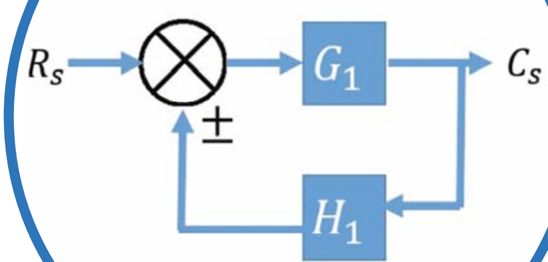
ELECTRONICS & COMMUNICATION ENGINEERING



GATE | PSUs

CONTROL SYSTEMS

Volume - I : Study Material with Classroom Practice Questions

Study Material with Classroom Practice solutions
To
Control Systems

CONTENTS

Chapter **1**

Basics of Control Systems

Class Room Practice Solutions

01. Ans: (c)

Sol: $2\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+4y(t) = r(t) + 2r(t-1)$

Apply LT on both sides

$2s^2\,Y(s)+3sY(s)+4Y(s) = R(s)+2e^{-s}R(s)$

$Y(s)(2s^2 + 3s+4) = R(s)(1+2e^{-s})$

$$\frac{Y(s)}{R(s)} = \frac{1+2e^{-s}}{2s^2+3s+4}$$

02. Ans: (b)

Sol: $I.R = 2.e^{-2t}u(t)$

Output response $c(t) = (1-e^{-2t})\,u(t)$

Input response $r(t) = ?$

$$T.F = \frac{C(s)}{R(s)}$$

$$T.F = L(I.R) = \frac{2}{s+2}$$

$$R(s) = \frac{C(s)}{T.F} = \frac{\frac{1}{s}-\frac{1}{s+2}}{\frac{2}{s+2}} = \frac{1}{s}$$

$$R(s) = \frac{1}{s}$$

$r(t) = u(t)$

03. Ans: (b)

Sol: Unit impulse response of unit-feedback control system is given

$$c(t) = t.e^{-t}$$

$$T.F = L(I.R)$$

$$= \frac{1}{(s+1)^2}$$

$$\text{Open Loop T.F} = \frac{\text{Closed Loop T.F}}{1-\text{Closed Loop T.F}}$$

$$= \frac{\frac{1}{(s+1)^2}}{1-\frac{1}{(s+1)^2}} = \frac{1}{s^2+2s}$$

04. Ans: (a)

Sol: G changes by 10%

$$\Rightarrow \frac{\Delta G}{G}\times 100 = 10\%$$

$C_1 = 10\%$ [$\because$ open loop] whose sensitivity is 100%]

%G change = 10%

$$\frac{\%\text{ of change in M}}{\%\text{ of change in G}} = \frac{1}{1+GH}$$

$$\%\text{ of change in M} = \frac{10\%}{1+(10)1} = 1\%$$

% change in C_2 by 1%

05.

Sol: $M = C/R$

$$\frac{C}{R} = M = \frac{GK}{1+GH}$$

$$S_K^M = \frac{\partial M}{\partial K}\times\frac{K}{M} = 1$$

[$\because$K is not in the loop $\Rightarrow$ sensitivity is 100%]

$$S_H^M = \frac{\partial M}{\partial H}\times\frac{H}{M}$$

$$= \frac{\partial}{\partial H}\left(\frac{GK}{1+GH}\right)\frac{H}{M}$$

$$= \left(\frac{GK(-G)}{(1+GH)^2}\right)\left[\frac{H}{\frac{GK}{1+GH}}\right]$$

$$S_H^M = \frac{-GH}{(1+GH)}$$

06.

Sol: Given data

$G = 2\times10^3,\ \partial G = 100$

$$\%\text{ change in } G = \frac{\partial G}{G}\times100 = 5\%$$

$$\%\text{ change in } M = 0.5\%$$

$$\frac{\%\text{ of change in } M}{\%\text{ of change in } G} = \frac{1}{1+GH}$$

$$\frac{0.5\%}{5\%} = \frac{1}{1+2\times10^3 H}$$

$$1+2\times10^3 H = 10$$

$$H = 4.5\times10^{-3}$$

07. Ans: (b)

Sol: $K = \dfrac{\text{output}}{\text{input}} = \dfrac{c(t)}{r(t)} = \dfrac{\text{mm}}{{}^0\text{c}}$

Chapter 2 Signal Flow Graph and Block Diagrams

Class Room Practice Solutions

01. Ans: (d)

Sol: No. of loops = 3

Loop1: $-G_1G_3G_4H_1H_2H_3$

Loop2: $-G_3G_4H_1H_2$

Loop3: $-G_4H_1$

No. of Forward paths = 3

Forward Path1: $G_1G_3G_4$

Forward Path 2: $G_2G_3G_4$

Forward Path 3: G_2G_4

$$=\frac{G_1G_3G_4+G_2G_3G_4+G_2G_4}{1+G_1G_3G_4H_1H_2H_3+G_3G_4H_1H_2+G_4H_1}$$

02. Ans: (a)

Sol: Number of forward paths = 2

Number of loops = 3

$$\frac{Y(s)}{R(s)}=\frac{\frac{1}{s}\times\frac{1}{s}\times\frac{1}{s}[1-0]+\frac{1}{s}}{1-\left[\frac{1}{s}\times(-1)\left(\frac{1}{s}\right)(-1)+\frac{1}{s}\times\frac{1}{s}(-1)+\left(\frac{1}{s}\times\frac{1}{s}(-1)\right)\right]}$$

$$=\frac{\frac{1}{s^3}+\frac{1}{s}}{1-\left[\frac{1}{s^2}-\frac{1}{s^2}-\frac{1}{s^2}\right]}=\frac{\frac{1+s^2}{s^3}}{1+\frac{1}{s^2}}=\frac{\frac{1+s^2}{s^3}}{\frac{s^2+1}{s^2}}$$

$$=\frac{1+s^2}{s}\times\frac{1}{s^2+1}=\frac{1}{s}$$

03.

Sol: Number of forward paths = 2

Number of loops = 5,

Two non touching loops = 4

$$TF=\frac{24[1-(-0.5)]+10[1-(-3)]}{1-[-24-3-4+(5\times2\times(-1)+(-0.5))]+[30+1.5+2]+\left(\left(\frac{-1}{2}\right)\times(-24)\right)}$$

$$=\frac{76}{88}=\frac{19}{22}$$

04.

