MECHANICAL ENGINEERING

THEORY OF MACHINES & VIBRATIONS

Volume-I & II: Study Material with Classroom and Self Practice Questions (Workbook)
Chapter 1
Analysis of Planar Mechanisms

01. Ans: (a, c)

02. Ans: (c)
Sol:

03. Ans: (d)
Sol: At toggle position velocity ratio is ‘zero’ so mechanical advantage is ‘∞’.

04. Ans: (d)
Sol: The two extreme positions of crank rocker mechanisms are shown below figure.

05. Ans: (a)
Sol:

06. Ans: (c)
Sol: Two extreme positions are as shown in figure below.
Let r = radius of crank = 20 cm
\( l = \text{length of connecting rod} = 40 \text{ cm} \)
\( h = 10 \text{ cm} \)

Stroke = \( S_1 - S_2 \)
\[
S_1 = \sqrt{(l + r)^2 - h^2} = \sqrt{60^2 - 10^2} = 59.16 \text{ cm}
\]
\[
S_2 = \sqrt{(l - r)^2 - h^2} = \sqrt{20^2 - 10^2} = 17.32 \text{ cm}
\]
Stroke = \( S_1 - S_2 = 59.16 - 17.32 = 41.84 \text{ cm} \)

07. Ans: (b)
Sol:
\[
\theta_1 = \sin^{-1}\left(\frac{h}{l + r}\right) = \sin^{-1}\left(\frac{10}{60}\right) = 9.55^\circ
\]
\[
\theta_2 = \sin^{-1}\left(\frac{h}{l - r}\right) = \sin^{-1}\left(\frac{10}{20}\right) = 30^\circ
\]
\( \alpha = \theta_2 - \theta_1 = 20.41^\circ \)
Quick return ratio
\[
(QRR) = \frac{180 + \alpha}{180 - \alpha} = 1.2558
\]

08. Ans: (c)
Sol:
\[
I_{13} \text{is obtained by joining} \ I_{12} \ I_{23} \text{and} \ I_{14} \ I_3
\]
\[
\frac{\omega_3}{\omega_2} = \frac{I_{12} \ I_{23}}{I_{13} \ I_{23}} = \frac{a}{2a}
\]
\[
= \frac{1}{2}
\]
\( \omega_3 = 1 \text{ rad/sec} \)

09. Ans: (c)
Sol: \( \angle O_2O_4P = 180^\circ \) sketch the position diagram for the given input angle and identify the Instantaneous Centers.
Alternate Method:
The position diagram is isosceles right angle triangle and the velocity triangle is similar to the position diagram.

V_{qp} = \omega_3 l_3 \Rightarrow \sqrt{2}a = \omega_3 \times \sqrt{2}a
\omega_3 = 1
V_q = l_4 \omega_4 \Rightarrow \sqrt{2}a = \sqrt{2}a \omega_4
\Rightarrow \omega_4 = 1 \text{ rad/sec}

10. Ans: (b)
Sol:

\begin{align*}
OC &= r \\
\text{Velocity of slider } V_S &= (12 - 24) \times \omega_2 \\
&= x \omega_2 \\
\frac{x}{\sin (\alpha + \beta)} &= \frac{r}{\sin (90 - \beta)}
\end{align*}

11. Ans: (a)
Sol:

\begin{align*}
V_S &= r \omega_3 \sin (\alpha + \beta) \times \sec \beta \\
&= V_C \sin (\alpha + \beta) \times \sec \beta
\end{align*}

Note: If input and coupler links are collinear, then output angular velocity will be zero.

12. Ans: (c)
Sol: In a four bar mechanism when input link and output links are parallel then coupler velocity(\omega_3) is zero.
\Rightarrow l_2 \omega_2 = l_4 \omega_4
l_4 = 2l_2 \text{ (Given) }
\Rightarrow \omega_4 = \omega_2 / 2 = 2/2 = 1 \text{ rad/s}
\omega_2, \omega_4 = \text{angular velocity of input and output link respectively.}

Fixed links have zero velocity.
At joint 1, relative velocity between fixed link and input link = 2 – 0 = 2
Rubbing velocity at joint 1 = Relative velocity \times radius of pin = 2 \times 10 = 20 \text{ cm/s}
At joint 2, rubbing velocity = (\omega_2 + \omega_3) \times r \\
= (2 + 0) \times 10 = 20 \text{ cm/s}
+ve sign means $\omega_2$ and $\omega_3$ are moving in opposite directions.

At joint 3, rubbing velocity $= (\omega_4 + \omega_3) \times r$
$= (1+0) \times 10 = 10 \text{ cm/s}$

At joint 4, rubbing velocity
$= (\omega_4 - 0) \times r$
$= (1 - 0) \times 10 = 10 \text{ cm/s}$

13. Ans: (a)
Sol:

Considering the four bar mechanism
ABCD, $l_2 \parallel l_4$

$\therefore \ell_2 \omega_2 = l_4 \omega_4 \Rightarrow \omega_4 = \frac{50 \times 3}{75} = 2 \text{ rad/sec}$

CDE being a ternary link angular velocity of DE is same as that of the link DC ($\omega_4$).

For the slider crank mechanism DEF, crank is perpendicular to the axis of the slider.

$\therefore$ Slider velocity $= DE \times \omega_4$
$= 50 \times 2$
$= 100 \text{ cm/sec (upward)}$

14. Ans: (a)
Sol: Here as angular velocity of the connecting rod is zero so crank is perpendicular to the line of stroke.

$V_s = \text{velocity of slider} = r \omega_2$
$\therefore 2 = 1 \times \omega_2 \Rightarrow \omega_2 = 2 \text{ rad/sec}$

15. Ans: (d)
Sol:

Here the crank is perpendicular to connecting rod

$\text{Velocity of rubbing} = (\omega_2 + \omega_3) \times r$

Where, $r = \text{radius of crank pin}$

From the velocity diagram $V_{AB} = ab = ?$

$oa = \omega_2 \times r = 10 \times 0.3 = 3 \text{ m/sec}$

$\Delta oab$ is right angle $\Delta$.

$\tan \theta = \frac{oa}{ab} = \frac{40}{30} \Rightarrow \theta = 53.13^\circ$

$\tan \theta = \frac{r \omega_2}{r \omega_3}$

where, $n = \frac{\ell}{r}$

$\omega_3 = \frac{\omega_2}{n^2} = \frac{10}{4} = \frac{90}{16} = 5.625 (\text{CW})$

$V_{rb} = (\omega_2 + \omega_3) \times r$
$= (10 + 5.625) \times 2.5 = 39 \text{ cm/s}$
16. **Ans:** (d)
**Sol:** As for the given dimensions the mechanism is in a right angle triangle configuration and the crank AB is perpendicular to the lever CD. The velocity of B is along CD only which is purely sliding component

\[ = \text{AB} \times \omega_{AB} = 10 \times 250 = 2.5 \text{ m/sec} \]

17. **Ans:** (a)
**Sol:**

\[ \text{QRR} = \frac{180 + 2\alpha}{180 - 2\alpha} = \frac{2}{1} \implies \alpha = 30^\circ \]

\[ \sin \alpha = \frac{OS}{OP} \implies OS = \frac{OP}{2} = 250 \text{mm} \]

18. **Ans:** (b)
**Sol:** Maximum speed during forward stroke occurs when PQ is perpendicular to the line of stroke of the tool i.e. PQ, OS & OQ are in straight line

\[ \implies V = 250 \times 2 = 750 \times \omega_{PQ} \]

\[ \implies \omega_{PQ} = \frac{2}{3} \]

19. **Ans:** (d)
**Sol:**

\[ V_Q = V_P + V_{PQ} \]

20. **Ans:** (a)
**Sol:** For rigid thin disc rolling on plane without slip. The ‘I’ centre lies on the point of contact.

21. **Ans:** (a)
**Sol:**

Here ‘O’ is the instantaneous centre

\[ V_P = \omega \times OP \]

\[ V_A = R\omega \]

In \( \triangle OAP \),

\[ \cos 120^\circ = \frac{R^2 + R^2 - OP^2}{2R \times R} 
\]

\[ -0.5 = \frac{2R^2 - OP^2}{2R^2} \]

\[ OP = \sqrt{3}R \]

\[ V_P = \sqrt{3}R \times \omega = \sqrt{3}V \]

or

\[ V_P = \vec{V}_O + \vec{V}_{PO} = \vec{V} + \vec{OP} \times \omega \]

\[ = \sqrt{V^2 + V^2 + 2V^2 \cos 60} = \sqrt{3}V \]

22. **Ans:** (d)
**Sol:**
By considering the links 1, 2 and 4 as for three centers in line theorem, $I_{12}$, $I_{14}$ and $I_{24}$ lies on a straight line $I_{12}$ is at infinity along the horizontal direction while $I_{14}$ is at infinity along vertical direction hence $I_{24}$ must be at infinity.

23. Ans: (a)
Sol:

![Position Diagram]

$V_a = 1 \text{ m/s}$
$V_a = \text{Velocity along vertical direction}$
$V_b = \text{Velocity along horizontal direction}$

So instantaneous center of link AB will be perpendicular to A and B respectively i.e at I

$I_A = OB = \cos \theta = 1 \times \cos 60^0 = \frac{1}{2} \text{ m}$

$I_B = OA = \sin \theta = 1 \times \sin 60^0 = \frac{\sqrt{3}}{2} \text{ m}$

$V_a = \omega \times I_A$

$\Rightarrow \omega = \frac{V_a}{I_A} = \frac{1}{V_2} = 2 \text{ rad/ sec}$

24. Ans: (a)
Sol:

![Velocity Diagram]

Let the angle between BC & CD is $\alpha$. Same will be the angle between their perpendiculars.

From Velocity Diagram, $\frac{\ell_2 \omega_2}{\ell_4 \omega_4} = \tan \alpha$

From Position diagram, $\tan \alpha = \frac{30}{40}$

$\therefore \omega_2 = \omega_4 \times \frac{\ell_4}{\ell_2} \times \tan \alpha = \frac{40}{20} \times \frac{30}{40} = 3$

$\omega_2 = 3 \text{ rad/ sec}$

Note: DC is the rocker (Output link) and AB is the crank (Input link).
25. Ans: (c)
Sol:
\[
\begin{align*}
I_{13} &= \text{Instantaneous center of link 3 with respect to link 1} \\
\text{As AED is a right angle triangle and the sides are being integers so } AE = 30 \text{ cm and } DE = 40 \text{ cm} \\
\text{BE = 3 cm and CE = 4 cm} \\
\text{By ‘I’ center velocity method,} \\
V_{23} &= \omega_2 \times (AB) = \omega_2 \times (BE) \\
\omega_3 &= \frac{1 \times 27}{3} = 9 \text{ rad/s}
\end{align*}
\]

26. Ans: (a)
Sol: Similarly, \( V_{34} = \omega_3 \times (EC) = \omega_4 \times (CD) \)
\[
\omega_4 = \frac{9 \times 4}{36} = 1 \text{ rad/s}
\]

27. Ans: (d)
Sol: Refer the figure shown below, By knowing the velocity directions instantaneous centre can be located as shown. By knowing velocity (magnitude) of Q we can get the angular velocity of the link, from this we can get the velocity of ‘P’ using sine rule.

28. Ans: (c)
Sol: Consider the three bodies the bigger spool (Radius 20), smaller spool (Radius 10) and the frame. They together have three I centers, I centre of big spool with respect to the frame is at its centre A. that of the small spool with respect to the frame is at its centre H. The I centre for the two spools P is to be located.
As for the three centers in line theorem all the three centers should lie on a straight line implies on the line joining of A and H. More over as both the spools are rotating in the same direction, P should lie on the same side of A and H. Also it should be close to the spool running at higher angular velocity. Implies close to H and it is to be on the right of H. Whether P belongs to bigger spool or smaller spool its velocity must be same. As for the radii of the spools and noting that the velocity of the tape is same on both the spools

\[ \omega_H = 2\omega_A \]

\[ \therefore \text{AP} \cdot \omega_A = \text{HP} \cdot \omega_H \text{ and} \]

\[ \text{AP} = \text{AH} + \text{HP} \Rightarrow \text{HP} = \text{AH} \]

**Note:**

(i) If two links are rotating in same directions then their Instantaneous centre will never lie in between them. The ‘I’ center will always close to that link which is having high velocity.

