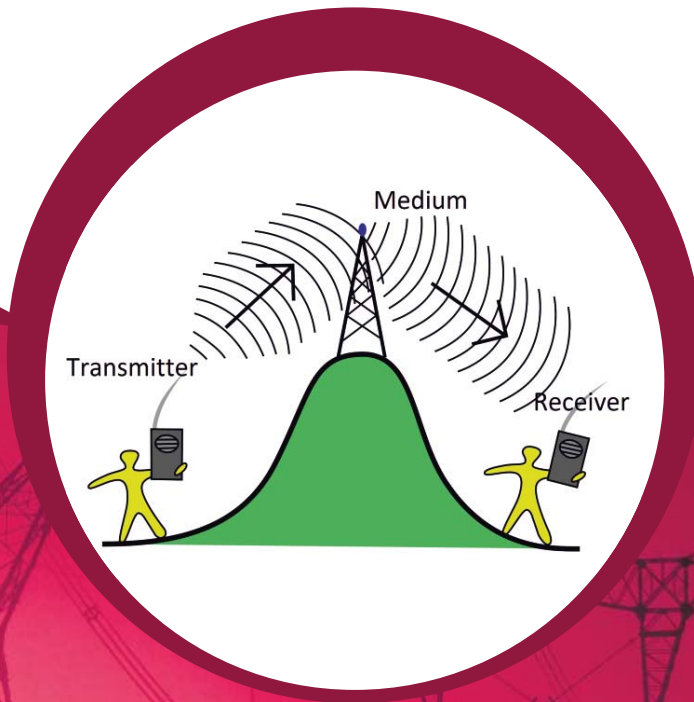




ESE | GATE | PSUs

ELECTRICAL ENGINEERING COMMUNICATION SYSTEMS

Volume - 1 : Study Material with Classroom Practice Questions



Communication Systems

(Solutions for Volume-1 Class Room Practice Questions)

1. Introduction

01. Ans: (b)

Sol: We know that

$$e^{-at}u(t) \xleftrightarrow{\text{F.T.}} \frac{1}{a + j\omega}$$

$$e^{at}u(-t) \xleftrightarrow{\text{F.T.}} \frac{1}{a - j\omega}$$

$$e^{-at}u(t) - e^{at}u(-t) \xleftrightarrow{\text{F.T.}} \frac{1}{a + j\omega} - \frac{1}{a - j\omega}$$

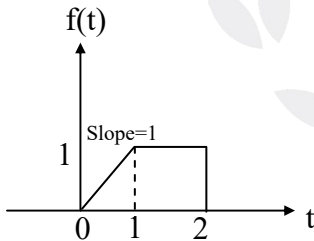
Put $a = 0$

$$u(t) - u(-t) \xleftrightarrow{\text{F.T.}} \frac{1}{j\omega} - \frac{1}{-j\omega}$$

$$\text{sgn}(t) \xleftrightarrow{\text{F.T.}} \frac{2}{j\omega}$$

02. Ans: (a)

Sol:



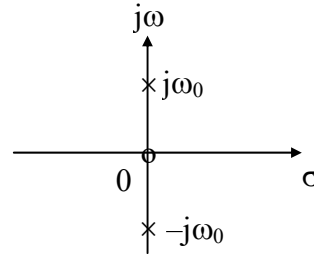
$$f(t) = r(t) - r(t-1) - u(t-2)$$

03. Ans: (a)

Sol: The convergence of Fourier transform is along the $j\omega$ -axis in s-plane.

04. Ans: (a)

Sol:

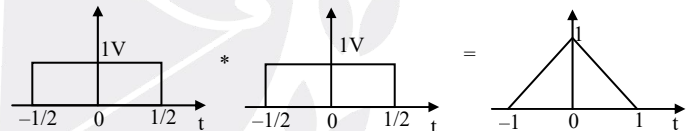


$$F(s) = \frac{s}{s^2 + \omega_0^2} \xleftrightarrow{\text{I.L.T.}} f(t) = \cos \omega_0 t$$

$$f(t) = \cos \omega_0 t \xleftrightarrow{\text{I.L.T.}} F(\omega) = \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

05. Ans: (d)

Sol:



06. Ans: (c)

Sol: Given $x(t) = e^{-at^2}$

Fourier transform of $x(t)$ is

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-at^2} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-(at^2 + j\omega t)} dt$$

$$= e^{-\frac{\omega^2}{4a}} \int_{-\infty}^{\infty} e^{-\left[\sqrt{at} + \frac{j\omega}{2\sqrt{a}}\right]^2} dt$$



$$\text{Let } p = \sqrt{a}t + \frac{j\omega}{2\sqrt{a}}$$

$$dp = \sqrt{a}dt$$

$$X(\omega) = \frac{e^{-\frac{\omega^2}{4a}}}{\sqrt{a}} \int_{-\infty}^{\infty} e^{-p^2} dp$$

$$\int_{-\infty}^{\infty} e^{-p^2} dp = \sqrt{\pi}$$

$$X(\omega) = \frac{e^{-\frac{\omega^2}{4a}}}{\sqrt{a}} \sqrt{\pi}$$

$$X(\omega) = \sqrt{\frac{\pi}{a}} e^{-\frac{\omega^2}{4a}}$$

07. Ans: (d)

Sol: The EFS expression of a periodic signal $x(t)$ is

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

where, ' c_n ' is EFS coefficient.

Apply F.T on both sides

$$X(\omega) = \sum_{n=-\infty}^{\infty} c_n \text{FT}[e^{jn\omega_0 t}]$$

$$1 \leftrightarrow 2\pi\delta(\omega)$$

$$e^{jn\omega_0 t} \leftrightarrow 2\pi\delta(\omega - n\omega_0)$$

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} c_n \delta(\omega - n\omega_0)$$

So, it is a train of impulse.

08. Ans: (a)

Sol: $V(j\omega) = e^{-j2\omega}; |\omega| \leq 1$

$$\text{Energy} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |V(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-1}^1 |e^{-j2\omega}|^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-1}^1 1 d\omega$$

$$= \frac{2}{2\pi}$$

$$= \frac{1}{\pi}$$

09. Ans: (b)

Sol: Parseval's theorem is used to find the energy of the signal in frequency domain.

$$\therefore \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

10. Ans: (a)

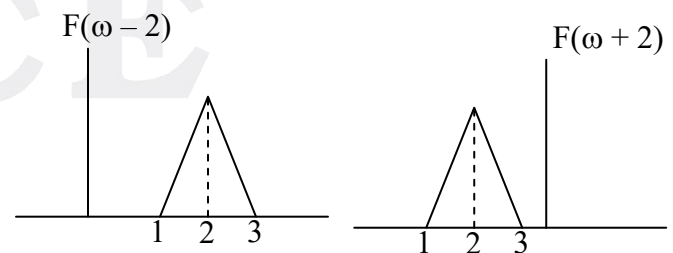
$$\text{Sol: } f(t) = A.e^{-a|t|} \xleftrightarrow{\text{F.T}} F(j\omega) = \frac{2Aa}{a^2 + \omega^2}$$

11. Ans: (d)

$$\text{Sol: } m(t) = f(t) \cos 2t$$

Apply Fourier transform

$$M(f) = \frac{1}{2} [F(\omega - 2) + F(\omega + 2)]$$



12. Ans: (b)

Sol: For band limited signals,

$$S(f) \neq 0; |f| < W$$

$$S(f) = 0; |f| > W$$



13. Ans: (a)

Sol: In a communication system, antenna is used to convert voltage variations to field variation and vice-versa.

14. Ans: (d)

Sol: Hilbert transform of $f(t)$ is

$$\text{H.T}\{f(t)\} = f(t) * \frac{1}{\pi t}$$

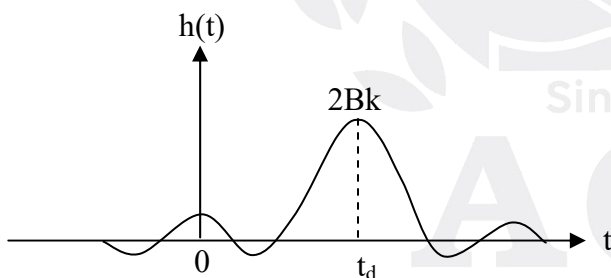
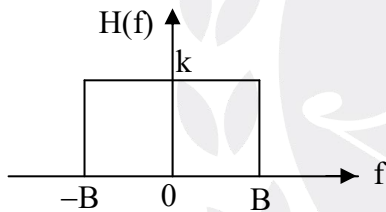
It is in the terms of 't'.

15. Ans: (a)

Sol: For an ideal LPF

$$H(f) = k e^{-j\omega t_0} \text{ for } -B < f < B$$

$$\begin{aligned} h(t) &= F^{-1}[H(f)] \\ &= 2Bk \text{ sinc } 2B(t-t_d) \end{aligned}$$



$$h(t) \neq 0 \text{ for } t < 0$$

Output exists before input is applied i.e. non-causal, which is physically impossible.

16. Ans: (b)

$$\text{Sol: } \delta(at) = \frac{1}{|a|} \delta(t)$$

$$\delta(2t) = \frac{1}{2} \delta(t)$$

17. Ans: (a)

Sol: By modulation we are translating the low frequency spectrum into high frequency spectrum.

18. Ans: (a)

Sol: We know that

$$P(\text{dBm}) = 10 \log(P \times 10^3)$$

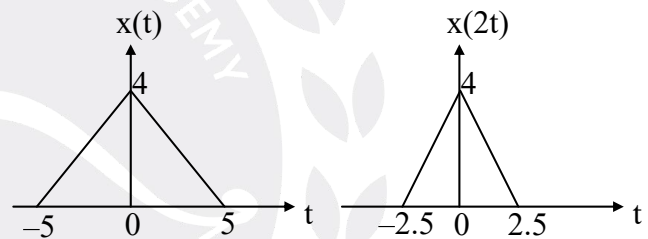
$$-10 = 10 \log(P \times 10^3)$$

$$P \times 10^3 = 10^{-1}$$

$$P = 10^{-4} = 100 \mu\text{W}$$

19. Ans: (a)

Sol: $x(2t)$ means signal time axis is compressed by 2



20. Ans: (b)

Sol: Audio frequency is between 20Hz to 20kHz

21. Ans: (d)

Sol: Telephone channel carries voice. Voice frequency is between 300 Hz to 3500 Hz. So bandwidth is 3200Hz. So we approximately consider 4kHz is the bandwidth requirement of a telephone channel.

22. Ans: (c)

Sol: From the signal spectrum $f_H = 530 \text{ kHz}$, $f_L = 50 \text{ kHz}$

$$\begin{aligned} \text{Bandwidth} &= f_H - f_L \\ &= 530 \text{ kHz} - 50 \text{ kHz} \\ &= 480 \text{ kHz} \end{aligned}$$



2. Amplitude Modulation

01. Ans: (a)

Sol: $V(t) = A_c \cos \omega_c t + 2 \cos \omega_m t \cdot \cos \omega_c t$

Comparing this with the AM-DSB-SC signal

$A \cos \omega_c t + m(t) \cos \omega_c t$, it implies that

$$m(t) = 2 \cos \omega_m t \Rightarrow E_m = 2$$

To implement Envelope detection,

$$A_c \geq E_m$$

$$\therefore (A_c)_{\min} = 2$$

02. Ans: (d)

Sol: $m(t) = (A_c + A_m \cos \omega_m t) \cos \omega_c t$

$$= A_c \left(1 + \frac{A_m}{A_c} \cos \omega_m t \right) \cos \omega_c t$$

Given

$$A_c = 2A_m$$

$$= A_c \left(1 + \frac{1}{2} \cos \omega_m t \right) \cos \omega_c t$$

$$P_T = \frac{A_c^2}{2} \left[1 + \frac{\mu^2}{2} \right], P_s = \frac{A_c^2}{2} \left[\frac{\mu^2}{4} \right]$$

$$\frac{P_T}{P_s} = \frac{1 + \frac{\mu^2}{2}}{\frac{\mu^2}{4}} = \frac{1 + \frac{1}{8}}{\frac{1}{16}} = \frac{9}{8} \times 16$$

$$P_T = 18 P_s$$

03. Ans: (a)

Sol: $m(t) = 2 \cos 2\pi f_1 t + \cos 2\pi f_2 t$

$$C(t) = A_c \cos 2\pi f_c t$$

$$S(t) = [A_c + m(t)] \cos 2\pi f_c t$$

$$S(t) = A_c \left[1 + \frac{1}{A_c} m(t) \right] \cos 2\pi f_c t$$

$$K_a = \frac{1}{A_c}$$

$$A_{m1} = 2, A_{m2} = 1$$

$$\mu_1 = K_a A_{m1} = \frac{2}{A_c}, \mu_2 = K_a A_{m2} = \frac{1}{A_c}$$

$$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

$$\Rightarrow 0.5 = \sqrt{\frac{4}{A_c^2} + \frac{1}{A_c^2}}$$

$$\Rightarrow A_c = \sqrt{20}$$

04. Ans: (c)

Sol: $m(t) = -0.2 + 0.6 \sin \omega_1 t, k_a = 1, A_c = 100$

$$S(t) = A_c [1 - 0.2 + 0.6 \sin \omega_1 t] \cos \omega_c t$$

$$= 100 [0.8 + 0.6 \sin \omega_1 t] \cos \omega_c t$$

$$V_{\max} = A_c [1 + \mu] = 100 [0.8 + 0.6] = 140 \text{ V}$$

$$V_{\min} = A_c [1 - \mu] = 100 [0.8 - 0.6] = 20 \text{ V}$$

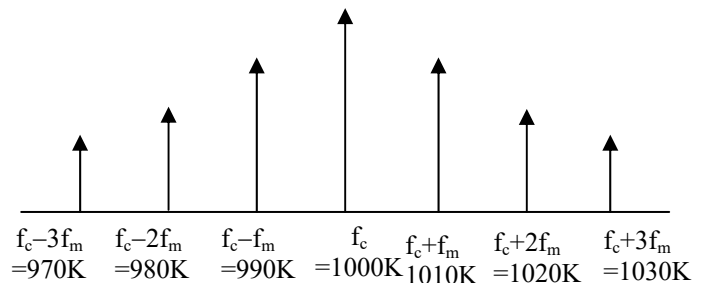
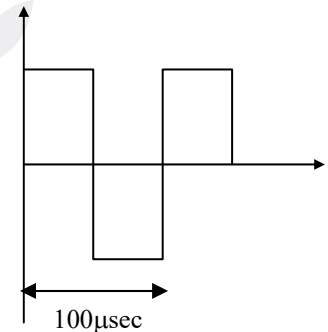
$$= 20 \text{ V to } 140 \text{ V}$$

