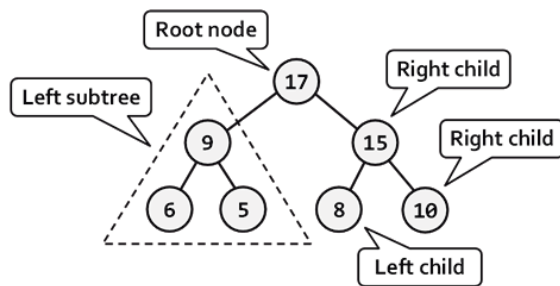




GATE | PSU_s

COMPUTER SCIENCE &

INFORMATION TECHNOLOGY



**COMPUTER SCIENCE &
INFORMATION TECHNOLOGY**

DATA STRUCTURES

Volume-1 : Study Material with Classroom Practice Questions

Data Structures

(Solutions for Vol-1_Classroom Practice Questions)

1. Arrays

01. Ans: 1010

Sol: Loc. of A (I) = $L_0 + (i-lb) * C$

$$\text{Loc of A [0]} = 1000 + (0+5) \times 2 = 1010$$

02. Ans: 1024 and 1024

Sol: (i) By RMO, the loc. of

$$A[i, j] = L_0 + [(i-b_1)(u_2-b_2+1) + (j-b_2)] * C$$

$$A[0, 5] = 1000 + [(0+2) \times 5 + (5-3)] \times 2 \\ = 1000 + 24 = 1024$$

(ii) By CMO, the loc of

$$A[i, j] = L_0 + [(j-b_2)(u_1-b_1+1) + (i-b_1)] * C$$

$$A[0, 5] = 1000 + [(5-3) \times 5 + (0+2)] \times 2 \\ = 1024$$

03. Ans: (a)

Sol: In general

$$\begin{aligned} \text{RMO} &= L_0 + (i-1)r_2 + (j-1) \\ &= 100 + (i-1)15 + (j-1) \\ &= 100 + 15i - 15 + j - 1 \\ &= 15i + j + 84 \end{aligned}$$

04. Ans: (c)

Sol: Lower triangular matrix

$$\begin{pmatrix} a & 0 & 0 & \dots & 0 \\ b & c & 0 & \dots & 0 \\ d & e & f & 0 & -0 \\ g & h & i & j & -0 \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix}$$

$$\text{RMO} = L_0 + \text{the number of elements in}$$

$$\begin{aligned} &(i-1) \text{ rows} + \text{one dimensional elements} \\ &= L_0 + (1 + 2 + \dots + i-1) + (j-1) \\ &= L_0 + i \frac{(i-1)}{2} + (j-1) \end{aligned}$$

05. Ans: (c)

Sol: CMO:

Storage:

$$\begin{array}{cccc|cccc|cccc} a_{11} & a_{21} & a_{31} & a_{41} & a_{22} & a_{32} & a_{42} & a_{33} & a_{43} & a_{44} \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array}$$

Retrieval:

$$\text{loc of } A[i, j] = L_0 + 2D + 1D$$

$$= L_0 + [(j-1) \text{ cols} + (i-1/b)]$$

In each col., $i/b = j$.

$$\text{Loc. of } A[i, j] = L_0 + [(j-1) \text{ cols} + (i-j)]$$

In (j-1) cols

The no. of elements is

$$\begin{aligned} &n + (n-1) + \dots + (n - (j-1-1)) \\ &= (j-1)n - [1 + 2 + \dots + j-2] \\ &= n(j-1) - \frac{(j-1)(j-2)}{2} \end{aligned}$$

loc. of $A[i, j]$

$$= L_0 + \left[n(j-1) - \frac{(j-1)(j-2)}{2} + (i-j) \right]$$

06. Ans: (d)

Sol: RMO:

Storage:

$$\begin{array}{cccc|cccc|cccc} a_{11} & a_{12} & a_{21} & a_{22} & a_{23} & a_{32} & a_{33} & a_{34} & a_{43} & a_{44} \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array}$$



Retrieval:

$$\text{loc of } A[i, j] = L_0 + 2D + 1D$$

$$= L_0 + \text{number of elements in } (i-1) \text{ rows} \\ + (j-1)b$$

Row **j/b**

4 3

3 2

2 1

except 1st row

ith (i-1)

$$\text{loc. of } A[i, j] = L_0 + [(3i-4) + j - (i-1)] \\ = L_0 + (2i + j - 3)$$

07. Ans: (a)

Sol: CMO:

Storage:

$$a_{11} \ a_{21} \mid a_{12} \ a_{22} \ a_{32} \mid a_{23} \ a_{33} \ a_{43} \mid a_{34} \ a_{44} \\ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9$$

Retrieval:

$$\text{loc. of } A[i, j] = L_0 + 2D + 1D \\ = L_0 + (j-1)\text{cols} + (i-1)b$$

Since i is Varying

Col **i/b**

4 3

3 2

2 1

except 1st column

jth j-1

$$\therefore \text{loc of } A[i, j] = L_0 + [3(j-1) - 1 + i - (j-1)] \\ = L_0 + [2j + i - 3]$$

08. Ans: (b)

Sol: Storage & Retrieval:

$$a_{21} \ a_{32} \ a_{43} \mid a_{11} \ a_{22} \ a_{33} \ a_{44} \mid a_{12} \ a_{23} \ a_{34} \\ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9$$

If $i - j = 1$

$$\text{loc of } A[i, j] = L_0 + 0 + (i - 1/b)$$

or

$$(j - j/b)$$

i.e.,

$$\text{loc. of } A[i, j] = L_0 + 0 \\ + (i - 2)$$

or

$$(j - 1)$$

If $i - j = 0$

$$\text{loc. of } A[i, j] = L_0 + (n - 1) \\ + (i - 1)$$

or

$$(j - 1)$$

If $i - j = -1$ // upper diagonal

$$\text{loc. of } A[i, j] = L_0 + 2n - 1 \\ + (i - 1)$$

or

$$(j - 2)$$

09. Ans: (a)

Sol: A sample 5×5 S-matrix is given below.

1	8	3	2	1
3	0	0	0	0
6	1	7	4	3
0	0	0	0	1
9	6	5	4	1

The compact representation is

$$[1, 8, 3, 2, 1, \ 6, 1, 7, 4, 3, \ 9, 6, 5, 4, 1, \ 3, 1].$$



10. Ans: 9

Sol: $2n - 1 = 10 - 1 = 9$

1	2	3	4
5	1	2	3
6	5	1	2
7	6	5	1

11. Ans: 190900

Sol: $n + (a - 1)(2n - a)$

$$1000 + (101 - 1)(2 \cdot 1000 - 101)$$

$$1000 + 100 \times (2000 - 101)$$

$$1000 + 100 \times 1899$$

$$1000 + 189900 = 190900$$

12. Ans: (a)

Sol: Square Band Matrix:

It is denoted by $D_{n,a}$ where n is size of matrix a is $(a-1)$'s diagonals exists above and below the matrix diagonal.

$$D_{5,4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix}_{5 \times 5}$$

Size:- N (diagonal elements)

$$+ 2[N-1+N-2+N-3+\dots+N-(a-1)]$$

$$= N+2[(a-1)N-(1+2+3+\dots+a-1)]$$

Total no. of elements

$$= n + 2[n-1 + n-2 + \dots + n-(a-1)]$$

$$= n + 2[(a-1)n - [1+2+\dots+(a-1)]]$$

$$= n + 2\left[(a-1)n - \frac{a(a-1)}{2}\right]$$

$$= n+2n(a-1) - a(a-1) = n + (a-1)[2n-a]$$

(b) $D_{6,4}$

In $D_{6,4} \rightarrow A(5,4)$ in 3rd diagonal

In $D_{6,5} \rightarrow A(5,4)$ in 4th diagonal

(i) $i - j$ value is remained constant throughout the diagonal

(ii) As 'a' value changes, accordingly K value also changes.

$$\therefore K \text{ value} = a - (i - j)$$

$\therefore$ loc. of $A[i,j] = L_0 + \text{no. of elements in } (k-1) \text{ dig} + 1D \text{ cross}$

loc. of $A[i,j] = L_0 + \text{no. of element in } (k-1) \text{ dig} + (i - i/b) \text{ or } (j - j/b)$

but here i/b is varying diagonal by diagonal so prefer j/b which is always 1 in each diagonal

$$= L_0 + \text{no. of elements in } (k-1) \text{ dig} + (j-1)$$

No. of elements in $(k-1)$ diagonals is

$$[n-(a-1)] + [n-(a-1)+1] + [n-(a-1)+2] + \dots + [n-a+(k-1)]$$

$[n-(a-1)] \rightarrow 1^{\text{st}}$ diagonal because it is $(a-1)^{\text{th}}$ diagonal

$(n-(a-1)+1) \rightarrow 2^{\text{nd}}$ diagonal

$[n-(a-1)+2] \rightarrow 3^{\text{rd}}$ diagonal

$n-a+(k-1) \rightarrow (k-1)^{\text{th}}$ term i.e. $(k-1)^{\text{th}}$

diagonal

$$= (k-1)(n-a) + 1 + 2 + \dots + k-1$$

$$= (k-1)(n-a) + \frac{k(k-1)}{2}$$

$\therefore$ loc. of $A[i,j]$

$$= L_0 + \left[(k-1)(n-a) + \frac{k(k-1)}{2} + (j-1) \right]$$



2. Stacks & Queues

01. (i). Ans: (a) (ii). Ans: (c)