(ii) If two links are rotating in different directions, their ‘I’ centre will lie in between the line joining the centres of the links.

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**29. Ans: (b)**

**Sol:** \( I_{23} \) should be in the line joining \( I_{12} \) and \( I_{13} \). Similarly the link 3 is rolling on link 2.

**Locus of \( I_{23} \)**

So the I – Center \( I_{23} \) will be on the line perpendicular to the link – 2. (\( I_{23} \) lies common normal passing through the contact point)

So the point C is the intersection of these two loci which is the center of the disc.

So \( \omega_2 (I_{12}, I_{23}) = \omega_3 (I_{13}, I_{23}) \)

\[ \Rightarrow \omega_2 \times 50 = 1 \times 5 \]

\[ \Rightarrow \omega_2 = 0.1 \text{ rad/ sec} \]

**30. Ans: 1 (range 0.95 to 1.05)**

**Sol:** Locate the I-centre for the link AB as shown in fig. M is the mid point of AB

Given, \( V_A = 2 \text{ m/sec} \)

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[Diagram of spools and centers]

[Diagram of link AB and velocities]
31. Ans: (a) & 32. Ans: (b)
Sol:
\[ a_T = r\alpha \]
\[ a_{TA} = r\alpha \]
\[ a_n \]
\[ f_c = 0.4 \text{ m/s}^2 \] acts towards the centre
Tangential acceleration, \( f_t = r\alpha = 0.2 \text{ m/s}^2 \) acts perpendicular to the link in the direction of angular acceleration. Linear deceleration = 0.5 m/s\(^2\) acts opposite to velocity of slider
As the link is rotating and sliding so coriolis component of acceleration acts
\[ f^c = 2V\omega = 2 \times 0.2 \times 1 = 0.4 \text{ m/s}^2 \]
To get the direction of coriolis acceleration, rotate the velocity vector by 90° in the direction of \( \omega \).
Resultant acceleration
\[ = \sqrt{0.6^2 + 0.1^2} = 0.608 \text{ m/sec}^2 \]
\[ \phi = \tan^{-1}\left(\frac{0.6}{0.1}\right) = 80.5^\circ \]
Angle of Resultant vector with reference to \( OX = 30 + \phi = 30 + 80.5 = 110.53^\circ \)

33. Ans: (d)
Sol:
\[ a_{TO} = r\alpha \]
\[ a_{TA} = r\alpha \]
\[ a_n \]
\[ f_R = r\omega^2 \]
\[ a_o = a_{TO} + a_{TA} + a_n \]
\[ a_{TO} \] and \( a_{TA} \) are linear accelerations with same magnitude and opposite in direction.
\[ \Rightarrow a_o^2 = a_n^2 = \frac{V^2}{r} = r\omega^2 \]
Resultant acceleration, \( f_R = r\omega^2 \)

34. Ans: (c)
Sol:
\[ V_B = OB \times \omega \]
\[ V_A = OA \times \omega \]
\[ V_{BA} = V_B - V_A = (OB - OA) \times \omega = \omega (r_B - r_A) \]
and direction of motion point ‘B’.
35. Ans: (d)
Sol: As uniform angular velocity is given, Tangential acceleration, \( \alpha = 0 \)
Centripetal acceleration, 
\[ f_{BA} = (r_B^2 - r_A^2) \times \omega \] from Z to ‘O’.

36. Ans: (a)
Sol:
\[ \text{Velocity Diagram} \]
From velocity diagram, \( V_C = V_B \)
\[ l_4 \omega_4 = l_2 \omega_2 \]
\[ 25 \times \omega_4 = 50 \times 0.2 \]
\[ \Rightarrow \omega_4 = 0.4 \text{ rad/sec} \]
From acceleration diagram, 
\[ l_4 \alpha_4 = l_2 \alpha_2 \]
\[ 25 \times \alpha_4 = 50 \times 0.1 \]
\[ \Rightarrow \alpha_4 = 0.2 \text{ rad/sec}^2 \]

37. Ans: (d)
Sol:
As links \( O_1A \) and \( O_2B \) are parallel then 
\[ V_A = V_B \]
\[ \Rightarrow 50 \times 2 = 50 \times \omega_2 \]
\[ \Rightarrow \omega_2 = 2 \text{ rad/sec} \]
As a \( O_2C \) and \( O_3D \) are parallel links then 
\[ V_C = V_D \]
\[ \Rightarrow 100 \times 2 = 100 \times \omega_1 \]
\[ \Rightarrow \omega_1 = 2 \text{ rad/sec} \]
Centripetal acceleration, 
\[ f = r \omega^2 \]
\[ = 100 \times (2)^2 = 400 \text{ mm/sec}^2 \]
Radial relative acceleration, \( f^{\text{linear}} = 0 \)

Centripetal acceleration, \( f^c = r\omega^2 \)
\[ = 1 \times 1^2 = 1 \text{ m/s}^2 \] (acts towards the center)

Tangential acceleration, \( f^t = r\alpha \)
\[ = 1 \times 0.732 = 0.732 \text{ m/sec}^2 \]

Coriolis acceleration, \( f^{\text{cor}} = 2V\omega \)
\[ = 2 \times 0.5 \times 1 = 1 \text{ m/sec}^2 \]

Resultant acceleration,
\[ f^r = \sqrt{1^2 + (1 + 0.732)^2} = 2 \text{ m/sec}^2 \]
\[ \phi = \tan^{-1} \left( \frac{1.732}{1} \right) = 60^\circ \]
\[ \theta_{\text{reference}} = 30 + 180 + 60 = 270^\circ \]

39. **Ans:** (d)

**Sol:** Angular acceleration of connecting rod is given by
\[ a = -\omega^2 \sin \theta \left[ \frac{(n^2-1)}{(n^2-\sin^2 \theta)^{3/2}} \right] \]
when \( n = 1, \ a = 0 \)

40. **Ans:** (b) & 41. **Ans:** (a)

**Sol:**

\[ F_p = 2 \text{ kN} \]
\[ l = 80 \text{ cm} = 0.8 \text{ m} \]
\[ r = 20 \text{ cm} = 0.2 \text{ m} \]

From the triangle

\[ \text{OAB} \]
\[ \cos \phi = \frac{\ell^2 + \ell^2 - r^2}{2 \ell^2} \]
\[ = \frac{2 \times 80^2 - 20^2}{2 \times 80^2} \Rightarrow \phi = 14.36 \]

\[ \cos \theta = \frac{20^2 + 80^2 - 80^2}{2 \times 20 \times 80} \Rightarrow \theta = 82.82 \]

Thrust connecting rod
\[ F_T = \frac{F_p}{\cos \phi} = \frac{2}{\cos 14.36} = 2.065 \text{ kN} \]

Turning moment,
\[ T = F_T \times r = \frac{F_p}{\cos \phi} \left( \sin(\theta + \phi) \right) \times r \]
\[ = \frac{2}{\cos 14.36} \times \sin(14.36 + 82.82) \times 0.2 \]
\[ = 0.409 \text{ kN-m} \]

42. **Ans:** (b)

**Sol:** Calculate AB that will be equal to 260 mm
\[ L = 260 \text{ mm}, \quad P = 160 \text{ mm} \]
\[ S = 60 \text{ mm}, \quad Q = 240 \text{ mm} \]
\[ L + S = 320 \]
\[ P + Q = 400 \]

\[ \therefore L+S < P+Q \]

It is a Grashof’s chain

Link adjacent to the shortest link is fixed
\[ \therefore \text{Crank – Rocker Mechanism.} \]
43. Ans: (b)
Sol: O₂A || O₄B
Then linear velocity is same at A and B.
∴ \( \omega_2 \times O₂A = \omega_4 \times O₄B \)
∴ \( 8 \times 60 = \omega_4 \times 160 \)
⇒ \( \omega_4 = 3 \text{ rad/sec} \)

44. Ans: (c)
Sol:
\[
\begin{align*}
\tan \theta &= \frac{100}{240} \Rightarrow \theta = 22.62^\circ \\
\text{As centre of mass falls at } O_2 \\
m\ddot{r} = 0 \quad (\because \dot{r} = 0) \\
\alpha = 0 \quad (\text{Given}) \\
\text{Inertia torque } = 0 \\
\text{Since torque on link } O₂A \text{ is zero, the resultant force at point } A \text{ must be along } O₂A. \\
\Rightarrow F \sin 22.62 = 30 \\
\Rightarrow F = \frac{30}{\sin 22.62} = 78 \text{ N} \\
\text{The magnitude of the joint reaction at } O_2 = F = 78 \text{ N}
\end{align*}
\]

45. Ans: (d)
Sol: \( I \frac{d^2 \theta}{dt^2} = T + f(\sin \theta, \cos \theta) \)
Where ‘T’ is applied torque, f is inertia torque which is function of \( \sin \theta \) & \( \cos \theta \)
\[ \frac{d\theta}{dt} = \frac{T}{I} + f'(\sin \theta, \cos \theta) + c_i \]
\[ \theta = \frac{T}{I} t^2 + c_1 t + f'(\sin \theta, \cos \theta) \]
\( \theta \) is fluctuating on parabola
and \( @ t = 0, \theta = 0, \dot{\theta}(\text{slope}) = 0 \) (because it starts from rest)

46. Ans: 1 (range 0.9 to 1.1)
Sol:
Given \( F_p = 5 \text{ kN} \)
\[ F_{rod} = \frac{F_p}{\cos \phi}, F_t = F_{rod} \cos \phi \]
∴ \( F_t = 5 \text{ kN} \)
Turning moment = \( F_t \cdot r = 5 \times 0.2 = 1 \text{ kN-m} \)
47. Ans: (a)
Sol:

\[ N = 120 \text{ rpm} \]

\[ \omega_2 = \frac{2\pi N}{60} = 4\pi \text{ rad/s} \]

\[ \therefore l_1 = O_1 O_2 = 50 \text{ cm} \]

\[ QRR = 1:2 = \frac{1}{2} \]

\[ \frac{1}{2} = \frac{180 - 2\alpha}{180 + 2\alpha} \Rightarrow 180 + 2\alpha = 360 - 4\alpha \]

\[ \Rightarrow 6\alpha = 180^\circ \]

\[ \Rightarrow \alpha = 30^\circ \]

\[ \sin 30 = \frac{O_2 A}{O_1 O_2} \]

\[ \Rightarrow \frac{1}{2} = \frac{O_2 A}{50} \Rightarrow O_2 A = 25 \text{ cm} \]

\[ \therefore l_2 = 25 \text{ cm} \]

At the position given above \((O_1 O_2 B)\) the tool post attains the maximum velocity.

At that given instant

\[ l_2 \omega_2 = l_4 \omega_4 \] & velocity of slider is zero.

\[ l_4 = O_1 B = l_1 + l_2 = 50 + 25 = 75 \text{ cm} \]

\[ \Rightarrow 25 \times 4\pi = 75 \times \omega_4 \]

\[ \omega_4 = \frac{100\pi}{75} = \frac{4\pi}{3} = 4.19 \text{ rad/s} \]

\[ \omega_4 = \text{angular velocity of slotted lever.} \]
03. Ans: (a)
Sol: When addendum of both gear and pinion are same then interference occurs between tip of the gear tooth and pinion.

