + 2^2)$

$n = 3 \Rightarrow a_3 = a_2 + 3(3^2)$

$\quad = a_0 + 3(1^2 + 2^2 + 3^2)$

$a_n = a_0 + 3(1^2 + 2^2 + \ldots + n^2)$

$= 7 + \frac{1}{2} n (n+1) (2n +1)$

$a_{20} = 7 + \frac{1}{2}$ (20) (21) (41) = 8617

69. Ans: (b)

Sol: The characteristic equation is $t^2 - t - 1 = 0$

$$\Rightarrow t = \frac{1 \pm \sqrt{5}}{2}$$

The solution is

$$a_n = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

Using the initial conditions, we get $C_1 = \frac{1}{\sqrt{5}}$

and $C_2 = -\frac{1}{\sqrt{5}}$

70. Ans: (d)

Sol: The recurrence relation can be written as

$$(E^2 - 2E + 1)\, a_n = 2^{n+2}$$

The auxiliary equation is

$$t^2 - 2t + 1 = 0$$

$$t = 1, 1$$

C.F. = $(C_1 + C_2 n)$

$$\text{P.S.} = \frac{2^{n+2}}{(E-1)^2} = 4\left[\frac{2^n}{(E-1)^2}\right]$$

$$= 4\frac{2^n}{(2-1)^2} = 2^{n+2}$$

∴ The solution is

$$a_n = C_1 + C_2 n + 2^{n+2}$$

71. Ans: (a)

Sol: **Case 1:** If the first digit is 1, then number of bit strings possible with 3 consecutive zeros, is a_{n-1}.

Case 2: If the first bit is 0 and second bit is 1, then the number of bit strings possible with 3 consecutive zeros is a_{n-2}.

Case 3: If the first two bits are zeros and third bit is 1, then number of bit strings with 3 consecutive zeros is a_{n-3}

Case 4: If the first 3 bits are zeros, then each of the remaining n–3 bits we can choose in 2 ways. The number of bit strings with 3 consecutive zeros in this case is 2^{n-3}.

∴ The recurrence relation for a_n is

$$a_n = a_{n-1} + a_{n-2} + a_{n-3} + 2^{n-3}.$$

72. Ans: (a)

Sol: Replacing n by n+1, the given relation can be written as

$$a_{n+1} = 4a_n + 3(n + 1)\, 2^{n+1}$$

$\Rightarrow (E - 4)\, a_n = 6\, (n+1)\, 2^n$ ……….(1)

The characteristic equation is

$$t - 4 = 0 \Rightarrow t = 4$$

complementary function = $C_1 4^n$

Let particular solution is

$a_n = 2^n(cn + d)$ where c and d are undetermined coefficients.

Substituting in the given recurrence relation, we have

$2^n (c n + d) - 4\ 2^{n-1}\ \{c(n-1)+d\} = 3n2^n$

$\Rightarrow (c n + d) - 2\{c(n-1)+d\} = 3n$

Equating coefficients of n and constants on both sides, we get

$c = -3$ and $d = -6$

$\therefore$ Particular solution $= 2^n (-3n - 6)$

Hence the solution is

$a_n = C_1 4^n - (3n + 6)\, 2^n$ …………(2)

$x = 0 \Rightarrow 4 = C_1 - 6 \Rightarrow C_1 = 10$

$a_n = 10(4^n) - (3n + 6)\, 2^n$

73. Ans: (a)

Sol: **Case(i):** If the first bit is 1, then the required number of bit strings is a_{n-1}

Case(ii): If the first bit is 0, then all the remaining bits should be zero

The recurrence relation for a_n is

$$a_n = a_{n-1} + 1$$

74. Ans: (b)

Sol: $a_n = a_{n-1} + (n-1)$

$n = 2 \Rightarrow a_2 = a_1 + 1 = 1$

$n = 3 \Rightarrow a_3 = a_2 + 2 = 1 + 2$

$n = 4 \Rightarrow a_4 = a_3 + 3 = 1 + 2 + 3$

.

.

.

$a_n = 1 + 2 + \ldots\ldots\ldots + n - 1$

$= \dfrac{n(n-1)}{2}$

75. Ans: 4

Sol: Let $a = 1 + x$, $b = 2 + y$, $c = 3 + z$, and $d = 4 + w$

The transformed system is

$x + y + z + w = 7$ where $0 \le x, y, z, w \le 2$

The generating function

$f(x) = (1 + x + x^2)^4$

$= \left(\dfrac{1-x^3}{1-x}\right)^4$

$= (1 - 4x^3 + 6x^6 - x^9)(1 - x)^{-4}$

$= (1 - 4x^3 + 6x^6 - x^9) \sum_{n=0}^{\infty} C(n+3, n)\, x^n$

Required number of solutions = Coefficient of x^7 in f(x)

$= C(10, 3) - 4\,C(7, 3) + 6\,C(4, 1)$

$= 120 - 140 + 24 = 4$

76. Ans: (a)

Sol: Required generating function

$= f(x) = 0 + x + 3x^2 + 9x^3 + 27x^4 + \ldots$

$= x(1 + 3x + 3^2 x^2 + 3^3 x^3 + \ldots\ \infty)$

$= x\sum_{n=0}^{\infty} 3^n x^n = x.(1-3x)^{-1}$

77. Ans: 861

Sol: The generating function for the given equation is

$F(X) = (X + X^2 + X^3 + X^4 + X^5 + X^6)^4$

The coefficient of X^{15} in F(X) is the answer in our problem

$F(X) = X^4 (1 + X + \ldots + X^5)^4$

$= X^4\left(\dfrac{1-X^6}{1-X}\right)^4$

$= X^4 (1-X^6)^4 (1-X)^{-4}$

$= (X^4 - 4X^{10} + 6X^{16} - 4X^{22} + X^{28}).$

$$\sum_{n=0}^{\infty} C(n+3, n) X^n$$

Coefficient of $X^{15} = C(14, 3) - 4\,C(8, 3)$

$= 140$

78. Ans: (d)

Sol: Required generating function

$f(x) = 0 + 0\,x + 1\,x^2 - 2x^3 + 3x^4 - 4x^5 + \ldots$

$= x^2 (1 - 2x + 3x^2 - 4x^3 + \ldots \infty)$

$= x^2(1 + x)^{-2}$ (Binomial theorem)

79. Ans: (a)

Sol: $(x^4 + 2x^5 + 3x^6 + 4x^7 + \ldots\ldots\infty)^5$

$= x^{20} (1 + 2x + 3x^2 + 4x^3 + \ldots\ldots\infty)^5$

$= x^{20}.\ [(1 - x)^{-2}]^5$

$= x^{20}\ [1 - x]^{-10}$

$$= x^{20} \sum_{n=0}^{\infty} C(n+9, n) x^n$$

Coefficient of $x^{27} = C(16, 7)$

$= C(16, 9)$

80. Ans: 85

Sol: The generating functions for the problem is

$f(x) = (x + x^2 + x^3 + x^4)^4$

$= x^4(1 + x + x^2 + x^3)^4$

$= x^4\left(\dfrac{1-x^4}{1-x}\right)^4$

$= x^4(1 - x^4)^4 (1 - x)^{-4}$

$= x^4(1 - 4x^4 + 6x^8 - 4x^{12} + x^{16})(1 - x)^{-4}$

$= (x^4 - 4x^8 + 6x^{12} - 4x^{16} + x^{20})$

$$\sum_{n=0}^{\infty} C(n+3, n) x^n$$

The required number of solutions

= coefficient of x^{12} in f(x)

$= C(11, 3) - 4\,C(7, 3) + 6$

$= 31$

81. Ans: (c)

Sol: The generating function is

$f(x) = 1 + 0.x + 1.x^2 + 0.x^3 + 1.x^4 + \ldots\ldots\ \infty$

$= 1 + (x^2) + (x^2)^2 + \ldots\ldots\ \infty$

$= (1 - x^2)^{-}$

3. Graph Theory

01. Ans: (a)

Sol: For any simple graph,

$$\delta(G)|V| \le 2|E| \le \Delta(G)|V|$$

$$\Rightarrow \delta(G)\,(10) \le 2(16)$$

$$\Rightarrow \quad \delta(G) \le 3.2$$

$$\Rightarrow \quad \delta(G) \le 3$$

02. Ans: 19

Sol: By sum of degrees of regions theorem, if degree of each vertex is k, then

$$k|V| = 2|E|$$

$$\Rightarrow 4|V| = 2(38)$$

$$\Rightarrow \quad |V| = 19$$

03. **Ans: (c)**

Sol: If degree of each vertex is k,

$$k|V| = 2|E|$$

$$\Rightarrow k|V| = 2\,(12)$$

$$\Rightarrow \quad |V| = \frac{24}{k} \quad (\,k = 1,2,3,4)$$

$$\Rightarrow |V| = 24 \text{ or } 12 \text{ or } 8 \text{ or } 6$$

$\therefore$ only option (c) is possible.

05. Ans: (e)

Sol: (a) {2, 3, 3, 4, 4, 5}

Here, sum of degrees

= 21, an odd number.

$\therefore$ The given sequence cannot represent a simple non directed graph

(b) {2, 3, 4, 4, 5}

In a simple graph with 5 vertices, degree of every vertex should be ≤ 4.

$\therefore$ The given sequence cannot represent a simple non directed graph.

(c) {1, 3, 3, 4, 5, 6, 6}

Here we have two vertices with degree 6. These two vertices are adjacent to all the other vertices. Therefore, a vertex with degree 1 is not possible.

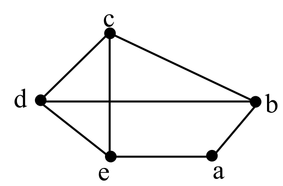
Hence, the given sequence cannot represent a simple non directed graph.

(d) {0, 1, 2,….., n –1}

Here, we have n vertices, with one vertex having degree n-1. This vertex is adjacent to all the other vertices. Therefore, a vertex with degree 1 is not possible.

Hence, the given sequence, cannot represent a simple non directed graph.

(e) A graph with the degree sequence {2, 3, 3, 3, 3} is shown below.



06. Ans: 8

Sol: Here, degree of each vertex is ≤ 3

By sum of degrees theorem,

$3 | V | \geq 2 | E |$

$\Rightarrow 3| V | \geq 2(12)$

$\Rightarrow \ | V | \geq 8$

$\therefore$ The minimum number of vertices G can have = 8

07. Ans: 12

Sol: G is a tree

By sum of degrees theorem,

$n.1 + 2 (2) + 4. (3) + 3. (4) = 2 |E|$

$\therefore\ n + 28 = 2 (|v| - 1)$

$= 2 (n + 2 + 4 + 3 - 1)$

$\Rightarrow n + 28 = 2n + 16$

$\Rightarrow n = 12$

08. Ans: 8

Sol: G has 8 vertices with odd degree.

For any vertex $v \in G$,

Degree of v in G + degree of v in $\overline{G} = 8$

If degree of v in G is odd, then degree of v in $\overline{G}$ is also odd. If degree of v in G is even, then degree of v in $\overline{G}$ is also even.

$\therefore$Number of vertices with odd degree in G = 8

09. Ans: 27

Sol: By sum of degrees theorem, if degree of each vertex is atmost K,

then $K|V| \geq 2 |E|$

$\Rightarrow 5 (11) \geq 2 |E|$

$\Rightarrow \quad |E| \leq 27.5$

$\Rightarrow \quad |E| \leq 27$

10. Ans: (d)

Sol: (a) Sum of the degree of the vertices

= 15(5) = 75 = An odd number.

$\therefore$ The graph is not a simple graph

(b) Maximum number of edges possible in a simple graph with 10 vertices

C(10, 2) = 45

(c) Sum of the degrees = 2n – 1

= An odd number

$\therefore$ The graph is not simple graph

(d) A connected graph with n vertices and n–1 edges is a tree. A tree is a simple graph.

11. Ans: 2

Sol: In the graph, all the cycles are of even length.

$\therefore$ G is a bipartite graph.

Chromatic number of any bipartite graph is 2.

12. Ans: 2

Sol: In the given graph, all the cycles are of even length.

$\therefore$ G is a bipartite graph and every bipartite graph is 2-colorable

$\therefore$ Chromatic number of G = 2.

13. Ans: 5

Sol: $\overline{G}$ is a disconnected graph with two components, one component is the complete graph K_5 and the other component is the trivial graph with only an isolated vertex

$\therefore$ Chromatic number of $\overline{G} = 5$

14. Ans: (b)

Sol: $\alpha = n - 2\lfloor n/2 \rfloor + 2$

$\beta = n - 2\lceil n/2 \rceil + 4$

$\alpha + \beta = 2n - 2\{\lfloor n/2 \rfloor + \lceil n/2 \rceil\} + 6$

$= 2n - 2n + 6 = 6$

15. Ans: (c)

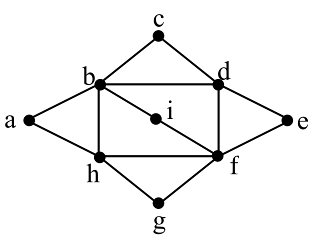
Sol: Chromatic number of $K_n = n$

If we delete an edge in K_{10}, then for the two vertices connecting that edge we can assign same color.

$\therefore$ Chromatic number = 9

16. Ans: 4

Sol:



The graph has 9 vertices. The maximum number of vertices we can match is 8. A matching in which we can match 8 vertices is { a – b, c – d, e – f, g – h}

$\therefore$ Matching number of the graph = 4

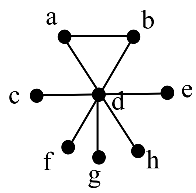
17. Ans: 2

Sol: The given graph is $K_{2,4}$

$\therefore$ Matching number = 2

18. Ans: 2

Sol: The given graph is

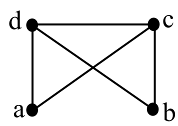


If we delete the edge {a,b} then the graph is a star graph. If we match a with b, then in the remaining vertices we can match only two vertices.

$\therefore$ Matching number = 2

19. Ans: 3

Sol: Let us label the vertices of the graph as shown below

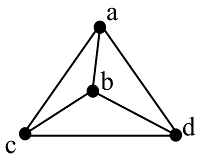


There are 3 maximal matchings as given below

{a–d, b–c}, {a–c, b–d} and { c–d}

20. Ans: 3

Sol: The given graph is



The maximal matchings are

{a–b, c–d}, {a–c, b–d}, {a–d, b–c}

21. Ans: 10

Sol: The graph has 3 maximal matchings, 6 matchings with one edge, and a matching with no edges.