05. Ans: (c)

Sol: $f_c = 1 \text{ MHz} = 1000 \text{ kHz}$

The given $m(t)$ is symmetrical square wave of period $T = 100 \mu\text{sec}$

$$f_m = \frac{1}{T_0} = 10 \text{ kHz}$$

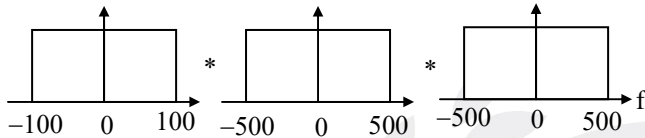




These frequencies 980k, 1020k are not present because the symmetrical square wave it consists of half wave symmetries only odd harmonics are present, even harmonics are dismissed

06. Ans: (d)

Sol: $m(t) = \text{sinc}(200t)\text{sinc}^2(1000t)$
 $= \text{sinc}(200t)\text{sinc}(1000t)\text{sinc}(1000t)$



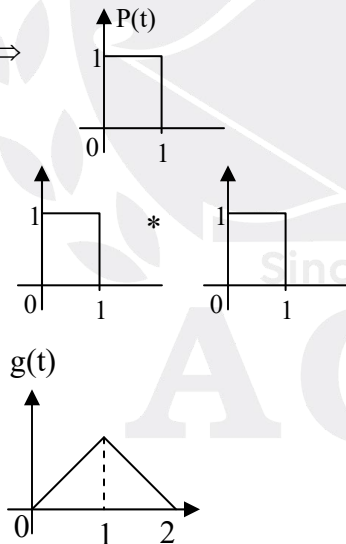
So, highest frequency component in the signal $m(t)$ is $100 + 500 + 500 = 1100$

$BW = 2 \times 1100$

$BW = 2200 \text{ Hz}$

07. Ans: (a)

Sol: $P(t) = u(t) - u(t-1) \Rightarrow$



$g(t) = P(t) * P(t) =$

$x(t) = 100(P(t) + 0.5g(t))\cos\omega_c t$
 $= 100(1 + 0.5t)\cos\omega_c t$
 $= A_c(1 + K_a m(t))\cos\omega_c t$

$k_a = 0.5, m(t) = t$

$\mu = k_a[m(t)]_{\max}$

$\mu = 0.5 \times 1 = 0.5$

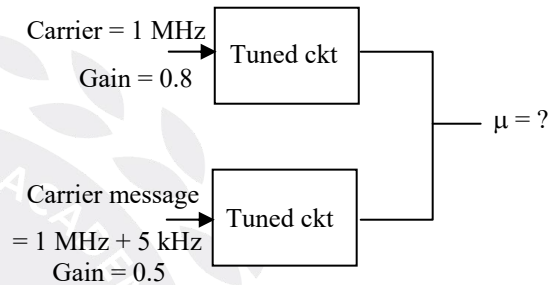
08. Ans: (d)

Sol: $R_L C \leq \frac{\sqrt{1-\mu^2}}{2\pi f_m \mu}$

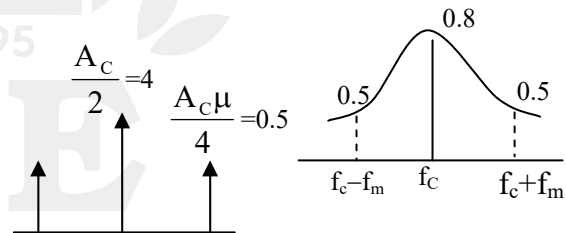
So it depends on depth of modulation and the highest modulation frequency.

09. Ans: (b)

Sol: $S(t) = 10\cos 2\pi 10^6 t + 8\cos 2\pi 5 \times 10^3 t \cos 2\pi 10^6 t$



$S(t) = 0.8 \times 10\cos 2\pi 10^6 t$
 $+ 0.5 \times 8\cos 2\pi 5000 t \cos 2\pi 10^6 t$
 $= 8\left(1 + \frac{4}{8}\cos 2\pi 5000 t\right)\cos 2\pi 10^6 t$
 $\mu = \frac{4}{8} = \frac{1}{2} = 0.5$



10. Ans: (d)

Sol: $A_{\max} = 10V$

$A_{\min} = 5V$

$\mu = 0.1$

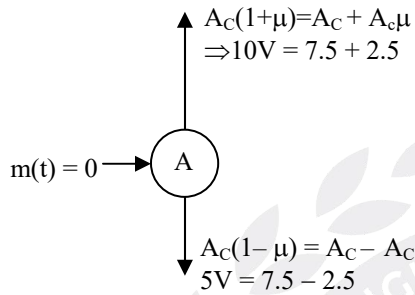
$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$



$$= \frac{1}{3} = 0.33$$

$$A_C = \frac{A_{\max} + A_{\min}}{2}$$

$$= \frac{10 + 5}{2} = 7.5 \text{ V}$$



$$\text{Amplitude deviation } A_C\mu = 7.5 \times \frac{1}{3} = 2.5 \text{ V}$$

$$\mu_2 = 0.1 \Rightarrow A_{C2}\mu_2 = 2.5$$

$$A_{C2} = 25 \text{ V}$$

Which must be added to attain = 17.5

11. Ans: (d)

Sol: Modulation index

$$\mu = k_a |m(t)|_{\max}$$

$$k_a = \frac{2b}{a} = \frac{2(\text{square term coefficient})}{\text{linear term coefficient}}$$

$$|m(t)|_{\max} = 1$$

$$\mu = 2\left(\frac{b}{a}\right)$$

$$P_{SB} = \frac{1}{2} P_C \Rightarrow P_C \frac{\mu^2}{2} = \frac{1}{2} P_C$$

$$\mu^2 = 1 \Rightarrow \left(2\frac{b}{a}\right)^2 = 1$$

$$\Rightarrow 2\frac{b}{a} = 1 \Rightarrow \frac{a}{b} = 2$$

12. Ans: 0.125

$$\text{Sol: } s(t) = \cos(2000\pi t) + 4\cos(2400\pi t) + \cos(2000\pi t)$$

Here $4\cos(2400\pi t)$ is the carrier signal.

$\cos(2000\pi t)$ and $\cos(2000\pi t)$ are the sideband message signals.

$$P_c = \frac{4^2}{2} = 8 \text{ W}$$

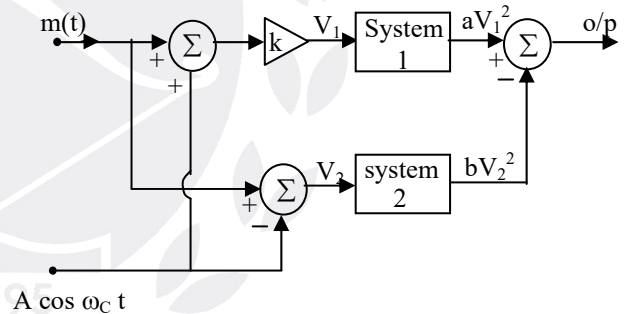
$$P_m = \frac{1}{2} + \frac{1}{2} = 1 \text{ W}$$

$$\frac{P_m}{P_c} = \frac{1}{8} = 0.125$$

3. Sideband Modulation Techniques

01. Ans: (c)

Sol:



$$V_1 = k [m(t) + c(t)]$$

$$V_2 = [m(t) - c(t)]$$

$$V_0 = aV_1^2 - bV_2^2$$

$$= ak^2[m(t) + c(t)]^2 - b[m(t) - c(t)]^2$$

$$= ak^2 [m^2(t) + c^2(t) + 2m(t)c(t)]$$

$$- b[m^2(t) + c^2(t) - 2m(t)c(t)]$$

$$= [ak^2 - b]m^2(t) + [ak^2 - b]c^2(t)$$

$$+ 2[ak^2 + b][m(t)c(t)]$$

$$\text{on verification if } k = \sqrt{\frac{b}{a}}$$

$$S(t) = 4bm(t)c(t) \rightarrow \text{DSBSC Signal}$$



02. Ans: (d)

Sol: Given $A = 10$

$$m(t) = \cos 1000\pi t$$

$$b = 1$$

B.W = ? and power = ?

$$\begin{aligned} s(t) &= 4b.A \cos 2\pi f_c t \cdot \cos 2\pi (500)t \\ &= 40 \cdot \cos 2\pi f_c t \cdot \cos 2\pi (500)t \end{aligned}$$

$$B.W = 2 f_m$$

$$= 2 (500) = 1 \text{ kHz}$$

$$\text{Power} = \frac{A_c^2 A_m^2}{4}$$

$$= \frac{1600 \times 1}{4} = 400 \text{ W}$$

03. Ans: (c)

Sol: Carrier = $\cos 2\pi (100 \times 10^6)t$

Modulating signal = $\cos(2\pi \times 10^6)t$

Output of Balanced modulator

$$= 0.5[\cos 2\pi (101 \times 10^6)t + \cos 2\pi (99 \times 10^6)t]$$

The Output of HPF is $0.5 \cos 2\pi (101 \times 10^6)t$

Output of the adder is

$$= 0.5 \cos 2\pi (101 \times 10^6)t + \sin 2\pi (100 \times 10^6)t$$

$$= 0.5 \cos 2\pi [(100+1)10^6 t] + \sin 2\pi (100 \times 10^6)t$$

$$= 0.5[\cos 2\pi (100 \times 10^6)t \cdot \cos 2\pi (10^6)t$$

$$- \sin 2\pi (100 \times 10^6)t \cdot \sin 2\pi (10^6)t]$$

$$+ \sin 2\pi (100 \times 10^6)t]$$

$$= 0.5 \cos 2\pi (100 \times 10^6)t \cdot \cos 2\pi (10^6)t$$

$$+ \sin 2\pi (100 \times 10^6)t [1 - 0.5 \sin 2\pi (10^6)t]$$

Let $0.5 \cos 2\pi (10^6)t = r(t) \cos \theta(t)$

$$1 - 0.5 \sin 2\pi (10^6)t = r(t) \cdot \sin \theta(t)$$

The envelope is

$$r(t) = [0.25 \cos^2 2\pi (10^6)t$$

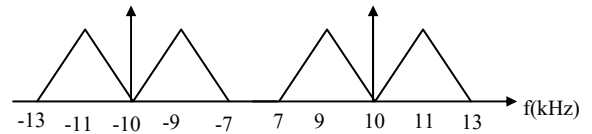
$$+ \{1 - 0.5 \sin 2\pi (10^6)t\}^2]^{1/2}$$

$$= [1.25 - \sin 2\pi (10^6)t]^{1/2}$$

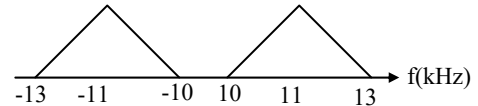
$$= \left[\frac{5}{4} - \sin 2\pi (10^6)t \right]^{1/2}$$

04. Ans: (b)

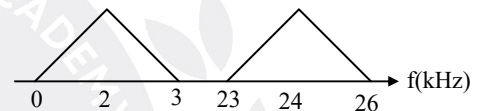
Sol: Output of 1st balanced modulator is



Output of HPF is



The Output of 2nd balanced modulator is consisting of the following +ve frequencies.



Thus, the spectral peaks occur at 2 kHz and 24 kHz

05. Ans: (c)

Sol: Given

$$f_{m1} = 100 \text{ Hz}, f_{m2} = 200 \text{ Hz}, f_{m3} = 400 \text{ Hz},$$

$$f_c = 100 \text{ kHz}, f_{cLo} = 100.02 \text{ kHz}$$

$$\begin{aligned} S(t)/T_x &= \frac{A_c A_m}{2} [\cos(f_c + f_{m1})t + \\ &\quad \cos(f_c + f_{m2})t + \cos(f_c + f_{m3})t] \end{aligned}$$

$$S(t)/R_x = [S(t)/T_x] A_c \cos 2\pi f_{cLo} t$$

$$\begin{aligned} \Rightarrow \frac{A_c^2 A_m}{4} &[\cos(f_c + f_{cLo} + f_{m1}) + \cos(f_{m1} - 20) + \\ &\quad \cos(f_c + f_{cLo} + f_{m2}) + \cos(f_{m2} - 20) + \\ &\quad \cos(f_c + f_{cLo} + f_{m3}) + \cos(f_{m3} - 20)] \end{aligned}$$

Detector output frequencies:

$$80 \text{ Hz}, 180 \text{ Hz}, 380 \text{ Hz}$$



06. Ans: (b)

Sol: Given

SSB AM is used, LSB is transmitted

$$f_{LO} = (f_c + 10)$$

$$S(t)/T_X = \frac{A_c A_m}{2} \cos 2\pi[f_c - f_m]t$$

$$S(t)/R_X = \left[\frac{A_c A_m}{2} \cos 2\pi(f_c - f_m)t \right] \cos 2\pi(f_c + 10)t$$

$$\Rightarrow \frac{A_c A_m}{4} [\cos 2\pi(2f_c + 10 - f_m)t + \cos 2\pi(10 + f_m)t]$$

i.e., from 310 Hz to 1010 Hz

07. Ans: (b)

Sol: BW of Basic group = $12 \times 4 = 48$ kHz

BW of super group = $5 \times 48 = 240$ kHz

08. Ans: (d)

Sol: Given 11 voice signals

B.W. of each signals = 3 kHz

Guard Band Width = 1 kHz

Lowest $f_c = 300$ kHz

Highest $f_c =$

$$\Rightarrow f_{c_H} + f_{m_{lost}} = 300 \text{ kHz} + 11(3 \text{ kHz}) + 10(1 \text{ kHz})$$

$$= 343 \text{ kHz}$$

$$f_{c_H} = 343 \text{ kHz} - 3 \text{ kHz}$$

$$= 340 \text{ kHz}$$

09. Ans: (b)

Sol: $f_{m1} = 5$ kHz $\rightarrow$ AM

$f_{m2} = 10$ kHz $\rightarrow$ DSB

$f_{m3} = 10$ kHz $\rightarrow$ SSB

$f_{m4} = 2$ kHz $\rightarrow$ SSB

$f_{m5} = 5$ kHz $\rightarrow$ AM

$f_g = 1$ kHz

$$BW = (2f_{m1} + 2f_{m2} + f_{m3} + f_{m4} + 2f_{m5} + 4f_g)$$

$$= 2 \times 5 + 2 \times 10 + 10 + 2 + 2 \times 5 + 4 \times 1$$

$$= 10 + 20 + 10 + 10 + 6$$

$$= 56 \text{ kHz}$$

$$\therefore BW = 56 \text{ kHz}$$

4. Angle Modulation

01. Ans: (a)

Sol: $s(t) = 10 \cos(20\pi t + \pi t^2)$

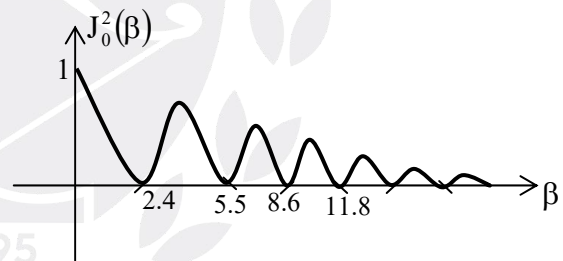
$$f_i = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$f_i = \frac{1}{2\pi} [20\pi + 2\pi t]$$

$$\frac{df_i}{dt} = \frac{1}{2\pi} \times 2\pi \times 1 = 1 \text{ Hz/sec}$$

02. Ans: (d)

$$\text{Sol: } P_{fc} = \frac{A_c^2 J_0^2(\beta)}{2}$$



So, $J_0^2(\beta)$ is decreasing first, becoming zero and then increasing so power is also behave like $J_0^2(\beta)$.