04. Ans: Decreases, Increases

05. Ans: (b)
Sol: For same addendum interference is most likely to occur between tip of the gear tooth and pinion i.e., at the beginning of the contact.

06. Ans: (b)
Sol: For two gears are to be meshed, they should have same module and same pressure angle.

07. Ans: (b)
Sol:

\[
\text{Centre distance between } P \text{ and } S \text{ is given by}
\]
\[
R_p + R_q + R_r + R_t
\]
\[
= m_p \frac{T_p}{2} + m_q \frac{T_Q}{2} + m_r \frac{T_R}{2} + m_s \frac{T_S}{2}
\]
\[
= 1.5 \left[ \frac{40 + 20}{2} \right] + 2 \left[ \frac{15 + 20}{2} \right]
\]
\[
= 45 + 35 = 80 \text{ mm}
\]

08. Ans: (c)
Sol: 
\[
\frac{N_2}{N_6} = \frac{N_3N_5N_6}{N_2N_4N_5} = \frac{N_3N_6}{N_2N_4}
\]
Wheel 5 is the only Idler gear as the number of teeth on wheel ‘5’ does not appear in the velocity ratio.

09. Ans: (a)
Sol:

\[
Z_1 = 16, \quad Z_3 = 15, \quad Z_2 = ?, \quad Z_4 = ?
\]
First stage gear ratio, \( G_1 = 4 \),
Second stage gear ratio, \( G_2 = 3 \),
\[
m_{12} = 3, \quad m_{34} = 4
\]
\[
Z_2 = 16 \times 4 = 64
\]
\[
Z_4 = 15 \times 3 = 45
\]
10. Ans: (b)
Sol: Centre distance
\[ m = \frac{m_2}{2} (Z_1 + Z_2) = \frac{m_3}{2} (Z_3 + Z_4) \]
\[ = \frac{4}{2} (15 + 45) = 120 \text{mm} \]

11. Ans: 5 rpm (CCW)
Sol:
\[ T_1 = 104, \quad N_1 = 0, \]
\[ T_2 = 96, \quad N_a = 60 \text{ rpm (CW, +ve)}, \quad N_2 = ? \]
\[ \frac{N_2 - N_a}{N_1 - N_a} = \frac{T_1}{T_2} = \frac{104}{96} \]
\[ N_2 - 60 = \frac{104}{96} \]
\[ N_2 = 60 \left[ 1 - \frac{104}{96} \right] = \frac{-60 \times 8}{96} = -5 \text{ rpm CW} \]
\[ = 5 \text{ rpm in CCW} \]

13. Ans: (d)
Sol: Data given:
\[ \omega_1 = 60 \text{ rpm (CW, +ve)} \]
\[ \omega_4 = -120 \text{ rpm [2 times speed of gear -1]} \]
We have, \[ \frac{\omega_1 - \omega_5}{\omega_4 - \omega_5} = 6 \]
\[ \Rightarrow \frac{60 - \omega_5}{-120 - \omega_5} = 6, \text{ simplifying} \]
\[ 60 - \omega_5 = -720 - 6\omega_5 \]
\[ \omega_5 = -156 \text{ rpm CW} \]
\[ \Rightarrow \omega_5 = 156 \text{ rpm CCW} \]

14. Ans: (c)
Sol: \[ \omega_2 = 100 \text{ rad/sec (CW+, ve)} \]
\[ \omega_{arm} = 80 \text{ rad/s (CCW)} = -80 \text{ rad/sec} \]
\[ \frac{\omega_5 - \omega_{arm}}{\omega_2 - \omega_{arm}} = \frac{-T_2}{T_3} \times \frac{T_4}{T_5} \]
\[ \frac{-80 - (-80)}{100 - (-80)} = \frac{-20 \times 32}{80} = \frac{1}{3} \]
\[ \Rightarrow \omega_5 = -140 \text{ CW} = 140 \text{ CCW} \]

15. Ans (c)

16. Ans: (c)
Sol: No. of Links, \( L = 4 \)
No. of class 1 pairs \( J_1 = 3 \)
No. of class 2 pairs \( J_2 = 1 \) (Between gears)
No. of dof = 3(\( L - 1 \)) - 2\( J_1 - J_2 \) = 2
17. Ans: (a)
Sol: \( r_b = \) base circle radius,
\( r_d = \) dedendum radius
\( r = \) pitch circle radius.
For the complete profile to be involute,
\( r_b = r_d \)
\( r_d = r - 1 \) module
\[ r = \frac{mT}{2} = \frac{16 \times 5}{2} = 40 \text{ mm} \]
\[ \therefore r_b = r_d = 40 - 1 \times 5 = 35 \text{ mm} \]
\[ r_b = r \cos \phi \Rightarrow \phi \approx 29^\circ \]

18. Ans: \(-3.33 \text{ N-m}\)
Sol:
\[ \frac{\omega_s - \omega_a}{\omega_p - \omega_a} = -\frac{Z_p}{Z_s} \]
\[ \Rightarrow \frac{0 - 10}{\omega_p - 10} = -\frac{20}{40} \]
\[ \Rightarrow \omega_p = 30 \text{ rad/sec} \]
By assuming no losses in power transmission
\[ T_p \times \omega_p + T_s \times \omega_s + T_a \times \omega_a = 0 \]
\[ \Rightarrow T_p \times 30 + T_s \times 0 + 5 \times 10 = 0 \]
\[ \Rightarrow T_p = \frac{-50}{30} = -1.67 \text{ N-m, } T_p + T_s + T_a = 0 \]
\[ \Rightarrow -1.67 + T_s + 5 = 0 \]
\[ \Rightarrow T_s = -3.33 \text{ N-m} \]

19. Ans: (a)
Sol: Train value = \( \frac{1}{\text{speed ratio}} \)

20. Ans: (d)
Sol:
\[ T_s + 2T_p = T_A \quad \text{-----(1)} \]
\[ \frac{N_A - N_a}{N_p - N_a} = \frac{T_p}{T_A} \quad \text{-----(2)} \]
and \[ \frac{N_p - N_s}{N_s - N_G} = -\frac{T_s}{T_p} \quad \text{-----(3)} \]
From (2) and (3)
\[ \frac{N_A - N_a}{N_s - N_G} = -\frac{T_B}{T_A} \]
\[ \Rightarrow 300 - 180 = -80 \]
\[ \Rightarrow 0 - 180 = \frac{T_s}{T_p} \]
\[ \therefore T_A = 120 \]
\[ 80 + 2T_p = 120 \]
\[ \Rightarrow T_p = 20 \]
Chapter- 3
Fly Wheels

01.
Sol: Given
\[ P = 80 \text{kW} = 80 \times 10^3 \text{W} = 80,000 \text{W} \]
\[ \Delta E = 0.9 \text{ Per cycle} \]
\[ N = 300 \text{rpm} \]
\[ C_s = 0.02 \]
\[ \omega = \frac{2\pi N}{60} = \frac{2\pi \times 30}{60} = 31.41 \text{rad/s} \]
\[ \rho = 7500 \text{ kg/m}^3 \]
\[ \sigma_c = 6 \text{ MN/m}^2 \]
\[ \sigma_c = \rho V^2 = \rho R^2 \omega^2 \]
\[ R = \sqrt{\frac{\sigma_c}{\rho \omega^2}} = \sqrt{\frac{6 \times 10^6}{7500 \times 31.41^2}} \]
\[ R = 0.9 \text{ m} \]
\[ D = 2R = 1.8 \text{m} \]
\[ N = 300 \text{rpm} = 5 \text{rps} \rightarrow 0.2 \text{ Sec/rev} \]
\[ 1 \text{ cycle} = 2 \text{ revolution (~4 stroke engine)} \]
\[ = 0.4 \text{ sec} \]
Energy developed per cycle
\[ = 0.4 \times 80 = 32 \text{ kJ} \]
\[ \Delta E = E \text{ per cycle} \times 0.9 \]
\[ = 32 \times 10^3 \times 0.9 \]
\[ \Delta E = 28800 \text{J} \]
\[ \Delta E = I \omega^2 C_s \]
\[ I = \frac{\Delta E}{\omega^2 C_s} \]
\[ I = 1459.58 \text{ kg-m}^2 \]

02.
Sol:
\[ \text{Given: } 1 \text{ cm}^2 = 1400 \text{ J} \]
Assume on x-axis 1 cm = 1 radian and on y-axis 1 cm = 1400 N-m
\[ a_1 = -0.5 \text{ cm}^2 \]
\[ a_2 = -1.7 \text{ cm}^2 \]
\[ a_3 = 9 \text{ cm}^2 \]
\[ a_4 = -0.8 \text{ cm}^2 \]
Work done per cycle
\[ = -a_1 - a_2 + a_3 - a_4 \]
\[ = -0.5 - 1.7 + 9 - 0.8 \]
\[ = 6 \text{ cm}^2 \]
Mean torque
\[ T_{mean} = \frac{\text{Work done per cycle}}{4\pi} \]
\[ = \frac{6}{4\pi} = \frac{1.5}{\pi} \text{ cm} \]
Area of the triangle (expansion)

\[ A = \frac{1}{2} \times \pi \times H = 9 \]

\[ H = 18 / \pi \]

Area above the mean torque line

\[ \Delta E = \frac{1}{2} \times b \times h \]

From the similar triangles,

\[ \frac{b}{B} = \frac{h}{H} \Rightarrow b = \frac{16.5}{18} \times \pi \]

\[ \Delta E = \frac{1}{2} \times b \times \frac{16.5}{\pi} \]

\[ = \frac{1}{2} \times \frac{16.5 \times 16.5}{18 \times \pi} = 7.56 \text{ cm}^2 \]

\[ \Delta E = 7.56 \times 1400 = 10587 \text{ N-m} \]

\[ N_1 = 102 \text{ rpm, } N_2 = 98 \text{ rpm,} \]

\[ \omega_1 = \frac{2\pi N_1}{60} = 10.68 \text{ rad/s} \]

\[ \omega_2 = \frac{2\pi N_2}{60} = 10.26 \text{ rad/s} \]

\[ \Delta E = \frac{1}{2} \times I \times (\omega_1^2 - \omega_2^2) \]

\[ I = \frac{2 \times \Delta E}{(\omega_1^2 - \omega_2^2)} \]

\[ I = 2405.6 \text{ kg-m}^2 \]

03.

Sol:

\begin{align*}
\text{Power} & \quad 8.5 \times 2639 = 22431 \text{ Nm} \\
\text{Time} & \quad 8.5 \text{ sec} \quad 10 \text{ sec} \\
\end{align*}

Given:

\[ d = 40 \text{ mm}, \quad t = 30 \text{ mm} \]

\[ E_1 = 7 \text{ N-m/mm}^2, \quad S = 100 \text{ mm} \]

\[ V = 25 \text{ m/s}, \quad V_1 - V_2 = 3\% V, \quad C_S = 0.03 \]

\[ A = \pi dt = \pi \times 40 \times 30 \]

\[ = 3769.9 = 3770 \text{ mm}^2 \]

Since the energy required to punch the hole is 7 Nm/mm² of sheared area, therefore the total energy required for punching one hole

\[ = 7 \times \pi dt = 26390 \text{ N-m} \]

Also the time required to punch a hole is 10 sec, therefore power of the motor required

\[ = \frac{26390}{10} = 2639 \text{ Watt} \]

The stroke of the punch is 100 mm and it punches one hole in every 10 seconds.