$\therefore$ Number of matchings = 10

22. Ans: 3

Sol: G is a complete graph on 7 vertices.

$\therefore$ Matching number of $K_n = \left\lfloor \frac{n}{2} \right\rfloor$

$\therefore$ Matching number of $K_7 = 3$

23. Ans: (a)

Sol: If n is even, then a bipartite graph with maximum number of edges is $k_{n/2,n/2}$

$\therefore$ Matching number of $G = \frac{n}{2}$

If n is odd, then a bipartite graph with maximum number of edges = $k_{m,n}$

Where $m = \frac{n-1}{2}$ and $n = \frac{n+1}{2}$

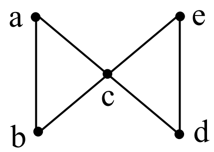
$\therefore$ Matching number of G

$$= \begin{cases} \frac{n}{2}, \text{ if } n \text{ is even} \\ \frac{n-1}{2}, \text{ if } n \text{ is odd} \end{cases}$$

$\therefore$ Matching number of $G = \left\lfloor \frac{n}{2} \right\rfloor$

24. Ans: 1, 2

Sol: The graph can be labeled as



c is a cut vertex of the graph G.

$\therefore$ vertex connectivity of G = K(G) = 1

G has no cut edge.

$\Rightarrow$ Edge connectivity = λ (G) $\geq$ 2 ….. (1)

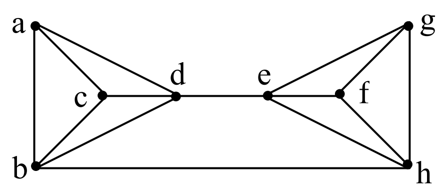
We have, λ (G) $\leq$ δ (G) = 2 …… (2)

From (1) and (2), we have

λ (G) = 2

25. Ans: 2, 2

Sol: The graph G can be labeled as



G has no cut edge and no cut vertex. By deleting the edges {d, e} and {b, h} we can disconnect G.

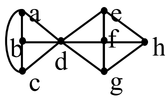
$\therefore \lambda(G) = 2$

By deleting the vertices b and d, we can disconnect G.

$\therefore K(G) = 2$

26. Ans: 1 & 3

Sol: The graph G can be labeled as



The vertex d is a cut vertex of G.

$\therefore K(G) = 1$

We have $\lambda(G) \le \delta(G) = 3$ ...... (1)

G has no cut edge and by deleting any two edges of G we cannot disconnect G.

$\therefore \lambda(G) = 3$

27. Ans: S_1, S_3 & S_4

Sol: **S_1:** This statement is true.

Proof:

Suppose G is not connected G has atleast 2 connected components.

Let G_1 and G_2 are two components of G.

Let u and v are any two vertices in G

We can prove that there exists a path between u and v in $\overline{G}$.

Case1: u and v are in different component of G.

Now u and v are not adjacent in G.

$\therefore$ u and v are adjacent in $\overline{G}$

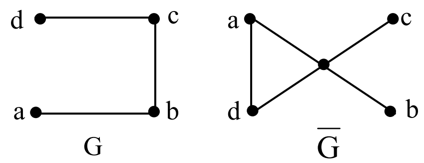
Case2: u and v are in same component G_1 of G. Take any vertex $w \in G_2$.

Now u and v are adjacent to w in G.

$\therefore$ There exists a path between u and v in G. Hence, $\overline{G}$ is connected.

S_2: The statement is false.

we can give a counter example.



Here, G is connected and $\overline{G}$ is also connected.

S_3: Suppose G is not connected

Let G_1 and G_2 are two connected components of G.

Let $v \in G_1$

$$\Rightarrow \deg(v) \ge \frac{n-1}{2} \qquad \left(\because \delta(G) = \frac{n-1}{2}\right)$$

$$\text{Now} \quad |V(G_1)| \ge \left(\frac{n-1}{2} + 1\right)$$

$$\text{Similarly, } |V(G_2)| \ge \frac{n+1}{2}$$

Now, $|V(G)| = |V(G_1)| + |V(G_2)|$

$$\Rightarrow |V(G)| \ge n+1$$

Which is a contradiction

$\therefore$ G is connected.

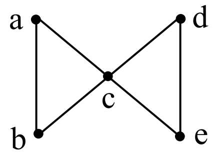
S_4: If G is connected, then the statement is true. If G is not connected, then the two vertices of odd degree should lie in the same component,

By sum of degrees of vertices theorem.

$\therefore$ There exists a path between the 2 vertices.

28. Ans: S_1, S_2 & S_4

Sol: The graph G can be labeled as



The number of vertices with odd degree is 0.

$\therefore$ S_1 and S_2 are true

C is a cut vertex of G.

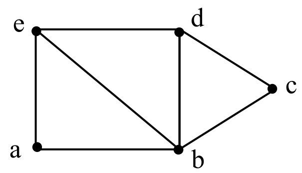
$\therefore$ Hamiltonian cycle does not exists.

By deleting the edges {a, c} and {c, e}, there exists a Hamiltonian path

a – b – c – d – e

29. Ans: S_1, S_3 & S_4

Sol: The graph G can be labeled as



The number of vertices with odd degree = 2

$\therefore$ Euler path exists but Euler circuit does not exist.

There exists a cycle passing through all the vertices of G.

a – b – c – d – e – a is the Hamiltonian cycle of G.

The Hamiltonian path is a – b – c – d – e

30. Ans: S_1 & S_2

Sol: The number of vertices with odd degree = 0

$\therefore$ S_1 and S_2 are true.

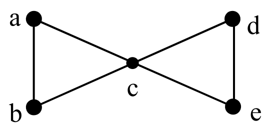
To construct Hamiltonian cycle, we have to delete two edges at each of the vertices a and f. Then, we are left with 4 edges and 6 vertices.

$\therefore$ G has neither Hamiltonian cycle nor Hamiltonian path.

31. Ans: (B)

Sol: S_1 is false. We can prove it by giving a counter example.

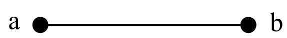
Consider the graph G shown below



'e' is a cut vertex of G. But, G has no cut edge

S_2 is false. We can prove it by giving a counter example.

For the graph K_2 shown below,



The edge {a, b} is a cut edge. But K_2 has no cut vertex.

32. Ans: 33

Sol: If G has K components, then

$$|E| = |V| - K$$

$$\Rightarrow \quad 26 = |V| - 7$$

$$\Rightarrow |V| = 33$$

33. Ans: (b)

Sol: A 2-regular graph G has a perfect matching iff every component of G is an even cycle.

$\therefore S_2$ and S_4 are true.

S_1 need not be true. For example the complete graph K_2 has a perfect matching but K_2 has no cycle.

S_3 need not be true. For example G can have two components where each component is K_2.

34. Ans: 21

Sol: In a simple graph with n vertices and K components,

$$|E| > \frac{(n-k)(n-k+1)}{2}$$

$\therefore$ Required minimum number of edges

$$= \frac{(n-k)(n-k+1)}{2} = 21$$

Where n = 10 and k = 4

35. Ans: (d)

Sol: G has exactly two vertices of odd degree. Therefore, Euler path exists in G but Euler circuit does not exist.

In Hamiltonian cycle, degree of each vertex is 2. So, we have to delete 2 edges at vertex 'd' and one edge at each of the vertices 'a' and 'g'. Then we are left with 8 vertices and 6 edges. Therefore, neither Hamilton cycle exists nor Hamiltonian path exists.

36. Ans: (b)

Sol: G has cycles of odd length

$\therefore$Chromatic number of

$$G = \chi(G) \geq 3 \;\ldots\ldots(1)$$

For the vertices c and h we can use same color C_1

The remaining vertices from a cycle of length 6.

A cycle of even length require only two colors for its vertex coloring.

For vertices a, d and f we can apply same color C_2

For the vertices {b, e, g} we can use same color C_3

$$\therefore \chi(G) = 3$$

A perfect matching of the graph is

a–b, c – d, e –f, g – h

$\therefore$Matching number = 4

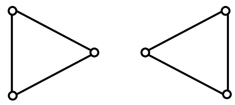
Hence, chromatic number of G

+ Matching number of G = 3 + 4

= 7

37. Ans: (c)

Sol: S_1 need not be true. Consider the graph



Here, we have 6 vertices with degree 2, but the graph is not connected.

S_2 need not be true. For the graph given above, Euler circuit does not exist, because it is not a connected graph.

A simple graph G with n vertices is necessarily connected if $\delta(G) \geq \frac{n-1}{2}$.

$\therefore$ S_3 is true.

38. Ans: (a)

Sol: Vertex connectivity of G = $k(G) \leq \delta(G)$

$$\Rightarrow \delta(G) \geq 3$$

By sum of degrees theorem

$3|V| \leq 2|E|$

$\Rightarrow |E| \geq 15$

$\therefore$ Minimum number of edges necessary = 15

39. Ans: (b)

Sol: Here, G is a complete graph with k+1 vertices.

$\therefore$ Number of edges = C(k+1, 2)

40. Ans: 14

Sol: If vertex connectivity of G is 3, then degree of each vertex in G is $\geq$ 3.

By sum of degrees theorem,

$3|V| \leq 2|E|$

$\Rightarrow |E| \geq 13.5$

$\Rightarrow |E| \geq 14$

4. Set Theory

01. Ans: (c)

Sol: (a) $A \vee (A \wedge B) = A$ (Absorption law)

$\therefore$ Option (a) is false

(b) $A \wedge (A \vee B) = A$ (Absorption law)

$\therefore$ Option (b) is false

(c) $(A \vee B) \wedge (A \vee \overline{B})$

$= A \vee (B \wedge \overline{B})$ Distribution law

$= A$

$\therefore$ Option (c) is true

02. Ans: (b)

Sol: If S = {ϕ} then

(a) $P(S) \wedge S = \{\phi\}$

$\therefore$ option (a) is false

(b) $P(S) \wedge P(S) = \{\phi, \{\phi\}\}$

$\therefore$ Option (b) is true

(c) If S = {a, b} then

$P(S) \wedge S = \phi$

$\therefore$ option (c) is false

(d) false

Refer option (b)

03. Ans: (c)

Sol: If $|A| = 3$ then

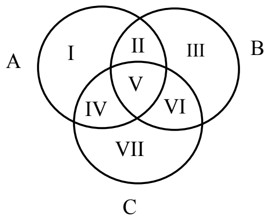
$|A| \times |A| = 9$

$|P(A \times A)| = 2^9$

$= 512$

04. Ans: (a, c & d)

Sol:



I, II,…….., VII are regions

(a) $(A - B) - C = \{I, IV\} - \{IV, V, VI, VIII\}$

$= \{I\}$

$A - (C–B) = \{I, II, IV, V\} - \{IV, VII\}$

$= \{I, II, V\}$

$\therefore$ Option (a) is false

(b) $A–(B \vee C) = (A–B) \wedge (A–C)$ is true by Demorgan's law

(c) $A–(B–C) = \{I, II, IV, V\} - \{II, III\}$

$= \{I, IV, V\}$

$A–(C–B) = \{I, II, IV, V\} - \{IV, VII\}$

$= \{I, II, V\}$

$\therefore$ Option (c) need not be true

(d) Similarly show that option (d) is not true

05. Ans: (c)

Sol: Let $x \in X$

Case 1: If x is even number then it can appear in two ways i.e., either $x \in A - B$ or $x \in B - A$

Case 2: If x is odd number then it can appear in two ways i.e., $x \in A \wedge B$ or $x \in \left(\overline{A \vee B}\right)$

$\therefore$ By product rule, required number of subsets $= 2^{100}$

06. Ans: (c)

Sol: If $\quad A \,\Delta\, B = (A \wedge B)^C$

Then $\left(\overline{A \vee B}\right) = \phi$

But $(A \vee B) \vee \left(\overline{A \vee B}\right) = U$

$\Rightarrow (A \vee B) = U$

Where U is universal set

07. Ans: (d)

Sol: (a) Let $A \oplus B = A$

$\Rightarrow A \oplus B = A \oplus \phi$

$\Rightarrow \quad B = \phi$

(b) $(A \oplus B) \oplus B$

$= A \oplus (B \oplus B)$

$= A \oplus \phi$

$= A$

(c) $A \oplus C = B \oplus C$

$\Rightarrow A = B \qquad$ (cancellation law)

(d) LHS = $A \oplus B = (A \vee B) - (A \wedge B)$

RHS = $(A \vee B) \vee (A \wedge B)$

= $(A \vee B)$

$\therefore$ L.H.S $\neq$ R.H.S

08. Ans: (c)

Sol: S_1) Let A = {1}, B = {2}, C = {3}

Now $(A \cap B) = (B \cap A) = \phi$

But $A \neq B$

$\therefore$ S_1 is not true

S_2) Let A = {1}, B = {2}, C = {1, 2}

Now $A \cup C = B \cup C = C$

But $A \neq B$

$\therefore$ S_2 is not true

S_3) Let $x \in A$.

Consider the two cases

Case1: $x \in C$

$\Rightarrow x \notin (A \Delta C)$ $\quad (\because x \in (A \Delta C))$

$\Rightarrow x \notin (B \Delta C)$ $\quad (\because A \Delta C = B \Delta C)$

$\Rightarrow x \in B$ ……….(1)

Case2: $x \notin C$

$\Rightarrow x \in (A \Delta C)$

$\Rightarrow x \in (B \Delta C)$

$\Rightarrow x \in B$ $\quad (\because x \notin C)$ ………(2)

$\therefore A \subseteq B$ $\quad$ (Form (1) and (2))

Similarly we can show that $B \subseteq A$.

$\therefore$ A = B

Hence, S_3 is true

09. Ans: 686

Sol: A symmetric relation on A with exact 4 ordered pairs can be in one of the following 3 ways.

i) The relation may contain 4 diagonal pairs. We can choose 4 diagonal pairs in C(7, 4) ways.

ii) The relation may contain 2 diagonal pairs and 2 non-diagonal pairs.
Number of symmetric relations in this case is C(7, 2). C(21, 1)

iii) The relation may contain 4 non-diagonal pairs.