Sol: Number of forward paths = 2

Number of loops = 5

$$T.F=\frac{G_1G_2G_3+G_1G_4}{1+G_2G_3H_2+G_1G_2H_1+G_1G_2G_3+G_4H_2+G_1G_4}$$

05. Ans: (c)

Sol: From the network

$$V_o(s)=\frac{1}{sC}I(s)\quad ....(1)$$

$$-V_i(s)+RI(s)+V_o(s)=0$$

$$I(s)=\frac{1}{R}V_i(s)+\left(\frac{-1}{R}\right)V_o(s)\,.............(2)$$

From SFG

$$V_o(s)=x.I(s)\quad ....(3)$$

$$I(s)=\frac{1}{R}V_i(s)+y\,V_o(s)\quad ....\ (4)$$

From equ(1) and (3)

$$x=\frac{1}{sC}$$

From equ(2) and (4)

$$y=-\frac{1}{R}$$

06. Ans: (a)

Sol: Use gain formula

$$\text{transfer function}=\frac{G(s)}{1-\left(G(s)\frac{1}{G(s)}+G(s)\right)}$$

$$=\frac{G(s)}{1-1-G(s)}$$

$$=-1$$

Chapter **3**

Time Response Analysis

Class Room Practice Solutions

01. Ans: (a)

Sol: $\dfrac{C(s)}{R(s)} = \dfrac{1}{1+sT}$, $R(s) = \dfrac{8}{s}$

$$C(s) = \frac{8}{s(1+sT)} \Rightarrow c(t) = 8\left(1-e^{-t/T}\right)$$

$$3.6 = 8\left(1-e^{\frac{-0.32}{T}}\right)$$

$$0.45 = 1 - e^{\frac{-0.32}{T}}$$

$$0.55 = e^{\frac{-0.32}{T}}$$

$$-0.59 = \frac{-0.32}{T}$$

T = 0.535 sec

02. Ans: (c)

Sol: $\cos\phi = \xi$

$\cos 60 = 0.5$

$\cos 45 = 0.707$

Poles left side $0.5 \le \xi \le 0.707$

Poles right side $-0.707 \le \xi \le -0.5$

$\therefore\ 0.5 \le |\xi| \le 0.707$

3 rad/s $\le \omega_n \le$ 5 rad/s

03. Ans: (c)

Sol: For R-L-C circuit:

$$T.F = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{1}{Cs} I(s)$$

$$= \frac{1}{Cs}\frac{V_i(s)}{R + Ls + \frac{1}{Cs}}$$

$$T.F = \frac{V_o(s)}{V_i(s)} = \frac{1}{RCs + LCs^2 + 1}$$

$$= \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}} \qquad 2\xi\omega_n = \frac{R}{L}$$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\xi = \frac{10}{2}\sqrt{\frac{10\times10^{-6}}{1\times10^{-3}}} = 0.5$$

$$M.P = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$= 16.3\% \approx 16\%$$

04. Ans: (b)

Sol: $$TF = \frac{8/s(s+2)}{1-\left(\frac{-8\,as}{s(s+2)} - \frac{8}{s(s+2)}\right)}$$

$$= \frac{8}{s(s+2) + 8as + 8}$$

$$= \frac{8}{s^2 + 2s + 8as + 8}$$

$$= \frac{8}{s^2 + (2+8a)s + 8}$$

$$\omega_n^2 = 8$$

$$\Rightarrow \omega_n = 2\sqrt{2}$$

$$2\xi\omega_n = 2 + 8a$$

$$\xi = \frac{1+4a}{2\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1+4a}{2\sqrt{2}} \Rightarrow a = 0.25$$

05. Ans: 4 sec

Sol: $\text{T.F} = \frac{100}{(s+1)(s+100)}$

$$= \frac{100}{s^2+101s+100}$$

$\omega_n^2 = 100$

$\omega_n = 10$

$2\xi\omega_n = 101$

$\xi = \frac{101}{20}$

$\xi > 1 \rightarrow$system is over damped i.e., roots are real & unequal.

Using dominate pole concept,

$\text{T.F} = \frac{100}{100(s+1)} = \frac{1}{s+1}$, Here $\tau = 1$ sec

$\therefore$ setting time for 2% criterion = 4τ

= 4 sec

06.

Sol: $M_p = \frac{C(t_p) - C(\infty)}{C(\infty)}$

$$= \frac{1.254 - 1.04}{1.04} = 0.2$$

$$\xi = \sqrt{\frac{(\ln M_P)^2}{(\ln M_P)^2 + \pi^2}}$$

$M_p = 0.2$

$\xi = 0.46$

07. Ans: (d)

Sol: Given data: $\omega_n = 2$, $\zeta = 0.5$

Steady state gain = 1

$\text{OLTF} = \frac{K_1}{s^2+as+2}$ and $H(s) = K_2$

$\text{CLTF} = \frac{G(s)}{1+G(s)}$

$$\frac{C(s)}{R(s)} = \frac{K_1}{s^2+as+2+K_1K_2}$$

DC or steady state gain from the TF

$\frac{K_1}{2+K_1K_2} = 1$

$K_1(1 - K_2) = 2$ ……… (1)

CE is $s^2 + as + 2 + K_1K_2 = 0$

$\omega_n = \sqrt{2+K_1K_2} = 2$

$4 = (2+K_1K_2)$

$K_1K_2 = 2$ ………. (2)

Solving equations (1) & (2) we get

$K_1 = 4$, $K_2 = 0.5$

$2\zeta\,\omega_n = a$

$2\times\frac{1}{2}\times 2 = a$

$a = 2$

08. Ans: A – T, B – S, C- P, D – R, E – Q

Sol:

(A) If the poles are real & left side of s-plane, the step response approaches a steady state value without oscillations.

(B) If the poles are complex & left side of s-plane, the step response approaches a steady state value with the damped oscillations.

(C) If poles are non-repeated on the $j\omega$ axis, the step response will have fixed amplitude oscillations.

(D) If the poles are complex & right side of s-plane, response goes to '∞' with damped oscillations.

(E) If the poles are real & right side of s-plane, the step response goes to '∞' without any oscillations.

09. Ans: (c)

Sol: If R $\uparrow$ damping $\uparrow$

$$\Rightarrow \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

(i) If R$\uparrow$, steady state voltage across C will be reduced (wrong)

(Since steady state value does not depend on ξ)

If $\xi \uparrow$, C (∞) = remain same

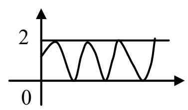
(ii) If $\xi\uparrow$, ($\omega_d = \omega_n\sqrt{1-\xi^2}$) $\omega_d \downarrow$

(iii) If $\xi \downarrow$, $t_s \uparrow \Rightarrow 3^{rd}$

Statement is false

(iv) If $\xi = 0$

True



$\Rightarrow$ 2 and 4 are correct

10.