Total punch travel = 200 mm

(upper stroke + down stroke)

Velocity of punch = (200/10) = 20 mm/s

Actual punching time = 30/20 = 1.5 sec

Energy supplied by the motor in 1.5 sec is

\[ E_2 = 2639 \times 1.5 = 3958.5 = 3959 \text{ N-m} \]

Energy to be supplied by the flywheel during punching or the maximum fluctuation of energy

\[ \Delta E = E_1 - E_2 \]

\[ = 26390 - 3959 = 22431 \text{ N-m} \]

Coefficient of fluctuation of speed

\[ C_S = \frac{V_1 - V_2}{V} = 0.03 \]
We know that maximum fluctuation of energy ($\Delta E$)

$$22431 = m V^2 C_S = m (25)^2 (0.03)$$

$m = 1196 \text{ kg}$

**04.**

**Sol:**

**Given:**

- $P = 2 \text{ kW}$
- $K = 0.5$
- $N = 260 \text{ rpm}$
- $\omega = 27.23 \text{ rad/s}$

Actual punching time = 1.5 sec

Work done per cycle = 10000 Joule per hole

Motor power = 2 kW

$\Delta N = 30 \text{ rpm}$

$\Delta \omega = 2\pi \times (30/60) = \pi \text{ rad/sec}$

600 holes/hr = 10 holes/min $\Rightarrow$ 6 sec/hole

Cycle time = 6 sec

Energy withdrawn from motor

$$= \frac{10000}{6} = 1666.67 \text{ J}$$

Energy stored in flywheel

$$= \frac{10000}{6} \times 4.5 = 7.5 \text{ kJ}$$

Fluctuation of Energy $\Delta E = 7500 \text{ J}$

$$\Delta E = I \omega \Delta \omega = mk^2 \omega \Delta \omega$$

$$m = \frac{\Delta E}{k^2 \omega \Delta \omega}$$

Where $k = $ radius of gyration

$$m = \frac{7500}{0.5^2 \times 27.23 \times \pi} = 349.5 \text{ kg}$$

**05.**

**Ans: (d)**

**Sol:**

Work done = $-0.5 + 1 - 2 + 25 - 0.8 + 0.5$

$$= 23.2 \text{ cm}^2$$

Work done per cycle = $23.2 \times 100 = 2320$

$$\left( : 1 \text{ cm}^2 = 100 \text{ N} \cdot \text{m} \right)$$

$$T_{\text{mean}} = \frac{W.D \text{ per cycle}}{4\pi}$$

$$= \frac{2320}{4\pi} = \frac{580}{\pi} N \cdot m$$

Suction = 0 to $\pi$,

Compression = $\pi$ to $2\pi$

Expansion = $2\pi$ to $3\pi$,

Exhaust = $3\pi$ to $4\pi$

**06.**

**Ans: (c)**

**Sol:**

$E_A = E$

$E_B = E + 60$

$E_C = E + 60 - 40 = E + 20$

$E_D = E + 20 + 80 = E + 100 = E_{\text{max}}$

$E_E = E + 100 - 100 = E$

$E_F = E + 60$

$E_G = E + 60 - 60 = E_{\text{min}}$

$\therefore R > P > Q > S$

**Correct answer is option (c).**
07. Ans: (b)
Sol: 
\[ I_{\text{disk}} = \frac{m r^2}{2} \]
\[ I_1 = \frac{m r^1_1}{2}, \quad C_{s1} = 0.04 \]
\[ I_2 = 4 \times m r^1_1 = 4I_1 \]
\[ C_{s2} = \frac{I_1}{I_2} \times C_{s1} = 0.01 \Rightarrow 1\% \text{ reduce} \]
Correct answer is option (b).

08. Ans: (b)
Sol: For same \( \Delta E \) and \( \omega \)
\[ C_s \propto I \]
\[ C_{s1} = I_{s1} = 2I, \quad C_{s2} = \frac{C_{s1}}{I_{s2}} = 0.04 \]
\[ C_{s2} = \frac{0.04}{2} = 0.02 \]

09. Ans: (a)
Sol: Let the cycle time = \( t \)
Actual punching time = \( t/4 \)
Energy required in actual punching
\[ W = 3W/4 \]
During \( 3t/4 \) time, energy consumed = \( W/4 \)
\[ E_{\text{max}} = \frac{3W}{4}, \quad E_{\text{min}} = \frac{E}{4} \]
\[ \Delta E = E_{\text{max}} - E_{\text{min}} = \frac{E}{2} \]
\[ \frac{\Delta E}{E} = 0.5 \]

10. Ans: (c)
Sol:
Correct answer is option (b).

11. Ans: 0.5625
Sol: The flywheel is considered as two parts \( \frac{m}{2} \) as rim type with Radius \( R \) and \( \frac{m}{2} \) as disk type with Radius \( \frac{R}{2} \)
\[ I_{\text{Rim}} = \frac{m}{2} R^2, \]
\[ I_{\text{disk}} = \frac{1}{2} \times \frac{m}{2} \times \left( \frac{R}{2} \right)^2 = \frac{mR^2}{16} \]
\[ I = \frac{mR^2}{2} + \frac{mR^2}{16} \]
12. Ans: 104.71
Sol: \( N = 100 \ \text{rpm} \)

\[
T_{\text{mean}} = \frac{1}{\pi} \int_{0}^{\pi} T \, d\theta = \frac{1}{\pi} \int_{0}^{\pi} (10000 + 1000\sin 2\theta - 1200\cos 2\theta) \, d\theta = \frac{1}{\pi} \left[10000\theta - 500\cos 2\theta - 600\sin 2\theta\right]_{0}^{\pi} = 10000 \ \text{Nm}
\]

Power = \[
\frac{2\pi NT}{60} = \frac{2 \times \pi \times 100 \times 10000}{60} = 104719.75 \ \text{W}
\]
\( P = 104.719 \ \text{kW} \)

01. Ans: (a)
Sol: As the governor runs at constant speed, force on the sleeve is zero.

02. Ans: (d)
Sol: At equilibrium speed, friction at the sleeve is zero.

03. Ans: (a)
Sol: \( m\rho \omega^2 = \frac{r}{h} \left( mg + M(1+k) \right) \)

\( k = 1 \)
\( \omega^2 = \frac{9.8}{2 \times 0.2 (10 + 2)} \)
\( \omega = 17.15 \ \text{rad/sec} \)

04. Ans: (a)
Sol: \( m\rho \omega^2 \times a = \frac{1}{2} \times 200 \times \delta \times a \)

\( \delta = \frac{1 	imes 20^2 \times 0.25 \times 2}{200} = 0.5 \times 2 = 1 \ \text{cm} \)

05. Ans: (a)
Sol: \( m\rho \omega^2 \times a = \left( \frac{F_s}{2} \right) \times a \)

\( F_s = 2m\rho \omega^2 \)

\( = 2 \times 1 \times 0.4 \times (20)^2 = 320 \ \text{N} \)
06. Ans: (c)
Sol: A governor is used to limit the change in speed of engine between minimum to full load conditions, the sensitiveness of a governor is defined as the ratio of difference between maximum and minimum speed to mean equilibrium speed, thus,

\[ \text{sensitiveness} = \frac{\text{Range of speed}}{\text{mean speed}} = \frac{N_1 - N_2}{\frac{N_1 + N_2}{2}} \]

Where, mean speed, \( N = \frac{N_1 + N_2}{2} \)

\( N_1 = \) maximum speed corresponding to no-load conditions.
\( N_2 = \) minimum speed corresponding to full load conditions.

07. Ans: (b)

08. Ans: (a)
Sol: \( r_1 = 50 \text{ cm}, \ F_1 = 600 \text{ N} \)

\[ F = a + rb \]

600 = \( a + 50b \)
700 = \( a + 60b \)
10 \( b = 100 \)
\( b = 10 \text{ N/cm} \)
\( a = 100 \text{ N} \)
\( F = 100 + 10r \)

This is unstable governor. It can be isochronous if its initial compression is reduced by 100 N.

09. Ans: (d)
Sol: By increasing the dead weight in a porter governor it becomes more sensitive to speed change.

10. Ans: (d)

11. Ans: (a)
Sol:

At radius, \( r_1 = F_1 < F_2 < F_3 \)

\( \therefore \) As Controlling force is less suitable 1 is for low speed and 2 for high speed ad 3 is for still high speed.

(1) is active after 40 cm
(2) is active after 20 cm
(3) is active after 10 cm

At given radius above 20 cm
\( F_3 > F_2 \)
\( m\omega_3^2 > m\omega_2^2 \)
\( \omega_3 > \omega_2 \)
12. Ans: (b) 
Sol: The vertical intercept $\frac{gh}{r}$ signifies that between the speeds corresponding to $\frac{gh}{r}$, the radius of the ball does not change while direction of movement of sleeve does. Between speeds $N_1$ and $N_2$, the governor is insensitive.

13. Ans: (c) 
Sol: A governor is said to be sensitive if for a given fractional change in speed, displacement of sleeve is high.

14. Ans: (c) 
Sol: A governor is stable if radius of rotation of ball is increases as the speed increases. Centripetal force, $F = mr\omega^2$  
\[ \Rightarrow \frac{F}{r} = m\omega^2 \]
Slope of the centripetal force represents speed. Higher the slope, higher will be the speed.

when $r = 2 \text{ cm}$; $F = 14 \text{ N}$  
\[ \therefore \frac{F}{r} = \frac{14}{2} = 7 \]

when $r = 6 \text{ cm}$; $F = 38 \text{ N}$  
\[ \frac{F}{r} = \frac{38}{6} = 6.33 \]
As the radius increases slope of the centripetal force curve decreases and therefore speed of the governor decreases. Thus the governor is unstable.

The vertical intercept $\frac{gh}{r}$ signifies that between the speeds corresponding to $\frac{gh}{r}$, the radius of the ball does not change while direction of movement of sleeve does. Between speeds $N_1$ and $N_2$, the governor is insensitive.
16. Ans:
Sol: Given, m = 8 kg
F₁ = 1500 N at r₁ = 0.2 m and
F₂ = 887.5 N at r₂ = 0.13 m,
For spring controlled governor, controlling force is given by
F = ar + b
1500 = a × 0.2 + b
887.5 = a × 0.13 + b
∴ a = 8750, b = −250

F = 8750r − 250
At r = 0.15 m,
F = 8750×0.15 − 250 = 1062.5 N
So, controlling force, F = 1062.5 m
F = mrω²
1062.5 = 8 × 0.15 ω²
∴ ω = 29.72 rad/s
N = \( \frac{60\omega}{2\pi} \) = 284 rpm

For isochronous speed
F = a r = 8750 × 0.15 = 1312.5 N
F = mrω²
1312.5 = 8 × 0.5 × ω²
⇒ ω = 33.07 rad/s
N = \( \frac{60\omega}{2\pi} \) = 316 rpm

The increase in tension is 250 N to make the governor isochronous.

17. Ans: (d)
Sol: The condition for the stability of governor is that the slope of the curve for the controlling force should be more than that of the line representing centripetal force at the speed considered. Therefore, if a centrifugal governor is stable at a particular position, it would be stable at all other position only if the condition of stability is satisfied.

Controlling force of porter governor suggests the it is stable through out its range of operation.

18. Ans: (a)
Sol: The problem of hunting becomes more acute when sensitiveness of governor is high. Isochronous governors are infinitely sensitive so hunting occurs if the speed deviates from isochronous.
Hunting in unstable governors is less than isochronous governor as it is more sensitive.
Chapter- 5
Balancing

01. Ans: (c)
Sol: unbalanced force \((F_{un}) \propto m r^2 \omega^2\)
Unbalance force is directly proportional to square of speed. At high speed this force is very high. Hence, dynamic balancing becomes necessary at high speeds.

