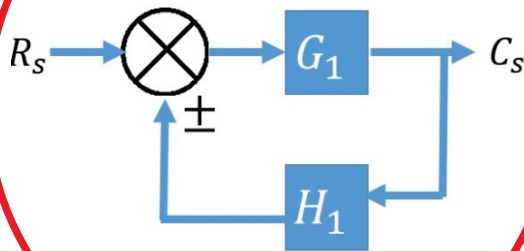




ELECTRICAL ENGINEERING



GATE | PSUs

**CONTROL
SYSTEMS**

Volume - I : Study Material with Classroom Practice Questions

Control Systems

1. Basics of Control Systems

01. Ans: (c)

Sol: $2 \frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 4y(t) = r(t) + 2r(t-1)$

Apply LT on both sides

$$2s^2 Y(s) + 3sY(s) + 4Y(s) = R(s) + 2e^{-s}R(s)$$

$$Y(s)(2s^2 + 3s + 4) = R(s)(1 + 2e^{-s})$$

$$\frac{Y(s)}{R(s)} = \frac{1 + 2e^{-s}}{2s^2 + 3s + 4}$$

02. Ans: (b)

Sol: I.R = $2 \cdot e^{-2t} u(t)$

Output response $c(t) = (1 - e^{-2t}) u(t)$

Input response $r(t) = ?$

$$T.F = \frac{C(s)}{R(s)}$$

$$T.F = L(I.R) = \frac{2}{s+2}$$

$$R(s) = \frac{C(s)}{T.F} = \frac{\frac{1}{s} - \frac{1}{s+2}}{\frac{2}{s+2}} = \frac{1}{s}$$

$$R(s) = \frac{1}{s}$$

$$r(t) = u(t)$$

03. Ans: (b)

Sol: Unit impulse response of unit-feedback control system is given

$$c(t) = t \cdot e^{-t}$$

$$T.F = L(I.R)$$

$$= \frac{1}{(s+1)^2}$$

$$\text{Open Loop T.F} = \frac{\text{Closed Loop T.F}}{1 - \text{Closed Loop T.F}}$$

$$= \frac{\frac{1}{(s+1)^2}}{1 - \frac{1}{(s+1)^2}} = \frac{1}{s^2 + 2s}$$

04. Ans: (a)

Sol: G changes by 10%

$$\Rightarrow \frac{\Delta G}{G} \times 100 = 10\%$$

$$C_1 = 10\% [\because \text{open loop, whose sensitivity is } 100\%]$$

$$\%G \text{ change} = 10\%$$

$$\frac{\% \text{ of change in } M}{\% \text{ of change in } G} = \frac{1}{1 + GH}$$

$$\% \text{ of change in } M = \frac{10\%}{1 + (10)1} = 1\%$$

$$\% \text{ change in } C_2 \text{ by } 1\%$$

05.

Sol: $M = C/R$

$$\frac{C}{R} = M = \frac{GK}{1 + GH}$$

$$S_K^M = \frac{\partial M}{\partial K} \times \frac{K}{M} = 1$$

$[\because K \text{ is not in the loop} \Rightarrow \text{sensitivity is } 100\%]$

$$S_H^M = \frac{\partial M}{\partial H} \times \frac{H}{M} = \frac{\partial}{\partial H} \left(\frac{GK}{1 + GH} \right) \frac{H}{M}$$



$$= \left(\frac{GK(-G)}{(1+GH)^2} \right) \left[\frac{H}{GK} \right]$$

$$S_H^M = \frac{-GH}{(1+GH)}$$

06.

Sol: Given data

$$G = 2 \times 10^3, \partial G = 100$$

$$\% \text{ change in } G = \frac{\partial G}{G} \times 100 = 5\%$$

$$\% \text{ change in } M = 0.5\%$$

$$\frac{\% \text{ of change in } M}{\% \text{ of change in } G} = \frac{1}{1+GH}$$

$$\frac{0.5\%}{5\%} = \frac{1}{1+2 \times 10^3 H}$$

$$1+2 \times 10^3 H = 10$$

$$H = 4.5 \times 10^{-3}$$

07. Ans: (b)

$$\text{Sol: } K = \frac{\text{output}}{\text{input}} = \frac{c(t)}{r(t)} = \frac{\text{mm}}{0_c}$$

2. Signal Flow Graph and Block Diagrams

01. Ans: (d)

Sol: No. of loops = 3

$$\text{Loop1: } -G_1 G_3 G_4 H_1 H_2 H_3$$

$$\text{Loop2: } -G_3 G_4 H_1 H_2$$

$$\text{Loop3: } -G_4 H_1$$

$$\text{No. of Forward paths} = 3$$

$$\text{Forward Path1: } G_1 G_3 G_4$$

$$\text{Forward Path 2: } G_2 G_3 G_4$$

$$\text{Forward Path 3: } G_2 G_4$$

$$= \frac{G_1 G_3 G_4 + G_2 G_3 G_4 + G_2 G_4}{1 + G_1 G_3 G_4 H_1 H_2 H_3 + G_3 G_4 H_1 H_2 + G_4 H_1}$$

02. Ans: (a)

Sol: Number of forward paths = 2

$$\text{Number of loops} = 3$$

$$\begin{aligned} \frac{Y(s)}{R(s)} &= \frac{\frac{1}{s} \times \frac{1}{s} \times \frac{1}{s} [1-0] + \frac{1}{s}}{1 - \left[\frac{1}{s} \times (-1) \left(\frac{1}{s} \right) (-1) + \frac{1}{s} \times \frac{1}{s} (-1) + \left(\frac{1}{s} \times \frac{1}{s} (-1) \right) \right]} \\ &= \frac{\frac{1}{s^3} + \frac{1}{s}}{1 - \left[\frac{1}{s^2} - \frac{1}{s^2} - \frac{1}{s^2} \right]} = \frac{\frac{1+s^2}{s^3}}{1 + \frac{1}{s^2}} = \frac{\frac{1+s^2}{s^3}}{\frac{s^2+1}{s^2}} \\ &= \frac{1+s^2}{s} \times \frac{1}{s^2+1} = \frac{1}{s} \end{aligned}$$

03.

Sol: Number of forward paths = 2

$$\text{Number of loops} = 5,$$

$$\text{Two non touching loops} = 4$$

$$\begin{aligned} \text{TF} &= \frac{24[1-(-0.5)] + 10[1-(-3)]}{1 - [-24 - 3 - 4 + (5 \times 2 \times (-1) + (-0.5))] + [30 + 1.5 + 2] + \left(\left(\frac{-1}{2} \right) \times (-24) \right)} \\ &= \frac{76}{88} = \frac{19}{22} \end{aligned}$$

04.