Sol: (i) Unstable system

$\therefore$ error = ∞

(ii) $G(s) = \dfrac{10(s+1)}{s^2}$

Step $\rightarrow$ R (s) = $\dfrac{1}{s}$

$k_p = \infty$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+\infty} = 0$$

Parabolic $\Rightarrow k_a = 10$

$$e_{ss} = \frac{1}{10} = 0.1$$

11.

Sol: $G(s) = 10/s^2$ (marginally stable system)

$\therefore$ Error can't be determined

12.

Sol: $e_{ss} = \dfrac{1}{11}$, $R(s) = \dfrac{1}{s}$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+k_p} = \frac{1}{11} = \frac{1}{1+10}$$

$k_p = \underset{s\to 0}{\text{Lt}}\ G(s)$

$10 = \underset{s\to 0}{\text{Lt}}\ G(s)$

k = 10

$R(s) = \dfrac{1}{s^2}$ (ramp)

$$e_{ss} = \frac{A}{k_v} = \frac{1}{k_v} = \frac{1}{10}$$

(System is increased by 1)

$\Rightarrow e_{ss} = 0.1$

13. Ans: (a)

Sol: $T(s) = \dfrac{(s-2)}{(s-1)\ (s+2)^2}$ (unstable system)

14. Ans: (b)

Sol: Given data: r(t) = 400tu(t) rad/sec

Steady state error =10°

i.e., $e_{ss} = \dfrac{\pi}{180^{\circ}}(10^{\circ})$ radians

$G(s) = \dfrac{20K}{s(1+0.1s)}$ and H(s) = 1

r(t) = 400tu(t) $\Rightarrow 400/s^2$

Error (e_{ss}) = $\dfrac{A}{K_V} = \dfrac{400}{K_V}$

$K_V = \underset{s\to 0}{\text{Lim}}\, s\,G(s)$

$K_V = \underset{s\to 0}{\text{Lim}}\, s\dfrac{20K}{s(1+0.1s)}$

$K_V = 20K$

$e_{ss} = \dfrac{400}{20K}$

$e_{ss} = \dfrac{20}{K} = \dfrac{\pi}{18}$

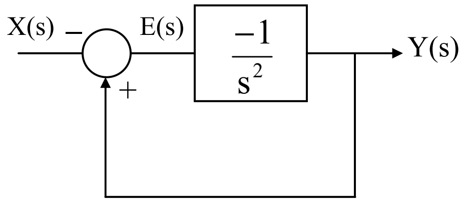
K = 114.5

15. Ans: (d)

Sol: $\dfrac{d^2y}{dt^2} = -e(t)$

$s^2\,Y(s) = -\,E(s)$

$x(t) = t\,u(t) \Rightarrow X(s) = \frac{1}{s^2}$



$$Y(s) = \frac{-1}{s^2}E(s)$$

$$\frac{Y(s)}{E(s)} = \frac{-1}{s^2}$$

$$\frac{E(s)}{X(s)} = \frac{-1}{1+\frac{1}{s^2}}$$

$$E(s) = \frac{-s^2}{1+s^2}X(s)$$

$$= \frac{-s^2}{1+s^2}\times\frac{1}{s^2} = \frac{-1}{1+s^2}$$

$$= L^{-1}\left[\frac{-1}{1+s^2}\right] = -\sin\ t$$

16. Ans: (a)

Sol: $e_{ss} = 0.1$ for step input

For pulse input = 10

time = 1 sec

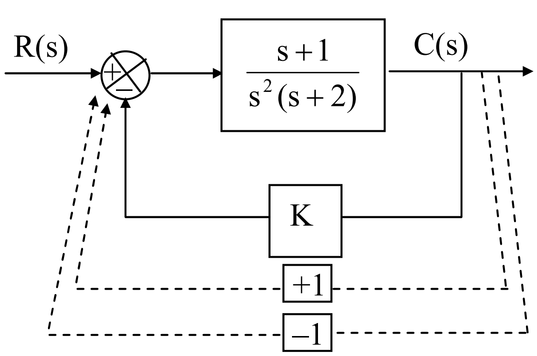
error is function of input

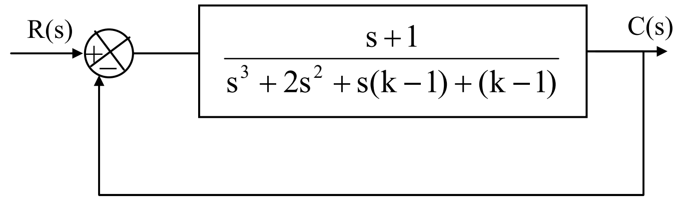
$t \rightarrow \infty$ input = 0

$\therefore$ Error = zero

17. Ans: (a)

Sol:





$\therefore$ Type –0 system

18. Ans: (a)

Sol: $K_p = \underset{s\to 0}{\text{lt}}\, G(s)$

$$K_p = \frac{1}{k-1}$$

$$e_{ss} = \frac{0.1}{100} = \frac{1}{1+\frac{1}{k-1}}$$

$$= \frac{k-1}{k} = \frac{1}{100\times 10}$$

$1000\,k - 1000 = k$

$$\Rightarrow k = \frac{1000}{999}$$

19. Ans: (c)

Sol:
$$\frac{C(s)}{R(s)} = \frac{\frac{100}{(s+1)(s+5)}}{1+\frac{100\times 0.2}{(s+1)(s+5)}}$$

$$= \frac{100}{(s+1)(s+5)+20}$$

$$= \frac{100}{s^2+6s+5+20}$$

$$= \frac{100}{s^2+6s+25}$$

$\omega_n^2 = 25$, $\omega_n = 5$

$2\xi\omega_n = 6$

$$\xi = \frac{6}{10} = \frac{3}{5}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 5\sqrt{1-\left(\frac{3}{5}\right)^2} = 5\times\frac{4}{5} = 4 \text{ rad/sec}$$

Chapter 4 Stability

Class Room Practice Solutions

01.

Sol: $CE = s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$

s^5	1	8	7	
s^4	4(1)	8(2)	4(1)	
s^3	6(1)	6(1)	0	
s^2	1	1	0	$\to$ Row of AE
s^1	0(2)	0	0	$\to$ Row of zero
s^0	1			

No. of AE roots = 2	No. of CE roots = 5
No. of sign changes Below AE = 0	No. of sign changes in 1st column = 0
No. of RHP = 0	$\therefore$ No .of RHP = 0
No. of LHP = 0	No. of $j\omega$p = 2
No. of $j\omega$p = 2	$\Rightarrow$ No .of LHP = 3

System is marginally stable.