02. Ans: (a)
Sol: Dynamic force = \(\frac{W}{g} e \omega^2\)
Couple = \(\frac{W}{g} e \omega^2 a\)
Reaction on each bearing = \(\pm \frac{W}{g} e \omega^2 a \frac{l}{l}\)
Total reaction on bearing
\[
= \left(\frac{W}{g} e \omega^2 a \frac{l}{l}\right) - \left(\frac{W}{g} e \omega^2 a \frac{l}{l}\right) = 0
\]

03. Ans: (b)
Sol: Since total dynamic reaction is zero the system is in static balance.

04. Ans: (a)

05. Ans: (b)
Sol:
\[
\begin{align*}
\theta_c &= \tan^{-1}\left[\frac{\frac{1}{2}(5+9 \sqrt{2})}{\sqrt{\frac{9}{9}}}\right] = 54.31^\circ \\
&= 90 - 54.31 = 35.69 \text{ w.r.t ‘A’} \\
m_c \cos \theta_c + m_d \cos \theta_d - 3 \sqrt{2} = 0 \\
\Rightarrow m_c \cos \theta_c + 10.91 \cos 54.31 - 3 \sqrt{2} = 0 \\
m_c \cos \theta_c = -2.122 \\
m_c \sin \theta_c + m_d \sin \theta_d - 3 \sqrt{2} + 5 = 0 \\
m_c \sin \theta_c + 10.91 \sin 54.31 - 3 \sqrt{2} + 5 = 0 \\
m_c \sin \theta_c = -9.618 \\
m_c = \sqrt{(-2.122)^2 + (-9.618)^2} = 9.85 \text{kg} \\
\tan \theta_c = \frac{-9.618}{-2.122} \\
\theta_c = 257.56 \text{ or } 257.56 - 90 \text{ w.r.t ‘A’} \\
= 167.56
\end{align*}
\]
### Common data Q. 06 & 07

#### 06. Ans: (a)

**Sol:** \( m_1 = 5 \text{ kg} \), \( m_2 = 5 \text{ kg} \), \( r_1 = 10 \text{ cm} \), \( r_2 = 20 \text{ cm} \), \( m_d = ? \), \( r_d = 10 \text{ cm} \)

- \( m_1 r_1 = 100 \text{ kg cm} \)
- \( m_2 r_2 = 100 \text{ kg cm} \)

#### 07. Ans: (d)

**Sol:**

- \( mr = 100 \text{ kg cm} = 1 \text{ kg m} \)
- \( N = 600 \text{ rpm} \Rightarrow \omega = \frac{2 \pi N}{60} = 20 \pi \text{ rad/s} \)

Couple ‘C’ = \( mr \omega^2 \times 0.2 = 1 \times (20\pi)^2 \times 0.2 = 789.56 \text{ Nm} \)

Reaction on the bearing

\[
= \frac{\text{couple}}{\text{distance between bearing}}
= \frac{789.56}{0.4} = 1973.92 \text{ N}
\]
08. Ans: (a)
Sol:
\[ r_1 = 10 \text{ cm}, \quad r_2 = 10 \text{ cm}, \quad m_1 = 52 \text{ kg} \]
\[ m_2 = 75 \text{ kg}, \quad \theta_1 = 0 \text{ (Reference)} \]
\[ \theta_2 = 90^\circ, \quad m = 2000\text{kg}, \quad e = ?, \quad \theta = ? \]
\[ m_1 e \cos \theta = m_1 r_1 = 520 \]
\[ m_2 e \sin \theta = m_2 r_2 = 750 \]

\[ m = \sqrt{(m_1 r_1)^2 + (m_2 r_2)^2} = \sqrt{520^2 + 750^2} = 913 \text{ kg-cm} \]
\[ e = \frac{913}{2000} = 0.456 \text{cm} \]
\[ \theta = \tan^{-1} \left( \frac{m_2 r_2}{m_1 r_1} \right) = \tan^{-1} \left( \frac{75}{52} \right) = 55.26^\circ \]
\[ = 180 + 55.26 = 235.26^\circ \text{ w.r.t mass ‘1’}. \]

09. Ans: (a)
Sol:

<table>
<thead>
<tr>
<th>Plane</th>
<th>( m ) (kg)</th>
<th>( r ) (m)</th>
<th>L (m) (reference Plane A)</th>
<th>( \theta )</th>
<th>( F_x ) (mr\cos\theta)</th>
<th>( F_y ) (mr\sin\theta)</th>
<th>( C_x ) (mr/\cos\theta)</th>
<th>( C_y ) (mr/\sin\theta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2 kg.m</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0.6</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>-m_a</td>
<td>0.5m</td>
<td>0</td>
<td>\theta_a</td>
<td>-0.5m_a \cos \theta_a</td>
<td>-0.5m_a \sin \theta_a</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>-m_b</td>
<td>0.5m</td>
<td>0.5</td>
<td>\theta_b</td>
<td>-0.5m_b \cos \theta_b</td>
<td>-0.5m_b \sin \theta_b</td>
<td>-\frac{m_b}{4} \cos \theta_b</td>
<td>-\frac{m_b}{4} \sin \theta_b</td>
</tr>
</tbody>
</table>

\[ C_x = 0 \Rightarrow \frac{m_b \cos \theta_b}{4} = 0.6 \]
\[ C_y = 0 \Rightarrow \frac{m_b \sin \theta_b}{4} = 0 \]
\[ \Rightarrow m_b = 2.4 \text{kg}, \quad \theta_b = 0 \]
\[ \Sigma F_x = 0 \]
\[ \Rightarrow 2 - 0.5 m_a \cos \theta_a - 0.5 m_b \cos \theta_b = 0 \]
\[ \Rightarrow \frac{m_a}{2} \cos \theta_a = 0.8 \]
\[ \Sigma F_y = 0 \Rightarrow \frac{m_a}{2} \sin \theta_a = 0 \]
\[ \therefore \theta_a = 0^\circ, \quad m_a = 1.6 \text{ kg} \]
(Note: mass is to be removed so that is taken as –ve).

10. Ans: (a)
Sol:

\[ \frac{F_x}{\omega^2} = m_1 r_1 + m_2 r_2 \cos \theta \]
\[ = 20 \times 15 + 25 \times 20 \cos 135 \]
\[ = -53.55 \text{ gm-cm} \]
\[ F_y = \frac{m_2 r_2 \sin \theta_2}{\omega^2} = 25 \times 20 \sin 135 \]
\[ = 353.553 \text{ gm-cm} \]

\[ m_b r_b = \sqrt{F_x^2 + F_y^2} \]

\[ \Rightarrow m_b = \frac{\sqrt{F_x^2 + F_y^2}}{r_b} \]

\[ = \frac{\sqrt{(-53.55)^2 + (353.553)^2}}{20} \approx 17.88 \text{ gm} \]

\[ \theta_b = \tan^{-1} \frac{F_y}{F_x} = \tan^{-1} \left( \frac{353.553}{-53.55} \right) = 98.7^\circ \]

11. **Ans:** 30 N

**Sol:**
- Crank radius = stroke/2 = 0.1 m,
- \( \omega = 10 \text{ rad/sec} \)
- \( m_b = 6 \text{ kg} \)

Unbalanced force along perpendicular to the line of stroke = \( m_b \omega^2 \cos \theta \sin 30^\circ \)

\[ = 6 \times (0.1) \times (10)^2 \sin 30^\circ \]
\[ = 30 \text{ N} \]

12. **Ans:** (b)

**Sol:**
- Primary unbalanced force = \( m \omega^2 \cos \theta \)
  At \( \theta = 0^\circ \) and \( 180^\circ \), primary force attains maximum.

Secondary force = \( \frac{m \omega^2}{n} \cos 2\theta \) where \( n \) is obliquity ratio. As \( n > 1 \), primary force is greater than secondary force.

- Unbalanced force due to reciprocating mass varies in magnitude. It is always along the time of stroke.

13. **Ans:** (b)

**Sol:** In balancing of single-cylinder engine, the rotating balance is completely made zero and the reciprocating unbalance is partially reduced.

14. **Ans:** (b)

**Sol:** \( m = 10 \text{ kg}, \quad r = 0.15 \text{ m}, \quad c = 0.6, \quad \theta = 60^\circ, \quad \omega = 4 \text{ rad/sec} \)

Residual unbalance along the line of stroke

\[ = (1 - c) m r \omega^2 \cos \theta \]
\[ = (1 - 0.6) \times 10 \times 0.15 \times 4^2 \cos 60^\circ \]
\[ = 4.8 \text{ N} \]

15. **Ans:** 2

**Sol:** By symmetric two system is in dynamic balance when

\[ m_{ea} = m_1 e_1 a_1 \]

\[ m_1 = m e \frac{a}{a_1} = 1 \times \frac{50}{20} \frac{2.5}{2.5} = 2 \text{ kg} \]

16. **Ans:** (a)

**Sol:**

\[ m_1 = \frac{mL_2}{L_1 + L_2} = \frac{100 \times 60}{100} = 60 \text{ kg} \]
17. Ans: (d)
Sol: For primary forces balances
\[ \sum r \cos \theta_i = 0 \]

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>( \theta )</th>
<th>( \cos \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \alpha )</td>
<td>( \cos \alpha )</td>
</tr>
<tr>
<td>2</td>
<td>180 + ( \beta )</td>
<td>( \cos (180 + \beta) )</td>
</tr>
<tr>
<td>3</td>
<td>180 - ( \beta )</td>
<td>( \cos (180 - \beta) )</td>
</tr>
<tr>
<td>4</td>
<td>360 - ( \alpha )</td>
<td>( \cos (360 - \alpha) )</td>
</tr>
</tbody>
</table>

\[ \therefore \sum m \cos \theta_i = R_1 \cos \alpha + R_2 \cos (180 + \beta) + R_2 \cos (180 - \beta) + R_1 \cos (360 - \alpha) = 0 \]

\[ = R_1 \cos \alpha - R_2 \cos \beta - R_2 \cos \beta + R_1 \cos \alpha = 0 \]

\[ \therefore 2 R_1 \cos \alpha = 2 R_2 \cos \beta \]

\[ \therefore R_1 \cos \alpha = R_2 \cos \beta \]

18. Ans: (a)
Sol:

Resultant primary unbalanced force is given by
\[ \sum F_p = m_1 \omega_1^2 \cos \theta_1 + m_2 \omega_2^2 \cos \theta_2 + m_3 \omega_3^2 \cos \theta_3 + m_4 \omega_4^2 \cos \theta_4 = 0 \]

Resultant secondary unbalanced force is given by
\[ \sum F_s = m_1 \omega_1^2 \cos \theta_1 + m_2 \omega_2^2 \cos \theta_2 + m_3 \omega_3^2 \cos \theta_3 + m_4 \omega_4^2 \cos \theta_4 = 0 \]

\[ \therefore \text{All primary and secondary forces are balanced.} \]

19. Ans: (d)
Sol:

- For primary direct crank total unbalanced mass is \( \frac{3W}{2g} \). Therefore primary direct force is equal to \( \frac{3W}{2g} \omega^2 \).
- As primary reverse crank is balanced, primary reverse force is equal to zero.
- Primary direct crank speed is \( \omega \).
- Primary reverse crank speed is equal and opposite to the primary direct crank speed.
20. Ans: (d)
Sol: 
If the shaft is statically balanced then
Reactions due to unbalanced couple
\[ R_A \times 1 \text{ m} = 300 \text{ N-m} \]
\[ R_A = 300 \text{ N} \]
\[ R_B = -300 \text{ N} \]

21. Ans: (a)

22. Ans: (a)
Sol: 
Let \( c \) be the fraction of reciprocating mass
Primary force balance by the mass
\[ = cmr^2 \cos \theta \]
Unbalance force along Line of action
\[ = (1 - c)mr^2 \cos \theta \]
Unbalance force perpendicular to line of action
\[ = cmr^2 \sin \theta \]
Resultant unbalanced force
\[ = \sqrt{(1-c)^2 mr^2 \cos^2 \theta + c^2 \sin^2 \theta (mr^2)} \]

Resultant unbalanced force is minimum when \( c = \frac{1}{2} \). But common practice is to use \( \frac{2}{3} \) of the reciprocating mass to minimize the effect of unbalanced force along line of stroke.