Sol: Number of forward paths = 2

$$\text{Number of loops} = 5$$

$$\text{T.F} = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_2 G_3 H_2 + G_1 G_2 H_1 + G_1 G_2 G_3 + G_4 H_2 + G_1 G_4}$$



05. Ans: (c)

Sol: From the network

$$V_o(s) = \frac{1}{sC} I(s) \dots\dots\dots(1)$$

$$-V_i(s) + RI(s) + V_o(s) = 0$$

$$I(s) = \frac{1}{R} V_i(s) + \left(\frac{-1}{R} \right) V_o(s) \dots\dots\dots(2)$$

From SFG

$$V_o(s) = x.I(s) \dots\dots\dots(3)$$

$$I(s) = \frac{1}{R} V_i(s) + y V_o(s) \dots\dots\dots(4)$$

From equ(1) and (3)

$$x = \frac{1}{sC}$$

From equ(2) and (4)

$$y = -\frac{1}{R}$$

06. Ans: (a)

Sol: Use gain formula

$$\begin{aligned} \text{transfer function} &= \frac{G(s)}{1 - \left(G(s) \frac{1}{G(s)} + G(s) \right)} \\ &= \frac{G(s)}{1 - 1 - G(s)} = -1 \end{aligned}$$

3. Time Response Analysis

01. Ans: (a)

Sol: $\frac{C(s)}{R(s)} = \frac{1}{1+sT}, R(s) = \frac{8}{s}$

$$C(s) = \frac{8}{s(1+sT)} \Rightarrow c(t) = 8(1 - e^{-t/T})$$

$$3.6 = 8 \left(1 - e^{\frac{-0.32}{T}} \right)$$

$$0.45 = 1 - e^{\frac{-0.32}{T}}$$

$$0.55 = e^{\frac{-0.32}{T}}$$

$$-0.59 = \frac{-0.32}{T}$$

$$T = 0.535 \text{ sec}$$

02. Ans: (c)

Sol: $\cos \phi = \xi$

$$\cos 60 = 0.5$$

$$\cos 45 = 0.707$$

$$\text{Poles left side } 0.5 \leq \xi \leq 0.707$$

$$\text{Poles right side } -0.707 \leq \xi \leq -0.5$$

$$\therefore 0.5 \leq |\xi| \leq 0.707$$

$$3 \text{ rad/s} \leq \omega_n \leq 5 \text{ rad/s}$$

03. Ans: (c)

Sol: For R-L-C circuit:

$$\text{T.F} = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = \frac{1}{Cs} I(s)$$

$$= \frac{1}{Cs} \frac{V_i(s)}{R + Ls + \frac{1}{Cs}}$$

$$\text{T.F} = \frac{V_o(s)}{V_i(s)} = \frac{1}{RCs + LCs^2 + 1}$$

$$= \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$



$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\omega_n = \frac{1}{\sqrt{LC}} \quad 2\xi\omega_n = \frac{R}{L}$$

$$\xi = \frac{R}{2} \sqrt{\frac{C}{L}}$$

$$\xi = \frac{10}{2} \sqrt{\frac{10 \times 10^{-6}}{1 \times 10^{-3}}} = 0.5$$

$$\text{M.P} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \\ = 16.3\% \approx 16\%$$

04. Ans: (b)

$$\text{Sol: TF} = \frac{8/s(s+2)}{1 - \left(\frac{-8as}{s(s+2)} - \frac{8}{s(s+2)} \right)} \\ = \frac{8}{s(s+2) + 8as + 8} \\ = \frac{8}{s^2 + 2s + 8as + 8} \\ = \frac{8}{s^2 + (2+8a)s + 8}$$

$$\omega_n^2 = 8 \Rightarrow \omega_n = 2\sqrt{2}$$

$$2\xi\omega_n = 2 + 8a$$

$$\xi = \frac{1+4a}{2\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{1+4a}{2\sqrt{2}} \Rightarrow a = 0.25$$

05. Ans: 4 sec

$$\text{Sol: T.F} = \frac{100}{(s+1)(s+100)}$$

$$= \frac{100}{s^2 + 101s + 100}$$

$$\omega_n^2 = 100$$

$$\omega_n = 10$$

$$2\xi\omega_n = 101$$

$$\xi = \frac{101}{20}$$

$\xi > 1 \rightarrow$ system is over damped i.e., roots are real & unequal.

Using dominate pole concept,

$$\text{T.F} = \frac{100}{100(s+1)} = \frac{1}{s+1}, \text{ Here } \tau = 1 \text{ sec}$$

$$\therefore \text{ setting time for 2\% criterion} = 4\tau$$

$$= 4 \text{ sec}$$

$$\text{06. } M_p = \frac{C(t_p) - C(\infty)}{C(\infty)} \\ = \frac{1.254 - 1.04}{1.04} = 0.2$$

$$\xi = \sqrt{\frac{(\ln M_p)^2}{(\ln M_p)^2 + \pi^2}}$$

$$M_p = 0.2$$

$$\xi = 0.46$$

07. Ans: (d)

Sol: Given data: $\omega_n = 2, \zeta = 0.5$

Steady state gain = 1

$$\text{OLTF} = \frac{K_1}{s^2 + as + 2} \text{ and } H(s) = K_2$$

$$\text{CLTF} = \frac{G(s)}{1+G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{K_1}{s^2 + as + 2 + K_1 K_2}$$



DC or steady state gain from the TF

$$\frac{K_1}{2 + K_1 K_2} = 1$$

$$K_1(1 - K_2) = 2 \quad \dots\dots\dots (1)$$

$$CE \text{ is } s^2 + as + 2 + K_1 K_2 = 0$$

$$\omega_n = \sqrt{2 + K_1 K_2} = 2$$

$$4 = (2 + K_1 K_2)$$

$$K_1 K_2 = 2 \quad \dots\dots\dots (2)$$

Solving equations (1) & (2) we get

$$K_1 = 4, \quad K_2 = 0.5$$

$$2\zeta \omega_n = a$$

$$2 \times \frac{1}{2} \times 2 = a$$

$$a = 2$$

08. Ans: A – T, B – S, C- P, D – R, E – Q

Sol:

(A) If the poles are real & left side of s-plane, the step response approaches a steady state value without oscillations.

(B) If the poles are complex & left side of s-plane, the step response approaches a steady state value with the damped oscillations.

(C) If poles are non-repeated on the $j\omega$ axis, the step response will have fixed amplitude oscillations.

(D) If the poles are complex & right side of s-plane, response goes to ' ∞ ' with damped oscillations.

(E) If the poles are real & right side of s-plane, the step response goes to ' ∞ ' without any oscillations.

09. Ans: (c)

Sol: If $R \uparrow$ damping $\uparrow$

$$\Rightarrow \xi = \frac{R}{2} \sqrt{\frac{C}{L}}$$

(i) If $R \uparrow$, steady state voltage across C will be reduced (wrong)

(Since steady state value does not depend on ξ)

If $\xi \uparrow$, $C(\infty)$ = remain same

(ii) If $\xi \uparrow$, $\omega_d \downarrow$ ($\omega_d = \omega_n \sqrt{1 - \xi^2}$)

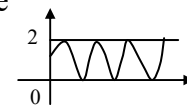
(iii) If $\xi \downarrow$, $t_s \uparrow \Rightarrow 3^{\text{rd}}$

Statement is false

(iv) If $\xi = 0$

True

$\Rightarrow 2$ and 4 are correct



10.