(ii) $s^2 + 1 = 0$

$s = \pm 1\, j = \pm j\omega_n$

$\omega_n = 1$ rad/ sec

Oscillating frequency $\omega_n = 1$ rad/sec

02.

Sol: (i) $s^5 + s^4 + s^3 + s^2 + s + 1 = 0$

$+ s^5$	1	1	1
$+ s^4$	1	1	1
$+ s^3$	0(2)	0(1)	0
$+ s^2$	$\frac{1}{2}$	1	
$(1) - s^1$	-3	0	
$(2) + s^0$	1		

$AE\ (1) = s^4 + s^2 + 1 = 0$

$$\frac{d(AE)}{ds} = 4s^3 + 2s = 0$$

$\Rightarrow 2s^3 + s = 0$

AE	**CE**
No. of sign changes below AE = 2	No. of sign changes in 1st column = 2
No. of AE roots = 4	No. of CE roots = 5
No .of RHP = 2	No. of RHP = 2
No .of LHP = 2	No. of LHP = 3
No. of $j\omega$p = 0	No. of $j\omega$p = 0

System is unstable

(ii) $s^6 + 2s^5 + 2s^4 + 0s^3 - s^2 - 2s - 2 = 0$

s^6	1	2	-1	-2
s^5	2(1)	0	$-2(-1)$	0
s^4	2(1)	+0	$-2(-1)$	0
s^3	0(4)	0	0	0
s^2	$0(\varepsilon)$	-1	0	0
s^1	$4/\varepsilon$			
$-s^0$	-1			

$AE = s^4 - 1 = 0$

$$\frac{dAE}{ds} = 4s^3 + 0 = 0$$

CE	AE
No. of CE roots = 6	No. of AE roots = 4
No. of sign changes in the 1st column= 1	No. of sign changes below AE = 1
No .of RHP = 1	No. of RHP = 1
No .of LHP = 3	No. of $j\omega p$ = 2
No. of $j\omega p$ = 2	No. of LHP = 1

03.

Sol: CE = $s^3 + 20s^2 + 16s + 16K = 0$

s^3	1	16
s^2	20	16K
s^1	$\frac{20(16)-16K}{20}$	0
s^0	16K	

(i) For stability $\frac{20(16)-16K}{20} > 0$

$\Rightarrow 20(16) - 16K > 0$

$\Rightarrow K < 20$ and $16K > 0 \Rightarrow K > 0$

Range of K for stability $0 < K < 20$

(ii) For the system to oscillate with ω_n it must be marginally stable

i.e., s^1 row should be 0

s^2 row should be AE

$$\boxed{\therefore \text{A.E roots} = \pm j\omega_n}$$

$\therefore s^1$ row $\Rightarrow 20(16) - 16K = 0$

$\Rightarrow K = 20$

AE is $20s^2 + 16K = 0$

$20s^2 + 16(20) = 0$

$\Rightarrow s = \pm j4$

$\omega_n = 4$ rad/sec

04.

Sol: CE = $1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$

$s^3 + as^2 + (K+2)s + K + 1 = 0$

$s^3 + as^2 + (K+2)s + (K+1) = 0$

s^3	1	K + 2
s^2	a	K+ 1
s^1	$\frac{a(K+2)-(K+1)}{a}$	0
s^0	K + 1	

Given,

$\omega_n = 2$

$\Rightarrow s^1$ row = 0

s^2 row is A.E

$a(K+2) - (K+1) = 0$

$a = \frac{K+1}{K+2}$

AE = $as^2 + K + 1 = 0$

$= \frac{K+1}{K+2}s^2 + K + 1 = 0$

$(k+1)\left(\frac{s^2}{k+2} + 1\right) = 0$

$s^2 + k + 2 = 0$

$s = \pm j\sqrt{(k+2)}$

$\omega_n = \sqrt{k+2} = 2$

$k = 2$

$a = \frac{k+1}{k+2} = \frac{3}{4}$

$= 0.75$

05.

Sol: $s^3 + ks^2 + 9s + 18$

s^3	1	9
s^2	K	18
s^1	$\frac{9K-18}{K}$	0
s^0	18	

Given that system is marginally stable,

Hence

s^1 row = 0

$\frac{9K-18}{K} = 0$

$9K = 18 \Rightarrow K = 2$

A.E is $9s^2 + 18 = 0$

$Ks^2 + 18 = 0$,

$2s^2 + 18 = 0$

$2s^2 = -18$

$s = \pm j3$

$\therefore\ \omega_n = 3$ rad/sec.

06. Ans: (d)

Sol: Given transfer function $G(s) = \frac{k}{(s^2+1)^2}$

Characteristic equation $1 - G(s).H(s) = 0$

$1 - \frac{k}{(s^2+1)^2} = 0$

$s^4 + 2s^2 + 1 - k = 0$ ….. (1)

RH criteria

s^4	1	2	1−K
s^3	4	4	-
s^2	1	1−K	
s^1	4K		
s^o	1−K		

$AE = s^4 + 2s^2 + 1-K$

$\frac{d}{ds}(AE) = 4s^3 + 4s$

$1-K > 0$ no poles are on RHS plane and LHS plane.

All poles are on $j\omega$- axis

$\therefore\ 0 < K < 1$ system marginally stable

Chapter 5

Root Locus Diagram

Class Room Practice Solutions

01. Ans: (a)

Sol: $S_1 = -1 + j\sqrt{3}$

$S_2 = -3 - j\sqrt{3}$

$$G(s).H(s) = \frac{K}{(s+2)^3}$$

$S_1 = -1 + j\sqrt{3}$

$$G(s).H(s) = \frac{K}{\left(-1+j\sqrt{3}+2\right)^3}$$

$$= \frac{K}{\left(1+j\sqrt{3}\right)^3}$$

$$= -3\tan^{-1}(\sqrt{3})$$

$$= -180°$$

It is odd multiples of 180°, Hence S_1 lies on Root locus

$S_2 = -3 - j\sqrt{3}$

$$G(s).H(s) = \frac{K}{\left(-3-j\sqrt{3}+2\right)^3}$$

$$= \frac{K}{\left(-1-j\sqrt{3}\right)^3}$$

$$= -3\,[180° + 60°] = -720°$$

It is not odd multiples of 180°, Hence S_2 is not lies on Root locus.