Number of symmetric relations in this case is C(21, 2)

$\therefore$ By sum rule,

Required number of symmetric relations

= C(7, 4) + C(7, 2) . C(21, 1) + C(21, 2)

= 35 + 441 + 210

= 686

10. Ans: (d)

Sol: **Case 1:**

If the relation R contains only diagonal pairs, then R is symmetric and transitive but not irreflexive.

Case 2:

If the relation R contains some non diagonal pairs

Let $(a, b) \in R$

$\Rightarrow$ (b, a) $\in$ R (∵ R is symmetric)

Now (a, b) $\in$ R and (b, a) $\in$ R

$\Rightarrow$ (a, a) $\in$ R (∵ R is transitive)

∴ R is not irreflexive.

Options (a) & (b) need not be true, for example. If A = {1, 2, 3} then the relation R = {(1, 2), {2, 1), (1, 1), (2, 2)} is symmetric and transitive but neither reflexive nor irreflexive.

Option (b) need not be true for example. If A = {1, 2, 3}, then the relation R= {(1,1), (2, 2), (3, 3)} is symmetric, transitive and not irreflexive.

11. Ans: (c)

Sol: We have, a *divides* a $\forall$ a $\in$ A

$\Rightarrow$ a R a $\forall$ a $\in$ A

∴ R is reflexive

Let a R b

$\Rightarrow$ a is a divisor of b or b is a divisor of a

$\Rightarrow$ b is a divisor of a or a is a divisor of b

$\Rightarrow$ b R a

∴ R is a symmetric

R is not transitive, for examples 2 R 6 and 6 R 3, but 2 is not related to 3.

Hence, R is a compatibility relation on A.

12. Ans: (c)

Sol: We have, g.c.d of (x, x) = x

∴ R is not reflexive

Let x R y

$\Rightarrow$ g. c. d of (x, y) = 1

$\Rightarrow$ g. c. d. of (y, x) = 1

$\Rightarrow$ y R x

$\Rightarrow$ R is symmetric

R is not transitive,

For example g. c. d. of (2, 3) = 1

and g. c. d. of (3, 4) = 1

but g. c. d. of (2, 4) $\neq$ 1

13. Ans: (c)

Sol: Let A = {1, 2, 3} and

R = {(1, 2), (2, 2)}

S = {(2, 1), (3, 3)}

R $\cup$ S = {(1, 2), (2, 1), (2, 2) (3, 3)}

Here, R and S are anti-symmetric relations on A, But (R $\cup$ S) is not anti-symmetric.

∴ (R $\cup$ S) need not be anti-symmetric.

Any subset of anti-symmetric relation is also anti-symmetric, and (R $\cap$ S) $\subseteq$ R.

∴ (R $\cap$ S) is anti-symmetric

Hence, only option (c) is true.

14. Ans: (D)

Sol: (a) R is not reflexive.

For example, –1 is not related to –1.

i.e., (–1, –1) $\notin$ R

(b) R is not irreflexive.

For example, 1 is related to 1.

i.e., (1, 1) $\in$ R

(c) R is not symmetric.

For example, –1 is related to 1 but 1 is not related to –1.

(d) R is anti-symmetric.

i.e., If (a R b and b R a) then

(a = b) $\forall$ a, b $\in$ A

i.e., For any two integers.

If {a = | b | and b = | a |} then a = b

15. Ans: (d)

Sol: Two positive integers a and b are relatively prime, if g. c. d of a and b is 1.

i) R is not reflexive. For example

g. c. d. of {2, 2} = 2

$\Rightarrow$ 2 is not related 2

ii) Let a ^{R} b

$\Rightarrow$ g. c. d. of {a, b} = 1

$\Rightarrow$ g. c. d. of {b, a} = 1

$\Rightarrow$ b ^{R} a

$\Rightarrow$ R is symmetric

iii) R is not transitive.

For example, 2 ^{R} 3 and 3 ^{R} 4 but 2 is not related 4.

iv) R is not anti-symmetric.

For example (2 ^{R} 3) and (3 ^{R} 2)

v) R is not irreflexive.

For example (1, 1) $\in$ R

$\Rightarrow$ 1 ^{R} 1

16. Ans: (a)

Sol: Symmetric closure of

R = {(2, 1), (1, 2), (3, 2), (2, 3)}

Transitive Symmetric closure of R = A × A

17. Ans: (d)

Sol: [2] = {x | (2 – x) is divisible by 4}

= {4k + 2 | k is integer}

= {…– 10, –6, –2, 2, 6, 10, 14, ….}

$\therefore$ [2] = [–6]

18. Ans: (d)

Sol: $|a - a| \neq 2 \quad \forall a \in Z$

$\Rightarrow$ a is not related to a $\forall a \in Z$

$\Rightarrow$ R is irreflexive

Let a ^{R} b

$\Rightarrow |a - b| = 2$

$\Rightarrow |b - a| = 2$

$\Rightarrow$ b ^{R} a $\Rightarrow$ R is symmetric

19. Ans: (b)

Sol: S = {1, 2, 3, 4, 6, 9, 12, 18, 36}

(a) R is a partial order on any set of positive integers.

$\therefore$ Option (a) is true

(b) In the set S, 2 and 3 are not comparable. Therefore R is not a total order on S.

(c) R is not symmetric. For example 2 is a divisor of 4, then 4 is not a divisor of 2.

$\therefore$ R is not an equivalence relation

(d) R is a partial order. Therefore R is reflexive and transitive.

20. Ans: (c)

Sol: If S is any set of positive integers, then R is R partial order on S.

$\therefore$ [A; R] is a poset

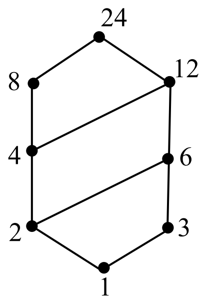
The greatest lower bound of 2 and 3 does not exist.

$\therefore$ [A ; R] is not a lattice.

In the poset all the prime numbers are minimal elements of A with respect to R.

21. Ans: 10

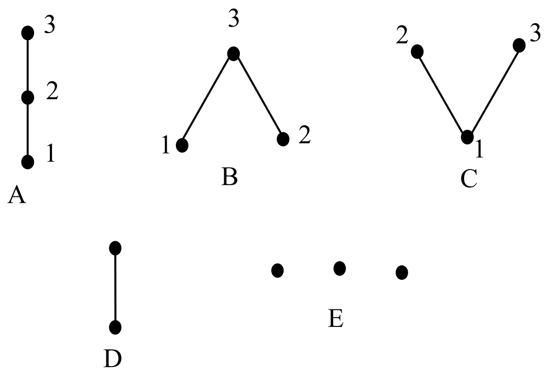
Sol: The Hasse diagram is



$\therefore$ Number of edges = 10

22. Ans: 19

Sol: The possible Hasse diagrams are



Number of partial orders of type A = 6

Number of partial orders of type B = 3

Number of partial orders of type C =3

Number of partial orders of type D = 6

Number of partial orders of type E = 1

$\therefore$ Required number of partial orders

$= 6 + 3 + 3 + 6 + 1 = 19$

23. Ans: 3

Sol: The partial orders are

$R_1 = \{(1, 1), (2, 2), (1, 2)\}$

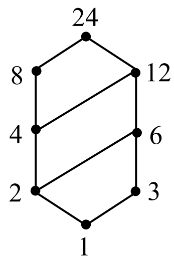
$R_2 = \{(1, 1), (2, 2), (2, 1)\}$

$R_3 = \{(1, 1), (2, 2)\}$

24. Ans: (a)

Sol: $D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$

The Hasse diagram of the poset is given below



S_1: Number of edges in the Hasse diagram is 10.

$\therefore$ S_1 is true

S_2: If a and b are complement of each other, then

Join of a and b = I = 24

and Meet of a and b = O = 1

Here Join of 2 and 12 = 12 $\neq$ I

$\therefore$ Complement of 2 $\neq$ 12

S_3: Join of 3 and 8 = 24 = I

Meet of 3 and 8 = 1 = O

$\therefore$ Complement of 3 = 8

S_4: Join of 4 and 6 = 12 $\neq$ I

Meet of 4 and 6 = 2 $\neq$ O

$\therefore$ Complement of 4 $\neq$ 6

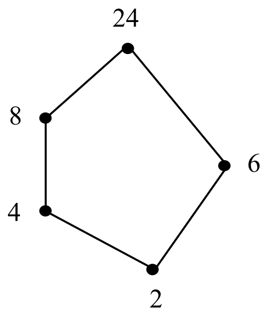
25. Ans: (b)

Sol: S = {1, 3, 9, 27, 81, 243}

(S, *) is a totally ordered set. A totally ordered set is always a distributive lattice but not a Boolean algebra

26. Ans: (c)

Sol: The Hasse diagram of the poset is



The poset is a lattice, because for every pair of elements lub and glb exist.

27. Ans: (a)

Sol: We have 385 = 5.7.11

= A product of distinct prime numbers

$\Rightarrow$ [D_{385}; |] is a Boolean algebra

28. Ans: (c)

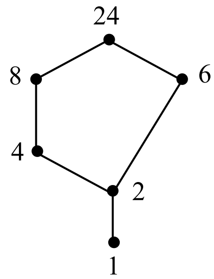
Sol: The join of d and g = h = Upper bound.

The meet of d and g = c $\neq$ Lower bound

$\therefore$ Complement of d $\neq$ g

29. Ans: (d)

Sol: The Hasse diagram is



If we delete the element 1, the resulting sub-lattice is not distributive.

$\therefore$ The given lattice is not distributive.

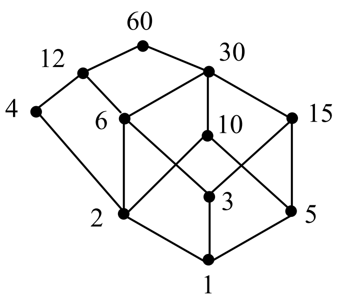
For the element 2, there is no complement.

$\therefore$ The given lattice is not a complemented lattice.

30. Ans: 17

Sol: The Hasse diagram is shown below.

D_{60} = {1, 2, 3, 4, 5, 6, 10, 12, 15, 30, 60}



Number of edges in the diagram = 17

31. Ans: 232

Sol: Number of functions possible on A = 4^4

Number of functions which are either 1-1 or on-to

$$= \angle 4 + \angle 4 - \angle 4 = 24$$

$\therefore$ Required number of functions = $4^4 - 24$

$$= 232$$

32. Ans: (c)

Sol: Let f (a,b) = f (c,d)

$\Rightarrow$ (a+b, a–b) = (c+d, c–d)

$\Rightarrow$ a+b = c+d ……(1)

a – b = c – d ……..(2)

Adding (1) and (2), we get a = c

Subtracting (2) from (1), we get b = d

$\Rightarrow$ (a,b) = (c,d)

$\therefore$ f is one-to-one

Let f (a,b) = (c, d)

$\Rightarrow$ (a+b, a–b) = (c,d)

$\Rightarrow$ a+b = c and a–b =d

$\Rightarrow$ a= $\frac{c+d}{2}$ and b = $\frac{c-d}{2}$

For each (c,d) $\in$B

We have,

$$(a,b) = \left(\frac{c+d}{2}, \frac{c-d}{2}\right) \in B \text{ such that}$$

f(a,b) = (c,d)

$\therefore$ f is on-to.

Hence, f is a bijection.

33. Ans: 1

Sol: The only equivalence relation on A which is also a surjection is the diagonal relation on A.

34. Ans: (b)

Sol: Given that f = f^{-1}

we have (f o f^{-1}) x = x

$\Rightarrow$ (f o f)x = x $\quad$ ($\because$ f = f^{-1})

$\Rightarrow$ f(f(x)) = x

$\Rightarrow$ f(1+kx) = x

$\Rightarrow$ 1+ k(1+kx) = x

$\Rightarrow$ 1 + k + k^2x = x

$\Rightarrow$ k = – 1

35. Ans: (a)

Sol: (f o g) (x) = f{g(x)}

$$= f\left(\frac{2x}{x-1}\right) = x$$

And (g o f)x = g{f(x)}

$$g\left(\frac{x}{x-2}\right) = x$$

But (f o g) = I_B and (g o f) = I_A

$\therefore$ (f o g) $\neq$ (g o f)

36. Ans: (d)

Sol: (a) Let (g o f) a = (g o f) b

$\Rightarrow$ g{f(a)} = g(f(b))

$\Rightarrow$ f(a) = f(b) $\quad$ ($\because$ g is 1–1)

$\Rightarrow$ a = b $\quad$ ($\because$ f is 1–1)

$\Rightarrow$ (g o f) is 1–1

(b) Let $c \in C$

Since g is on-to, then exists an element $b \in B$, such that

$g(b) = c$ ………(i)

Since f is on-to, there exists an element $a \in A$, such that

$f(a) = b$ ……..(ii)

From (i) and (ii), we have

To each element $c \in C$, there exists an element $a \in A$, such that

$(g \circ f)\, a = c$

$\Rightarrow (g \circ f)$ is on-to

(c) If $(g \circ f)$ is 1–1, then f is also 1–1. Otherwise g is not a function.

(d) If (gof) is on-to then f need not be on-to.

$$A \xrightarrow{f} B \xrightarrow{g} C$$

$$\begin{array}{ccccc} a_1 & \to & b_1 & \to & c_1 \\ a_2 & \to & b_2 & \to & c_2 \\ & & b_3 & \nearrow & \end{array}$$

Here (gof) is on-to, but f is not on-to.

37. Ans: (c)

Sol: (a) We have, $4 \otimes_9 7 = 1$ = Identity element

$\therefore$ Inverse of $4 = 7$

(b) The set $\{1, 4, 7\}$ is closed with respect to $\otimes_9$. Therefore it is a sub group of G.

(c) We have, $2 \otimes_9 8 = 7 \neq$ Identity element

$\therefore$ Inverse of $2 \neq 8$

(d) The set $\{1, 8\}$ is closed with respect to $\otimes_8$. Therefore, it is a sub group.

38. Ans: 1

Sol: Let e be the identity element.