Sol: (i) Unstable system

$$\therefore \text{error} = \infty$$

$$(ii) G(s) = \frac{10(s+1)}{s^2}$$

$$\text{Step} \rightarrow R(s) = \frac{1}{s}$$

$$k_p = \infty$$

$$e_{ss} = \frac{A}{1 + k_p} = \frac{1}{1 + \infty} = 0$$

$$\text{Parabolic} \Rightarrow k_a = 10$$

$$e_{ss} = \frac{1}{10} = 0.1$$

11.

Sol: $G(s) = 10/s^2$ (marginally stable system)

$\therefore$ Error can't be determined



12.

Sol: $e_{ss} = \frac{1}{11}$, $R(s) = \frac{1}{s}$

$$e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+k_p} = \frac{1}{11} = \frac{1}{1+10}$$

$$k_p = \lim_{s \rightarrow 0} s G(s)$$

$$10 = \lim_{s \rightarrow 0} s G(s)$$

$$k = 10$$

$$R(s) = \frac{1}{s^2} \text{ (ramp)}$$

$$e_{ss} = \frac{A}{k_v} = \frac{1}{k_v} = \frac{1}{10}$$

(System is increased by 1)

$$\Rightarrow e_{ss} = 0.1$$

13. **Ans: (a)**

Sol: $T(s) = \frac{(s-2)}{(s-1)(s+2)^2}$ (unstable system)

14. **Ans: (b)**

Sol: Given data: $r(t) = 400tu(t)$ rad/sec

Steady state error = 10°

$$\text{i.e., } e_{ss} = \frac{\pi}{180^\circ} (10^\circ) \text{ radians}$$

$$G(s) = \frac{20K}{s(1+0.1s)} \text{ and } H(s) = 1$$

$$r(t) = 400tu(t) \Rightarrow 400/s^2$$

$$\text{Error } (e_{ss}) = \frac{A}{K_v} = \frac{400}{K_v}$$

$$K_v = \lim_{s \rightarrow 0} s G(s)$$

$$K_v = \lim_{s \rightarrow 0} \frac{20K}{s(1+0.1s)}$$

$$K_v = 20K$$

$$e_{ss} = \frac{400}{20K}$$

$$e_{ss} = \frac{20}{K} = \frac{\pi}{18}$$

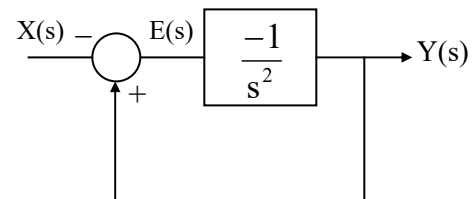
$$K = 114.5$$

15. **Ans: (d)**

Sol: $\frac{d^2 y}{dt^2} = -e(t)$

$$s^2 Y(s) = -E(s)$$

$$x(t) = t u(t) \Rightarrow X(s) = \frac{1}{s^2}$$



$$Y(s) = \frac{-1}{s^2} E(s)$$

$$\frac{Y(s)}{E(s)} = \frac{-1}{s^2}$$

$$\frac{E(s)}{X(s)} = \frac{-1}{1 + \frac{1}{s^2}}$$

$$E(s) = \frac{-s^2}{1+s^2} X(s)$$

$$= \frac{-s^2}{1+s^2} \times \frac{1}{s^2} = \frac{-1}{1+s^2}$$

$$= L^{-1} \left[\frac{-1}{1+s^2} \right] = -\sin t$$

16. **Ans: (a)**

Sol: $e_{ss} = 0.1$ for step input

For pulse input = 10



time = 1 sec

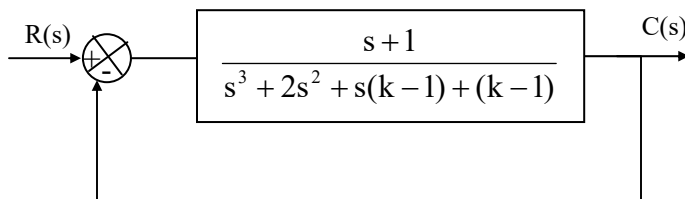
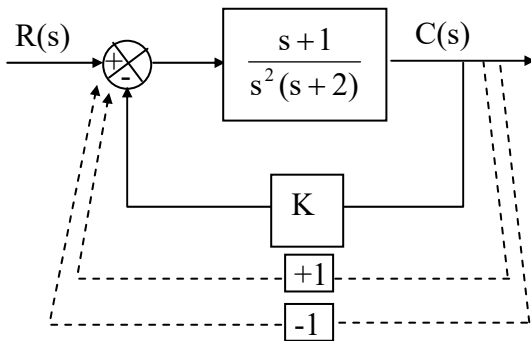
error is function of input

$t \rightarrow \infty$ input = 0

$\therefore$ Error = zero

17. Ans: (a)

Sol:



$\therefore$ Type -0 system

18. Ans: (a)

Sol: $K_p = \lim_{s \rightarrow 0} G(s)$

$$K_p = \frac{1}{k-1}$$

$$e_{ss} = \frac{0.1}{100}$$

$$= \frac{1}{1 + \frac{1}{k-1}}$$

$$\frac{k-1}{k} = \frac{1}{100 \times 10}$$

$$1000k - 1000 = k$$

$$\Rightarrow k = \frac{1000}{999}$$

19. Ans: (c)

$$\begin{aligned} \text{Sol: } \frac{C(s)}{R(s)} &= \frac{100}{(s+1)(s+5)} \\ &= \frac{100 \times 0.2}{1 + \frac{100 \times 0.2}{(s+1)(s+5)}} \\ &= \frac{100}{(s+1)(s+5) + 20} \\ &= \frac{100}{s^2 + 6s + 5 + 20} \\ &= \frac{100}{s^2 + 6s + 25} \end{aligned}$$

$$\omega_n^2 = 25, \omega_n = 5$$

$$2\xi\omega_n = 6$$

$$\xi = \frac{6}{10} = \frac{3}{5}$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 5 \sqrt{1 - \left(\frac{3}{5}\right)^2} = 5 \times \frac{4}{5} = 4 \text{ rad/sec}$$

4. Stability

01. $CE = s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$

s^5	1	8	7	
s^4	4(1)	8(2)	4(1)	
s^3	6(1)	6(1)	0	
s^2	1	1	0	→ Row of AE
s^1	0(2)	0	0	→ Row of zero
s^0	1			



No. of AE roots = 2	No. of CE roots = 5
No. of sign changes	No. of sign changes
Below AE = 0	in 1 st column = 0
No. of RHP = 0	∴ No. of RHP = 0
No. of LHP = 0	No. of jωp = 2
No. of jωp = 2	⇒ No. of LHP = 3

System is marginally stable.