Sol: Given array size m, say 9

Number of stacks n, say 3

$$0 \leq i < n \quad T[i] = B[i] = i \left\lfloor \frac{m}{n} \right\rfloor - 1$$

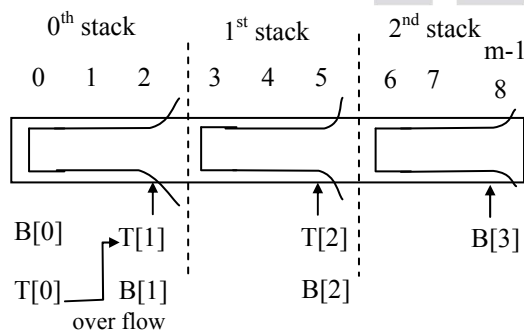
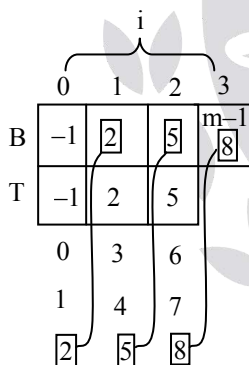
$$i = 0 \Rightarrow T[0] = B[0] = 0 \left\lfloor \frac{9}{3} \right\rfloor - 1 = 0 - 1 = -1$$

$$i = 1 \Rightarrow T[1] = B[1] = 1 \left\lfloor \frac{9}{3} \right\rfloor - 1 = 3 - 1 = 2$$

$$i = 2 \Rightarrow T[2] = B[2] = 2 \left\lfloor \frac{9}{3} \right\rfloor - 1 = 6 - 1 = 5$$

$$\text{when } i = 3 \Rightarrow B[3] = m - 1 = 9 - 1 = 8$$

(i) Push = overflow = size

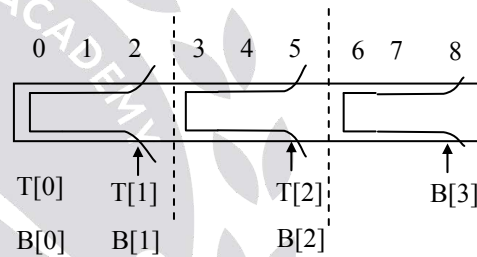


$$\left. \begin{aligned} T[0] &= B[1] \\ T[1] &= B[2] \\ T[2] &= B[3] \end{aligned} \right\} \text{over flow cases}$$

$$\therefore T[i] = B[i + 1]$$

(ii) POP = underflow = initial

	0	1	2
B	-1	2	5
T	-1	2	5



$$\left. \begin{aligned} T[0] &= B[0] \\ T[1] &= B[1] \\ T[2] &= B[2] \end{aligned} \right\} \text{under flow cases} \therefore T[i] = B[i]$$



02. Ans: (b)

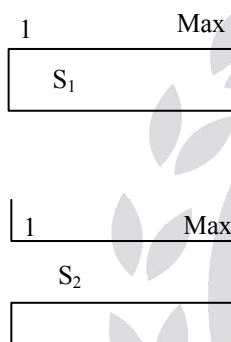
Sol:

Stack operation	Push (10)	Push (20)	Pop	Push (10)	Push (20)	Pop	Pop	Pop	Push (20)	Pop
Stack	10	10 20	10	10 10	10 10 20	10 10	10		20	
Pop list			20	20	20	20, 20	20, 20, 10	20, 20, 10, 10	20, 20, 10, 10	20, 20, 10, 10, 20

The sequence of popped out values $\Rightarrow 20, 20, 10, 10, 20$

03. Ans: (d)

Sol:



An instance of array having two stacks is shown above. Stack1 occupied from 1 to MAXSIZE and stack2 occupied from MAXSIZE to 1. Above shown array is filled completely. So condition for 'stack full' is

$$\text{Top 1} = \text{Top 2} - 1$$

04. Ans: (c)

Sol: Stack S is

2	let S ₁	S ₂	S ₃	S ₄
-1	-1			
2	2	2		
-3	-3	-3	-3	
2	2	2	2	2

$$f(0) = 0$$

$$f(S_4) = \max \{f(0), 0\} + 2 = 2$$

$$f(S_3) = \max \{f(S_4), 0\} + -3 = -1$$

$$\{ \because S_3 = \text{push}(S_4, -3) \}$$

$$f(S_2) = \max \{f(S_3), 0\} + 2 = 2$$

$$f(S_1) = \max \{f(S_2), 0\} + (-1) = 1$$

$$f(S) = \max \{f(S_1), 0\} + 2 = 3$$

$$f(S) = 3$$

05. Ans: (b)

Sol: Stack insertion order $\Rightarrow 1, 2, 3, 4, 5$. The only possible output sequence 3, 4, 5, 2, 1

That occurs when

Push (1)

Push (2)

Push (3)

Pop (3) $\rightarrow 3$

($\because$ There is no constraint on the order of deletion operations)

Push (4)

Pop (4) $\rightarrow 3, 4$

Push (5)

Pop (5) $\rightarrow 3, 4, 5$

Pop (2) $\rightarrow 3, 4, 5, 2$

Pop (1) $\rightarrow 3, 4, 5, 2, 1$

Other remaining combinations are not possible



06. Ans: 321

Sol: invocation tail (3)

$$T(3) = 3$$

$$T(2) = 2$$

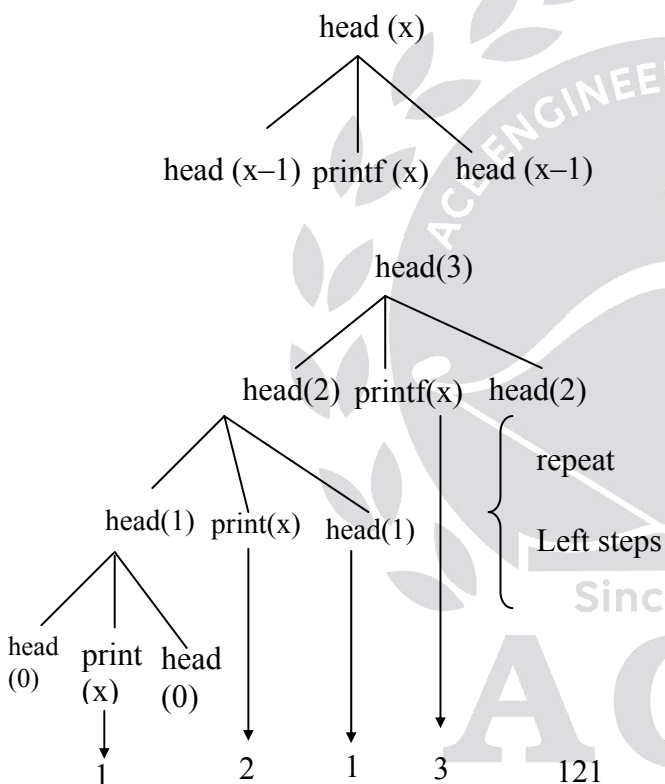
$$T(1) = 1$$

$$T(0) = \text{stop}$$

Output: 3, 2, 1

07. Ans: 1213121

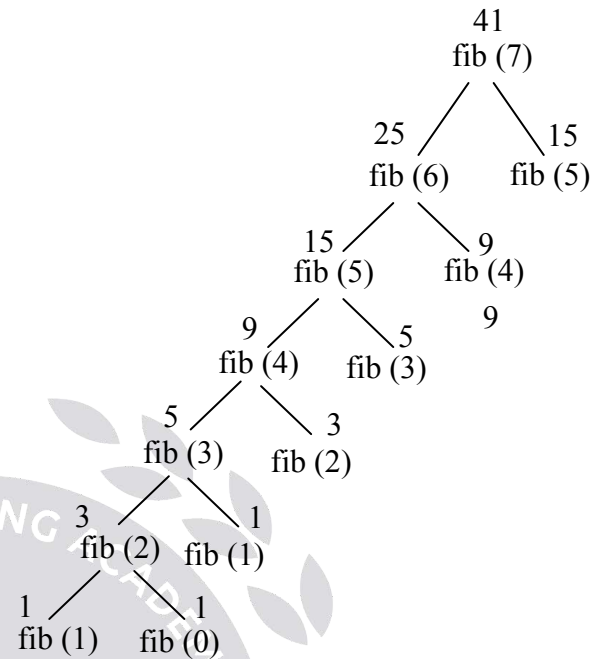
Sol:



Output: 1213121

08.

Sol: (i) Ans: 41



(ii) Ans: 67

Sol: Number of calls for evaluating

$$f(n) = 2 \times f(n+1) - 1$$

The total number of calls in

$$\text{Fibonacci}(8) = 2 f(9) - 1$$

$$= 2 \times 34 - 1 = 68 - 1 = 67$$

(iii) Ans: 54

n	0	1	2	3	4	5	6	7	8	9	10
Fib(n)	0	1	1	2	3	5	8	13	21	34	55
Call:	1	1	3	5	9			41	35		
Add:				2	4						

$$\text{Additions} = f(n+1) - 1$$

$$f(9) = f(10) - 1 = 55 - 1 = 54$$



09.