$\Rightarrow a * e = a \ \ \forall\, a \in A$

$\Rightarrow 2\,ae = a$

$\Rightarrow e = \frac{1}{2}$

Let b = inverse of $\frac{1}{4}$

$\Rightarrow b * \frac{1}{4} = e$

$\Rightarrow 2b\,\frac{1}{4} = \frac{1}{2}$

$\Rightarrow \quad b = 1$

$\therefore$ Inverse of $\frac{1}{4} = 1$

39. Ans: (c)

Sol: Order of the cyclic group = 10

Number of generators in G $=\phi(10)=$ Number of positive integers which are less than 10 and coprime to 10 = 4

40. Ans: (c)

Sol: (a) The set $\{1, 4\}$ is closed w.r.t. the given binary operation.

$\therefore$ It is subgroup of G

(b) The set $\{1, 11\}$ is closed w.r.t. the given binary operation.

$\therefore$ It is a subgroup of G.

(c) The set {1, 13} is not closed w.r.t. the given binary operation.

$\therefore$ The set is not a subgroup of G.

(d) The set {1, 14} is closed w.r.t. the given binary operation.

$\therefore$ It is a sub group of G.

41. Ans: (c)

Sol: We have $(A*B) \in P(S) \quad \forall A,B \in P(S)$

$\therefore$ * is a closed operation.

The symmetric difference operation * is associative.

The empty set $\phi \in P(S)$ and ϕ is identity element.

We have $A^{-1} = A \quad \forall A \in P(S)$

$\therefore (P(S),*)$ is a group.

42. Ans: (c)

Sol: From the table

$b * b = b$

$\Rightarrow$ b is identity element

$\Rightarrow$ The second row is a b c and

The second column is a b c

Now c cannot appear in the first row and 3rd column.

$\Rightarrow a * a = c$ and $a * c = b$

$\Rightarrow$ The first row is c a b

$\therefore$ Third row is b c a

43. Ans: (c)

Sol: {1,3} is closed with respect to *

$\therefore$ {1,3} is a group.

44. Ans: (c)

Sol: $(a*b)^2 = (a*b)*(a*b)$

$= a*(b*b)*a$

$= (a*a)*(b*b) \quad (\because$ G is abelian)

$= a^2 * b^2$

45. Ans: (a)

Sol: Let e be the identity element

$a * e = a + e - ae = a$

$\Rightarrow e(1-a) = 0 \quad \forall\, x \in Q-\{1\}$

$\Rightarrow e = 0$

46. Ans: (b)

Sol: $a * a^{-1} = e$

$\Rightarrow a + a^{-1} - aa^{-1} = 0$

$\Rightarrow a^{-1} = \dfrac{a}{a-1}$

47. Ans: (a)

Sol: (a) $2 \otimes_{13} 7 = 1$ (e is identity element)

$\therefore$ inverse of 2 = 7

48. Ans: (b)

Sol: We have

$a * b = \text{g.c.d of } \{a, b\} \in D_{12} \quad \forall\, a, b \in D_{12}$

$\therefore$ * is a closed operation on D_{12}

* is associative on D_{12}

We have $a * 12 = a \quad \forall a$

The identity element = 12

The inverse of any element in D_{12} except 12 does not exist.

$\therefore (D_{12};*)$ is a monoid but not a group.

49. Ans: (c)

Sol: Let e be the identity element.

$\Rightarrow a * e = a \quad \forall a \in R$

$\Rightarrow a + e + 2 = a$

$\Rightarrow e = -2$

We have, $a * a^{-1} = e$

$\Rightarrow a + a^{-1} + 2 = -2$

$\Rightarrow a^{-1} = (-4 - a)$

$\therefore$ Inverse of $2 = (-4 - 2) = -6$

50. Ans: (d)

Sol: We have $x * y = x^y \in N$

$\Rightarrow *$ is a closed operation

$(a*b)*c = a^b * c = \left(a^b\right)^c = a^{bc}$

$a*(b*c) = a * b^c = a^{\left(b^c\right)}$

$\therefore *$ is not associative on N.

Hence, (N, *) is not a semi-group.

51. Ans: (d)

Sol: The identity element = 6

We have, $2^2 = 2 \otimes_{10} 2 = 4$

$2^3 = 2^2 \otimes_{10} 2 = 4 \otimes_{10} 2 = 8$

$2^4 = 2^3 \otimes_{10} 2 = 6$

$\therefore$ 2 is a generator

52. Ans: (d)

Sol: (a) sum and product of any two even numbers are also even.

Further, addition and multiplication operations are associative.

$\therefore$ (A, .) and (A, +) are semi-groups.

Sum of 2 odd numbers is always even.

$\therefore$ + is not a closed operation on B.

$\therefore$ (B, +) is not a semi-group.

53. Ans: (a)

Sol: $f(x + y) = 2^{x+y} = 2^x . 2^y = f(x). f(y)$

$\therefore$ f is homomorphism.

Further $f(x) = 2^x$ is a bijection.

$\therefore$ f is an isomorphism.

5. Probability

01. Ans: (c)

Sol: Four numbers can be selected out of 40 in $^{40}C_4 = 37 \times 38 \times 65$ ways.

E: Event that the four numbers are consecutive.

Favourable cases to E: (1 2, 3,4), (2,3,4,5), (3,4,5,).......(37,38,39,40) whose number is 37

$$\therefore P(E) = \frac{37}{^{40}C_4} = \frac{1}{2470}$$

$$\therefore \text{Required probability} = P\left(\overline{E}\right)$$

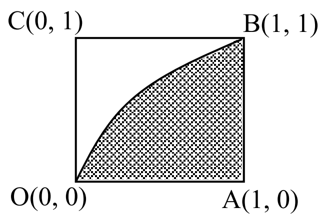
$$= 1 - P(E)$$

$$= 1 - \frac{1}{2470}$$

$$= \frac{2469}{2470}$$

02. Ans: (c)

Sol: The sample space is a square whose sides are unit segments of the coordinate axes. The figure whose set of points correspond to the outcomes favourable to the event $y^2 \le x$ is bounded by the graphs of the function and $y^2 = x$, $y = 0$ and $x = 1$ is shown below.



Required Probability = area of the shaded region $= \int_0^1 \sqrt{x}\, dx = \frac{2}{3}$

03. Ans: (c)

Sol: Let $x \in S$. Then either

$x \in P$, $x \in Q$, or $x \notin P$, $x \in Q$, or $x \in P$, $x \notin Q$ or $x \notin P$, $x \notin Q$. Out of the above four cases, three cases are favourable to the event $P \cap Q = \phi$.

$\therefore$ The required probability $= \left(\frac{3}{4}\right)^{20}$

04. Ans: (b)

Sol: Let

A = The event that 5 appears in first throw

B = The event that sum is 6

The cases favourable to B are

$\{(5, 5, 6), (5, 6, 5), (6, 5, 5), (4, 6, 6), (6, 4, 6), (6, 6, 4)\}$

$A \cap B = \{(5, 5, 6), (5, 6, 5)\}$

Required probability $= P(A|B)$

$$= \frac{n(A \cap B)}{n(B)} = \frac{2}{6} = \frac{1}{3}$$

05. Ans: (a)

Sol: The total number of five digit numbers formed by 1, 2, 3, 4 and 5 (without repetition) $= \angle 5 = 120$

A number is divisible by 4 if the last two digit number (i.e., tens and unit place) is divisible 4.

$\therefore$ The last two digit number must be: 12, 24, 32 and 52 (4 cases). With last two digits fixed, the other three places can be arranged in $\angle 3 (=6)$ ways.

$\therefore$ The number of favourable cases $= \angle 3 \times 4$

$= 24$

$\therefore$ Probability $= \frac{24}{120} = \frac{1}{5}$

06. Ans: (c)

Sol: Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c and d can take values 0 or 1.

$\therefore$ Total number of such matrices = $2^4 = 16$

Let E be the event that A is non singular.

$\therefore$ det A $\neq 0$.

i.e., atleast one of the two numbers a & d is zero or atleast one of the two numbers b & c is zero.

The matrices whose determinants are non zero are:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$\therefore$ $P(E) = \frac{6}{16} = \frac{3}{8}$

07. Ans: (d)

Sol: Total number of triangles that can be formed by using the vertices of a regular hexagon = ${}^6C_3 = 20$.

Among these, there are only two equilateral triangles.

$\therefore$ Required probability $= \frac{2}{20}$

$= \frac{1}{10}$

08. Ans: 0.4

Sol: If A and B are independent then

$P(A \cap B) = P(A).\ P(B) = 0.16$ .... (1)

By Addition theorem of probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.64 = P(A) + P(B) - 0.16$

$\Rightarrow P(A) + P(B) = 0.8$ .... (2)

From (1) & (2), we get

$P(A) = P(B)$

$= 0.4$

09. Ans: (a)

Sol: We have

$P(A) = P(B) = P(C) = \frac{18}{16} = \frac{1}{2}$

$P(A \cap B) = P(A \cap C) = P(B \cap C)$

$= \frac{9}{36}$

$= \frac{1}{4}$

Thus $P(A \cap B) = \frac{1}{4} = P(A)\ P(B)$

$P(A \cap C) = \frac{1}{4} = P(A)P(C)$

$P(B \cap C) = \frac{1}{4} = P(B)P(C)$

which indicates that A, B, and C are pair wise independent. However, since the sum of two numbers is even,

$\{A \cap B \cap C) = \phi$ and $P(A)\ P(B)\ P(C) = \frac{1}{8}$

$\therefore$ $P(A \cap B \cap C) \neq P(A)P(B)P(C)$

which shows that A, B, and C are not independent.

10. Ans: 0.46

Sol: Let

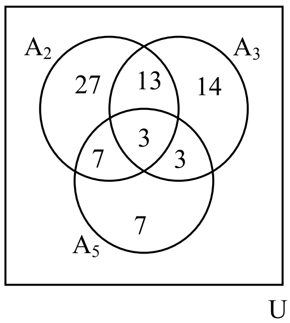
A_2 = event that the number is divisible by 2

A_3 = event that the number is divisible by 3

A_5 = event that the number is divisible by 5

Then the required probability

$$= P\{(A_3 \cup A_5) \mid A_2\}$$



$$= \frac{n[(A_3 \cup A_5) \cap A_2]}{n(A_2)}$$

$$= \frac{23}{50} = 0.46$$

11. Ans: (c)

Sol: Let E_1, E_2, E_3 be the events of selecting urns U_1, U_2, U_3 respectively and W be the event of the drawn ball is white.

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

By Total theorem of probability P(W)=

$$= P(E_1)\,P\left(\frac{W}{E_1}\right) + P(E_2)\,P\left(\frac{W}{E_2}\right) + P(E_3)\,P\left(\frac{W}{E_3}\right)$$

$$= \frac{1}{3}\left(\frac{2}{5}\right) + \frac{1}{3}\left(\frac{3}{5}\right) + \frac{1}{3}\left(\frac{4}{5}\right)$$

$$= \frac{9}{15} = \frac{3}{5}$$

12. Ans: (a)

Sol: Let A, B and C denote events of a bolt manufactured by A, B and C.

Let D be the event of the drawn bolt is defective.

By Total theorem of probability P(D)

$$= P(A)\,P\left(\frac{D}{A}\right) + P(B)\,P\left(\frac{D}{B}\right) + P(C)\,P\left(\frac{D}{C}\right)$$

$$= \frac{25}{100}\left(\frac{5}{100}\right) + \frac{35}{100}\left(\frac{4}{100}\right) + \frac{40}{100}\left(\frac{2}{100}\right)$$

$$= \frac{69}{2000}$$

13. Ans: (c)

Sol: E : Correct diagnosis

$\overline{E}$: Wrong diagnosis

D : Event of death.

$$P(E) = \frac{60}{100} = \frac{3}{5}, \quad P(\overline{E}) = \frac{2}{5}$$

$$P\left(\frac{D}{E}\right) = \frac{70}{100} = \frac{7}{10}, \quad P\left(\frac{D}{\overline{E}}\right) = \frac{80}{100} = \frac{4}{5}$$

By Baye's theorem,

Required probability $= P\left(\frac{E}{D}\right)$

$$= \frac{P(E)P\left(\frac{D}{E}\right)}{P(E)P\left(\frac{D}{E}\right) + P(\overline{E})P\left(\frac{D}{\overline{E}}\right)}$$

$$= \frac{\frac{3}{5}\times\frac{7}{10}}{\frac{3}{5}\times\frac{7}{10}+\frac{2}{5}\times\frac{4}{5}}$$

$$= \frac{21}{37}$$

14. Ans: (b)

Sol: Let E_1 bet the event of guessing, E_2 the event of copying and E_3 the event of knowing the answer.

$\therefore$ $P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{1}{6}$, $P(E_3)$

$$= 1 - \left(\frac{1}{3}+\frac{1}{6}\right) = \frac{1}{2}$$

Let E be the event of writing correct answer.

$P\left(\frac{E}{E_1}\right) = \frac{1}{4}$, $P\left(\frac{E}{E_2}\right) = \frac{1}{8}$ (Given)

$P\left(\frac{E}{E_3}\right) = 1$

By Baye's theorem,

Required probability $= P\left(\frac{E_3}{E}\right)$

$$= \frac{P(E_3)P\left(\frac{E}{E_3}\right)}{\sum_{j=1}^{3} P(E_j)\,P\left(\frac{E}{E_j}\right)}$$

$$= \frac{\frac{1}{2}(1)}{\frac{1}{3}\times\frac{1}{4}+\frac{1}{6}\times\frac{1}{8}+\frac{1}{2}\times 1}$$

$$= \frac{24}{29}$$

15. Ans: (c)

Sol: Let E_j be the event that the bag contains j number of red balls (j = 1, 2, 3, 4)

$\therefore$ $P(E_j) = \frac{1}{4}$ (j = 1, 2, 3, 4)

Let E be the event of drawing a red ball.

$P\left(\frac{E}{E_1}\right) = \frac{1}{4}$, $P\left(\frac{E}{E_2}\right) = \frac{2}{4}$, $P\left(\frac{E}{E_3}\right) = \frac{3}{4}$

$P\left(\frac{E}{E_4}\right) = \frac{4}{4} = 1$

$\therefore$ By Baye's theorem,

$$P\left(\frac{E_4}{E}\right) = \frac{P(E_4)\,P\left(\frac{E}{E_4}\right)}{\sum_{j=1}^{4} P(E_j)P\left(\frac{E}{E_j}\right)}$$

$$= \frac{\frac{1}{4}\times 1}{\frac{1}{4}\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+\frac{4}{4}\right)}$$

$$= \frac{2}{5}$$

16. Ans: (d)

Sol: **E_1:** Event of letter coming from LONDON.