(ii) $s^2 + 1 = 0$

$s = \pm 1 j = \pm j\omega_n$

$\omega_n = 1 \text{ rad/sec}$

Oscillating frequency $\omega_n = 1 \text{ rad/sec}$

02. (i) $s^5 + s^4 + s^3 + s^2 + s + 1 = 0$

$+s^5$	1	1	1
$+s^4$	1	1	1
$+s^3$	0(2)	0(1)	0
$+s^2$	$\frac{1}{2}$	1	
$(1) - s^1$	-3	0	
$(2) + s^0$	1		

AE (1) $= s^4 + s^2 + 1 = 0$

$\frac{d(AE)}{ds} = 4s^3 + 2s = 0$

$\Rightarrow 2s^3 + s = 0$

AE	CE
No. of sign changes below AE = 2	No. of sign changes in 1 st column = 2
No. of AE roots = 4	No. of CE roots = 5
No. of RHP = 2	No. of RHP = 2
No. of LHP = 2	No. of LHP = 3
No. of jωp = 0	No. of jωp = 0

System is unstable

(ii) $s^6 + 2s^5 + 2s^4 + 0s^3 - s^2 - 2s - 2 = 0$

s^6	1	2	-1	-2
s^5	2(1)	0	-2(-1)	0
s^4	2(1)	+0	-2(-1)	0
s^3	0(4)	0	0	0
s^2	0(ε)	-1	0	0
s^1	4/ε			
$-s^0$	-1			

AE $= s^4 - 1 = 0$

$\frac{dAE}{ds} = 4s^3 + 0 = 0$

AE	CE
No. of CE roots = 6	No. of AE roots = 4
No. of sign changes in the 1 st column = 1	No. of sign changes below AE = 1
No. of RHP = 1	No. of RHP = 1
No. of LHP = 3	No. of jωp = 2
No. of jωp = 2	No. of LHP = 1

03. CE $= s^3 + 20s^2 + 16s + 16K = 0$

s^3	1	16
s^2	20	16K
s^1	$\frac{20(16) - 16K}{20}$	0
s^0	16K	



(i) For stability $\frac{20(16) - 16K}{20} > 0$
 $\Rightarrow 20(16) - 16K > 0$
 $\Rightarrow K < 20$ and $16K > 0 \Rightarrow K > 0$
 Range of K for stability $0 < K < 20$

(ii) For the system to oscillate with ω_n it must be marginally stable

i.e., s^1 row should be 0

s^2 row should be AE

$$\therefore \text{A.E roots} = \pm j\omega_n$$

$$\therefore s^1 \text{ row} \Rightarrow 20(16) - 16K = 0$$

$$\Rightarrow K = 20$$

$$\text{AE is } 20s^2 + 16K = 0$$

$$20s^2 + 16(20) = 0$$

$$\Rightarrow s = \pm j4$$

$$\omega_n = 4 \text{ rad/sec}$$

04.

Sol: $CE = 1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$

$$s^3 + as^2 + (K+2)s + K+1 = 0$$

$$s^3 + as^2 + (K+2)s + (K+1) = 0$$

s^3	1	$K+2$
s^2	a	$K+1$
s^1	$\frac{a(K+2) - (K+1)}{a}$	0
s^0	$K+1$	

Given,

$$\omega_n = 2$$

$$\Rightarrow s^1 \text{ row} = 0$$

s^2 row is A.E

$$a(K+2) - (K+1) = 0$$

$$a = \frac{K+1}{K+2}$$

$$\text{AE} = as^2 + K + 1 = 0$$

$$= \frac{K+1}{K+2} s^2 + K + 1 = 0$$

$$(k+1) \left(\frac{s^2}{k+2} + 1 \right) = 0$$

$$s^2 + k + 2 = 0$$

$$s = \pm j\sqrt{(k+2)}$$

$$\omega_n = \sqrt{k+2} = 2$$

$$k = 2$$

$$a = \frac{k+1}{k+2} = \frac{3}{4} = 0.75$$

05.

Sol: $s^3 + ks^2 + 9s + 18$

s^3	1	9
s^2	K	18
s^1	$\frac{9K-18}{K}$	0
s^0	18	

Given that system is marginally stable,

Hence

$$s^1 \text{ row} = 0$$

$$\frac{9K-18}{K} = 0$$

$$9K = 18 \Rightarrow K = 2$$



$$A.E \text{ is } 9s^2 + 18 = 0$$

$$Ks^2 + 18 = 0,$$

$$2s^2 + 18 = 0$$

$$2s^2 = -18$$

$$s = \pm j3$$

$$\therefore \omega_n = 3 \text{ rad/sec.}$$

06. Ans: (d)

Sol: Given transfer function $G(s) = \frac{k}{(s^2 + 1)^2}$

Characteristic equation $1 - G(s).H(s) = 0$

$$1 - \frac{k}{(s^2 + 1)^2} = 0$$

$$s^4 + 2s^2 + 1 - k = 0 \dots (1)$$

RH criteria

s^4	1	2	1-K
s^3	4	4	-
s^2	1	1-K	
s^1	4K		
s^0	1-K		

$$AE = s^4 + 2s^2 + 1 - K$$

$$\frac{d}{ds}(AE) = 4s^3 + 4s$$

$1 - K > 0$ no poles are on RHS plane and LHS plane.

All poles are on $j\omega$ - axis

$\therefore 0 < K < 1$ system marginally stable

5. Root Locus Diagram

01. Ans: (a)

Sol: $S_1 = -1 + j\sqrt{3}$

$$S_2 = -3 - j\sqrt{3}$$

$$G(s).H(s) = \frac{K}{(s+2)^3}$$

$$S_1 = -1 + j\sqrt{3}$$

$$G(s).H(s) = \frac{K}{(-1 + j\sqrt{3} + 2)^3}$$

$$= \frac{K}{(1 + j\sqrt{3})^3}$$

$$= -3 \tan^{-1}(\sqrt{3})$$

$$= -180^\circ$$

It is odd multiples of 180° , Hence S_1 lies on Root locus

$$S_2 = -3 - j\sqrt{3}$$

$$G(s).H(s) = \frac{K}{(-3 - j\sqrt{3} + 2)^3}$$

$$= \frac{K}{(-1 - j\sqrt{3})^3}$$

$$= -3 [180^\circ + 60^\circ] = -720^\circ$$

It is not odd multiples of 180° , Hence S_2 is not lies on Root locus.

02. Ans: (a)

Sol: Over damped – roots are real & unequal

$$\Rightarrow 0 < k < 4$$

(b) $k = 4$ roots are real & equal

$$\Rightarrow \text{Critically damped } \xi = 1$$



(c) $k > 4 \Rightarrow$ roots are complex
 $0 < \xi < 1 \Rightarrow$ under damped

03. Ans: (a)

Sol: Asymptotes meeting point is nothing but centroid

$$\text{centroid } \sigma = \frac{\sum \text{poles} - \sum \text{zeros}}{p - z}$$

$$= \frac{-3 - 0}{3 - 0} = -1$$

$$\text{centroid} = (-1, 0)$$

04. Ans: (b)

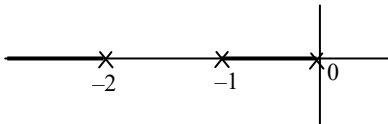
Sol: break point = $\frac{dK}{ds} = 0$

$$\frac{d}{ds}(G_1(s).H_1(s)) = 0$$

$$\frac{d}{ds}[s(s+1)(s+2)] = 0$$

$$3s^2 + 6s + 2 = 0$$

$$s = -0.422, -1.57$$



But $s = -1.57$ do not lie on root locus

So, $s = -0.422$ is valid break point.