Sol: Ackerman(m, n) =

$$\begin{cases} n+1 & \text{if } m=0 \\ \text{Ackerman}(m-1, 1) & \text{if } n=0 \\ \text{Ackerman}(m-1, \text{Ackerman}(m, n-1)) & \text{otherwise} \end{cases}$$

(i) **Ans: 9**

Sol: Ackerman(2, 3) = A(1, A(2, 2)) = A(1, 7)

$$A(2, 2) = A(1, A(2, 1)) = A(1, 5) = 7$$

$$A(2, 1) = A(1, A(2, 0)) = A(1, 3) = 5$$

$$A(2, 0) = A(1, 1) = 3$$

$$A(1, 1) = A(0, A(1, 0)) = A(0, 2) = 2 + 1 = 3$$

$$A(1, 0) = A(0, 1) = 2$$

$$A(0, 1) = 1 + 1 = 2$$

$$A(1, 3) = A(0, A(1, 2)) = A(0, 4) = 4 + 1 = 5$$

$$A(1, 2) = A(0, A(1, 1)) = A(0, 3) = 3 + 1 = 4$$

$$\begin{aligned} A(1, 5) &= A(0, A(1, 4)) \\ &= A(0, A(0, A(1, 3))) \\ &= A(0, A(0, 5)) \\ &= A(0, 6) = 6 + 1 = 7 \end{aligned}$$

$$\begin{aligned} A(1, 7) &= A(0, A(1, 6)) \\ &= A(0, A(0, A(1, 5))) \\ &= A(0, A(0, 7)) \\ &= A(0, 8) = 9 \end{aligned}$$

$$\text{Ackerman}(2, 3) = 9$$

(ii) **Ans: 13**

Sol: Ackerman(2, 5) = A(1, A(2, 4))

$$= A(1, A(1, A(2, 3)))$$

$$= A(1, A(1, 9))$$

$$A(1, 9) = A(0, A(1, 8))$$

$$= A(0, A(0, A(1, 7)))$$

$$= A(0, A(0, 9))$$

$$= A(0, 10)$$

$$= 11$$

$$\begin{aligned} A(1, 11) &= A(0, A(1, 10)) \\ &= A(0, A(0, A(1, 9))) \\ &= A(0, A(0, 11)) \\ &= A(0, 12) \\ &= 13 \end{aligned}$$

$$\text{Ackerman}(2, 5) = 13$$

(iii) **Ans: 4**

Sol: Ackerman(0, 3) = 4

(iv) **Ans: 5**

Sol: Ackerman(3, 0) = A(2, 1)

$$A(2, 1) = A(1, A(2, 0))$$

$$A(2, 0) = A(1, 1)$$

$$A(1, 1) = A(0, A(1, 0))$$

$$A(1, 0) = A(0, 1) = 2$$

$$A(1, 1) = A(0, 1) = 3$$

$$A(2, 0) = 3$$

$$A(2, 1) = A(1, 3)$$

$$A(1, 3) = 5 \quad \text{from (i)}$$

$$A(2, 1) = 5$$

$$\text{Ackerman}(3, 0) = 5$$

10.

Sol: (a) After $N+1$ calls we have the first move.

So after 4 calls we have the first move.

(b) After total calls -1 calls, we have the last move.

(c) Total moves $2^N - 1 = 7$

(d) Total invocations $= 2^{N+1} - 1$
 $= 2^4 - 1 = 15$

11. **Ans: (b)**

Sol: Post fix expression A B C D + * F / + DE * +



12. Ans: (a)

Sol: $a = -b + c * d / e + f \uparrow g \uparrow h - i * j$

Prefix:

$a = -b + c * d / e + (\uparrow f \uparrow gh) - i * j$

$a = -b + *cd/e + (\uparrow f \uparrow gh) - i * j$

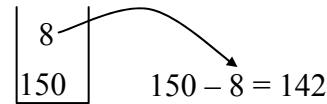
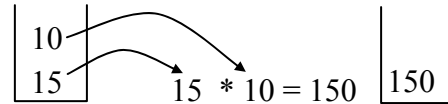
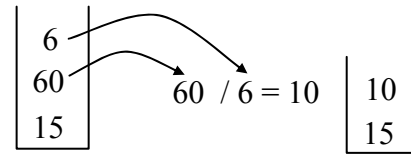
$a = -b + /*cde + \uparrow f \uparrow gh - *ij$

$a = +-b/*cde + \uparrow f \uparrow gh - *ij$

$a = ++-b/*cde \uparrow f \uparrow gh - *ij$

$a = -++-b/*cde \uparrow f \uparrow gh *ij$

$\Rightarrow a = -++-b/*cde \uparrow f \uparrow gh *ij$



Value of the postfix expression is 142

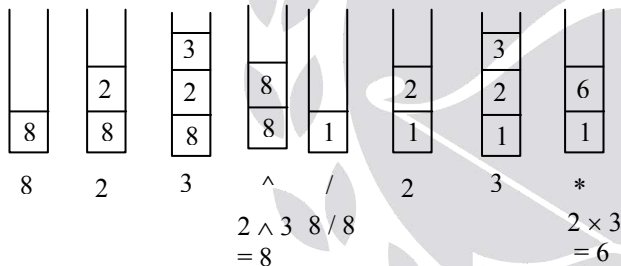
13. Ans: (a)

Sol: Infix expression: $[a+(b \times c)] - (d \wedge (e \wedge f))$

Postfix expression: $abc \times + def \wedge \wedge -$

14. Ans: (a)

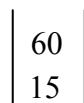
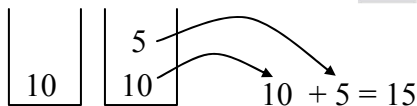
Sol:



$\Rightarrow$ The top two elements are 6, 1

15. Ans: (c)

Sol: 10, 5, +, 60, 6, /, *, 8, -



16. Ans: (c)

Sol: (i) ab

(ii) b

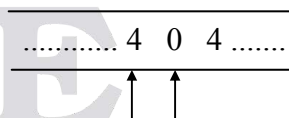
(iii) byz

(iv) yz

Output is $\Rightarrow$ yz

17. Ans: (b)

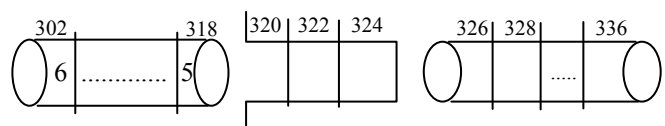
Sol:



Queue rear Replace with predecessor

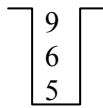
18. Ans: (i) 322 and (iii) 324

Sol:



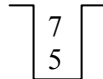


Until first '0' is encountered, stack contains



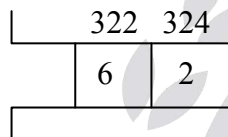
So $5+6+9 = 20$ is enqueued in Q_2 @ loc 326

Until second '0' is encountered, stack contains



So $5+7 = 12$ is enqueued in Q_2 @ loc 328

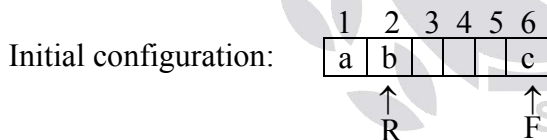
Then simply 2 and 6 are pushed in stack



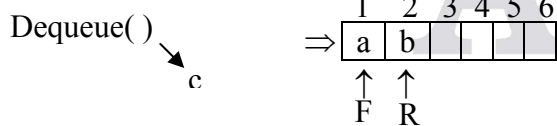
So the location of 6 and 20 are 322 and 324

19. Ans (c)

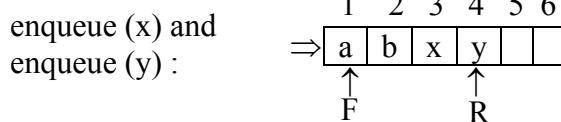
Sol: Suppose that array contains



Delete element



Now two added



$\therefore (R, F) = (4, 1)$

Option (c).

20. Ans: (b)

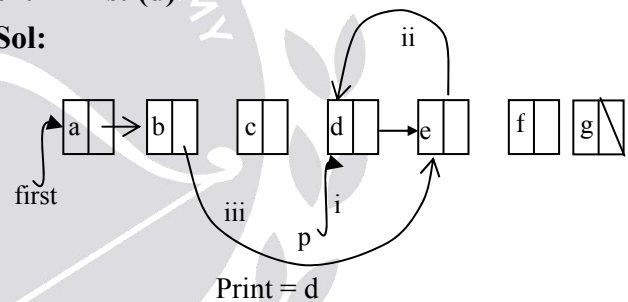
Sol: The given recursive procedure simply reverses the order of elements in the queue. Because in every invocation the deleted element is stored in 'i' and when the queue becomes empty.