03. Ans: (d)

$$\text{Sol: } S(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + nf_m)t$$

$$\Delta f = 3(2f_m) = 12 \text{ kHz}$$

$$\beta = \frac{\Delta f}{f_m} = 6$$



$$\therefore S(t) = \sum_{n=-\infty}^{\infty} 5.J_n(6) \cos 2\pi(f_c + nf_m)t$$

$$f_c = 1000 \text{ kHz}, f_m = 2 \text{ kHz}$$

$$= \cos 2\pi(1000 \times 10^3)t$$

$$= \cos 2\pi(1000 + 4 \times 2) \times 10^3 t$$

$$\text{i.e., } n = 4$$

The required coefficient is $5.J_4(6)$

04. Ans: (c)

Sol: $2\pi f_m = 4\pi \times 10^3$

$$\Rightarrow f_m = 2 \text{ kHz}$$

$$J_0(\beta) = 0 \text{ at } \beta = 2.4$$

$$\beta = \frac{k_f A_m}{f_m} \Rightarrow 2.4 = \frac{k_f \times 2}{2 \text{ kHz}}$$

$$k_f = 2.4 \text{ kHz/V}$$

$$\text{at } \beta = 5.5$$

$$5.5 = \frac{2.4 \text{ kHz} \times 2}{f_m}$$

$$\Rightarrow f_m = 872.72 \text{ Hz}$$

05. Ans: (d)

Sol: In frequency modulation the spectrum contains

$$f_c \pm nf_1 \pm mf_2, \text{ where } n \text{ \& } m = 0, 1, 2, 3, \dots$$

06. Ans: (c)

Sol: Given $f_c = 1 \text{ MHz}$

$$f_{\max} = f_c + k_f A_m$$

$$k_p = 2\pi k_f$$

$$k_f = \frac{k_p}{2\pi} = \frac{\pi}{2\pi}$$

$$= \frac{1}{2}$$

$$= \left(10^6 + \frac{1}{2} \times 10^5\right) = (10^6 + 0.5 \times 10^5)$$

$$= (10^6 + 5 \times 10^4)$$

$$= (10^3 + 50) \times 10^3$$

$$= (10^3 + 50) \text{ kHz}$$

$$= 1050 \text{ kHz.}$$

$$f_{\min} = f_c - k_f A_m$$

$$= \left(10^6 - \frac{1}{2} \times 10^5\right)$$

$$= (10^6 - 0.5 \times 10^5)$$

$$= (10^6 - 5 \times 10^4)$$

$$= (10^3 - 50) \times 10^3$$

$$= (10^3 - 50) \text{ kHz}$$

$$= 950 \text{ kHz}$$

07. Ans: (d)

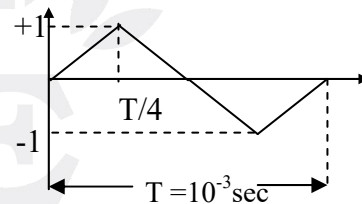
Sol: $\beta = \frac{\Delta f}{f_m}$

$$\Delta \phi = \frac{\Delta f}{f_m}$$

$$\Delta f = \Delta \phi f_m = k_p A_m f_m$$

08. Ans: (c)

Sol: Given



$$f_c = 100 \times 10^3 \text{ Hz}$$

$$k_f = 10 \times 10^3 \text{ Hz}$$

$$m(t)|_{\max} = +1, m(t)|_{\min} = -1$$

$$f_i = f_c \pm \Delta f$$

$$= f_c \pm k_f A_m$$

$$= 100 \times 10^3 \pm 10 \times 10^3 (m(t))$$

$$= 110 \text{ kHz \& } 90 \text{ kHz}$$



09. Ans: (c)

Sol: $S(t) = A_c \cos (2\pi f_c t + k_p m(t))$

$$f_i = \frac{1}{2\pi} \frac{d}{dt} \theta_i(t)$$

$$= \frac{1}{2\pi} \frac{d}{dt} (2\pi f_c t + k_p m(t))$$

$$= f_c + \frac{1}{2\pi} k_p \frac{d}{dt} m(t)$$

$$f_{\max} = f_c + \frac{k_p}{2\pi} \left(\frac{10^{-3}}{4} \right)$$

$$= f_c + \frac{k_p}{2\pi} \times 4 \times 10^3$$

$$= 100 \text{ kHz} + \frac{\pi}{2\pi} \times 4 \times 10^3$$

$$= 102 \text{ kHz}$$

$$f_{\min} = f_c - k_p \left(\frac{10^{-3}}{4} \right)$$

$$= f_c - 2 \text{ kHz}$$

$$f_{\min} = 98 \text{ kHz}$$

10. Ans: (c)

Sol: Given,

$$S(t) = A_c \cos (\theta_i(t))$$

$$= A_c \cos (\omega_c t + \phi(t))$$

$$m(t) = \cos (\omega_m t)$$

$$f_i(t) = f_c + 2\pi k(f_m)^2 \cos \omega_m t$$

$$f_i = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$\theta_i(t) = \int 2\pi f_i(t) dt$$

$$\theta_i(t) = \int 2\pi [f_c + 2\pi k(f_m)^2 \cos \omega_m t] dt$$

$$\theta_i(t) = 2\pi f_c t + (2\pi f_m)^2 k \frac{\cos \omega_m t}{\omega_m t}$$

$$\theta_i(t) = \omega_c t + \omega_m k \sin \omega_m t$$

11. Ans: (b)

Sol: $\Delta f_{\max} = K_f |m(t)|_{\max}$

$$= \frac{100}{2\pi} \times [10]$$

$$\Delta f_{\max} = \left(\frac{500}{\pi} \right) \text{ Hz}$$

12. Ans: (a)

Sol: $\beta_p = k_p \max [|m(t)|] = 1.5 \times 2 = 3$

$$\beta_f = \frac{k_f \max [|m(t)|]}{f_m}$$

$$= \frac{3000 \times 2}{1000} = 6$$

13. Ans: (a)

Sol: Using Carson's rule we obtain

$$\begin{aligned} BW_{PM} &= 2 (\beta_p + 1) f_m \\ &= 8 \times 1000 = 8000 \text{ Hz} \end{aligned}$$

$$\begin{aligned} BW_{FM} &= 2 (\beta_f + 1) f_m \\ &= 14 \times 1000 = 14000 \text{ Hz} \end{aligned}$$

5. Radio Receivers

01. Ans: (d)

Sol: The image channel selectivity of super heterodyne receiver depends upon Pre selector and RF amplifier only.

02. Ans: (b)

Sol: The image (second) channel selectivity of a super heterodyne communication receiver is determined by the pre selector and RF amplifier.

03. Ans: (d)

Sol: Given $f_s = 4$ to 10 MHz

$$IF = 1.8 \text{ MHz}$$



$$f_{si} = ?$$

$$f_{si} = f_s + 2 \times IF$$

$$= 7.6 \text{ MHz to } 13.6 \text{ MHz}$$

04. Ans: (a)

Sol: Image frequency $f_{si} = f_s + 2 \times IF$

$$= 700 \times 10^3 + 2(450 \times 10^3)$$

$$= 1600 \text{ kHz}$$

Local oscillator frequency, $f_l = f_s + IF$

$$(f_l)_{\max} = (f_s)_{\max} + IF = 1650 + 450$$

$$= 2100 \text{ kHz}$$

$$(f_l)_{\min} = (f_s)_{\min} + IF = 550 + 450$$

$$= 1000 \text{ kHz}$$

$$R = \frac{C_{\max}}{C_{\min}} = \left(\frac{f_{l\max}}{f_{l\min}} \right)^2$$

$$= \left(\frac{2100}{1000} \right)^2 = 4.41$$

05. Ans: (a)

Sol: $f_s(\text{range}) = 88 - 108 \text{ MHz}$

Given condition $f_{IF} < f_{LO}, f_{si} > 108 \text{ MHz}$

$$f_{si} = f_s + 2 \times IF$$

$$f_{si} > 108 \text{ MHz}$$

$$f_s + 2IF > 108 \text{ MHz}$$

$$88 \text{ MHz} + 2 \times IF > 108 \text{ MHz}$$

$$IF > 10 \text{ MHz}$$

Among the given options $IF = 10.7 \text{ MHz}$

06. Ans: (a)

Sol: Range of variation in local oscillator frequency is

$$f_{L\min} = f_{s\min} + IF$$

$$= 88 + 10.7$$

$$f_{L\min} = 98.7 \text{ MHz}$$

$$f_{L\max} = f_{s\max} + IF$$

$$= 108 + 10.7$$

$$f_{L\max} = 118.7 \text{ MHz}$$

07. Ans: 5

Sol: $f_s = 58 \text{ MHz} - 68 \text{ MHz}$

When $f_s = 58 \text{ MHz}$

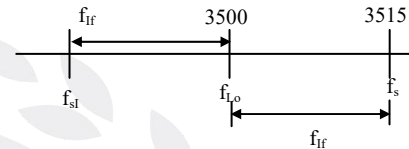
$$f_{si} = f_s + 2IF > 68 \text{ MHz}$$

$$2IF > 10 \text{ MHz}$$

$$IF \geq 5 \text{ MHz}$$

08. Ans: 3485 MHz

Sol:



$$f_{if} = 15 \text{ MHz}$$

$$f_{Lo} = 3500 \text{ MHz}$$

$$f_s - f_{Lo} = f_{IF}$$

$$f_s = f_{Lo} + f_{IF} = 3515 \text{ MHz}$$

$$f_{si} = \text{image frequency} = f_s - 2 f_{IF}$$

$$= 3515 - 2 \times 15$$

$$= 3485 \text{ MHz}$$

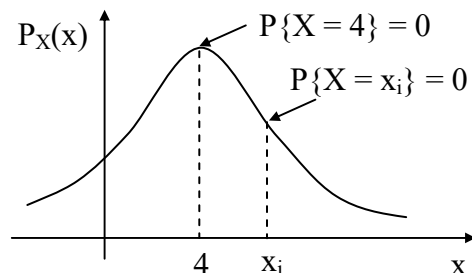
6. Random Variables and Noise

01. Ans: (c)

Sol: A continuous Random variable X takes every value in a certain range, the probability that $X = x$, is zero for every x in that range.

Given $P_X(x) = \frac{1}{3\sqrt{2\pi}} e^{-\frac{(x-4)^2}{18}}$ is a continuous

Random variable therefore probability of the event $\{X = 4\}$ is zero.





02. Ans: (b)

Sol: Given,

X & Y are two Random Variables

$$Y = \cos \pi x$$

$$f(x) = 1 \quad -\frac{1}{2} < x < \frac{1}{2}$$

$$= 0 \quad \text{else where}$$

$$f(y) = ?$$

$$f(y) = f(x) \left| \frac{dx}{dy} \right|$$

$$x = \frac{1}{\pi} \cos^{-1}(y)$$

$$dx = \frac{1}{\pi} \times \frac{-1}{\sqrt{1-y^2}} dy$$

$$\Rightarrow \frac{dx}{dy} = \frac{-1}{\pi \sqrt{1-y^2}}$$

$$f(y) = \frac{1}{\pi \sqrt{1-y^2}}$$

$$\sigma_y^2 = E[y^2] - [E[y]]^2$$

03. Ans: (d)

Sol: The probability density function of the envelope of a sinusoidal plus narrow band noise is Rician.

$$f_R(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right)$$

04. Ans: (a)

Sol: Given,

Differential equation of a system is

$$\frac{dy(t)}{dt} + y(t) = \frac{dx(t)}{dt} - x(t)$$

Applying Fourier transform,

$$\Rightarrow Y(f)(1 + jf) = X(f)(jf - 1)$$

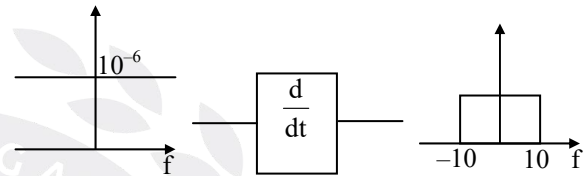
$$\frac{Y(f)}{X(f)} = \frac{-1 + jf}{1 + jf}$$

The transform function of system is a All pass filter

$$\therefore S_y(f) = S_x(f)$$

05. Ans: (a)

Sol:



$$S_{YY}(f) = |H(f)|^2 S_{XX}(f)$$

$$H(f) = j2\pi f$$

$$|H(f)|^2 = 4\pi^2 f^2$$

$$S_{YY}(f) = 4\pi^2 f^2 S_{XX}(f)$$

The Noise power at the output of the LPF is

$$N_o = \int_{-10}^{10} S_{YY}(f) df$$

$$N_o = \int_{-10}^{10} 4\pi^2 f^2 \times 10^{-6} df$$

$$= 2 \times 4\pi^2 \times 10^{-6} \int_0^{10} f^2 df$$

$$= 2 \times 4\pi^2 \times 10^{-6} \times \frac{10^3}{3}$$

$$\therefore N_o = 0.0263 \text{ W}$$

06. Ans: (a)

Sol: Given,

$$\text{PSD of Noise} = \frac{\eta_0}{2}$$

$$T = 27^\circ \text{C} = 300\text{K}$$



$$P_n = K.T.B$$

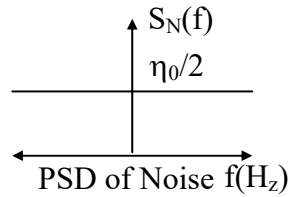
$$\eta_0 = KT$$

$$= 1.38 \times 10^{-23} \times 300$$

$$\text{PSD} = \frac{\eta_0}{2}$$

$$= 1.38 \times 10^{-23} \times 150$$

$$= \frac{207}{10^{23}}$$



07. Ans: (b)

Sol: $P_n = K.T.B$

$$= \left(\frac{1}{2} \times 1.38 \times 10^{-23} \times 300 \right) \times 2 \times 10^6 \times 2$$

$$= 8.28 \times 10^{-15} \text{ W}$$

08. Ans: (b)

$$\text{Sol: } E(X) = \int_{-1}^3 x \cdot p(x) dx = \frac{1}{4} \left[\frac{x^2}{2} \right]_{-1}^3 = 1$$

$$E(X^2) = \int_{-1}^3 x^2 p(x) dx = \frac{1}{4} \left[\frac{x^3}{3} \right]_{-1}^3 = 7/3$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{7}{3} - 1 = \frac{4}{3}$$