Point of intersection wrt $j\omega$ axis

$$s^3 + 3s^2 + 2s + k = 0$$

s^3	1	2
s^2	3	k
s^1	$\frac{6-k}{3}$	0
s^0	k	

As s^1 Row = 0

$$k = 6$$

$$3s^2 + 6 = 0$$

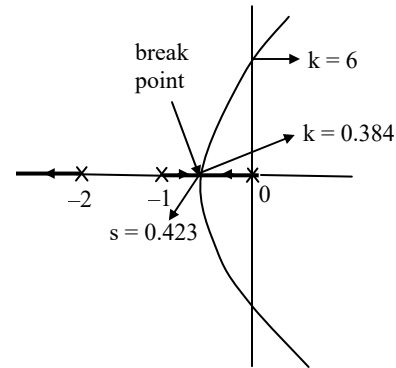
$$s^2 = -2$$

$$s = \pm j\sqrt{2}$$

Point of intersection: $s = \pm j\sqrt{2}$

05. Ans: (b)

Sol:



$$\frac{K}{s(s+1)(s+2)}$$

substitute $s = -0.423$ and apply the magnitude criteria.

$$\left| \frac{K}{(-0.423)(-0.423+1)(-0.423+2)} \right| = 1$$

$$K = 0.354$$

when the roots are complex conjugate then the system response is under damped.

From $K > 0.384$ to $K < 6$ roots are complex conjugate then system to be under damped the values of k is $0.384 < K < 6$.

06. Ans: (c)

Sol: If the roots are lies on the real axis then system exhibits the non-oscillatory response.

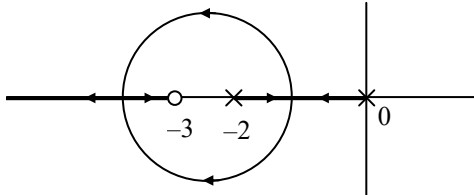
from $K \geq 0$ to $K \leq 0.384$ roots lies on the



real axis. Hence for $0 \leq K \leq 0.384$ system exhibits the non-oscillatory response.

07. Ans: (a)

Sol:



$$\frac{d}{ds}[G(s)H(s)] = \frac{d}{ds}\left[\frac{k(s+3)}{s(s+2)}\right]$$

$$s^2 + 6s + 6 = 0$$

break points $-1.27, -4.73$

$$\text{radius} = \frac{4.73 - 1.27}{2} = 1.73$$

center $= (-3, 0)$

08. Ans: (c)

Sol: $G(s)H(s) = \frac{K(s+3)}{s(s+2)}$

$$k|_{s=-4} = \left| \frac{(-4)(-4+2)}{(-4+3)} \right|$$

$$= \left| \frac{(-4)(-2)}{(-1)} \right| = 8$$

09. Ans: (a)

Sol: $s^2 - 4s + 8 = 0 \Rightarrow s = 2 \pm 2j$ are two zeroes

$s^2 + 4s + 8 = 0 \Rightarrow s = -2 \pm 2j$ are two poles

$$\phi_A = 180 - \angle GH|_{s=2 \pm 2j}$$

$$GH = \frac{k[s - (2 + 2j)][s - (2 - 2j)]}{[s - (-2 + 2j)][s - (-2 - 2j)]}$$

$$\angle GH|_{s=2+2j} = \frac{\angle k \angle 4j}{\angle 4 \angle 4 + 4j}$$

$$= 90^\circ - 45^\circ = 45^\circ$$

$$\phi_A = 180^\circ - 45^\circ = \pm 135^\circ$$

10. Ans: (b)

Sol: $s^2 - 4s + 8 = 0 \Rightarrow s = 2 \pm 2j$ are two zeroes

$s^2 + 4s + 8 = 0 \Rightarrow s = -2 \pm 2j$ are two poles

$$\phi_d = 180^\circ + \angle GH|_{s=-2 \pm 2j}$$

$$\angle GH|_{s=-2+2j} = \angle \frac{k[s - (2 + 2j)][s - (2 - 2j)]}{[s - (-2 + 2j)][s - (-2 - 2j)]}|_{s=-2+2j}$$

$$= \frac{\angle k(-4)(-4 + 4j)}{\angle 4j}$$

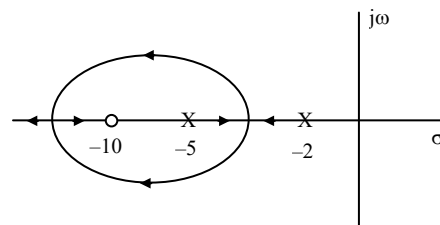
$$= 180^\circ + 180^\circ - 45^\circ - 90^\circ = 225^\circ$$

$$\phi_d = 180^\circ + 225^\circ = 405^\circ$$

$$\therefore \phi_d = \pm 45^\circ$$

11. Ans: (d)

Sol: Poles $s = -2, -5$; Zero $s = -10$



$\therefore$ breakaway point exist between -2 and -5



6. Frequency Response Analysis

01. Ans: (c)

$$\text{Sol: } G(s).H(s) = \frac{100}{s(s+4)(s+16)}$$

Phase crossover frequency (ω_{pc}):

$$\angle G(j\omega).H(j\omega)/\omega = \omega_{pc} = -180^\circ$$

$$-90^\circ - \tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -180^\circ$$

$$-\tan^{-1}(\omega_{pc}/4) - \tan^{-1}(\omega_{pc}/16) = -90^\circ$$

$$\tan[\tan^{-1}(\omega_{pc}/4) + \tan^{-1}(\omega_{pc}/16)] = \tan(90^\circ)$$

$$\frac{\frac{\omega_{pc}}{4} + \frac{\omega_{pc}}{16}}{1 - \frac{\omega_{pc}}{4} \cdot \frac{\omega_{pc}}{16}} = \frac{1}{0}$$

$$\omega_{pc}^2 = 16 \times 4 \Rightarrow \omega_{pc} = 8 \text{ rad/sec}$$

02. Ans: (d)

$$\text{Sol: } G(s).H(s) = \frac{100}{s(s+2)(s+16)}$$

$$\text{Gain margin (G.M)} = \frac{1}{|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}}}$$

$$|G(j\omega).H(j\omega)|_{\omega=\omega_{pc}} = \frac{100}{\omega_{pc} \sqrt{\omega_{pc}^2 + 16} \sqrt{\omega_{pc}^2 + 16^2}}$$

$$= \frac{5}{64}$$

$$\text{G.M} = \frac{64}{5} = 12.8$$

03. Ans: (c)

$$\text{Sol: } G(s).H(s) = \frac{2e^{-0.5s}}{(s+1)}$$

gain crossover frequency,

$$\omega_{gc} = |G(j\omega).H(j\omega)|_{\omega=\omega_{gc}} = 1$$

$$\frac{2}{\sqrt{\omega_{gc}^2 + 1}} = 1$$

$$\omega_{gc}^2 + 1 = 4 \Rightarrow \omega_{gc} = \sqrt{3} \text{ rad/sec}$$

04. Ans: (b)

$$\text{Sol: } \omega_{gc} = \sqrt{3} \text{ rad/sec}$$

$$\text{P.M} = 180^\circ + \angle G(j\omega).H(j\omega)/\omega = \omega_{gc}$$

$$\angle G(j\omega).H(j\omega)/\omega=\omega_{gc} = -0.5 \omega_{gc} - \tan^{-1}(\omega_{gc})$$

$$= -109.62^\circ$$

$$\text{P.M} = 70.39^\circ$$

05. Ans: (a)

$$\text{Sol: } M_r = 2.5 = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$2\xi\sqrt{1-\xi^2} = \frac{1}{2.5}$$