23. Ans: (a)
Sol: For Four cylinders:
For five cylinders:

\[ \theta = \frac{2\pi}{5} = 72^\circ \]
Chapter- 6
Cams

01. Ans: (d)
Sol: Pressure angle is given by
\[
\tan \phi = \frac{\frac{dy(\theta)}{d\theta} - e}{y(\theta) + \sqrt{r_p^2 - e^2}}
\]
where, 
\( \phi \) is pressure angle , 
\( \theta \) is angle of rotation of cam 
e is eccentricity 
r_p is pitch circle radius 
y is follower displacement

02. Ans: (d)
Sol: Cycloidal motion
\[
y = \frac{h}{2\pi} \left( \frac{2\pi \theta}{\phi} - \sin \left( \frac{2\pi}{\phi} \theta \right) \right)
\]
\[
\dot{y}_{\text{max}} = \frac{2h\omega}{\phi}
\]
Simple harmonic motion :
\[
\dot{y}_{\text{max}} = \left( \frac{\pi}{2} \frac{h\omega}{\phi} \right)
\]
Uniform velocity :
\[
\dot{y} = \frac{h\omega}{\phi}
\]
From (1), (2) and (3) we observe that 
\( V_{\text{cycloidal}} > V_{\text{SHM}} > V_{\text{UV}} \)

03. Ans: (b)
04. Ans: (b)
Sol: \( L = 4 \text{ cm}, \quad \phi = 90^\circ = \pi/2 \text{ radian}, \)
\[ \omega = 2 \text{ rad/sec}, \quad \theta = \frac{2}{3} \times 90 = 60^\circ \]
\[ \frac{\theta}{\phi} = \frac{2}{3} \]
\[ s(t) = \frac{L}{2} \left( 1 - \cos \frac{\pi \theta}{\phi} \right) \]
\[ = 2(1 - \cos 120) = 3 \text{ cm} \]
\[ V(t) = \frac{L}{2} \times \frac{\pi}{\phi} \times \omega \times \sin \left( \frac{\pi \theta}{\phi} \right) \]
\[ = \frac{4}{2} \times 2 \times 2 \sin(120) = 7 \text{ cm/s} \]
\[ a(t) = \frac{L}{2} \left( \frac{\pi}{\phi} \right)^2 \times \omega^2 \times \cos \left( \frac{\pi \theta}{\phi} \right) \]
\[ = \frac{4}{2} \times 2^2 \times 2^2 \times \cos(120) = -16 \text{ cm/sec}^2 \]

05. Ans: (b)
Sol: 
\[ x = 15 \cos \theta, \quad y = 10 + 5 \sin \theta \]
\[ \tan \phi = \frac{dy}{dx} = \frac{dy}{d\theta} = \frac{5 \cos \theta}{-15 \sin \theta} \]
\[ \tan \theta = \frac{y}{x} = \frac{10 + 5 \sin \theta}{15 \cos \theta} = \frac{10 + 5 \sin 30}{15 \cos 30} \]
\[ \theta = 43.897^\circ \]
Pressure angle is angle between normal and radial line = 16.10°.

\[ \tan \phi = \frac{5 \times \sqrt{3}}{-15 \times \frac{1}{2}} = -\frac{1}{\sqrt{3}} \Rightarrow \phi = 150^\circ \]
\[ \tan \theta = \frac{y}{x} = \frac{10 + 5 \sin \theta}{15 \cos \theta} = \frac{10 + 5 \sin 30}{15 \cos 30} \]
\[ \theta = 43.897^\circ \]

At \( \theta = 30^\circ \),
\[ \tan \phi = \frac{5 \times \sqrt{3}}{-15 \times \frac{1}{2}} = -\frac{1}{\sqrt{3}} \Rightarrow \phi = 150^\circ \]
\[ \tan \theta = \frac{y}{x} = \frac{10 + 5 \sin \theta}{15 \cos \theta} = \frac{10 + 5 \sin 30}{15 \cos 30} \]
\[ \theta = 43.897^\circ \]

Then normal makes with x-axis
\[ \tan^{-1} (\sqrt{3}) = 60^\circ \]
\[ \tan \theta = \frac{y}{x} = \frac{10 + 5 \sin \theta}{15 \cos \theta} = \frac{10 + 5 \sin 30}{15 \cos 30} \]
\[ \theta = 43.897^\circ \]

With follower axis angle made by normal (pressure angle) = 60° - 43.897° = 16.10°
06. Ans: (a)
Sol:

Let \( \alpha \) be the angle made by the normal to the curve

\[
\left( \frac{dy}{dx} \right)_{(4,2)} = 9
\]

\[
\tan \alpha = \frac{dy}{dx} = 4x - 7
\]

At \( x = 4 \) & \( y = 2 \),
\[
\alpha = \tan^{-1}(9) = 83.7^\circ
\]

The normal makes an angle
\[
= \tan^{-1}\left( \frac{-1}{9} \right) = 6.3^\circ \text{ with } x \text{ axis}
\]

\[
\theta = \tan^{-1}\left( \frac{2}{4} \right) = 26.52^\circ
\]

Pressure angle is angle between normal and radial line = 26.52 + 6.3 = 32.82°

07. Ans: (b)
Sol: For the highest position the distance between the cam center and follower

\( = (r + 5) \text{ mm} \)

For the lowest position it is \( (r - 5) \text{ mm} \)

So the distance between the two positions

\( = (r + 5) - (r - 5) = 10 \text{ mm} \)

08. Ans: (a)
Sol:

When ‘c’ move about ‘o’ through ‘\( \theta \)’, point ‘p’ moves to ‘p’.

‘\( \phi \)’ is angle between normal drawn at point of contact which always passes through centre of circle and follower axis. So this is pressure angle.

From \( \Delta p'oc' \)

\[
\frac{r}{\sin (\pi - \theta)} = \frac{e}{\sin \phi}
\]

\[
\sin \phi = \frac{e}{r}
\]

\( \phi \) is maximum \( \theta = 90^\circ \)

\[
\sin \phi = \frac{e}{r}
\]

Pressure angle is maximum at pitch point

\[
\phi = \sin^{-1}\left( \frac{e}{r} \right) = 30^\circ
\]

09. Ans: (c)
Sol: For a cycloidal motion displacement is given by

\[
s = \frac{h}{\pi} - \frac{\pi \theta}{2} - \frac{1}{2} \sin 2\pi \theta
\]

Velocity,

\[
\frac{ds}{dt} = h \omega \left( 1 - \cos \frac{2\pi \theta}{\phi} \right)
\]
Chapter - 7
GYROSCOPE

01. Ans: (c)
Sol: Due to gyroscopic couple effect and centrifugal force effect the inner wheels tend to leave the ground.

02. Ans: (d)
Sol: Pitching is angular motion of ship about transverse axis.

03. Sol: \( m = 100 \text{ kg}, \quad r_k = 200 \text{ mm} \)

---

10. Ans: (b)
Sol: By providing offset in a radial cam, translating follower pressure angle is decreased during ascent of the follower.

\[ \text{let}, \phi \text{ is pressure angle, } \theta \text{ is angle of rotation of cam, } e \text{ is eccentricity, } r_p \text{ is pitch circle radius, } y \text{ is follower displacement} \]

11. Ans: (d)
Sol: The cam in contact with a follower is case of successful constraint.

12. Ans: (a)
Sol: By providing offset to the follower, pressure angle decreases. As a result side thrust reduces and prevents jamming of follower in its guide. Wear between follower and cam surface also decreases.

Acceleration, \( f = \frac{dv}{dt} = \frac{2h\pi \omega^2}{\phi} \sin \frac{2\pi \theta}{\phi} \)

\[ \therefore \text{Shape of acceleration curve is a sine curve as shown below:} \]

\[ \begin{array}{c}
\text{f} \\
\text{f_{max}} \\
\theta
\end{array} \]

Due to pitching gyroscopic couple acts about vertical axis.
I = 1000 \times (0.2)^2 = 40 \text{ kg-m}^2 \\
N = 5000 \text{ rpm (CCW)} \text{ looking from stern} \\
\omega = \frac{2\pi \times 5000}{60} = 523.33 \text{ rpm} \\
\tilde{\omega} = -523.33 \hat{j} \\
\text{Precession velocity} \\
\omega_p = \frac{V}{r} = \frac{25 \times 0.514}{400} = 0.032125 \text{ rad/s} \\
\tilde{\omega}_p = 0.0312 \hat{k} \\
\text{Gyroscopic couple} = I(\tilde{\omega} \times \tilde{\omega}_p) \\
G = 40(-523.33 \hat{j} \times 0.032125 \hat{k}) \\
= -672 \hat{i} \text{ N-m} \\
\text{Now,} \\
R_1 = -R_2 = \frac{M}{L} = \frac{1672}{1.5} = 448 \text{ N} \\
R_1 = 448 \text{ N (Acting downwards)} \\
R_2 = 448 \text{ N (Acting upwards)} \\
\text{Now reaction due to weight} \\
R_1' = \frac{9810 \times 900}{1500} = 5886 \text{ N (upwards)} \\
R_2' = \frac{9810 \times 600}{1500} = 3924 \text{ N (upwards)} \\
\text{Total bearing reaction at A} \\
= R_A + R_A' \\
= 5886 - 448 = 5438 \text{ N} \\
\text{Total bearing reaction at B} \\
= R_B + R_B' \\
= 3924 + 448 = 4372 \text{ N} \\
\text{Bow falls and stern rises.} \\

04. \text{ Sol:} \\
\omega = \frac{2\pi \times 1800}{60} = 1884.95 \text{ rad/s} \\
\omega_p = \frac{V}{R} = \frac{0.2}{100} = 0.002 \text{ rad/s} \\
M = I \omega \omega_p \\
= 10.164 \times 0.002 \times 1884.95 \\
= 1681 \text{ N-m} \\
\text{Ans: 200} \\
\text{Sol:} \ R = 100 \text{ m}, \ \nu = 20 \text{ m/sec}, \\
\omega_p = \frac{V}{R} = 0.2 \text{ rad/sec} \\
\omega_z = 100 \text{ rad/sec} \\
I = 10 \text{ kg-m}^2
06. Ans:
Sol:
(i)
\[
\begin{align*}
\sum M_\theta &= 0 \\
2aT \sin \theta + I_\omega \Omega &= mg \times a \\
\frac{2aTb}{\sqrt{4a^2 + b^2}} + \frac{mr^2}{2} \omega \Omega &= mg \times a
\end{align*}
\]
\[
T = \frac{\sqrt{4a^2 + b^2}}{2ab} \left( mga - \frac{mr^2}{2} \omega \Omega \right)
\]
For clockwise rotation of precession
(ii)
\[
\sum M_\theta = 0 \\
2aT \sin \theta - I_\omega \Omega &= mg \times a \\
T &= \left( \frac{mga + \frac{1}{2} mr^2 \omega \Omega}{2ab} \right) \left( b^2 + 4a^2 \right)^{\frac{1}{2}}
\]

01. Ans: (b)
Sol:
\[
T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow 0.5 = 2\pi \times \sqrt{\frac{L}{9.81}}
\]
\[
\Rightarrow L = 62.12 \text{ mm}
\]

02. Ans: (d)
Sol:
Let the system is displaced by \( \theta \) from the equilibrium position. It’s position will be as shown in figure.