E_2: Event of the letter coming from CLIFTON.

E: Event of two consecutive letters ON.

$P(E_1) = P(E_2) = \frac{1}{2}$.

Word LONDON consists of 5 pairs of consecutive letters

(LO, ON, ND, DO, ON) out of which there are 2 ON's.

CLIFTON consists of 6 pairs of consecutive letters

(CL, LI, IF, FT, TO, ON) out of which there is only one 'ON'.

$$\therefore\ P\left(\frac{E_1}{E}\right) = \frac{P(E_1)P\left(\frac{E}{E_1}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{2}{5}}{\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{1}{6}}$$

$$= \frac{12}{17}$$

17. Ans: (c)

Sol: Total probability $= \sum_{d=1}^{4} C\left(\frac{2^d}{\angle d}\right) = 1$

$$\Rightarrow C\ (2 + 2 + \frac{4}{3} + \frac{2}{3}) = 1$$

$$\Rightarrow C = \frac{1}{6}$$

Expected demand = E (D)

$$= \sum_{d=1}^{4} d.P(D = d)$$

$$= 1\left(\frac{2}{6}\right) + 2\left(\frac{2}{6}\right) + 3\left(\frac{4}{8}\right) + 4\left(\frac{2}{18}\right)$$

$$= \left(\frac{19}{9}\right)$$

18. Ans: 5

Sol: Let X = Amount the player wins in rupees

The probability distribution for X is given below

Number of heads	0	1	2
X	x	1	3
P(X)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

For the game to be fair we have to find x, so that E(X) = 0

$$\Rightarrow x.\left(\frac{1}{4}\right) + 1.\left(\frac{2}{4}\right) + 3.\left(\frac{1}{4}\right) = 0$$

$\Rightarrow x = 5$

$\therefore$ Number of rupees, the player has to lose, if no heads occur = 5.

19. Ans: (b)

Sol: P (X is even) = P (X = 2) + P (X = 4) + P (X = 6) + ….∞

$$= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + ...\infty$$

$$=\frac{1}{2^2}\left[1+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^3+.......\infty\right]$$

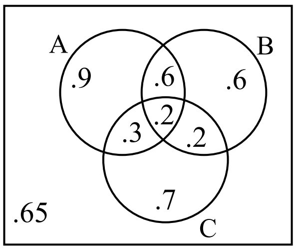
$$=\frac{1}{4}\left(1-\frac{1}{4}\right)^{-1}=\frac{1}{3}$$

20. Ans: 0.62857 Range(0.62 to 0.63)

Sol: Let E_1 = The selected reader is reading only one news papers

E_2 = The selected reader is reading atleast one of the newspapers

The Venn diagram for the given data is



Required probability = $P(E_1|E_2)$

$$=\frac{P(E_1\cap E_2)}{P(E_2)}=\frac{P(E_1)}{P(E_2)}$$

$$=\frac{0.22}{0.35}=0.62857$$

21. Ans: (b)

Sol: Here f (x) is an even function

$$E(X)=\int_{-\infty}^{\infty}x.f(x)\,dx=0$$

($\because$ x f(x) is an odd function)

$$E(X^2)=\int_{-\infty}^{\infty}x^2 f(x)\,dx$$

$$=\int_{-1}^{0}x^2(1+x)\,dx+\int_{0}^{1}x^2(1+x)\,dx=\frac{1}{6}$$

Variance of X = $E(X^2)-(E(X))^2=\frac{1}{6}$

22. Ans: (a)

Sol: Required probability

$$=C(10,5).\left(\frac{1}{2}\right)^5.\left(\frac{1}{2}\right)^5.1$$

$$=\frac{{}^{10}C_5}{2^{10}}$$

23. Ans: (c)

Sol: If the person is one step away, then we have two cases:

Case1: 6 forward steps and 5 backward

or

Case2: 6 backward steps and 5 forward.

Required Probability

$= C(11,6)(0.4)^6 + C(11,5)(0.6)^6\,(0.4)^5$

$= C(11,5)\,(0.4)^5\,(0.6)^5\,(0.4 +0.6)$

$= 462 \times (0.24)^5$

24. Ans: (d)

Sol: E_1 = Event of writing good book

E_2 = Event of not writing a good book

E = Probability of publication

$P(E_1) = P(E_2) = \frac{1}{2}$, $P\left(\frac{E}{E_1}\right)=\frac{2}{3}$,

$$P\left(\frac{E}{E_2}\right) = \frac{1}{4}$$

$P(E) = P(E \cap E_1) + P(E \cap E_2)$

$$= \frac{1}{2}.\frac{2}{3} + \frac{1}{2}.\frac{1}{4}$$

$$= \frac{11}{24}$$

X denote the number of books published.

$\therefore$ Required probability =

$P(X = 1) + P(X = 2)$

$$= {}^2C_1 \frac{11}{24} \times \frac{13}{24} + {}^2C_2 \left(\frac{11}{24}\right)^2$$

$$= 2 \times \frac{11}{24} \times \frac{13}{24} + \left(\frac{11}{24}\right)^2$$

$$= \frac{407}{576}$$

25. Ans: (a)

Sol: $P(A) = P(B) = P(C) = \frac{1}{3}$

E = Event of getting 2 heads and 1 tail

$$P\left(\frac{E}{A}\right) = {}^3C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right) = \frac{3}{8}$$

$$P\left(\frac{E}{B}\right) = {}^3C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right) = \frac{4}{9}$$

$$P\left(\frac{E}{C}\right) = {}^3C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right) = \frac{2}{9}$$

Required probability

$$P\left(\frac{A}{E}\right) = \frac{P(A)P\left(\frac{E}{A}\right)}{P(A)P\left(\frac{E}{A}\right) + P(B)P\left(\frac{E}{B}\right) + P(C)P\left(\frac{E}{C}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{3} \times \frac{3}{8} + \frac{1}{3} \times \frac{4}{9} + \frac{1}{3} \times \frac{2}{9}} = \frac{27}{75} = \frac{9}{25}$$

26. Ans: (b)

Sol: Let X = Number of times we have to toss a pair of dice.

P = probability of getting 7 in one throw

$$= \frac{1}{6}$$

$q = 1 - P$

= probability of not getting 7 in one throw

$$= \frac{5}{6}$$

q^6 = probability of not getting a 6 in 6 throws

$P(X \le 6)$ = Probability that it take less than 6 tosses to get a 7

$$= 1 - \left(\frac{5}{6}\right)^6$$

Required probability = $P(X > 6)$

$$= 1 - \left\{1 - \left(\frac{5}{6}\right)^6\right\} = \left(\frac{5}{6}\right)^6$$

$$\approx 0.335$$

27. Ans: 7

Sol: The probability of missing the target is $q = 1 - p = 0.7$. Hence the probability that n missiles miss the target is $(0.7)^n$. Thus, we seek the smallest n for which

$1 - (0.7)^n > 0.90$ or

equivalently $(0.7)^n < 0.10$

Compute

$(0.7)^1 = 0.7$, $(0.7)^2 = 0.49$, $(0.7)^3 = 0.343$,

$(0.7)^4 = 0.240$, $(0.7)^5 = 0.168$

$(0.7)^6 = 0.118$, $(0.7)^7 = 0.0823$

Thus, atleast 7 missiles should be fired.

28. Ans: (b)

Sol: Let X = number of accidents between 5 P.M and 6 P.M.

For Poisson distribution,

$\lambda = np = (1000)(0.0001) = 0.1$

$$P(X = x) = \frac{e^{-\lambda}.\lambda^x}{\angle x} \qquad (x = 0, 1, 2, \ldots\ldots)$$

Required Probability $= P(X \geq 2)$

$= 1 - P(X < 2)$

$= 1 - \{P(X = 0) + P(X = 1)\}$

$= 1 - e^{-0.1}(1 + 0.1)$

$= 0.0045$

29. Ans: 0.122

Sol: We view the number of misprints on one page as the number of successes in a sequence of Bernoulli trials. Here n = 300 since there are 300 misprints, and $p = \frac{1}{500}$, the probability that a misprint appears on a given page. Since p is small, we use the Poisson approximation to the binomial distribution with $\lambda = np = 0.6$.

We have

P(0misprint) = f(0; 0.6)

$$= \frac{(0.6)^0 e^{-0.6}}{0!} = e^{-0.6} = 0.549$$

P(1 misprint) = f(1; 0.6)

$$= \frac{(0.6)^1 e^{-0.6}}{1!} = (0.6)(0.549)$$

$= 0.329$

Required probability

$= 1 - (0.549 + 0.329)$

$= 0.122$

30. Ans: 0.1353

Sol: Given that λ = 900 vehicles/hour

= 1 vehicle/ 4 sec = 2 vehicles/8 sec

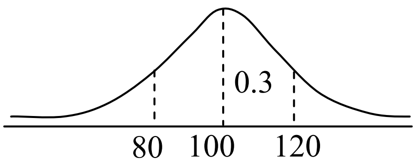
Probability for k vehicles in a time gap of 8 seconds $= P(X = k) = \frac{\lambda^k e^{-\pi}}{k!}$

Required probability $= P(X = 0) = e^{-\lambda} = e^{-2}$

$= 0.1353$

31. Ans: (c)

Sol: The area under normal curve is 1 and the curve is symmetric about mean.



$\therefore$ P(100 < X < 120) = P(80 < X < 120)

= 0.3

Now, P(X < 80) = 0.5 – P(80 < X < 120)

= 0.5 – 0.3 = 0.2

32. Ans: 0.7939

Sol: This is a binomial experiment B(n, p) with n = 3500, p = 0.04, and q = 1 – p

= 0.96.

Then $\mu = np = (3500)\,(0.04) = 140$,

$\sigma^2 = npq = (3500)(0.04)\,(0.96)$

= 134.4,

$\sigma = \sqrt{134.4} = 11.6$

Let X denote the number of people with Alzheimer's disease.

We seek BP(X < 150) or, approximately, NP (X ≤ 149.5). (BP denote Binomial Probability and NP denote Normal Probability)

We have 149.5 in standard units

$$= \frac{(149.5 - 140)}{11.6}$$

= 0.82

Therefore,

Required Probability = NP (X ≤ 149.5)

= NP(Z ≤ 0.82) = 0.5000 + ϕ(0.82)

= 0.5000 + 0.2939

= 0.7939

33. Ans: 2

Sol: $f(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-z^2}{2}}$ is the probability density function of Normal Distribution

$$\therefore \int_{-\infty}^{\infty} f(z)\, dz = 1$$

$\therefore$ The value of given integral

$$= 2\int_{-\infty}^{\infty} f(z)\, dz$$

= 2

34. Ans: 0.3085

Sol: Let X = diameter of cable in inches

mean = μ = 0.80

Standard deviation = $\sigma = \sqrt{0.0004}$ = 0.02

The standard normal variable $Z = \frac{X - \mu}{\sigma}$

When X = 0.81, $Z = \frac{0.81 - 0.80}{0.02} = \frac{1}{2}$

Required probability = P (X > 0.81)

$= P(Z > \frac{1}{2})$

=1 – (Area under the normal curve to the left of Z = 0.5)

= 1 – 0.6915 = 0.3085

35. Ans: (i) 28 (ii) 28 (iii) 205

Sol: The parameters of normal distribution are $\mu = 68$ and $\sigma = 3$

Let X = weight of student in kgs

Standard normal variable = $Z = \dfrac{X-\mu}{\sigma}$

(i) When X = 72, we have Z = 1.33

Required probability = P(X > 72)

= Area under the normal curve to the right of Z = 1.33

= 0.5 – (Area under the normal curve between Z = 0 and Z = 1.33)

= 0.5 – 0.4082

= 0.0918

Expected number of students who weigh greater than 72 kgs = 300 × 0.0918 = 28

(ii) When X = 64, we have Z = –1.33

Required probability = $P(X \le 64)$

= Area under the normal curve to the left of Z = – 1.33

= 0.5 – (Area under the normal curve between Z = 0 and Z = 1.33)

(By symmetry of normal curve)

= 0.5 – 0.4082

= 0.0918

Expected number of students who weigh less than 68 kgs = 300 × 0.0918

= 28

(iii) When X = 65, we have Z = –1

When X = 71, we have Z = +1

Required probability = P(65< X < 71)

= Area under the normal curve to the left of Z = – 1 and Z = +1

= 0.6826

(By Property of normal curve)

Expected number of students who weighs between 65 and 71 kgs

= 300 × 0.6826

≈ 205

36. Ans: (b)

Sol: If X has uniform distribution in [a, b] then

$$\text{variance} = \frac{(b-a)^2}{12}$$

$$= \frac{[3a-(-a)]^2}{12} = \frac{16a^2}{12} = \frac{4a^2}{3}$$

37. Ans: (b)

Sol: Let X be a uniformly distributed random variable defined on [a, b].