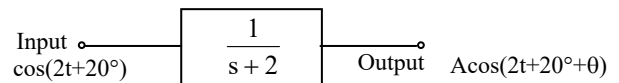
$$\xi^4 - \xi^2 + 0.04 = 0$$

$$\xi^2 = 0.958 \quad \xi^2 = 0.0417$$

$$\xi = 0.204 \quad (M_r > 1)$$

06. Ans: (a)

$$\text{Sol: Closed loop T.F} = \frac{1}{s+2}$$



$$A = \frac{1}{\sqrt{\omega^2 + 4}} = \frac{1}{\sqrt{4 + 4}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

$$\phi = -\tan^{-1}\omega/2$$

$$= -\tan^{-1}2/2$$

$$\Rightarrow \phi = -\tan^{-1}(1) = -45^\circ$$



$$\begin{aligned}\text{output} &= \frac{1}{2\sqrt{2}} \cos(2t + 20^\circ - 45^\circ) \\ &= \frac{1}{2\sqrt{2}} \cos(2t - 25^\circ)\end{aligned}$$

07. Ans: (c)

Sol: Initial slope = -40 dB/dec

Two integral terms $\left(\frac{1}{s^2}\right)$

$$\therefore \text{Part of TF} = G(s)H(s) = \frac{K}{s^2}$$

at $\omega = 0.1$

$$\begin{aligned}\text{change in slope} &= -20 - (-40) \\ &= 20^\circ\end{aligned}$$

$$\text{Part of TF} = G(s)H(s) = \frac{K\left(1 + \frac{s}{0.1}\right)}{s^2}$$

At $\omega = 10$ slope changed to -60 dB/dec

$$\begin{aligned}\text{Change in slope} &= -60 - (-20) \\ &= -40 \text{ dB/dec}\end{aligned}$$

$$\text{TF } (G(s)H(s)) = \frac{K\left(1 + \frac{s}{0.1}\right)}{s^2\left(\frac{s}{10} + 1\right)^2}$$

$$20 \log K - 2(20 \log 0.1) = 20 \text{ dB}$$

$$20 \log K = 20 - 40$$

$$20 \log K = -20$$

$$K = 0.1$$

$$\begin{aligned}G(s)H(s) &= \frac{(0.1)\left(1 + \frac{s}{0.1}\right)}{s^2\left(1 + \frac{s}{10}\right)^2} \\ &= \frac{(0.1) \times 10^2 (s + 0.1)}{(0.1)s^2 (s + 10)^2}\end{aligned}$$

$$G(s)H(s) = \frac{100(s + 0.1)}{s^2(s + 10)^2}$$

08. Ans: (b)

$$\text{Sol: } G(s)H(s) = \frac{Ks}{\left(1 + \frac{s}{2}\right)\left(1 + \frac{s}{10}\right)}$$

$$12 = 20 \log K + 20 \log 0.5$$

$$12 = 20 \log K + (-6)$$

$$20 \log K = 18 \text{ dB} = 20 \log 2^3$$

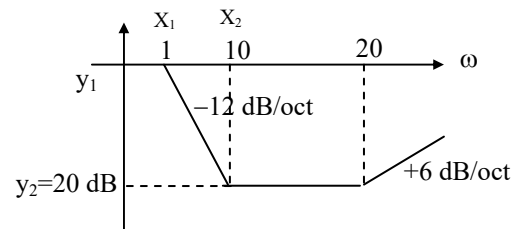
$$K = 8$$

$$G(s)H(s) = \frac{8s \times 2 \times 10}{(2 + s)(10 + s)}$$

$$G(s)H(s) = \frac{160s}{(2 + s)(10 + s)}$$

09. Ans: (b)

Sol:



$$G(s)H(s) = \frac{K\left(1 + \frac{s}{10}\right)^2\left(1 + \frac{s}{20}\right)}{(1 + s)^2}$$

$$\frac{y_2 - y_1}{x_2 - x_1} = -40 \text{ dB/dec}$$

$$\frac{20 - y_1}{\log 10 - \log 1} = -40$$

$$y_1 = +60 \text{ dB} \Big|_{\omega \leq 1}$$

$$\Rightarrow 20 \log K = 60$$

$$K = 10^3$$

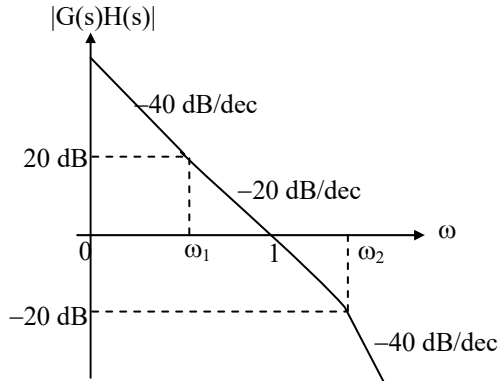


$$G(s)H(s) = \frac{10^3(s+10)^2(s+20)}{10^2 \times 20 \times (s+1)^2}$$

$$= \frac{(s+10)^2(s+20)}{2(s+1)^2}$$

10. Ans: (d)

Sol:



ω_1 calculation:

$$\frac{0 - 20}{\log 1 - \log \omega_1}$$

$$= -20 \text{ dB/dec}$$

$$\omega_1 = 0.1$$

ω_2 calculation:

$$\frac{-20 - 0}{\log \omega_2 - \log 1}$$

$$= -20 \text{ dB/dec}$$

$$\omega_2 = 10$$

$$G(s)H(s) = \frac{K \left(1 + \frac{s}{0.1}\right)}{s^2 \left(1 + \frac{s}{10}\right)}$$

$$20 \log K - 2(20 \log 0.1) = 20$$

$$20 \log K = 20 - 40$$

$$K = 0.1$$

$$G(s)H(s) = \frac{0.1 \times \frac{1}{0.1} (0.1 + s)}{s^2 \frac{1}{10} (10 + s)}$$

$$= \frac{10(0.1 + s)}{s^2 (10 + s)}$$

11.

Sol: $\frac{200}{s(s+2)} = \frac{100}{s \left(1 + \frac{s}{2}\right)}$

$$x = -KT \Rightarrow -(100) \times \frac{1}{2} = x = -50$$

12. Ans: (c)

Sol: For stability $(-1, j0)$ should not be enclosed by the polar plot.

For stability

$$1 > 0.01 K$$

$$\Rightarrow K < 100$$

13.