02. Ans: (a)

Sol: Over damped – roots are real & unequal $\Rightarrow 0 < k < 4$

(b) k = 4 roots are real & equal
$\Rightarrow$ Critically damped $\xi = 1$

(c) $k > 4 \Rightarrow$ roots are complex
$0 < \xi < 1 \Rightarrow$ under damped

03. Ans: (a)

Sol: Asymptotes meeting point is nothing but centroid

$$\text{centroid } \sigma = \frac{\sum \text{poles} - \sum \text{zeros}}{p - z}$$

$$= \frac{-3-0}{3-0} = -1$$

centroid = (–1, 0)

04. Ans: (b)

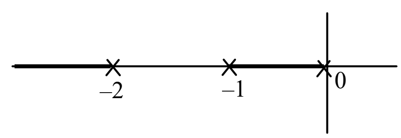
Sol: break point $= \dfrac{dK}{ds} = 0$

$$\frac{d}{ds}\left(G_1(s).H_1(s)\right) = 0$$

$$\frac{d}{ds}\left[s(s+1)(s+2)\right] = 0$$

$3s^2 + 6s + 2 = 0$

$s = -0.422, -1.57$



But s = – 1.57 do not lie on root locus
So, s = –0.422 is valid break point.

Point of intersection:

$s^3 + 3s^2 + 2s + k = 0$

s^3	1	2
s^2	3	k
s^1	$\dfrac{6-k}{3}$	0
s^0	k	

As s^1 Row = 0

k = 6

$3s^2 + 6 = 0$

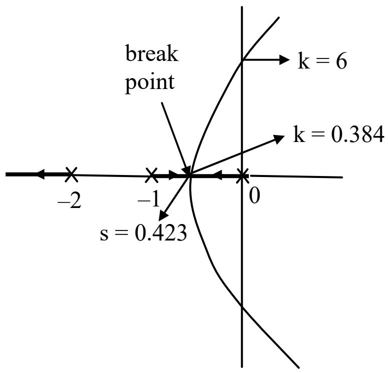
$s^2 = -2$

$s = \pm j\sqrt{2}$

point of inter section: $s = \pm j\sqrt{2}$

05. Ans: (b)

Sol:



$$\frac{K}{s(s+1)(s+2)}$$

substitute s = – 0.423 and apply the magnitude criteria.

$$\left|\frac{K}{(-0.423)(-0.423+1)(-0.423+2)}\right| = 1$$

K = 0.354

when the roots are complex conjugate then the system response is under damped.

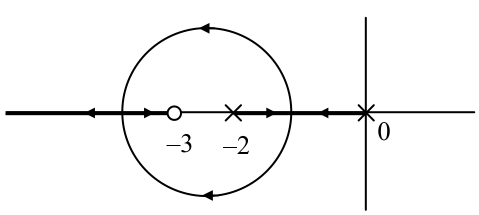
From K > 0.384 to K < 6 roots are complex conjugate then system to be under damped the values of k is 0.384 < K < 6.

06. Ans: (c)

Sol: If the roots are lies on the real axis then system exhibits the non-oscillatory response. from $K \geq 0$ to $K \leq 0.384$ roots lies on the real axis. Hence for $0 \leq K \leq 0.384$ system exhibits the non-oscillatory response.

07. Ans: (a)

Sol:



$$\frac{d}{ds}[G(s).H(s)] = \frac{d}{ds}\left[\frac{k(s+3)}{s(s+2)}\right]$$

$s^2 + 6s + 6 = 0$

break points – 1.27, – 4.73

$$\text{radius} = \frac{4.73-1.27}{2} = 1.73$$

center = (–3, 0)

08. Ans: (c)

Sol: $$G(s).H(s) = \frac{K(s+3)}{s(s+2)}$$

$$k|_{s=-4} = \left|\frac{(-4)(-4+2)}{(-4+3)}\right|$$

$$= \left|\frac{(-4)(-2)}{(-1)}\right| = 8$$

09. Ans: (a)

Sol: $s^2 - 4s + 8 = 0 \Rightarrow s = 2 \pm 2j$ are two zeroes

$s^2 + 4s + 8 = 0 \Rightarrow s = -2 \pm 2j$ are two poles

$$\phi_A = 180 - \angle GH|_{S=2\pm2j}$$

$$GH = \frac{k[s-(2+2j)][s-(2-2j)]}{[s-(-2+2j)][s-(-2-2j)]}$$

$$\angle GH|_{S=2+2j} = \frac{\angle k \angle 4j}{\angle 4 \angle 4+4j}$$

$$= 90^\circ - 45^\circ = 45^\circ$$

$$\phi_A = 180^\circ - 45^\circ = \pm 135^\circ$$

10. Ans: (b)

Sol: $s^2 - 4s + 8 = 0 \Rightarrow s = 2 \pm 2j$ are two zeroes

$s^2 + 4s + 8 = 0 \Rightarrow s = -2 \pm 2j$ are two poles

$$\phi_d = 180^0 + \angle GH|_{S=-2\pm2j}$$

$$\angle GH|_{S=-2\pm2j} = \angle \left.\frac{k[s-(2+2j)][s-(2-2j)]}{[s-(-2+2j)][s-(-2-2j)]}\right|_{S=-2\pm2j}$$

$$= \frac{\angle k(-4)(-4+4j)}{\angle 4j}$$

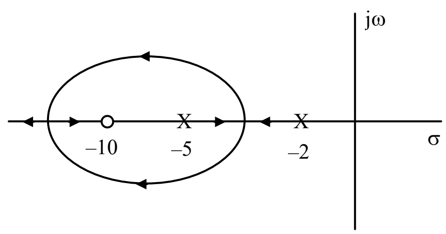
$$= 180^\circ + 180^\circ - 45^\circ - 90^\circ = 225^\circ$$

$$\phi_d = 180^\circ + 225^\circ = 405^\circ$$

$$\therefore \phi_d = \pm 45^\circ$$

11. Ans: (d)

Sol: Poles s = –2, –5 ; Zero s = –10



∴ breakaway point exist between –2 and –5

Chapter 6 Frequency Response Analysis

Class Room Practice Solutions

01. Ans: (c)

Sol: $G(s).H(s) = \dfrac{100}{s(s+4)(s+16)}$

Phase crossover frequency (ω_{pc}):

$\angle G(j\omega).H(j\omega)/_{\omega=\omega_{pc}} = -180°$

$-90° - \tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -180°$

$-\tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -90°$

$\tan[\tan^{-1}(\omega_{pc}/4) + \tan^{-1}(\omega_{pc}/16)] = \tan(90°)$

$$\frac{\frac{\omega_{pc}}{4}+\frac{\omega_{pc}}{16}}{1-\frac{\omega_{pc}}{4}.\frac{\omega_{pc}}{16}} = \frac{1}{0}$$