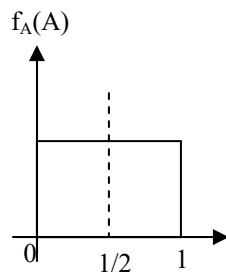
09. Ans: (d)

Sol: $R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)]$

$$= E[A \cos \omega t_1 A \cos \omega t_2]$$

$$= \cos \omega t_1 \cos \omega t_2 E[A^2] \quad [\because E[A^2] = 1/3]$$

$$= \frac{1}{3} \cos \omega t_1 \cos \omega t_2$$



$$\sigma^2 = \frac{(1)^2}{12} \rightarrow \text{variance}$$

$$E[A^2] = \sigma^2 + [E[A]]^2$$

$$= \frac{1}{12} + \frac{1}{4}$$

$$E[A^2] = \frac{4}{12} = \frac{1}{3}$$

10. Ans: (b)

Sol: $R_{XY}(t_1, t_2) = E[X(t_1)Y(t_2)]$

$$\text{Let } t_2 - t_1 = \tau$$

$$E[(A \cos \omega t_1 + B \sin \omega t_1)(B \cos \omega t_2 - A \sin \omega t_2)]$$

$$\because E[AB] = E[A] E[B]$$

$$E[AB] = 0$$

$$E[BA] = 0$$

$$E[A^2] = \sigma^2$$

$$E[B^2] = \sigma^2$$

$$= \cos \omega t_1 \cos \omega t_2 E[AB] - \sin \omega t_1 \sin \omega t_2 E[BA]$$

$$- E[A^2] \cos \omega t_1 \sin \omega t_2 + E[B^2] \sin \omega t_1 \cos \omega t_2$$

$$= 0 - 0 - \sigma^2 \cos \omega t_1 \sin \omega t_2 + \sigma^2 \sin \omega t_1 \cos \omega t_2$$

$$= -\sigma^2 (\cos \omega t_1 \sin \omega t_2 - \sin \omega t_1 \cos \omega t_2)$$

$$= -\sigma^2 \sin \omega (t_2 - t_1) \quad (\because \tau = (t_2 - t_1))$$

$$= -\sigma^2 \sin \omega \tau$$

11. Ans: (b)

Sol: $X(t) =$ positive frequencies required

$$E[X^2(t)] \text{ and } E[X(t)]$$

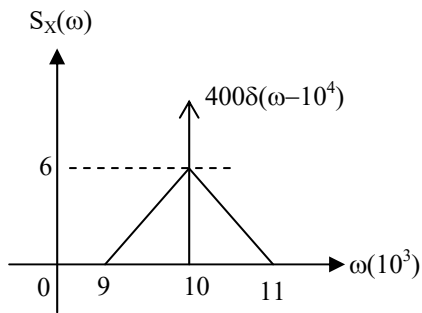
$$E[X^2(t)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega) d\omega$$

$$= \frac{1}{\pi} \left(400 + \frac{1}{2} (2000) \times 6 \right)$$

$$= \frac{6400}{\pi}$$

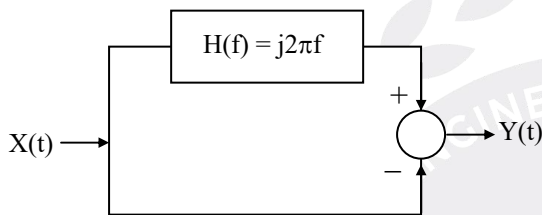
$$E[X(t)] = 0$$

$[\because \text{The given function is periodic function}]$



12. Ans: (a)

Sol:



Overall $H(f) = j2\pi f - 1$

$$R_X(\tau) = e^{-\pi\tau^2}$$

$$Y(t) = X(t) * h(t)$$

$$|H(f)|^2 = (4\pi^2 f^2 + 1)$$

$$R_{XX}(T) \xleftrightarrow{FT} S_{XX}(f)$$

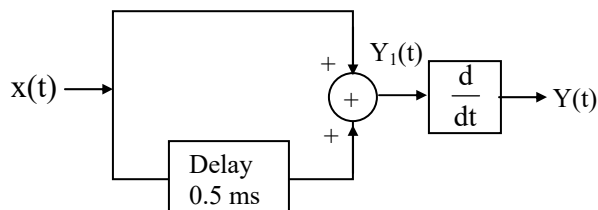
$$e^{-\pi\tau^2} \xleftrightarrow{FT} e^{-\pi f^2}$$

Normalized Gaussian function

$$S_{YY}(f) = |H(f)|^2 S_{XX}(f) = (4\pi^2 f^2 + 1) e^{-\pi f^2}$$

13. Ans: (d)

Sol:



$$Y(t) = \frac{d}{dt} (X(t) + X(t - t_d))$$

$$Y(f) = j2\pi f (1 + e^{-j2\pi f t_d}) X(f)$$

$$H(f) = \frac{Y(f)}{X(f)}$$

$$= j2\pi f (1 + e^{-j2\pi f t_d})$$

$$|H(f)|^2 = 4\cos^2 \pi f t_d$$

$$S_{YY}(f) = |H(f)|^2 S_{XX}(f) \\ = 4\pi^2 f^2 (2\cos(\pi f t_d))^2 S_{XX}(f)$$

$$\text{At } S_{YY}(f) = 0$$

$$\pi f t_d = (2n+1) \frac{1}{2t_d}$$

$$f = (2n+1) \frac{1}{2 \times 0.5 \times 10^{-3}}$$

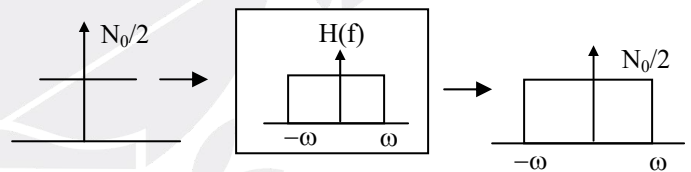
$$f = (2n+1) 10^3$$

$$f = (2n+1) f_0$$

$$f_0 = 1 \text{ kHz}$$

14. Ans: (b)

Sol:



$$\text{Uncorrelated} \Rightarrow \text{cov}(\tau) \Rightarrow R_{XX}(\tau) - \mu^2 \times (\tau)$$

$$\text{cov}(\tau) = R_{XX}(\tau) \Rightarrow R_{n_0}(\tau) = 0$$

$$\Rightarrow N\omega_0 \sin(2\omega\tau) = 0, \sin Cx = 0; x \text{ is an integer}$$

$$2\omega\tau = m$$

$$\tau = \frac{m}{2\omega}, \text{ integer } m = 1, 2, 3, \dots$$

15. Ans: (b)

Sol: We know that,

$$\text{ACF} \xleftrightarrow{F.T} S_x(f)$$



Taking Inverse Fourier Transform

$$F^{-1}[S_y(t)] = \int_{-\infty}^{\infty} S_y(t) e^{j2\pi f\tau} df$$

$$R_y(\tau) = \int_{-B_0}^{B_0} \frac{N_0}{2} e^{j2\pi f\tau} df = \frac{N_0}{2} \left[\frac{e^{j2\pi f\tau}}{j2\pi\tau} \right]_{-B_0}^{B_0}$$

$$= \frac{N_0}{2\pi\tau} \left[\frac{e^{j2\pi B_0\tau} - e^{-j2\pi B_0\tau}}{2j} \right]$$

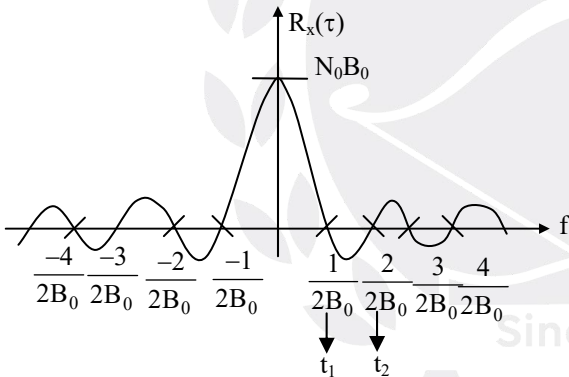
$$= \frac{N_0}{2\pi\tau} \sin(2\pi B_0\tau)$$

$$= N_0 B_0 \frac{\sin(2\pi B_0\tau)}{2\pi B_0\tau}$$

$$R_y(\tau) = N_0 B_0 \text{sinc}(2B_0\tau)$$

16. Ans: (b)

Sol:



$$|t_1 - t_2| = \text{multiple of } \frac{1}{2B}$$

17.

Sol: Since

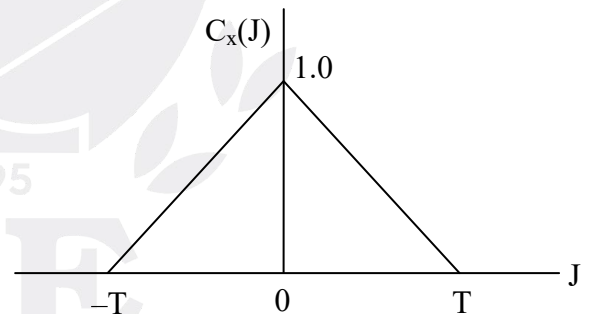
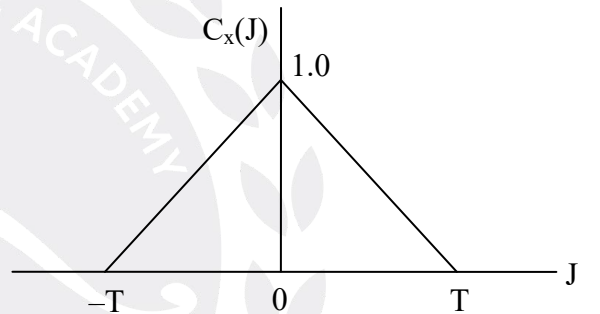
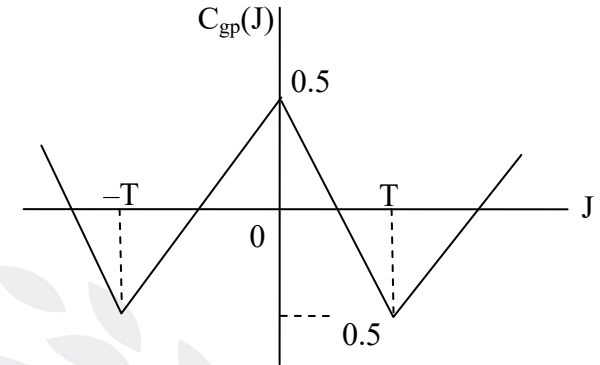
$$y(t) = g_p(t) + X(t) + \sqrt{3}/2$$

and $g_p(t)$ and $X(t)$ are uncorrelated, then

$$C_Y(\tau) = C_{g_p}(\tau) + C_X(\tau).$$

Where $C_{g_p}(\tau)$ is the auto covariance of the periodic component and $C_X(\tau)$ is the auto covariance of the random component $C_Y(\tau)$ is

the plot figure shifted down by $3/2$, removing the DC component $C_{g_p}(\tau)$ and $C_X(\tau)$ are plotted below



Both $g_p(t)$ and $X(t)$ have zero mean,

Average

(a) The power of the periodic component $g_p(t)$ is therefore,

$$\frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g_p^2(t) dt = C_{g_p}(0) = \frac{1}{2}$$

(b) The average power of the random component $x(t)$ is

$$E[X^2(t)] = C_X(0) = 1$$



18.

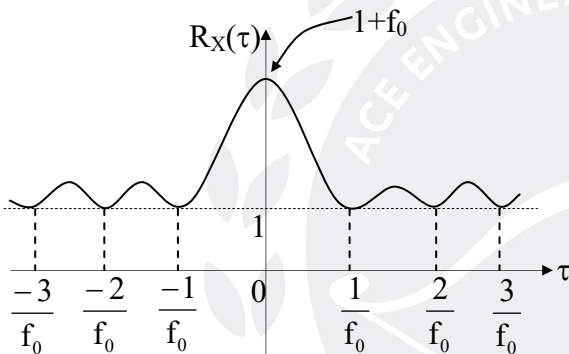
Sol: (a) The power spectral density consists of two components:

1. A delta function $\delta(t)$ and the origin, whose inverse Fourier transform is one.
2. A triangular component of unit amplitude and width $2f_0$, centered at the origin; the inverse Fourier transform of this component is $f_0 \text{sinc}^2(f_0\tau)$

Therefore, the autocorrelation function of $X(t)$ is

$$R_X(\tau) = 1 + f_0 \text{sinc}^2(f_0\tau)$$

Which is sketched below:



(b) Since $R_X(\tau)$ contains a constant component of amplitude 1. It follows that the dc power contained in $X(t)$ is 1.

(c) The mean-square value of $X(t)$ is given by

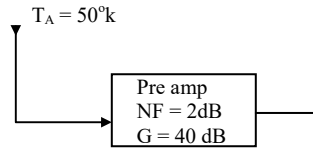
$$\begin{aligned} E[X^2(t)] &= R_X(0) \\ &= 1 + f_0 \end{aligned}$$

The ac power contained in $X(f)$ is therefore equal to f_0 .

(d) If the sampling rate is f_0/n , where n is an integer, the samples are uncorrelated. They are not, however, statistically independent. They would be statistically independent if $X(t)$ were a Gaussian process.