Sol: $GM = -40 \text{ dB}$

$$20 \log \frac{1}{a} = -40 \Rightarrow a = 10^2$$

$$POI = 100$$

14.

Sol: (i) $GM = \frac{1}{0.1} = +10 = 20 \text{ dB}$

$$PM = 180^\circ - 140^\circ = 40^\circ$$

(ii) $PM = 180 - 150^\circ = 30^\circ$

$$GM = \frac{1}{0} = \infty \quad POI = 0$$



(iii) ω_{pc} does not exist

$$GM = \frac{1}{0} = \infty \quad PM = 180^\circ + 0^\circ = 180^\circ$$

(iv) ω_{gc} not exist

$$\omega_{pc} = \infty$$

$$GM = \frac{1}{0} = \infty$$

$$PM = \infty$$

(v) $GM = \frac{1}{0.5} = 2$

$$PM = 180 - 90 = 90^\circ$$

15. Ans: (d)

Sol: For stability $(-1, j0)$ should not be enclosed by the polar plot. In figures (1) & (2) $(-1, j0)$ is not enclosed.

$\therefore$ Systems represented by (1) & (2) are stable.

16. Ans: (b)

Sol: Open loop system is stable, since the open loop poles are lies in the left half of s-plane
 $\therefore P = 0$.

From the plot $N = -2$.

No. of encirclements $N = P - Z$

$N = -2, P = 0$ (Given)

$\therefore N = P - Z$

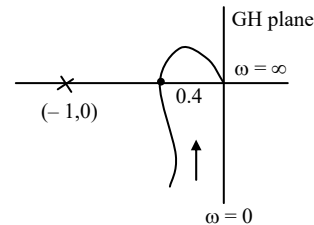
$$-2 = 0 - Z$$

$$Z = 2$$

Two closed loop poles are lies on RH of s-plane and hence the closed loop system is unstable.

17. Ans: (c)

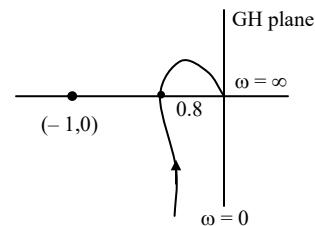
Sol:



$$\frac{K_c}{K} = 0.4 \quad \text{When } K = 1$$

Now, K double, $\frac{K_c}{K} = 0.4$

$$K_c = 0.4 \times 2 = 0.8$$



Even though the value of K is double, the system is stable (negative real axis magnitude is less than one)

Oscillations depends on ' ξ '

$$\xi \propto \frac{1}{\sqrt{K}} \quad \text{as } K \text{ is increased } \xi \text{ reduced, then}$$

more oscillations.



7. Controllers & Compensators

01. Ans: (a)

$$\begin{aligned}\text{Sol: } G_C(s) &= (-1) \left(-\frac{Z_2}{Z_1} \right) \\ &= (-1)(-1) \left(\frac{R_2 + \frac{1}{sC}}{R_1} \right) \\ G_C(s) &= \frac{(100 \times 10^3) + \frac{1}{s \times 10^{-6}}}{10^6}\end{aligned}$$

$$G_C(s) = \frac{1 + 0.1s}{s}$$

02. Ans: (c)

$$\begin{aligned}\text{Sol: CE} \Rightarrow 1 + G_C(s) G_P(s) &= 0 \\ &= 1 + \frac{1 + 0.1s}{s} \times \frac{1}{(s+1)(1+0.1s)} \\ &= 1 + \frac{1 + 0.1s}{s(s+1)(1+0.1s)} = 0 \\ \Rightarrow s^2 + s + 1 &= 0 \Rightarrow \omega_n = 1, \\ e^{\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \right]}_{\zeta=0.5} &= 0.163 \Rightarrow M_p = 16.3\%\end{aligned}$$

03. Ans: (b)

$$\begin{aligned}\text{Sol: T.F} &= \frac{k(1+0.3s)}{1+0.17s} \\ T = 0.17, aT = 0.3 \Rightarrow a &= \frac{0.3}{0.17} \\ C = 1 \mu F \\ T = \frac{R_1 R_2}{R_1 + R_2} C, a &= \frac{R_1 + R_2}{R_2} \\ \frac{R_1 R_2}{R_1 + R_2} &= \frac{0.17}{1 \times 10^{-6}} = 170000\end{aligned}$$

$$\frac{R_1 + R_2}{R_2} = 1.764$$

$$aT = R_1 C$$

$$R_1 = \frac{aT}{C} = \frac{0.3}{C} = (0.3)(10^6) = 300 \text{ k}\Omega$$

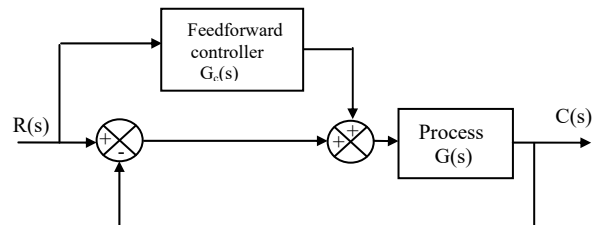
$$\frac{300 + R_2}{R_2} = 1.76$$

$$300 \text{ k} + R_2 - 1.76 R_2 = 0$$

$$R_2 = \frac{300}{0.70} = 394.736 = 400 \text{ k}\Omega$$

04. Ans: (b)

Sol: The feed forward controller $G_c(s)$ is placed in series with closed loop system as shown in fig. below. The poles, zeros of $G_c(s)$ may be selected to add or cancel the poles and zeros of the closed loop system transfer function.



$$\text{Given } G(s) = \frac{1}{s^2 + 2s + 2}, H(s) = 1$$

$$\text{If } G_c(s) = \frac{s+2}{s+1} \text{ is taken the closed loop}$$

transfer function with controller is

$$\frac{C(s)}{R(s)} = \frac{G(s)[1 + G_c(s)]}{1 + G(s)}$$

$$\frac{C(s)}{R(s)} = \frac{2s+3}{(s+1)(s^2+2s+3)}$$

$$\text{Given } R(s) = \frac{1}{s}$$



The eigen values are distinct i.e., -1 & -2 .