Mean is $\dfrac{a+b}{2} = 1 \Rightarrow a + b = 2$ ……. (1)

Variance is $\dfrac{(b-a)^2}{12} = \dfrac{1}{3} \Rightarrow b - a = 2$ …..(2)

On solving, we get a = 0, b = 2

$\therefore$ The PDF of f(x) is = $\dfrac{1}{b-a}$, $a \le x \le b$

$= \dfrac{1}{2}$, $0 \le x \le 2$

$$P(X < \frac{1}{2}) \int_0^{\frac{1}{2}} f(x)dx = \int_0^{\frac{1}{2}} \frac{1}{2} dx = \frac{1}{4}$$

38. Ans: (c)

Sol: $f(x) = \frac{1}{4}, -2 \le X \le 2$

$|X-1| \ge \frac{1}{2}$

$= \left(\frac{-1}{2} \le (X-1) < -2\right) \cup \left(\frac{1}{2} \le (X-1) < 2\right)$

$|X-1| \ge \frac{1}{2}$

$= \left(-1 < X \le \frac{1}{2}\right) + \left(\frac{3}{2} \le X < 3\right)$

$P\left(|X-1| \ge \frac{1}{2}\right)$

$= P\left(-1 < X \le \frac{1}{2}\right) + P\left(\frac{3}{2} \le X < 3\right)$

$= \int_{-1}^{\frac{1}{2}} f(x)dx + \int_{\frac{3}{2}}^{3} f(x)dx$

$= \int_{-1}^{\frac{1}{2}} \frac{1}{4} dx + \int_{\frac{3}{2}}^{3} \frac{1}{4} dx$

$= \frac{1}{4}\left(\frac{1}{2} + 1 + 3 - \frac{3}{2}\right)$

$= \frac{3}{4}$

39. Ans: (b)

Sol: The probability density function of X is

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0 & , x \le 0 \end{cases}$$

$$P(X<5) = \int_0^5 f(x)\,dx$$

$$= \int_0^5 \frac{1}{10} e^{\frac{-x}{10}} dx \approx 0.393$$

40. Ans: (a)

Sol: Mean $\frac{1}{\theta} = 0.5 \Rightarrow \theta = \frac{1}{0.5} = 2$

The probability function of exponential distribution is $f(x) = 2e^{-2x}, x \ge 0$.

$$P(X > \frac{1}{2}) = \int_{\frac{1}{2}}^{\infty} 2e^{-2x} dx = \left(-e^{-2x}\right)_{\frac{1}{2}}^{\infty}$$

$$= (0) - \{-e^{-1}\} = e^{-1}$$

41. Ans: (d)

Sol: The density function $f(x) = \frac{1}{5} e^{\frac{-1}{5}x}$

We require $P(x>8) = \int_8^{\infty} f(x)\,dx = e^{-8/5}$

$= 0.2$

42. Ans: Mean = 34, Median = 35,
Modes = 35, 36 & SD = 4.14

Sol: Mean $= \frac{\sum x_i}{n} = 34$

Median is the middle most value of the data by keeping the data points in increasing order or decreasing order.

Mode = 36

S.D = 4.14

43. Ans: (b)

Sol: Mean = $\Sigma x_i P_i = 3$

Variance = $\sum x_i^2 p_i - \mu^2 = 10.2 - 9 = 1.2$

44. Ans: k = 6, Mean = $\frac{1}{2}$, Median = $\frac{1}{2}$,

Mode = $\frac{1}{2}$ and S.D = $\frac{1}{2\sqrt{5}}$

Sol: we have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_0^1 k(x - x^2)\,dx = 1$$

$$\Rightarrow k\left[\left(\frac{x^2}{2}\right)_0^1 - \left(\frac{x^3}{3}\right)_0^1\right] = 1$$

$$\Rightarrow k\left(\frac{1}{2} - \frac{1}{3}\right) = 1$$

$$\Rightarrow k\left(\frac{3-2}{6}\right) = 1$$

$$\Rightarrow k = 6$$

$$\text{Mean} = \int_{-\infty}^{\infty} x f(x)\,dx = \int_0^1 6(x^2 - x^3)\,dx$$

$$= 6\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 6\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{1}{2}$$

Median is that value 'a' for which

$$P(X \le a) = \frac{1}{2}$$

$$\int_0^a 6(x - x^2)\,dx = \frac{1}{2}$$

$$\Rightarrow 6\left(\frac{a^2}{2} - \frac{a^3}{3}\right) = \frac{1}{2}$$

$$\Rightarrow 3a^2 - 2a^3 = \frac{1}{2} \Rightarrow a = \frac{1}{2}$$

Mode a that value at which f(x) is max/min

$\therefore$ $f(x) = 6x - 6x^2$

$f^1(x) = 6 - 12x$

For max or min $f^1(x) = 0 \Rightarrow 6 - 12x = 0$

$$\Rightarrow \quad x = \frac{1}{2}$$

$f^{11}(x) = -12 \qquad f^{11}\left(\frac{1}{2}\right) = -12 < 0$

$\therefore$ maximum at $x = \frac{1}{2}$

$\therefore$ mode is $\frac{1}{2}$

S.D = $\sqrt{E(x^2) - (E(x))^2}$

$= \frac{1}{2\sqrt{5}}$

6. Linear Algebra

01. Ans: (b)

Sol: Given that P is 10×5 matrix.

Q is 5×20 matrix

and R is 20×10 matrix

Now PQR is 10×10 matrix. Total number of elements in PQR = 100. Here, we can find the product PQR only in two ways i.e., (PQ)R and P(QR) because $PQ \neq QP$.

So, to find the product matrix PQR first we find PQ and then find (PQ)R (or) we, first find QR and then find P(QR).

For the product $(PQ)_{10\times 20}$

Number of elements in PQ = 200.

To compute each element of the matrix PQ, we require '5' multiplications.

$\therefore$ Number of multiplications = 200×5

= 1000

For the product $[(PQ)R]_{10\times10}$

Number of elements in (PQ) R = 100

To compute each element of the matrix (PQ)R, we require 20 multiplications.

$\therefore$ Number of multiplications = 100×20

= 2000

Hence, the total number of multiplication operations to find the product $[(PQ)R]_{10\times10}$

= 1000+2000

= 3000

Similarly, if we find the product $[P(QR)]_{10\times10}$ by above method, the total number of multiplication operations to find the product $[P(QR)]_{10\times10}$ = 1000 + 500

= 1500

$\therefore$ The minimum number of multiplication operations to find PQR= 1500.

02. Ans: (d)

Sol: Giving that

$(I - A + A^2 - ....+ (-1)^n A^n) = O$ .... (i)

multiplying by A^{-1}

$A^{-1} - I + A - A^2 + ...+ (-1)^{n-1}A^{n-1} = 0$ ...... (ii)

Adding (i) & (ii), we get

$$A^{-1} + (-1)^n A^n = O$$

$\therefore A^{-1} = (-1)^{(n-1)}.\ A^n$

03. Ans: (b)

Sol: Here determinant of A = –8

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$\Rightarrow c = \dfrac{-1}{8}$ (cofactor of the element 6 in A)

$$= \frac{-1}{8}.\left(-1^{3+1}\right)\begin{vmatrix} 2 & 3 \\ 0 & 4 \end{vmatrix} = -1$$

04. Ans: 324

Sol: Det $M_r = 2r - 1$

Det M_1 + Det M_2 + ... Det M_{18}

= 1 + 3 + 5 + ...+ 37

= 324

05. Ans: –3

Sol: Given that $|A|^{10} = 2^{10}$

$\Rightarrow |A| = \pm 2$

$\Rightarrow -\alpha^3 - 25 = \pm 2$

$\Rightarrow \alpha^3 = -27 \quad \text{or} \quad \alpha^3 = -23$

$\Rightarrow \alpha = -3 \quad \text{or} \quad \alpha = (-23)^{\frac{1}{3}}$

06. Ans: 8

Sol: Given that $\sum_{n=1}^{k} A_n = 72$

$$\Rightarrow \begin{vmatrix} k & k & k \\ k^2+k & k^2+k+1 & k^2+k \\ k^2 & k^2 & k^2+k+1 \end{vmatrix} = 72$$

$C_2 \to (C_2 - C_1)$, $C_3 \to C_3 - C_1$

$$\Rightarrow \begin{vmatrix} k & 0 & 0 \\ k^2+k & 1 & 0 \\ k^2 & 0 & k+1 \end{vmatrix} = 72$$

$\Rightarrow k(k+1) = 72u$

$\Rightarrow k = 8$

07. Ans: $\frac{1}{2}$

Sol: Given that $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

$R_2 - R_1$, $R_3 - R_1$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & \sin\theta & 1 \\ 0 & 0 & \cos\theta \end{vmatrix}$$

$= \sin\theta . \cos\theta$

$= \frac{\sin 2\theta}{2}$

$\therefore$ maximum value of $\Delta = \frac{1}{2}$

08. Ans: 0

Sol: Given that

$$f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & x \end{vmatrix}$$

applying $\frac{R_2}{x}$ and $\frac{R_3}{x}$

$$\frac{f(x)}{x^2} = \begin{vmatrix} \cos x & x & 1 \\ \frac{2\sin x}{x} & x & 2 \\ \frac{\tan x}{x} & 1 & 1 \end{vmatrix}$$

$$\underset{x\to 0}{\text{Lt}} \frac{f(x)}{x^2} = \begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

09. Ans: (d)

Sol: In a symmetric matrix, the diagonal elements are zero and $a_{ij} = -a_{ji}$ for $i \neq j$.

Each element above the principal diagonal, we can choose in 3 ways (0, 1, –1).

Number of elements above the principal diagonal $= \frac{n(n-1)}{2}$

$\therefore$ By product rule,

Required number of skew symmetric matrices $= 3^{\frac{n(n-1)}{2}}$.

10. Ans: $\frac{3}{16}$

Sol: Number of 2×2 determinants possible with each entry as 0 or 1 = $2^4 = 16$.

Let $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

If $\Delta > 0$ then a = d = 1 and atleast one of the entries b or c is 0.

$\therefore$ The determinants whose value is +ve are

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} \text{ and } \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}$$

$\therefore$ Required probability = $\frac{3}{16}$

11. Ans: 1

Sol: If the vectors are linearly dependent, then

$$\begin{vmatrix} 1-t & 0 & 0 \\ 1 & 1-t & 0 \\ 1 & 1 & 1-t \end{vmatrix} = 0$$

$\Rightarrow (1-t)^3 = 0$

$\Rightarrow t = 1$

12. Ans: 1

Sol: If the vectors are linearly independent, then

$$\begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & t \\ 0 & 0 & 1 & 0 \end{vmatrix} \neq 0$$

Expanding by third column

$$\Rightarrow (-1)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & t \end{vmatrix} \neq 0$$

$\Rightarrow (-1).\ (1-(t-1)-1) \neq 0$

$\Rightarrow t \neq 1$

13. Ans: (c)

Sol: $\begin{pmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{pmatrix} \sim \begin{pmatrix} -1 & 2 & 5 \\ 0 & 0 & a+6 \\ 0 & 0 & a+6 \end{pmatrix}$

$= \begin{pmatrix} -1 & 2 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ if a = –6 and Rank = 1

If a ≠ –6 then Rank of the matrix is 2

$\therefore$ Option (c) is correct.

14. Ans: (d)

Sol: The characteristic equation is

$$|A - \lambda I| = 0$$

$\Rightarrow (\lambda^2 - 4)(\lambda^2 + 4) = 0$

$\Rightarrow \lambda^4 = 16$

By Caley Hamilton's Theorem

$A^4 = 16I$

15. Ans: 4

Sol: $A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$

$R_4 \to R_4 + R_1$

$$A \sim \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$A \sim \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & +1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_4 \to R_4 + R_2$

$$A \sim \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & +1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

$$A \sim \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_5 \to R_5 - R_4$

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

= Echelon form of A

$\therefore$ Rank of A = number of non-zero rows in Echelon form of 'A' = 4

16. Ans: (b)

Sol: The augmented matrix of the given system is

$$[A|B] = \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & -2 & -1 \end{array}\right]$$

$R_2 - R_1$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 1 \\ 0 & 1 & -1 & 2 & -1 \\ 0 & -1 & 1 & -2 & -1 \end{array}\right]$$

$R_3 + R_2$

$$\sim \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 1 \\ 0 & 1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -2 \end{array}\right]$$

Rank of coefficient matrix A = 2

Rank of [A|B] = 3

$\therefore$ The system has no solution

17. Ans: (c)

Sol: Let the given system be AX = B

The augmented matrix of the system

$$= [A|B] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$R_3 - R_1$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$R_3 - R_2$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

$R_4 + R_3$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

Here $\rho[A] = 3$ and

$\rho[A|B] = 4$

$\therefore$ The system has no solution.

18. Ans: (d)

Sol: $A = \begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$

$R_2 - 5R_1$

$R_3 + 2R_1$

$$\sim \begin{pmatrix} 1 & 1 & 3 \\ 0 & -3 & -9 \\ -0 & 1 & 3 \end{pmatrix}$$

$3R_3 + R_2$

$$\sim \begin{pmatrix} 1 & 1 & 3 \\ 0 & -3 & -9 \\ 0 & 0 & 0 \end{pmatrix}$$

Here $\rho[A] = 2$

If B is a linear combination of columns of A, then $\rho[A] = \rho[A|B]$

$\therefore$ The system has infinitely many solutions

19. Ans: (c)

Sol: If the system has non trivial solution, then

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix} = 0$$

$R_2 - R_1$, $R_3 - R_1$

$$\Rightarrow \begin{vmatrix} a+b+c & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0$$

$$\Rightarrow (a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0$$

$\Rightarrow a + b + c = 0$ or

$a = b = c$

20. Ans: (b)

Sol: Let the given system be AX = B

The augmented matrix of the system

$$= [A|B] = \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 2 & 3 & 3 & b \\ 5 & 9 & -6 & c \end{array}\right]$$

$R_2 - 2R_1$

$R_3 - 5R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & -1 & 9 & c-5a \end{array}\right]$$

$R_3 - R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & -3 & a \\ 0 & -1 & 9 & b-2a \\ 0 & 0 & 0 & c-b-3a \end{array}\right]$$

The system is inconsistent

if $c - b - 3a \neq 0$

$\Rightarrow 3a + b - c \neq 0$

21. Ans: (c)

Sol: Let the given system be AX = B

The augmented matrix of the system =

$$[A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

$R_2 - R_1$, $R_2 - R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

$R_3 - R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-6 \end{array}\right]$$

The system has unique solution if $\lambda \neq 3$.

22. Ans: (d)

Sol: If the system has non trivial solution then

$$\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} = 0$$

$\Rightarrow 1(k - 3) + k.2k + (k–9) = 0$

$\Rightarrow 2k^2 + 2k - 12 = 0$

$\Rightarrow k = 2, \; -3$

23. Ans: 2

Sol: The characteristic equation is

$|A - \lambda I| = 0$

A real eigen value of A is $\lambda = 5$

The eigen vectors for $\lambda = 5$ are given by

$[A - 5I]\, X = 0$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_4 = 0, x_3 = 0$

$\Rightarrow \rho[A] = 2$ and n = 4=number of variables

$\therefore$ The number of linear independent eigen vectors corresponding to $\lambda = 5$ are 2.