By considering moment equilibrium about the axis of rotation (Hinge)
\[
I \theta + mg \ell \sin (\alpha + \theta) - mg \ell \sin (\alpha - \theta) = 0
\]
\[
1 = m\ell^2 + m\ell^2 = 2m\ell^2
\]
After simplification
\[
2m\ell^2 \ddot{\theta} + 2mg\ell \cos \alpha \sin \theta = 0
\]
For small oscillations (\( \theta \) is small) \( \sin \theta = \theta \)
\[
\therefore 2m\ell^2 \ddot{\theta} + 2mg\ell \cos \alpha \cdot \theta = 0
\]
\[
\omega_n = \sqrt{\frac{2mg \ell \cos \alpha}{2m\ell^2}} = \sqrt{\frac{g \cos \alpha}{\ell}}
\]
03. **Ans:** (c)  
**Sol:** Let, \( V_0 \) is the initial velocity,  
‘\( m \)’ is the mass  
Equating Impulse = momentum  
\[ mV_0 = 5 \text{kN} \times 10^{-4} \text{ sec} \]  
\[ = 5 \times 10^3 \times 10^{-4} = 0.5 \text{ sec} \]  
\[ \therefore V_0 = \frac{0.5}{m} = 0.5 \text{ m/sec} \]  
\[ \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{10000}{1}} = 100 \text{ rad/ sec} \]  
When the free vibrations are initiate with initial velocity,  
The amplitude  
\[ X = \frac{V_0}{\omega_n} \text{(Initial displacement )} \]  
\[ \therefore X = \frac{V_0}{\omega_n} = \frac{0.5 \times 10^3}{100} = 5 \text{ mm} \]  

04. **Ans:** (a)  
**Sol:** Note: \( \omega_n \) depends on mass of the system not on gravity  
\[ \therefore \omega_n \propto \frac{1}{\sqrt{m}} \]  
If \( \omega_n = \sqrt{\frac{g}{\delta}} \), \( \delta = \frac{mg}{K} \)  
\[ \therefore \omega_n = \sqrt{\frac{g}{\delta}} = \sqrt{\frac{K}{m}} \]  
\[ \therefore \omega_n \text{ is constant every where.} \]  

05. **Ans:** (c)  
**Sol:**  
\[ K = 300 \text{N/m} \]  

By energy method  
\[ E = \frac{1}{2} I \theta^2 + \frac{1}{2} Kx^2 = \text{constant} \]  
\[ E = \frac{1}{2} I \theta^2 + \frac{1}{2} K \frac{\ell}{2} \theta^2 = \text{cons tan} \theta \]  
Differentiating w.r.t ‘\( \theta \)’  
\[ \frac{dE}{dt} = 10 + \frac{K}{2} \frac{\ell^2}{4} \times 20 = 0 \]  
\[ I = \frac{m \ell^2}{12} \]  
\[ \frac{m \ell^2}{12} + \frac{K \ell^2}{4} \theta = 0 \]  
\[ \Rightarrow \theta + \frac{3K}{m} \theta = 0 \]  
\[ \Rightarrow \omega_n = \sqrt{\frac{3K}{m}} = 30 \text{ rad/ sec} \]  

06. **Ans:** (a)  
**Sol:**
Assume that in equilibrium position mass M is vertically above ‘A’. Consider the displaced position of the system at any instant as shown above figure.

If \( \Delta_{st} \) is the static extension of the spring in equilibrium position, its total extension in the displaced position is \( (\Delta_{st} + a\theta) \).

From the Newton’s second law, we have

\[
I_0 \ddot{\theta} = Mg(L + b\theta) - k(\Delta_{st} + a\theta)a... (1)
\]

But in the equilibrium position

\[
MgL = k\Delta_{st}a
\]

Substituting the value in equation (1), we have

\[
I_0 \ddot{\theta} = (Mgb - ka^2)\theta
\]

\[
\Rightarrow I_0 \ddot{\theta} + (ka^2 - Mgb)\theta = 0
\]

\[
\omega_n = \sqrt{\frac{ka^2 - Mgb}{I_0}}
\]

\[
\tau = 2\pi \sqrt{\frac{I_0}{ka^2 - Mgb}}
\]

The time period becomes an imaginary quantity if \( ka^2 < Mgb \). This makes the system unstable. Thus the system to vibrate the limitation is

\[
ka^2 > Mgb
\]

\[
b < \frac{ka^2}{Mg}
\]

Where \( W = Mg \)

07. Ans: (a)

08. Sol:

\[
\begin{align*}
KE &= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{4}i\dot{\theta}^2 \\
&= \frac{1}{2}mr^2\dot{\theta}^2 + \frac{1}{4}mr^2\dot{\theta}^2 \\
&= \frac{3}{4}mr^2\dot{\theta}^2 \\
PE &= \frac{1}{2}Kx^2 + \frac{1}{2}Kx^2 = Kx^2 \\
x &= (r + a)\theta \\
\Rightarrow PE &= K\{(r + a)\theta\}^2 \\
\frac{dKE}{dt} + \frac{dPE}{dt} &= 0
\end{align*}
\]

Substituting in the above equation

\[
\frac{3}{2}mr^2\ddot{\theta} + 2K(r + a)^2 \theta = 0
\]

Natural frequency

\[
f_n = \frac{1}{2\pi} \sqrt{\frac{4K(r + a)^2}{3mr^2}}
\]

So \( f_n = 47.74 \text{ Hz.} \)
Taking the moment about the instantaneous centre ‘A’.

\[ I_A \ddot{\theta} + 2K (r+a) \theta (r+a) = 0 \]

\[ I_A = \frac{mr^2}{2} + mr^2 = \frac{3}{2} mr^2 \]

\[ \frac{3}{2} mr^2 \ddot{\theta} + 2k(r+a)^2 \theta = 0 \]

\[ \omega_n = \sqrt{\frac{k_{el}}{m_{el}}} = \sqrt{\frac{2k(r+a)^2}{3 mr^2}} = \sqrt{\frac{4k(r+a)^2}{3mr^2}} \]

09. Ans: (b)

Sol:

By considering the equilibrium about the pivot ‘O’

\[ I_0 \ddot{\theta} + mg \times \frac{L}{6} \sin \theta + \frac{L}{3} \theta \times \frac{L}{3} = 0 \]
13. Ans: 0.0658 N.m²

Sol: For a Cantilever beam stiffness, \( K = \frac{3EI}{\ell^3} \)

Natural frequency, \( \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{3EI}{m\ell^3}} \)

Given \( f_n = 100 \text{ Hz} \)

\[ \Rightarrow \omega_n = 2\pi f_n = 200\pi \]

\[ 200\pi = \sqrt{\frac{3EI}{m\ell^3}} \]

Flexural Rigidity

\[ EI = \left( \frac{200\pi}{3} \right)^2 m\ell^3 = 0.0658 \text{ N.m}^2 \]

14. Ans: (d)

Sol: Free body diagram

By taking the moment about ‘O’, \( \Sigma m_o = 0 \)

\( (m2a\ddot{o} \times 2a) + (ka\dot{o} \times a) = 0 \)

\[ \Rightarrow 4a^2 m\ddot{o} + ka^2 \dot{o} = 0 \]

Where, \( m_{eq} = 4a^2 m, \ k_{eq} = ka^2 \)

Natural frequency, \( \omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} \)

\[ = \sqrt{\frac{ka^2}{4a^2 m}} = \sqrt{\frac{k}{4m}} \text{ rad} \]

[∵ \( \omega_n = 2\pi f \) ]

16. Ans: (a)

Sol: Moment equilibrium above instantaneous centre (contact point)

\[ -k(a+d)\ddot{o}(a+d) = I_c \ddot{o} \]

\[ I_c = \frac{3}{2} Ma^2, \]

\[ \omega_n = \sqrt{\frac{k(a+d)^2}{\frac{3}{2} Ma^2}} \]

\[ \omega_n = \sqrt{\frac{2k(a+d)^2}{3Ma^2}} \]
17. Ans: 10
Sol:  
\[ KE = \frac{1}{2} mx^2 + \frac{1}{2} I\dot{\theta}^2 \]

\[ m = 5 \text{ kg}, \quad \theta = \frac{x}{r} \]

\[ I = \frac{20 \times r^2}{2} = 10r^2 \]

\[ KE = \frac{1}{2} 5x^2 + \frac{1}{2} 10r^2 \cdot \dot{x}^2 = \frac{1}{2} (15)x^2 \]

\[ \therefore m_{eq} = 15 \]

\[ PE = \frac{1}{2} kx^2 \]

\[ \therefore k_{eq} = k = 1500 \text{ N/m} \]

Natural frequency

\[ \omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{1500}{15}} = 10 \text{ rad/sec} \]

18. Ans: (b)
Sol: In damped free vibrations the oscillatory motion becomes non-oscillatory at critical damping. Hence critical damping is the smallest damping at which no oscillation occurs in free vibration.

19. Ans: (a)
Sol: \( \omega_n = 50 \text{ rad/sec} = \frac{5}{\sqrt{m}} \]

If mass increases by 4 times

\[ \omega_{n_1} = \sqrt{\frac{k}{4m}} = \frac{1}{2} \times \sqrt{\frac{k}{m}} = \frac{50}{2} = 25 \text{ rad/sec} \]

Damped frequency natural frequency,

\[ \omega_d = \sqrt{1 - \xi^2} \times \omega_n \]

\[ \Rightarrow 20 = \sqrt{1 - \xi^2} \times 25 = 0.6 = 60\% \]

20. Ans: (a)
Sol: \( K_1, K_2 = 16 \text{ MN/m} \)

\[ K_3, K_4 = 32 \text{ MN/m} \]

\[ K_{eq} = K_1 + K_2 + K_3 + K_4 \]

\[ m = 240 \text{ kg} \]

\[ \omega_n = \sqrt{\frac{K_{eq}}{m}} \]

\[ K_{eq} = \left( (16 \times 2) + (32 \times 2) \right) \times 10^6 = 96 \times 10^6 \text{ N/m} \]

\[ \omega_n = \sqrt{\frac{96 \times 10^6}{240}} = 632.455 \text{ rad/sec} \]

\[ N = \frac{\omega_n \times 60}{2\pi} = 6040 \text{ rpm} \]

21. Ans: (a)
Sol: For slender rod, \( I_o = \left[ \frac{\rho x^3}{3} \right]_{\ell}^{2\ell} \)

\[ = \frac{\rho}{3} \times \left( 8\ell^3 + 2\ell^3 \right) = \frac{9\rho\ell^3}{3} = 3\rho\ell^3 = m\ell^2 \]

Where, \( \rho = m/3\ell \)

Considering the equilibrium at hinge ‘O’.
22. Ans: (b)  
Sol: Damping ratio, \( \zeta = \frac{c}{c_c} = \frac{c_{eq}}{2 \sqrt{k_{eq}m_{eq}}} \)
\[ = \frac{4c^2}{2 \times \sqrt{k_c^2 \times m^2}} = \frac{2c}{\sqrt{mk_c^4}} = \frac{2c}{\sqrt{km}} \]

23. Ans: (a)  
Sol:
\[ I = m(2l)^2 + m^2 = 5m^2 \]
The equation motion is
\[ \left( m \times (2l)^2 + m^2 \right) \ddot{\theta} + \frac{c^2}{4} \dot{\theta} + \frac{k_c^2}{4} \theta + \frac{mg}{\theta} = 0 \]
\[ = 5m^2 \ddot{\theta} + \frac{c^2}{4} \dot{\theta} + \frac{k_c^2}{4} \theta + \frac{mg}{\theta} = 0 \]
\[ \omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{k_c^2 + mg\ell}{5m^2}} \]
\[ = \frac{400}{\sqrt{5 \times 10}} = 3.162 \text{ rad/s} \]

24. Ans: (a)  
Sol:
\[ \zeta = \frac{c_{eq}}{2 \sqrt{k_{eq}m_{eq}}} = \frac{\left( \frac{c^2}{4} \right)}{2 \sqrt{(k_c^2 + mg\ell) \times 5m^2}} \]
\[ = \frac{400 \times 1^2}{4} = \frac{4}{2 \sqrt{(400 \times 1^2 + 10 \times 9.81 \times 1) \times 5 \times 10 \times 1^2}} = 0.316 \]