$\therefore$ Corresponding normal form is called as diagonal canonical form

DCF – SSR

$$\frac{Y(s)}{U(s)} = \frac{b_1}{s+1} + \frac{b_2}{s+2}$$

$$b_1 = 1, b_2 = -1$$

$$Y(s) = \underbrace{\frac{b_1}{s+1}}_{x_1} U(s) + \underbrace{\frac{b_2}{s+2}}_{x_2} U(s)$$

$$\text{Let } Y(s) = X_1(s) + X_2(s)$$

$$\text{Where } y(t) = x_1(t) + x_2(t) \dots\dots\dots (1)$$

$$\text{Where } X_1(s) = \frac{b_1}{s+1} U(s)$$

$$s X_1(s) + X_1(s) = b_1 U(s)$$

Take Laplace Inverse

$$\dot{x}_1 + x_1 = b_1 u(t) \dots\dots\dots (2)$$

$$X_2(s) = \frac{b_2}{s+2} U(s)$$

$$s X_2(s) + 2 X_2(s) = b_2 U(s)$$

Laplace Inverse

$$\dot{x}_2 + 2x_2 = b_2 u(t)$$

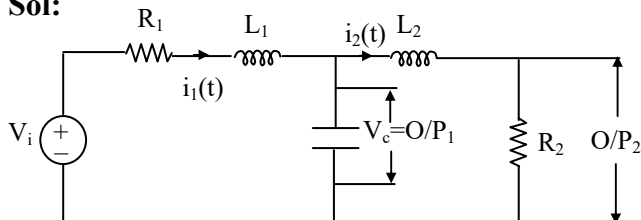
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

From(1) output equation.

$$y(t) = [1 \ 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

06. Ans: (c)

Sol:



$$O/P_1 \Rightarrow y_1 = V_c$$

$$O/P_2 \Rightarrow y_2 = R_2 i_2$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} V_c \\ i_1 \\ i_2 \end{bmatrix}$$

$$y = C X$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & R_2 \end{bmatrix}$$

07. Ans: (a)

$$\text{Sol: T.F} = C[sI - A]^{-1}B + D$$

$$= [1 \ 0] \begin{bmatrix} s+4 & 1 \\ 3 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= [1 \ 0] \frac{1}{s^2 + 5s + 1} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2 + 5s + 1} [1 \ 0]_{1 \times 2} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2 \times 1}$$

$$= \frac{1}{s^2 + 5s + 1} [s+1 \ -1]_{1 \times 2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}_{2 \times 1}$$

$$= \frac{1}{s^2 + 5s + 1} [s+1-1]$$

$$= \frac{s}{s^2 + 5s + 1}$$

08. Ans: (c)

$$\text{Sol: State transition matrix } \phi(t) = L^{-1}[(sI - A)^{-1}]$$

$$sI - A = \begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}$$



$$= \begin{bmatrix} \frac{1}{s+3} & \frac{1}{(s+2)(s+3)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$L^{-1}[[sI - A]^{-1}] = \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix}$$

09. Ans: (b)

Sol: Controllability

$$[M] = [B \quad AB \quad A^2B \dots A^{n-1}B]$$

$$AB = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$$|M| = -1 \neq 0 \text{ (Controllable)}$$

Observability

$$[N] = [C^T \quad A^T C^T \dots (A^T)^{n-1} C^T]$$

$$A^T C^T = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$$|N| = 0 \text{ (Not observable)}$$

10. Ans: (c)

Sol: According to Gilberts test the system is controllable and observable.

11. Ans: (c)

Sol: $\frac{Y(s)}{U(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$

at node $\dot{x}_1$

$$\dot{x}_1 = -a_1 x_1 - a_2 x_2 - a_3 x_3$$

at $\dot{x}_2 = x_1$ & $\dot{x}_3 = x_2$

$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

9. Mathematical Modeling of Mechanical Systems

01. Ans: (c)

Sol: $M \frac{d^2 x_0}{dt^2} + B_2 \frac{dx_0}{dt} + B_1 \frac{d}{dt}(x_0 - x_i) = 0$

$$(Ms^2 + B_2 s + B_1 s)X_0 = B_1 s X_i$$

$$\frac{X_0(s)}{X_i(s)} = \frac{B_1}{Ms + B_1 + B_2}$$

02. Ans: (a)

Sol: $\frac{\theta_1}{\theta_2} = \frac{N_2}{N_1} = \frac{T_2}{T_1}$

$$T_2 = J_2 \ddot{\theta}_2 + B_2 \dot{\theta}_2$$

$$T_2 = J_2 \left(\frac{N_1}{N_2} \right) \ddot{\theta}_1 + B_2 \frac{N_1}{N_2} \dot{\theta}_1$$

$$T_1 = \frac{N_1}{N_2} T_2$$

$$= \frac{N_1}{N_2} \left[J_2 \left(\frac{N_1}{N_2} \right) \ddot{\theta}_1 + B_2 \left(\frac{N_1}{N_2} \right) \dot{\theta}_1 \right]$$

$$= J_2 \left(\frac{N_1}{N_2} \right)^2 \ddot{\theta}_1 + B_2 \left(\frac{N_1}{N_2} \right)^2 \dot{\theta}_1$$



03. Ans: (c)

Sol: Nodal diagram

Mechanical system → Force Current Analogy

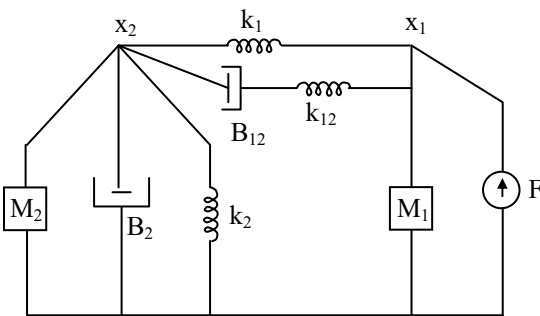
Mass (M) Capacitance (c)

Damper (B) Conductance (1/R)

Spring co-efficient(k) Inductance (1/L)

Force Current

Mechanical network is shown below



04. Ans: (c)

Sol: $\delta(t) = m \frac{d^2 x}{dt^2} + kx$

$$1 = ms^2 X(s) + kX(s)$$

$$1 = (ms^2 + k)X(s)$$

$$X(s) = \frac{1}{ms^2 + k}$$

$$X(s) = \frac{1}{m} \cdot \frac{1}{s^2 + \frac{k}{m}} = \frac{1}{m} \sqrt{\frac{k}{m}} \cdot \frac{1}{s^2 + \left(\sqrt{\frac{k}{m}}\right)^2}$$

$$x(t) = \frac{1}{\sqrt{mk}} \sin\left(\sqrt{\frac{k}{m}} \cdot t\right)$$

05. Ans: (b)

Sol: $M \frac{d^2 x}{dt^2} + f \frac{dx}{dt} + (K_1 + K_2)x = F$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + fs + K_1 + K_2}$$

06. Ans: (c)

Sol: $f(t) = \frac{Md^2 x}{dt^2} + B \frac{dx}{dt} + Kx(t)$

Applying Laplace transform on both sides, with zero initial conditions

$$F(s) = Ms^2 X(s) + BsX(s) + KX(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

Characteristic equation is $Ms^2 + Bs + K = 0$

$$s^2 + \frac{B}{M}s + \frac{K}{M} = 0$$

Compare with $s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$

$$2\zeta\omega_n = \frac{B}{M}$$

$$\xi = \frac{B}{2\sqrt{MK}} \quad \omega_n = \sqrt{\frac{K}{M}}$$

$$\text{Time constant } T = \frac{1}{\zeta\omega_n} = \frac{1}{B} \times 2M$$

$$T = \frac{2M}{B}$$

Hence, statements 2 & 3 are correct

07. Ans: (b)

Sol: In force current analogy

$$F \rightarrow i$$

$$M \rightarrow C$$

$$B \rightarrow \frac{1}{R}$$

$$K \rightarrow \frac{1}{L}$$