Then the insert () function call will be executed from the very last invoked function call. So, the last deleted element will be inserted first and the procedure goes on

3. Linked Lists

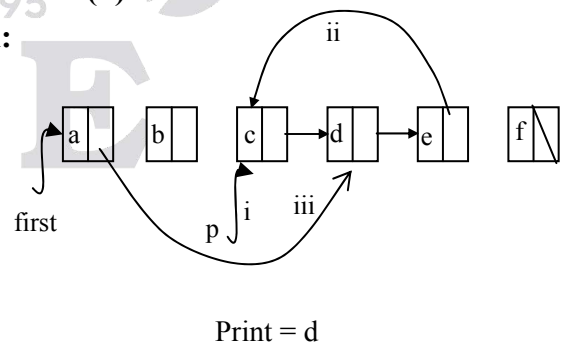
01. Ans: (d)

Sol:



02. Ans: (d)

Sol:



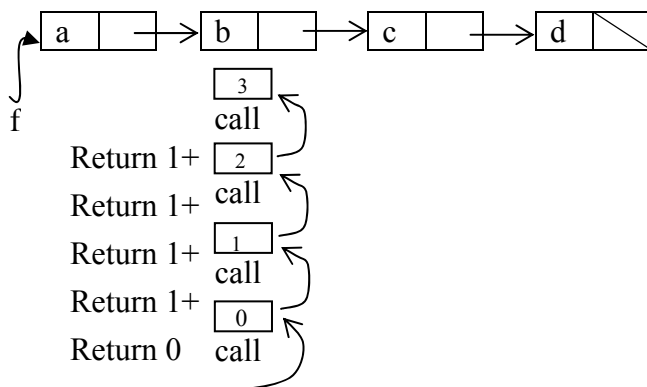
03. Ans: (a)

Sol: while (P) or while (P!= Null)
while P is pointing to somebody



04. Ans: (d)

Sol: Recursive routine for 'Count'



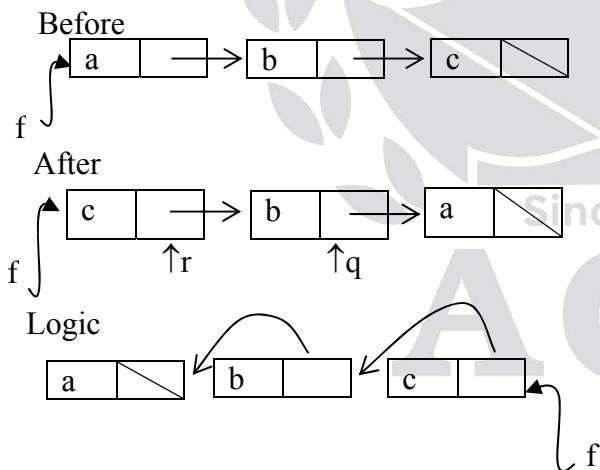
No. of nodes = 4

05. Ans: (b)

Sol: either causes a null pointer dereference or append list m to the end of list n.

06. Ans: (b)

Sol:

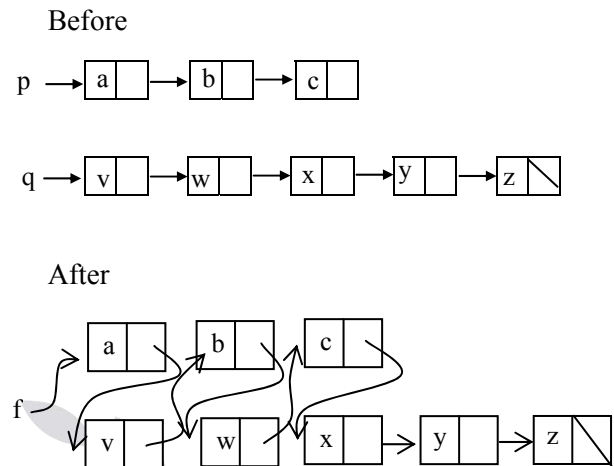


07. Ans: (b)

Sol: This is recursive routine for reversing a SLL.

08. Ans: (a)

Sol:



concatenation of two single linked lists by choosing alternative nodes.

09. Ans: (d)

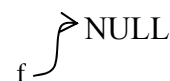
Sol: Cur. Next = New Node (X, Cur. Next)

- (i) Struct Node * n = Get Node () ;
- (ii) n -> data = X ;
- (iii) n -> Next = Cur -> Next ;
- (iv) Cur -> Next = n

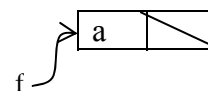
10. Ans: (a)

Sol: Linked stack push () = insert front ()

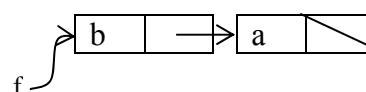
Initial Do ()



Do(a)



Do(b)





11.

Sol:

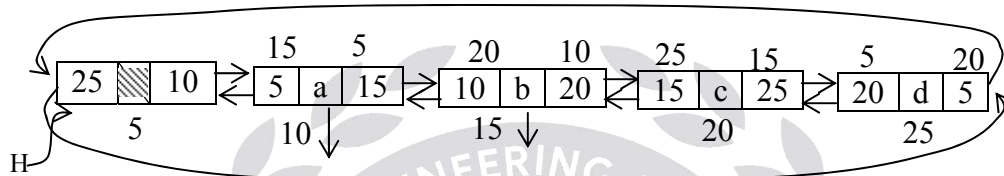
Operation	Left most	Right most	Middle
Insert	3	3	4
Delete	1	1	2

12. Ans: (b)

Sol: Inserts to the left of middle node in doubly linked list.

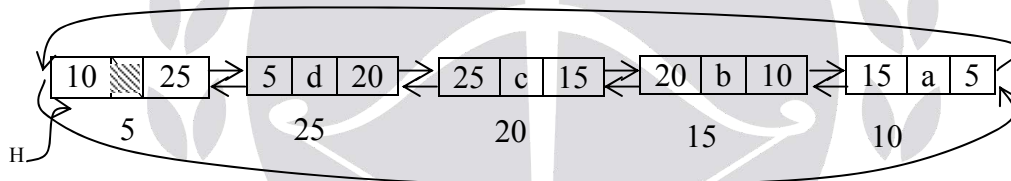
13. Ans: (a)

Sol: Before reverse:



$t : t \rightarrow \text{Llink}$

After reverse:



4. Trees

01. Ans: (d)

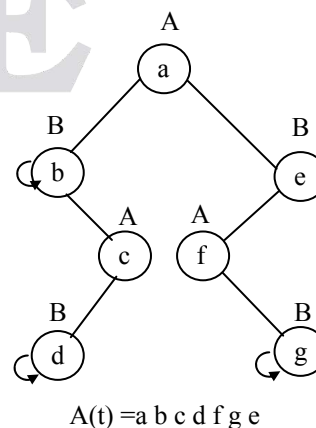
Sol: 1. Traverse the left subtree in postorder.
2. Traverse the right subtree in postorder.
3. Process the root node

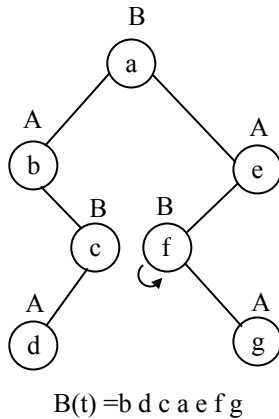
02. Ans: (c)

Sol: 1. Traverse the right sub tree in postorder.
2. Traverse the left sub tree in postorder.
3. Process the root node

03. Ans: (c)

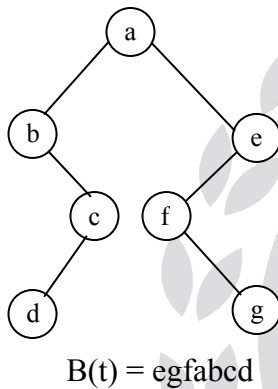
Sol:





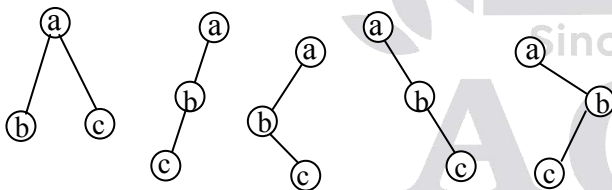
04. Ans: (b)

Sol:



05. Ans: 5

Sol:



∴ Totally 5 distinct trees possible

Note: The number of binary trees can be formulated with unlabeled nodes are

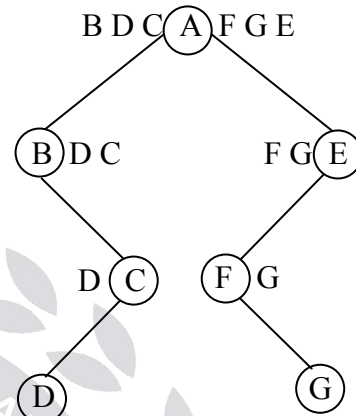
$$\frac{2^n C_n}{n+1}$$

06. Ans: (c)

Sol: Preorder : A B C D E F G

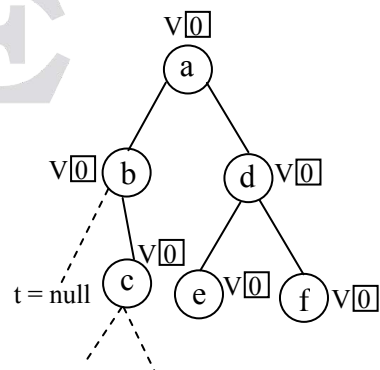
In-order : B D C A F G E

Post-order: D C B G F E A



07. Ans: 3

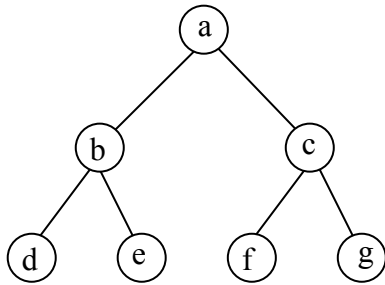
Sol: Note: If pre-order is given, along with terminal node information & all right child information the unique pattern can be found. If post-order is given along with terminal information and all left child information the unique pattern can be identified.