19. **Ans: (a)**

Sol:



$$10 \log_{10} NF = 2 \text{ dB}$$

$$\log_{10} NF = 0.2$$

$$NF = 10^{0.2}$$

$$\text{Noise temperature} = (F - 1) T_0$$

$$= (10^{0.2} - 1) 290$$

$$= 169.36 \text{ K}$$

$$\text{Noise power i/p} = k T_e B$$

$$= 1.38 \times 10^{-23} \times (169.36 + 50) \times 12 \times 10^6$$

$$\text{Noise power at o/p} = (3.632 \times 10^{-14}) \times 10^4$$

$$= 3.73 \times 10^{-10} \text{ watts}$$

20. **Ans: 100 W**

$$\begin{aligned} \text{Sol: } E[x^2(t)] &= E[(3V(t) - 8)^2] \\ &= E[(9V(t)^2 + 64 - 2 \times 3V(t) \times 8)] \\ &= E[(9V^2(t) + 64 - 48V(t))] \\ &= 9E[V^2(t)] + E[64] - 48E[V(t)] \end{aligned}$$

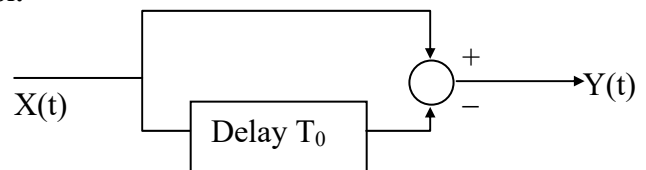
$$[E[V(t)] = 0, E[V^2(t)] = MS = R(0) = 4e^{-5(0)} = 4,$$

$$E[\text{constant}] = \text{constant}]$$

$$E[x^2(t)] = 9 \times 4 + 64 = 36 + 64 = 100$$

21. **Ans: (b)**

Sol:



$$Y(t) = X(t) - X(t - T_0)$$

$$\text{ACf of o/p} = R_y(\tau) = E[y(t) Y(t + \tau)]$$

$$\begin{aligned} R_y(\tau) &= E[(X(t) - X(t - T_0)) [X(t + \tau) \\ &\quad - X(t + \tau - T_0)]] \end{aligned}$$



$$R_y(\tau) = E [(X(t) X(t+\tau) - X(t)X(t+\tau-T_o) - X(t-T_o) X(t+\tau) + X(t-T_o) X(t+\tau-T_o))]$$

$$R_y(\tau) = [R_x(\tau) - R_x(\tau-T_o) - R_x(\tau+T_o) + R_x(\tau)]$$

$$R_y(\tau) = 2 R_x(\tau) - R_x(\tau-T_o) - R_x(\tau+T_o)$$

7. Noise in Analog Communication

01. Ans: (d)

Sol: Output of the multiplier

$$= m(t) \cdot \cos \omega_o t \cos(\omega_o t + \theta)$$

$$= \frac{m(t)}{2} [\cos(2\omega_o t + \theta) + \cos \theta]$$

$$\text{Output of LPF } V_o(t) = \frac{m(t)}{2} \cos \theta$$

$$= \frac{1}{2} \cos \theta m(t)$$

$$\text{Power of o/p signal} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{<T>} v_o^2(t) dt$$

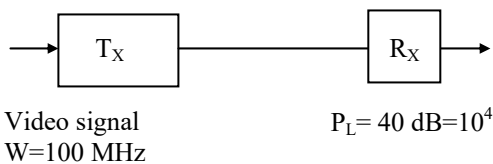
$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{<T>} \left(\frac{1}{2} \cos \theta m(t) \right)^2 dt$$

$$= \frac{1}{4} \cos^2 \theta \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_{<T>} m^2(t) dt \right]$$

$$= \frac{1}{4} \cos^2 \theta P_m$$

02. Ans: (a)

Sol:



$$n_i = n_0$$

$$n_i = n_0 \times W = 10^{-20} \times 100 \times 10^6$$

$$S_i = \frac{P_t}{P_L} = \frac{1 \text{ mw}}{10^4} = 1 \times 10^{-7}$$

$$n_i = 10^{-20} \times 100 \times 10^6$$

$$\frac{S_i}{n_i} = \frac{10^{-7}}{10^{-12}} = 10^5 = 50 \text{ dB}$$

$$\frac{S_i}{n_i} = 50 \text{ dB}$$

03. Ans: (b)

Sol: $\Delta f = 75 \text{ kHz}$

$$f_m = 15 \text{ kHz}$$

$$\left(\frac{S}{N} \right)_0 = 40 \text{ dB} = 10^4$$

$$\text{FOM} = \frac{3}{2} \beta^2; \beta = \frac{\Delta f}{f_m}$$

$$\left(\frac{S_0}{N_0} \right) = \frac{3}{2} \beta^2$$

$$\left(\frac{S}{N} \right)_i = \left(\frac{S}{N} \right)_0 \times \frac{2}{3} \times \frac{1}{\beta^2}$$

$$\left(\frac{S}{N} \right)_{i(\text{dB})} = 24 \text{ dB}$$

04. Ans: (c)

$$\text{Sol: } \left(\frac{S}{N} \right)_i = 10 \text{ dB}; \text{ FOM} = \frac{1}{3}$$

$$\left(\frac{S}{N} \right)_0 = \frac{1}{3} \times 10 = 3.33$$

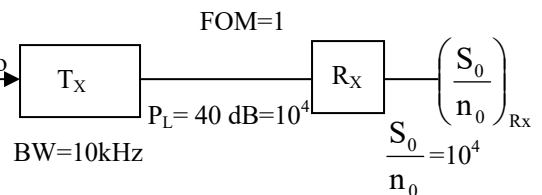
05. Ans: (a)

Sol:

DSB

$P_t = ?$

Audio





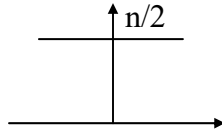
06. Ans: (a)

Sol: For SSB modulation

$$\Rightarrow \frac{S_0}{N_0} = \frac{S_i}{N_i} = 10^4$$

(Only SSB modulation in one sided $n/2$)

$P_t = ?$



$$\frac{S_i}{n_i} = \frac{S_0}{n_0} = 10^4$$

$$S_i = 10^4 \times 10 \times 10^3 \times 2 \times 10^{-9} \text{ w/Hz}$$

$$S_i = 20 \times 10^{-2}$$

$$(S_i)_{dB} = (P_t)_{dB} - (P_L)_{dB}$$

$$(P_t)_{dB} = (S_i)_{dB} + (P_L)_{dB}$$

$$P_t = S_i P_L = 20 \times 10^{-2} \times 10^4$$

$$P_L = 2 \text{ kW}$$

07. Ans: (c)

Sol: For AM

$$\text{FOM} = \frac{1}{3} \text{ (if } \mu = 1)$$

$$\frac{S_0}{N_0} = \left(\frac{1}{3}\right) \frac{S_i}{N_i} \Rightarrow S_i = 3 \left(\frac{S_0}{N_0}\right) \times N_i$$

$$= 3 \times 10^4 \times 2 \times 10^{-9} \times 10 \text{ kHz} = 0.6$$

$$\therefore P_t = S_i \times P_L = 0.6 \times 10^4 = 6 \text{ KW}$$

08. Ans: (b)

Sol: Noise figure = $\frac{(SNR)_{I/P}}{(SNR)_{O/P}}$

$$Nf_{dB} = SNR_{i,dB} - SNR_{o/p,dB}$$

$$SNR_{o/p,dB} = SNR_{I/P,dB} - Nf_{dB}$$

$$= 37 - 3$$

$$= 34 \text{ dB}$$

8. Baseband Data Transmission

01. Ans: (d)

Sol: $\Delta = \frac{V_{\max} - V_{\min}}{2^n}$

$$\Delta \propto \frac{1}{2^n} ; \frac{\Delta_1}{\Delta_2} = \frac{2^{n_2}}{2^{n_1}}$$

$$\frac{0.1}{\Delta_2} = \frac{2^{n+3}}{2^n}$$

$$\Delta_2 = 0.1 \times \frac{1}{8} = 0.0125$$

02. Ans: (3)

Sol: $(BW)_{PCM} = \frac{n f_s}{2}$

Where 'n' is the number of bits to encode the signal and $L = 2^n$, where 'L' is the number of quantization levels.

$$L_1 = 4 \Rightarrow n_1 = 2$$

$$L_2 = 64 \Rightarrow n_2 = 6$$

$$\frac{(BW)_2}{(BW)_1} = \frac{n_2}{n_1} = \frac{6}{2} = 3$$

$$(BW)_2 = 3 (BW)_1$$

03. Ans: (c)

Sol: Given,

Two signals are sampled with $f_s = 44100$ s/sec and each sample contains '16' bits

Due to additional bits there is a 100% overhead.

Out put bit rate = ?

$$R_b = n^l f_s^l$$

$$f_s^l = 2 f_s = 2 [44100]$$

($\because$ two signals sampled simultaneously)

$$n^l = 2n$$



($\therefore$ due to overhead by additional bits)

$$R_b = 4 (nf_s) = 2.822 \text{ Mbps}$$

04. Ans: (c)

Sol: Number of bits recorded over an hour

$$= R_b \times 3600 = 10.16 \text{ G.bits}$$

05. Ans: (c)

Sol: $p(t) = \frac{\sin(4\pi W t)}{4\pi W t (1 - 16W^2 t^2)}$

At $t = \frac{1}{4W}$; $P\left(\frac{1}{4W}\right) = \frac{0}{0}$

Use L-Hospital Rule

$$\begin{aligned} \lim_{t \rightarrow \frac{1}{4W}} p(t) &= \lim_{t \rightarrow \frac{1}{4W}} \frac{4\pi W \cos(4\pi W t)}{4\pi W - 64\pi W^3 (3t^2)} \\ &= \frac{4\pi W (-1)}{4\pi W - 64\pi W^3 3\left(\frac{1}{16W^2}\right)} \\ &= \frac{-4\pi W}{-8\pi W} = 0.5 \end{aligned}$$

06. Ans: 35

Sol: Given bit rate $R_b = 56 \text{ kbps}$, Roll off factor $\alpha = 0.25$

BW required for base band binary PAM system

$$BW = \frac{R_b}{2} [1 + \alpha] = \frac{56}{2} [1 + 0.25] \text{ kHz} = 35 \text{ kHz}$$

07. Ans: 16

Sol: $R_b = nf_s = 8 \text{ bit/sample} \times 8 \text{ kHz} = 64 \text{ kbps}$

$$\begin{aligned} (B_T)_{\min} &= \frac{R_b}{2 \log_2 M} \\ &= \frac{R_b}{2 \log_2 4} = \frac{R_b}{2 \times 2} \\ &= \frac{R_b}{4} = \frac{64}{4} = 16 \text{ kHz} \end{aligned}$$

08. Ans: (b)

Sol: Given $f_s = 1/T_s = 2 \text{ k symbols/sec}$

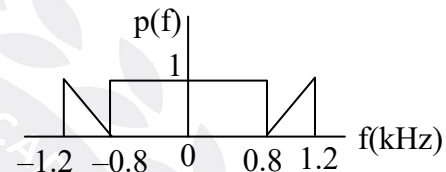
If $P(f) \xleftrightarrow{\text{F.T}} p(t)$,

Condition for zero ISI is given by

$$\frac{1}{T_s} \sum_{n=-\infty}^{\infty} P(f - n/T_s) = p(0)$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f - n/T_s) = p(0)T_s$$

$p(0) = \text{area under } P(f)$



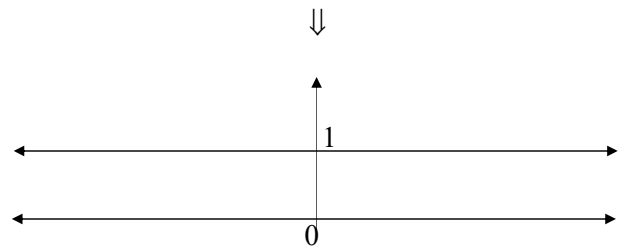
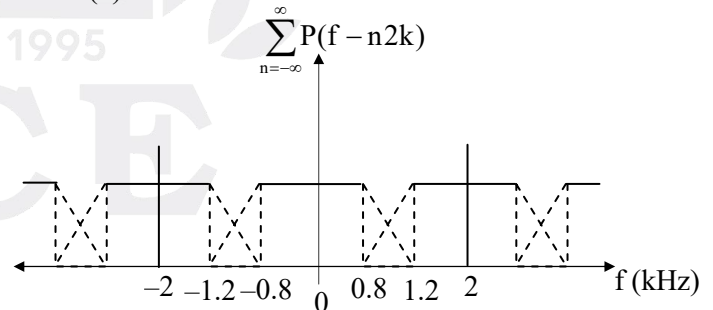
$$\text{Area} = 2 \times \frac{1}{2} (1)(0.4) \text{ k} + 2 \times 0.8 \text{ k} = 2 \text{ k}$$

$$p(0) T_s = 2 \text{ k} \times \frac{1}{2 \text{ k}} = 1$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} P(f - n/T_s) = 1$$

The above condition is satisfied by only option

(b)





$$\therefore \sum_{n=-\infty}^{\infty} P(f - n2k) = 1$$

Option (a) is correct if pulse duration is from -1 to $+1$

Option (c) is correct if the transition is from 0.8 to 1.2 , -0.8 to -1.2

Option (d) is correct if the triangular duration is from -2 to $+2$

09. Ans: 200

Sol: $m(t) = \sin 100\pi t + \cos 100\pi t$

$$= \sqrt{2} \cos [100\pi t + \phi]$$

$$\Delta = 0.75 = \frac{V_{\max} - V_{\min}}{L}$$

$$= \frac{\sqrt{2} - (-\sqrt{2})}{L} = \frac{2\sqrt{2}}{L}$$

$$L = \frac{2\sqrt{2}}{0.75} \approx 4 = 2^n$$

So $n = 2$

$f = 50 \text{ Hz}$ so Nyquist rate = 100

So, the bit rate = $100 \times 2 = 200 \text{ bps}$

10. Ans: (b)

Sol: Given

$$f_{m_1} = 3.6 \text{ kHz} \Rightarrow f_{s_1} = 7.2 \text{ kHz}$$

$$f_{m_2} = f_{m_3} = 1.2 \text{ kHz}$$

$$\Rightarrow f_{s_2} = f_{s_3} = 2.4 \text{ kHz}$$

$$f_s = f_{s_1} + f_{s_2} + f_{s_3}$$

$$= 12 \text{ kHz}$$

No. of Levels used = 1024

$\Rightarrow n = 10 \text{ bits}$

$$\therefore \text{Bit rate} = nf_s$$

$$= 10 \times 12 \text{ kHz}$$

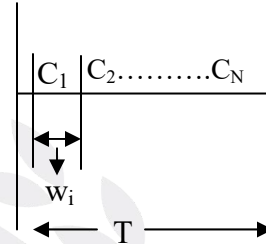
$$= 120 \text{ kbps}$$

11. Ans: (a)

$$\begin{aligned} \text{Sol: } (f_s)_{\min} &= (f_{s_1})_{\min} + (f_{s_2})_{\min} + (f_{s_3})_{\min} + (f_{s_4})_{\min} \\ &= 200 + 200 + 400 + 800 \\ &= 1600 \text{ Hz} \end{aligned}$$

12. Ans: (c)

Sol:



Minimum B.W of TDM is $\sum_{i=1}^N w_i$

13. Ans: (b)

Sol: Number of patients = 10

ECG signal B.W = 100 Hz

$$(Q_e)_{\max} \leq (0.25) \% V_{\max}$$

$$\frac{2V_{\max}}{2 \times 2^n} \leq \frac{0.25}{100} V_{\max}$$

$$2^n \geq 400$$

$$n \geq 8.64$$

$$n = 9$$

$$\begin{aligned} \text{Bit rate of transmitted data} &= 10 \times 9 \times 200 \\ &= 18 \text{ kbps} \end{aligned}$$

14. Ans: (a)

Sol: Peak amplitude $\rightarrow A_m$

Peak to peak amplitude A_m

$$\frac{-\Delta}{2} \leq Q_e \leq \frac{\Delta}{2}$$

$$\text{PCM maximum tolerable } \frac{\Delta}{2} = 0.2\% A_m$$

$$\Delta = \frac{\text{Peak to peak}}{L}$$



$$\Rightarrow \frac{2A/m}{2L} = \frac{0.2}{100} A_m \quad (\because \Delta = \frac{2A_m}{L})$$

$$\Rightarrow L = 500$$

$$2^n = 500 \Rightarrow n = 9$$

$$R_b = n(f_s)_{TDM} + 9$$

$$f_s = R_N + 20\%R_N$$

$$= R_N + 0.2R_N$$

$$f_s = 1.2R_N = 1.2 \times 2 \times \omega$$

$$f_s = 2.4 \text{ K samples/sec}$$

$$(f_s)_{TDM} = 5(f_s) = 5 \times 2.4 \text{ K} = 12 \text{ K sample/sec}$$

$$R_b = (nf_s) + 0.5\%(nf_s)$$

$$= (9 \times 12k) + \frac{0.5}{100} (9 \times 12k) = 108540 \text{ bps}$$

15. Ans: (b)

Sol: To avoid slope over loading, rate of rise of the o/p of the Integrator and rate of rise of the Base band signal should be the same.