24. Ans: 0

Sol: Let $a = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

The characteristic equation is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 4 \\ 4 & -3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda = \pm 5$$

The eigen vectors for $\lambda = 5$ are given by

$[A - 5I]\, X = 0$

$$\Rightarrow \begin{bmatrix} -2 & 4 \\ 4 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x - 2y = 0$$

$$\therefore X_1 = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

The eigen vectors for $\lambda = -5$ are given by

$[A + 5I]\, X = 0$

$$\Rightarrow \begin{bmatrix} 8 & 4 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 2x + y = 0$$

$$\therefore X_2 = c_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\therefore a + b = 0$$

25. Ans: (a)

Sol: Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$

The eigen values of A are 1, 2

The eigen vectors for $\lambda = 1$ are given by

$$[A - I]\, X = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow y = 0$$

$$\therefore X_1 = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The eigen vectors for $\lambda = 2$ are given by

$[A - 2I]\, X = 0$

$$\Rightarrow \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -x + y = 0$$

$$\therefore X_2 = c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$\therefore$ The eigen vector pair is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

26. Ans: (c)

Sol: If A is singular then 0 is an eigen value of A.

$\therefore$ The minimum eigen value of A is 0.

The eigen vectors corresponding to the eigen value $\lambda = 0$ is given by

$$[A - 0I]X = 0$$

$$\Rightarrow \begin{bmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying cross multiplication rule for first and second rows of A, we have

$$\Rightarrow \frac{x}{11} = \frac{y}{-11} = \frac{z}{11}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$$

$\therefore$ The eigen vectors are

$$X = k\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

27. Ans: (b)

Sol: Here, A is the elementary matrix obtained given I_3 with elementary operation $R_1 \leftrightarrow R_3$

$$\therefore A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

The characteristic equation is

$$|A - \lambda I| = 0$$

$$\Rightarrow \lambda \begin{vmatrix} -\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1 - \lambda)(\lambda^2 - 1) = 0 \Rightarrow \lambda = 1, 1, -1$$

28. Ans: –6

Sol: The given matrix has rank 2

$\therefore$ There are only 2 non zero eigen values

The characteristic equation is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 0 & 0 & 0 & 1 \\ 0 & -1-\lambda & -1 & -1 & 0 \\ 0 & -1 & -1-\lambda & -1 & 0 \\ 1 & -1 & -1 & -1-\lambda & 0 \\ 1 & 0 & 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$R_1 \to R_1 + R_5$ and $R_2 \to R_2 + R_3 + R_4$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 0 & 0 & 0 & 2-\lambda \\ 0 & -3-\lambda & -3-\lambda & -3-\lambda & 0 \\ 0 & -1 & -1-\lambda & -1 & 0 \\ 1 & -1 & -1 & -1-\lambda & 0 \\ 1 & 0 & 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow (2 - \lambda).(-3 - \lambda).$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & -1-\lambda & -1 & 0 \\ 1 & -1 & -1 & -1-\lambda & 0 \\ 1 & 0 & 0 & 0 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda = 2, -3$

$\therefore$ product of the non zero eigen values = –6

29. Ans: 3

Sol: If λ is eigen value then $AX = \lambda X$

$$\Rightarrow \begin{bmatrix} 4 & 1 & 2 \\ 17 & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ k \end{bmatrix} = \lambda\begin{bmatrix} 1 \\ 2 \\ k \end{bmatrix}$$

$\Rightarrow 6 + 2k = \lambda$

$21 + k = 2\lambda$

$\Rightarrow 42 + 2k = 4\lambda$

$\lambda = 12$ and $k = 3$

30. Ans: 3

Sol: Sum of the eigen values = Trace of A = 14

$\Rightarrow a + b + 7 = 14$ ...... (i)

product of eigen values = $|A|$ = 100

$\Rightarrow 10ab = 100$

$\Rightarrow ab = 10$ .....(ii)

solving (i) & (ii), we have

$\Rightarrow a = 5$ and $b = 2$

$\therefore |a - b| = 3$

31. Ans: 1

Sol: Product of eigen values = $|A| = 0$

$$\Rightarrow \begin{vmatrix} 3 & 4 & 2 \\ 9 & 13 & 7 \\ -6 & -9+x & -4 \end{vmatrix} = 0$$

$R_2 - 3R_1$, $R_3 + 2R_1$

$$\Rightarrow \begin{vmatrix} 3 & 4 & 2 \\ 0 & 1 & 1 \\ 0 & x-1 & 0 \end{vmatrix} = 0$$

$\Rightarrow 3(1 - x) = 0$

$\Rightarrow x = 1$

32. Ans: (d)

Sol: The characteristic equation is

$\lambda^4 = \lambda$

$\Rightarrow \lambda^4 - \lambda = 0$

$\Rightarrow \lambda(\lambda^3 - 1) = 0$

$\Rightarrow \lambda = 0, 1, -1 \pm \sqrt{3}\, i$

$\Rightarrow \lambda = 0, 1, -0.5 \pm (0.866)i$

33. Ans: 3

Sol: Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

Here, A is upper triangular matrix

The eigen values are $\lambda = 2, 2, 3$

The eigen vectors for $\lambda = 2$ are given by

$[A - 2I]X = 0$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ Here Rank of $[A - 2I] = 1$

$\therefore$ Number of Linearly independent eigen vectors for $\lambda = 2$ is $n - r$

$= 3 - 1 = 2$

For since, $\lambda = 3$ is not a repeated eigen value, there will be only one independent eigen vector for $\lambda = 3$.

$\therefore$ The number of linearly independent eigen vectors of A = 3.

34. Ans: (d)

Sol: The characteristic equation is

$(\lambda^3 - 6\lambda^2 + 9\lambda - 4) = 0$

$|A|$= product of the roots of the characteristic equation = 4

Trace of A = sum of the roots of characteristic equation = 6

35. Ans: (b)

Sol: A is symmetric matrix.

The eigen vectors of A are orthogonal.

For the given eigen vector, only the vector given in option (b) is orthogonal.

$\therefore$ option (b) is correct.

36. Ans: (c)

Sol: The characteristic equation is

$\lambda^3 - 18\lambda^2 + 45\lambda = 0$

$\Rightarrow \lambda = 0, 3, 15$

The eigen vector for $\lambda = 15$ are given by

$[A - 15I]\, X = 0$

$$\Rightarrow \begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \frac{x}{40} = \frac{y}{-40} = \frac{z}{20}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{-2} = \frac{z}{1}$$

$\therefore$ The eigen vectors for $\lambda = 15$ are

$$X = k \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \quad (k \neq 0)$$

37. Ans: (c)

Sol: The characteristic equation is

$$\begin{vmatrix} a-\lambda & 1 & 0 \\ 1 & a-\lambda & 1 \\ 0 & 1 & a-\lambda \end{vmatrix} = 0$$

$\Rightarrow (a - \lambda)\, [\{(a - \lambda)^2 - 1\} - (a - \lambda)] = 0$

$\Rightarrow \lambda = a, a \pm \sqrt{2}$

38. Ans: $\lambda^2 - 3\lambda + 2$

Sol: The characteristic equation is

$$\begin{vmatrix} 5-\lambda & -6 & -6 \\ -1 & 4-\lambda & 2 \\ 3 & -6 & -4-\lambda \end{vmatrix} = 0$$

$\Rightarrow (\lambda - 1)\, (\lambda - 2)^2 = 0$

$\therefore$ Either $(\lambda - 1)\, (\lambda - 2)$ or $(\lambda - 1)\, (\lambda - 2)^2$ is the minimal polynomial

$(A - I)\, (A - 2I)$

$$= \begin{bmatrix} 4 & -6 & -6 \\ -1 & 3 & 2 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 3 & -6 & -6 \\ -1 & 2 & 2 \\ 3 & -6 & -6 \end{bmatrix} = O$$

$\therefore$ The minimal polynomial of A

$= (\lambda - 1)\, (\lambda - 2)$

$= \lambda^2 - 3\lambda + 2$

LU Decomposition

39. Ans: (b)

Sol: The coefficient matrix

$$A = \begin{bmatrix} 4 & 5 \\ 12 & 14 \end{bmatrix}$$

$R_2 - 3R_1$

$$\sim \begin{bmatrix} 4 & 5 \\ 0 & -1 \end{bmatrix}$$

$$\therefore U = \begin{bmatrix} 4 & 5 \\ 0 & -1 \end{bmatrix}$$

40. Ans: (b)

Sol: $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ -3 & -10 & 2 \end{bmatrix}$

$R_2 - 2R_1$

$R_3 + 3R_1$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & -4 & 5 \end{bmatrix}$$

$R_3 - 4R_2$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore U = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

41. Ans: (a)

Sol: $A = \begin{bmatrix} 1 & -3 & 5 \\ 2 & -4 & 7 \\ -1 & -2 & 1 \end{bmatrix}$

$R_2 - 2R_1$

$R_3 + R_1$

$$\sim \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & -3 \\ 0 & -5 & 6 \end{bmatrix}$$

$R_3 + \frac{5}{2}R_2$

$$\sim \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & -3 \\ 0 & 0 & \frac{-3}{2} \end{bmatrix}$$

From the elementary operations used above, we can write

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & \frac{-5}{2} & 1 \end{bmatrix}$$

7. Calculus

01. Ans: 1

Sol: Put $x = \frac{1}{y}$. Then $y \to 0$ as $x \to \infty$.

$$\text{Given Lt} = \underset{y\to 0}{\text{Lt}}\left[\sqrt{\frac{1}{y^2}+\frac{2}{y}-1}-\frac{1}{y}\right]$$

$$= \underset{y\to 0}{\text{Lt}}\left[\frac{\sqrt{1+2y-y^2}-1}{y}\right]$$

$$= \underset{y\to 0}{\text{Lt}}\frac{1}{2\sqrt{1+2y-y^2}}(2-2y)$$

$$= 1$$

02. Ans: 1

Sol: $\underset{x\to a}{\text{Lt}} = \frac{a^x - x^a}{x^x - a^a} = -1$

$$\Rightarrow \underset{x\to a}{\text{Lt}}\frac{a^x \log a - a x^{a-1}}{x^x(1+\log x)} = -1$$

(By L' Hospital's Rule)

$$\Rightarrow \frac{a^a \log a - a.a^{a-1}}{a^a(1+\log a)} = -1$$

$$\Rightarrow \frac{\log a - 1}{\log a + 1} = -1$$

$\Rightarrow \log a - 1 = -\log a - 1$

$\Rightarrow \log a = 0$

$\Rightarrow a = 1$

03. Ans: 7

Sol: $\underset{n\to\infty}{\text{Lt}}\left(7^n+5^n\right)^{\frac{1}{n}} = \underset{n\to\infty}{\text{Lt}}\ 7\left(1+\left(\frac{5}{7}\right)^n\right)^{\frac{1}{n}}$

$$= 7$$

$$\left[\because \underset{n\to\infty}{\text{Lt}}\ r^n = 0,\ |r|<1\right]$$

04. Ans: –2

Sol: $\underset{x\to\frac{\pi}{2}}{\text{Lt}}\left(\sec x - \frac{1}{1-\sin x}\right)$

$$= \underset{x\to\frac{\pi}{2}}{\text{Lt}}\left[\frac{1-\sin x-\cos x}{\cos x(1-\sin x)}\right] \qquad \left[\frac{0}{0}\text{ form}\right]$$

$$= \underset{x\to\frac{\pi}{2}}{\text{Lt}}\left[\frac{1-\cos x+\sin x}{-\sin x-\cos 2x}\right]$$

(by L' Hospital's Rule)

$$= -2$$

05. Ans: 1

Sol: $\underset{x\to 0}{\text{Lt}}\left[\frac{e^x - e^{\sin x}}{x-\sin x}\right] \qquad \left[\frac{0}{0}\text{ form}\right]$

$$= \underset{x\to 0}{\text{Lt}}\left[\frac{e^x - e^{\sin x}.\cos x}{1-\cos x}\right]$$

(by L' Hospital's Rule)

= 1 (applying L' Hospital's Rules two times)

06. Ans: (a)

Sol: $\underset{x\to 0}{\text{Lt}} \dfrac{\sin 2x + a\sin x}{x^3} = b$

By L' Hospital's rule

$$\underset{x\to 0}{\text{Lt}} \frac{2\cos 2x + a\cos x}{3x^2} = b$$

$\Rightarrow 2 + a = 0 \quad (\because \text{ b is finite})$

$\therefore \quad a = -2$

By L' Hospital's rule

$$\underset{x\to 0}{\text{Lt}} \frac{-4\sin 2x + 2\sin x}{6x} = b$$

again, by L' Hospital's rule

$$\underset{x\to 0}{\text{Lt}} \frac{-8\cos 2x + 2\cos x}{6} = b$$

$\Rightarrow b = -1$

$\therefore \quad a = -2 \;\&\; b = -1$

07. Ans: 1

Sol: $\underset{x\to 0}{\text{Lt}} \left(\dfrac{1}{x}\right)^{\tan x} \quad (\infty)^0$

Let $y = \underset{x\to 0}{\text{Lt}} \left(\dfrac{1}{x}\right)^{\tan x}$

Taking Logarithms on both sides

$$\log y = \underset{x\to 0}{\text{Lt}} \tan x \log\left(\frac{1}{x}\right) \text{ .... } 1 \; (0\times\infty)$$

$$= \underset{x\to 0}{\text{Lt}} \frac{-\log x}{\cot x} \qquad \left(\frac{\infty}{\infty}\right)$$

By L' Hospital's rule

$$= \underset{x\to 0}{\text{Lt}} \frac{\frac{-1}{x}}{-\cos^2 x}$$

$$= \underset{x\to 0}{\text{Lt}} \left(\frac{\sin x}{x}\right) \underset{x\to 0}{\text{Lt}} \sin x = 0$$