08. Ans: 3

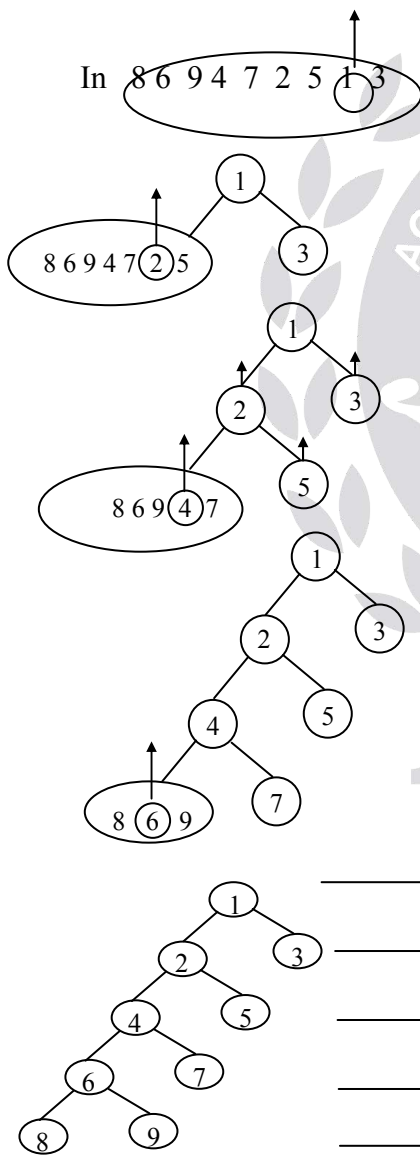
Sol:



09. Ans: 4

Sol: Post 8 9 6 7 4 5 2 3 1

In 8 6 9 4 7 2 5 1 3



Post 8 9 6 7 4 5 2 3 1

In 8 6 9 4 7 2 5 1 3

Height = 4

10. (a) Ans: 19

Sol: Leaf nodes (L) = Total nodes – internal nodes

$$L = In + 1 - I$$

$$L = I(n-1) + 1$$

$$L = 20$$

$$I = ?$$

$$20 = I(2 - 1) + 1$$

$$20 = I + 1$$

$$I = 19$$

10. (b) Ans: 199

Sol: $L = I(n-1) + 1$

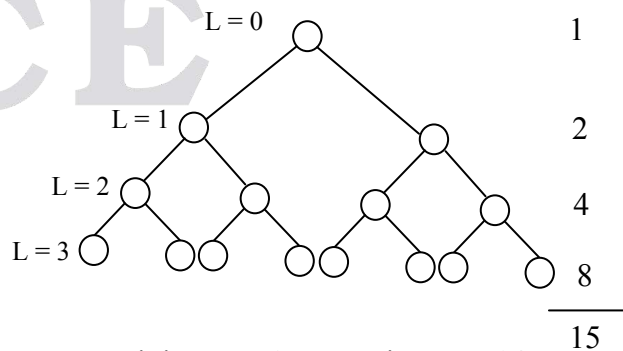
$$L = 200$$

$$200 = I + 1$$

$$I = 199$$

11. Ans: (b)

Sol:

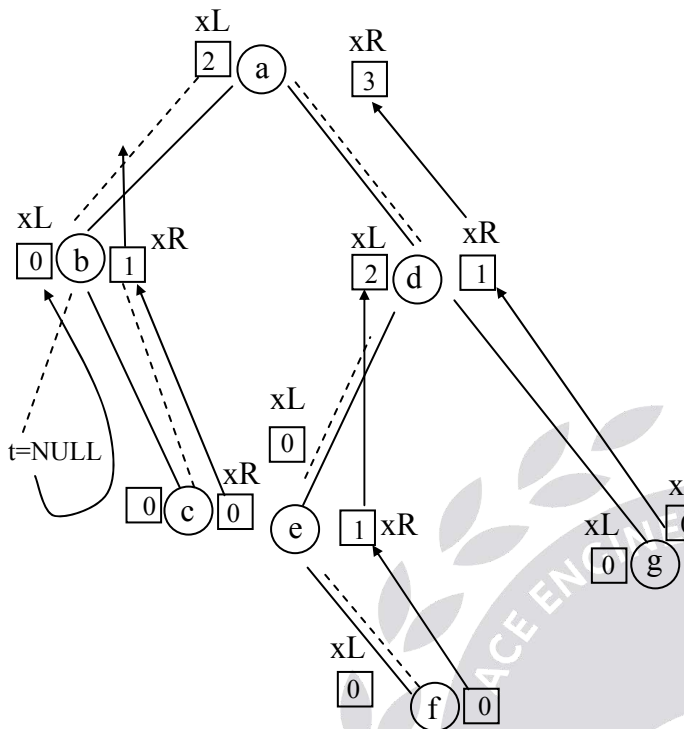


Minimum = 3, Maximum = 14



12. Ans: 2 & 1

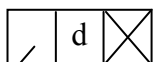
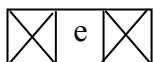
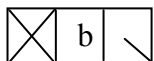
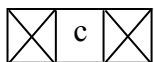
Sol:



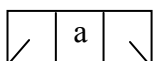
13. Ans: (a)

Sol:

Before Swap



e

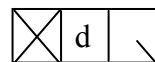
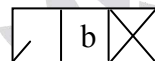
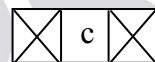


b

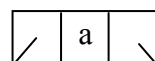


c

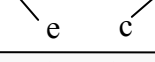
After swap



e



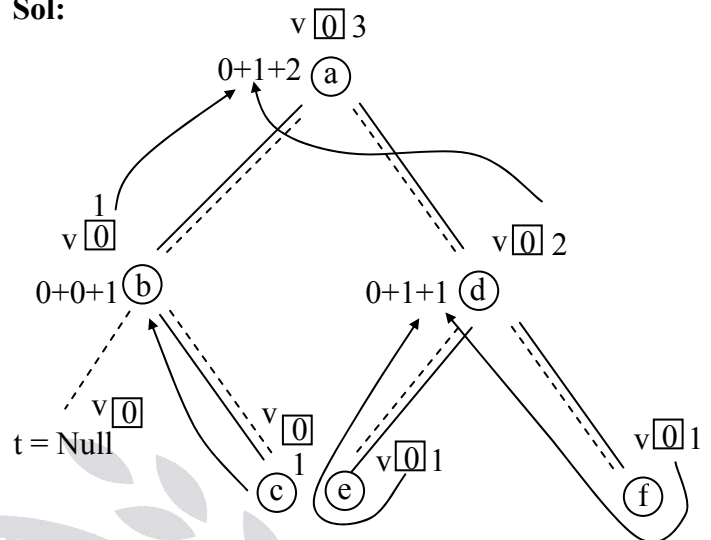
d



e

14. Ans: (d)

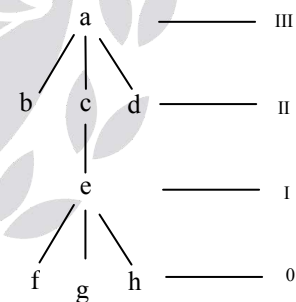
Sol:



15. Ans: 3

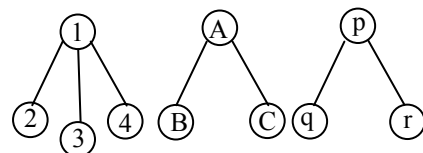
Sol: a (b, c (e (f, g, h)), d)

Parent of f, g, h is e. i.e. internal parenthesis has children of parent which is out of parenthesis.