$\omega_{pc}^2 = 16 \times 4 \Rightarrow \omega_{pc} = 8$ rad/sec

02. Ans: (d)

Sol: $G(s).H(s) = \dfrac{100}{s(s+2)(s+16)}$

Gain margin (G.M) $= \dfrac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$

$$|G(j\omega).H(j\omega)|_{\omega=\omega_{pc}} = \frac{100}{\omega_{pc}\sqrt{\omega_{pc}^2+16}\sqrt{\omega_{pc}^2+16^2}}$$

$$= \frac{5}{64}$$

$$\text{G.M} = \frac{64}{5} = 12.8$$

03. Ans: (c)

Sol: $G(s).H(s) = \dfrac{2e^{-0.5s}}{(s+1)}$

gain crossover frequency,

$\omega_{gc} = |G(j\omega).H(j\omega)|_{\omega=\omega_{gc}} = 1$

$$\frac{2}{\sqrt{\omega_{gc}^2+1}} = 1$$

$\omega_{gc}^2 + 1 = 4 \Rightarrow \omega_{gc} = \sqrt{3}$ rad/sec

04. Ans: (b)

Sol: $\omega_{gc} = \sqrt{3}$ rad/sec

P.M = $180° + \angle G(j\omega).H(j\omega)/\omega = \omega_{gc}$

$\angle G(j\omega).H(j\omega)/_{\omega=\omega_{gc}} = -0.5\,\omega_{gc} - \tan^{-1}(\omega_{gc})$

$= -109.62°$

P.M = 70.39°

05. Ans: (a)

Sol: $M_r = 2.5 = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$

$2\xi\sqrt{1-\xi^2} = \dfrac{1}{2.5}$

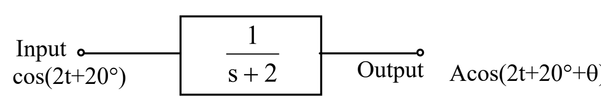
$\xi^4 - \xi^2 + 0.04 = 0$

$\xi^2 = 0.958 \qquad \xi^2 = 0.0417$

$\xi = 0.204 \qquad (M_r > 1)$

06. Ans: (a)

Sol: Closed loop T.F $= \dfrac{1}{s+2}$



$$A = \frac{1}{\sqrt{\omega^2+4}} = \frac{1}{\sqrt{4+4}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

$\phi = -\tan^{-1}\omega/2$

$= -\tan^{-1}2/2$

$\Rightarrow \phi = -\tan^{-1}(1) = -45°$

output $= \dfrac{1}{2\sqrt{2}}\cos(2t + 20° - 45°)$

$= \dfrac{1}{2\sqrt{2}}\cos(2t - 25°)$

07. Ans: (c)

Sol: Initial slope = –40 dB/dec

Two integral terms $\left(\frac{1}{s^2}\right)$

$\therefore$ Part of TF $= G(s)H(s) = \frac{K}{s^2}$

at $\omega = 0.1$

change in slope = –20 – (–40)

= 20°

Part of TF = G(s) H(s) $= \frac{K\left(1+\frac{s}{0.1}\right)}{s^2}$

At $\omega = 10$ slope changed to –60 dB/dec

Change in slope = –60–(–20)

= – 40dB/dec

$$\text{TF }(G(s)H(s)) = \frac{K\left(1+\frac{s}{0.1}\right)}{s^2\left(\frac{s}{10}+1\right)^2}$$

20 log K– 2 (20 log 0.1) = 20 dB

20 log K = 20 – 40

20 log K = –20

K = 0.1

$$G(s)H(s) = \frac{(0.1)\left(1+\frac{s}{0.1}\right)}{s^2\left(1+\frac{s}{10}\right)^2}$$

$$= \frac{(0.1)\times 10^2(s+0.1)}{(0.1)s^2(s+10)^2}$$

$$G(s)H(s) = \frac{100(s+0.1)}{s^2(s+10)^2}$$

08. Ans: (b)

Sol: $G(s)H(s) = \dfrac{Ks}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$

12 = 20 log K+ 20 log 0.5

12 = 20logK + (−6)

20 log K = 18 dB = 20 log 2^3

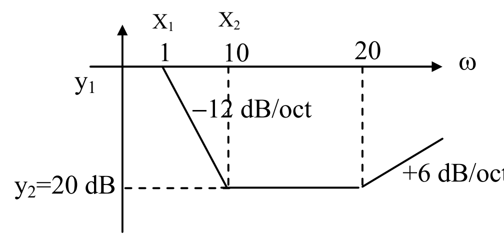
K = 8

$$G(s)H(s) = \frac{8s\times 2\times 10}{(2+s)(10+s)}$$

$$G(s)H(s) = \frac{160s}{(2+s)(10+s)}$$

09. Ans: (b)

Sol:



$$G(s)H(s) = \frac{K\left(1+\frac{s}{10}\right)^2\left(1+\frac{s}{20}\right)}{(1+s)^2}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = -40\,\text{dB/dec}$$

$$\frac{20 - y_1}{\log 10 - \log 1} = -40$$

$$y_1 = +60\,\text{dB}\Big|_{\omega\le 1}$$

$\Rightarrow$ 20 log K = 60

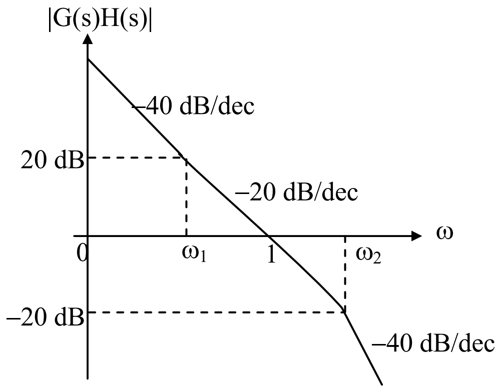
K = 10^3

$$G(s)H(s) = \frac{10^3(s+10)^2(s+20)}{10^2\times 20\times(s+1)^2}$$

$$= \frac{(s+10)^2(s+20)}{2(s+1)^2}$$

10. Ans: (d)

Sol:



ω_1 calculation:

$$\frac{0-20}{\log 1-\log \omega_1}=-20\text{ dB/dec}$$

$\omega_1 = 0.1$

ω_2 calculation:

$$\frac{-20-0}{\log \omega_2-\log 1}=-20\text{dB/dec}$$

$\omega_2 = 10$

$$G(s)H(s)=\frac{K\left(1+\frac{s}{0.1}\right)}{s^2\left(1+\frac{s}{10}\right)}$$

20logK–2 (20 log 0.1) = 20

20 logK = 20–40

K = 0.1

$$G(s)H(s)=\frac{0.1\times\frac{1}{0.1}(0.1+s)}{s^2\frac{1}{10}(10+s)}$$

$$=\frac{10(0.1+s)}{s^2(10+s)}$$

11.