$\log y = 0$

$\Rightarrow y = e^0 = 1$

08. Ans: $\dfrac{-1}{2}$

Sol: $\underset{x\to 0^-}{\text{Lt}} f(x) = \underset{x\to 0^-}{\text{Lt}} f(x) = f(0)$

$$\underset{x\to 0^-}{\text{Lt}} f(x) = \underset{h\to 0^-}{\text{Lt}} f(0-h) = \underset{h\to 0^-}{\text{Lt}} f(-h)$$

$$= \underset{h\to 0^-}{\text{Lt}} \frac{\sqrt{1-ph} - \sqrt{1+ph}}{-h}$$

$$= \underset{h\to 0^-}{\text{Lt}} \frac{(1-ph)-(1+ph)}{-h\left(\sqrt{1-ph}+\sqrt{1+ph}\right)}$$

$$= \underset{h\to 0^-}{\text{Lt}} \frac{2p}{\sqrt{1-ph}+\sqrt{1+ph}} = \frac{2p}{2}$$

$= p$

Now $f(0) = \dfrac{2(0)+1}{0-2} = \dfrac{-1}{2}$

$\therefore\ p = \dfrac{-1}{2}$

09. Ans: (d)

Sol: $f'(1+) = \underset{x\to 1^+}{\text{Lt}} \dfrac{|x-3|-|-2|}{x-1}$

$$= \underset{x\to 1^+}{\text{Lt}} \frac{-x+3-2}{x-1} = -1$$

$$f'(1-) = \underset{x\to 1^-}{\text{Lt}} \frac{\frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} - 2}{x-1}$$

$$= \frac{1}{4} \underset{x\to 1^-}{\text{Lt}} \frac{(x-1)(x-5)}{(x-1)} = -1$$

$\therefore$ f is continuous and differentiable at x = 1

$$f'(3^+) = \underset{x\to 3^+}{\text{Lt}} \frac{|x-3|-0}{(x-3)} = 1$$

$$f'(3^-) = \underset{x\to 3^-}{\text{Lt}} \frac{-(x-3)}{(x-3)} = -1$$

$\therefore$ f is not differentiable at x = 3

$$\underset{x\to 3}{\text{Lt}}\, f(x) = \underset{x\to 3}{\text{Lt}}\, |x-3| = 0 = f(3)$$

$\therefore$ f is continuous at x = 3

10. Ans: (a)

Sol: If f(x) is continuous at x = 0, then

$$\underset{x=0}{\text{Lt}}\, f(x) = f(0)$$

$$\Rightarrow \underset{x\to 0}{\text{Lt}} \left(\frac{1-x}{1+x}\right)^{\frac{1}{x}} = f(0)$$

$$\Rightarrow \underset{x\to 0}{\text{Lt}} \left[\frac{(1-x)^{\frac{1}{x}}}{(1+x)^{\frac{1}{x}}}\right] = f(0)$$

$$\Rightarrow \frac{e^{-1}}{e} = f(0)$$

$\Rightarrow f(0) = e^{-2}$

11. Ans: (c)

Sol: $f^1(x) = 2ax,\ x \le 1$

$= 2x + a,\ x > 1$

$f^1(1^-) = f'(1^+)$

($\because$ since f(x) is differentiable at x = 1)

$2a = a + 2 \Rightarrow a = 2$

$f(1^-) = f(1^+)$ ($\because$ f(x) is continuous at x = 1)

$a + 1 = 1 + a + b \Rightarrow b = 0$

12. Ans: (c)

Sol: (a) Let f(x) = (x–2) in [1, 3]

Here, $f(1) \ne f(3)$

$\therefore$ Roll's theorem is not applicable

(b) Let $f(x) = 1 - (1-x)^{-1}$ in [0, 2]

Here, f(x) is not continuous in [0, 2]

$\therefore$ Roll's theorem is not applicable

(c) Let f(x) = sin x in [0, π]

Here, f(x) is continuous in [0, π] and differentiable in (0, π).

Further, f(0) = f(π)

$\therefore$ Roll's theorem is applicable

(d) Let f(x) = Tan x in [0, 2π]

Here, f(x) is not continuous in [0, 2π]

$\therefore$ Roll's theorem is not applicable

13. Ans: (d)

Sol: Here, f(x) is neither continuous nor differentiable in the interval [–1, +1].

$\therefore$ Option (d) is correct.

14. Ans: 1.732

Sol: By Cauchy's mean value theorem

$$\frac{f'(d)}{g'(d)} = \frac{f(3)-f(1)}{g(3)-g(1)}$$

$$\Rightarrow -d = \left[\frac{\sqrt{3}-1}{\frac{1}{\sqrt{3}}-1}\right]$$

$$\Rightarrow d = \sqrt{3}$$

15. Ans: (d)

Sol: Let $f(x) = x^{\frac{1}{x}}$

$$f'(x) = x^{\frac{1}{x}}\left[\frac{1-\log x}{x^2}\right]$$

$f'(x) = 0 \quad \Rightarrow \quad x = e$

Further $f''(e) < 0$

$\therefore$ f(x) has maximum at x = 0

The maximum value = $f(e) = e^{\frac{1}{e}}$

16. Ans: 0.785

Sol: Let y = f(x)

$$= \tan^{-1}\left[\frac{1-x}{1+x}\right]$$

$$f'(x) = \frac{-1}{(1+x^2)}$$

f(x) has no stationary points.

Further $f(0) = \frac{\pi}{4}$ and $f(1) = 0$

$\therefore$ The maximum value of $y = \frac{\pi}{4}$

17. Ans: (b)

Sol: $f'(t) = (t-2)^2 (t-1)$

$f'(t) = 0$

$\Rightarrow t = 1, 2$

$f''(t) = (t-2)^2 + 2(t-1)(t-2)$

$f''(1) = 1$ and $f''(2) = 0$

$\therefore$ f(t) has a minimum at t = 1

18. Ans: (c)

Sol: $y = a \log |x| + bx^2 + x$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{x=-1} = 0$$

$\Rightarrow -a - 2b + 1 = 0$ .... (1)

$$\Rightarrow \left[\frac{dy}{dx}\right]_{x=2} = 0$$

$\Rightarrow \frac{a}{2} + 4b + 1 = 0$ .... (2)

solving (1) & (2), we have $a = 2$, $b = \frac{-1}{2}$

19. Ans: 2

Sol: $f_1(x) = 6x^2 - 18ax + 12a^2$

$= 6(x-a)(x-2a)$

$\therefore f'(x) = 0 \Rightarrow x = a$ or $2a$

If $x_1 = a$ then $x_2 = 2a$

$x_2 = x_1^2 \Rightarrow 2a = a^2 \Rightarrow a = 0$ or 2

Clearly f has a local maximum at x = 2 and a local minimum at x = 4

20. Ans: 5

Sol: $z = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= a^2 - 2a + 6$

$= (a-1)^2 + 5 \geq 5$

$\therefore$ z is least iff a = 1

least value of $z = [z]_{a=1}$

$= 5$

21. Ans: (a)

Sol: $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$

consider $f_x = 4x - 4x^3 = 0$

$\Rightarrow x = 0, 1, -1$

$f_y = -4y + 4y^{3==0}$

$\Rightarrow y = 0, 1, -1$

$r = f_{xy} = 4 - 12x^2$

$s = f_{xy} = 0$

$t = f_{yy} = -4 + 12y^2$

At (0,1), we have $r > 0$ and $(rt - s^2) > 0$

$\therefore$ f(x, y) has minimum at (0,1)

At (–1, 0), we have $r < 0$ and $(rt - s^2) > 0$

$\therefore$ f(x, y) has a maximum at (–1, 0)

22. Ans: 0.523

Sol: We have,

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Here, $f(x) = \dfrac{1}{1+\tan^4 x}$

$$= \frac{\cos^4 x}{\cos^4 x + \sin^4 x}$$

$$f(a+b-x) = f\left(\frac{\pi}{2} - x\right)$$

$$= \frac{\sin^4 x}{\sin^4 x + \cos^4 x}$$

Let $I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} f(x)\,dx$

$$= \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\cos^4 x}{\cos^4 x + \sin^4 x}dx$$

again

$$I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} f(a+b-x)\,dx$$

$$= \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}dx$$

adding

$$2I = \int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} dx = \frac{4\pi}{12}$$

$$\therefore I = \frac{\pi}{6}$$

23. Ans: 6.28

Sol: On substituting $x = 7\sin^2\theta + 3\cos^2\theta$,

We get $7 - x = 4\cos^2\theta$,

$x - 3 = 4\sin^2\theta$ and $dx = 8\sin\theta\cos\theta\,d\theta$.

Also $x \to 3 \Rightarrow \theta \to 0$

and $x \to 7 \Rightarrow \theta \to \dfrac{\pi}{2}$

The given integral becomes

$$\therefore I = 8.\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta$$

$$= 2\pi = 6.28$$

24. Ans: 16.15

Sol: Put $1 + x = t^2$

then $I = \int_1^2 \left(t^2 - 1\right)^2 .\ t.\ 2t\,dt$

$$2\int_1^2 \left(t^6 - 2t^4 + t^2\right)dt = 2\left[\frac{t^7}{7} - \frac{2t^5}{5} + \frac{t^3}{3}\right]_1^2$$

$$= \frac{1696}{105}$$

25. Ans: 39

Sol: $\int_4^{10} [x]\,dx$

$= \int_4^5 [x]\,dx + \int_5^6 [x]\,dx + \ldots\ldots.. + \int_9^{10} [x]\,dx$

$= \int_4^5 4\,dx + \int_5^6 5\,dx + \ldots\ldots.. + \int_9^{10} 9\,dx$

$= 4 + 5 + \ldots\ldots + 9$

$= 39$

26. Ans: (a)

Sol: $\int_0^{\pi} x \sin^4 x \cos^6 x\,dx$

$I = \frac{\pi}{2}\int_0^{\pi} \sin^6 x \cos^4 x\,dx$ (property 9)

$= 2\frac{\pi}{2}\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x\,dx$ (property 6)

$I = \pi\left[\frac{5.3.1.3.1}{10.8.6.4.2}\,\frac{\pi}{2}\right] = \frac{3\pi^2}{512}$

27. Ans: 4

Sol: $\int_0^{2\pi} [x \sin x]\,dx = k\pi$

$\Rightarrow \int_0^{\pi} x \sin x\,dx + \int_{\pi}^{2\pi} -x \sin x\,dx = k\pi$

$\Rightarrow [x(-\cos x + \sin x)]_0^{\pi} - [-x\cos x + \sin x]_{\pi}^{2\pi} = k\pi$

$\Rightarrow \pi - [-3\pi] = k\pi$

$\Rightarrow k = 4$

28. Ans: (a)

Sol: $\int_{-\infty}^{0} \sin hx\,dx = |\cos hx|_{-\infty}^{0}$

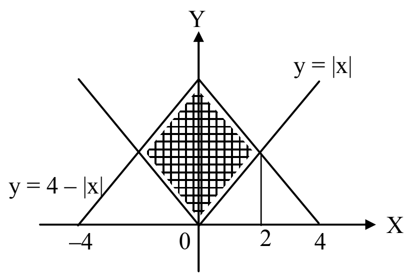
$= \left|\frac{e^x + e^{-x}}{2}\right|_{-\infty}^{0}$

$= \frac{2}{2} - \left(\frac{e^{-\infty} + e^{\infty}}{2}\right)$

$= 1 - 0 - \frac{e^{\infty}}{2} = -\infty$

29. Ans: 8

Sol:



On solving the two curves in the first Quadrant, we get x = 2. Therefore, the area bounded by the curves is

$= 2\left(\int_0^2 (4 - x)\,dx - \int_0^2 x\,dx\right)$

$= 2\left(\left(4x - \frac{x^2}{2}\right)_0^2 - \left(\frac{x^2}{2}\right)_0^2\right)$

$= 2(8 - 2 - 2)$

$= 8$ sq. units

30. Ans: 25.12

Sol: Volume = $\int_0^4 \pi y^2 \, dx$

$= \int_0^4 \pi x \, dx = 8\pi$ cubic units

31. Ans: (d)

Sol: Length = $\int_0^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2} \, dx$

$= \int_0^3 \sqrt{1+x} \, dx$

$= \frac{2}{3}\left[(1+x)^{\frac{3}{2}}\right]_0^3$

$= \frac{14}{3}$

32. Ans: (b)

Sol: $\int_{-\infty}^{\infty} x e^{-x^2} \, dx = 0 \quad f(x) = xe^{-x^2}$

(0 dx) $\quad f(-x) = -xe^{-x^2} = -f(x)$

Converges to 0

33. Ans: (a)

Sol: $\int_{-1}^{1} \frac{dx}{x^2} = 2\int_0^1 \frac{dx}{x^2}$ $\quad$ ($\because \frac{1}{x^2}$ is even function)

$= 2 \underset{x\to 0^+}{\text{Lt}} \int_0^1 \frac{dx}{x^2}$ (since $\frac{1}{x^2}$ is not defined)

$= 2\left(\frac{-1}{x}\right)_0^1$

$= 2\{(-1) - (-\infty)\}$

$= \infty$(Divergent)

34. Ans: (a)

Sol: $\int_1^3 \frac{\sqrt{1+x^2}}{(x-1)^2} dx$ $\quad$ at x =1

$f(x) = \frac{\sqrt{1+x^2}}{(x-1)^2} \quad f(x) \to \infty$ as $x \to 1$

$g(x) = \frac{1}{(x-1)^2} = \frac{\sqrt{1+x}}{x^2\left(1-\frac{1}{x}\right)^2}$

$\underset{x\to 1}{\text{Lt}} \frac{f(x)}{g(x)} = \frac{\sqrt{1+x}}{(x-1)^2} \times (x-1)^2 = \sqrt{2}$

$\int_1^3 \frac{1}{(x-1)^2} = \left(\frac{-1}{x-1}\right)_1^3$

$= \frac{-1}{2} + \frac{1}{0} = \infty$

35. Ans: (a)

Sol: $\int_1^2 \frac{x^3+1}{\sqrt{2-x}} dx$ at x = 2 point of infinite

$f(x) \to \infty$ an $x \to 2$

$\underset{x\to 2}{\text{Lt}} \frac{f(x)}{g(x)} = \frac{x^3+1}{\sqrt{2-x}} \times \sqrt{2-x} = 9$ finite

$\int_1^2 g(x) = \int_1^2 \frac{1}{\sqrt{2-x}} = 2\left|\sqrt{t}\right|_1^2 = 2$ convergent

36. Ans: (d)

Sol: $\int_1^{\infty} \frac{e^{-x}}{x^2} dx$

$\int_1^{\infty} g(x) \, dx = \int_1^{\infty} \frac{1}{x^2} dx = -1$