16. Ans: 4

Sol: Given are 3 trees



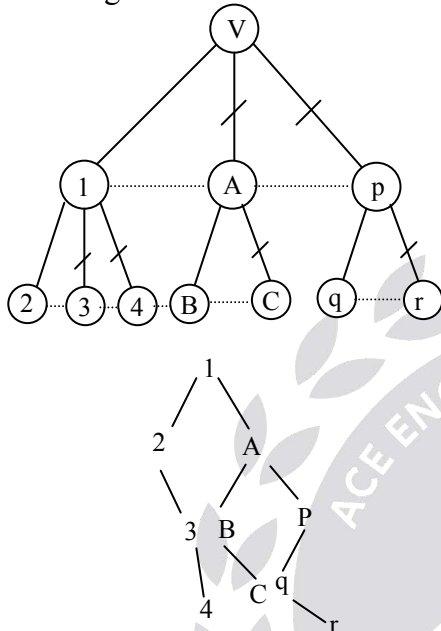
To get the converted binary tree of these given trees



∴ parent is not given we have to assume virtual parent

Among siblings

- Keep the leftmost as it is,
- Cut and connect right siblings as shown in diagram

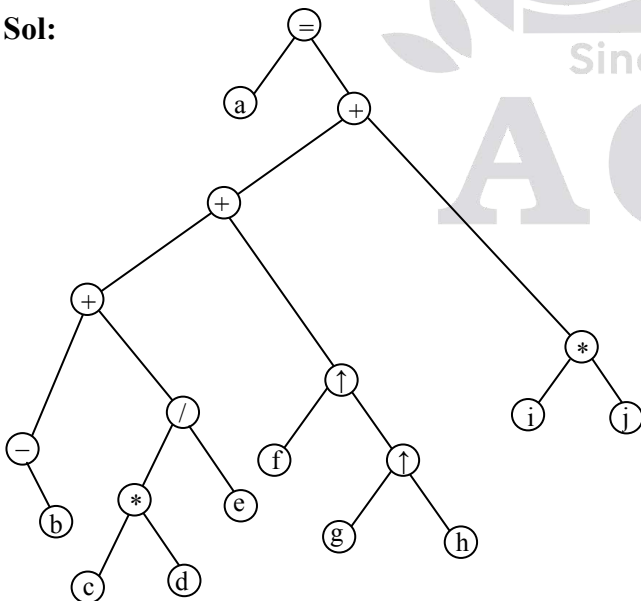


17. Ans: (d)

Sol: Count the number of trees in forest.

18. Ans: (b)

Sol:



19. Ans: 6

Sol: Expanded as

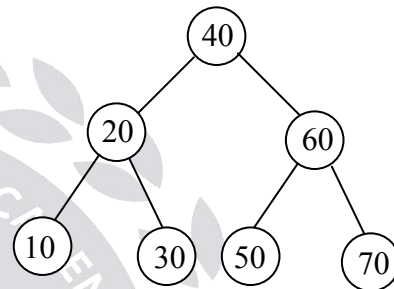
$$((1+1) - (0 - 1)) + ((1 - 0) + (1+1)) \\ = 3 + 3 = 6$$

20. Ans: -2

Sol: $(0 + 0) - (1 - 0) + (0 - 1) + (0 + 0)$
 $= -1 + (-1) = -2$

21. Ans: 4

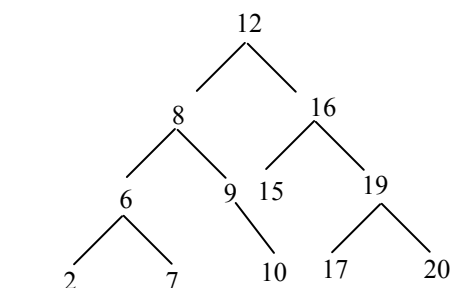
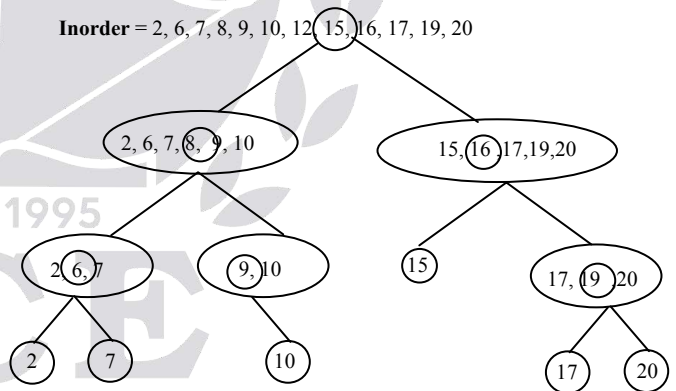
Sol:



22. Ans: (b)

Sol: Preorder = 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20

Inorder = 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20

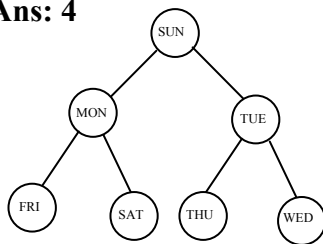


$$= 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12$$



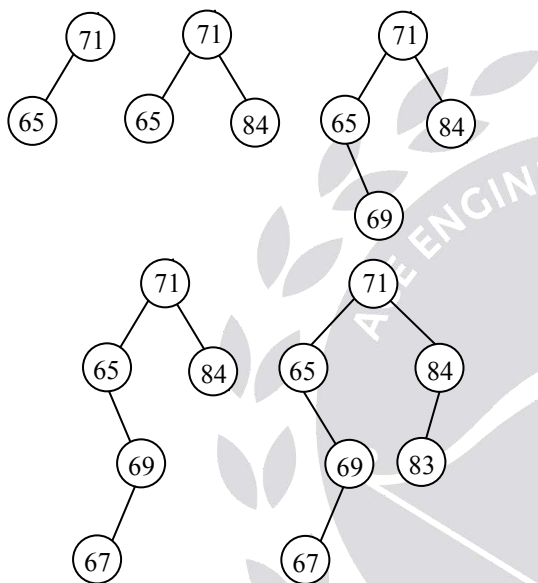
23. Ans: 4

Sol:



24. Ans: 67

Sol: 71, 65, 84, 69, 67, 83 insert into empty binary search tree

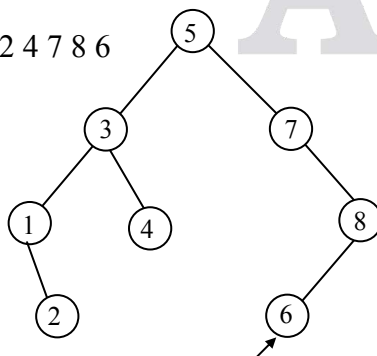


∴ Element in the lowest level is 67

25. Ans: 30

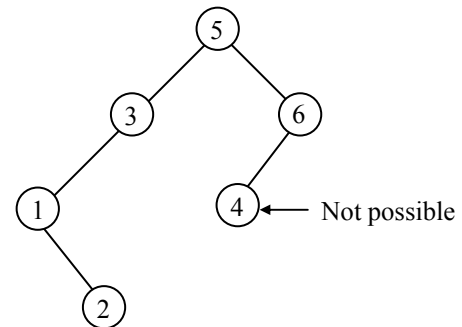
26. Ans: (d)

Sol: (a) 5 3 1 2 4 7 8 6

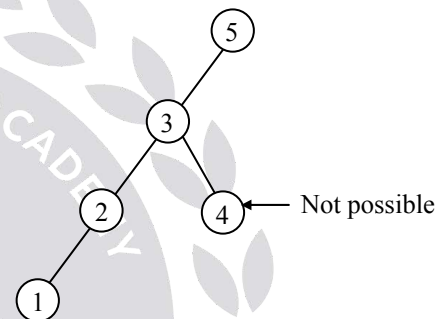


Not possible
IN: not sorted order

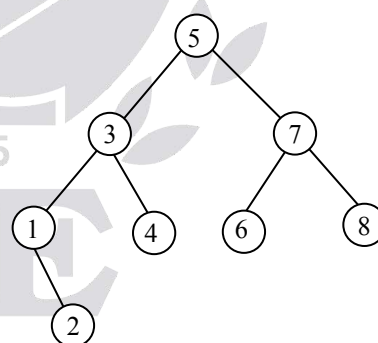
(b) 5 3 1 2 6 4 8 7



(c) 5 3 2 4 1 6 7 8



(d) 5 3 1 2 4 7 6 8



27. Ans: 15

Sol: 1. Jump right

2. Go on descend left



28. Ans: 88

$$\text{Sol: } N(H) = \begin{cases} 1 & L=H=0 \\ 2 & L=H=1 \\ 1+N(H-1)+N(H-2) & (L=H)>1 \end{cases}$$

$$N(H) = 1 + N(H-1) + N(H-2)$$

$$N(2) = 1 + N(1) + N(0)$$

$$= 1 + 2 + 1$$

$$= 4$$

$$N(3) = 1 + N(2) + N(1)$$

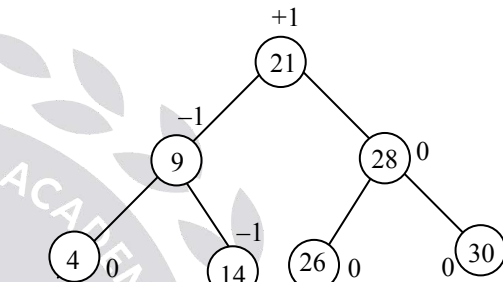
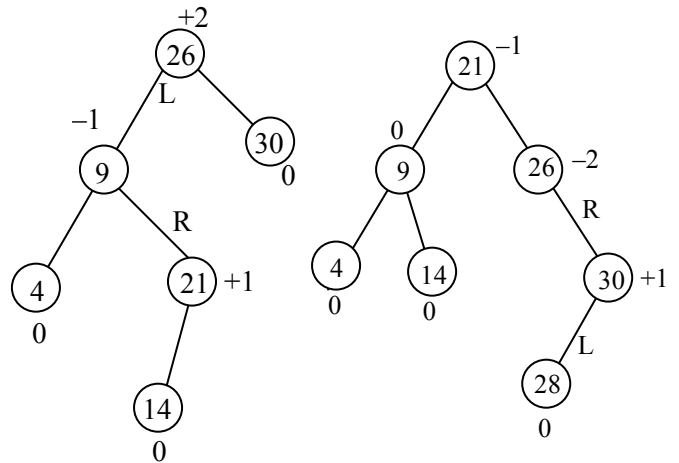
$$= 1 + 4 + 2$$