$\therefore \Delta f_s = \text{slope of base band signal}$

$$\Delta \times 32 \times 10^3 = 125$$

$$\Delta = 2^{-8} \text{ Volts.}$$

16. Ans: (b)

Sol: $x(t) = E_m \sin 2\pi f_m(t)$

$$\frac{\Delta}{T_s} < \left| \frac{dm(t)}{dt} \right| \rightarrow \text{slope overload distortion takes place}$$

$$\Delta f_s < E_m 2\pi f_m$$

$$\Rightarrow \frac{\Delta f_s}{2\pi} < E_m f_m \quad (\because \Delta = 0.628)$$

$$\Rightarrow \frac{0.628 \times 40K}{2\pi} < E_m f_m$$

$$f_s = 40 \text{ kHz} \Rightarrow 4 \text{ kHz} < E_m f_m$$

Check for options

$$(a) E_m \times f_m = 0.3 \times 8 \text{ K} = 2.4 \text{ kHz}$$

$$(4K \not< 2.4 \text{ K})$$

$$(b) E_m \times f_m = 1.5 \times 4K = 6 \text{ kHz}$$

$$(4K < 6 \text{ K}) \text{ correct}$$

$$(c) E_m \times f_m = 1.5 \times 2 \text{ K} = 3 \text{ kHz}$$

$$(4K \not< 3K)$$

$$(d) E_m \times f_m = 30 \times 1 \text{ K} = 3 \text{ kHz}$$

$$(4K \not< 3K)$$

17. Ans: (a)

Sol: Given

$$m(t) = 6 \sin (2\pi \times 10^3 t) + 4 \sin (4\pi \times 10^3 t)$$

$$\Delta = 0.314 \text{ V}$$

$$\text{Maximum slope of } m(t) = \frac{d}{dt}(m(t)) / t = \frac{\pi}{2}$$

$$= 2\pi \times 10^3 (6) + 4\pi \times 10^3 [4] = 28\pi \times 10^3$$

18. Ans: (c)

Sol: Pulse rate which avoid distortion

$$\Delta f_s = \frac{d}{dt} m(t)$$

$$f_s = \frac{28\pi \times 10^5}{0.314}$$

$$f_s = 280 \times 10^3 \text{ pulses/sec}$$

9. Band pass Data Transmission

01. Ans: (c)

$$\text{Sol: } (BW)_{BPSK} = 2f_b = 20 \text{ kHz}$$

$$(BW)_{QPSK} = f_b = 10 \text{ kHz}$$

02. Ans: (b)

$$\text{Sol: } f_H = 25 \text{ kHz ; } f_L = 10 \text{ kHz}$$

$\therefore$ Center frequency

$$= \left(\frac{25+10}{2} \right) \text{ kHz} = 17.5 \text{ kHz}$$

$\therefore$ Frequency offset,

$$\Omega = 2\pi (25 - 17.5) \times 10^3$$

$$= 2\pi (7.5) \times 10^3$$

$$= 15 \times 10^3 \pi \text{ rad/sec.}$$



The two possible FSK signals are orthogonal, if

$$2\Omega T = n\pi$$

$$\Rightarrow 2(15\pi) \times 10^3 \times T = n\pi$$

$$\Rightarrow 30 \times 10^3 \times T = n \text{ (integer)}$$

This is satisfied for, $T = 200\mu\text{sec}$.

03. Ans: (a)

Sol: $r_b = 8 \text{ kbps}$

Coherent detection

$$\Delta f = \frac{nr_b}{2}$$

Best possible $n = 1$

$$\Delta f = \frac{8K}{2} = 4K$$

To verify the options $\Delta f = 4k$

i.e. $f_{C2} - f_{C1} = 4K$

(a) $20 K - 16 K = 4 K$

(b) $32 K - 20 K = 12 K$

(c) $40 K - 20 K = 20 K$

(d) $40 K - 32 K = 8 K$

04. Ans: (a) & (c)

Sol: Non coherent detection of PSK is not possible.

So to overcome that, DPSK is implemented. A coherent carrier is not required to be generated at the receiver.

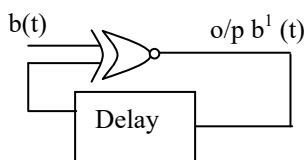
05. Ans: (c)

Sol: In QPSK baud rate = $\frac{\text{bit rate}}{2}$

$$= \frac{34}{2} = 17 \text{ Mbps}$$

06. Ans: (d)

Sol:



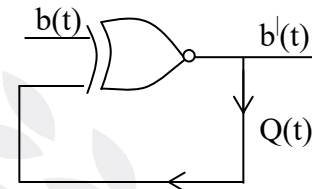
$b(t)$	0	1	0	0	1
$b^1(t)_{(\text{Ref.bit})}$	0	0	1	0	0
Phase	π	π	0	π	π

07. Ans: (b)

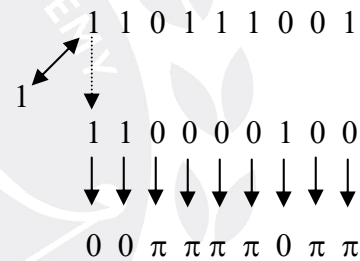
Sol: Given

Bit stream 110 111001

Reference bit = 1



$$b^1(t) = b(t) \odot Q(t)$$



08. Ans: (d)

Sol: $r_b = 1.544 \times 10^6$, $\alpha = 0.2$

$$\begin{aligned} BW &= \frac{r_b}{\log_2 M} (1 + \alpha) \\ &= \frac{1.544 \times 10^6}{2} (1 + 0.2) \quad (\because M = 4) \end{aligned}$$

$$BW = 926.4 \times 10^3 \text{ Hz}$$

09. Ans: 0.25

Sol: $BW = 1500 \text{ Hz}$

BW required for M-ary PSK is

$$\frac{R_b [1 + \alpha]}{\log_2 16} = 1500 \text{ Hz}$$

$$\Rightarrow R_b [1 + \alpha] = 1500 \times 4 = 6000$$

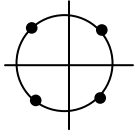
$$\Rightarrow (1 + \alpha) = \frac{6000}{4800}$$



$$\text{Roll off factor, } \alpha = \frac{6000}{4800} - 1 = 0.25$$

10. Ans: (d)

Sol:



Here only phase is changing.

From options (d) is the optimum answer.

11. Ans: (b)

Sol: Here 16-points are available in constellation which are varying in both amplitude and phase. So, it 16QAM.

12. Ans: (d)

$$\text{Sol: } BW = \frac{r_b}{\log_2 M} (1 + \alpha)$$

$$36 \times 10^6 = \frac{r_b}{2} (1 + 0.2) (\because M = 4, \text{QPSK})$$

$$r_b = 60 \times 10^6 \text{ bps}$$

10. Noise in Digital Communication

Noise Ratio

01. Ans: (a)

Sol: Signal to quantization noise ratio only depends on no. of quantization levels (L) and no. of bits per sample(n)

$$\begin{aligned} \text{For sinusoidal input SQNR} &= 1.76 + 6n \text{ dB} \\ &= 1.76 + 6 \times 12 \\ &= 73.76 \text{ dB} \end{aligned}$$

$$\begin{aligned} \text{For uniform distributed signal} &= 6n \text{ dB} \\ &= 6 \times 12 \\ &= 72 \text{ dB} \end{aligned}$$

02. Ans: (a)

Sol: For Bipolar pulses,

$$\text{PSD} = \frac{|P(\omega)|^2}{T_b} \cdot \sin^2 \left(\frac{\omega T_b}{2} \right)$$

The zero magnitude occurs for

$$f = n/T_b.$$

$$\therefore \text{The width of the major lobe} = 1/T_b$$

$$= f_b$$

$$\therefore (B.W)_{\min} = f_b$$

Here, Data rate = $n f_s$

$$= 8(8 \text{ kHz}) = 64 \text{ kbps}$$

$$\therefore (B.W)_{\min} = 64 \text{ kHz}$$

03. Ans: (c)

Sol: Since the signal is uniformly distributed,

$$f(x) = \frac{1}{10} \quad \text{for } -5 \leq x \leq 5$$

$$= 0 \quad : \text{ else where.}$$

$$\text{Signal Power} = \int_{-5}^5 x^2 f(x) dx = \frac{25}{3} \text{ volts}^2$$

$$\text{Step size} = \frac{V_{p-p}}{L} = \frac{10}{2^8} = 0.039 \text{ V}$$

$$N_q = \frac{\Delta^2}{12} = 0.126 \text{ mW}$$

Signal to noise ratio, SNR in dB is

$$\text{SNR} = 10 \log \left(\frac{\text{signal power}}{\text{Noise power}} \right)$$

$$= 10 \log \left(\frac{25/3}{0.126 \times 10^{-3}} \right)$$

$$= 48 \text{ dB}$$

04. Ans: (b)

Sol: For every one bit increase in data word length,

quantization Noise Power becomes $\frac{1}{4}$ th of the



original. Hence, Data word length for $n = 9$ bits is,

$$\therefore L = 2^n = 2^9 = 512$$

05. Ans: (c)

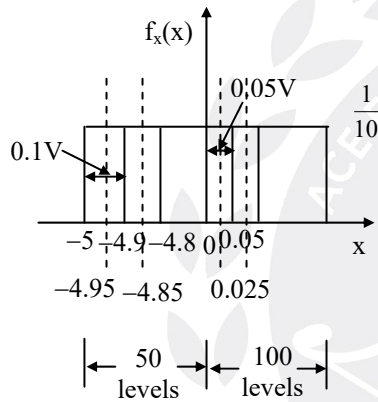
Sol: $V_{P-P} = -5V$ to $5V$; $20\log L = 43.5$

$$L = 10^{2.175} = 149.6$$

$$\Rightarrow \Delta = \frac{V_H - V_L}{L} = \frac{5 - (-5)}{10^{2.175}} \Rightarrow \Delta = 0.06683$$

06. Ans: (c)

Sol:



$$\begin{aligned} \text{Signal power } E[X^2] &= \int_{-5}^5 x^2 \left(\frac{1}{10} \right) dx \\ &= \frac{1}{10} \left(\frac{x^3}{3} \right)_{-5}^5 = \frac{1}{30} (250) = \frac{25}{3} \text{ W} \end{aligned}$$

Quantization Noise power

$$\begin{aligned} &= E[(X - Q(X))^2] \\ &= \int_{-5}^5 [X - Q(x)]^2 f_X(x) dx \\ &= \int_{-5}^{-4.9} [x - (-4.95)]^2 \frac{1}{10} dx + \dots (50 \text{ times}) \\ &\quad + \int_{-4.9}^{-4.85} [x - (-4.85)]^2 \frac{1}{10} dx + \dots (50 \text{ times}) \\ &\quad + \int_0^{0.05} (x - 0.025)^2 \frac{1}{10} dx \end{aligned}$$

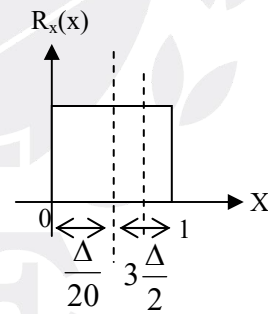
$$\begin{aligned} &+ \int_{0.05}^{0.1} [(x - 0.075)^2] \frac{1}{10} dx + \dots (100 \text{ times}) \\ &= 50 \int_{-5}^{-4.9} (x + 4.95)^2 \frac{1}{10} dx + 100 \int_0^{0.05} (x - 0.025)^2 \frac{1}{10} dx \\ &= 5 \left[\frac{(x + 4.95)^3}{3} \right]_{-5}^{-4.9} + 10 \left[\frac{(x - 0.025)^3}{3} \right]_0^{0.05} \\ &= \frac{5}{3} [(0.05)^3 + (0.05)^3] + \frac{10}{3} [(0.025)^3 + (0.025)^3] \\ &= \frac{5}{3} (125 \times 10^{-6} + 125 \times 10^{-6}) + \frac{10}{3} [(0.025)^3 + (0.025)^3] \\ &= \frac{5}{3} (125 \times 10^{-6} + 125 \times 10^{-6}) + \frac{10}{3} (3.125 \times 10^{-5}) \end{aligned}$$

$$\begin{aligned} &= \frac{1250}{3} \times 10^{-6} + \frac{312.5}{3} \times 10^{-6} \\ &= 520.83333 \times 10^{-6} \end{aligned}$$

$$(SNR)_{dB} = 10 \log \left(\frac{25}{3 \times 5.2 \times 10^{-4}} \right)$$

$$= 42.04 \text{ dB}$$

$$\approx 42 \text{ dB}$$



07. Ans: (b)

Sol: $E[X - Q(x)]^2$

$$\begin{aligned} &= \int_0^{0.3} (X - 0)^2 (1) dx + \int_{0.3}^1 (X - (0.7))^2 (1) dx \\ &= \left[\frac{x^3}{3} \right]_0^{0.3} + \left[\frac{(x - 0.7)^3}{3} \right]_{0.3}^1 \\ &= \frac{(0.3)^3}{3} + \frac{(0.3)^3}{3} + \frac{(0.4)^3}{3} = 0.198 \end{aligned}$$