Sol: $\frac{200}{s(s+2)}=\frac{100}{s\left(1+\frac{s}{2}\right)}$

$x=-KT \Rightarrow -(100)\times\frac{1}{2}=x=-50$

12. Ans: (c)

Sol: For stability (–1, j0) should not be enclosed by the polar plot.

For stability

$1 > 0.01$ K

$\Rightarrow$ K < 100

13.

Sol: GM = –40 dB

$20\log\frac{1}{a}=-40 \Rightarrow a=10^2$

POI = 100

14.

Sol: (i) $GM=\frac{1}{0.1}=+10=20\,dB$

PM = 180°– 140°= 40°

(ii) PM = 180–150°= 30°

$GM=\frac{1}{0}=\infty$ POI = 0

(iii) ω_{PC} does not exist

$GM=\frac{1}{0}=\infty$ PM = 180° + 0° = 180°

(iv) ω_{gc} not exist

$\omega_{pc}=\infty$

$GM=\frac{1}{0}=\infty$

$PM=\infty$

(v) $GM=\frac{1}{0.5}=2$

PM = 180 − 90

$=90^0$

15. Ans: (d)

Sol: For stability (–1, j0) should not be enclosed by the polar plot. In figures (1) & (2) (–1, j0) is not enclosed.

$\therefore$ Systems represented by (1) & (2) are stable.

16. Ans: (b)

Sol: Open loop system is stable, since the open loop poles are lies in the left half of s-plane

$\therefore$ P = 0.

From the plot N = –2 .

No.of encirclements N = P – Z

N = –2, P = 0 (Given)

$\therefore$ N = P – Z

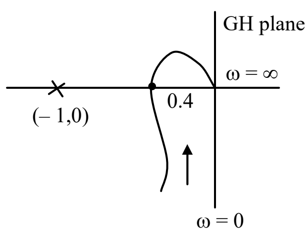
–2 = 0 – Z

Z = 2

Two closed loop poles are lies on RH of s-plane and hence the closed loop system is unstable.

17. Ans: (c)

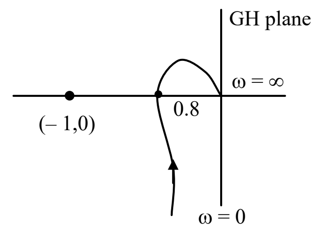
Sol:



$\frac{K_c}{K} = 0.4$ When K = 1

Now, K double, $\frac{K_c}{K} = 0.4$

$K_c = 0.4 \times 2 = 0.8$



even though the value of K is double, the system is stable (negative real axis magnitude is less than one)

Oscillations depends on ' ξ'

$\xi \propto \frac{1}{\sqrt{K}}$ as K is increased ξ reduced, then more oscillations.

Chapter **7**

Controllers & Compensators

Class Room Practice Solutions

01. Ans: (a)

Sol: $G_C(s) = (-1)\left(-\frac{Z_2}{Z_1}\right)$

$$= (-1)(-1)\left(\frac{R_2 + \frac{1}{sC}}{R_1}\right)$$

$$G_c(s) = \frac{(100\times10^3) + \frac{1}{s\times10^{-6}}}{10^6}$$

$$G_c(s) = \frac{1+0.1s}{s}$$

02. Ans: (c)

Sol: CE $\Rightarrow 1 + G_c(s)\,G_p(s) = 0$

$$= 1 + \frac{1+0.1s}{s}\times\frac{1}{(s+1)(1+0.1s)}$$

$$= 1 + \frac{1+0.1s}{s(s+1)(1+0.1s)} = 0$$

$\Rightarrow s^2 + s + 1 = 0 \Rightarrow \omega_n = 1,$

$$e^{\left[\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right]_{\xi=0.5}} = 0.163$$

$M_p = 16.3\%$

03. Ans: (b)

Sol: T.F $= \frac{k(1+0.3s)}{1+0.17s}$

$T = 0.17,\ aT = 0.3 \Rightarrow a = \frac{0.3}{0.17}$

$C = 1\ \mu F$

$$T = \frac{R_1R_2}{R_1+R_2}C,\quad a = \frac{R_1+R_2}{R_2}$$

$$\frac{R_1R_2}{R_1+R_2} = \frac{0.17}{1\times10^{-6}} = 170000$$

$$\frac{R_1+R_2}{R_2} = 1.764$$

$aT = R_1 C$

$$R_1 = \frac{aT}{C} = \frac{0.3}{C} = (0.3)(10^6)$$

$= 300\ k\Omega$

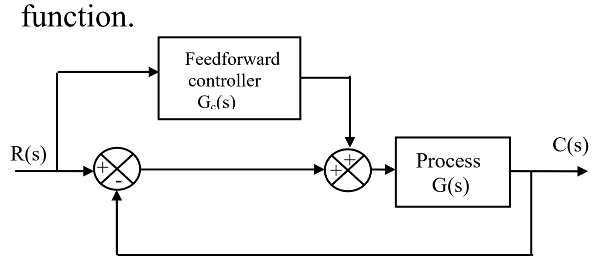
$$\frac{300+R_2}{R_2} = 1.76$$

$300\ k + R_2 - 1.76\ R_2 = 0$

$$R_2 = \frac{300}{0.70} = 394.736 = 400\ k\Omega$$

04. Ans: (b)

Sol: The feed forward controller $G_c(s)$ is placed in series with closed loop system as shown in fig. below. The poles, zeros of $G_c(s)$ may be selected to add or cancel the poles and zeros of the closed loop system transfer function.