$$= 7$$

$$N(8) = 1 + N(7) + N(6)$$

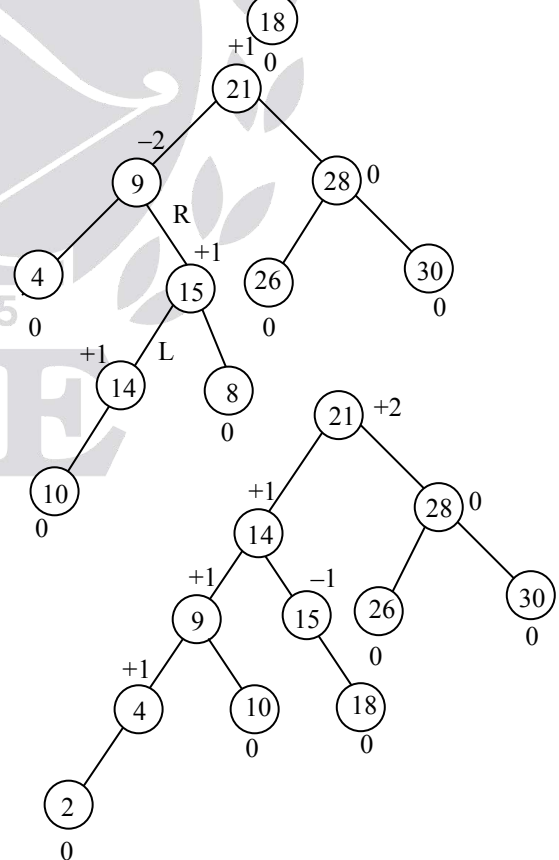
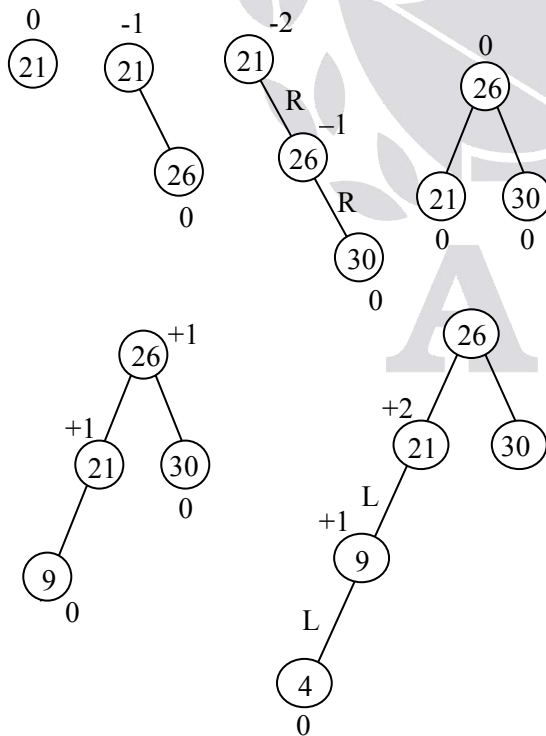
$$= 1 + 54 + 33 = 88$$

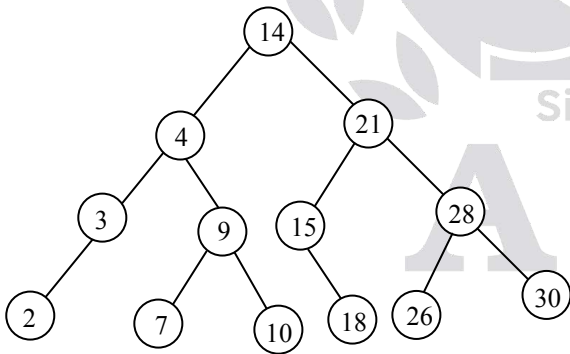
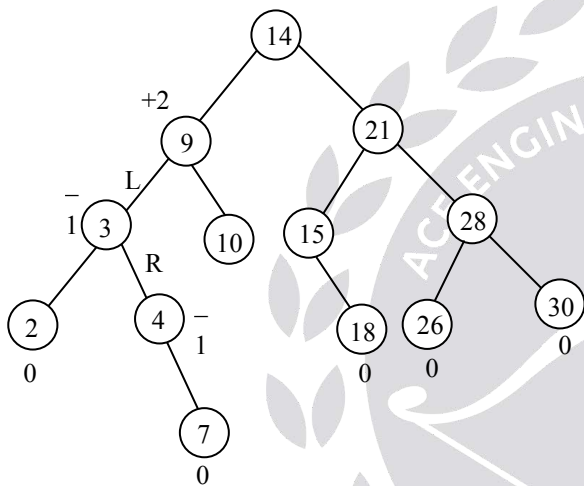
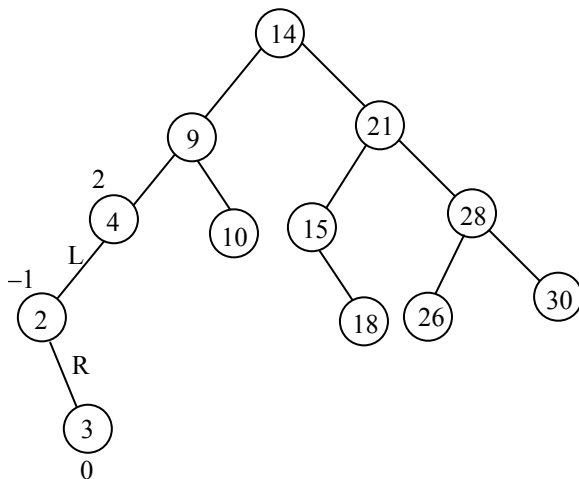
H	0	1	2	3	4	5	6	7	8
N(H)	1	2	4	7	12	20	33	54	88



29. Ans: 14

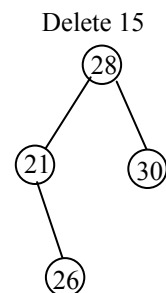
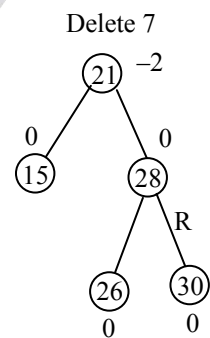
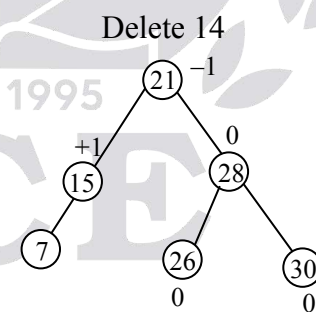
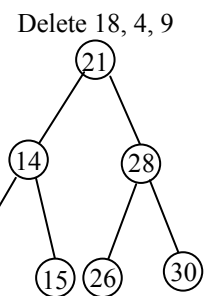
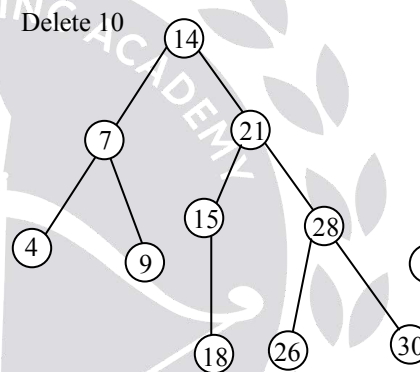
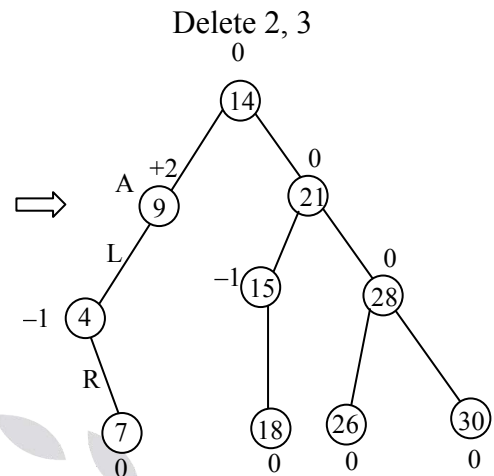
Sol: 21, 26, 30, 9, 4, 14, 28, 18, 15, 10, 2, 3, 7





30. Ans: 28

Sol:

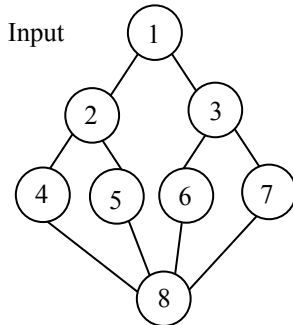




5. Graphs

01. Ans: (b)

Sol: V_8 is pushed in for two times



Output: 1, 2, 4, ⑧, 5 step back
⑧, 6, 3, 7.