08. Ans: (b)

Sol: Since, all the quantization levels are equiprobable,

$$\int_{-a}^a \frac{1}{4} dx = \frac{1}{3}$$

$$\Rightarrow a = \frac{2}{3}$$

09. Ans: (a)

Sol: $\int_{-2/3}^{2/3} x^2 f(x).dx = \frac{1}{4} \int_{-2/3}^{2/3} x^2 .dx = \frac{4}{81}$

Matched Filter

01. Ans: (d)

Sol: The time domain representation of the o/p of a Matched filter is proportional to Auto correlation function of the i/p signal, except for a time delay

$$R_{ss}(\tau) = \int_0^{10^{-4}} S(t).S(t+\tau)dt$$

$$= \int_0^{10^{-4}} 10\sin(2\pi \times 10^6 t).10\sin(2\pi \times 10^6 (t+\tau))dt$$

$$= 50 \int_0^{10^{-4}} [\cos(2\pi \times 10^6 \tau) - \cos(4\pi \times 10^6 t + 2\pi \times 10^6 \tau)] dt$$

$$= 50 \times 10^{-4} \cos(2\pi \times 10^6) \tau$$

$\therefore$ The Peak is 5mV

02. Ans: (b)

Sol: The matched filter has maximum value of output at $t = T$ is energy of the signal

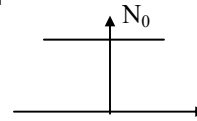
$$\Rightarrow E_s = \int_0^1 A^2 dt + \int_2^3 A^2 (1) dt$$

$$= A^2 + A^2 = 2A^2$$

03. Ans: (d)

Sol: $(SNR)_0 = \frac{E_s}{N_0} = \frac{B^2}{2} \cdot \frac{T}{N}$

$$= \frac{B^2 T}{2N}$$



04. Ans: (b)

Sol: Given,

$$\frac{S_{02}(t)}{N} = \frac{S_{01}(t)}{N} \Rightarrow \frac{2E_{s_1}}{N} = \frac{2E_2}{N}$$

$$A^2 T = \frac{B^2}{2} T \Rightarrow A = \frac{B}{\sqrt{2}}$$

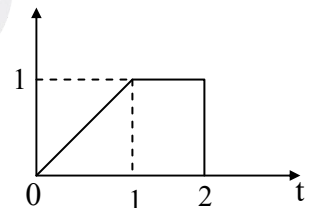
05. Ans: (d)

Sol: Output of the matched filter is maximum which is equal to the energy in the signal

$$E = \int_0^1 1.t^2 dt + \int_1^2 (1)dt$$

$$= \left[\frac{t^3}{3} \right]_0^1 + 1[t]_1^2$$

$$= \frac{1}{3} + 1 = \frac{4}{3}$$



The time instant which occurs the maximum value is its time period $T = 2$

06. Ans: (c)

Sol: Given,

$$H(f) = \frac{1 - e^{-j\omega t}}{j\omega}$$

$$H(f) = \frac{1}{j\omega} - \frac{e^{-j\omega t}}{j\omega}$$

Applying I.F.T



$$h(t) = 0.5(\text{sgn}(t) - \text{sgn}(t - T_0))$$

$$\left(\because F(\text{sgn}(t)) = \frac{2}{j\omega} \right)$$

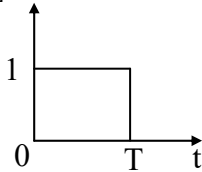
$$= 0.5[2u(t) - 1 - [2u(t - T_0) - 1]]$$

$$= [u(t) - u(t - T_0)]$$

We know that

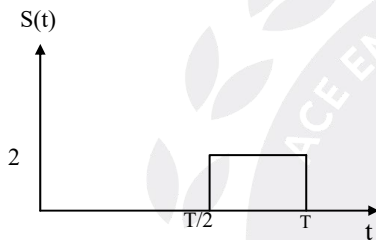
$$h(t) = s^*(t - T)$$

$$\therefore S_i(t)$$



07. Ans: (d)

Sol: The maximum value in the output is energy inside the signal

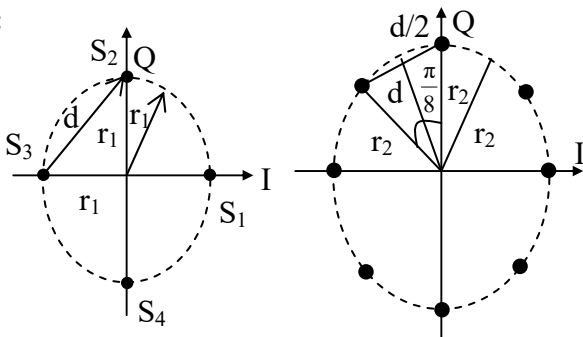


$$\begin{aligned} \Rightarrow S_0(t)_{\max} &= \int_{T/2}^T 2^2 \cdot dt \\ &= 4 \int_{T/2}^T 1 \cdot dt \\ &= 4[T - T/2] \\ &= 2T \end{aligned}$$

Probability of Error

01. Ans: (d)

Sol:



$$d = \sqrt{2} r_1 \quad \sin \frac{\pi}{8} = \frac{(d/2)}{r_2}$$

$$\Rightarrow r_1 = \frac{d}{\sqrt{2}} = 0.707 d$$

$$\Rightarrow r_2 = \frac{d}{2 \sin \frac{\pi}{8}} = 1.307 d$$

02. Ans: (d)

Sol: 4-PSK, 8-PSK both have same error probability when both signals have same minimum distance between pairs of signal points.

$$P_e = Q \left(\frac{\sqrt{d_{\min}^2}}{2N_0} \right)$$

$$P_e = 2Q \left(\sqrt{\frac{2E_s}{N_0} \sin^2 \left(\frac{\pi}{M} \right)} \right)$$

Where E_s is the average symbol energy

Given both constellation $d_{\min}$ is same i.e., 'd'

Average Symbol Energy:

$$(E_s)_{4\text{PSK}} = \frac{E_{s_1} + E_{s_2} + E_{s_3} + E_{s_4}}{4}$$

Where E_{s_k} is the symbol ' S_k ' Energy

$$= (\text{distance from the origin to the symbol } 'S_k')^2$$

$$(E_s)_{4\text{PSK}} = \frac{r_1^2 + r_1^2 + r_1^2 + r_1^2}{4} = r_1^2$$

Similarly, For 8 PSK

$$(E_s)_{8\text{PSK}} = r_2^2$$

$$\frac{(E_s)_{8\text{PSK}}}{(E_s)_{4\text{PSK}}} = \left(\frac{r_2}{r_1} \right)^2 = \left(\frac{1.307d}{0.707d} \right)^2$$

In dB,

$$\begin{aligned} (E_s)_{8\text{PSK (dB)}} - (E_s)_{4\text{PSK (dB)}} &= 10 \log \left(\frac{1.307}{0.707} \right)^2 \\ &= 5.33 \text{ dB} \end{aligned}$$

$$(E_s)_{8\text{PSK}} = (E_s)_{4\text{PSK}} + 5.33 \text{ dB}$$

8 PSK required additional 5.33 dB



03. Ans: (b)

Sol: Constellation 1:

$$s_1(t) = 0 ;$$

$$s_2(t) = -\sqrt{2} a \phi_1 + \sqrt{2} a \phi_2$$

$$s_3(t) = -2\sqrt{2} a \phi_1 ;$$

$$s_4(t) = -\sqrt{2} a \phi_1 - \sqrt{2} a \phi_2$$

$$\text{Energy of } S_1(t) = E_{S1} = 0 ; E_{S2} = 4a^2 ;$$

$$E_{S3} = 8a^2 ; E_{S4} = 4a^2$$

Average Energy of constellation 1

$$= \frac{E_{S1} + E_{S2} + E_{S3} + E_{S4}}{4} = 4a^2$$

Constellation 2:

$$s_1(t) = a\phi_1 \Rightarrow E_{S1} = a^2$$

$$s_2(t) = a\phi_2 \Rightarrow E_{S2} = a^2$$

$$s_3(t) = -a\phi_1 \Rightarrow E_{S3} = a^2$$

$$s_4(t) = -a\phi_2 \Rightarrow E_{S4} = a^2$$

Average Energy of constellation 2

$$= \frac{E_{S1} + E_{S2} + E_{S3} + E_{S4}}{4} = a^2$$

The required Ratio is 4

04. Ans: (a)

Sol: The distance between the two closest points in constellation 1 is $d_1 = 2a$.

The same in constellation 2,

$$d_2 = \sqrt{2} a$$

Since $d_1 > d_2$, Probability of symbol error for constellation 1 is lower

05. Ans: (a)

$$\begin{aligned} \text{Sol: } S(t) &= \sqrt{\frac{2E}{T_b}} \left[\cos(\omega_c t + \frac{2\pi}{m}(i-1)) \right] \\ &= \sqrt{\frac{2E}{T_b}} \left[\cos\omega_c t \cdot \cos\left(\frac{2\pi}{m}(i-1)\right) - \sin\omega_c t \cdot \sin\frac{2\pi}{m}(i-1) \right] \\ &= \sqrt{\frac{2}{T_b}} \cos\omega_c t \sqrt{E} \cos\left(\frac{2\pi}{m}(i-1)\right) - \sqrt{\frac{2}{T_b}} \sin\omega_c t \sqrt{E} \sin\frac{2\pi}{m}(i-1) \end{aligned}$$

Given binary digital communication $m = 2$

$$\sqrt{\frac{2}{T_b}} \cos\omega_c t \sqrt{E} \cos\pi$$

$$\therefore \text{basic function} = 2 \cos\omega_c t$$

$$\Rightarrow T_b = \frac{1}{2}$$

$$2 \cos\omega_c t (\sqrt{E} \cos\pi(f-1)) - [2 \sin\omega_c t] \sqrt{E} \sin\pi(i-1)$$



Distance between two points is:

$$\sqrt{(\sqrt{E} + \sqrt{E})^2 + 0}$$

$$\sqrt{4E} = 2\sqrt{E}$$

Energy of the signal:

$$\int_0^{T_b} (A \cos\omega_c t)^2 dt = \frac{A^2 T}{2}$$

$$\Rightarrow d = 2\sqrt{\frac{A^2 T_b}{2}}$$

$$= 2\sqrt{\frac{A^2 \times T_b}{2}} = A$$

$$(\because T_b = \frac{1}{2}) \quad \therefore d = A$$

06. Ans: (c)

$$\text{Sol: } P_e = Q \left[\sqrt{\frac{E_b}{N_o}} \right]$$

$$E_b = \frac{\alpha^2 T_b}{2} = \frac{\alpha^2}{2R_b}$$

$$\alpha = 4mV, R_b = 500 \text{ kbps,}$$

$$N_o = 10^{-12} \text{ W/Hz.}$$



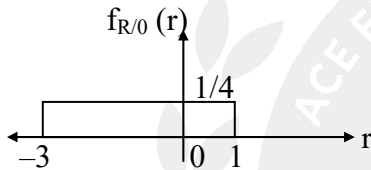
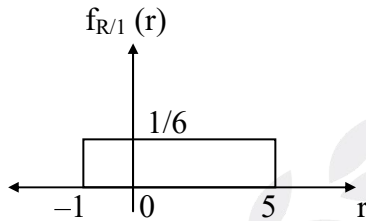
$$\frac{E_b}{N_o} = \frac{16 \times 10^{-6}}{2 \times 500 \times 10^3 \times 10^{-12}}$$

$$= 16$$

$$P_e = Q[\sqrt{16}] = Q[4]$$

07. Ans: (d)

Sol:

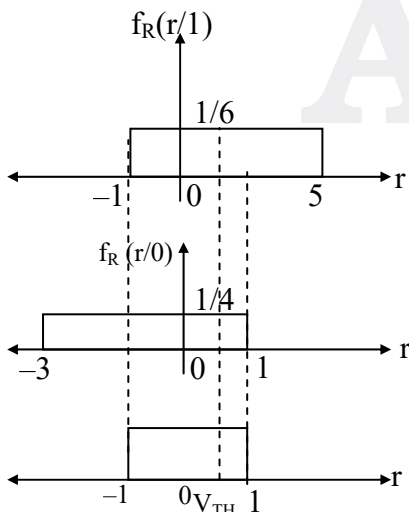


$$P(0) = 1/3; P(1) = 2/3$$

The probability of error of the symbols 0 & 1 are not the same.

∴ The intersection point of the two pdf's is not the threshold of detection.

Assume the threshold value to be V_{TH}



For minimum error the V_{TH} should lie in the area of intersection of the 2 pdf's.

$$P_{e_1} = \int_{-1}^{V_{TH}} \left(\frac{1}{6}\right) dr = \frac{1}{6}(V_{TH} + 1)$$

$$P_{e_0} = \int_{V_{TH}}^1 \left(\frac{1}{4}\right) dr = \frac{1}{4}(1 - V_{TH})$$

Decision error probability

$$= P_{e_0} P(0) + P_{e_1} P(1)$$

$$= \frac{1}{4}(1 - V_{TH})\left(\frac{1}{3}\right) + \frac{1}{6}(1 + V_{TH})\left(\frac{2}{3}\right)$$

$$P_e = \frac{1 - V_{TH}}{12} + \frac{2(1 + V_{TH})}{18}$$

For minimum decision error probability,

$$-1 \leq V_{TH} \leq 1$$

For $V_{TH} = -1$

$$BER = \frac{1 - (-1)}{12} = \frac{1}{6} \text{ (min value)}$$

∴ Decision error probability = 1/6

08. Ans: (c)

Sol: The optimum threshold value is

$$\hat{x} = \frac{\sigma^2}{x_1 - x_2} \left[\ln \frac{P(x_2)}{P(x_1)} + \frac{x_1^2 - x_2^2}{2\sigma^2} \right]$$

$$x_1 = 1, x_2 = -1$$

$$P(x_1) = 0.75, \quad P(x_2) = 0.25$$

$$\hat{x} = \frac{\sigma^2}{2} \left[\ln \frac{0.25}{0.75} \right] = -\frac{\sigma^2}{2}$$

So $\hat{x}$ should be strictly negative.