Given $G(s) = \frac{1}{s^2+2s+2}, H(s) = 1$

If $G_c(s) = \frac{s+2}{s+1}$ is taken the closed loop transfer function with controller is

$$\frac{C(s)}{R(s)} = \frac{G(s)[1+G_c(s)]}{1+G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{2s+3}{(s+1)(s^2+2s+3)}$$

Given $R(s) = \frac{1}{s}$

$$C(s) = \frac{2s+3}{s(s+1)(s^2+2s+3)}$$

Steady state value

$$\underset{t\to\infty}{\text{Lt}}\, c(t) = \underset{S\to 0}{\text{Lt}}\, s\, C(s)$$

$$= \underset{S\to 0}{\text{Lt}} \frac{s(2s+3)}{s(s+1)(s^2+2s+3)} = 1$$

Steady State Error (e_{ss}) = $\underset{t\to\infty}{\text{Lt}}\left[r(t) - c(t)\right]$

$= 1 - 1 = 0$

05. Ans: (d)

Chapter 8 State Variable Analysis

Class Room Practice Solutions

01. Ans: (a)

Sol: $TF = \dfrac{1}{s^2+5s+6}$

$= \dfrac{1}{(s+2)(s+3)}$

$= \dfrac{1}{s+2} + \dfrac{-1}{s+3}$

$\therefore \; A = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$C = [1 \;\; 1]$

02. Ans: (c)

Sol: Given problem is Controllable canonical form.

(or)

$TF = C[sI - A]^{-1}B + D$

$= [6 \;\; 5 \;\; 1] \begin{bmatrix} s & 1 & 0 \\ 0 & s & 1 \\ -5 & -3 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

$= \dfrac{3s^2+15s+18}{s^3+6s^2+3s+5}$

03. Ans: (d)

Sol: $\dfrac{d^2y}{dt^2} + \dfrac{3dy}{dt} + 2y = u(t)$

2^{nd} order system hence two state variables are chosen

Let $x_1(t)$, $x_2(t)$ are the state variables

CCF − SSR

Let $x_1(t) = y(t)$ ……….. (1)

$x_2(t) = \dot{y}(t)$ ……….. (2)

Differentiating (1)

$\dot{x}_1(t) = \dot{y}(t) = x_2(t)$ ……….. (3)

$\dot{x}_2(t) = \ddot{y}(t) = u(t) - 3y^1(t) - 2y(t)$

$= u(t) - 3x_2(t) - 2x_1(t)$ ……. (4)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} u(t)$$

From equation 1. The output equation in matrix form

$$y(t) = \underbrace{[1 \;\; 0]}_{C} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, D = 0$$

04. Ans: (b)

Sol: OCF − SSR

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [0 \;\; 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

05. Ans: (c)

Sol: Normal form − SSR

$TF = \dfrac{Y(s)}{G(s)} = \dfrac{1}{s^2+3s+2} = \dfrac{1}{(s+1)(s+2)}$

$\Rightarrow$ Diagonal canonical form

The eigen values are distinct i.e., − 1 & − 2.

$\therefore$ Corresponding normal form is called as diagonal canonical form

DCF − SSR

$\dfrac{Y(s)}{U(s)} = \dfrac{b_1}{s+1} + \dfrac{b_2}{s+2}$

$b_1 = 1$, $b_2 = -1$

$Y(s) = \underbrace{\dfrac{b_1}{s+1}U(s)}_{x_1} + \underbrace{\dfrac{b_2}{s+2}U(s)}_{x_2}$

Let $Y(s) = X_1(s) + X_2(s)$

Where y(t) = $x_1(t) + x_2(t)$ ………. (1)

Where $X_1(s) = \dfrac{b_1}{s+1} U(s)$

$S X_1(s) + X_1(s) = b_1 U(s)$

Take Laplace Inverse

$\dot{x}_1 + x_1 = b_1 u(t)$ ……… (2)

$X_2(s) = \dfrac{b_2}{s+2} U(s)$

$S X_2(s) + 2 X_2(s) = b_2 U(s)$

Laplace Inverse

$\dot{x}_2 + 2x_2 = b_2 u(t)$

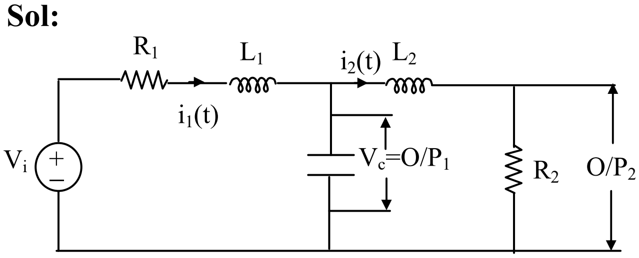
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

From(1) output equation.

$$y(t) = [1 \ \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

06. Ans: (c)

Sol:



$O/P_1 \Rightarrow y_1 = V_c$

$O/P_2 \Rightarrow y_2 = R_2 i_2$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} V_c \\ i_1 \\ i_2 \end{bmatrix}$$

y = C X

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix}$$

07. Ans: (a)

Sol: T.F = $C[sI-A]^{-1}B + D$

$$= [1 \ \ 0] \begin{bmatrix} s+4 & 1 \\ 3 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= [1 \ \ 0] \frac{1}{s^2+5s+1} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+5s+1} [1 \ \ 0]_{1\times 2} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix}_{2\times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+5s+1} [s+1 \ \ -1]_{1\times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2\times 1}$$

$$= \frac{1}{s^2+5s+1} [s+1-1]$$

$$= \frac{s}{s^2+5s+1}$$

08. Ans: (c)

Sol: State transition matrix $\phi(t) = L^{-1}[(sI-A)^{-1}]$

$$sI - A = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{s+3} & \dfrac{1}{(s+2)(s+3)} \\ 0 & \dfrac{1}{s+2} \end{bmatrix}$$

$$L^{-1}[[sI - A]^{-1}] = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

09. Ans: (b)

Sol: Controllability

$$[M] = [B \quad AB \quad A^2B.. \quad A^{n-1}B]$$

$$AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$|M| = -1 \neq 0$ (Controllable)

Observability

$$[N]=\left[C^T \;\; A^TC^T \;....\; (A^T)^{n-1}C^T\right]$$

$$A^TC^T=\begin{bmatrix}0 & -2\\ 1 & -3\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}=\begin{bmatrix}-2\\ -2\end{bmatrix}$$

$$N=\begin{bmatrix}1 & -2\\ 1 & -2\end{bmatrix}$$

$|N| = 0$ (Not observable)

10. Ans: (c)

Sol: According to Gilberts test the system is controllable and observable.

11. Ans: (c)

Sol: $\dfrac{Y(s)}{U(s)}=\dfrac{b_1s^2+b_2s+b_3}{s^3+a_1s^2+a_2s+a_3}$

at node $\dot{x}_1$

$\dot{x}_1 = -a_1x_1 - a_2x_2 - a_3\,x_3$

at $\dot{x}_2 = x_1$ & $\dot{x}_3 = x_2$

$$\therefore \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-a_1 & -a_2 & -a_3\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}=\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$\therefore A=\begin{bmatrix}-a_1 & -a_2 & -a_3\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}$$