25. Ans: (a)  
Sol:

By moment equilibrium
\[ I\ddot{\theta} + C_{eq} \theta + K_{eq} \theta + K_{eq} \dot{\theta} = 0 \]
\[ \frac{mL^2}{3} \ddot{\theta} + C_{eq} \theta + (K_{eq}^2 + \frac{K_{eq}}{2}) \theta = 0 \]
\[ \omega_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{K_{eq}^2 + \frac{K_{eq}}{2}}{m_{eq} \times \frac{L^2}{3}}} \]
\[ = \sqrt{\frac{1500}{0.833}} = 42.26 \text{ rad/sec} \]

26. Ans: (c)  
Sol: Refer to the above equilibrium equation
\[ C_{eq} = C_a^2 \]
\[ = 500 \times 0.4^2 = 80 \text{ Nm/ rad/ sec} \]
\[ \Rightarrow C = 80 \text{ Nm/ rad} \]
Note: For angular co-ordinate

Unit of Equivalent inertia = \( \frac{N\cdot m}{rad^2} \) = kg\cdot m^2

Unit of equivalent damping coefficient = \( \frac{N\cdot m}{rad\cdot s} \)

Unit of equivalent stiffness = N\cdot m/rad

27. Ans: (a)
Sol: Given length of cantilever beam,
\( l = 1000 \text{ mm} = 1 \text{ m}, \quad m = 20 \text{ kg} \)

[Diagram of cantilever beam with mass and moment of inertia]

Cross section of beam = square
\( W = mg \)

Moment of inertia of the shaft,
\( I = \frac{1}{12} bd^3 = \frac{25 \times (25)^3}{12} = 3.25 \times 10^{-8} \text{ m}^4 \)

\( E_{\text{steel}} = 200 \times 10^9 \text{ Pa} \)

Mass, \( M = 20 \text{ kg} \)

Stiffness, \( K = \frac{3EI}{l^3} \)

Critical damping coefficient,
\( C_c = 2\sqrt{Km} = 1250 \text{ Ns/m} \)

28. Ans: (c)

29. Ans: (d)
Sol: \( x = 10 \text{ cm at } \frac{\omega}{\omega_n} = 1; \)
\( \xi = 0.1 \)

At resonance \( x = \frac{x_0}{2\xi} = 10 \text{ cm} \)

\( \Rightarrow x_0 = 2 \times 0.1 \times 10 = 2 \text{ cm} \)

\( x_0 = \text{ static deflection} \)

At \( \frac{\omega}{\omega_n} = 0.5, \)

\( x = \frac{x_0}{\sqrt{1 - \left(\frac{\omega}{\omega_n}\right)^2} + 2\xi \frac{\omega}{\omega_n}} \)

\( x = \frac{2}{\sqrt{1 - (0.5)^2} + (2 \times 0.1 \times 0.5)^2} = 2.64 \text{ cm} \)

30. Ans: (a)
Sol: \( m\ddot{x} + Kx = F \cos \omega t \)
\( m = ? \)

\( K = 3000 \text{ N/m}, \)

\( X = 50 \text{ mm} = 0.05 \text{ m} \)

\( F = 100 \text{ N}, \)

\( \omega = 100 \text{ rad/sec} \)

\( X = \frac{F}{K - m\omega^2} \)

\( \Rightarrow m = \frac{K}{\omega^2} - \frac{F}{X\omega^2} = 0.1 \text{ kg} \)
31. Ans: (a)
Sol: \( \delta = \ln \left( \frac{x_1}{x_2} \right) = \ln 2 = 0.693 \)
\[ \zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \]
\[ = \frac{0.693}{\sqrt{4\pi^2 + 0.693^2}} = 0.109 \]
c = \( 2\zeta\sqrt{k_m} = 2 \times 0.109 \times \sqrt{100 \times 1} \)
\[ = 2.19 \text{ N-sec/m} \]

32. Ans: (b)
Sol: \( x_{\text{static}} = 3 \text{mm}, \quad \omega = 20 \text{ rad/sec} \)
As \( \omega > \omega_n \)
So, the phase is 180°.
\[ -x = x_{\text{static}} \times \frac{\omega^2}{\omega_n^2} \]
\[ = \frac{3}{1 - \left( \frac{20}{10} \right)^2} \times \left( 2 \times 0.109 \times \frac{20}{10} \right)^2 \]
\[ = 1 \text{ mm opposite to F.} \]

33. Ans: (c)
Sol: At resonance, magnification factor = \( \frac{1}{2\zeta} \)
\[ \Rightarrow 20 = \frac{1}{2\zeta} \]
\[ \Rightarrow \zeta = \frac{1}{40} = 0.025 \]

34. Ans: (c)
Sol: \( M = 100 \text{ kg}, \quad m = 20 \text{ kg}, \quad e = 0.5 \text{ mm} \)
\( K = 85 \text{ kN/m}, \quad C = 0 \) or \( \zeta = 0 \)
\( \omega = 20\pi \text{ rad/sec} \)
Dynamic amplitude
\[ X = \frac{m\omega^2}{\pm (k - M\omega^2)} = \frac{20 \times 5 \times 10^{-4} \times (20\pi)^2}{\pm (8500 - 100 \times (20\pi)^2)} \]
\[ = 1.27 \times 10^{-4} \text{ m} \]

35. Ans:
Sol:
\( x(t) = X \sin(\omega t - \phi) \)
\( m = 50 \text{ kg} \)
\[ k = \frac{m \omega^2}{\left( \frac{-0.01}{0.2} \right)} \]
\[ = \frac{939.96 \text{ kN/m}}{0.2} \]
\[ \Rightarrow k = 939.96 \text{ kN/m} \]

36. Ans: (b)
Sol: \( m = 5 \text{ kg}, \quad c = 20, \quad k = 80, \quad F = 8, \quad \omega = 4 \)
\( x = \frac{F}{\sqrt{(k - m\omega^2) + (cm)^2}} \)
\[ = \frac{8}{\sqrt{(80 - 5 \times 4^2) + (20 \times 4)^2}} = 0.1 \]
Magnification factor = \[ \frac{x}{x_{\text{static}}} \]

\[ x_{\text{static}} = \frac{F}{k} = \frac{8}{80} = 0.1 \]

Magnification factor = \[ \frac{0.1}{0.1} = 1 \]

37. Ans: (c)

Sol: Given, \( m = 250 \text{ kg} \)
\( K = 100,000 \text{ N/m} \)
\( N = 3600 \text{ rpm} \)
\( \xi = 0.15 \)
\( \omega_n = \sqrt{\frac{K}{m}} = 20 \text{ rad/sec} \)
\( \omega = \frac{2\pi \times N}{60} = 377 \text{ rad/sec} \)

\[ TR = \frac{\omega - \omega_n}{\omega + \omega_n} = 0.0162 \]

38. Ans: 10 N.sec/m

Sol: Given systems represented by
\[ m\ddot{x} + c\dot{x} + kx = F \cos \omega t \]
For which, \( X = \frac{F}{\sqrt{(K - m\omega^2)^2 + (C\omega)^2}} \)

Given, \( K = 6250 \text{ N/m}, m = 10 \text{ kg}, F = 10 \text{ N} \)
\( \omega = 25 \text{ rad/sec}, \quad X = 40 \times 10^{-3} \)
\( \omega_n = \sqrt{\frac{K}{m}} = 25 \text{ rad/sec} \)
\( \omega t = 25t \Rightarrow \omega = 25 \text{ rad/sec} \)

39. Ans: (b)

Sol: Transmissibility (T) reduces with increase in damping up to the frequency ratio of \( \sqrt{2} \).
Beyond \( \sqrt{2} \), T increases with increase in damping

40. Ans: (c).

Sol: Because \( f = 144 \text{ Hz} \) execution frequency.
\( f_{R_n} \) (Natural frequency) is 128.
\( \frac{\omega}{\omega_{R_n}} = \frac{f}{f_{R_n}} = \frac{144}{128} = 1.125 \)

It is close to 1, which ever sample for which \( \frac{\omega}{\omega_n} \) close to 1 will have more response, so sample R will show most perceptible to vibration

41. Ans: (b)

Sol: Given Problem of the type
\[ m\ddot{x} + c\dot{x} + kx = F \cos \omega t \]

or \( X = \frac{F}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right) + \left(2\xi\frac{\omega}{\omega_n}\right)^2}} \)
Given $F = 10, \ \omega_n = 10\omega$

$$k = 150 \text{ N/m} \text{ or } \frac{\omega}{\omega_n} = \frac{1}{10} = 0.1$$

$$\xi = 0.2$$

$$X = \frac{10/150}{\sqrt{(1-0.1)^2 + (2 \times 0.2 \times 0.1)^2}}$$

$$= 0.0669 \approx 0.07$$

42. Ans: 6767.7 N/m

Sol: Given $f = 60 \text{ Hz}, \ m = 1 \text{ kg}$

$$\omega = 2\pi f = 120\pi \text{ rad/sec}$$

Transmissibility ratio, $\text{TR} = 0.05$

Damping is negligible, $C = 0, \ K = ?$

We know $\text{TR} = 0.05$ when $C = 0$

As $\text{TR}$ is less than 1 $\Rightarrow \omega/\omega_n >> \sqrt{2}$

$\text{TR}$ is negative

$$\therefore -0.05 = \frac{K}{K - m\omega^2}$$

Solving we get $K = 6767.7 \text{ N/m}$

43. Ans: (c)

Sol:

$$\begin{align*}
\text{ma} &= \text{mg} \\
T \cos \alpha &= \text{mg} \\
T \sin \alpha &= \text{ma} \\
tan \alpha &= \frac{\text{ma}}{\text{mg}}
\end{align*}$$

44. Ans: (a)

45. Ans: (b)

Sol: $e = 2 \text{ mm} = 2 \times 10^{-3} \text{ m},$

$$\omega_n = 10 \text{ rad/s},$$

$N = 300 \text{ rpm}$

$$\omega = \frac{2\pi N}{60} = 10\pi \text{ rad/sec}$$

$$X = \frac{m \omega^2}{k - m \omega^2} = \frac{\omega^2}{\omega_n^2 - \omega^2}$$

$$e \left( \frac{\omega}{\omega_n} \right)^2 = 2 \times 10^{-3} \times \left( \frac{10\pi}{10} \right)^2$$

$$\pm \left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 \right]$$

$$= 2.25 \times 10^{-3} \text{ m} = 2.25 \text{ mm}$$

46. Ans: (a)

Sol: Number of nodes observed at a frequency of 1800 rpm is 2

$$n = \frac{\text{rpm}}{60} = \frac{1800}{60} = 30$$

$n$-mode number

$$\begin{align*}
\text{Number of nodes} &= 2 \\
\text{Frequency} &= 1800 \text{ rpm}
\end{align*}$$
The whirling frequency of shaft,
\[ f = \frac{\pi}{2} \times n^2 \sqrt{\frac{gEI}{WL^4}} \]

For 1st mode frequency, \( f_1 = \frac{\pi}{2} \times \sqrt{\frac{gEI}{WL^4}} \)

\[ f_n = n^2 f_1 \]

As there are two nodes present in 3rd mode,
\[ f_3 = 3^2 f_1 = 1800 \text{ rpm} \]

\[ \therefore f_1 = \frac{1800}{9} = 200 \text{ rpm} \]

\[ \therefore \text{The first critical speed of the shaft} = 200 \text{ rpm} \]

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47. Ans: (b)

Sol: Critical or whirling speed
\[ \omega_c = \omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{g}{\delta}} \text{ rad/sec} \]

If \( N_c \) is the critical or whirling speed in rpm

then \[ \frac{2\pi N_c}{60} = \sqrt{\frac{9.81 \text{ m/s}^2}{1.8 \times 10^{-3} \text{ m}}} \]

\[ \Rightarrow N_c = 705.32 \text{ rpm} \approx 705 \text{ rpm} \]