09. Ans: (c)

Sol: $Y = X + Z$

Z is Gaussian RV with mean βx

$$x \in \{-a, +a\}$$

$$\text{when } \beta = 0 \quad E[y] = E[x] + E[z]$$



$$E[y] = E[x] = +a$$

$$= a$$

$$BER = Q(a) = 1 \times 10^{-8}$$

$$Q(v) = \frac{1}{\sqrt{2\pi}} \int_v^{\infty} e^{-\frac{u^2}{2}} du \approx e^{-\frac{v^2}{2}}$$

$$Q(a) = 1 \times 10^{-8} \approx e^{-\frac{a^2}{2}}$$

$$a = 6$$

$$\text{when } \beta = -0.3 \text{ mean} = 6 \times -0.3 = -1.8$$

$$\text{so } E(y) = E(x) + E(z)$$

$$= 6 - 1.8 = 4.2$$

$$\text{so } BER = Q(4.2) \approx e^{-\frac{(4.2)^2}{2}}$$

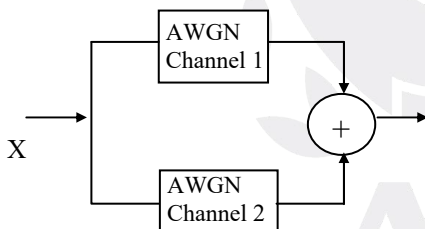
$$\approx 0.0001$$

$$\approx 10^{-4}$$

10. Ans: 1.414

Sol: When the signal is transmitted through a channel

$$BER = Q[\sqrt{r}].$$



At the input of the receiver signal amplitude is doubled. But when two independent Gaussian Random Variables are added, the resultant random variables is also a Gaussian random.

The pdf is the convolution of individual pdf's.

The variance indicates the noise power

But the variance is doubled.

Signal power increased by a factor of 4 (mean is doubled).

But the noise increases by a factor of 2

So the signal to noise increases by a factor of 2

$$\text{So } b = \sqrt{2} = 1.414$$

$$BER = Q[\sqrt{2}r] = Q[\sqrt{2}\sqrt{r}]$$

$$= Q[1.414\sqrt{r}]$$

$$\text{So } b = 1.414$$

11. Ans: (a)

Sol: Probability of error for an AWGN channel for binary transmission is given as

$$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$$

$$\text{Where } E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt$$

$$\text{Given } s_1(t) = g(t)$$

$$s_2(t) = -g(t)$$

$$E_d = \int_0^T [g(t) - (-g(t))]^2 dt$$

$$= 4 \int_0^T g^2(t) dt$$

$$E_{d,a} = 4 \int_0^1 (1)^2 dt = 4$$

$$E_{d,b} = 4 \left[\int_0^{1/2} (2t)^2 dt + \int_{1/2}^1 (-2t + 2)^2 dt \right]$$

$$= \frac{4}{6} + \frac{4}{6} = \frac{4}{3}$$

$$E_{d,c} = 4 \int_0^1 (1-t)^2 dt = \frac{4}{3}$$

$$E_{d,d} = 4 \int_0^1 (t)^2 dt = \frac{4}{3}$$

P_e is minimum when E_d is maximum

E_d of signal (a) is more when compared to E_d of other signals.

∴ Probability of error is minimum for signal (a).



12. Ans: (b)

Sol: o/p Noise Power = o/p PSD $\times$ B.W

$$= 10^{-20} \times 2 \times 10^6$$

$$= 2 \times 10^{-14} \text{ W}$$

Since mean square value = Power

$$\frac{2}{\alpha^2} = 2 \times 10^{-14} \Rightarrow \alpha = 10^7$$

13. Ans: (d)

Sol: When a 1 is transmitted:

$$Y_k = a + N_k$$

$$\text{Threshold } Z = \frac{a}{2} = 10^{-6}$$

$$\Rightarrow a = 2 \times 10^{-6}$$

For error to occur, $Y_k < 10^{-6}$

$$2 \times 10^{-6} + N_k < 10^{-6}$$

$$N_k < -10^{-6}$$

$$\therefore P(0/1) = \int_{-\infty}^{-10^{-6}} P(n) dn$$

$$= \int_{-\infty}^{-10^{-6}} (0.5)\alpha \cdot e^{-\alpha n} \cdot dn, \text{ with } \alpha = 10^7$$

$$= 0.5 \times e^{-10}$$

When a '0' is Transmitted:

$$Y_k = N_k$$

For error to occur, $Y_k > 10^{-6}$

$$\therefore P(1/0) = \int_{10^{-6}}^{\infty} P(n) dn = 0.5 \times e^{-10}$$

Since, both bits are equiprobable, the Probability of bit error

$$= \frac{1}{2} [P(0/1) + P(1/0)]$$

$$= 0.5 \times e^{-10}$$

14. Ans: (a)

Sol: $P(0/1) = P(1/0) = p$

$$\Rightarrow P(1/1) = P(0/0) = 1-p.$$

Reception with error means getting at most one 1.

$$\therefore P(\text{reception with error})$$

$$= P(X=0) + P(X=1)$$

$$= {}^3C_0 (1-p)^0 p^3 + {}^3C_1 (1-p)^1 p^2$$

$$= p^3 + 3p^2(1-p)$$

15. Ans: (d)

Sol: p = probability of a bit being in error = 10^{-3}

q = probability of the bit not being in error

$$= 1 - p = 1 - 10^{-3}$$

$$= 0.999$$

1. Total number of bits = 10;

P_e = probability of error

$$= 1 - P(X=0)$$

$P(X=0)$ = Probability of no error

$$\therefore P_e = 1 - \left[{}^{10}C_0 (10^{-3})^0 (1 - 10^{-3})^{10} \right]$$

$$= 0.00995$$

2. Total number of bits = 100

$$P_e = 1 - \left[{}^{100}C_0 (10^{-3})^0 (1 - 10^{-3})^{100} \right]$$

$$= 0.0952$$

3. Total number of bits = 1000

$$P_e = 1 - \left[{}^{1000}C_0 (10^{-3})^0 (0.999^{1000}) \right]$$

$$P_e = 0.632$$

4. If total number of bits = 10,000

$$= 1 - \left[\left({}^{10,000}C_0 \right) (1 - 10^{-3})^0 (0.999)^{10,000} \right]$$

$$= 0.9999$$

Conclusion: As the number of bits increases, the probability of error increases and it approaches unity.

16. Ans: (a)

Sol: Higher modulation techniques requires more power i.e., to achieve same probability of error,



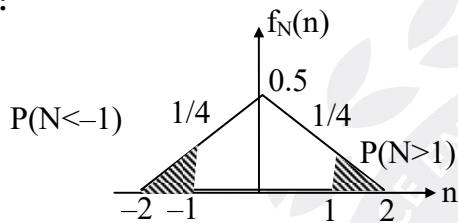
bit energy has to be increased. So, power also increased.

17. Ans: (a)

Sol: Higher modulation techniques requires more power i.e., to achieve same probability of error, bit energy has to be increased. So, power also increased.

18. Ans: 0.125

Sol:

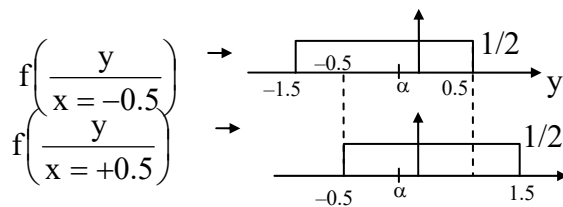


$$\begin{aligned}
 P(E) &= P(x = -1)P\left(\frac{R}{x = -1} > 0\right) + P(x = 1)P\left(\frac{R}{x = +1} < 0\right) \\
 &= 0.5P(x+N > 0) + 0.5P(x+N < 0) \\
 &= 0.5P(-1+N > 0) + 0.5P(1+N < 0) \\
 &= 0.5P(N > 1) + 0.5P(N < -1) \\
 &= 0.5\left[\frac{1}{2} \cdot \frac{1}{4}\right] + 0.5\left[\frac{1}{2} \cdot \frac{1}{4}\right] \\
 &= \frac{1}{8} = 0.125
 \end{aligned}$$

19. Ans: -0.5

Sol: $x = \{-0.5, 0.5\}$

$$P(x = -0.5) = 1/4, P(x = 0.5) = 3/4$$



P_e in the overlap region $-0.5 < \alpha < 0.5$

$$\begin{aligned}
 P_e &= \frac{1}{4} \cdot \frac{1}{2} (0.5 - \alpha) + \frac{3}{4} \left(\frac{1}{2}\right) (\alpha + 0.5) \\
 &= \frac{0.5}{8} + \frac{1.5}{8} + \left(\frac{3}{8} - \frac{1}{8}\right) \alpha \\
 &= \frac{2}{8} + \frac{2}{8} \alpha
 \end{aligned}$$

$\therefore P_e$ is minimum for $\alpha = -0.5$

11. Information Theory & Coding

01. Ans: (b)

Sol: Huffman encoder is the most efficient source encoder

$$\begin{array}{cc|cc}
 0.5 & 1 & 0.5 & 0 \\
 0.25 & 00 & 0.5 & 1 \\
 0.25 & 01 & &
 \end{array}$$

$$\begin{aligned}
 \bar{L} &= 1 \times 0.5 + 2 \times 0.25 + 2 \times 0.25 \\
 &= 1.5 \text{ bits/symbol}
 \end{aligned}$$

$$\begin{aligned}
 \text{Average bit rate} &= 3000 \times 1.5 \\
 &= 4500 \text{ bps}
 \end{aligned}$$

02. Ans: (c)

Sol: Assuming all the 64 levels are equiprobable,

$$H = \log_2 64 = 6 \text{ bits/pixel}$$

$$\begin{aligned}
 \text{Total No. of pixels} &= 625 \times 400 \times 400 \\
 &= 100 \text{ M pixels/sec}
 \end{aligned}$$

$$\begin{aligned}
 \text{Data rate} &= 6 \text{ bits/pixel} \times 100 \times 10^6 \text{ pixel/sec} \\
 &= 600 \text{ Mbps}
 \end{aligned}$$

03. Ans: (b)

$$\text{Sol: } C = B \log \left(1 + \frac{S}{N}\right)$$

$$\text{Since } \frac{S}{N} \gg 1, 1 + \frac{S}{N} \cong \frac{S}{N}$$



$$\therefore C_1 = B \log \frac{S}{N}$$

$$C_2 = B \log \left(2 \cdot \frac{S}{N} \right)$$

$$= B \log 2 + B \log \left(\frac{S}{N} \right) = C_1 + B$$

04. Ans: (b)

Sol: Given

$$B, W = 3 \text{ kHz}$$

$$\text{SNR} = 10\text{dB}$$

$$\Rightarrow 10 \log_{10} (\text{SNR}) = 10$$

$$\text{SNR} = 10^1 = 10$$

$$\text{Number of characters} = 128$$

$$\text{Channel capacity} = B \log_2 \left(1 + \frac{S}{N} \right)$$

$$= 3 \times 10^3 \log_2 (1 + 10)$$

$$= 10378\text{bps}$$

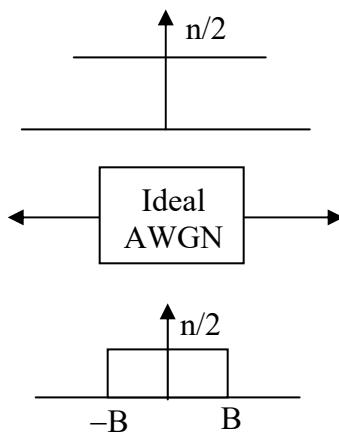
05. Ans: (b)

Sol: Number of characteristics can be sent without

$$\text{any error} = \frac{c}{\log_2 M} = \frac{c}{7} = 1482.\text{cps}$$

06. Ans: (c)

Sol:



$$C = B \log_2 \left(1 + \frac{S}{N} \right)$$

$$\lim_{B \rightarrow \infty} C = \lim_{B \rightarrow \infty} \frac{S}{n} \times \frac{n}{S} B \log_2 \left(1 + \frac{S}{nB} \right)$$

$$\lim_{B \rightarrow \infty} C = \frac{S}{n} \log_2 e$$

$$\left(\because \lim_{n \rightarrow \infty} x \log \left(1 + \frac{1}{Q} \right) = \log e \right)$$

$$\lim_{B \rightarrow \infty} C = 1.44 \frac{S}{n}$$

07. Ans: (b)

$$\text{Sol: Max. entropy} = 512 \times 512 \times \log_2 8$$

$$= 786432 \text{ bits}$$

08. Ans: (d)

Sol: Maximum entropy of a binary source:

$$H(x)_{\text{max}} = \log_2 M$$

$$H(x)_{\text{max}} = \log_2 2 = 1 \text{ bit/symbol}$$

09. Ans: 0.4

$$\text{Sol: } P\left(\frac{x=1}{y=0}\right) = \frac{P(x=1, y=0)}{P(y=0)}$$

$$= \frac{P(x=1)P\left(\frac{y=0}{x=1}\right)}{P(x=1)P\left(\frac{y=0}{x=1}\right) + P(x=0)P\left(\frac{y=0}{x=0}\right)}$$

$$= \frac{0.8 \times \frac{1}{7}}{0.8 \times \frac{1}{7} + 0.2 \times \frac{6}{7}} = 0.4$$



11. Optical Fiber Communication

01. Ans: (d)

Sol: $NA = 0.25$

$$n_2 = \frac{C}{V}$$

$$n_2 = \frac{C}{\frac{C}{\sqrt{\epsilon_r}}}$$

$$n_2 = \sqrt{\epsilon_r} = \sqrt{2.4375}$$

$$n_2 = 1.56$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$(NA)^2 = n_1^2 - n_2^2$$

$$n_1 = \sqrt{NA^2 + n_2^2}$$

$$= \sqrt{(0.25)^2 + 2.4375}$$

$$= \sqrt{\frac{10}{4}} = \sqrt{2.5}$$

02. Ans: (d)

Sol: Number of modes $M = \frac{V^2}{2} \frac{\alpha}{\alpha + 2}$

$$= \frac{1}{2} \left(\frac{2\pi a}{\lambda} (NA)^2 \right) \frac{\alpha}{\alpha + 2}$$

Here a = core radius

λ = wavelength

α = refractive index profile.

03. Ans: (b)

Sol: Power loss = 0.25dB/km

$$\text{For 100km, the power loss} = 100 \times 0.25 \\ = 25 \text{ dB}$$

$$\text{The optical power at 100km} \\ = 10 \log 0.1 \times 10^{-3} - 25 \\ = -65 \text{ dB}$$

In dBm

$$\rightarrow -65 \text{ dB} + 30 = -35 \text{ dBm.}$$

04. Ans: (c)

Sol: Numerical Aperture is used to describes light gathering (or) light collecting ability of an optical fiber.

05. Ans: (c)

Sol: The refractive index of the cladding material should be less than that of the core.

06. Ans: (d)

Sol: Fibers with higher numerical aperture exhibit greater losses and lower bandwidth.