V_8 : Two times

02. Ans: 2

Sol: Sequence of exploration

$V_5 \rightarrow V_2 \rightarrow V_1 \rightarrow V_3 \rightarrow$

$V_6 \rightarrow V_8 \rightarrow V_7 \rightarrow V_4$

Sequence of stack contents

Not pushed vertices are $\rightarrow V_4, V_7$

Vertices are pushed in more than once

$\rightarrow V_1, V_2, V_3, V_6, V_5$

V_8
V_8
V_6
V_3
V_1
V_2
V_5

03. Ans: 3

Sol: Sequence of exploration

$V_8 \rightarrow V_4 \rightarrow V_2 \rightarrow V_1 \rightarrow V_3 \rightarrow V_6 \rightarrow V_7 \rightarrow V_5$

Sequence of stack contents

V_2
V_3
V_3
V_1
V_2
V_4
V_8

Not pushed vertices are $\rightarrow V_6, V_7, V_5$

Vertices are not pushed in more than once

$\rightarrow V_1, V_4, V_8$

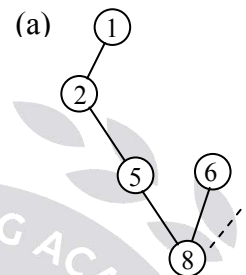
04. Ans: (a)

Sol: (a) invalid

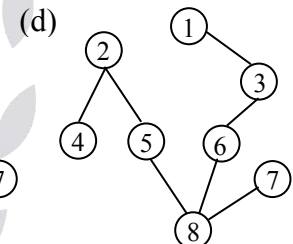
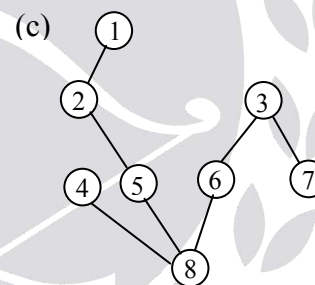
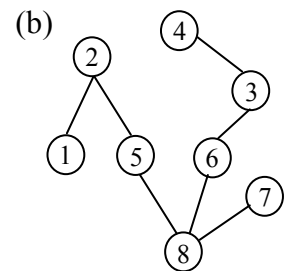
(b) valid

(c) valid

(d) valid



Step back only when already explored vertices are there



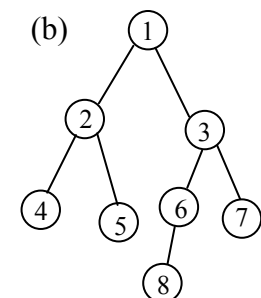
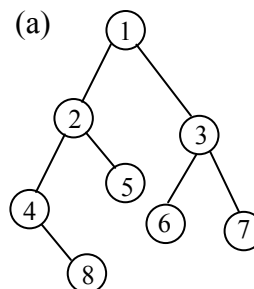
05. Ans: (c)

Sol: (a) valid

(b) valid

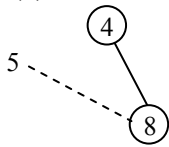
(c) invalid

(d) valid



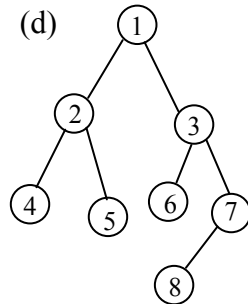


(c)



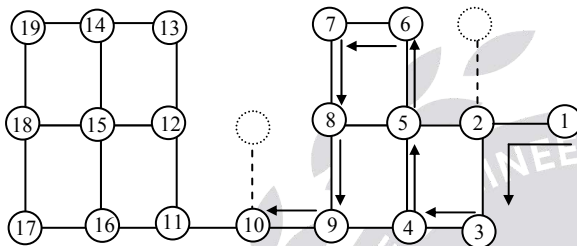
Step back only when
already explored vertices
are there

(d)



06. Ans: 19

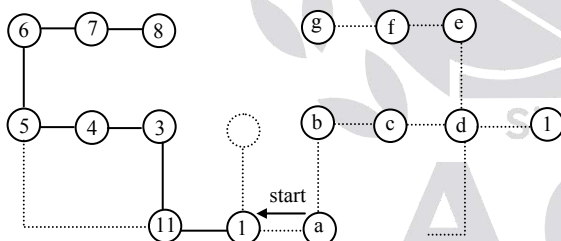
Sol:



Maximum possible recursion depth = 19
(The dashed link 'nodes' are explored while
stepping backward.)

07. Ans: 8

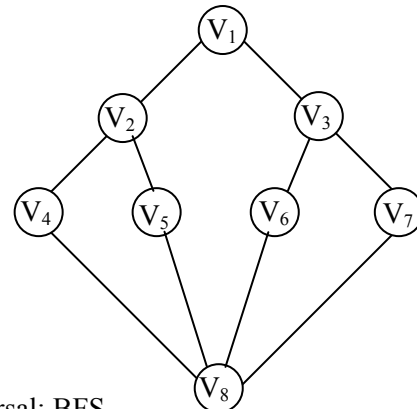
Sol:



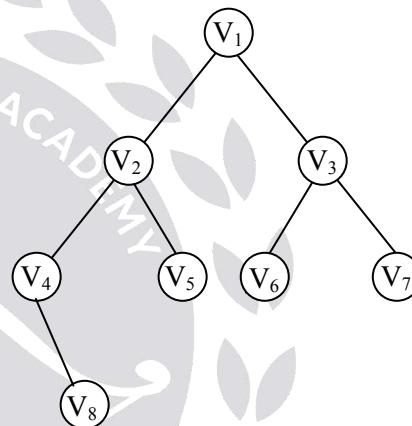
Minimum possible recursion depth = 8
(The dashed link 'nodes' are explored while
stepping backward.)

08. Ans: (d)

Sol:



Traversal: BFS



queue

V ₂	V ₃	
----------------	----------------	--

Dequeue

queue

V ₃	V ₄	V ₅	
----------------	----------------	----------------	--

Dequeue

queue

V ₄	V ₅	V ₆	V ₇
----------------	----------------	----------------	----------------

Dequeue

queue

V ₅	V ₆	V ₇	V ₈
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09. Ans: (c)

Sol: (a) valid (b) valid
(c) invalid (d) valid

10. Ans: (d)



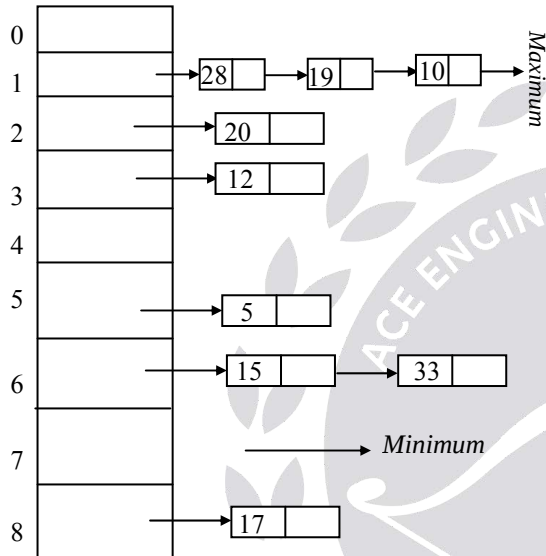
6. Hashing

01. Ans: (d)

Sol: 1 to 1000

02. Ans: (a)

Sol:



$$\frac{3+1+1+1+2+1}{9} = 1 \text{ (average)}$$

03. Ans: 80

Sol: Slots = 25

Elements = 2000

$$\begin{aligned} \text{Load factor} &= \frac{\text{elements}}{\text{slots}} \\ &= \frac{2000}{25} = 80 \end{aligned}$$

04. Ans: (b)

Sol: Hash function

$$h(x) = (3x + 4) \% 7$$

$$h(1) = (3+4) \% 7 = 0$$

$$h(3) = (9+4) \% 7 = 6$$

$$h(8) = (24+4) \% 7 = 0$$

$$h(10) = (30+4) \% 7 = 6$$

Assume Linear probing for collision resolution

The table will be like

①	⑧	⑩				③
0	1	2	3	4	5	6

05. Ans: (d)

Sol: After inserting all keys, the hash table is

Key	43	36	92	87	11	4	71	13	14
Loc	10	3	4	10	0	4	5	2	3

0	1	2	3	4	5	6	7	8	9	10
87	11	13	36	92	4	71	14			43

Last element is stored at the position 7

06. Ans: (c)

Sol: Resultant hash table.

In linear probing, we search hash table sequentially starting from the original location. If a location is occupied, we check the next location. We wrap around from the last table location to the first table location if necessary.



07. Ans: (c)

Sol:

	×	×	✓	×
	A	B	C	D
0				
1				
2	42	42	42	42
3	52	23	23	23
4	34	34	34	23
5	23	52	52	34
6	46	33	46	46
7	33	46	33	52
8				
9				

08. Ans: (c)

Sol: Case (I): To store 52

Variable part			Fixed part		
42	23	34	52	46	33
42	34	23	52	46	33
23	42	34	52	46	33
23	34	42	52	46	33
34	42	23	52	46	33
34	23	42	52	46	33

$$3! = 6$$

Case (II): To store 33

Variable part			Fixed part		
42	23	34	52	46	33
42	23	34		46	

$$4! = 24$$

Since 46 is not getting collided with any other key, it can be moved to the variable part.

Case (I) & Case (II) are mutually exclusive

$$\text{Case (I)} + \text{Case (II)} = 24 + 6 = 30$$

Total 30 different insertion sequences